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Can Children use Numerical Reasoning to Compare Odds in Games?

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Abstract

Children can represent, compute, and manipulate numbers from very early in development. Additionally, beginning in infancy, children appear to have intuitions about probability, correctly anticipating the outcomes of simple sampling events. In two experiments, we examined 3- to 7-year-olds' (N=196) ability to compare the number of items across sets in games of chance. In Experiment 1, children were asked to select between two games with different numbers of hiding locations to either find or hide a gold coin. Using a similar set up, in Experiment 2, they were asked to select the game that would make it easy or hard for another player to find the coin. Results from both experiments suggest that by around age 5, children can use numerical reasoning to compare odds: they were more likely to select the game with more cups when asked to help hide the gold coin than find it (Experiment 1) and when asked to make the game hard rather than easy (Experiment 2).

Keywords: Numerical reasoning; Probability; Numerical comparisons; Cognitive development

Introduction

Games of chance have been a source of entertainment for centuries. The oldest evidence of board games dates to the Neolithic Age 6,000 to 10,000 years ago and card games first appeared in Europe in the late 1300s, migrating from the east (Husband, 2016). These games have stood the test of time; they continue to provide entertainment for children and adults alike. Not only are they enjoyable, they also provide excellent means for practicing social skills (cooperation and competition), linguistic skills, and numerical reasoning. For instance, take the simple card game War, often played by children growing up in Western cultures. The objective of this game is to win all the cards in the deck. During each round, each player flips over one card from their deck and the player with the higher card takes them all. This is repeated until one player has won all the cards. This simple card game requires that children understand and follow rules and make numerical comparisons. Indeed, from Chess to Go Fish, the games we play involve elements of numbers, probabilities and odds.

Given these properties, games of chance also provide a way to explore children's competence in the cognitive and social-cognitive abilities they hinge on. We examine 3- to 7-year-old children's intuitive ability to assess the odds of a game of chance that requires comparing discrete items across sets. Specifically, we informed children of the goal of a game, for example, to find or hide a gold coin hidden under one cup in a set of cups. Then we presented them with two game environments to choose from on each trial, say, one with 5 cups and the other with 10 cups. We examine whether children will choose games with more cups when the goal is to hide a coin and games with fewer cups when the goal is to find it.

Children in this age range may have the pre-requisite abilities to succeed in such a task. Past research has demonstrated that children have an intuitive number sense long before they even learn to count. For example, 6-month-old infants perceive and discriminate between large numbers, if the ratios are sufficiently large (i.e., 1:2 but not 2:3; Xu & Spelke, 2000). More directly pertinent to our task, infants and young children are able to reason about relative amounts of items among sets. That is, they are sensitive to the ordinal relationships among items in sets (Brannon & Van de Walle, 2001; Brannon, 2002), and beginning around age 2.5, and improving throughout the early years, they can compare sets of discrete items such as 1 versus 3 or 6 versus 9 items to indicate which contains more (even when controlling for variables, such as total surface area, that correlate with number; e.g., Cheung & Le Corre, 2018).

Finally, infants and young children can make statistical inferences using the basic principles of probability, suggesting that they should be able to grapple with both the uncertainty and the elements of randomness involved in the present task. That is, when infants are presented with a distribution of items in a population, they expect that a randomly drawn, blind sample should be similar in distribution (e.g., Denison et al., 2013; Kayhan et al., 2017; Xu & Garcia., 2008). Further, when infants and young children are presented with tasks that require they reason about the likely contents of a single blind draw from a population, they tend to anticipate that the item will be of the majority type (Denison & Xu, 2014; Yost et al., 1962). Indeed, 12-month-old infants have been shown to make inferences on the probabilities of single sampling events by comparing proportions of objects. However, this ability was not found in 3- and 4-year-olds (Girotto et al., 2016; Placì et al., 2020). Therefore, it is possible that younger children would find our experimental task difficult. Nevertheless, our task does not require comparisons of ratios or proportions, instead, it requires that children compare absolute quantities across sets when inferring odds, so a potential lack of ability to compare proportions may not influence performance.

Thus, children have intuitions about probability and randomness from an early age, and in the absence of formal education and explicit teaching. Consistent with this view, prenumerate Mayan adults can solve a range of probabilistic problems as well. Their abilities to make correct probabilistic evaluations matches the performance of Mayan school children and Western controls. These findings suggest that regardless of schooling and culture, humans possess basic probabilistic knowledge (Fontanari et al., 2014).

Together, this literature suggests that children have the pre-requisite abilities to reason successfully about the individual components of the current tasks. Thus, we examine here whether they can put this knowledge together to decide which games of chance are more advantageous, depending on the goals of the games.

Experiment 1

Experiment 1 asks whether children can use numerical information to select a game that will increase the odds of a character successfully finding or hiding an item. Three- to 6-year-olds were presented with a character who either wanted to find or hide a gold coin under one of the cups. In the *Finder* condition, participants were asked to help the character find the gold coin. In the *Hider* condition, participants were asked to help the character hide the gold coin. They were presented with three trials: 2vs4, 3vs6 and 5vs10. All trials used the same ratios (1:2), but absolute number was varied, in order to increase the number of trials each child could respond to without exact repetitions and to see whether the absolute numbers would impact performance, particularly at younger ages, as this sometimes impacts numerical comparisons for young children (e.g., Cheung & Le Corre, 2018).

Methods

Participants 130 three- to six-year-olds (M_{age} =4.5 years; 60.5 months; 56 girls) participated. 68 participants were in the Finder condition and 62 were in the Hider condition. We initially planned to test 20 children per age in years per condition, but data collection was discontinued due to the COVID-19 pandemic (resulting in: 35 3-year-olds, 28 4-year-olds, 31 5-year-olds, and 36 6-year-olds, distributed approximately evenly across the two conditions, which were randomly assigned at each age). No additional children were tested and excluded. Participants were recruited and individually tested in local daycares and elementary schools in [blinded] region. In both experiments, we obtained informed consent from guardians and oral assent from children.

Design and Procedure See Figure 1 for illustrations of the slides and the scripts, which were run using PowerPoint, narrated live by an experimenter. Children were introduced to a character, Alex, (gender matched to each child participant). They were instructed, based on their between-subjects condition (Finder or Hider) to help Alex find or hide gold coins. Following these instructions, all children completed three trial types (2 vs 4 cups, 3 vs 6 cups and 5 vs 10 cups) where they chose which of the two games Alex should either try to find or try to hide a gold coin in, with the order of their presentation counterbalanced. The visual stimuli in both the Finder and Hider conditions were identical; only the verbal scripts were modified to describe hiding or finding. Whether the higher number of cups appeared on the left or right was also counterbalanced.

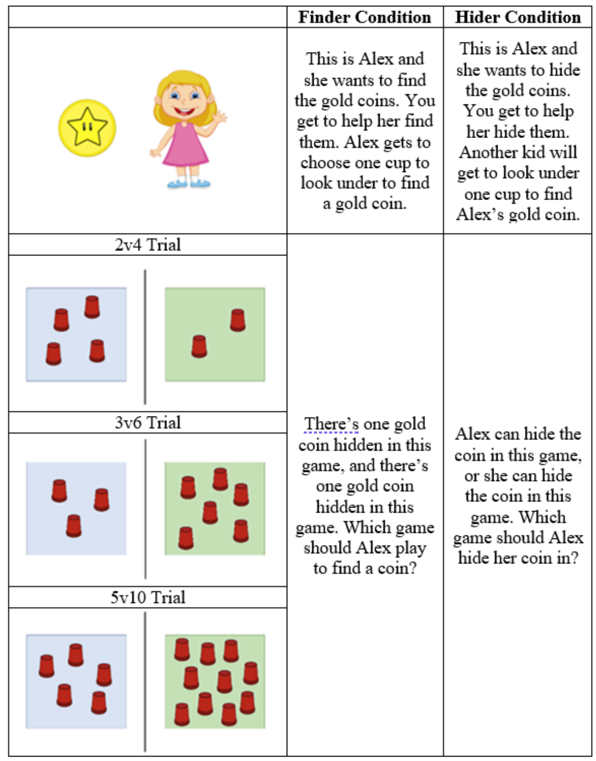

Figure 1: Experiment 1 stimuli & script.

Results and Discussion

See anonymized data for both experiments here: https://osf.io/vhwk3/?view_only=bf36f5062a6749afb3b9ce dfc5cf97a8. Participants received a score of 1 when they picked the game with more cups and 0 when they picked the game with fewer cups on each trial, regardless of condition, resulting in scores ranging from 0 to 3. Thus, if children were reasoning correctly, they should score higher in the Hider condition than in the Finder condition.

We used a Generalized Estimating Equations (GEE) model with condition (Hider, Finder) as a between-subjects factor, age in months (mean-centered) and entered as a continuous covariate and trial type (2v4, 3v6, 5v10) as a within-subjects factor and their interactions.

We found a significant main effect of condition, *Wald* X^2 (1) = 7.63, p = .006, in that children scored higher in the Hider than the Finder condition.

There was also a significant interaction between condition and age (see Figure 2), *Wald* X^2 (1) = 13.23, p <.001. To further investigate this interaction, we conducted a median split for age in months (Median = 61.41 months) and examined the differences in responses in the two conditions between the younger versus older age groups (see Table 1 for means and standard deviations). We found that the older participants were more likely to select the game with more cups in the Hider condition than in the Finder condition, *Wald* X^2 (1) = 10.64, p =.001. The younger half of the sample did not show a significant difference in their responses between the two conditions, *Wald* X^2 (1) = 0.13, p = .720. We originally planned to break this down by age in years but used a median split due to the disrupted data collection, which resulted in uneven numbers of children at each age and a smaller total sample.

We also examined each condition separately. In the Hider condition, with age, children were more likely to select the game with more cups, *Wald* X^2 (1) = 12.94, p < .001. In the Finder condition, age did not affect the responses, *Wald* X^2 (1) = 2.62, p = .106.

Finally, there was a significant main effect of trial type, *Wald* X^2 (2) = 7.84, p = .020. Pairwise comparisons using a Wilcoxon Signed-Ranks test revealed an unexpected and slightly unusual pattern: children chose the higher number of cups significantly more often in the 2v4 trial than in the 3v6 trial, Z = -2.65, p = .008. The responses between the 2v4 trial and the 5v10 trial, 3v6 trial and 5v10 trial did not differ; Z = -0.67, p = .500 and Z = -1.69, p = .091, respectively.

Table 1: Average Score (out of 1) by median split.

Months	Condition	$M_{Total\ Score}$	$SD_{Total\ Score}$
<61.41	Finder	0.54	0.34
	Hider	0.59	0.32
>61.41	Finder	0.44	0.30
	Hider	0.71	0.33

Older children used numerical reasoning to optimally choose between games but younger children did not. That is, older children were more likely to select the game with fewer cups when asked to help find the gold coin (which makes the game easier for the player) and to select the game with more cups when asked to help hide the gold coin (making the game harder). Thus, at approximately age five, children appear able to determine how the number of locations in an environment impacts the likelihood of succeeding or failing at a hiding and finding game.

We also found a main effect of trial type. Somewhat unexpectedly, pairwise comparisons revealed that children were significantly more likely to select more cups in the 2v4 trial rather than the 3v6 trial, regardless of condition. However, there were no significant differences between the 3v6 and 5v10 trial or the 2v4 and 5v10 trial. This pattern of responding is difficult to explain, and the most reasonable explanation might be that this is a random fluctuation that may have smoothed out with the larger sample size. If the absolute number of cups truly affects performance on this task, we would probably expect to find an interaction between condition and trial type. For example, children would be more successful in the 2v4 trial than the 3v6 trial and the 5v10 trial, since 2- to 4- year-old children have sometimes been more successful in making numerical comparisons when there are fewer items involved (Cheung & Le Corre, 2018).

One potential interpretation of our findings is that children may be better at the hiding condition than at the finding condition. In observing the graph, it is evident that, at least in the older half of the sample, children are diverging from what might be considered "chance" performance (a score of 1.5) when hiding but not finding. Indeed, comparisons to chance confirm this: in the older half, children performed above chance in the Hider condition (t(30) = 3.53, p = .001) and no different from chance in the Finder condition (t(33) = 1.13, p = .267). This could represent an interesting difference, wherein children are more competent hiders than finders early in development. However, we are cautious in interpreting our data this way because, in this task, children are required to choose the game with fewer cups to produce a correct response in the Finder condition. This may require that children inhibit a desire to choose the side with more items (cups), which could be a tempting choice at baseline for children.

Overall, around age 5, children were able to compare the odds of different games to determine game difficulty. Given our disrupted data collection, the surprising trial-type finding, and the move to online testing, we attempted to conceptually replicate this finding in Experiment 2.

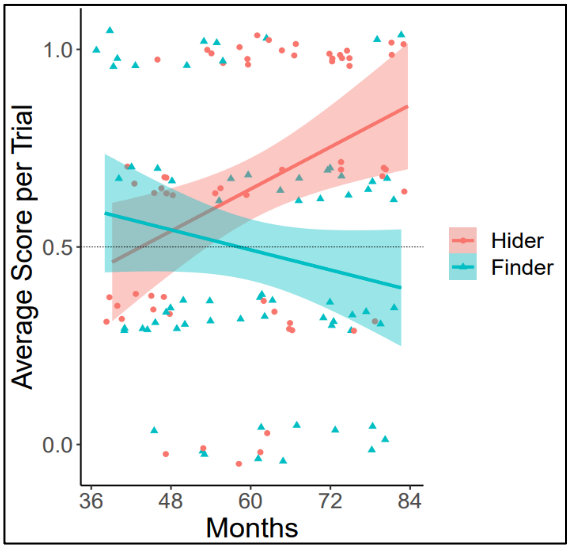

Figure 2: Children's average scores in Experiment 1 with age in months and condition. Dots are jittered for visibility.

Experiment 2

In Experiment 2, we simplified our procedure slightly, which allowed us to use a within-subject design and made the task particularly well-suited for online testing.

Instead of asking children to help a character find or hide gold coins, they were asked to select a game that would either be easy or hard for another person to find a gold coin. This design allowed us to simplify our script and make the scripts across the two conditions even better equated. Essentially, in Experiment 2, just one word [easy/hard] changes across conditions.

In the Easy condition, children were instructed to make the game easy for another player to find the gold coin. In the Hard condition, they were asked to make the game hard for another player to find the gold coin. We decided to test 4- to 7-year-olds in this experiment, largely because it wasn't until at least age 5 that children made correct inferences in Experiment 1 but also because this is the current age range that we are including in our online lab.

Methods

Participants Data collection was conducted online via live zoom calls using screen sharing. 80 four- to seven-year-olds participated in the study (20 children per age in years, *M*age = 6.01 years; 72.14 months; 38 females). Eight additional children were tested but excluded for failing the comprehension question twice (see Procedure);. The sample size, experimental procedures, statistical analyses and exclusion criteria were pre-registered (https://aspredicted.org/blind.php?x=yt3di9).

Design and Procedure In this experiment, both condition (Easy, Hard) and trial type (2v4, 3v6, 5v10) were tested within-subjects. Trial type order was counterbalanced as in Experiment 1 and condition order was blocked and counterbalanced (i.e., the easy block appeared first for half the children).

Participants were told that they had one gold coin to hide under a cup (see Figure 3 for the procedure). In counterbalanced order, they were told to make the game easy or hard for another player to find. After being told the task, they were asked the comprehension question: "Do I want it to be easy or hard for another kid to find the gold coin?" For those who answered incorrectly the first time, the task instructions were repeated, and the comprehension question was asked a second time. If children failed the question a second time, the experimenter continued with the task, but these children's data were excluded from analyses, as planned in the pre-registration.

After the comprehension check, the three trials for that block proceeded and children were asked to select the side they would like to hide their coin in. Then the experimenter proceeded with the next block by saying, "now I want to make it very [easy/hard] for other kids to find the gold coin in my game." They again went through the comprehension check with the new task (again, giving children two attempts at it) and children completed the next block of three trials.

The visual stimuli in both the Easy and Hard condition were identical and scripts were identical other than the words "easy"/"hard".

Results and Discussion

Coding was identical to Experiment 1. Thus, if children reasoned correctly, they should have scored higher in the Hard condition than in the Easy condition.

We used a GEE (binary logistic, independent correlation matrix) with condition (Easy, Hard) and trial type (2v4, 3v6, 5v10) as within-subjects factors, age in months was mean-centered and entered as a continuous covariate and interactions were included in the model.

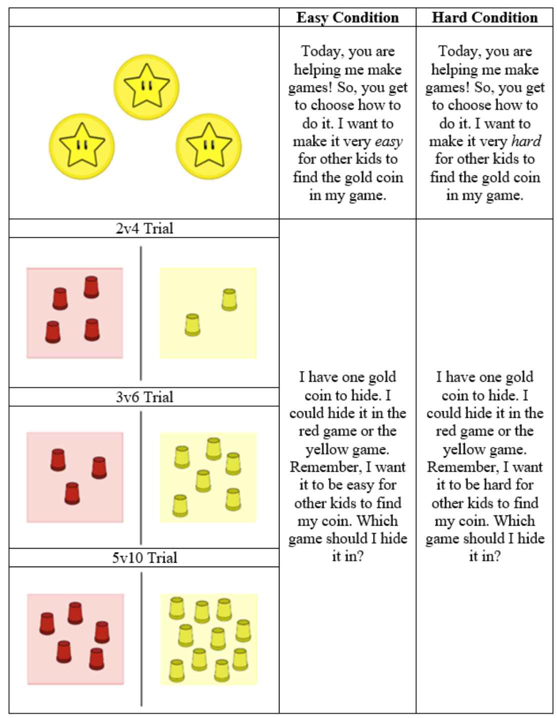

Figure 3: Experiment 2 stimuli & script. The order of trial-type and whether participants saw the Easy or Hard condition first was counterbalanced.

There was a significant main effect of condition, *Wald* X^2 (1) = 43.57, p <.001, in that children scored higher in the Hard condition than in the Easy condition. There was also a significant Condition x Age interaction (see Figure 4), *Wald* X^2 (1) = 18.04, p = <.001. There was no main effect of trial type, *Wald* X^2 (2) = 1.00, p = .606 (see Table 2 for means and standard deviations at each age).

To further investigate the interaction, we examined the differences in responses in the two conditions for each age group. Because we pre-registered the analyses (and we finished data collection for this experiment, as opposed to Experiment 1, which we do not intend on revisiting), we examined each age separately. Among the 4-year-olds, there was no significant effect of condition, *Wald* X^2 (1) = 1.32, p

= .250. In contrast, there was a significant effect of condition for the 5-, 6- and, 7-year-olds, *ps* < .001. The 5- to 7-year-olds were more likely to select the game with more cups in the Hard condition. For consistency with the Experiment 1 analyses, we conducted analyses with a median split on age in months (Median = 72.40 months). We found that younger and older children showed a significant effect of condition, *Wald* X^2 (1) = 10.342, *p* = .001 and *Wald* X^2 (1) = 35.275, *p* < .001, respectively, selecting the game with more cups in the Hard condition than in the Easy condition.

We also examined each condition separately. In the Easy condition, with age, children were more likely to select the game with fewer cups, *Wald* X^2 (3) = 19.50, *p* < .001. In the Hard Condition, with age, children were more likely to select the game with more cups, *Wald* X^2 (3) = 9.61, *p* = .022.

Table 2: Average Score (out of 1) by age in years.

Age	Condition	$M_{Average\ Score}$	$SD_{Average\ Score}$
4	Easy	0.48	0.35
	Hard	0.63	0.37
5	Easy	0.32	0.38
	Hard	0.80	0.33
6	Easy	0.12	0.20
	Hard	0.88	0.22
7	Easy	0.07	0.23
	Hard	0.92	0.24

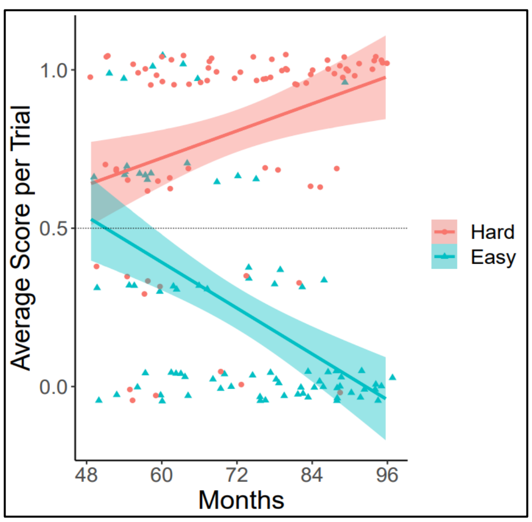

Figure 4: Children's average scores in Experiment 2 with age in months and condition. Dots are jittered for visibility.

In Experiment 2, we again found that children's ability to use numerical reasoning to determine game difficulty improved with age. Consistent with the findings from Experiment 1, 4-year-olds did not succeed at this task, while children 5-years and older did succeed.

Again, in thinking about comparisons to chance, we see that children appear to perform slightly better in the Hard condition, where choosing the higher number of cups is correct: Four-year-olds performed no differently than chance in either condition (*ps* > .126), but 5-year-olds performed above chance in the Hard condition ($t(19) = 4.05$, $p < .001$) and marginally different from chance in the Easy condition ($t(19) = 2.15$, $p = .045$). Six- and 7-year-olds scored well beyond chance in both conditions (all *p*'s < .001).

General Discussion

We investigated children's use of numerical reasoning to compare odds in games. We presented children with two games side-by-side that varied in the number of locations to see if they can use this information to determine which game increases the odds of successfully finding or hiding a coin. Around age 5, children made appropriate inferences about game difficulty.

To be successful on these tasks, children must be able to make numerical comparisons, evaluate and compare the probability of success on each of the games and remember the goal of the game. Young children appear to be capable of making numerical comparisons that involve set sizes and ratio differences (1:2) that are similar to the ones included here (Cheung & Le Corre, 2018). They are also capable of evaluating probabilities and making predictions about single draws from distributions, thus showing some sensitivity to the concepts involved in games of chance (Denison et al., 2006; Girotto et al., 2016; Yost et al., 1962). However, the present task requires that children make inferences about the probability of locating a hidden object based on the number of hiding locations and using that information to manipulate difficulty. It is possible that young children find it difficult to coordinate the three aspects of the task that they must keep in mind (the specific instruction, the numerical comparison, and the idea of a blind choice). Our findings suggest that this ability might not be present until around age 5.

It is not particularly surprising that older children perform better than younger children on this task. First, children's numerical reasoning, including their ANS acuity, which is likely implicated in this task, is drastically improving across these ages (e.g., Odic, 2018). Further, children are learning about quantifier words and their associated concepts at these ages (e.g., Odic et al., 2013), such as *more* and *less*, and better mastery of these concepts would surely aid them in this task. Finally, children would likely have had much more experience with games of all kinds, including games of chance, as they progress through this period. In a study conducted by Ramani and Siegler (2008), low-income preschoolers (M_{age} = 5.4 years) became more proficient on various numerical tasks (i.e., numerical comparisons, number line estimation, counting, and numeral identification) after playing a number board game for an hour. Participants who played an identical game that focused on colors instead of numbers did not show improvement. Thus, children's experience with number board games appears to improve their numerical knowledge.

Moreover, it is likely that performance on our task is associated with improvements in children's general cognitive development. For example, in order to produce the correct responses in the "Finder" and "Easy" conditions, children must select a smaller number of cups. As revealed in our comparisons to chance, young children appear slightly more pulled towards choosing the game with more cups, regardless of condition. It is possible that they are struggling to inhibit a desire to simply select more cups in our task, as significant improvements in executive functioning, including inhibition, are well documented throughout this period (Diamond, 2013). We also speculated, in Experiment 1, that children might be better at tasks that require hiding than those that require finding. This is an interesting idea, and in some ways, our results from Experiment 2 could support this: in comparing results across experiments, 5- and 6-year-old children appeared to perform better in the Easy condition of Experiment 2 than in the somewhat analogous Finder condition of Experiment 1. This suggests that inhibiting choosing the larger number of items is perhaps not *solely* responsible for weaker performance in the Finder than Hider conditions of Experiment 1. But, there are many other differences between Experiments 1 and 2, including simplified instructions, an online format, and within-subjects design in Experiment 2. Future work can examine whether there is truly an asymmetry in children's abilities in hiding versus finding, as it may have implications for the developmental roots of these and related abilities.

It is also worth noting that in our experiments so far, we ask children to compare numbers of items in two sets, without controlling for other variables that correlate with number such as surface area. We plan to conduct follow-up studies wherein the cups in the more numerous sets are half the size of those in the less numerous sets. We currently cannot be certain that the children who succeed on this task do so by truly considering the discrete number of locations and the single chance at uncovering a coin, rather than using a heuristic like, "searching is easier when there's less stuff".

Our main interest in designing this experiment is to use the set-up in future studies to examine children's application of numerical knowledge for exploring their reasoning about knowledge and ignorance, and social inferences. More specifically, we are interested in children's inferences about deception, cheating, and lying, based on their intuitions about probability, and these games could provide a potentially fruitful paradigm. If children can compare the odds of games, they might also become suspicious of a character who is winning too frequently based on odds, and start to wonder whether they had surreptitiously gained knowledge about the hiding locations. Young children can use probability to make a number of other social inferences, including people's preferences (Diesendruck et al., 2015; Kushnir et al., 2010; Ma & Xu, 2011) and their emotions (Doan et al., 2018; 2020), suggesting this could be extended to examine other kinds of social inferences involved in cooperation and competition.

In sum, findings from two experiments suggest that around age 5, children can use numerical reasoning to compare odds in a game of chance. Future work will determine whether children make these comparisons based on the discrete number of locations and whether they can apply this ability to make social inferences.

Acknowledgments

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