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Authors

Yu, L.-H.
Sessler, A
Whittum, D H

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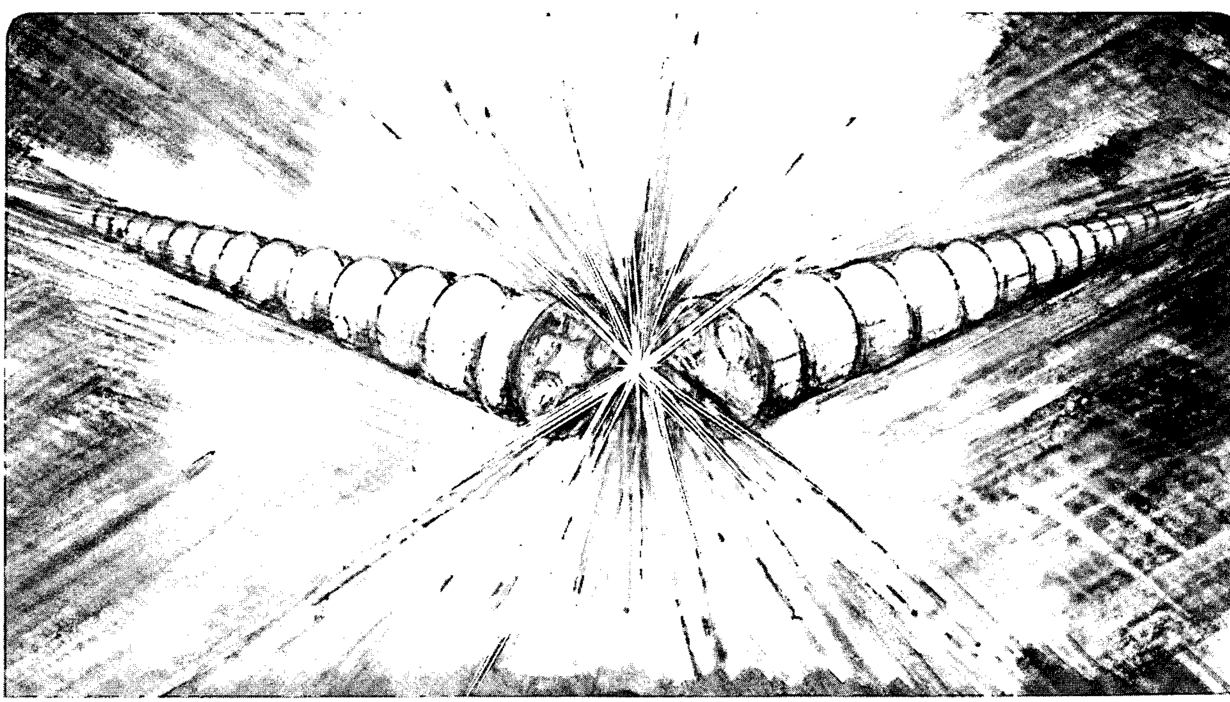

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Free-Electron Laser Generation of VUV and X-Ray Radiation Using a Conditioned Beam and Ion-Channel Focusing*

Li-Hua Yu
Brookhaven National Laboratory, Upton, NY 11973

A. Sessler and David H. Whittum**
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

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** Present address: Accelerator Division, National Laboratory for High Energy Physics, KEK, 1-1 Oho, Tsukuba, Ibaraki, 305, Japan.

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Li-Hua Yu

Brookhaven National Laboratory, Upton, N Y 11973

Andrew M. Sessler and **David H. Whittum**[(a)]

Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720

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The use of ion-focusing and a conditioned beam greatly enhances FEL gain in the VUV and Soft X-Ray range. The equations governing FEL amplification are derived and results of a linear analysis are noted. Numerical results, including 3D effects and having an order of magnitude improvement in gain, are presented for a 30 Å example.

(a) Present address: Accelerator Division, National Laboratory for High Energy Physics, KEK,1-1 Oho, Tsukuba, Ibaraki, 305 Japan.

I. Introduction

The performance of an FEL is limited by a spread in longitudinal velocity of beam particles and, in order to bound this variation, restrictions are imposed upon both the energy spread and emittance of the beam. Recently, a method has been proposed for "conditioning" a beam to enhance FEL gain and diminish the restriction on emittance [1]. This is accomplished by introducing a correlation between energy and amplitude of transverse oscillations in just such a way as to reduce the spread in longitudinal velocity. As shown in Fig. 1 the device consists of microwave cavities operated in the TM_{210} mode, situated within a FODO channel. As discussed in Ref.[1], conditioning relies on the oscillations between vertical and horizontal amplitudes characteristic of the FODO lattice, together with the characteristic coupling of the TM_{210} mode to the electron beam.

In Ref.[1] the theory of an FEL with a conditioned beam and regular or curved pole faces is presented following closely the work of Yu, Krinsky, and Gluckstern [2]. It is also suggested in Ref. [1] that gain may be significantly enhanced by stronger focusing, in contrast to the result for an unconditioned beam. In this work we analyze FEL performance with a conditioned beam, and strong "ion-focusing" [3]. In Sec. II the equations governing the ion-channel FEL are noted and a linear analysis of the gain is presented. In Sec. III, theory is confirmed with numerical simulation (including 3D effects), and examples are presented. In Sec. IV conclusions are offered and problems for future work are noted.

II. Equations

We need to do two things to the usual FEL theory in order to make the theory available for study of a conditioned beam subject to ion channel focusing. First, we need to derive the appropriate equations for an FEL with an ion channel, and that is done in the first half of this section Second, we need to derive the appropriate initial conditions for a conditioned beam; that is done in the second half of this section.

1. FEL Equations

Ion channel focusing is, to good approximation, electrostatic focusing and is simply characterized by adding to the usual FEL Hamiltonian an electrostatic potential. Thus the starting point of the calculation is the Hamiltonian

$$H = \left[1 + (\tilde{p}_x - a_x)^2 + (\tilde{p}_y - a_y)^2 + (\tilde{p}_z - a_z)^2\right]^{1/2} + \varphi \ , \tag{1}$$

where the canonical momentum of a particle is $mc\tilde{p} = mc(\vec{p} + \vec{A})$, the vector potential is $\frac{mc}{e}\underset{\sim}{a}$ and the electrostatic potential is $mc^2\varphi$. Changing variables so that the independent variable is z and the dependent variables are x, y, ct, one finds that the new Hamiltonian, h, is $-\tilde{p}_z$:

$$h = -\tilde{p}_z = -\left[(H - \varphi)^2 - 1 - (\tilde{p}_x - a_x)^2 - (\tilde{p}_y - a_y)^2\right]^{1/2} - a_z \tag{2}$$

where H is a constant.

Introducing the vector potential for a wiggler,

$$\begin{aligned} a_x &= a_0 \cosh \frac{k_w x}{\sqrt{2}} \cosh \frac{k_w y}{\sqrt{2}} \sin k_w z \ , \\ a_y &= a_0 \sinh \frac{k_w x}{\sqrt{2}} \sinh \frac{k_w y}{\sqrt{2}} \sin k_w z \ , \\ a_z &= 0 \ , \end{aligned} \tag{3}$$

and making various approximations on Hamilton's equations, we arrive at the wiggle-averaged equations for the transverse motion

$$\begin{aligned} p_x' &= (\gamma x')' = -\frac{1}{2\gamma}\frac{\partial a_w^2}{\partial x} - \frac{\partial \varphi}{\partial x} \ , \\ p_y' &= (\gamma x')' = -\frac{1}{2\gamma}\frac{\partial a_w^2}{\partial y} - \frac{\partial \varphi}{\partial y} \ , \end{aligned} \tag{4}$$

where $'$ denotes derivative with respect to z, ($\mathbf{p} = \tilde{\mathbf{p}} - \mathbf{a}$ is momentum)

$$\gamma = \left[1 + p_x^2 + p_y^2 + p_z^2\right]^{1/2} \approx p_z \, ,$$
$$a_w = \frac{a_0}{\sqrt{2}}\left[1 + \frac{1}{4} k_w^2 \left(x^2 + y^2\right)\right] \, . \tag{5}$$

For the longitudinal motion we must include the interaction with the signal wave, which we characterize by vector potential a in the x-direction. (The affect of the signal on transverse motion is negligible.) One finds

$$\frac{d\gamma}{dz} = -\frac{x'}{c}\frac{\partial a}{\partial t} - \frac{d\varphi}{dz} \, ,$$
$$\frac{d(ct)}{dz} = \frac{1}{\beta_{11}} \, . \tag{6}$$

The quantity β_{11} is given by

$$\beta_{11} = \left[\beta^2 - \beta_\perp^2\right]^{1/2} \tag{7}$$

and keeping only the slowly varying terms (averaging over a wiggler period) we obtain

$$\beta_{11} = 1 - \frac{1 + a_w^2 + p_x^2 + p_y^2}{2\gamma^2} \, . \tag{8}$$

Introducing the phase, θ, by $\theta = (k_s + k_w)\, z - \omega_s t$ where we have characterized the signal wave by k_s, ω_s; i.e., taken where a_s, φ_s are the amplitude and phase of the signal wave, we find

$$\frac{d\theta}{dz} = k_w - k_s \frac{1 + a_w^2 + p_x^2 + p_y^2}{2\gamma^2} . \tag{10}$$

Taking the potential, φ, as z independent (uniform axial focusing) we have

$$\frac{d\varphi}{dz} = \frac{\partial\varphi}{\partial x} x' + \frac{\partial\varphi}{\partial y} y' . \tag{11}$$

As in the usual formulation for an FEL, one can show that

$$\left(\frac{x'}{c}\right)\frac{\partial a}{\partial t} = \frac{k_s a_s a_w f_B}{\gamma} \sin\left(\theta + \varphi_s\right) , \tag{12}$$

where $f_B = J_0(b) - J_1(b)$, and $b = a_{w0}^2/[2(1+ a_{w0}^2)]$, is the usual Bessel function factor.

Thus we arrive at the particle equations:

$$\frac{d\gamma}{dz} = -\frac{f_B k_s a_s a_w}{\gamma} \sin\left(\theta + \varphi_s\right) - \underline{\frac{p_x \frac{\partial\varphi}{\partial x}}{\gamma} - \frac{p_y \frac{\partial\varphi}{\partial y}}{\gamma}} ,$$

$$\frac{d\theta}{dz} = k_w \left(\frac{\gamma^2 - \gamma_r^2}{\gamma_r^2}\right) ,$$

$$\gamma_r^2 = \frac{1}{2}\frac{k_s}{k_w}\left[1 + a_w^2 + p_x^2 + p_y^2\right] ,$$

$$\frac{dp_x}{dz} = -\frac{1}{2\gamma}\frac{\partial a_w^2}{\partial x} - \underline{\frac{\partial \varphi}{\partial x}} ,$$

$$\frac{dp_y}{dz} = -\frac{1}{2\gamma}\frac{\partial a_w^2}{\partial y} - \underline{\frac{\partial \varphi}{\partial y}} ,$$

$$\frac{dx}{dz} = \frac{p_x}{\gamma} ,$$

$$\frac{dy}{dz} = \frac{p_y}{\gamma} . \tag{13}$$

The underlined terms are the new terms. These are then combined with the usual equations for the signal field a_s, φ_s so as to completely describe the FEL.

For a wide ion channel the electrostatic well is harmonic

$$\varphi = \frac{k}{2}\left(x^2 + y^2\right) , \tag{14}$$

and thus the transverse particle motion is (for x or y):

$$\frac{d^2x}{dz^2} \cong -\frac{k}{\gamma}x - k_{\beta n}^2 x , \tag{15}$$

with

$$k_{\beta n} = \frac{a_0}{2\gamma} k_w , \tag{16}$$

so the particle oscillates with a betatron wavelength $\lambda_B = 2\pi/k_\beta$ and

$$k_\beta^2 = \frac{k}{\gamma} + k_{\beta n}^2 \ . \tag{17}$$

For our applications the ion-focusing term (the first term) is much larger than the natural focusing term (the second term). The focusing constant k is just $\omega_p{}^2/(2c^2)$ of the channel; i.e., in terms of the background gas density

$$\omega_p^2 = \frac{4\pi n e^2}{m} \ . \tag{18}$$

2. Conditioner Loading

A conditioner prepares an electron beam so that there is a correlation between energy and transverse oscillation amplitude. The criterion for the conditioner comes from the phase equation Eq. (6.3). If we define the on axis resonance energy as γ_0:

$$\gamma_0^2 \cong \frac{k_s}{2k_w}\left(1 + \frac{a_0^2}{2}\right) , \tag{19}$$

then it is easy to verify that the phase equation can be written in the form (for j^{th} particle)

$$\frac{d\theta_j}{d(k_w z)} \cong 2\,\frac{\gamma_j - \gamma_0}{\gamma_0} - \frac{1}{\left(1 + \frac{a_0^2}{2}\right)}\left[p_{xj}^2 + p_{yj}^2 + \gamma_0^2\, k_{\beta n}^2 \left(x_j^2 + y_j^2\right)\right] \ . \tag{20}$$

The spread of the phase change rate (proportional to longitudinal velocity) causes gain reduction. The first term of the right hand side is from the energy spread, the second term is from the emittance.

For an FEL without ion channel focusing, $k_\beta = k_{\beta n}$, and the emittance term is a time constant, so if the conditioner introduces correlation between the initial $\Delta\gamma_j$ and p_{xj0}, p_{yj0} x_{j0}, y_{j0}:

$$\frac{\Delta\gamma_j}{\gamma_0} = \frac{1}{2\left(1 + \frac{a_0^2}{2}\right)}\left[p_{xj0}^2 + p_{yj0}^2 + \gamma_0^2 k_{\beta n}^2 \left(x_{j0}^2 + y_{j0}^2\right)\right] , \tag{21}$$

then the emittance's contribution to the spread in the phase changing rate is cancelled as pointed out in Ref. [1].

For an ion channel focused FEL, $k_\beta > k_{\beta n}$, and the second term is no longer a time-constant. But $p_{xj}^2 + \gamma_0^2 k_\beta^2 x_j^2$ is still constant, and it is:

$$p_{xj}^2 + \gamma_0^2 k_\beta^2 x_j^2 = p_{xj0}^2 + \gamma_0^2 k_\beta^2 x_{j0}^2 = 2\overline{p_{xj}^2} = 2\gamma_0^2 k_\beta^2 \overline{x_j^2} \tag{22}$$

where $\overline{p_{xj}^2}$ and $\overline{x_j^2}$ are the values averaged over a β oscillation period. A similar expression holds for $\overline{p_{yj}^2}$ and $\overline{y_j^2}$.

Therefore we can select the conditioner criterion such that the phase changing rate, averaged over a betatron period, becomes independent of the initial transverse condition, i.e., free from emittance spread. It is easy to show that, after averaging over a betatron period,

$$\frac{\overline{d\theta_j}}{d(k_w z)} = 2\frac{\gamma_j - \gamma_0}{\gamma_0} - \frac{1}{2\left(1 + \frac{a_0^2}{2}\right)}\left(1 + \frac{k_{\beta n}^2}{k_\beta^2}\right)\left[p_{xjo}^2 + p_{yjo}^2 + \gamma_0^2 k_\beta^2 \left(x_{jo}^2 + y_{jo}^2\right)\right] . \tag{23}$$

Thus, if the initial loading is

$$\frac{\Delta\gamma_j}{\gamma_0} = \frac{1}{4\left(1 + \frac{a_0^2}{2}\right)}\left(1 + \frac{k_{\beta n}^2}{k_\beta^2}\right)\left[p_{j0}^2 + \gamma_0^2 k_\beta^2 r_{j0}^2\right] \tag{24}$$

then, although for each individual particle β_{11} still fluctuates, all the particles with the same energy will have the same $\overline{\beta_{11}}$, averaged over a betatron period. Our simulation shows that when the conditioner correlates the energy with emittance according to the last equation, the gain reduction due to emittance is minimized; deviation from this formula causes a large gain reduction.

3. Linear Theory

A first estimate of linear gain is provided by the theory of Ref. [1]. This theory ignores the longitudinal oscillations in v_z, caused by electrons oscillating transversely in the potential well created by the ion channel.

The simplest way to incorporate v_z oscillation is to perform a linear analysis on the wiggler-averaged equations neglecting modifications to the optical mode. In fact, one expects mode-profile modifications to be fairly negligible, since in the limit $\lambda_\beta << L_g$, the beam appears uniform to the evolving signal. We find that the effect of betatron jitter for a perfectly conditioned beam is to scale the Pierce parameter, $\rho \rightarrow \rho\eta$, where

$$\eta^3 = \left\langle \sum_{n=-\infty}^{\infty} J_n(\theta_x)^2 J_n(\theta_y)^2 \right\rangle . \tag{25}$$

The brackets indicate an average over the beam distribution,

$$\theta_{x,y} = \frac{1}{8}\left(\frac{\omega}{c}\right) k_\beta A_{x,y}^2 = \left(\frac{\pi}{4}\right)\left(\frac{\varepsilon_{x,y}}{\lambda_s}\right), \tag{26}$$

with $A_{x,y}$ the amplitude of the betatron motion in x,y.

A useful lower bound on η is obtained by including only the n = 0 term. Taking a symmetric gaussian distribution one can evaluate the requisite average in terms of an elliptic integral. The result predicts only a gradual reduction in gain, with increasing emittance, from that given by the theory of Ref. [1].

III. Numerical Simulation Results

We have applied the theory presented in this paper to a 30 Å FEL employing the parameters given in Table 1. Theoretical evaluation of the growth distance can be readily made using the theory presented in Ref [1]; i.e., ignoring the longitudinal modulation of velocity. In that case the power growth length is 1.58 m. The numerical simulation shows that the optimum growth length occurs when the betatron wavelength is 0.62 m (whereas the natural betatron wavelength is 83 m). The optimum is when the betatron wavelength is about 1/3 of the growth length. At the optimum, the power growth length is 2.13 m, which is not very different than that obtained from the very simple theory.

By comparison, an unconditioned beam in an FEL with only "natural focusing" would have a growth length of 25.6 m; i.e., be completely impossible.

We have studied bandwidth by varying the wavelength of the input seed, while keeping everything else fixed. We find that the FEL amplifies over a variation of 10^{-3} ; i.e., the width is about given by the Pierce parameter ρ (as usual). We have also studied the sensitivity to conditioner performance by changing the conditioner parameters. A 1% deviation from Eq. (2.5) makes no difference, but a 10% variation completely stops the FEL. Thus a few percent error in conditioning is allowed, but not more than that.

Building the requisite conditioner is a difficult, but not impossible, task.

IV. Conclusions

We have seen that the combination of ion-channel focusing and a conditioned beam allows one to operate an FEL where it would otherwise be impossible to do so. Thus it is important to check the theoretical work by experiment. Ion-channel focusing has been explored in non-FEL situations for a good number of years, and in an FEL in the pioneering work at KEK. However, much more remains to be done. Conditioners have not been studied experimentally at all. The first thing is to construct and measure beam properties, thus showing the intended introduction of correlation in the beam. Then one needs to show that FEL performance is really enhanced by a conditioner. (Note that this might first be done at long wavelengths, where the conditions are easier and less expensive.) Finally, both advances must be put into operation at the same time.

In conclusion, we note that the conditioner concept can be applied to other fast wave devices and, might have a proof-of-principle and first application there. In an FEL, the conditioner concept, plus ion-focusing, allows possibilities that previously were not open. For example, one can imagine a very powerful FEL having a low energy beams, subject to a wiggler of small strength, but then making an intense wiggled channel which a high energy beam must follow. Thus one achieves very short wavelengths with a not-so-high energy beam (because of the very small wiggler wavelength) and a not-so-intense beam (because the effective magnetic strength of the wiggler is very great).

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Table I. Parameters for a λ_s = 30 Å FEL using a conditioned beam of 80 A and 1,240 MeV and normalized emittance of 2×10^{-6} πm.

electron energy (γ)	2426
wiggler parameter (max) K	1.24
Bessel factor J_0-J_1	0.880
magnetic wiggler gap (mm)	6
maximum magnetic field Bw (Tesla)	0.663
wiggler period (cm)	2.00
normalized rms emittance/pi (m)	2.00×10^{-6}
radiation wavelength (Å)	30
fractional energy spread (rms)	4.35×10^{-4}
betatron wavelength (m)	0.62
peak current (Amp)	80.0
ion channel density (cm^{-3})	1.5×10^{13}
beam density (cm^{-3})	1.1×10^{15}
$\Delta\gamma_c$	33.9

Figure Captions

Fig. 1. A diagram showing the location of the beam conditioner (1a), and one period of the conditioner (1b). A period consists of two focusing lenses (each of strength $f/2$), two defocusing lenses (each of strength $-f/2$), and two RF cavities each operating in the TM_{210} mode.

Fig. 2. Numerical simulation result showing power as a function of longitudinal distance, for a $\lambda_s = 30$ Å FEL, with parameters as given in Table I.

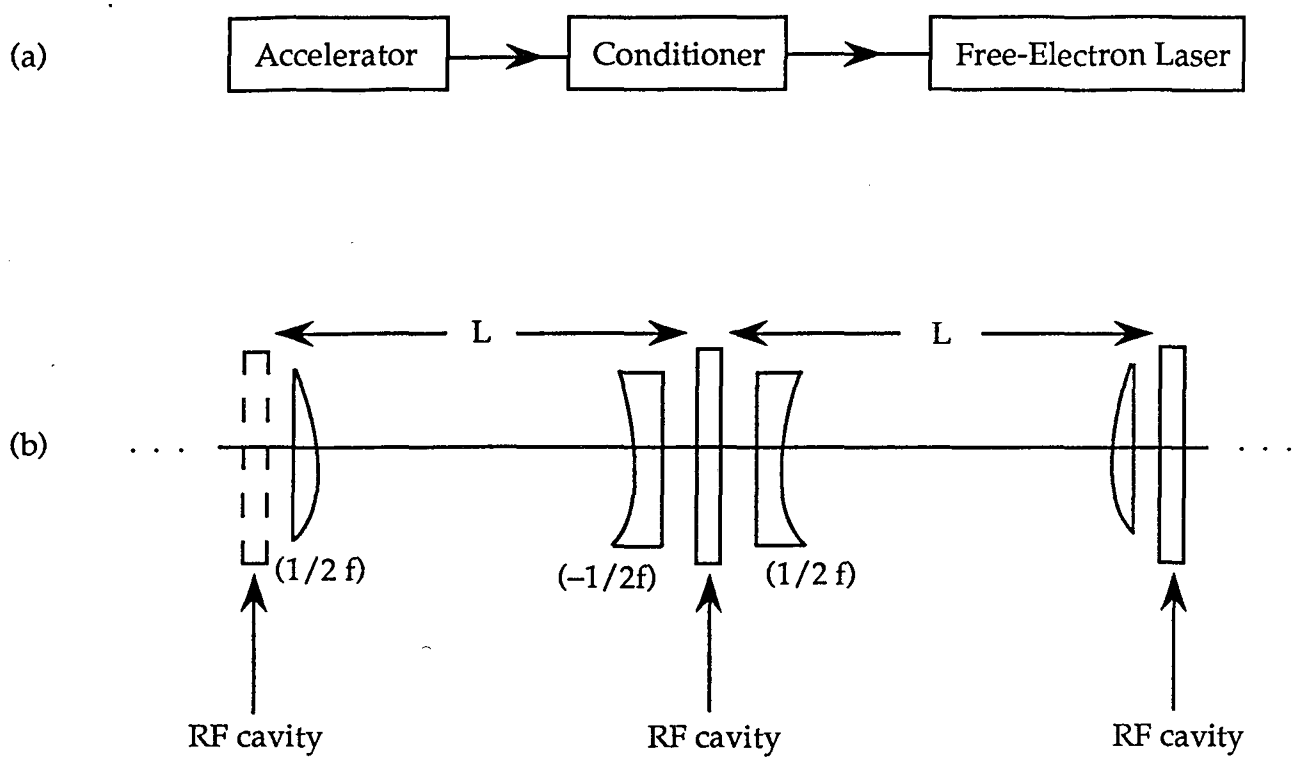
(a)
Accelerator
Conditioner
Free-Electron Laser
(b)
L
L
(1/2 f)
(–1/2f)
(1/2 f)
RF cavity
RF cavity
RF cavity

Fig. 1

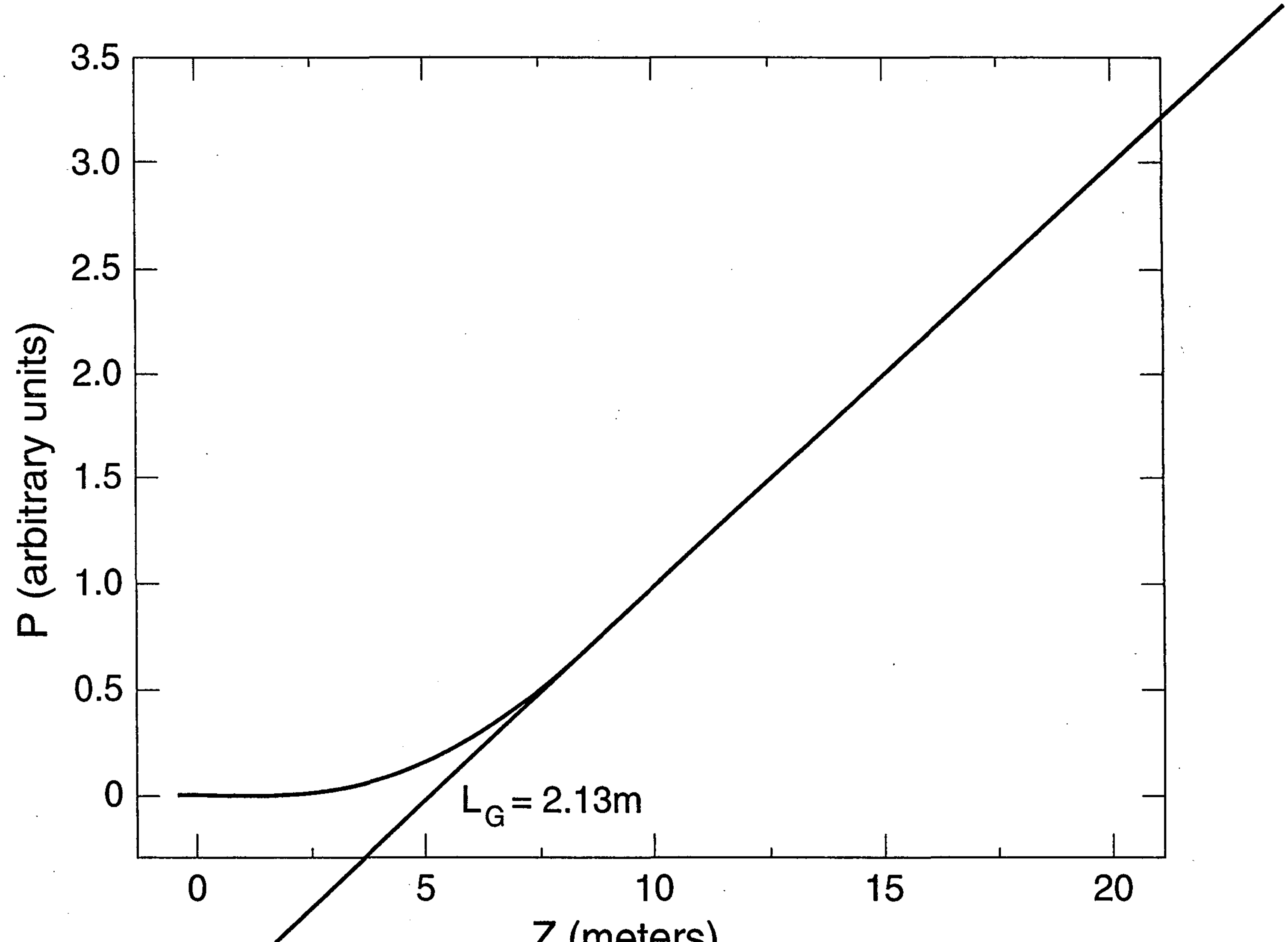

Fig. 2

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INFORMATION RESOURCES DEPARTMENT
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