**Title**

Nonparametrical Statistical Techniques for Location Discovery-Friendly Deployment



**Authors**

Feng, Jessica
Goudar, Vishwa
Girod, Lewis
et al.

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# Nonparametrical Statistical Techniques for Location Discovery – Friendly Deployment

Jessica Feng, Vishwa Goudar, Lewis Girod, Miodrag Potkonjak

## Introduction: Accuracy of Error Model Critical to Solution of Localization Problem

Although a great variety of *centralized* and *localized* algorithms have been proposed as solutions to the localization problem, their effectiveness is constrained by the accuracy of the underlying error model. None of available error models, which range from closed form parametric models to sophisticated kernel estimation-based non-parametric models, is a-priori applicable in new environments.

For example, consider the localization of the 10[th] node in a deployed network of 10 nodes where the location of 9 nodes are known and real distances of these to the 10th node are given. A comparison of the location errors of solutions from an exhaustive search following the maximum likelihood principle based on different data sets and error models are shown. Finally, we explore the problem of positioning nodes/beacons to maximally reduce error in location discovery.

| | GAUSSIAN | STAT1 | STAT 2 | CONSISTENCY |
|---|---|---|---|---|
| **GAUSSIAN** | ***0.0208*** | 7.993 | 4.258 | 0.0302 |
| **STAT 1** | 8.179 | ***0.0117*** | 5.275 | 0.0215 |
| **STAT 2** | 7.658 | 6.042 | ***0.0303*** | 0.0310 |

## Problem Description: Non-Linear Program Formulation of Localization Problem

We formulate the location discovery problem as a nonlinear function minimization instance with the objective function,

$$\mathbf{F = M(\varepsilon_{ij})}$$

where $\varepsilon_{ij}$ denotes the discrepancy between the calculated distance and the measured distance.

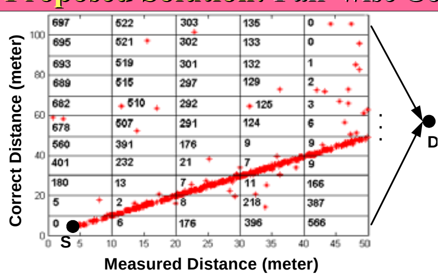
$$\varepsilon_{ij} = \sqrt{\sum_{l=1}^{k} (x_{li} - x_{lj})^2} - d_{ij}$$

M can be a norm subject to minimization, or an error distribution function, such as the pair-wise consistency-based error model, subject to maximization. As a function of the error distribution, M is formulated as

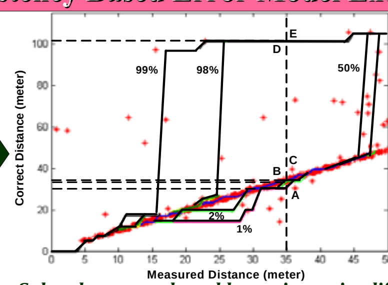
$$\mathbf{M} = \prod P_{ij}$$

where $P_{ij}$ is the probability, according to the error model, that error $\varepsilon_{ij}$ is detected when the measured distance between sensors i and j is $d_{ij}$. We would like to maximize the likelihood of the proposed solution.

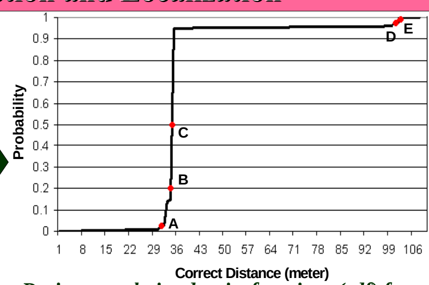
## Proposed Solution: Pair-wise Consistency Based Error Model Extraction and Localization




*Map continuous instance to the discrete domain and transform this instance to a graph format where the most consistent monotonic regression function of the instance is equivalent to the shortest path of the graph.*




*Solve shortest path problem using a simplified dynamic programming-based shortest path algorithm. Derive multiple regression functions based on statistically selected subsets of data.*




*Derive cumulative density functions (cdf) for different measured distances from resulting regression curves. Derive probability density function (pdf) from resulting cdf.*

### Simultaneous Location Discovery and Error Model Construction

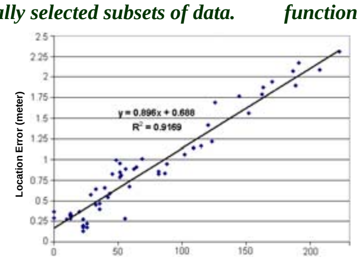
1. Location discovery by minimization of non-linear function developed based on pair-wise consistency principle.

   Objective function: $F2 = F + \sum \varepsilon_s$

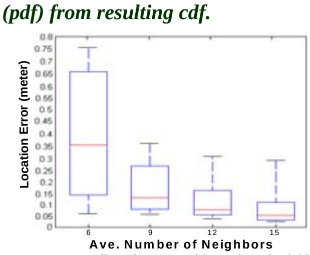
$$F+ = \begin{cases} [(-(c_{ij} - c_{kl})\cdot(d_{ij} - d_{kl}))] & \text{if } (c_{ij} - c_{kl})\cdot(d_{ij} - d_{kl}) < 0 \\ H & \text{otherwise} \end{cases}$$

   $\varepsilon_s$ - location error of hidden beacons, H - negative real constant, $c_{ij}$ - calculated distance between nodes i,j, $d_{ij}$ - measured distance between nodes i,j

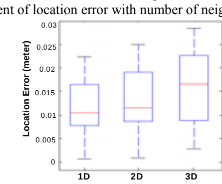
2. Obtaining error model based on measured and calculated distances.
3. Iterative usage of the error model as the objective function for localization, and the resultant locations for the construction of the error model.



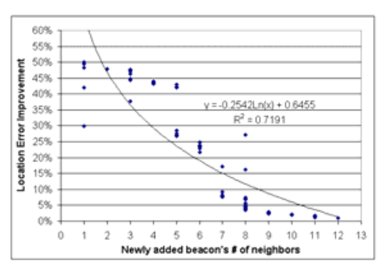

Consistency of objective function with location error



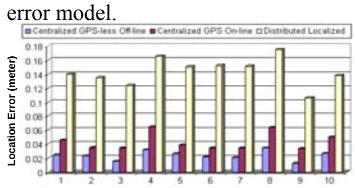

Improvement of location error with number of neighbors



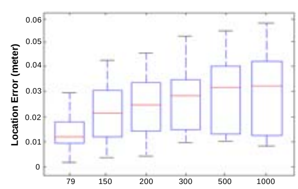

Location error for different dimensions



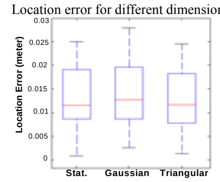

Improvement in location error with beacon placement




Location error based on different data sets




Location error increase with network size




Location error for different error distributions