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ABSTRACT

The basic investigation of transistor feedback amplifiers has proven mathematically simple and of great practical value. The behavior of single-stage common-emitter amplifiers is described and provides a building block with which cascaded feedback amplifiers can be analyzed and designed.

The problem of designing these amplifiers is complex, and what is felt to be the most important phases of the problem are discussed.

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I. INTRODUCTION

It is not necessary to discuss the importance of precision high-gain pulse amplifiers. They are perhaps the backbone of nuclear instrumentation. Despite this, little has been written about design for optimum performance, and it is probably due to the nature of the problem. They are used for many different purposes and a specification important to one application may not be important to another. Rise-time, gain, overshoot, noise, sensitivity, linearity, and overload are the main problems involved. Clearly not all of these may be optimized in a single design, therefore one must understand the effect of each factor upon the others in order to find an optimum design for a given set of specifications.

The scope of this study is as follows:

(a) Analysis of the behavior of the single-stage common-emitter amplifier with and without local feedback.

(b) Analysis of the behavior of cascaded amplifiers with loop feedback.

(c) Determining a practical method of shaping the feedback network to produce the desired pulse response.

(d) Description of the variations (sensitivity) of the ac gain for local and loop feedback.

(e) Comparison of the types of amplifiers in order to predict which type will give the best performance.

Basically, the question is: What benefit will be derived by using different types of simple feedback loops rather than cascaded single stages, and will such benefits by appreciable?

II. SINGLE STAGES

Sensitivity may be described simply for local and loop feedback and for cascaded stages. First, however, the gain equations for the single stage must be discussed. From the equivalent circuits of Fig. 1 and the amplifiers of Fig. 2, the following may be shown simply.

Case I

$$\text{Current gain} = A_i = \frac{\beta\, R_S/R'_S}{S} \tag{1}$$

$$\text{Upper 3 db frequency} = BW = \frac{f_\beta}{D}\ S, \quad [\text{Roll-off} = \text{Gdb/octave}] \tag{2}$$

where

$$S = 1 + (\beta + 1)\, R'_e/R'_S\ , \tag{3}$$

$$D = 1 + 2\pi\, f'_\alpha\, (R_L + R'_e)\, C_c\ , \tag{4}$$

$$R'_S = R_S + r'_b\ , \tag{5}$$

$$R'_e = R_e + r_e\ , \tag{6}$$

$$f'_\alpha = \beta f_\beta\ . \tag{7}$$

The quantity f_β is therefore the frequency at which A falls 3 db for $R_L = R_e = 0$ and $R_S = \infty$ (β cut-off frequency).

$$\text{Gain-bandwidth product} = GBW_i = \frac{f'_a}{D} \quad \frac{R_S}{R'_S} \qquad (8)$$

Notice that r'_b and C_c are the effects which reduce the GBW "figure of merit." Also, the GBW cannot be considered constant as the gain is changed, because R'_e affects the D factor. It is of course necessary to make D close to one for best results.

Case II

$$\text{Voltage gain} = A_V = \frac{\beta R_L/R'_S}{S} , \qquad (9)$$

$$BW = \frac{f_\beta}{D} S \qquad (10)$$

The same definitions as for Case I hold.

$$GBW_V = \frac{f'_a}{D} \quad \frac{R_L}{R'_S} . \qquad (11)$$

These formulas are of great practical aid and much easier to use than may be supposed at first glance.

III. DOUBLETS, TRIPLETS, AND PULSE SHAPING

The discussion of drift reduction will assume that any desired amount of loop gain (ℓ) can be achieved. In every case some shaping of the frequency response of the feedback network is necessary to produce the time response desired. A further comparison must therefore be made between singles, doublets, and triplets in order to see which is best for rise time and overshoot. The amplifier must be shown in detail, and the types discussed here are shown in Figs. 3 and 4. There are other possible configurations of course.

Doublets

The gains spoken of from now on are current gains and apply to Case I of Fig. 1. A denotes the gain without a loop, and K denotes the gain of the same unit after the loop is introduced; $\ell = \mu A$ is still the loop gain. From Fig. 3 we have

$$A_0 = \frac{\beta^2 R_S/R'_S \;\; R_L/R'_L}{\left[1+(\beta+1)\frac{r_e}{R'_S}\right]\left[1+(\beta+1)\frac{r_e}{R'_L}\right]} = \text{low-frequency gain;}$$

$$A_0 \cong \beta^2 H,$$

where $H = R_S/R'_S$ or R_L/R'_L, and $(\beta+1)\frac{r_e}{R'_S} << 1$, $(\beta+1)\frac{r_e}{R'_e} << 1$.

The local internal feedback is therefore considered negligible.

From here on, A is the gain function including frequency

$$A = A(f) = \frac{A_0 \;\omega_1\omega_2}{(p+\omega_1)(p+\omega_2)} ,$$

where $p = j\omega$ and ω_1, ω_2 are the 3-db cutoff frequencies of each stage. Local feedback (S) is negligible, and good design demands $D \cong 1$:

$$A \cong \frac{A_0\,\omega_\beta^{\,2}}{(p+\omega_\beta)^2} ,$$

$$K = \frac{A}{1+\mu A} \cong \frac{A_0\;\omega_\beta^{\,2}}{(p+\omega_\beta)^2 + \mu A_0\,\omega_\beta^{\,2}} .$$

$$K = A_0\omega_\beta^{\,2}/p^2 + 2p\omega_\beta + \omega_\beta^{\,2} + \mu A_0\omega_\beta^{\,2} .$$

When μ is constant with frequency one has

$$K = \frac{A_0 \omega_\beta^2}{p^2 + 2p\omega_\beta + \omega_\beta^2 (1 + \mu A_0)}$$

and

$$K_0 = \frac{A_0}{1 + \mu A_0} \quad .$$

Here K is of the form

$$K = \frac{A_0 \omega_\beta^2}{(p-a)(p-b)} \quad ,$$

where the roots a and b cause the step function response to have overshoot and ringing as shown in Fig. 5.

$$a, b = \omega_\beta \left[1 \pm \sqrt{1 - (1 + \mu A_0)}\right]$$

$$a, b = -\omega_\beta \left[1 \pm j\sqrt{\ell}\right] .$$

Complex roots always cause overshoot. The angle of the complex roots is $\phi = \tan^{-1} \sqrt{\ell}$, and the overshoot is a function only of this angle, as shown in Fig. 6.

Complex roots may be eliminated by causing μ to be a function of frequency. From the basic definitions and approximations for feedback ratio, we have $\mu = i_f/i_L$ (Fig. 3). The following assumes that the input impedance of the first transistor does not affect the feedback ratio. This assumption is not good, but the problem may be partially overcome by inserting an impedance-matching stage in the loop (See Fig. 3):

$$\mu = \frac{i_f}{i'_f} \cdot \frac{i'_f}{i_e} \cdot \frac{i_e}{i_L} = a \cdot \frac{R_e}{Z_f} ; \quad \frac{1}{a} \doteq \frac{R_e}{Z_f} ;$$

Z_f then must be shaped to eliminate the complex roots for any given ℓ.

The elimination of complex roots requires that Z_f become a parallel RC combination where

$$R = \frac{R_e}{\mu_0}$$

and

$$C \cong \frac{2}{\omega_\beta \sqrt{\ell}\; R_f} \quad .$$

The K equation for this shaping becomes

$$K \cong \frac{A_0\, \omega_\beta^{\;2}}{(p + \omega_\beta \sqrt{\ell})^2} \quad , \quad A_0 = \beta^2 H \; .$$

This looks like two identical cascaded singles with gains of $\beta \frac{\sqrt{H}}{\sqrt{\ell}}$ and bandwidths of $f_\beta \sqrt{\ell}$.

Normally C is adjustable so the shaping may be adjusted to fit the circumstances. This analysis is of course no great revelation for feedback capacitors have been used for years. This result allows a calculation of the capacitor value, which is quite worthwhile and this value agrees closely with experiment, which serves to strengthen faith in the over-all analysis.

When n transistors are cascaded in n/2 doublets, the total bandwidth is

$$BW = f_\beta \sqrt{\ell} \sqrt{2^{1/n} - 1} \; .$$

Total gain is $G = \left(\frac{\beta^2 H_2}{\ell}\right)^{n/2}$.

Hence, we have

$$\ell = \frac{\beta^2 H_2}{G^{2/n}} , \qquad \sqrt{\ell} = \frac{\beta}{G^{1/n}} \sqrt{H_2}$$

and

$$BW = f_a^{\ell} \frac{\sqrt{H_2}}{G^{1/n}} \sqrt{2^{1/n} - 1} .$$

When n transistors are cascaded in n singles, the total bandwidth is

$$BW = f_\beta S \sqrt{2^{1/n} - 1} .$$

Since we have

$$G = \left(\frac{\beta H_1}{S}\right)^n ,$$

we can write

$$BW = \frac{f_a H_1}{G^{1/n}} \sqrt{2^{1/n} - 1} .$$

The rise times will therefore be the same except for differences in coupling efficiencies (H factors) for singles of doublets when they are adjusted for no overshoot.

Triplets

For three stages one has

$$A \cong \frac{\beta^3 H \omega_\beta^3}{(p + \omega_\beta)^3} = \frac{A_0 \omega_\beta^3}{(p + \omega_\beta)^3} ,$$

which neglects local feedback (S).

The K equation becomes therefore

$$K = \frac{A}{1 + \mu A} = \frac{A_0 \omega_\beta^3}{(p + \omega_\beta)^3 + \mu A_0 \omega_\beta^3} = \frac{A_0 \omega_\beta^3}{(p-a)(p-b)(p-c)} .$$

When μ is not frequency-sensitive, complex roots appear for $\mu > 0$. In order to eliminate them, techniques may be employed which bring about results similar to the doublet analysis. In Fig. 4, Z_L is a series L-R, and Z_L is a parallel R-C. It is not possible to completely eliminate the imaginary parts of the roots unless an S factor is introduced in one of the internal stages. This naturally reduces the ℓ achievable for specified gain or bandwidth and much of the sensitivity advantage of the triplet is lost.

III. SENSITIVITY

The variations in gain are of primary importance in nuclear amplifiers because they effect the precision of the whole system, of which the amplifier is only a part. It is not necessary to stress this point for anyone who has worked with such systems.

From the discussion of pulse response, it is evident that this will not dictate the form of the amplifier. For singles and doublets, it will be an easy matter to shape the pulse response, and the bandwidth requirements will not be helpful in choosing between them. At this point the triplet is not favored much because it is somewhat tedious to shape its pulse response, and it will not have as great an advantage in sensitivity over the doublet as might be expected because of the presence of local feedback in one of the three stages in each loop.

Sensitivity problems usually result from two things--variations due to temperature and bias.

Bias-point shift

There are several publications on this topic.[1] We shall therefore pass over it now and take it up later in general terms where it seems more appropriate.

Temperature effects

Values of α and r_e change with temperature and must be considered in a sensitivity analysis. The first consideration is, of course, the sensitivity of the single-stage amplifier.

From Fig. 2, case 1, and equations of section II, we have

$$A = \frac{\beta R_g / R_g'}{S} . \quad (1)$$

[1] For example, see R. F. Shea, Ed., Transistor Circuit Engineering (Wiley, New York, 1957).

From this we derive

$$\frac{dA}{dT} = \frac{\beta R_S/R'_S}{S}\left[\frac{d\beta}{\beta dT} - \frac{dS}{SdT}\right] .$$

For $dA/dT = 0$, we have

$$\frac{d\beta}{\beta dT} = \frac{dS}{SdT}$$

and therefore

$$\frac{d\beta}{\beta} = \frac{dS}{S} .$$

Accordingly we find

$$\frac{dS}{S} = \left(\frac{S-1}{S}\right)\left[\frac{d\beta}{(\beta+1)} + \frac{d R_e'}{R_e'}\right] ,$$

if we assume $\beta \gg 1$.

The necessary condition for zero rate of change of A with temperature is therefore

$$\frac{d\beta}{dR_e'} = \frac{\beta^2}{\left[R_S' + R_e'\right]} . \qquad (12)$$

From the following three familar relations,

$$d R_e' = d r_e, \quad r_e = \frac{KT}{qI_e} \quad , \text{ and } \quad \frac{dr_e}{dT} = \frac{K}{qI_e} = \frac{r_e}{T},$$

it is evident that

$$\frac{d\beta}{d R_e'} = \frac{d\beta}{dT} \cdot \frac{1}{\frac{d r_e}{d T}} = \frac{d\beta}{dT} \frac{T}{r_e} . \qquad (13)$$

Using Eqs. (12) and (13), we obtain

$$\frac{d\beta}{d\,R_e'} = \frac{\beta^2}{R_S' + R_e'} = \frac{\beta^2}{R_e'\left[A+1\right]} ,$$

Since $A \cong R_S'/R_e'$ when $S \gg 1$.

The result is

$$\frac{d\beta}{dT}\;\frac{T}{r_e} = \frac{\beta^2}{R_e'\left[A+1\right]} ,$$

which reduces to

$$\frac{R_e}{r_e} = \frac{\beta^2}{\left(\frac{d\beta}{dT}\right)\;T\,(A+1)} - 1 ,$$

where T is temperature in degrees Kelvin.

These values turn out to be practical, and it appears that two temperature effects in the same stage may be used to balance one another.

The action can be explained easily from Eq. (1).

$$A = \frac{\beta\, R_S/R_S'}{S} \qquad (1)$$

If β increases with temperature, the gain will go up. The sensitivity to temperature will be reduced by about the factor S if R_e' and R_s' are constant. The value of R'_e varies (increases) with temperature, and this gives the possibility of increasing S at the same rate that β increases and thereby cancelling out the change in gain. This requires adjustment of R_e, and good results may be difficult to achieve, so it is important to study doublets and triplets with loop feedback and make comparisons.

Cascaded variations

Assume two stages cascaded with gains A and B. We have

$$G = A\,B.$$

Therefore, when variations occur the change in G is $G_2 - G_1 = \Delta G$, where $G_2 = A_2 B_2$ and $G_1 = A_1 B_1$.

The (fractional) change of G is

$$\frac{\Delta G}{G_1} = D_G .$$

Subscript 1 denotes gains before change and subscript two denotes gains after the change.

From the above definition, we have

$$D_G = \frac{A_2 B_2 - A_1 B_1}{A_1 B_1} = \frac{(A_1 + \Delta A)(B_1 + \Delta B) - A_1 B_1}{A_1 B_1},$$

$$D_G = \frac{\Delta A}{A_1} + \frac{\Delta B}{B_1} + \frac{\Delta A \Delta B}{A_1 B_1},$$

and

$$D_G = D_A + D_B + D_A D_B . \qquad (14)$$

since by definition

$$D_A = \frac{\Delta A}{A_1} \qquad \text{and} \qquad D_B = \frac{\Delta B}{B_1}$$

For very small variations the effect is essentially addative. This approximation will be used from now on.

Loop feedback in small-signal stages

Assume an amplifier of gain K contained in one loop of negative feedback and the total gain G made up of these loops. The stages in each loop have a total gain of A. Therefore we can write $K = A/(1 + \ell)$ where ℓ is the loop gain. Assume no local feedback because most desensitivity is achieved when it is all done in the loop.

From our previous definitions

$$D_K = \frac{K_2 - K_1}{K_1} = \frac{\dfrac{A_2}{1 + \ell_2} - \dfrac{A_1}{1 + \ell_1}}{\dfrac{A_1}{1 + \ell_1}}$$

and

$$D_K = \frac{(A_2 - A_1) + A_2 \ell_1 - A_1 \ell_2}{A_1 (1 + \ell_2)} .$$

Since

$$A_2 \ell_1 = A_2 \mu A_1 = A_1 \ell_2$$

then

$$A_2 \ell_1 - A_1 \ell_2 = 0$$

and it follows that

$$D_K = D_A / (1 + \ell_2)$$

Desensitivity is achieved in the amount of $(1 + \ell_2)$ assuming that the feedback ratio (μ) is constant. As A increases ℓ also increases and the compensation increases. This effect is graphed in Fig. 7.

Optimum design for loops

When a great deal of feedback is used in each loop it will require many loops. These are two opposing effects. When few loops are used there is little feedback in each. It is possible to have too much feedback or too many stages. The optimum number of loops for minimum variation of the total gain G is the natural logarithm of G. If the variation D_G is plotted as a function of the number of loops for a certain variation of the single stages D_A, the curve reaches a minimum at $n = \ln G$. But this is a broad curve and not of much practical value. The main reason it is not usable is that the feedback ratio μ cannot be held constant.

Variations in μ

The variation of feedback ratio with temperature has already been discussed for singles. The effect was shown to be helpful in balancing out drift of transistor parameters. For doublets the problem is not so simple. The value of R_e must be kept down so the local feedback of the second stage is negligible. If this is not true the amount of desensitivity resulting from use of the loop is reduced by about the feedback factor S of the second stage. Because R_e is small, the feedback resistor R_f must be small. A normal value for R_f is a few thousand ohms, and the input impedance of the first stage is therefore of the same order of magnitude as R_f. This input impedance varies linearly with temperature (^{o}K). Therefore, when the gain K becomes $1/\mu$ the variations in gain are due chiefly to variations in the input impedance of the first stage. The same will be true for triplets, but R_f may be much higher and the effect is much less. Since both the internal gain and $1/\mu$ are increasing with temperature, there does not seem to be much possibility of balancing out the effects. For doublets, the use of a buffer (grounded-base) stage in the loop will reduce the sensitivity to that of the α of the buffer transistor. This promises to be very useful. If a buffer is used in the triplet, as shown in Fig. 4, it is outside the loop, and the desensitivity is limited by α of the buffer.

Singles then look good as small-signal amplifiers because of the possibility of trimming R_e to achieve great reductions in temperature effects over a usable range. Over or under compensation may result in balancing out effects caused by drift of the bias point. Doublets require no trimming and their performance is very predictable, but their sensitivity cannot be reduced to less than that of the α of a buffer transistor for normal conditions. It takes only a few simple calculations to show that the triplet will produce very stable amplifiers without the trimming required of singles. However there is a great deal of trimming required in shaping the triplets pulse response. Also some local feedback required to do this will reduce the possible desensitivity. The triplet also requires a buffer in order to avoid μ variations which limits the sensitivity to that of the buffer transistor α.

If a large R_e is used for singles and the balancing effect ignored; if there were no variations in the buffer of the doublet or the triplet, and if the optimum number of stages and loops were used ($\ell n G$), the sensitivity of the triplets would be better than that of the doublets by the factor $\frac{2}{3}\beta$ and doublets would be better than singles by the factor $\frac{1}{2}\beta$. This is definitely not the case, however, and the nice analysis that gave such simple results must be forgotten. The singles come close to obeying such an analysis if $R_e \gg r_e$.

IV. OTHER PROBLEMS

Input Impedance

It is generally preferable to use a very low input impedance and add a high-value resistor when high input impedances are needed. The transistor is not a good high-to-low-impedance buffer. The input impedance of an emitter follower has a rather high capacity which causes the impedance to fall 3db at a little less than f_β. Normally this is not good. The grounded-emitter stage with a series input resistor R_s will have as much current, voltage, and power gain as a common-emitter stage with an R_e when both input impedances are made R_s. The first type will have only stray capacities at the input, while the second will have a large capacity.

Linearity

This is of course no problem until the last few stages are reached. The value of β increases with collector voltage and for low emitter currents but falls off gradually at higher emitter currents. Most of the circuits an amplifier must drive can probably be designed with very low input impedances. This will have to be done many times because of the voltage limitations of most transistors, but it should be done anyway to get away from the nonlinear effect on the gain of the last stage. The voltage variation of β is not consistent among units, but the current variation is a broad curve and much more dependable. Basically linearity problems are the same as sensitivity problems because they are both changes in gain. The local-and loop-feedback analysis are therefore applicable. It is better to use a loop rather than local feedback in the output stages. Suppose the last stage and the one before it have 10% and 2% nonlinearity in their gains over the appropriate input pulse range. If a value of S = 10 were used in each, the net nonlinearity would be about $\frac{10}{10} + \frac{2}{10} = 1.2\%$. The reduction in their cascaded gain would be 100. If, however, this amount of gain reduction were obtained from a loop around both, the nonlinearity would be about $\frac{10 + 2}{100} = 0.12\%$, and the gain and bandwidth the same. The last few stages will therefore be biased where the nonlinear effects will be the least, and have a negative feedback loop around them.

Overload Effects

When pulses are present which drive the transistors beyond cutoff or saturation, care must be exercised so that dead time between pulses does not result. Dead time results when coupling capacitors charge through a small impedance and discharge through a large impedance. It is possible to cascade PNP and NPN units alternately so that each is being driven on by the pulse, but if there is undershoot at the end of the pulse it may cause dead time owing to some of the units' being cut off. It may seem better to design such an arrangement that each unit will be driven toward cutoff normally and undershoot at the end of a pulse will not cause any dead time. This, however, could require more stand-by current than can be tolerated.

Dead time may usually be eliminated by careful biasing. The output stage should be made to be driven further into conduction by the pulse, but it must be biased into conduction enough to absorb the undershoot without causing dead time. Of course, stages that are normally driven toward cutoff need only be biased on enough for the normal pulse to be passed.

V. CONCLUSIONS

The design for high-gain linear pulse amplifiers is obviously complicated, but no more so than most other important circuits.

Let us piece together some sort of consistent guiding statements by which the problem may be approached.

1. Sensitivity. There is an advantage in using singles rather than doublets. The triplets have a better drift performance than singles, but most of this is lost when the pulse overshoot has to be eliminated. Doublets will have a sensitivity determined by the drift of the input impedance of the first stage and the values of R_e and R_f. If a buffer is used in the loop, the sensitivity will essentially be that of its current gain a.

2. Biasing. Bias stability is of primary concern because β is sensitive to operating point as well as to temperature. The bias point must be carefully selected so that the most linear (distortionless) operation may be obtained in the large signal stages. Since the S factor of each stage depends upon r_e, and $r_e \propto \frac{1}{I_e}$, the emitter bias current I_e must sometimes be high enough to insure that local feedback is negligible. Bias will also determine the response under overload conditions. A stage must not be allowed to be cut off while the coupling capacitor is recovering from the pulse.

3. Pulse Response. Singles, doublets, or triplets may be designed so that the ringing of the pulse response is eliminated for any specified gain. For singles this ringing cannot occur unless there are peaking coils, of course; for doublets it is a small problem; for triplets and S factor must be used in one of the latter two stages, and this results in a loss of the triplets' drift-response advantage. The rise times are about the same for each type for a given over-all gain G. The rise times are equal when the number of transistors (not counting buffers) is kept the same for each type in accomplishing the over-all gain G.

4. Input Stage. The important things about the first stage are noise and input impedance. Noise can only be minimized by keeping the current down as low as

possible and the voltage from exceeding some value which is determined by the transistor. Drift transistors may present some problems along this line because of their limited operating range of current and voltage. Normally a V_{ce} of less than 5 or 6 volts is sufficient. Despite these precautions the input noise current is usually too large to be tolerated. Another transistor as a preamplifier will not help. There are two alternatives-- a tube or a transformer. The signal-to-noise ratio will be improved by the turns ratio of a transformer (step-down) when the high-input resistance is matched to the low-input impedance of the transistor instead of the input resistance simply being put in series with the transistor. The tube will of course be the best for noise, but it presents other problems also.

In order to get a low-input impedance in the first transistor so a resistor may be used in series with it to get rid of temperature variations, the singles spoken of above cannot be used. The input will therefore be a grounded-base stage or a common-emitter stage (or stages) with a negative-feedback loop.

5. Small-Signal (intermediate) Stages. The singles look good for the intermediate stages. They may be somewhat tedious to trim for temperature response, but they will give good pulse response without any trimming.

6. Output Stages. The last few stages that show appreciable nonlinearity will be in a loop. The circuit that these stages drive will be low impedance in order not to require much voltage swing.

VI. RESULTS (EXPERIMENTAL)

The descriptions of single-stage and loop drift behavior in Section II are in good agreement with experiment. Since the whole mathematical development is built upon Eqs. (1) and (14) and a knowledge of d_β, the drift formulas should be accurate. That is, once Eqs. (1) and (14) are substantiated all the rest is simple, rigid algebra and differential calculus. These equations have also been substantiated by simple experiment.

During the course of this study an amplifier has been built with the following characteristics. It consists of three doublets and one single-stage inverter.

Current Gain - 7,000

Rise time - 0.5 μsec

Overshoot - Negligible on a 'scope. This may be adjusted, resulting in about 0.3 μsec rise times for 20% overshoots.

Input Impedance - Series resistor of 1K.

Input Pulse - Negative

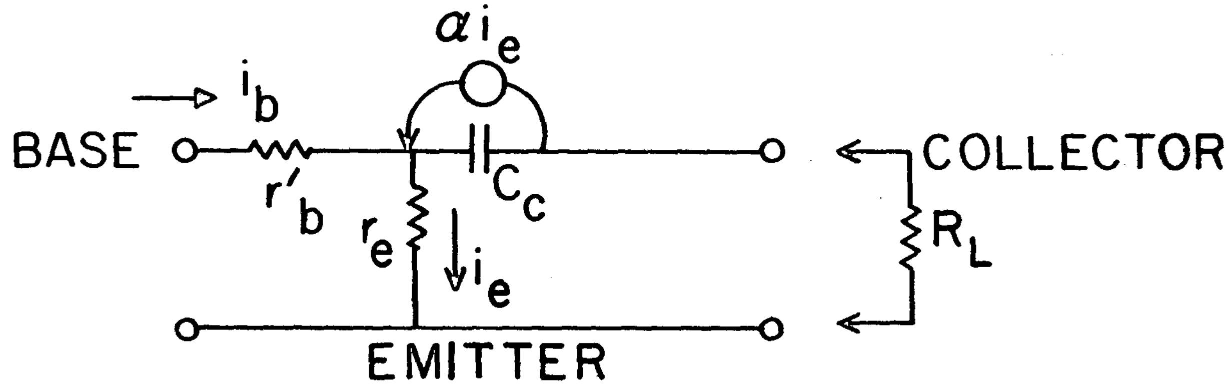
Output - Up to 8 or 10 ma positive. Voltage swings less than 2 v.

Sensitivity - 4.5% per $10^{\circ}C$ rise (fairly linear).

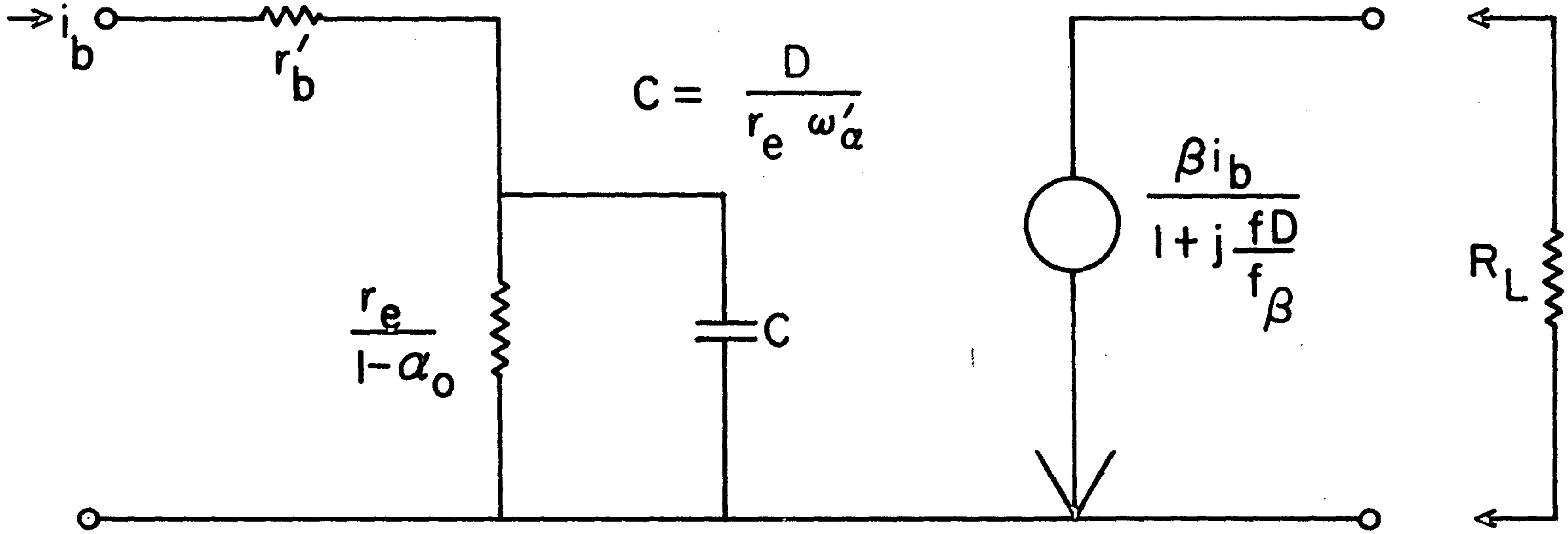
These doublets did not have buffers and it is expected the drift would go down by about an order of magnitude if they were used. The output works into a forward-biased diode subtractor, therefore only very small voltages are required from the amplifier. This amplifier is not an optimum design for drift response, but it gives a good idea of the results that may be achieved. The sensitivity is due to the variations in μ spoken of before.

We have tried to balance the singles temperature response since the possibility became evident after the above amplifier was made. The observations with two singles showed that with values of R_e, R_s, and r_e calculated from previous equations the reduction of sensitivity was quite good when the measured value of $d\beta/dt$ (about 1/2% per $^{\circ}C$) was used. The experiment did not produce sufficiently accurate results to prove the validity of the equations, but it did demonstrate the possibility of balancing the temperature effects to produce much less sensitivity than could be achieved with only simple local feedback.

FIGURE LEGENDS



"T"-EQUIVALENT



"π"-EQUIVALENT

Fig. 1

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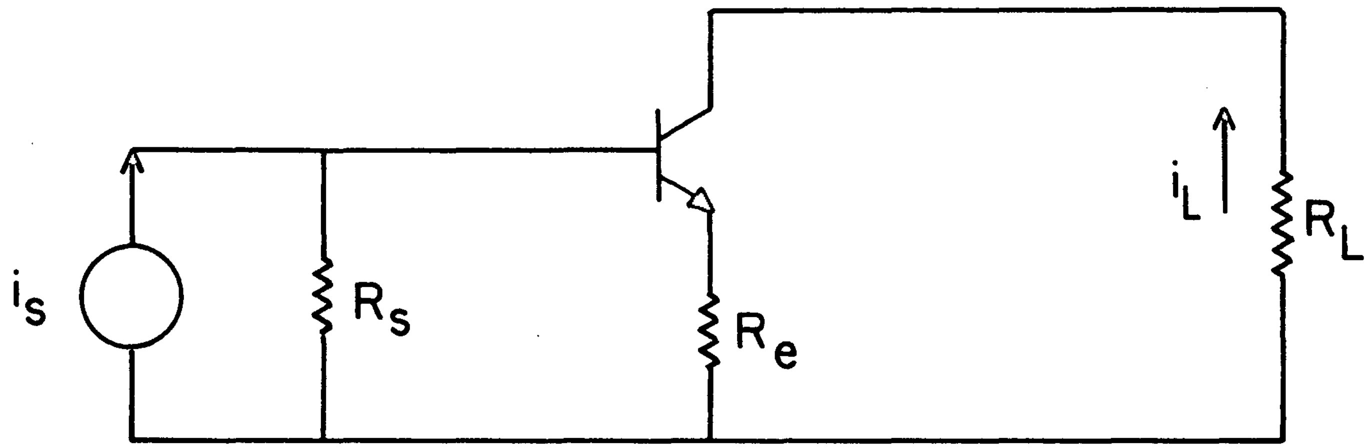
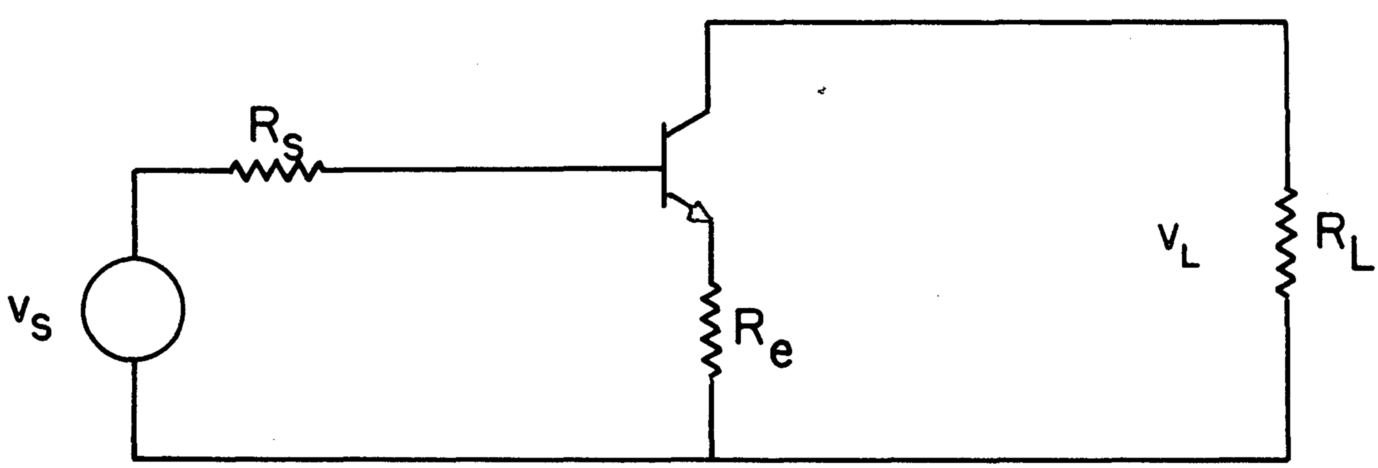
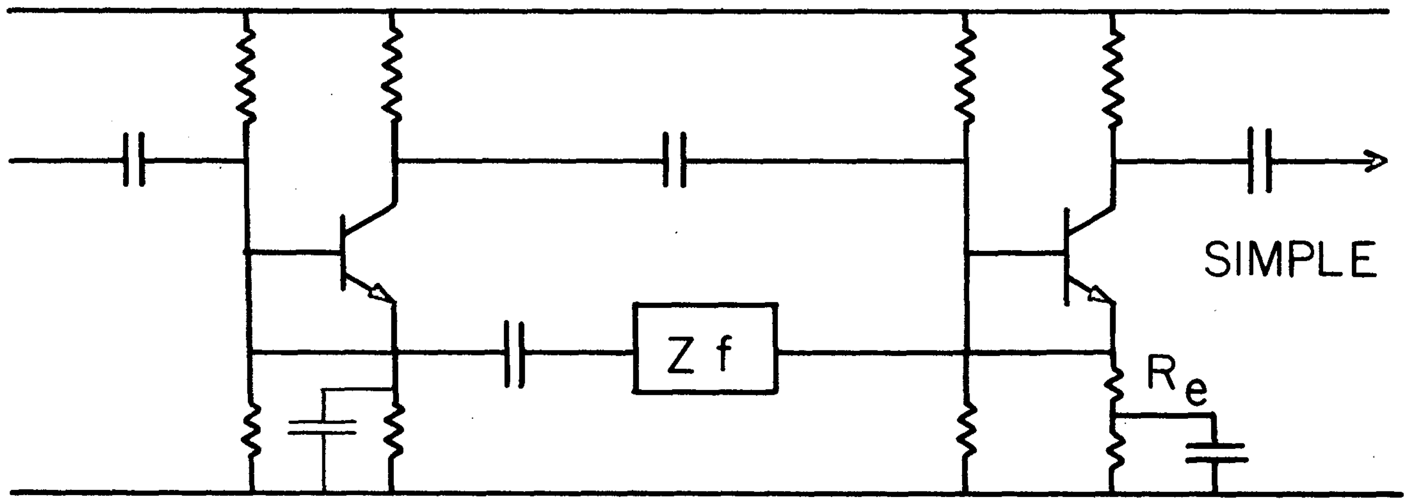


CASE ONE

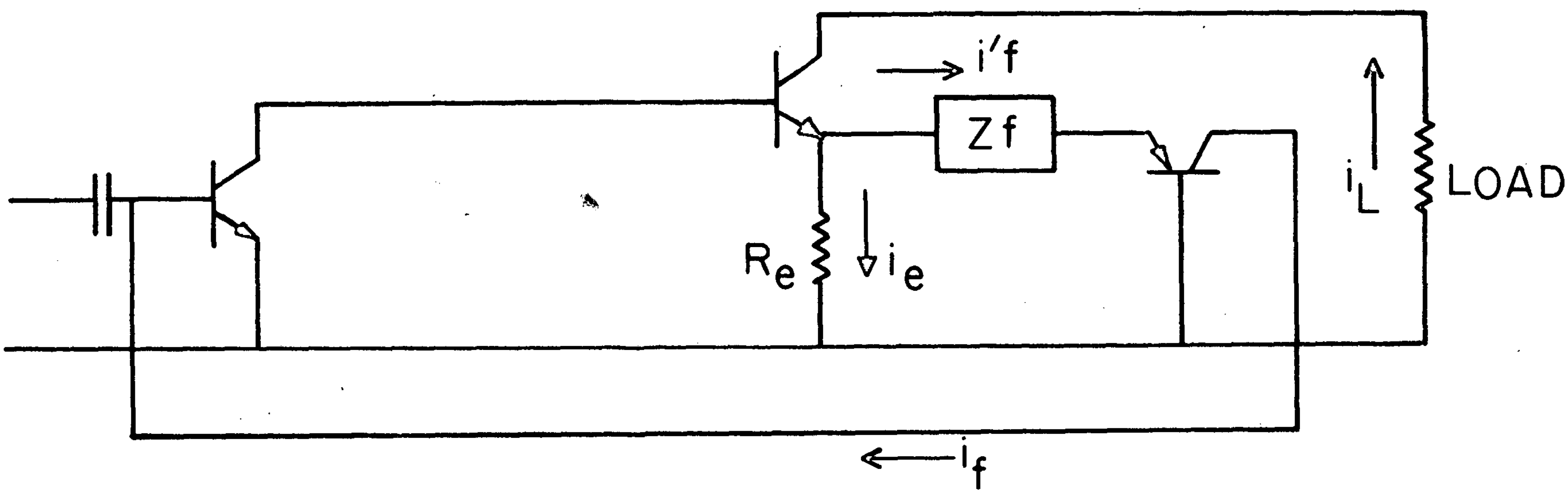


CASE TWO

Fig. 2
51321-1



SIMPLE DOUBLET

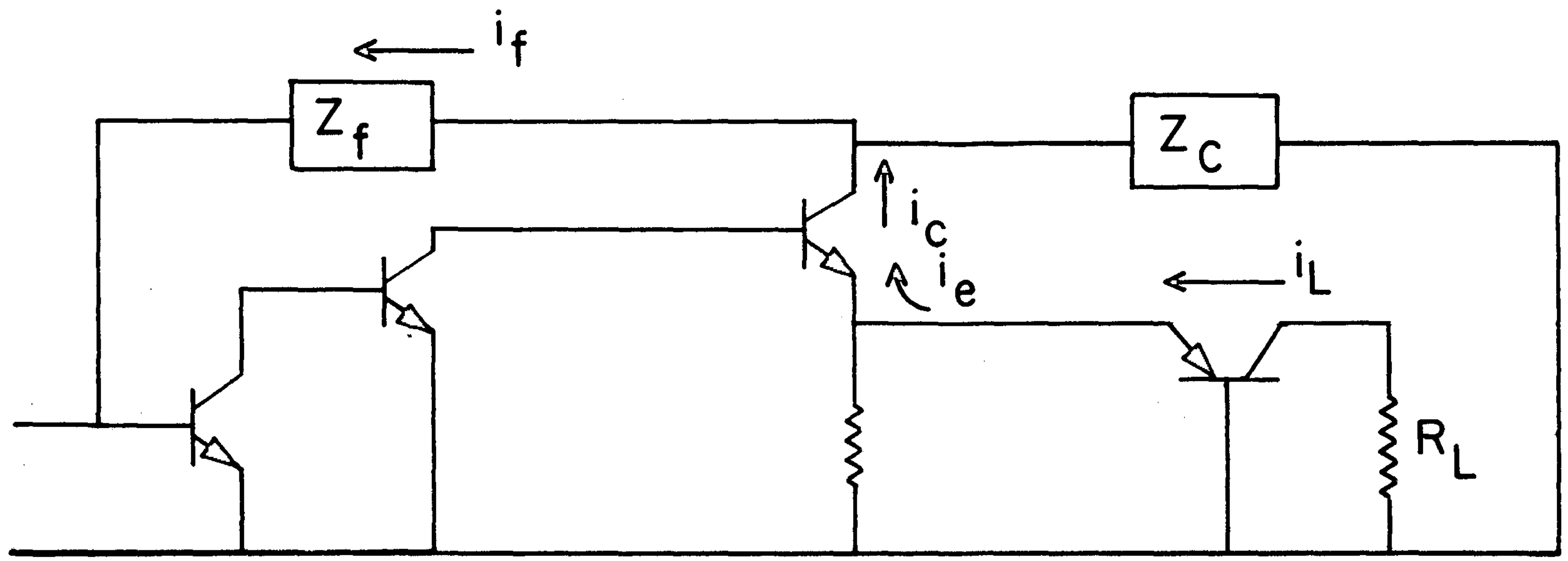


DOUBLET WITH FEEDBACK BUFFER

51,319-1

Fig. 3

TRIPLET WITH LOAD BUFFER



TRIPLET BIAS CIRCUIT AND BUFFER

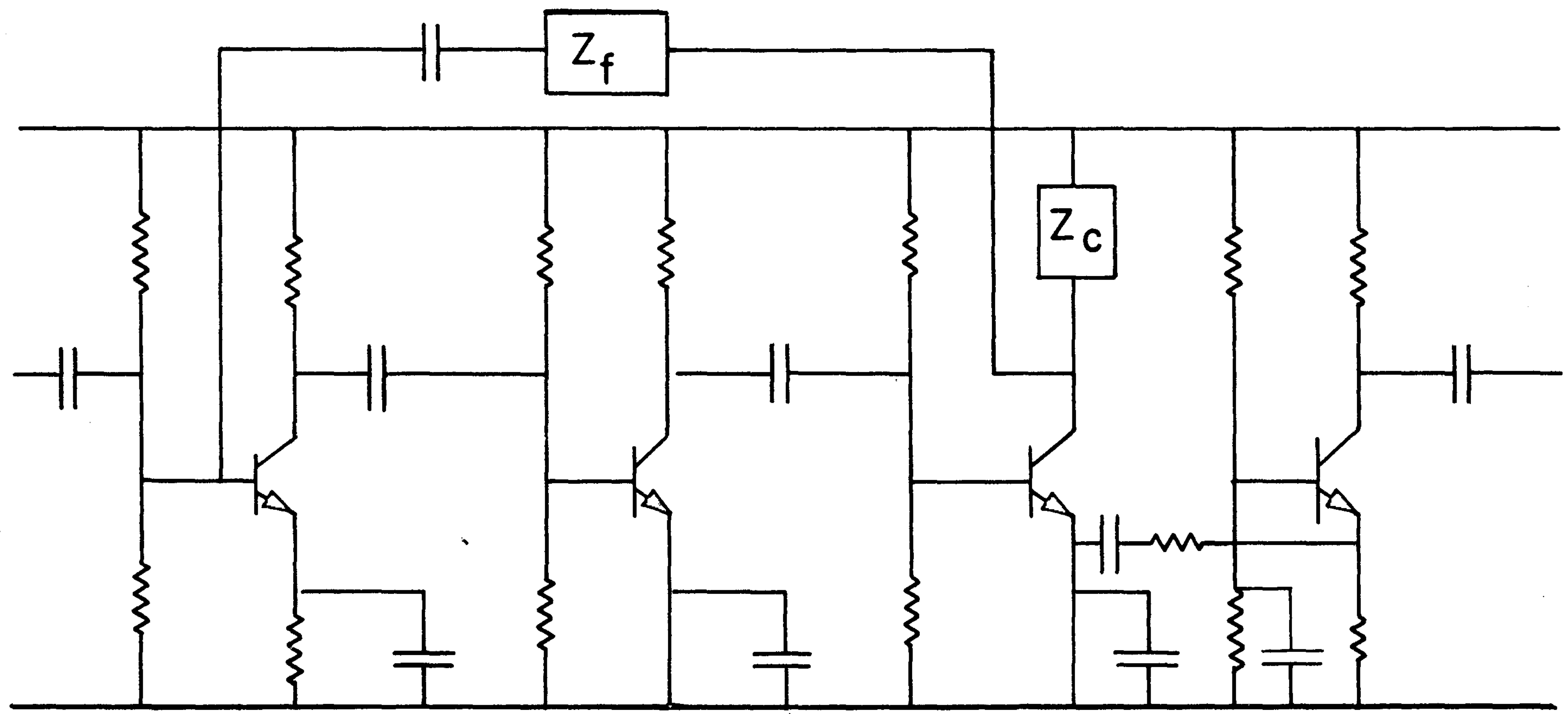


Fig. 4

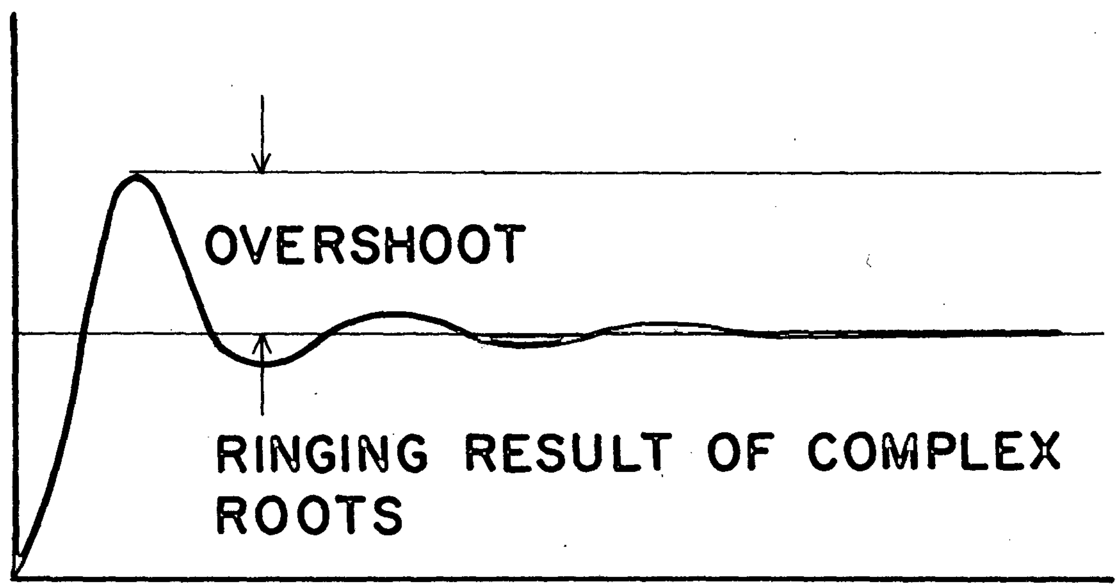


Fig. 5.

100

51,318-1

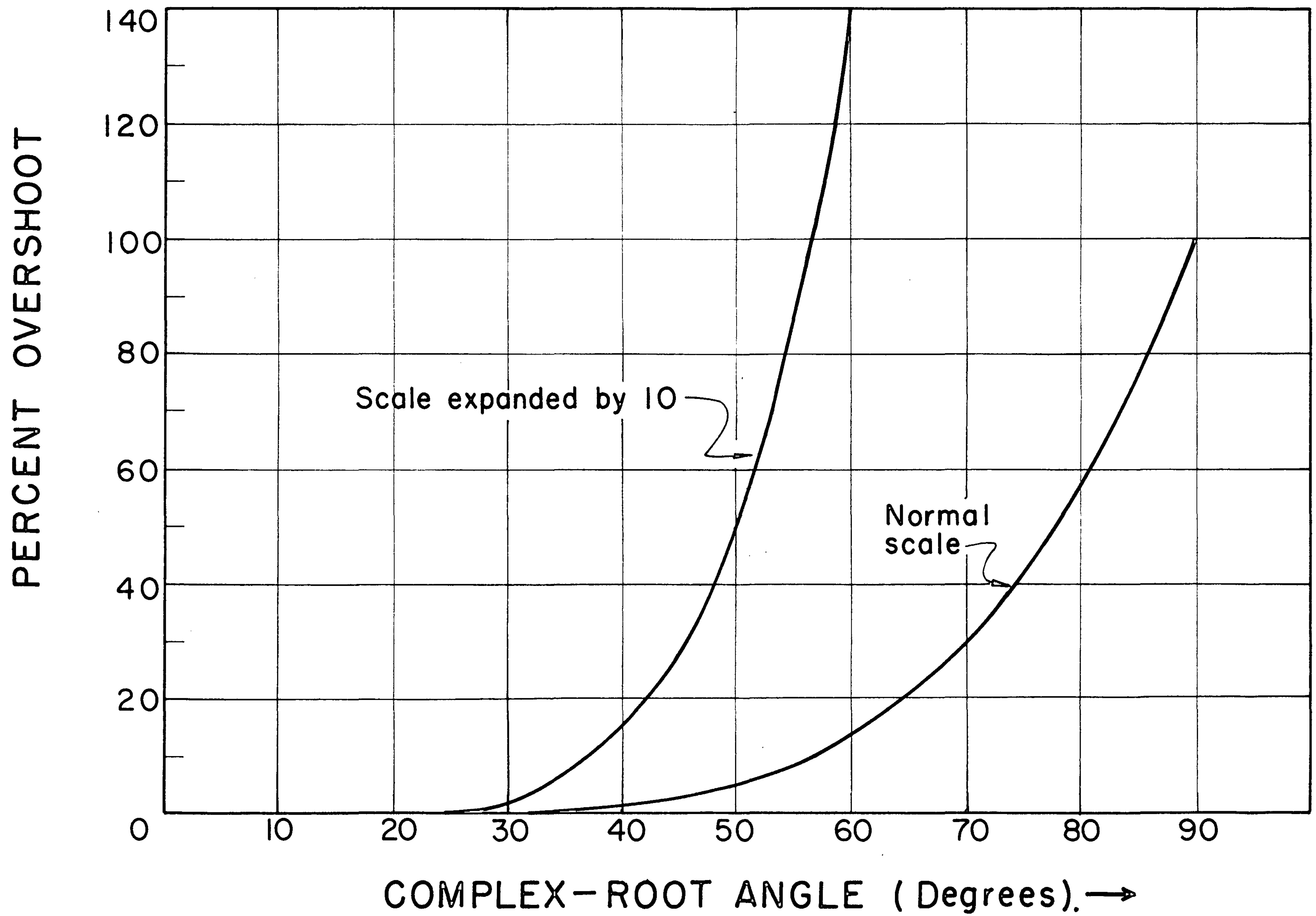
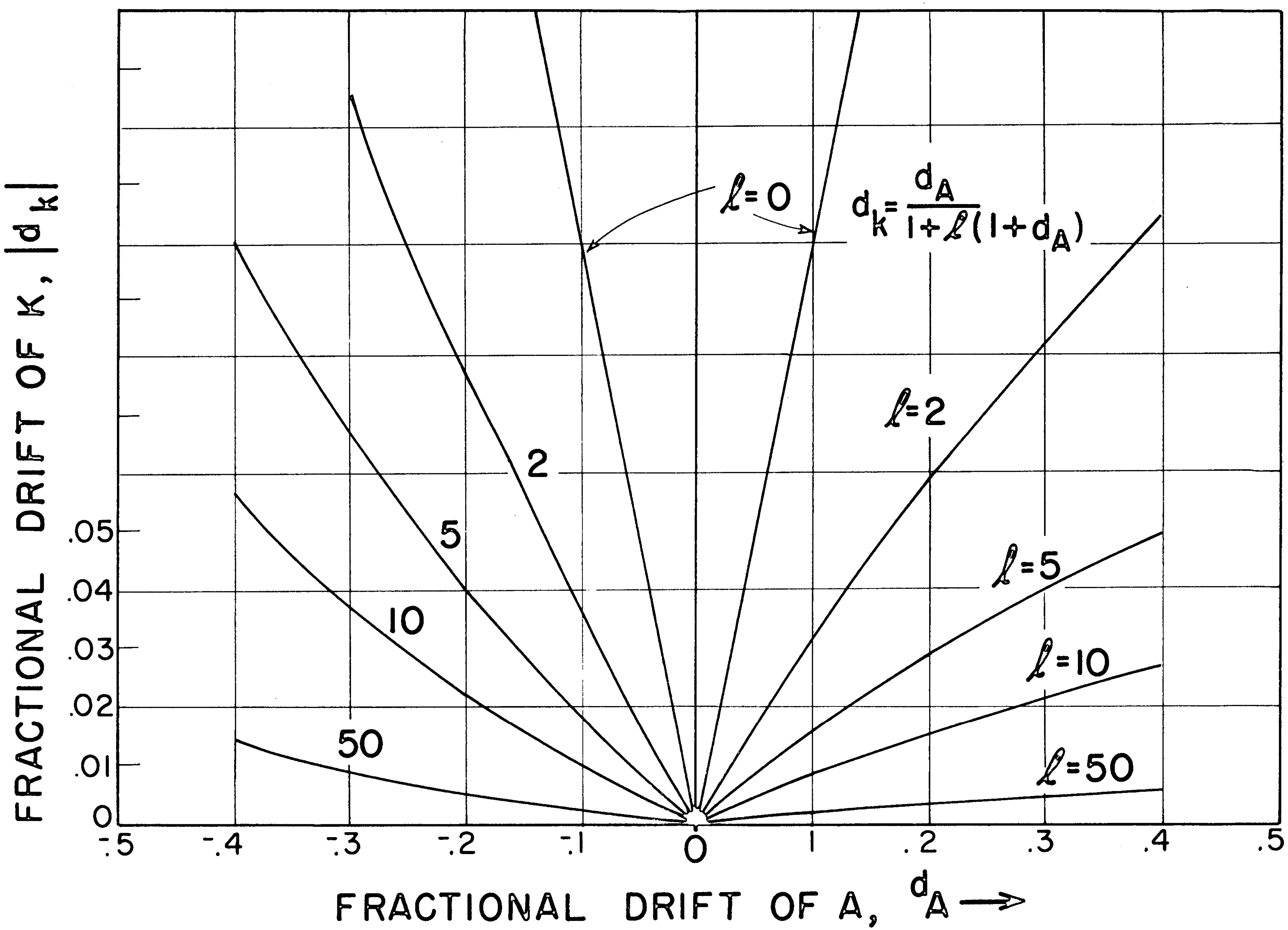

PERCENT OVERSHOOT
140
120
100
80
60
40
20
0
10
20
30
40
50
60
70
80
90
Scale expanded by 10
Normal scale
COMPLEX-ROOT ANGLE (Degrees).

Fig. 6
51,327-2



FRACTIONAL DRIFT OF K, |d_k|
.05
.04
.03
.02
.01
0
ℓ=0
$d_k = \frac{d_A}{1+\ell(1+d_A)}$
ℓ=2
ℓ=5
ℓ=10
ℓ=50
2
5
10
50
-.5
-.4
-.3
-.2
-.1
0
.1
.2
.3
.4
.5
FRACTIONAL DRIFT OF A, d_A →

Fig. 7