Title

EVIDENCE FOR THE EXCHANGE OF THE p AND A2 REGGE TRAJECTORIES IN THE REACTIONS n+p u> n|A++ -> n+p -> n| A++ , AND K+ pu>K|A++

Author

Mathews, Robert D.

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Robert D. Mathews

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University of California
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ABSTRACT

A study is made of three reactions involving Δ^{++} production, $\pi^+ p \to \pi^0 \Delta^{++}$, $\pi^+ p \to \eta^0 \Delta^{++}$, and $K^+ p \to K^0 \Delta^{++}$ which because of t-channel quantum number restrictions allow the exchange of only a few high-lying Regge trajectories. These trajectories are ρ, A_2, and $\rho + A_2$ respectively. In this respect these Δ production reactions are similar to the "elastic" reactions $\pi^- p \to \pi^0 n$, $\pi^- p \to \eta^0 n$, and $K^- p \to \bar{K}^0 n$, whose high-lying Regge exchanges are also restricted to ρ, A_2, and $\rho + A_2$. Effective α plots for the ρ and A_2 trajectories are obtained from the Δ reaction data, which are consistent with those obtained from the "elastic" reactions. Differential cross sections for corresponding "elastic" and inelastic reactions are compared and the observation made that similar exchange particles yield similarly shaped

differential cross sections. The reactions in which only the ρ contributes have narrow forward peaks, dips at $t \approx -0.6$, and secondary maxima; those with A_2 exchanges have much broader angular distributions; and those with both ρ and A_2 exchanges have forward peaks of intermediate width. The intermediate character of the K reactions is consistent with SU(3), which would predict roughly equal proportions of ρ and A_2 exchange. Exchange degeneracy is also discussed, especially with regard to the remarkable experimental evidence that the decay correlations of the Δ^{++} appear to be the same in all three reactions.

INTRODUCTION

When analyzed from the Regge pole phenomenological point of view, most reactions have a priori several Regge poles which can be exchanged. Fortunately, a few reactions exist which, because of quantum number restrictions in a crossed channel, would seem to permit the exchange of only one known, high-lying Regge pole. These are interesting because of the simplicity of the comparison of the Regge formulas to the data, the relatively unambiguous and straightforward determination of the Regge parameters, and the feeling that in these reactions one has isolated and is observing the effects of one pure Regge pole exchange.

Two such reactions are $\pi^- p \to \pi^0 n$ and $\pi^- p \to \eta^0 n$. The t channel of the first is restricted to $I = 1$, $G = +$, $P = (-)^J$ "normal parity" and the second to $I = 1$, $G = -$, $P = (-)^J$, which imply the exchange of only the ρ and A_2 (R) trajectories respectively. A reaction often considered along with the first two is $K^- p \to \bar{K}^0 n$, which has similar crossed-channel quantum number restrictions except that G parity is unrestricted so that both ρ and A_2 can be exchanged. This offers a chance to understand a third, slightly more complicated reaction in terms of the two "pure" reactions. There are quite good counter data available on these reactions, and they have been analyzed extensively from the Regge point of view.

In this paper we wish to analyze three inelastic reactions which are closely related to the previous "elastic" reactions, namely $\pi^+ p \to \pi^0 \Delta^{++}$, $\pi^+ p \to \eta^0 \Delta^{++}$, and $K^+ p \to K^0 \Delta^{++}$.

By "elastic" and "inelastic" reactions I will mean those listed with a nucleon or Δ in the final state, respectively. Also for brevity I shall often refer to reactions by their final state. As before, the t-channel quantum numbers are restricted to normal parity $P = (-)^J$, and $I^G = (1^+$ and $2^+)$, 1^-, and $1^{\text{arbitrary}}$, respectively, which implies the exchange of ρ, A_2, and $\rho + A_2$, respectively. Data are available only from bubble chambers for these reactions, which means there are problems with paucity of data at different energies, poor statistics, bin sizes, and normalization from one experiment to another. Nevertheless because of the theoretical interest in these reactions, and the recent and more accurate data on $\pi^0\Delta^{++}$ by Gidal et al. (see Table I for all data sources), we were motivated to see what could be said about these reactions.

In Section I we review for our purposes the Regge model and consider what complications spin does or does not introduce. Section II discusses total cross sections for these reactions and some problems of normalization. Section III gives the effective α determinations for ρ and A_2. Sections IV, V, and VI discuss shapes of $\frac{d\sigma}{dt}$, SU(3) considerations, and density matrices, respectively.

I. REGGE MODEL

Figure 1 shows the Regge exchanges under consideration. Notice that the meson-meson-(ρ and A_2) couplings are identical, after I spin rotation, for the corresponding elastic and inelastic reactions, so that any differences come from replacing an n with a Δ on the baryon side.

There may be some worry about the effect of spin on the Regge formulas and the fact that there are four independent amplitudes for the inelastic reactions and two for the elastic. However, for our purposes there is really not that much problem. If we let $f_\lambda^{\ s}$ and $f_\lambda^{\ t}$ be s- and t-channel helicity amplitudes, we have

$$\frac{d\sigma}{dt} = \frac{1}{p_{lab}^{\ 2}} \sum_\lambda |f_\lambda^{\ s}|^2 = \frac{1}{p_{lab}^{\ 2}} \sum_\lambda |f_\lambda^{\ t}|^2 .$$

Now for reactions where one Regge pole exchange dominates and $t_{minimum}$ is small (say of the order of 0.01 GeV^2 or less) the t channel amplitudes can be approximated by[1]

$$f_\lambda^{\ t} = g_\lambda(t) \left(\frac{s - u}{2}\right)^{\alpha(t)} .$$

That is, every helicity amplitude has the <u>same</u> $(s - u)^\alpha$ behavior, and the only difference between the amplitudes is in the t dependence of $g_\lambda(t)$. Now $g_\lambda(t)$ contains the signature factor and may contain factors of t and α, etc., but we will not concern ourselves at the moment with this. Here we have assumed daughter trajectories will take care of any unequal-mass problems near $t = 0$ and maintain the $(s - u)^\alpha$ asymptotic behavior of the amplitudes. Thus

$$\frac{d\sigma}{dt} = \frac{1}{p_{lab}^2} \left(\frac{s-u}{2}\right)^{2\alpha(t)} \sum_\lambda |g_\lambda(t)|^2$$

$$= \frac{1}{p_{lab}^2} g_{eff}^2(t) \left(\frac{s-u}{2}\right)^{2\alpha(t)} ,$$

where

$$g_{eff}^2(t) \equiv \sum_\lambda |g_\lambda(t)|^2$$

and

$$\sigma = \frac{1}{p_{lab}^2} \int_{-\infty}^{0} dt \; g_{eff}^2(t) \left(\frac{s-u}{2}\right)^{2\alpha(t)} .$$

In other words for some purposes one can analyze $\frac{d\sigma}{dt}$ and σ data in terms of an equivalent spinless problem. Of course spin density matrices, $\rho_{mm'}(t)$, depend only on relative magnitudes of the amplitudes and thus depend in our formulas only on the relative magnitudes of the $g_\lambda(t)$ and contain no s dependence. For two Regge poles, one merely adds the respective amplitudes

$$f_\lambda^t = g_\lambda^{(1)}(t) \left(\frac{s-u}{2}\right)^{\alpha_1(t)} + g_\lambda^{(2)}(t) \left(\frac{s-u}{2}\right)^{\alpha_2(t)} ,$$

but then the inverse problem of determining the Regge parameters from the data becomes more difficult in general.

II. CROSS SECTIONS

In Fig. 2, cross sections have been plotted versus p_{lab} for the elastic and inelastic reactions. The first observation is that the cross sections of the corresponding reactions are of the same order of magnitude and that the slopes of corresponding cross sections on the log-log plot are roughly parallel. More accurately the $\eta^0\Delta^{++}$ cross section is almost the same as the $\eta^0 n$; the $\pi^0\Delta^{++}$ cross section is consistently about 70-80% higher than the $\pi^0 n$; and the $K^0\Delta^{++}$ is consistently about 35-45% higher than the $\bar{K}^0 n$. Thinking now of the Regge exchange model and the equivalent spinless problem we can say that the net, "integrated over all t," coupling of A_2 to $\Delta^{++}\bar{p}$ is approximately the same as to $n\bar{p}$; the net coupling of ρ to $\Delta^{++}\bar{p}$ is stronger than to $n\bar{p}$; and that it is reasonable that the cross section for $K^0\Delta^{++}$ be larger than for $\bar{K}^0 n$ because both ρ and A_2 are exchanged for this reaction. The comparison of one elastic reaction with another and one inelastic reaction with another will be discussed in Section V below.

We come to some difficult but important problems of normalization. The $\eta^0\Delta^{++}$ cross section at 2.08 GeV/c was renormalized downward from 64 mb to 39 mb before being plotted.[2] It was felt to be more accurate to use the branching ratio

$$\frac{\Gamma(\eta \to \text{all})}{\Gamma(\eta \to \pi^+\pi^-\pi^0)} \approx 4.0$$

to determine the total cross section, as was done for the 3.65 GeV/c and the 3-4 GeV/c data, rather than use that group's original method.

For $\pi^0\Delta^{++}$ the data are plotted with the original normalizations to show the dilemma faced before the recent data of Gidal et al. were available (the five high open square points). Previously there appeared to be a considerable "kink" in the cross section curve at around 3.5 GeV/c. But the new data seem to join on smoothly to the higher energy points 4 and 8 GeV/c, and seem to directly contradict the normalizations of the 2.75 GeV/c and 3.54 GeV/c data. Accordingly, for the effective α determinations, the 2.75 and 3.54 GeV/c cross sections were renormalized to corresponding points on a straight line drawn through the new data.

For the $K^0\Delta^{++}$ data the only problem is perhaps a low normalization at 10 GeV/c.

III. EFFECTIVE α's

Figure 3 shows $\frac{d\sigma}{dt} \cdot p_{lab}^2$ versus $\left(\frac{s - u}{2}\right)$ plotted on a log-log graph for each fixed bin in t, for $\pi^0\Delta^{++}$ and $\eta^0\Delta^{++}$. The ρ and A_2 trajectories were determined by fitting straight lines through the data points, the resulting slopes being equal to $2\alpha(t)$. This particular way of plotting has the virtue of making it visually dramatic when the trajectory $\alpha(t)$ goes through zero because the slope changes from positive to negative.

There is some question how to reconcile the different published bin sizes at the different energies. One approach is to take the binning choice of the highest energy, which usually has the least statistics and which because of its "lever arm" significance should be treated more accurately, and make the bins of the other energies conform to it. This can be done by making the approximation that events in a given bin are uniformly distributed in t. Another approach is to say that the differential cross section histograms are statistically ragged approximations to some smooth form, and then to compare fitted smooth forms at several values in t. This has the virtue of smoothing out statistical fluctuations, and yielding an infinite number of t values to determine $\alpha(t)$at, but has a drawback in that the meaning of the error bars for α is more obscure. We analyzed the data by the first method. The errors for this "conforming bin size" method were determined by how much the slope could be changed in Fig. 3 to raise χ^2 by 1.

In Fig. 4 are plotted the trajectories for ρ and A_2 determined from the inelastic data, along with a linear fit to those trajectories. Also shown are plots from the elastic data by Phillips and Rarita[3] and Hohler et al.[4] A word of caution should be injected before the inelastic results are compared with the much more accurate elastic results. The inelastic results are determined mainly by the 3-4 GeV/c and 8 GeV/c experiments in spite of our efforts to add information from other experiments, and so will be sensitive to any errors in those two experiments. Also the statistical errors are much larger than for elastic reactions. With this in mind we can say that the inelastic results appear to be consistent with the elastic results. The ρ trajectory looks much the same as for the elastic data except for a strange "dip" in the forward direction. The A_2 trajectory, however, appears to pass through zero much sooner than found for the elastic data. This might give some support to the theory of "exchange degeneracy"[5], which holds that the trajectories and residues for the ρ and A_2 Regge poles should be identical. Thews[6] has also determined the ρ trajectory from $\pi^0\Delta^{++}$ data, but because of the previously mentioned "kink" in the cross section, his trajectory is higher than the one given here.

IV. SHAPES OF $\frac{d\sigma}{dt}$

Figure 5 compares the shapes of the differential cross sections of the inelastic and elastic reactions at similar energies for which good data are available. The main observation is that the reactions with the same exchanged particle have similarly shaped differential cross sections. The ρ exchanges both give narrow distributions with dips at $t = -0.6$; the A_2 exchanges give rather broad distributions with the $\eta^0\Delta^{++}$ being slightly broader than the $\eta^0 n$; and the K data are intermediate in width and have no dip at $t = -0.6$, consistent with the idea of ρ and A_2 exchanges of roughly comparable magnitudes. Because the $\eta^0\Delta^{++}$ data seem slightly broader than the $\eta^0 n$, one can predict that $K^0\Delta^{++}$ should be slightly broader than $\bar{K}^0 n$. However, it is such a small effect that it would be hard to see without very good statistics.

It must be pointed out that perhaps the most convincing evidence for the exchange of a ρ in the $\pi^0\Delta^{++}$ reaction is the shape of $\frac{d\sigma}{dt}$ for Gidal et al.'s 3-4 GeV/c high-statistics data (400 events). It has the same beautiful dip at $t = -0.6$ as the elastic data, and like the elastic data can be explained by saying that $t = -0.6$ is just the point where one would expect α_ρ to go through zero.

Perhaps a digression at this point would be elucidating. There are four independent helicity amplitudes for the $\pi^0\Delta^{++}$ reaction:

$$f_{00,\lambda_\Delta\lambda_N} : \quad f_{00,\frac{1}{2}\frac{1}{2}} \propto 1 \quad ,$$

$$f_{00,\frac{1}{2}-\frac{1}{2}} \propto t^{\frac{1}{2}} \alpha \quad ,$$

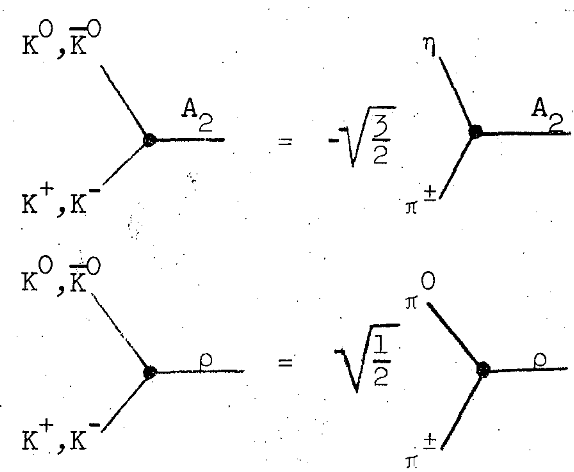
$$f_{00,\frac{3}{2}\frac{1}{2}} \propto t^{\frac{1}{2}} \alpha \quad ,$$

$$f_{00,\frac{3}{2}-\frac{1}{2}} \propto t\, \alpha(\alpha - 1) \quad .$$

Three of the four have a factor of α in them and thus go to zero when α goes to zero. These α's arise naturally from the Reggeization of helicity amplitudes[7], which instead of having angular dependence like $P_\alpha(-z)$ may have another dependence involving the derivatives of these P_α's, such as, for example, $z \frac{d}{dz} P_\alpha(-z)$. The leading power of z is then $\alpha \cdot z^\alpha$ instead of z^α, and the amplitude would go to zero at $\alpha = 0$. The data for A_2 exchange do not show a dip, presumably because either α never gets to zero or because such factors of α are required in the numerator to cancel out the $\sin \pi\alpha$ in the denominator and prevent a pole from occurring for negative t.

V. SU(3) CONSIDERATIONS

It is interesting to see if SU(3) considerations regarding coupling coefficients are consistent with the data for these six reactions. Rarita and Schwarzschild,[8] Barger and Olsson,[9] Derem and Smadja,[10] and Ahmadzadeh and Chan[11] have already tested SU(3) on the elastic reactions and found it satisfactory. For the elastic reactions, this is a repetition of their work and observations, but it is included for completeness. First notice that SU(3) cannot say anything about the baryon end of the Regge exchange because the overall scale of the meson-$\bar{N}N$ vertex to the meson-$\bar{N}\Delta$ vertex is not predicted by SU(3). This is because the Δ and the nucleon are in different multiplets. The pseudoscalar mesons, π, η, K, are in the same octet, however,[12] so that SU(3) can make the coupling coefficient predictions indicated schematically below[13]



Thus SU(3) gives us a prediction of the ρ and A_2 contributions to the K reaction amplitude. Now if we assume exchange degeneracy,

$\alpha_\rho = \alpha_{A_2}$, then the Regge phase and signature factors

$$1 - e^{-i\pi\alpha_\rho} \quad \text{and} \quad 1 + e^{-i\pi\alpha_{A_2}}$$

are 90° out of phase and for cross sections we can add the contributions incoherently. If we have only approximate exchange degeneracy then it still might not be too bad an approximation to add the contributions incoherently (e.g., $|\alpha_\rho - \alpha_{A_2}| = 0.06$ leads to 10% error). Let's see what this idea of incoherence implies, for example, at 10 GeV for the elastic cross section. Using Fig. 2 for rough estimates we have

$$\sigma^{\pi^0 n} = 0.050 \pm 0.005 \text{ mb} , \qquad \sigma^{\eta^0 n} = 0.025 \pm 0.003 \text{ mb} .$$

The contribution of ρ alone to $\sigma^{\overline{K}^0 n} = \frac{1}{2} \sigma^{\pi^0 n} = 0.025 \pm 0.003$ mb

The contribution of A_2 alone to $\sigma^{\overline{K}^0 n} = \frac{3}{2} \sigma^{\eta^0 n} = 0.038 \pm 0.005$ mb

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Approx. of incoherent addition = 0.063±0.008 mb

Actual $\sigma^{\overline{K}^0 n} = 0.064 \pm 0.008$ mb

The general agreement seems quite good. In fact $\frac{1}{2} \sigma^{\pi^0 n} + \frac{3}{2} \sigma^{\eta^0 n}$ passes within the error bars of $\sigma^{\overline{K}^0 n}$ from 4-13 GeV/c. SU(3) thus says that the ρ and A_2 contributions to the $\overline{K}^0 n$ reaction are somewhat comparable in magnitude, with the perhaps the A_2 contribution being somewhat larger. For the inelastic reactions the ρ exchange yields a 70% larger cross section which, using SU(3), implies about a 28% larger cross section for $K^0\Delta^{++}$, which seems to be roughly true.

SU(3) also seems to work for the differential cross sections. Still using the incoherence approximation, SU(3) predicts[11]

$$\frac{d\sigma^K}{dt} = \frac{1}{2}\frac{d\sigma^\pi}{dt} + \frac{3}{2}\frac{d\sigma^\eta}{dt} .$$

Since this leads to roughly comparable amounts of ρ and A_2 it fits the data fairly well. Figure 6 shows how the broad A_2 and narrow ρ can add up to give an intermediate width $\rho + A_2$. This is shown for one elastic energy and one inelastic energy for which data existed conveniently for all three reactions. The new observation of this paper is that SU(3) also works for the inelastic reactions. The agreement is quite good. What small discrepancies remain might be explained by breaking exchange degeneracy and introducing some coherence or breaking SU(3), or both.

If we accept this picture of ρ exchange in approximately SU(3) proportions we can make the observation that for $0.3 \leqslant |t| \leqslant 0.65$, $\frac{d\sigma^K}{dt}$ comes almost entirely from A_2 exchange. This would mean for the $K^0\Delta^{++}$ reaction, for example, that the density matrix elements for the Δ decay would be those for A_2 exchange, i.e., the same as for the $\eta^0\Delta^{++}$ reaction. It is also interesting to speculate whether the fast-rising secondary maximum of ρ exchange could be responsible for the possible "dip" in $\bar{K}^0n$ at $|t| \approx 0.9$. Such a fast-rising ρ might at least be able to make a gentle plateau, though probably not a dip.

VI. $\rho_{mm'}$ DATA

Density matrices for the produced Δ^{++} depend on the relative magnitudes of the four helicity amplitudes[14]

$$\rho_{33} = \frac{|f_3|^2 + |f_4|^2}{2\ \mathrm{Norm}} ,$$

$$\mathrm{Re}\ \rho_{31} = -\ \mathrm{Re}\ \frac{f_1^* f_3 + f_2^* f_4}{2\ \mathrm{Norm}} ,$$

$$\mathrm{Re}\ \rho_{3-1} = \mathrm{Re}\ \frac{f_2^* f_3 - f_1^* f_4}{2\ \mathrm{Norm}} ,$$

where

$$f_1 \equiv f^t_{00,\frac{1}{2}\frac{1}{2}} , \quad \text{where } f^t \text{ means } f^t_{00,\lambda_\Delta\lambda_{\overline{N}}} ,$$

$$f_2 \equiv f^t_{00,\frac{1}{2}-\frac{1}{2}} ,$$

$$f_3 \equiv f^t_{00,\frac{3}{2}\frac{1}{2}} ,$$

$$f_4 \equiv f^t_{00,\frac{3}{2}-\frac{1}{2}} \qquad \mathrm{Norm} \equiv \sum |f_i|^2 \propto \frac{d\sigma}{dt} .$$

As was remarked in Section I, over almost the entire range of momentum transfer each of these helicity amplitudes has the same energy dependence in the Regge model, that is

$$f_i(s,\ t) = g_i(t)\ s^{\alpha(t)} \quad ,$$

so that the energy dependence, $s^{\alpha(t)}$, can be canceled out completely from the expressions for $\rho_{mm'}(t)$. The "average $\rho_{mm'}$" often quoted in experiments usually means

$$\rho_{mm'} \equiv \frac{\int_{-\infty}^{0} \frac{d\sigma}{dt}\, \rho_{mm'}(t)\, dt}{\int_{-\infty}^{0} \frac{d\sigma}{dt}\, dt} \quad .$$

If $\rho_{mm'}(t)$ is slowly varying in the region where $\frac{d\sigma}{dt}$ is large, then there is virtually no change in $\rho_{mm'}$ with energy.

Figure 7 shows the density matrices of the Δ^{++} plotted against p_{lab} for the three inelastic reactions. It can be seen that they all are virtually constant with energy. ρ_{33} for $\pi^0\Delta^{++}$ is large, showing that the f_3 and/or f_4 amplitudes are large compared with f_1 and f_2, and lends credence to the $\alpha_\rho = 0$ interpretation of the dip. Notice that A_2 exchange produces the same density matrices as ρ exchange! The error bars are rather large, of course, but nevertheless the similarity is rather striking. One can regard this either as a remarkable coincidence or else as evidence for some deeper underlying symmetry. Stodolsky and Sakurai[15] have proposed a simple model for the ρ exchange which explains the density matrix elements in terms of a ρ-photon analogy.

The Stodolsky-Sakurai model:

$$f_1 \text{ and } f_4 = 0 \quad , \qquad f_2 = \frac{1}{\sqrt{3}} f_3 \quad ,$$

$$\rho_{33} = \frac{3}{8} \approx 0.38 \quad ,$$

$$\text{Re } \rho_{31} = 0 \quad ,$$

$$\text{Re } \rho_{3-1} = \frac{\sqrt{3}}{8} \approx 0.22 \quad .$$

However, it's hard to picture a similar A_2-photon analogy. Perhaps the idea of exchange degeneracy, which regards the ρ and A_2 Regge poles as being very similar, can explain it in terms of similar residues for the two poles in each helicity amplitude.

Given a set of Δ density matrix elements for the $\pi^0\Delta^{++}$ reaction and for the $\eta^0\Delta^{++}$ reaction, what can be said about the $K^0\Delta^{++}$ reaction? If we make the approximation that the exchange contributions are 90° out of phase, then it can be shown that the $K^0\Delta^{++}$ density matrix elements are just <u>weighted averages</u> of the $\pi^0\Delta^{++}$ and $\eta^0\Delta^{++}$ density matrix elements,

$$\rho_{33}^{K\Delta}(t) = \frac{a(t)\ \rho_{33}^{\pi\Delta}(t) + b(t)\ \rho_{33}^{\eta\Delta}(t)}{a(t) + b(t)} \quad ,$$

and similarly for Re ρ_{31} and Re ρ_{3-1}, where $a(t)$ and $b(t)$ are the ρ and A_2 exchange contributions to $\frac{d\sigma^{K\Delta}}{dt}$, i.e.

$$\frac{d\sigma^{K\Delta}}{dt} = a(t) + b(t)$$

and by SU(3)

$$a(t) \approx \frac{1}{2}\frac{d\sigma^{\pi\Delta}}{dt} ,$$

$$b(t) \approx \frac{3}{2}\frac{d\sigma^{\eta\Delta}}{dt} .$$

The above "average rules" come out so nicely because if ρ and A_2 are $90°$ out of phase all the interference terms go into the imaginary parts of $\rho_{mm'}$. Figure 7 shows that the $K^0\Delta^{++}$ density matrix elements are <u>indeed</u> the same as for $\pi^0\Delta^{++}$ and $\eta^0\Delta^{++}$. From at least one point of view, though, it is unfortunate that the density matrix elements for $\pi^0\Delta^{++}$ and $\eta^0\Delta^{++}$ are so nearly identical, for otherwise one might have had a more dramatic test of the ρ and A_2 exchange hypothesis. For example, one might have expected a "mottled" effect in $\rho_{mm'}{}^{K\Delta}(t)$, with first $\rho_{mm'}{}^{K\Delta} \approx \rho_{mm'}{}^{\pi\Delta}$, for $|t| = 0.0\text{-}0.1$, and then $\rho_{mm'}{}^{K\Delta} \approx \rho_{mm'}{}^{\eta\Delta}$, for $|t| = 0.3\text{-}0.65$, as first ρ dominates the amplitude and then A_2 (see Fig. 6e). Notice that in deriving the above "average rules" we have <u>not</u> imposed that the relative proportions of ρ and A_2 in each of the four helicity amplitudes be the same. Different relative proportions for ρ and A_2 will at most make $\text{Im}\,\rho_{31}$ and $\text{Im}\,\rho_{3-1} \neq 0$.

VIII. ACKNOWLEDGMENT

I would like to express my appreciation to Professor J. D. Jackson and Dr. G. Gidal for their many helpful discussions and encouragement.

ADDENDUM

While this work was in the final stages of preparation we received a preprint by G. H. Renninger and K. V. L. Sarma, "Hypothesis of M-1 Dominance in 3/2+ -Isobar Production Reactions." They also use a model of ρ and A_2 Regge pole exchange to explain the Δ production reactions we considered in this paper. They assume known Regge trajectory parameters and determine residues for invariant amplitudes for these reactions by fitting the data. The conclusion is made that the Stodolsky-Sakurai M-1 model is a good first-order approximation to the density matrix elements and to the angular distributions. As in this paper they also conclude that SU(3) works well and they also observe that the decay correlations for the Δ are remarkably the same for the three reactions.

FOOTNOTES AND REFERENCES

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Berkeley and Los Angeles, p. 115.) This could lead to a 10-20% effect on what we take to be the A_2 component in the K reactions, and would have to be considered in a more detailed test of SU(3). The author would like to thank Dr. G. Alexander for this observation.

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Table I. Data sources

$\underline{\pi^+ p \to \pi^0 \Delta^{++}}$	
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Table I (Continued)

$\pi^+ p \to \pi^0 \Delta^{++}$	
8.0	Aachen-Berlin-CERN Collaboration, Phys. Letters 19, 608 (1965); Aachen-Berlin-CERN Collaboration "Structure in Differential Cross Section Distributions of Two-Body Reactions in 8 GeV/c π^+p Interactions," CERN/D. Ph. II/ PHYSICS 68-20, submitted to Nuclear Physics; density matrices for above experiment, Aachen-Berlin-CERN collaboration, private communication.
$\pi^+ p \to \eta^0 \Delta^{++}$	
2.08	F. E. James and H. Kraybill, Phys. Rev. 142, 896 (1966).
3.65	G. H. Trilling, J. L. Brown, G. Goldhaber, S. Goldhaber, J. A. Kadyk, and J. Scanio, Phys. Letters 19, 427 (1965).
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8	Aachen-Berlin-CERN Collaboration, Phys. Letters 19, 608 (1965); Aachen-Berlin-CERN Collaboration "Structure in Differential Cross Section Distributions of Two-Body Reactions in 8 GeV/c π^+p Interactions," CERN/D. Ph.II/ PHYSICS 68-20, Submitted to Nuclear Physics; density

Table I (Continued)

$\pi^+p \rightarrow \eta^0\Delta^{++}$	
	matrices for above experiment, Aachen-Berlin-CERN Collaboration, private communication.
$K^+p \rightarrow K^0\Delta^{++}$	
0.86, 0.96, 1.2, 1.36	R. W. Bland, Lawrence Radiation Laboratory Report UCRL-18131, March 1968 (unpublished).
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Table I (Continued)

$\underline{K^- p \rightarrow \overline{K}^0 n}$	
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FIGURE CAPTIONS

Fig. 1. The Regge exchanges considered. "Inelastic" and "elastic" refer to those with a Δ or a nucleon in the final state respectively.

Fig. 2. Cross sections for the six reactions.

(a) $\pi^+ p \to \pi^0 \Delta^{++}$ and $\pi^- p \to \pi^0 n$.

(b) $\pi^+ p \to \eta^0 \Delta^{++}$ and $\pi^- p \to \eta^0 n$.

(c) $K^+ p \to K^0 \Delta^{++}$ and $K^- p \to \bar{K}^0 \Delta^{++}$.

The lines represent linear fits of the form $\sigma = A\, p_{lab}^{-n}$. For $\pi^0 \Delta^{++}$, $A = 2.56$, $n = 1.48$; $\pi^0 n$, $A = 0.95$, $n = 1.27$; $\eta^0 \Delta^{++}$, $A = 0.99$, $n = 1.44$; $\eta^0 n$, $A = 1.03$, $n = 1.62$; $K^0 \Delta^{++}$, $A = 6.19$, $n = 1.88$; $\bar{K}^0 n$, $A = 4.12$, $n = 1.86$.

Fig. 3. $\frac{d\sigma}{dt} \cdot p_{lab}^2$(mb) versus $\left(\frac{s - u}{2}\right)$ (GeV^2) for $\pi^+ p \to \pi^0 \Delta^{++}$ (Fig. 3a) and $\pi^+ p \to \eta^0 \Delta^{++}$ (Fig. 3b). The lines shown are linear fits to the data sets and the slope of a line is $2\,\alpha(t)$. The bin in t (GeV^2) is indicated for each data set. The lab momenta in GeV/c have been written above the data points for the first data set of each reaction.

Fig. 4. Effective α versus t.

(a) $\alpha_\rho(t)$ as determined from $\pi^+ p \to \pi^0 \Delta^{++}$. The line shown is a linear fit, $\alpha_\rho(t) \approx (0.41 \pm 0.10) \pm (0.96 \pm 0.18)t$.

(b) $\alpha_{A_2}(t)$ from $\pi^+ p \to \eta^0 \Delta^{++}$.

$\alpha_{A_2}(t) \approx (0.71 \pm 0.24) + (1.62 \pm 0.58)t$.

(c) $\alpha_\rho(t)$ from $\pi^- p \to \pi^0 n$ taken from Ref. 4.

$\alpha_\rho(t) \approx (0.57 \pm 0.01) + (0.91 \pm 0.06)t$.

(d) $\alpha_{A_2}(t)$ from $\pi^- p \to \eta^0 n$ taken from Ref. 3.

$\alpha_{A_2}(t) \approx (0.34 \pm 0.03) + (0.35 \pm 0.08)t$.

Fig. 5. Comparison of "elastic" and "inelastic" shapes of $\frac{d\sigma}{dt}$.

(a) $\pi^+ p \to \pi^0 \Delta^{++}$ at 3-4 GeV/c and $\pi^- p \to \pi^0 n$ at 3.67 GeV/c.

(b) $\pi^+ p \to \eta^0 \Delta^{++}$ at 3-4 GeV/c and $\pi^- p \to \eta^0 n$ at 3.72 GeV/c.

(c) $K^+ p \to K^0 \Delta^{++}$ and $K^- p \to \overline{K}^0 n$ both at 5.0 GeV/c.

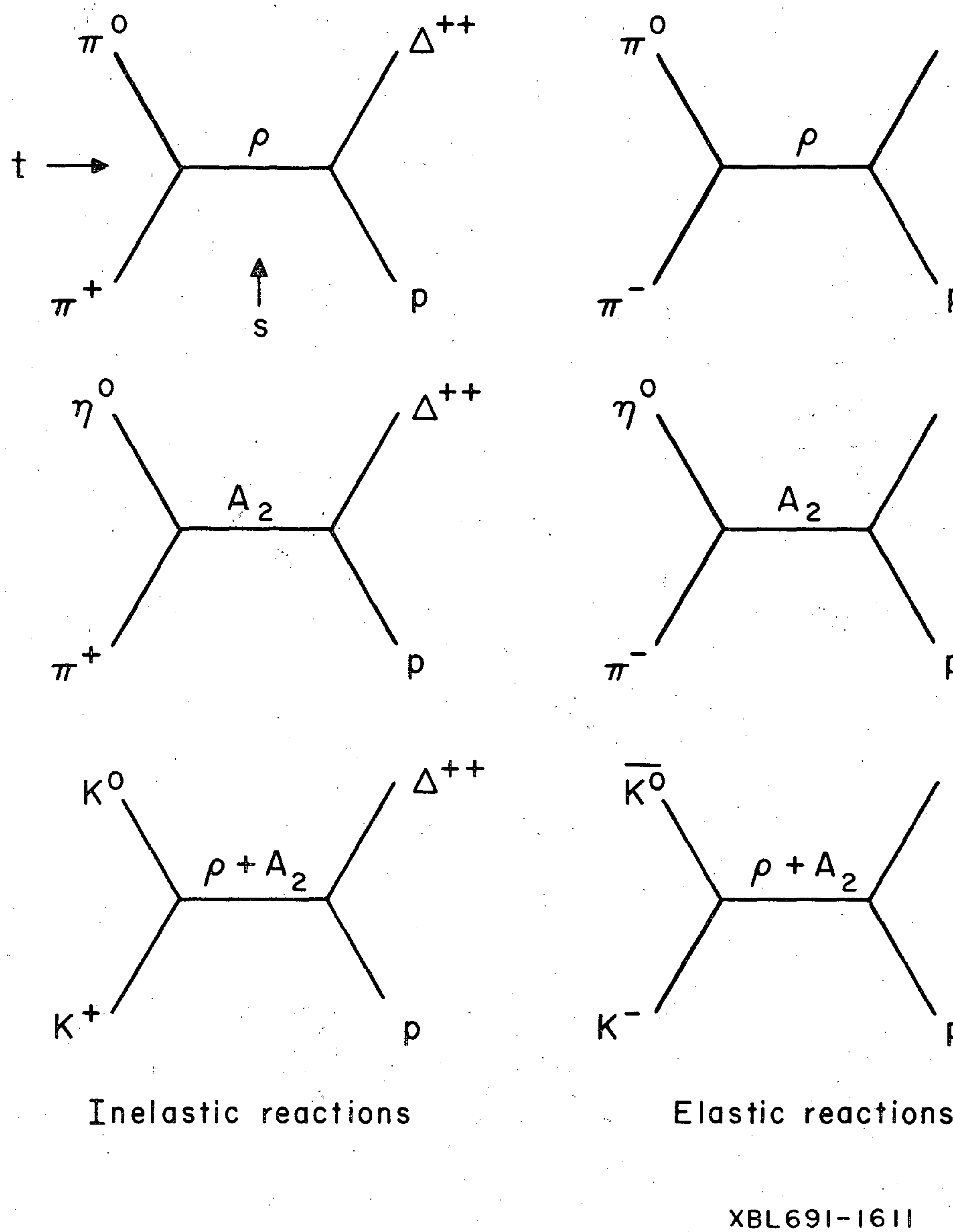
Fig. 6. (a) Differential cross sections for the elastic reactions at 9.8, 9.8, and 9.5 GeV/c for $\pi^0 n$, $\eta^0 n$, and $\overline{K}^0 n$ respectively. The curves shown here as for all Fig.6 were drawn by hand to guide the eye only.

(b) The "components" of $\overline{K}^0 n$. Shown are $\frac{1}{2}\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n)$ and $\frac{3}{2}\frac{d\sigma}{dt}(\pi^- p \to \eta^0 n)$ for the data of Fig. 6a.

(c) The SU(3) + Exchange Degeneracy test for $\frac{d\sigma}{dt}$ shapes. Shown are $\frac{d\sigma}{dt}(K^- p \to K^0 n)$ and $\frac{1}{2}\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n) + \frac{3}{2}\frac{d\sigma}{dt}(\pi^- p \to \eta^0 n)$ for the data of Fig. 6a. SU(3) plus exchange degeneracy would predict these two quantities to be equal. The curve shown is for $\overline{K}^0 n$ only.

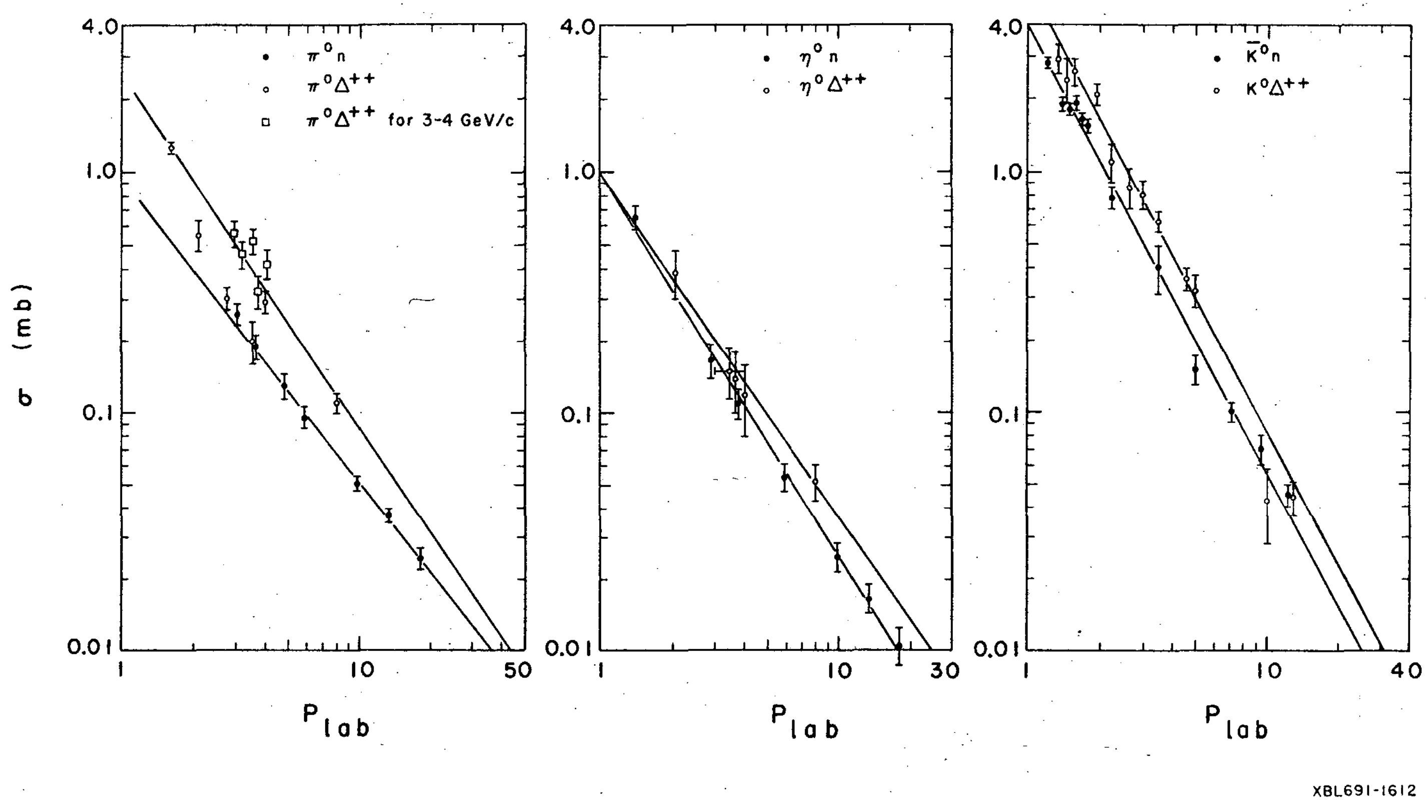
(d), (e), (f) Similar to Figs. 6a,b,c only for the inelastic reactions at energies of 3-4, 3-4, and 3.5 GeV/c respectively.

Fig. 7. Density matrices for Δ^{++} production versus beam momentum for the three inelastic reactions (a) $\pi^+ p \to \pi^0 \Delta^{++}$; (b) $\pi^+ p \to \eta^0 \Delta^{++}$; (c) $K^+ p \to K^0 \Delta^{++}$. The dashed line indicates the Stodolsky-Sakurai predictions: $\rho_{33} = \frac{3}{8} \approx 0.38$, Re $\rho_{3,-1} = \sqrt{3}/8 \approx 0.22$, and Re $\rho_{3,1} = 0$.



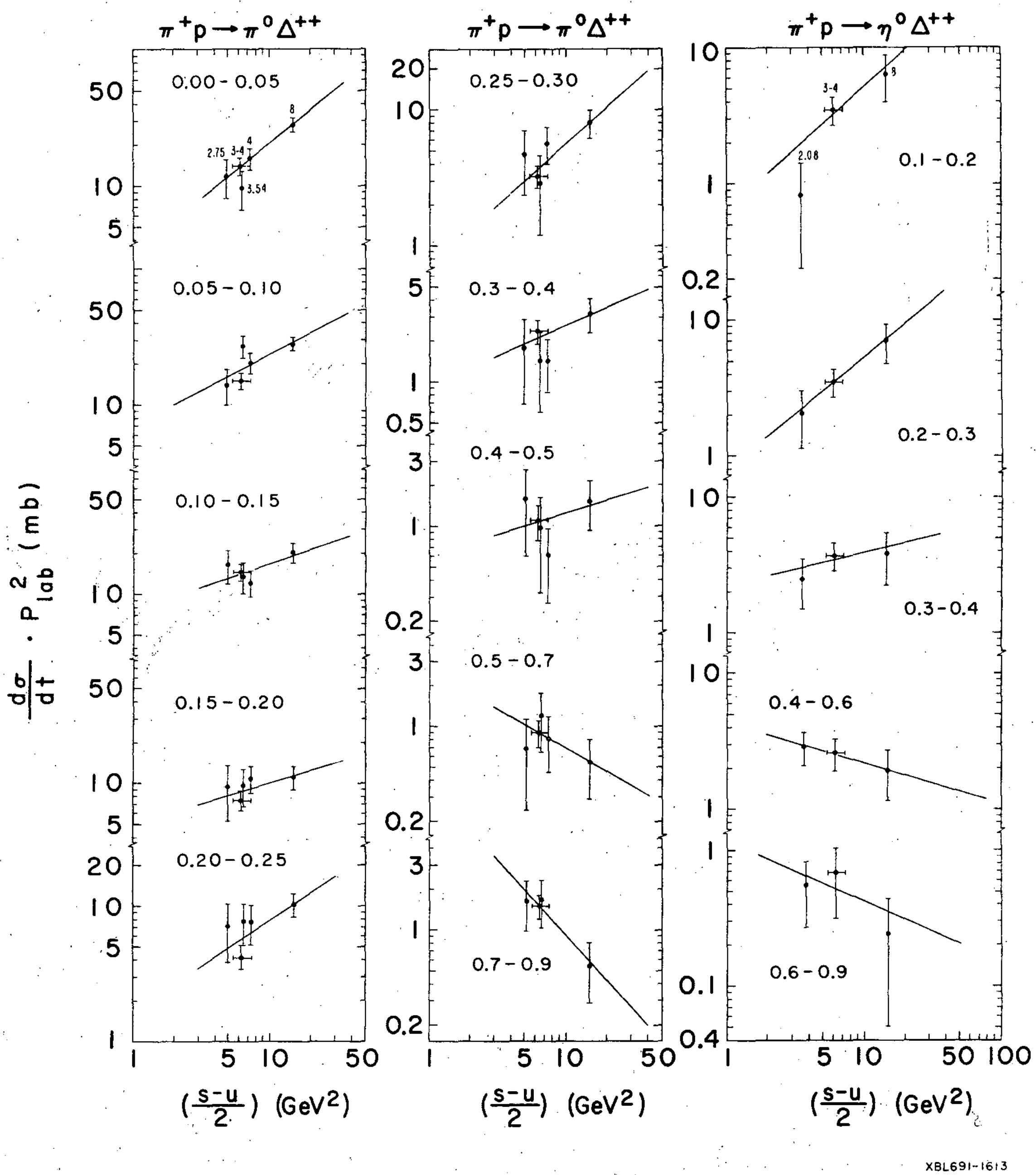
π⁰
Δ++
t
ρ
π+
s
p
π⁰
n
ρ
π−
p
η⁰
Δ++
A2
π+
p
η⁰
n
A2
π−
p
K⁰
Δ++
ρ + A2
K+
p
K̄⁰
n
ρ + A2
K−
p
Inelastic reactions
Elastic reactions
XBL691-1611

Fig. 1



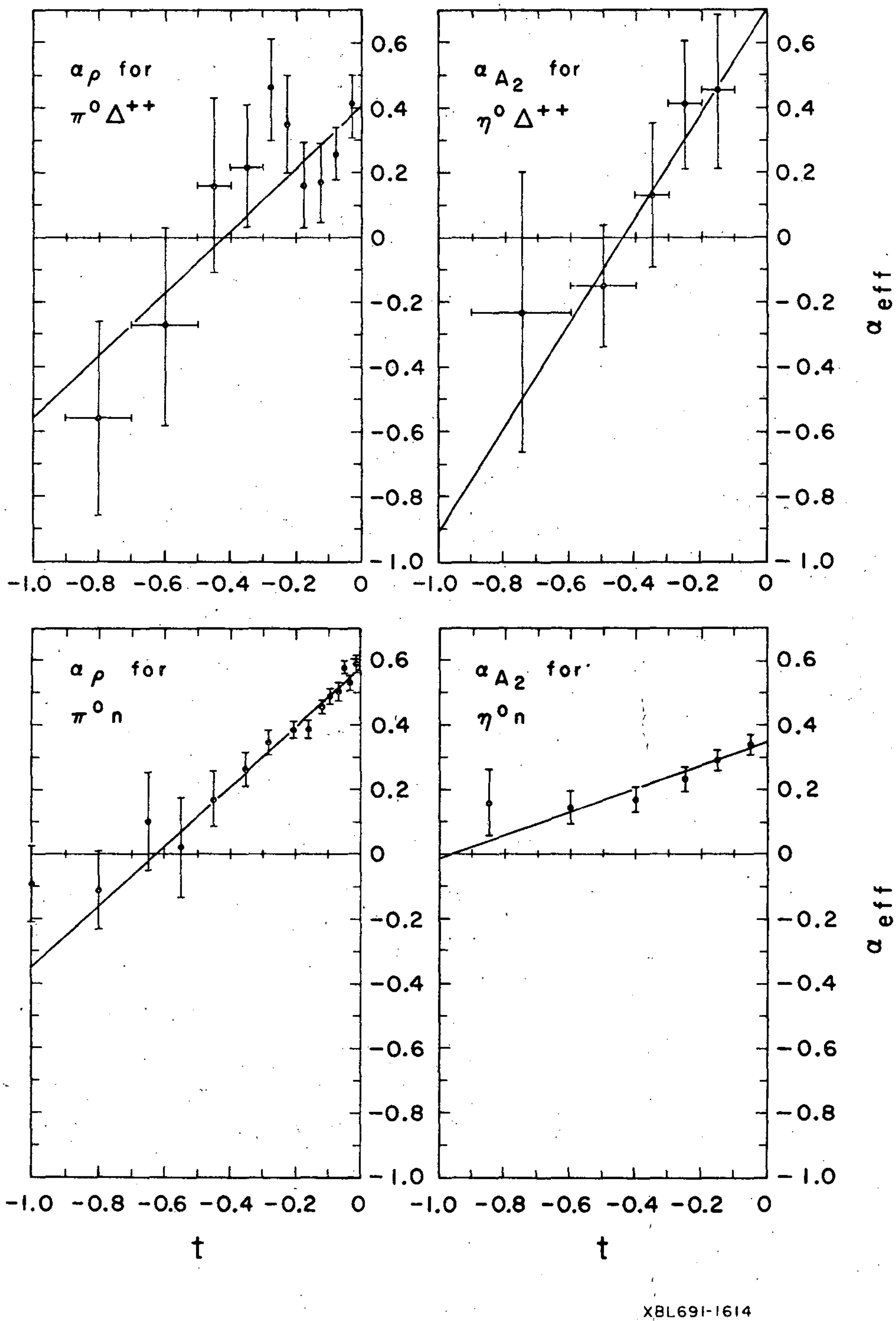
σ (mb)
4.0
1.0
0.1
0.01
π⁰n
π⁰Δ⁺⁺
π⁰Δ⁺⁺ for 3-4 GeV/c
η⁰n
η⁰Δ⁺⁺
K̄⁰n
K⁰Δ⁺⁺
1
10
50
30
40
P lab
XBL691-1612

Fig. 2



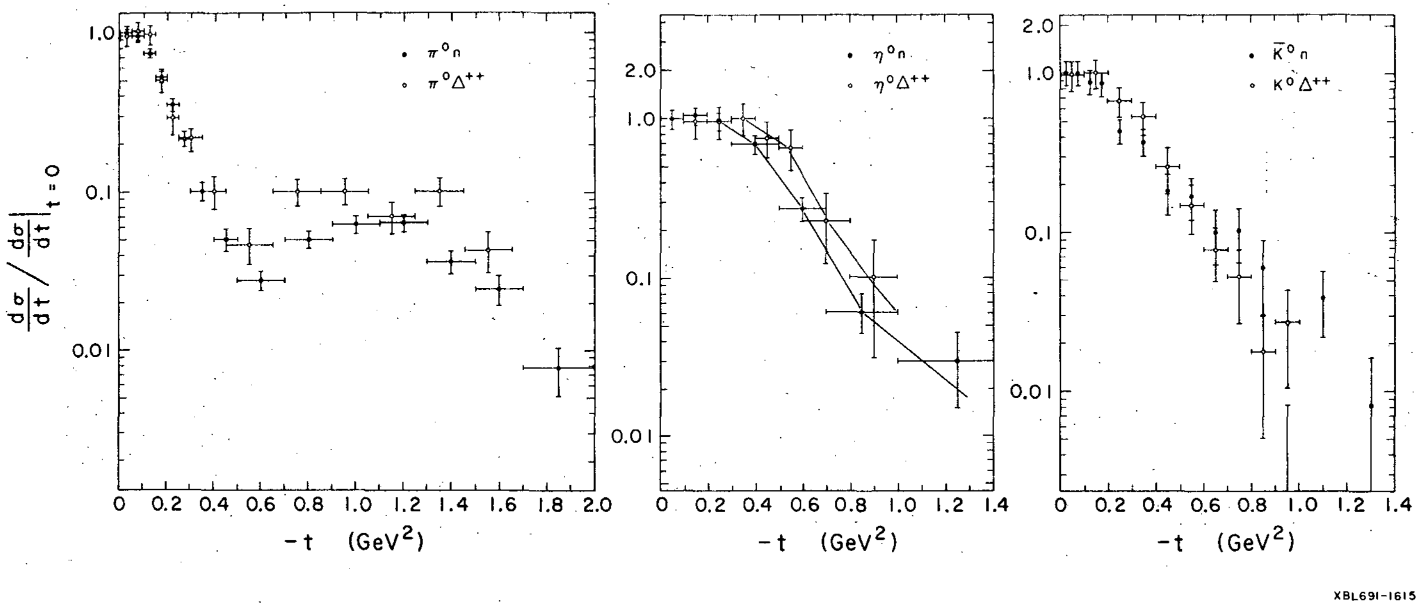
π⁺p → π⁰Δ⁺⁺
π⁺p → π⁰Δ⁺⁺
π⁺p → η⁰Δ⁺⁺
0.00 - 0.05
0.05 - 0.10
0.10 - 0.15
0.15 - 0.20
0.20 - 0.25
0.25 - 0.30
0.3 - 0.4
0.4 - 0.5
0.5 - 0.7
0.7 - 0.9
0.1 - 0.2
0.2 - 0.3
0.3 - 0.4
0.4 - 0.6
0.6 - 0.9
dσ/dt · P²lab (mb)
((s-u)/2) (GeV²)
XBL691-1613

Fig. 3



α ρ for
π⁰ Δ⁺⁺
α A2 for
η⁰ Δ⁺⁺
α ρ for
π⁰ n
α A2 for
η⁰ n
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.0 −0.8 −0.6 −0.4 −0.2 0
α eff
t
XBL691-1614

Fig. 4



$\frac{d\sigma}{dt} / \frac{d\sigma}{dt}\Big|_{t=0}$
1.0
0.1
0.01
2.0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
-t (GeV^2)
• $\pi^0 n$
◦ $\pi^0 \Delta^{++}$
• $\eta^0 n$
◦ $\eta^0 \Delta^{++}$
• $\bar{K}^0 n$
◦ $K^0 \Delta^{++}$
XBL691-1615

Fig. 5

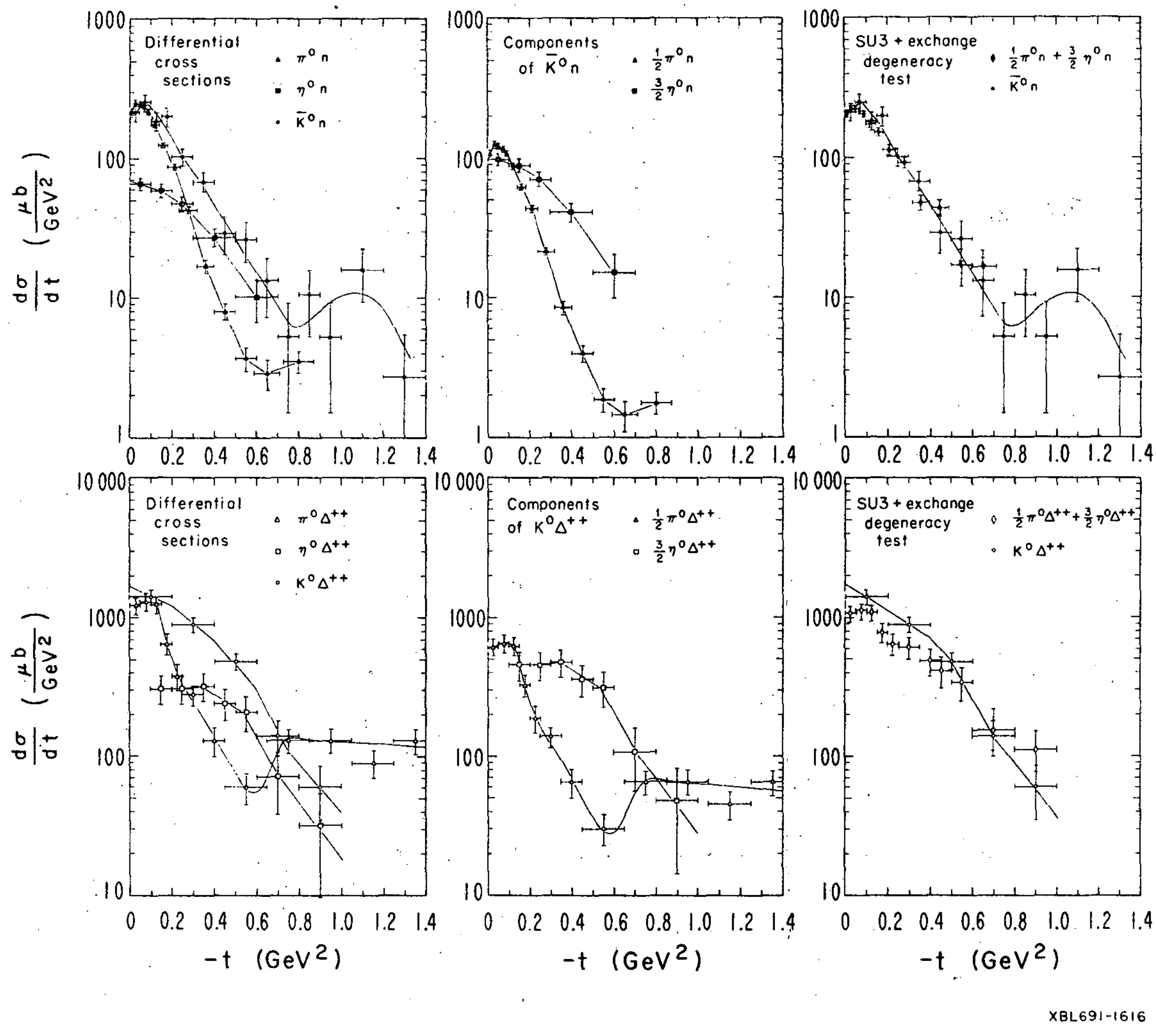
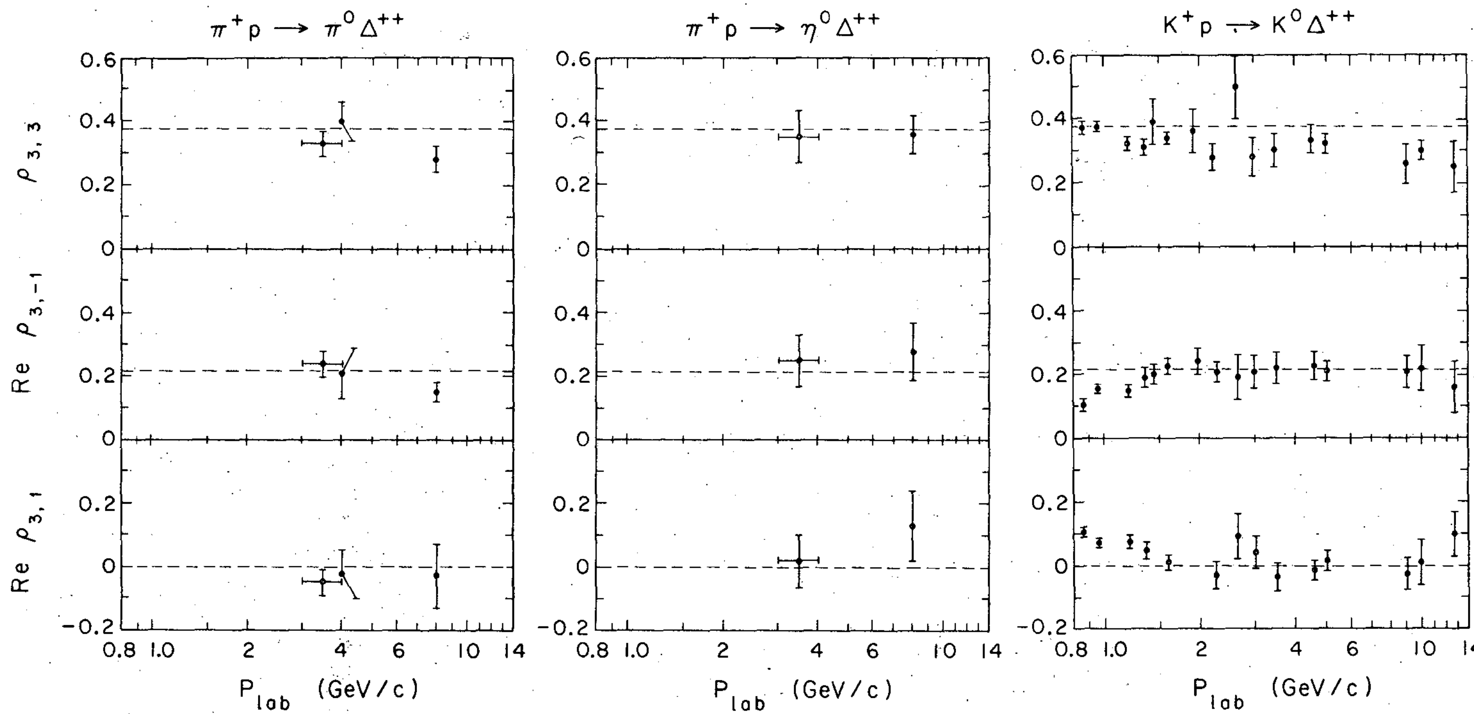

Differential cross sections
$\pi^0 n$
$\eta^0 n$
$\bar{K}^0 n$
Components of $\bar{K}^0 n$
$\frac{1}{2}\pi^0 n$
$\frac{3}{2}\eta^0 n$
SU3 + exchange degeneracy test
$\frac{1}{2}\pi^0 n + \frac{3}{2}\eta^0 n$
$\bar{K}^0 n$
Differential cross sections
$\pi^0 \Delta^{++}$
$\eta^0 \Delta^{++}$
$K^0 \Delta^{++}$
Components of $K^0 \Delta^{++}$
$\frac{1}{2}\pi^0 \Delta^{++}$
$\frac{3}{2}\eta^0 \Delta^{++}$
SU3 + exchange degeneracy test
$\frac{1}{2}\pi^0 \Delta^{++} + \frac{3}{2}\eta^0 \Delta^{++}$
$K^0 \Delta^{++}$
$\frac{d\sigma}{dt}$ $\left(\frac{\mu b}{GeV2}\right)$
$-t$ (GeV^2)
XBL691-1616

Fig. 6



XBL691-1617

Fig. 7