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## Coulomb Blockade and Transport in a Chain of One-Dimensional Quantum Dots

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A long one-dimensional wire with a finite density of strong random impurities is modeled as a chain of weakly coupled quantum dots. At low temperature  $T$  and applied voltage  $V$  its resistance is limited by breaks: randomly occurring clusters of quantum dots with a special length distribution pattern that inhibit the transport. Because of the interplay of interaction and disorder effects the resistance can exhibit  $T$  and  $V$  dependences that can be approximated by power laws. The corresponding two exponents differ greatly from each other and depend not only on the intrinsic electronic parameters but also on the impurity distribution statistics.

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Recently much interest has been attracted by the observations of algebraic current-voltage and current-temperature dependences in one-dimensional (1D) electron systems. The power laws,  $I \propto V^{b+1}$  at high  $V$  and  $I \propto T^a V$  at low  $V$ , appear in a variety of conductors including carbon nanotubes [1–4], nanowires [5–9], and polymer nanofibers [10]. This implies that the electron transport in such systems is effectively blocked by some barriers at  $T, V \rightarrow 0$ . It is a question of both fundamental and practical interest to determine what these barriers are and what controls the exponents  $a$  and  $b$ .

In the model with a single opaque barrier the power law can indeed appear if electron interactions are strong, in which case the effective transparency of the barrier is governed by the  $T$  and  $V$ -dependent suppression of electron tunneling into a correlated many-body state [11]. Similar physics may operate in the quasi-1D case [12–15]. Barring some difference in computational methods, the existing theories agree that the single-barrier exponents should be equal. For an interior barrier in a 1D wire they are given by  $a = b = 2(K^{-1} - 1)$ , where  $K < 1$  is determined by the interaction strength [11]. Experiments with wires a few  $\mu\text{m}$  or shorter in length [1–3,7,9] support these predictions. Yet in wires of length  $L \gtrsim 10 \mu\text{m}$  the ratio  $a/b$  is almost always larger than unity. It is usually between two and five [5,8–10]. To explain this behavior we propose that for long wires the single-barrier model may be unrealistic. Indeed, observations of Coulomb blockade (CB) at low  $T$  [1,2,10] suggest that a long wire typically contains multiple impurities that convert it into a chain of weakly coupled 1D quantum dots.

We show that for such a system there indeed exists a broad parameter regime where the function  $I(V, T)$  can be approximated by power laws with  $a \gg b$ . They originate from the statistics of impurity distribution *not* from

Luttinger-liquid effects [11]. Although in strictly 1D wires these power laws would be somewhat obscured by mesoscopic fluctuations, they may be observable in a cleaner manner in quasi-1D systems [5,8–10]. Under certain realistic conditions (discussed below) different transverse channels of such systems can act as independent 1D conductors in parallel, so that our results are fully relevant yet mesoscopic fluctuations are averaged out.

The model we study is similar to that of Refs. [16–18] and is depicted in Fig. 1(a). Identical impurities (e.g., dopants of the same chemical species) are positioned along the wire according to a Poissonian distribution with the mean spacing  $l \gg 1/n$ , where  $n$  is the average electron density. The tunneling transparency of each impurity is  $e^{-s} \ll 1$ . Electrons occupy a partially filled band with a generic (e.g., parabolic) dispersion. Their spin is ignored. Electron-electron interactions are short-range, e.g., due to

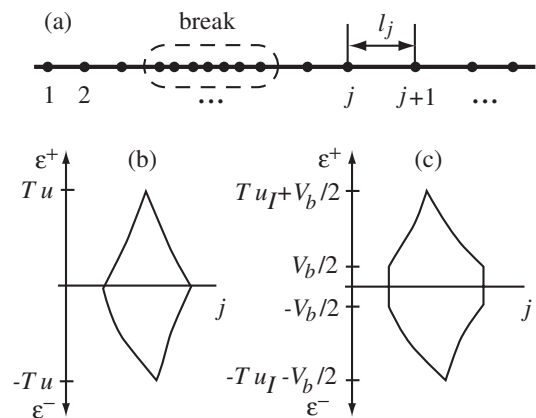


FIG. 1. (a) Illustration of the model. The consecutively numbered dots represent strong random impurities. (b) Typical shape of an Ohmic break. (c) A typical non-Ohmic break.

screening by a nearby gate, so that the system has a finite (geometry-dependent) capacitance per unit length  $C$ . The possibility of thermal activation and a finite conductance in the CB regime is ensured by weak coupling of electrons to the thermal bath of phonons. (The conductance depends on this coupling through a subleading prefactor, if at all [19]. We ignore such prefactors here because they are non-universal.)

The ground state and the low-energy charge excitations of this model are well approximated by the classical limit in which the number of electrons in each dot is restricted to be an integer. In the ground state it is the integer  $q_j$  closest to the “background charge”  $Q_j = nl_j + (\xi/\Delta_j)$ , where  $\xi$  is the electrochemical potential and  $l_j$  is the length of the dot. The charging energy  $\Delta_j = (d\mu/dn + C^{-1})/l_j$  includes contributions both due to a finite thermodynamic density of states  $dn/d\mu$  (the analog of the level-spacing in bulk quantum dots) and due to electrostatics, as usual. Here  $\mu = \xi - U$  and  $U$  are the chemical and the electrostatic potentials, respectively, and the units  $e = k_B = 1$  are used. The typical charging energy is  $\Delta = (d\mu/dn + C^{-1})/l$ .

Changing the electron number in the dot by  $m$  incurs the energy cost of  $E_j^m = (1/2)\Delta_j m^2 + \Delta_j m(q_j - Q_j)$ . Hence, the equilibrium probability of having  $q_j + m$  electrons in the  $j$ th dot is given by  $f_j^m = \exp(-\beta E_j^m)/Z$ , where  $\beta = 1/T$  and  $Z = \sum_m \exp(-\beta E_j^m)$ . The dot is in the CB state if  $|q_j - Q_j| - 1/2 \gg T/\Delta_j$ . In this case only  $m = 0, \pm 1$  are important. As common in CB literature, one can view  $\epsilon_j^\pm \equiv \mu \pm E_j^{\pm 1}$  as the energy of the lowest unoccupied (highest occupied) “single-particle” orbital. Important in the following is the probability  $P(\epsilon^+, \epsilon^-)$  of having no charge excitations within a given energy interval,  $E_j^{\pm 1} > \epsilon^\pm$  [20]. Function  $P$  depends only on  $\epsilon = \epsilon^+ + \epsilon^-$  and can be easily calculated:

$$P(\epsilon) = \int \frac{dl_j}{l} e^{-l_j/l} \left(1 - \frac{\epsilon}{\Delta_j}\right) = 1 - \frac{\epsilon}{\Delta} (1 - e^{-\Delta/\epsilon}) \quad (1)$$

(the integration is constrained to  $\Delta_j > \epsilon$ ). It behaves as  $\Delta/(2\epsilon)$  at  $\epsilon \gg \Delta$ , and so the average charge excitation gap  $\langle \epsilon \rangle = -\int \epsilon dP(\epsilon)$  is logarithmically divergent [21]. Below we turn to the study of transport and show that this divergence and the constraints of the 1D geometry lead to new conductivity laws.

We focus on the case of low voltages,  $V < V_c = L\Delta/(2l)$ , where the wire is in the CB state [22]. Electron transitions between any two dots  $j$  and  $k$  occur by the combination of quantum tunneling and thermal activation. For  $|j - k| = 1$  the tunneling is said to be sequential; otherwise, it is referred to as cotunneling [14]. In prior studies of random quantum dot arrays [17,23–25] the analogy between this transport mechanism and the variable-range hopping [26,27] (VRH) has been noted. Thus, the typical length of a hop  $|j - k|$  is established from the competition between activation and tunneling. Longer hops allow electrons to find transitions of lower

activation energy but involve a higher tunneling action  $s|j - k|$ . The crucial point is then as follows. It is known that 1D VRH (of noninteracting electrons) is special: the conductance is limited not by typical hops but by rare highly resistive spots—“breaks” [28–31]. Therefore, one may anticipate that some type of breaks operate in our model as well. Indeed, the calculation below shows that at not too low  $T$  the role of breaks is played by rare clusters of densely spaced impurities, see Fig. 1(a) [32].

The calculation closely parallels those of Refs. [30,31]. It starts by noting that if breaks exist, they act as transport bottlenecks where the most of the applied voltage drops. In contrast, the dots between two adjacent breaks are in a quasiequilibrium. Their electrochemical potentials  $\xi_j$  are nearly equal while the occupation factors of their charge excitations are given by  $f_j^{\pm 1} \simeq \exp[\pm\beta(\mu_j - \epsilon_j^\pm)]$ , where  $\mu_j$  is the *local* chemical potential. This allows one to get a simple formula for the net current  $I_{jk}$  from a dot  $j$  to another dot  $k$ . For the most relevant case of a large activation energy,  $E_{jk} \gg T$ , it coincides with the standard expression of the VRH theory:

$$I_{jk} \sim I_0 e^{-s|j-k|} e^{-\beta E_{kj}} \sinh\beta(\xi_j - \xi_k), \quad (2)$$

$$E_{kj} = \frac{1}{2} \min_{\sigma, \tau = \pm} (|\epsilon_j^\sigma - \mu_j| + |\epsilon_k^\tau - \mu_k| + |\epsilon_j^\sigma - \epsilon_k^\tau|), \quad (3)$$

where  $I_0 = \text{const}$  is some prefactor [33]. Consider first the Ohmic regime. Here  $\mu_j \simeq \mu = \text{const}$  and the effective resistance  $R_{jk} = (\xi_j - \xi_k)/I_{jk}$  for each link of the hopping network can be defined:  $R_{jk} = R_0 \exp(\beta E_{jk} + s|j - k|)$ , with  $R_0 = T/I_0$ . Denote by  $P_u(u)$  the probability per unit length of finding a break of resistance at least  $R_0 e^u$ , i.e., such a configuration of dots that  $R_{jk} \geq R_0 e^u$  regardless of how  $j$  to the left of the break and  $k$  to the right of it are chosen. For  $u \gg s$  this break contains many, at least  $u/s$ , dots. We can then talk about its *shape*, i.e., smooth envelope functions  $\epsilon^\pm(z) > 0$  such that  $\pm(\epsilon_j^\pm - \mu) \geq \epsilon^\pm(j)$  for all  $j$  in the break, see Fig. 1(b). The probability in question can be approximated as follows:

$$P_u(u) \sim \max_j \prod_j P[\epsilon^+(j) + \epsilon^-(j)], \quad (4)$$

where the maximum is sought under the constraint  $R_{jk} \geq R_0 e^u$  for all  $(j, k)$  pairs. Verbatim repetition of the steps taken in Ref. [30] establishes that the break has a familiar diamondlike shape [30,31], see Fig. 1(b).

It requires some effort to finish the derivation of  $P_u$  for an arbitrary  $u$ . However, for  $Tu \gg \Delta$  the final result can be obtained if the actual shape of the break is approximated by a rectangle of the same width and height, i.e., if we consider a break of  $N = u/s$  dots with the charging energy  $Tu$  each. This gives  $P_u(u) \sim [P(Tu)]^N$ , and so

$$\ln P_u(u) \simeq -(u/s) \ln(Tu/\Delta), \quad Tu \gg \Delta. \quad (5)$$

A non-Ohmic break, where  $V_b \equiv \xi_j - \xi_k \gg T$  for  $j$  and  $k$

on the opposite sides of the break, can be treated similarly. The necessary steps are spelled out in Ref. [31]. Namely, consider a state of a given fixed current  $I = I_0 \exp(-u_l)$ , so that the free parameter is the voltage  $V_b$  generated by a break. Denote by  $P_V(V_b)$  the probability per unit length of finding a non-Ohmic break with the voltage drop of at least  $V_b$ . Once again approximating the break shape by a rectangle  $\varepsilon^\pm = Tu_l + (V_b/\mathcal{L})$  with  $\mathcal{L} \sim 1$  [34] and using Eq. (4), we find this time

$$\ln P_V(V_b) \simeq \ln P_u(u_l) - V_b/(s\mathcal{L}T). \quad (6)$$

This entails that the average voltage drop  $V_*$  across a non-Ohmic break is  $V_* = s\mathcal{L}T$ . Thus, if  $V \gg V_*$ , many breaks must contribute. The derivation of  $I(V)$  is especially simple in this case, so we prefer to finish it first and turn to the Ohmic one later.

The total voltage  $V$  is the sum of voltages generated by all non-Ohmic breaks (contribution of Ohmic ones is sub-leading). For  $V \gg V_*$  (many breaks) the sum can be replaced by the integral  $V = L \int dV_b P_V(V_b)$ . Using Eqs. (5) and (6) we get a transcendental equation for  $u_l$

$$u_l = s \ln(c_1 LT/Vl) / \ln(Tu_l/\Delta), \quad V_* \ll V \ll V_c, \quad (7)$$

where  $c_1$  is a slow dimensionless function of  $V$  and  $T$ . We conclude that the dependence of  $I$  on  $V$  is close to the power law with a weakly  $T$  and  $V$ -dependent exponent

$$b \simeq s / \ln(Tu_l/\Delta), \quad T \gg \Delta/s. \quad (8)$$

This is the first of our two main results. As  $V$  decreases to a number ca.  $V_*$  the resistance of a wire becomes dominated by the single largest break, and so it exhibits strong ensemble fluctuations. To illustrate how to handle this case, let us consider next the Ohmic regime where such fluctuations are the strongest.

In this regime the resistance  $R$  of the wire is the sum of Ohmic resistances  $R_j = R_0 \exp(u_j)$ , where  $u_j$ 's form a set of  $N_b \sim L/l_b$  independent random numbers with the probability distribution function (PDF)  $l_b P_u(u)$  each [Eq. (5)]. Parameter  $l_b^{-1} \equiv \int P_u(u) du$  enters the final result only under the logarithm, so it need not be specified precisely.

Following [30], our goal is to compute  $P_R$ , the PDF of  $R$ . We begin by noting that the PDF  $p$  of  $R_j$  behaves as

$$p(R_j) \sim R_j^{-1-\alpha}, \quad \alpha = s^{-1} \ln[(T/\Delta) \ln(R_j/R_0)], \quad (9)$$

cf. Eq. (5). Since  $\alpha(R_j)$  is a very slow function,  $p(R_j)$  is basically a power law. Hence, the random variable  $R > 0$  is a Lévy random walk. A celebrated theorem of the probability theory immediately tells us that in the limit of large  $N_b$  the PDF of  $R$  approaches the stable Lévy distribution [35] (often encountered in plasma, astro, and atomic physics, as well as in biology, economics, and reliability theory). For  $\alpha \ll 1$  it becomes identical to Fréchet extreme-value statistics  $P_R(R) \propto R^{-1-\alpha_*} \exp[-(R_*/R)^{\alpha_*}]$ . In turn, the PDF of the conductance  $G = 1/R$  has the form

$$P_G(G) = \text{const} \times G^{\alpha_*-1} \exp[-(R_*G)^{\alpha_*}], \quad (10)$$

so that its first two moments are given by

$$\langle G \rangle R_* = \Gamma(1 + \alpha_*^{-1}), \quad \langle G^2 \rangle R_*^2 = \Gamma(1 + 2\alpha_*^{-1}), \quad (11)$$

$\Gamma(z)$  being the Euler gamma function.

The equations for  $\alpha_*$  and  $R_*$  are obtained by the standard procedure [35] (see also Ref. [30]):

$$\alpha_* \ln(R_*/R_0) = \ln(N_b/\alpha_*), \quad \alpha_* = \alpha(R_*). \quad (12)$$

Combined with Eq. (9), they yield

$$\ln(R_*/R_0) \simeq \frac{s \ln(L/\alpha_* l_b)}{\ln[(Ts/\Delta) \ln(L/\alpha_* l_b)]}. \quad (13)$$

Not surprisingly, the form of Eq. (10) is identical to that found in Ref. [30] for another model; however, the  $T$  dependence of  $R_*$  in that case is different from Eq. (13). If only a limited range of  $T$  is accessible, which is often the case in the experiment, the latter can be approximated by a power law with a weakly varying exponent. In turn, this entails  $\langle G \rangle \propto T^a$ , where  $a$  is given by

$$a = \frac{d \ln \langle G \rangle}{d \ln T} \simeq b \frac{\ln(c_2 L/l)}{\ln(Ts/\Delta)} \quad (14)$$

and  $c_2$  is an algebraic function of  $T$  [36]. This is our second main result. As emphasized above,  $a \gg b \gg 1$ .

Equation (14) applies at  $T > \Delta/[s \ln(L/l)]$  and for wires of length  $l \ll L \ll l \exp(e^s)$ . The lower bound guarantees that the wire contains a large number of impurities. The upper bound, which is likely to be satisfied in practice, ensures  $\alpha_* \ll 1$ , so that Eq. (10) holds.

The PDF of Eq. (10) describes the statistics of  $G$  in an ensemble of wires at a given  $T$ . However, fluctuations of comparable magnitude would appear in the *same* wire as  $T$  varies [29,30,37]. They would be superimposed on the backbone dependence  $G \propto T^a$ . According to Eq. (11), the amplitude of such fluctuations is large in the limit we considered,  $\alpha_* \ll 1$ . Let us however estimate how small  $\alpha_*$  can really be. For the experimentally accessible regime of  $\ln(R_*/R) \lesssim 10$  and  $N_b, L/l$  of a few hundred, Eq. (12) gives  $\alpha_* \sim 0.5$ . In this case, the fluctuations of  $G$  are still significant [cf. Eq. (11)], and so verification of our predictions in truly 1D systems would require statistical analysis of the data [37] and/or ensemble averaging.

At this point we draw attention to the fact that the cited experiments [5,8–10] were actually performed on quasi-1D systems with a large number  $N$  of transverse channels. Suppose that conductances of different channels are additive and statistically independent, then for  $\alpha_* = 0.5$  the relative fluctuations of the total  $G$  will be  $\sqrt{5/N}$ . This amounts to 10% for a representative number of  $N = 500$ , and so the power laws should no longer be obscured by fluctuations. The above scenario certainly needs an experimental verification. It will not hold for materials where transverse channels are coupled. However, there are systems in which it is reasonable, e.g., those with a *large*

concentration of neutral (in general, short-range) impurities and *strong* anisotropy of longitudinal and transverse transport; see Ref. [17].

Finally, let us comment on the expected values of  $a$ ,  $b$ , and  $s$ . For a 1D wire the result  $s = -\ln t + (K^{-1} - 1) \times \ln(E_F^2/\delta_j\delta_{j-1})$  can be obtained [17,18,25]. Here  $t \ll 1$  is the bare tunneling transparency of the impurity and  $E_F$ ,  $\delta_j$  are, respectively, the high and the low-energy cutoffs for the Luttinger-liquid effects. The latter cutoff  $\delta_j = K\Delta_j \propto 1/l_j$  is the energy spacing of the collective neutral excitations of the  $j$ th dot. In comparison, for an impurity in an infinite wire, the low-energy cutoff is set by either  $V$  or  $T$  [11]. Since  $s$  depends on  $l_j$ 's only logarithmically, our approximation that  $s$  is the same for all impurities is justified. Note that  $\delta_j \sim KTu \propto T \ln(L/l)$  for important Ohmic breaks, so that, nominally,  $s$  has the same logarithmic  $T$  dependence as in the single-barrier case [11]. Nevertheless, due to the (modestly) large logarithmic factors  $\ln(E_F/\delta_j)$  and  $\ln(L/l)$  both predicted exponents  $a$  and  $b$  would noticeably exceed the single-impurity one, in this case,  $2(K^{-1} - 1)$ .

Semiconductor nanowires, polymer nanofibers, and carbon nanotubes appear promising for a continuing investigation of the physics we discussed. We call for a systematic study of transport in such wires.

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