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Essays on Financial Frictions and Aggregate Dynamics

A dissertation submitted in partial satisfaction
of the requirements for the degree of
Doctor of Philosophy in Economics

by

David Laszlo Zeke

2016

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ABSTRACT OF THE DISSERTATION

Essays on Financial Frictions and Aggregate Dynamics

by

David Laszlo Zeke

Doctor of Philosophy in Economics

University of California, Los Angeles, 2016

Professor Andrew Granger Atkeson, Chair

This dissertation studies the effects of firm debt and financing frictions on the macroeconomy. Chapter 1 investigates the role of changes in firms' idiosyncratic risk and their cost of default in driving changes in employment and credit spreads, both over the business cycle and in the cross-section. I use firm-level panel data and a structural model of financial frictions and volatility shocks to assess the effects of shocks to firm volatility and default costs. I find that volatility shocks alone can only generate modest declines in aggregate employment. However, simultaneous shocks to firm volatility and default costs can interact to generate large employment declines.

Chapter 2, co-authored with Robert Kurtzman, investigates the role of changes in the allocation of labor and capital between firms in driving productivity dynamics. This chapter presents accounting decompositions of changes in aggregate labor and capital productivity. Our simplest decomposition breaks changes in an aggregate productivity ratio into two components: A mean component, which captures common changes to firm factor productivity ratios, and a dispersion component, which captures changes in the variance and higher order moments of their distribution. We demonstrate that in standard models of production with heterogeneous firms, our dispersion component reflects changes in distortions to the alloca-

tion of labor and capital between firms. We find, for public firms in the United States and Japan, that the dispersion component plays a minor role in productivity changes over the business cycle.

Chapter 3, co-authored with Robert Kurtzman, investigates the role of debt overhang, an agency problem between firms' equity holders and creditors, in distorting firm growth and aggregate welfare. This chapter addresses this question through the lens of a general equilibrium model of firm dynamics and endogenous innovation in which debt overhang affects the firm innovation decision and subsequent firm growth. The estimated model implies that while the private gains to a firm from resolving debt overhang can be large if it faces sufficient default risk, the social gains to long-run productivity and output are relatively modest. The time-varying distribution of firm default risk suggests social gains may be greater during recessions.

The dissertation of David Laszlo Zeke is approved.

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Chapter 1: Financial Frictions, Volatility, and Skewness

1.1 Introduction

The countercyclical behavior of the cross-sectional dispersion of economic variables has been well documented by economists, and convincing arguments have been made that this reflects the underlying volatility of firm-level idiosyncratic shocks.¹ A recent strand of literature has generated aggregate fluctuations by using the interaction of volatility shocks with firm-level financial market frictions to distort firm-level employment or investment decisions.² Several of these papers find that idiosyncratic volatility shocks, operating through financial frictions, are important for explaining business cycle dynamics.³ Financial frictions affect not only firm investment but also firm employment decisions if labor markets are not frictionless; several recent papers document that firm employment does in fact respond to changes in financial constraints.⁴

In this paper, I use firm-level panel data, together with a structural model, to assess how firm employment responds to volatility shocks in the presence of financial frictions. I use cross-sectional patterns in the data directly related to the response of firm employment and credit spreads to volatility shocks in order to discipline parameters key to the magnitude of this response. One parameter which crucially affects the response of firm employment to volatility shocks is the cost of default. This is true in a large number of models, including

¹Recent papers include Eisfeldt and Rampini (2006) and Bloom (2009). Additionally, Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2014) document that idiosyncratic equity volatility obeys a strong factor structure and spikes during recessions.

²See Christiano, Motto, and Rostagno (2013), Arellano, Bai, and Kehoe (2012), and Gilchrist, Sim, and Zakrajsek (2014).

³For instance, Christiano et al. (2013) find that changing volatility is the most important shock over the business cycle. Similarly, Arellano, Bai, and Kehoe (2012) find that most of the decline in output and employment during the recent recession can be explained by idiosyncratic volatility shocks.

⁴See Benmelech, Bergman, and Seru (2013) and Chodorow-Reich (2014).

papers using the financial accelerator framework of Bernanke, Gertler, and Gilchrist (1999) (hereafter BGG).⁵ If default is costless, corresponding to the assumptions of Modigliani and Miller (1958), then the heightened default risk caused by an increase in volatility does not affect firm decisions. As the cost of default increases, the magnitude of its effect on firm employment increases as well. I argue that the cost of default is important not only for the impact of volatility shocks on firm employment over the business cycle, but also for the strength of many cross-sectional relationships relating credit spreads, default rates, and firm employment. I use firm-level panel data to identify shocks to the cost of default and to the skewness of firm idiosyncratic risk, and evaluate their impact on firm decisions and aggregates over the business cycle.

Many of the models in this literature would generate cross-sectional implications if firms were to receive heterogeneous shocks to the level of idiosyncratic volatility.⁶ These cross-sectional implications arise from the same channels that lead to aggregate fluctuations over the business cycle. Therefore, I argue that the cross-section can be used to calibrate parameters which affect business cycle dynamics.

The specific structural model I use to assess the role of volatility shocks and financial frictions on firm employment is based on the model of Arellano, Bai, and Kehoe (2012) (hereafter, ABK). Firms face idiosyncratic shocks which have a persistent effect on the profitability of the firm, and the stochastic volatility of these shocks are time varying.⁷ The main mechanism at play in the model is the following: An increase in the firm's idiosyncratic volatility increases its probability of default. This, in turn, increases credit spreads and decreases firm employment. The decline in employment occurs because firms must choose their level of employment before realizing their idiosyncratic shocks, receiving proceeds from production, and paying the wage bill. Thus, choosing higher employment is risky and increases the probabil-

⁵In a BGG framework, the monitoring cost can be interpreted as the cost of default.

⁶These include papers using the financial accelerator framework following Bernanke, Gertler, and Gilchrist (1999), notably Christiano, Motto, and Rostagno (2013).

⁷These shocks are motivated as *demand shocks*, but they are isomorphic to productivity shocks.

ity of default, because the heightened wage bill increases the firm's liabilities the following period (therefore, the realized level of the idiosyncratic shock needed to pay off both debt and the higher wage bill increases). This effect of firm employment on default risk can be thought of as increasing the *operating leverage* of the firm, as the additional risk of a higher wage bill due in the future increases the fixed cost due similar to how it is increased by debt (financial leverage). A key property of this mechanism is that it is operational at the firm level. Therefore, an increase in the idiosyncratic volatility faced by a firm should increase its credit spread and reduce employment, even if the increase in volatility only affects that firm in isolation. Thus, this mechanism generates testable cross-sectional implications.

I base my analysis on the model of ABK as it is tractable and has meaningful heterogeneity in firm productivity and leverage. Additionally, their model focuses on the effect of volatility shocks and financial frictions on labor, rather than investment, and is able to generate a large decline in employment with volatility shocks calibrated for the 2007-2009 recession while matching important business cycle facts.⁸ I expand this model by adding heterogeneity in volatility and generalizing the parameterization of the default cost.⁹ I use this rich heterogeneity to generate cross-sectional implications. The same approach should work for an array of other models, notably those using the BGG framework and volatility shocks (such as Christiano, Motto, and Rostagno (2013)). The main mechanism in these models is effectively the same: in order to lower the risk of costly default firms/entrepreneurs reduce the scale of their operations. Increases to the likelihood or cost of default amplifies this reduction in scale.¹⁰

⁸ABK are able to generate a large decline in employment with volatility shocks consistent with observed sales growth dispersion during the 2007-2009 recession. Their model is able to do so without declining aggregate labor productivity.

⁹There is also heterogeneity in innovations to firm productivity, leverage, and volatility in my model. This heterogeneity is important because structural models of default risk and credit spreads, such as Merton (1974), use firm leverage and volatility as the key explanatory variables.

¹⁰BGG style models do not have meaningful heterogeneity in firms/entrepreneurs who are exposed to default, they are ex-ante identical. This allows the distribution of these agents to be summarized by the total wealth they hold. To implement my approach on such a model, meaningful heterogeneity would have to be added to generate cross-sectional implications.

I use firm-level panel data from U.S. public firms on credit spreads, equity prices, and accounting statements to test the presence and magnitude of cross-sectional patterns predicted by the model. I document that there is considerable cross-sectional variation in innovations to firm asset volatility, even during non-recession years.¹¹ The variation in innovations to volatility in the cross-section is significant relative to the average increase in asset volatility during the crisis. I show that innovations to asset volatility are associated with significant increases in credit spreads and are predictive of decreases in employment. These results are robust to a large number of controls, year effects, and substituting sales for employment.¹² This validates the key qualitative implications of the model, and stresses that the association of volatility shocks with credit spreads and employment is present not only in aggregate time-series, but also in explaining patterns between firms in panel data.

I calibrate the model with moments relating the probability of default and credits spreads, and using the joint distribution of innovations to employment, credit spreads, and firm-level volatility measures. I show that the slope of the relationship between the probability of default and credit spreads depends primarily on the cost of default; credit spreads are a function of both the probability of default and recovery rates conditional on default, which depends crucially on the costs of default. When I set the cost of default to 100%, as in ABK where defaulting firms exit and lose all firm value, the relationship between the probability of default and credit spreads in the model is very steep. Similarly, if default is costless, the slope of this relationship is very flat. The slope implied by historical default probabilities and credit spreads by ratings class (for speculative grade firms) is much different than either of these two extremes. The slopes corresponding to estimates of the cost of default from the corporate finance literature, ranging from 8.4–30% of firm value lost, generate a relationship

¹¹I consider innovations, as opposed to the level, of most of the measures I examine. This is because firm capital structure, given sufficient time, can adjust in response to differences in the level of volatility. Additionally, looking at innovations in volatility is the natural approach given the desire to examine the response to volatility *shocks*.

¹²There are some known measurement issues associated with firm employment in Compustat (see Bloom (2009)). I use sales, which is measured more accurately, to confirm that this measurement error is not significantly distorting my results.

much more in line with the data.¹³

With such default costs, my calibrated model generates a modest decline in employment in response to common shocks to firm idiosyncratic volatility. Feeding in volatility shocks corresponding to the 2007-2009 recession generates at most a 2.5% decline in employment under the upper bound calibration. This is much lower than that implied by the calibration of ABK, where all firm value is lost upon default. This suggests that idiosyncratic volatility shocks, by themselves, have trouble explaining much of the employment dynamics in the recession of 2007-2009 through this mechanism.

However, there is empirical evidence of other important changes during the great recession that affect employment in this model. First, Bloom, Guvenen, and Salgado (2015) document substantial evidence of a large negative skewness shock to the growth rates of U.S. public firms. This suggests that something may be lost by modeling the change in firm idiosyncratic risk only as a second moment shock, as skewness affects the relative amounts of left and right tail risk. Second, recovery rates, the fraction of debt obligations (principal and accrued interest) debtholders receive upon firm default, are procyclical and fell significantly during the 2007-2009 recession, which may be indicative of an increase in the cost of default.¹⁴ I show that the cross-sectional relationship between default probabilities and credit spreads among U.S. public firms changes in a way consistent with a heightened cost of default during this recession. Specifically, I estimate that it became around 2.5 times steeper, corresponding to a significant increase in the cost of default.¹⁵ Feeding in either of these shocks together with volatility shocks can amplify the decline in employment. If firm upside and downside volatility are parameterized separately, differing shocks to these volatil-

¹³See Davydenko, Strebulaev, and Zhao (2012) (which includes a summary of estimates in the literature), Kaplan and Andrade (1998), and Hennessy and Whited (2007).

¹⁴See Altman (2006) and Moody's (2015).

¹⁵For the purpose of the model, I consider the cost of default to be exogenous. One could microfound such a shock through disruptions in financial markets which reduce the liquidation value of firm assets, see Shleifer and Vishny (1992) and Shleifer and Vishny (2011). Additionally, Gilchrist et al. (2014) consider shocks to the liquidation value of capital, which can make default more expensive as some capital is liquidated upon default in their model.

ities consistent with micro data from the 2007-2009 recession (using the measures of upside and downside dispersion in Bloom, Guvenen, and Salgado (2015)) amplify the fall in employment by around 1% of employment. Shocks to the cost of default, when interacted with volatility shocks, can generate an enormous decline in employment. Simultaneous shocks to volatility and default cost, calibrated to micro data, explain the majority of employment losses during the 2007-2009 recession. These losses are substantially larger than if the cost of default were permanently high (calibrated to the maximum cost of default realized during the 2007-2009 in my parameterization). This suggests the interaction of these two shocks is key in explaining large employment losses.

Related Literature There are a number of papers which use volatility shocks to drive business cycle dynamics. Several of these papers use the increased volatility of firm-level shocks interacted with adjustment costs in capital or labor to generate business cycle fluctuations; see, for example Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2014a), and Bachmann and Bayer (2013). Schaal (2015) uses the interaction of volatility shocks with labor market frictions in a search and matching model to generate significant declines in employment. The papers in this literature closest to mine are those which study the interplay between volatility shocks and financial frictions, notably Arellano et al. (2012), Christiano et al. (2013), and Gilchrist et al. (2014). I focus on volatility shocks operating through financial frictions for two reasons. First, there is evidence suggesting that this interaction may be able to deliver larger declines in output and employment than the interaction of volatility shocks with other frictions.¹⁶ Second, the interaction of volatility shocks with financial frictions leads to clear cross-sectional implications which can be compared to available firm-level data.

My main contributions to this literature are as follows: First, I use cross-sectional patterns

¹⁶Gilchrist et al. (2014) find, in a model with volatility shocks interacting with both adjustment costs and financial frictions, that the quantitatively important channel is through financial frictions. Bachmann and Bayer (2013) also call into question the quantitative relevance of adjustment costs interacting with volatility shocks.

to discipline key parameters that control the impact of volatility shocks on aggregates over the business cycle. While some papers in this literature use the cross-section to parameterize volatility shocks, my paper focuses on other key relationships to parametrize the cost of default as well. Second, my paper is novel in the shocks I consider, notably the role of changing skewness of firm idiosyncratic shocks and its impact for aggregates in this setting. There is a strand of the finance literature which discusses the asset pricing implications of idiosyncratic skewness; see, for example Amaya, Christoffersen, Jacobs, and Vasquez (2015), Feunou, Jahan-Parvar, and Okou (2015), Kraus and Litzenberger (1976), and Barberis and Huang (2008). However, these changes have been under-explored in macroeconomics — idiosyncratic risk shocks have typically been modeled only as volatility shocks.¹⁷ Bloom, Guvenen, and Salgado (2015) provide convincing evidence in firm-level data of significant changes in idiosyncratic skewness over the business cycle, and my paper investigates its implications in a business cycle model.

My paper also contributes to this body of literature through my analysis of shocks to the cost of default. My paper is novel in that I stress the interaction of this default cost shock with volatility shocks and show that this interaction leads to amplification effects. While Gilchrist, Sim, and Zakrajsek (2014) do consider both idiosyncratic volatility and capital liquidity (related to the cost of default) shocks, my paper differs in that it stresses the interaction between the two happening simultaneously.¹⁸ Additionally, I use the change in a key cross-sectional relationship to help parameterize the magnitude of the shock to the cost of default.

My paper is also related to a strand of literature which aims to disentangle the effects of financial and volatility (or uncertainty) shocks. The challenge faced by this literature is

¹⁷There are some papers in macroeconomics related to possibly time-varying skewness, but they differ substantially from my approach. A notable example is Orlik and Veldkamp (2014), who provide a micro-foundation for time-varying uncertainty about aggregate shocks by having agents update beliefs about the skewness of aggregate shocks.

¹⁸Additionally, a large literature looks at the effect of a variety of financial shocks, including the cost of default or liquidation, on the macroeconomy.

that the predictions of the two shocks are quite similar for the time series. Caldara, Fuentes-Albero, Gilchrist, and Zakrajsek (2014) state that “distinguishing between these two types of shocks... is difficult because increases in uncertainty are frequently associated with a widening of credit spreads, an indication of a tightening in financial conditions” and use a penalty function approach to assess the individual impact of these two shocks on the economy. Stock and Watson (2012) use a dynamic factor model and conclude that uncertainty and financial shocks were the primary drivers of the recent recession, although they note the high correlation between the shocks and question whether they are distinct. Christiano, Motto, and Rostagno (2013) use time-series data and a structural model following BGG with volatility and a number of other shocks to estimate the key drivers of business cycle dynamics.¹⁹ My paper’s contribution relative to this literature is the use of firm-level panel data to pin down shocks and their effects. For instance, while it is difficult to separately identify shocks to the cost of default as opposed to idiosyncratic volatility using only time series data on macroeconomic and financial aggregates, there are some rather stark cross-sectional implications of these shocks. Namely, in the framework I consider, the slope of the relationship between credit spreads and probability of default is very sensitive to the cost of default but not to volatility. This is because the effect of volatility on credit spreads occurs primarily through its effect on the probability of default.

Finally, my paper is related to a number of studies which aim to quantify the cost of default. The main challenge lies not in estimating the direct costs (such as legal bills) but rather the indirect costs (for example, the loss of key employees/customers due to greater risk of unemployment/discontinued support). This is a difficult task due to potential selection effects and the anticipation of default by markets. Andrade and Kaplan (1998) is the most widely used of these studies, which looks at a sample of highly leveraged transactions which became distressed and finds costs of default from 10-23%. Other studies include Davydenko,

¹⁹They do not consider shocks to the cost of default/monitoring; indeed it would be difficult to separately identify them in a BGG-style model. They do, however, estimate the constant level of default/monitoring; their estimate of 20% is consistent with estimates from the finance literature.

Strebulaev, and Zhao (2012), which take advantage of the fact that default is only partially anticipated to estimate cost of default from the valuation of firm equity and debt before and after default. They produce estimates larger than previously found, finding that default destroy around 30% of firm value while debt renegotiations destroy around 15%. On the low end of estimates, Hennessy and Whited (2007) use structural estimation to find costs of default as low as 8.4% for large firms. My results are consistent with the range of estimates in this literature. My paper shows that this range of estimates not only corresponds to patterns in the pricing of debt (as others have), but is also consistent with firm dynamics, namely the magnitude of the decline in employment or sales predicted by innovations to credit spreads.

Road Map The rest of the paper follows as such: Section 1.2 introduces a simplified version of the model to introduce the key mechanism along with the strategy by which cross-sectional relationships are used to discipline model parameters. Section 1.3 describes the data I use and establishes facts about the cross-sectional relationships of innovations to asset volatility, credit spreads, and firm input decisions. Section 1.4 describes the model of firm volatility and financing frictions and characterizes an equilibrium. Section 1.5 describes the parameterization of the model and the resulting implications for both the cross-section and the business cycle. Section 1.6 documents the implications of negative idiosyncratic skewness shocks and shocks to the cost of default. Section 1.7 concludes.

1.2 Simple Model

I begin with a simplified version of the model to illustrate the mechanism of interest, through which firm-level volatility shocks can generate business cycle dynamics. Specifically, I show how changes in the distribution of a firm idiosyncratic shock can, in the presence of financial frictions, lead to fluctuations in the level of employment and credit spreads. The simple model will demonstrate how the costliness of default plays a key role in determining how

large of an impact fluctuations in firm-level risk have on firm employment. Also, the simple model generates cross-sectional implications, which can be compared to the data to discipline the theory.

The primary mechanism in this model is based on employment *operating leverage*, where firms' labor decisions affect their probabilities of default. Firms make their employment decision before realizing a firm-level demand shock, affecting the profits generated by a given number of workers. They then receive revenues from production (which depend on the shock) less the wage bill (which does not depend on the shock) and any debt outstanding. If a firm chooses a greater amount of labor, it takes on additional risk due to greater operating leverage (a higher fixed cost next period) in exchange for higher expected profits. A higher labor choice hurts firm net revenues if the realized shock is low, and thus can increase the probability of default. Default is costly, so firms have an incentive to reduce their labor demand in order to reduce their default risk.

This section also demonstrates a few key points of the paper. First, I show that the cost firms face upon default has an enormous impact on the magnitude of the distortion to firm labor, as well as the magnitude of labor losses due to a volatility shock. Second, I show that since the mechanism is operational in partial equilibrium, the cross-sectional implications of the model can be used to discipline the parameterization. For example, the slope of the relationship between the probability of default and credit spreads is effectively pinned down by the cost of default and can be used to discipline parameter choices controlling the magnitude of the cost.

The simple model not only provides intuition about the mechanism of interest; in the following subsections I use it to demonstrate the key points made in this paper. In subsection 1.2.1, I introduce the simple model and outline how labor is distorted due to the presence of financial frictions (there are incomplete markets, as equity holders can only raise revenues with a one period state-uncontingent debt which can lead to costly default) and idiosyncratic shocks. In subsection 1.2.2, I characterize the distortion to labor and show how the cost of

default and the distribution of idiosyncratic shocks affect the distortion. Namely, I show that the distortion is roughly proportional in the cost of default and the marginal probability of default (which naturally depends on the distribution of idiosyncratic shocks). In subsection 1.2.3, I show the cross-sectional implications of this mechanism and argue that they can be used for calibrating key parameters, such as the cost of default, to ensure the model is consistent with the data.

1.2.1 Model Setup and Characterization

Consider a two-period model of operating leverage and financial frictions. In the first period, firms issue uncontingent debt and choose the amount of labor input l to hire. Equity holders receive cash flows from debt issuance in that first period, with the debt priced as the expected discounted present value of cash flows to debtholders. In the second period, a firm-level demand shock which augments revenues, z , is realized. If the firm does not default, equity holders receive revenues from production, zl^α , less the cost of labor, wl , and the amount of debt due, b . They also receive a continuation value V . If the firm defaults, equity holders receive nothing while debt holders receive net revenues from production, $zl^\alpha - wl$, as well as a portion $1 - c$ of the continuation value V . The parameter c represents how costly default is. Thus, if $c = 1$, then all remaining firm value above and beyond current operating profit is lost upon default. If $c = 0$, then default is costless. Firms default if $zl^\alpha - wl - b + V < 0$, that is if the value to equity holders in the second period (net cash flows from production less the debt payment plus the continuation value) are less than zero.²⁰ The default threshold can be characterized by the level of the shock z below which a firm defaults, denoted as $\bar{z}(l, b)$.

²⁰In the full model, default occurs if net cash flows from production less debt due are low enough such that revenues raised through debt issuance are insufficient to cover the firm's obligations. This is an endogenous threshold solved in an infinite-horizon model, so I use this simple default threshold for illustration here.

The revenues the firm receives for issuing debt are:

$$bq(l, b) = \underbrace{(1 - F(\bar{z}(l, b)))\beta b}_{\text{prob. no default}} + \underbrace{F(\bar{z}(l, b))\beta}_{\text{prob. default}} \left(\underbrace{\mathbb{E}[z|z < \bar{z}(l, b)] l^\alpha - wl}_{\text{exp. operating profit}} + \underbrace{(1 - c)V}_{\text{recovery}} \right), \quad (1.1)$$

where $F(z)$ is the cumulative density function of the shock z . The optimization problem governing the labor demand of equity holders can be expressed as follows:

$$\max_l \underbrace{bq(l, b)}_{\text{debt issuance}} + \underbrace{(1 - F(\bar{z}(l, b)))\beta}_{\text{prob. no default}} \left(\underbrace{\mathbb{E}[z|z > \bar{z}(l, b)] l^\alpha - wl}_{\text{exp. operating profit}} - \underbrace{b}_{\text{debt due}} + \underbrace{V}_{\text{cont. val}} \right). \quad (1.2)$$

Plugging (1.1) into (1.2) yields:

$$\max_l \beta \left(\underbrace{\mathbb{E}[z] l^\alpha - wl}_{\text{exp. operating profit}} + \underbrace{V}_{\text{cont. val}} - \underbrace{F(\bar{z}(l, b))}_{\text{prob. default}} \times \underbrace{cV}_{\text{default cost}} \right). \quad (1.3)$$

The first order condition of (1.3) with respect to firm employment is the following:

$$l^* = \left(\frac{\alpha E[z]}{w + V \times c \times f(\bar{z}(l^*, b)) \frac{\partial \bar{z}(l^*, b)}{\partial l^*}} \right)^{\frac{1}{1-\alpha}}. \quad (1.4)$$

As a contrast, if default is costless (corresponding to the assumptions in Modigliani and Miller (1958)), the first order condition reduces to the following:²¹

$$l_{MM}^* = \left(\frac{\alpha E[z]}{w} \right)^{\frac{1}{1-\alpha}} \quad (1.5)$$

The key difference between these two cases is the term $Vc f(\bar{z}(l^*, b)) \frac{\partial \bar{z}(l^*, b)}{\partial l^*}$ in the denominator. In this expression, Vc represents the cost of default, while $f(\bar{z}(l^*, b)) \frac{\partial \bar{z}(l^*, b)}{\partial l^*}$ represents the derivative of the probability of default with respect to firm's labor demand. This second term can be split into $f(\bar{z}(l^*, b))$, which represents the probability density func-

²¹(1.5) is also the first order condition which would arise if I relaxed market incompleteness and allowed the debt payments to be conditional on the realized shock z .

tion of the shock z at the default threshold, and $\frac{\partial \bar{z}(l^*, b)}{\partial l^*}$, which represents the effect of the firms labor decision on the default threshold. While V , c , and $f(\bar{z}(l^*, b))$ are non-negative by definition, $\frac{\partial \bar{z}(l^*, b)}{\partial l^*}$ depends on the parametrization as well as firm debt and labor. The following proposition formalizes the conditions under which this derivative is positive and its effect on firm employment:

Proposition 1.1. *The following are equivalent:*

1. $wl^* \geq \alpha(wl^* + b - V)$
2. $\frac{\partial \bar{z}(l^*, b)}{\partial l^*} \geq 0$
3. $Vcf(\bar{z}(l^*, b)) \frac{\partial \bar{z}(l^*, b)}{\partial l^*} \geq 0$ if $V, c, f(\bar{z}(l^*, b)) > 0$
4. $l^* \leq l_{MM}^*$ if $V, c, f(\bar{z}(l^*, b)) > 0$

Proof. See Appendix A.

The parameter restriction $wl \geq \alpha(wl + b - V)$ can be understood as requiring that the wage bill is at least as large as a fraction α of the fixed cost less continuation value. To violate this condition, firms need to have sufficiently high levels of debt relative to their operating costs. Intuitively, if the wage bill is a small part of the fixed cost, a high labor choice increases the potential revenues from production but does little to the total fixed cost due. If firms have sufficiently high debt, the firm will default anyway for low realizations of z , and a higher labor choice helps the firm generate enough revenues for higher realizations of z to avoid default in some cases. However, firms with sufficient levels of debt to violate the condition in Proposition 1.1 are rare, both in the full model and the data. For most firms, the prospect of risky default distorts firm employment decisions downwards.

1.2.2 Distortion to Firm Employment

In this subsection, I demonstrate how changes to the distribution of z , such as volatility or skewness shocks, can decrease employment. Further, I discuss why the magnitude of the

change in employment in response to these distributional changes depends crucially on the cost of default, c . Finally, I discuss why changing the cost of default in itself can reduce firm labor demand and why the combined effect of shocks to volatility and the cost of default can interact to generate large declines in employment.

In this simple model, the distribution of z affects only the term $f(\bar{z}(l^*, b))$ in (1.4).²² The following proposition demonstrates that if a threshold \bar{z} is sufficiently below (or above) the mean, $f(\bar{z})$ is increasing in volatility:

Proposition 1.2. *Let $f_1(z)$ and $f_2(z)$ denote normal probability density functions with identical means μ and variances σ_1 and σ_2 , respectively.*

1. *If $|\mu - \bar{z}| > \sigma_1$, then $\frac{\partial f_1}{\partial \sigma_1}(\bar{z}) > 0$*
2. *If $|\mu - \bar{z}| > \sigma_2$ and $\sigma_2 > \sigma_1$, then $f_2(\bar{z}) > f_1(\bar{z})$*

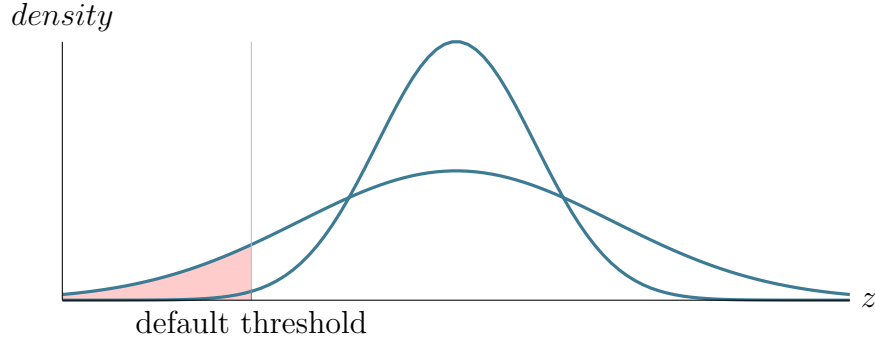
Proof. See Appendix A.

Figure 1.1 illustrates how changing the distribution of the shock z by increasing the variance can raise the probability density function of the shock at a given default threshold. Naturally, other changes to the distribution also change the probability density function and this term. Notably, a decrease in skewness of the distribution following the empirical findings of Bloom, Guvenen, and Salgado (2015) leads to substantially more dispersion of outcomes in the left tail than the right; this higher variance of the left tail can significantly raise the probability density function at the default threshold for many firms.

The cost of default, c , crucially affects the magnitude of the impact of volatility shocks (and other changes to the distribution of z) on firm employment and output. First, note that c multiplicatively enters the term which leads to the distortion from the case without the possibility of default in (1.4). In a world consistent with Modigliani and Miller (1958),

²²In the full model, where the continuation value and default threshold are endogenous, the distribution of z does affect other components in the problem. Quantitatively, however, the first-order effect still comes through $f(\bar{z}(l^*, b))$.

Figure 1.1: Effect of volatility shock on $f(\bar{z}(l^*, b))$



$c = 0$ and changes to the distribution of z have no impact on employment at all, as long as they do not affect the expected value of z . I can illustrate this by finding an expression for the distortion from the no-default case. (1.4) and (1.5) can be re-arranged to express the distortion from the case without default as the following:

$$\log(l^*) - \log(l_{MM}^*) = -\frac{1}{1-\alpha} \log\left(1 + \frac{cVf(\bar{z}(l^*, b)) \frac{\partial \bar{z}(l^*, b)}{\partial l^*}}{w}\right).$$

As the marginal cost of labor demand is primarily driven by wages, $\frac{cVf(\bar{z}(l^*, b)) \frac{\partial \bar{z}(l^*, b)}{\partial l^*}}{w}$ will be relatively small. Thus the following is a good approximation:

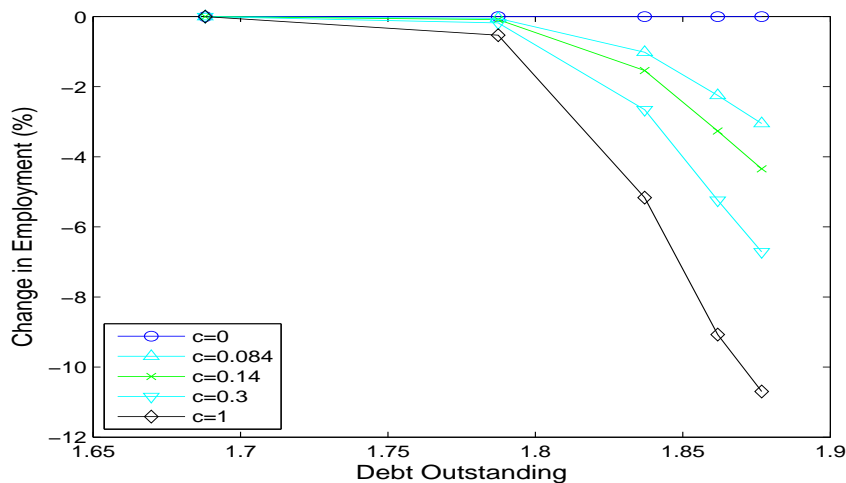
$$\frac{l^* - l_{MM}^*}{l_{MM}^*} \approx -\frac{c}{1-\alpha} \frac{Vf(\bar{z}(l^*, b)) \frac{\partial \bar{z}(l^*, b)}{\partial l^*}}{w}. \quad (1.6)$$

If the right hand side terms in (1.6) do not change very much in the firm's labor decision, then the percent deviation in labor choice from the costless default (Modigliani-Miller) case is roughly proportional in both how costly default is, c , as well as the probability density function at the default threshold, $f(\bar{z}(l^*, b))$. This suggests that the level of c will roughly proportionally affect the magnitude of the impact of shocks to the distribution, which change $f(\bar{z}(l^*, b))$, on employment. Also note that changing the level of c , all else fixed, will also affect firm employment – thus shocks to the cost of default can affect employment on their

own.

This can be numerically demonstrated in this simple model, which can be done with a simple parameterization. I normalize the wage w to 1, and parameterize z as a lognormal distribution with the mean chosen so that $\mathbb{E}[z] = 1$. I set $V = 1.81$ to hit the median ratio of market capitalization to sales, and set $\alpha = .6091$, the degree of decreasing returns to scale in ABK.²³ I allow c , the volatility of z , and the debt outstanding b to vary. The ranges of volatility and leverage are chosen to be reasonable given the distribution of firms sales growth and leverage. If default leads to firm death ($c = 1$), as in ABK, the induced distortions are much larger than if the higher estimates from the corporate finance literature ($c = 0.3$) are used. In this simple model, this will increase the magnitude of distortions by almost 70%. Figure 1.2 details the impact of a doubling in volatility of employment for a range of values of c .

Figure 1.2: Volatility shock



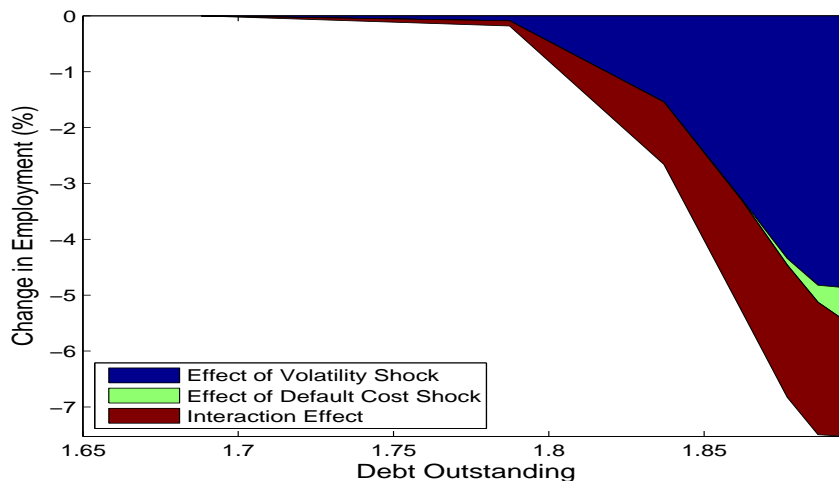
* Lines represent the implied effect of a doubling in the volatility of z on firm employment in the simple model, for a variety of values of c .

I can also demonstrate that there are significant interaction effects between shocks to the cost of default and volatility (or other distributional) shocks. For instance, if c and $f(\bar{z}(l^*, b))$ were each hit with s_1 and s_2 percent positive shocks, then (1.6) implies that they individually

²³This number accounts for both the decreasing returns to scale in their intermediate good production function and the extent of decreasing returns to scale inherent in the technology through which intermediate goods are aggregated into output.

would amplify the distortion in labor demand from the no-default case by roughly s_1 and s_2 percent, respectively (thus employment fall by roughly $s_1 s_0$ and $s_2 s_0$ percent, where s_0 is the percent distortion of labor demand before the shocks). If they arrived together they would amplify the distortion in labor demand by $s_1 + s_2 + s_1 s_2$ percent (and thus labor demand falls by $s_0(s_1 + s_2 + s_1 s_2)$ percent). The calibrated shocks to the equivalents of these objects during the recent recession are quite large; the increase in idiosyncratic volatility can lead to a more than doubling of $f(\bar{z}(l^*, b))$, and I also find that c increases substantially. Therefore, the additional term due to the interaction, $s_1 s_2$, may be quite large.²⁴ Figure 1.3 details the implied employment response to a doubling of volatility and a doubling of the cost of default (from $c = .15$ to $.3$), demonstrating that the interaction term is significant and potentially large.

Figure 1.3: Volatility shock



* Lines represent the implied effect of a doubling in the volatility of z or a doubling in the cost of default on firm employment in the simple model, denoting the individual and interaction effects of the two shocks.

²⁴The reason why the two shocks do not separate if looking at log changes in (1.4) is that w is quite large relative to $cVf(\bar{z}(l^*, b)) \frac{\partial \bar{z}(l^*, b)}{\partial l^*}$. If w were small, then the approximation in (1.6) would be poor and the interaction effects would be negligible.

1.2.3 Implications for the Joint Distribution of Credit Spreads, Leverage, and Observable Volatility

The above theory has several implications for the cross-sectional behavior of firms that can be used to discipline its application over the business cycle. A key property of the mechanism detailed above is that it is operational in partial equilibrium. In other words, a rise in firm business risk (changing the distribution $f(z)$) will distort a firm's behavior even if it happens only to that single firm alone. This implies that there is a link between the cross-sectional implications of this mechanism and its business cycle implications. Therefore, cross-sectional patterns, even outside of business cycle episodes, can be used to calibrate the model. This is useful, as many significant changes to conditions over the business cycle (such as changes to financial or product market conditions) are omitted from the model that likely affect observables over the business cycle and make calibration with data during recessions more challenging.

One of the key parameters for governing the strength of this operating leverage mechanism distorting firm labor is the costliness of default, c . In this section I demonstrate a key observable implication of this parameter: the derivative of credit spreads with respect to the probability of default is increasing in c . I argue that values of this parameter which correspond to the range of values found in the corporate finance literature (generally between 8.4-30% of firm value) generate more reasonable dynamics than the assumption of 100% of firm value lost.

I also show that the model has implications for the joint distribution of equity volatility, credit spreads, and employment. Namely, innovations to equity volatility should move together with credit spreads and predict employment declines.

1.2.3.1 Probability of Default and Credit Spreads

The probability of default, $F(\bar{z}(l, b))$, is the cumulative probability of receiving a shock below the default threshold implied by firm decisions. The credit spread faced by a firm can

be written as $\frac{b}{bq(l,b)} - \frac{1}{\beta}$. This can be expanded to find:

$$cs(b, l) = \frac{b}{\beta (b - (b + wl - V(1 - c)) F(\bar{z}(l, b))) + \beta \left(\int_0^{\bar{z}(l, b)} z dz \right) l^\alpha} - \frac{1}{\beta}. \quad (1.7)$$

Note that the credit spread is increasing in $F(\bar{z}(l, b))$, as it decreases the denominator in the expression of $cs(b, l)$ above. The two are positively correlated, as greater leverage or volatility of z both lead to an increased probability of default, and therefore a lower probability of full repayment of debt and thus higher credit spreads. The strength of the relationship between the two, however, depends crucially on c . When c is high, default is very costly and recoveries conditional on default are low. Thus, as the probability of default increases, credit spreads increase substantially. Conversely, if c is low, debtholders are able to recover most of the firm's value (and more of the debt face value) upon default, and thus credit spreads rise less in the probability of default. Assuming that l and b remain fixed, I can characterize the slope of credit spreads relative to default probability as the following:

$$\frac{\partial cs}{\partial P(def)} = \frac{b\beta (b + wl - V) + bVc}{\beta^2 \left((b - (b + wl - V(1 - c)) F(wl^{1-\alpha} + (b - d)l^{-\alpha})) + \left(\int_0^{\bar{z}(l^*, b)} z dz \right) l^\alpha \right)^2}.$$

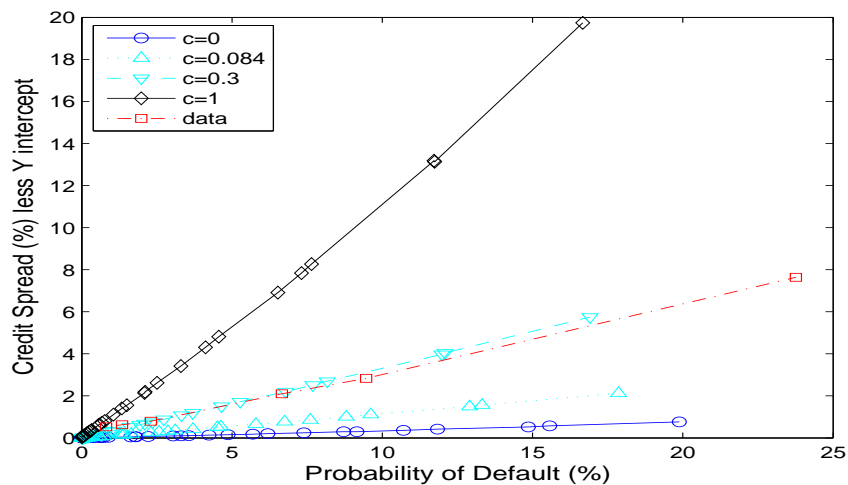
In the above equation, increasing the cost of default, c , increases the slope for reasonable parameterizations, as increasing the term in the numerator dominates the term in the denominator. In practice (allowing l , b , and the volatility of z to vary), in the simple model the relationship between the probability of default and credit spreads is approximately linear for reasonable probabilities of default.

Figure 1.4 shows how credit spreads vary with the probability of default (due to heterogeneity in both leverage and asset volatility) for a range of values for c .²⁵ Note that the slope of credit spreads with respect to default probability is increasing in c — it is near zero for $c = 0$, and in excess of one for $c = 1$. Also plotted on the figure are credit spreads and

²⁵I follow the parameterization outlined above for V , α , and $f(z)$. Volatility and debt outstanding are both allowed to vary — they are the reasons firms have differential credit spreads and default probabilities.

one-year realized default probabilities for speculative grade firms by rating class.²⁶ The teal lines ($c=0.084, 0.3$) represent the range of estimates for the cost of default from the corporate finance literature. Note that, as expected, the relationship in the data lies between these two lines.

Figure 1.4: Credit spreads vs. probability of default



* The red dashed line reflects data from speculative grade firms, by rating class, for the time period 1984-2012 as reported by Standard and Poor's. The remaining lines are implied by the simple model, for varying values of c . All series have the y-intercept from their line of best fit subtracted from them so the slopes can be easily compared.

What this plot suggests is that the assumption that default leads to firm death ($c = 1$) leads to much too strong of a relationship between default probabilities and credit spreads compared to the data. On the other hand, using c in a range of values found in the corporate finance literature, where default is much less costly, leads to a relationship much more in line with the data. As the choice of c is very important for the magnitude of the distortion to employment, this has significant implications for the effect of volatility shocks over the business cycle.

Correcting for Risk Premia I also account for the potential bias of risk premia on this relationship. Credit spreads are often decomposed into two parts: compensation for default

²⁶I consider only speculative grade firms because investment grade firms default incredibly rarely within one year, and when they do it is almost always during a recession. Speculative grade firms, on the other hand, have sufficient observed short-term default rates outside of recessions to allow for meaningful comparison across rating classes.

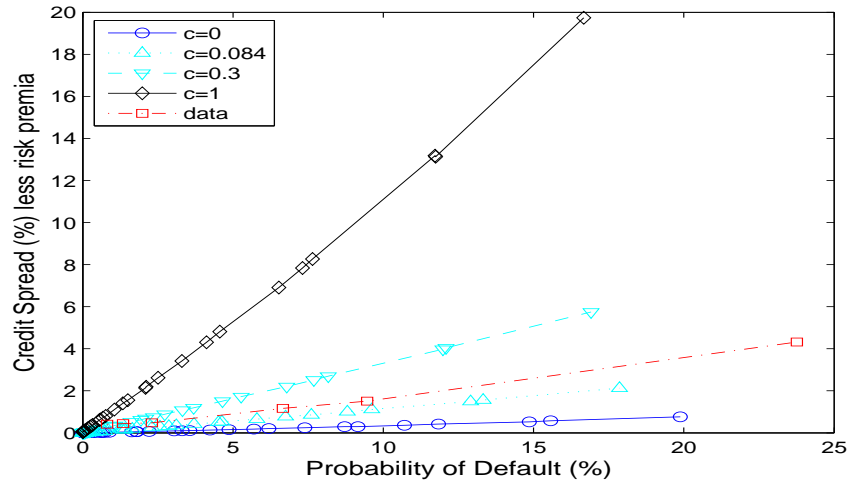
losses and the risk premium. The simple model corresponds only to the first part. Cross-sectional patterns in risk premia may affect this cross-sectional relationship and potentially bias calibration strategies reliant on it. A number of recent studies have estimated cross-sectional patterns in risk premia by computing excess bond returns, namely Choi and Kim (2015) and Chordia, Goyal, Nozawa, Subrahmanyam, and Tong (2015). Their results indicate that while firms with a low probability of default indeed do have a greater *proportion* of their credit spreads explained by risk premia, the *level* of the risk premium is increasing as firms face a higher risk of default. Both of these studies find that excess bond returns are decreasing in firm credit ratings (a worse credit rating implies higher risk premia). Chordia et al. (2015) find that mean excess bond returns are decreasing in distance to default, a common proxy for default risk. This implies that firms with lower distance-to-default (and thus higher default probabilities) have higher risk premia.

These results suggest that the model-implied relationship between the probability of default and credit spreads would be steeper if we were to perfectly account for cross-sectional patterns in risk premia. This will reduce model-implied estimates of c . For the sake of tractability, I perform this correction using the data. I compute the slope of the risk premium with respect to default probability using estimates by rating class of excess bond returns from Choi and Kim (2015) and historical default probabilities from Standard and Poor's. I find that the slope of the annualized risk premium with respect to the annual probability of default is 0.14.²⁷ This can be interpreted as the following: C/CCC rated firms, with an annual default rate of roughly 20–25%, should have annual risk premia of around 3% greater than firms with no probability of default. Figure 1.5 demonstrates the relationship between default probability and credit spreads less risk premia. The range of estimates discussed earlier, $c \in [.084, .3]$, still bounds the relationship in the data from below and above. With this correction, the lower end of this range corresponds better to the data, whereas without

²⁷I compute the slope using default probabilities and excess bonds returns for AAA and C/CCC rated firms.

it the upper end of this range matches the data better.

Figure 1.5: Credit spreads vs. probability of default, controlling for risk premia



* The red dashed lines reflects data from speculative grade firms, by rating class, for the time period 1984-2012 as reported by Standard and Poor's, less a risk premia effect implied by estimates from Choi and Kim (2015) by ratings class. The remaining lines are implied by the simple model, for varying values of c . All series have the y-intercept from their line of best fit subtracted from them so the slopes can be easily compared.

1.2.3.2 Innovations to Asset Volatility, Credit Spreads, and Employment

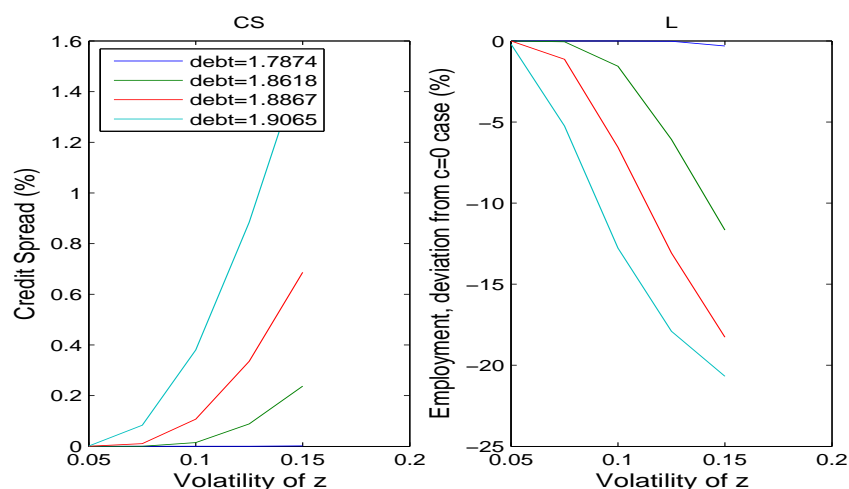
The model also has implications for how innovations to the level of firm fundamental idiosyncratic risk (the volatility of z) affects asset volatility, credit spreads, and firm's employment decisions. In the model, an increase in the volatility of z increases the volatility of the value of the firm's assets and the probability of default. This raises credit spreads and decreases firm employment decisions.²⁸ Therefore the model has two clear predictions: First, innovations in measures of firm volatility should co-move with credit spreads on the firm's debt. Second, innovations in measures of firm volatility should predict (negative) innovations in firm employment.

Figure 1.6 shows the implied credit spreads and labor choice generated in the simple model, conditional on the amount of debt due, for varying levels of volatility of z . The simple model is parameterized as before. Note that credit spreads are increasing in volatility, but are higher if the debt due is greater. The firm's labor choice is decreasing in volatility, and

²⁸In (1.4) and (1.7), this manifests itself by raising $f(\bar{z}(l^*, b))$ and $F(\bar{z}(l^*, b))$.

depends on the indebtedness of the firm. Figure 1.2 shows the implied impact of volatility doubling on firm employment in the simple model for varying values of c . It is clear that higher c leads to a greater decline in firm employment in response to volatility shocks. Note that this effect should exist in both the cross-section and over the business cycle.

Figure 1.6: Effect of volatility



* These results are generated from the simple model, with $c = 0.3$.

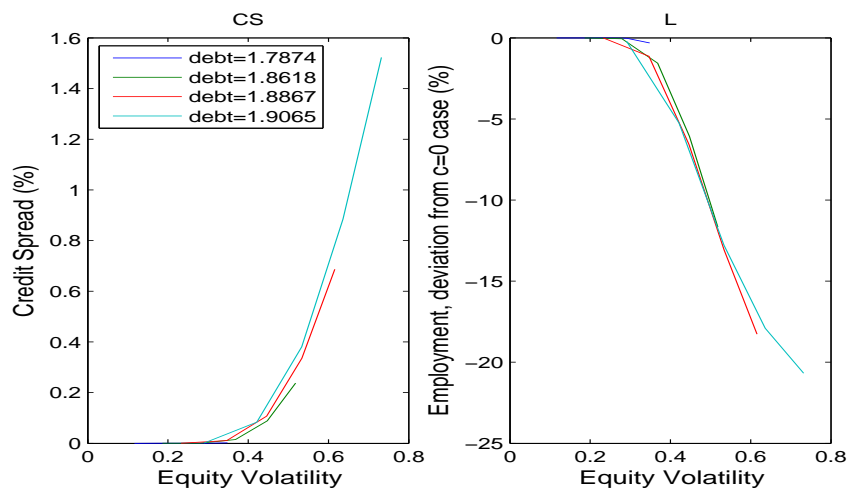
Implications for Financial Market Measures of Volatility As the volatility of firm fundamentals is difficult to measure at the firm level (due to the low frequency of observations), it is common to use financial market measures of volatility, particularly measures of realized or expected equity volatility.²⁹ The draw of z , reflecting stochastic firm fundamentals or demand, affects the market value of claims on the firm. Therefore, the volatility of z should be reflected in the volatility of equity returns. I can easily compute the model-implied equity volatility in this two period case introduced above, or a measure of instantaneous volatility implied by a Merton model related to this two-period case. Both expressions are functions of not only the volatility of z , but also capital structure considerations. If a firm is highly leveraged, a small volatility of z can imply larger volatility of equity returns because small movements in z can have large percent changes in the value of assets less liabilities. The

²⁹For instance, see Gilchrist et al. (2014).

presence of default itself also affects the volatility of equity returns. Appendix A outlines the computation of equity volatility in this simple model.

Relationship Between Equity Volatility, Credit Spreads, Employment Figure 1.7 shows the relationship between equity volatility, credit spreads, and employment in the simple model.³⁰ This is quite similar to the relationship with theoretical volatility — the model predicts that equity volatility will co-vary with credit spreads and predict employment losses.

Figure 1.7: Volatility shock



* These results are generated from the simple model, with $c = 0.3$.

This is a key qualitative prediction of the model I test in the data. As the relationship between equity volatility and the volatility of z relies crucially on assumptions about the variance of the continuation value and its correlation with z , I do not use the magnitude of this relationship to calibrate the model.³¹ Instead, I rely on the relationship between the probability of default and credit spreads, as well as the relationship between innovations to credit spreads and innovations to year-ahead employment.

³⁰The relationship here is plotted for equity volatility from a Merton model, however, the relationship is similar if I use the full model or measures of the volatility of the value of the firm as a whole.

³¹There is likely significant variation in equity returns driven by factors other than near-term shocks. This could be addressed in the model by making V stochastic (which it is in the full model), but its correlation with z would affect how much equity volatility responds to changes in the volatility of z and thus these cross-sectional relationships.

1.3 Empirical Evidence

In this section, I describe the dataset and the measures of volatility I derive from asset prices, and document key aspects of the cross-sectional relationship between volatility, credit spreads, and firm labor decisions. The main findings of this section are the following: Cross-sectional evidence suggests that an increase in measures of firm idiosyncratic risk are significantly correlated with changes in credit spreads, and are predictive of changes in firm employment. This is consistent with the main qualitative predictions of the model for the cross-section. I also document how predictive innovations in credit spreads are of employment, as this relationship can be compared to the (full) model.

1.3.1 Data and Measurement

I use data on public firm equity prices, financial statements, analyst sales forecasts, and credit spreads from 1984-2013. Equity price data is drawn from the Center for Research in Security Prices (CRSP), accounting data from Compustat, and data on credit spreads from the Lehman-Warga (1984-1998) and Merrill Lynch (1997-2013) databases.

1.3.1.1 Measuring Volatility

A key empirical challenge is constructing a panel dataset of measures of the time-varying idiosyncratic risk at the firm-level. The primary measure I consider is a measure of idiosyncratic equity volatility following Gilchrist, Sim, and Zakrajsek (2014), though I do consider alternative measures for robustness.

Measuring Volatility from Asset Returns Asset prices, in principle, should summarize all of the publicly available information relevant for a firm's future prospects. This corresponds closely to theory — a stochastic volatility shock is an innovation in the distribution of non-forecastable shocks affecting future firm cash flows. I thus follow the literature and

use a measure of the idiosyncratic component of daily return volatility as one of my measures of volatility.

I use a standard linear factor model to remove the portion of excess returns attributable to aggregate factors:

$$R_{it_d} - r_{t_d}^f = \alpha_i + \beta'_i f_{it_d} + u_{it_d}. \quad (1.8)$$

$R_{it_d} - r_{t_d}^f$ is the daily excess equity return on stock i in trading day d of year t . f_{it_d} corresponds to a vector of aggregate factors — in this case a four-factor model consisting of the Fama and French (1992) factors along with a momentum measure. I use u_{it_d} to denote the variation in excess returns beyond the four aggregate factors, for reasons one can consider idiosyncratic. The residual of running an OLS regression on (1.8), \hat{u}_{it_d} , is the idiosyncratic component of excess returns, and greater variation in this corresponds to higher levels of idiosyncratic volatility. The standard deviation of this component can thus be computed over an annual frequency as:

$$\sigma_{it}^{EI} = \sqrt{\frac{250}{D_t} \sum_{d=1}^{D_t} (\hat{u}_{it_d} - \bar{u}_{it})^2}. \quad (1.9)$$

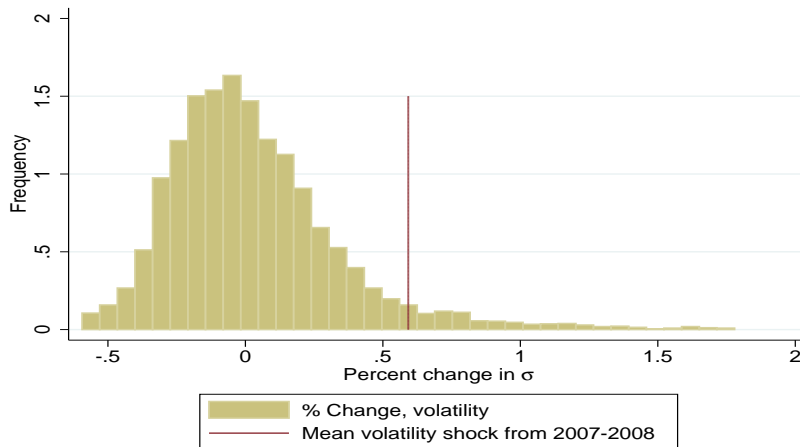
\bar{u}_{it} denotes the average of \hat{u}_{it_d} over the year, and D_t the number of trading days. Further details of this measure can be found in Gilchrist, Sim, and Zakrajsek (2014).

As measures of equity volatility are affected by leverage, I also compute a measure of asset volatility from returns. I follow a procedure consistent with Bharath and Shumway (2008) and Gilchrist and Zakrajsek (2012) in measuring firm asset volatility, and further derive a measure of idiosyncratic asset volatility, detailed in Appendix B.

Measure of Volatility from Sales Growth To allay concerns about variation in financial market returns representing factors other than changes in firm fundamentals or expectations of them, I construct measures of sales growth volatility over controls as a measure of funda-

mental volatility. This data is available at no more than a quarterly frequency, so I use a GARCH process to estimate the volatility of this process. I consider only firms with at least 30 quarters of consecutive sales data. I first remove all forecastable variation in firm growth rates by running a regression of firm sales growth on lagged firm fundamentals (investment rate, size, profitability) with time-industry and firm-season fixed effects.³² The residuals of this regression represent the variation in firm sales growth beyond those explained by predictive factors, and also help eliminate any seasonal trends, including those which exist only for given industries or firms. I then run a GARCH(1,1) procedure to estimate a volatility series from the time series of residuals for each firm. I test each firm’s residual series individually for ARCH effects, and keep only those which reject the null (support the presence of ARCH effects). This results in a panel dataset of quarterly firm-level volatility.

Figure 1.8: Cross-section of shocks to asset volatility during non-recession years



*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$.

1.3.1.2 Other Firm Measures

Firm measures of leverage, profitability, and Tobin’s Q are all computed from Compustat annual data. I define the book value of debt as the sum of short term debt outstanding

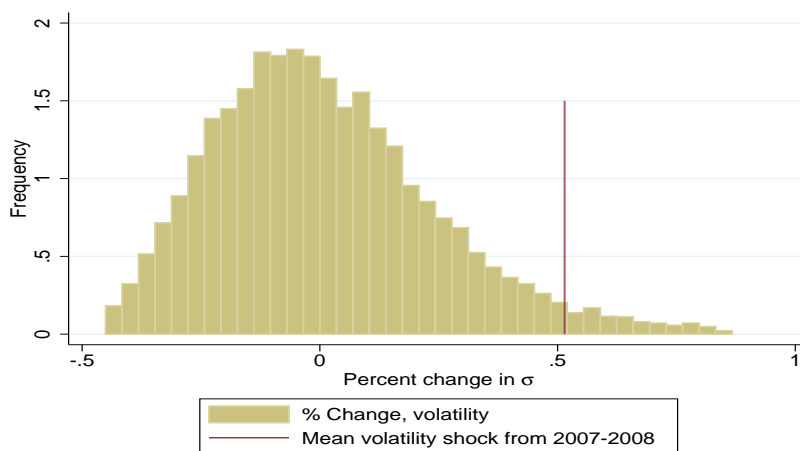
³²By time-industry fixed effects I mean a dummy for every 1 digit sic code for every year-quarter. By firm-season fixed effects I mean a fixed effects factor for every firm in a given season/quarter (i.e. Microsoft quarter 1).

and $\frac{1}{2}$ of long term debt outstanding (following Gilchrist and Zakrajsek (2012)). Leverage is computed as the ratio of the book value of debt and the sum of the value of equity and the book value of debt. Profitability is defined as the ratio of operating profits to the book value of assets (from the balance sheet). Tobin's Q is defined as the ratio of the sum of the value of equity and the book value of debt over the book value of assets (from the balance sheet). I compute credit spreads from the databases following the procedure outlined in Gilchrist et al. (2014). I take employment from Compustat as well, and allay concerns about measurement error by replicating my results with sales.

1.3.2 Empirical Results

1.3.2.1 Cross-section of Innovations to Volatility

Figure 1.9: Cross-section of shocks to idiosyncratic equity volatility during non-recession years

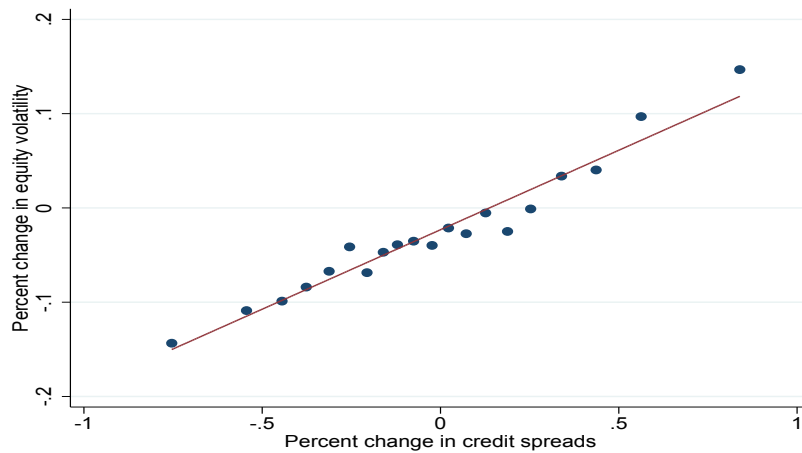


*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$.

There is a large dispersion of innovations (year-over-year percent changes) to firm-level measures of volatility, even during non-recession years. The magnitude of this dispersion is significant compared to the size of the average innovation in the equivalent measures of volatility during the great recession of 2007-2009. Figures 1.8 and 1.9 show histograms of the frequency of innovations to measures of volatility, measured as year-over-year percent

changes, during non-recession years. Overlaid on the histogram is the average year-over-year percent change that occurs during the rise in volatility during the great recession.³³ The dispersion in innovations to my measures of volatility outside of recessions is substantial. In terms of the volatility of assets, the mean increase in volatility in 2007/2008 represents approximately two standard deviations of the non-recession distribution. Thus, while the mean innovation to volatility during the crisis is clearly large, the cross-sectional variation during non-recession years is comparably large enough that cross-sectional relationships are empirically relevant for understanding the effect of volatility shocks on firm decisions.

Figure 1.10: Innovations in credit spreads vs. volatility



*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

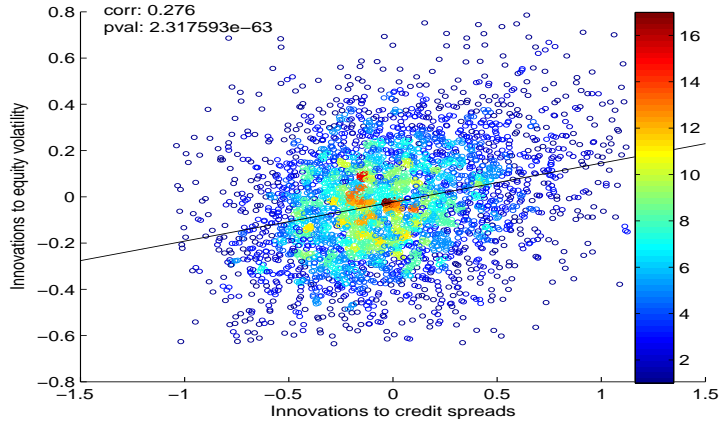
1.3.2.2 Relationship to Credit Spreads

I now look at the response of credit spreads to innovations to financial market measures of volatility. If greater volatility in asset prices reflects increases in volatility about future firm prospects, it will increase the probability of default and raise credit spreads.

In the data, credit spreads increase significantly in response to firm level innovations in volatility, suggesting that the financial market measure of volatility reflects economically

³³The change from 2007-2008 is used because it is by far the greatest average annual increase in volatility that occurs in the time series.

Figure 1.11: Innovations in credit spreads vs. volatility



*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$. The color denotes the number of observations at a given point.

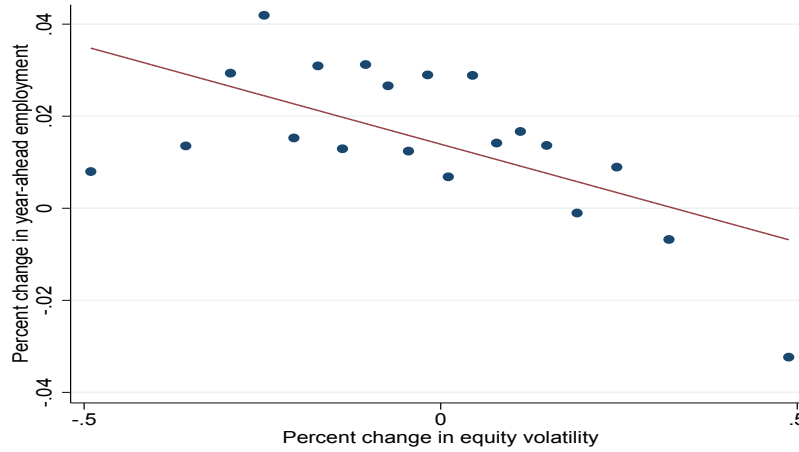
significant changes in volatility. Figure 1.10 shows this relationship of year-over-year innovations of credit spreads against idiosyncratic equity volatility, for non-recession years.³⁴ The observations are binned by the x-axis since there are thousands of observations. Figure 1.11 shows the entire scatter in a heatmap and a line of best fit confirming the positive correlation. Table 1.1 shows that this positive relationship is robust to controlling for firm characteristics which are known determinants of credit spreads, specifically innovations in leverage, profitability, and equity values. This is consistent with the findings of Gilchrist, Sim, and Zakrajsek (2013), who find that an increase in idiosyncratic volatility is associated with a significant widening of credit spreads. They also find that increases in idiosyncratic volatility also has statistically significant effects on capital expenditures. I therefore assess the impact of idiosyncratic volatility on employment, my corresponding variable of interest.

1.3.2.3 Relationship with Firm Employment

I evaluate the magnitude of a decrease in firm employment predicted by increases in the level of idiosyncratic volatility facing the firm. I find that the relationship is negative and robust,

³⁴The relationship between idiosyncratic asset volatility and credit spreads is very similar.

Figure 1.12: Innovations in firm employment vs. volatility



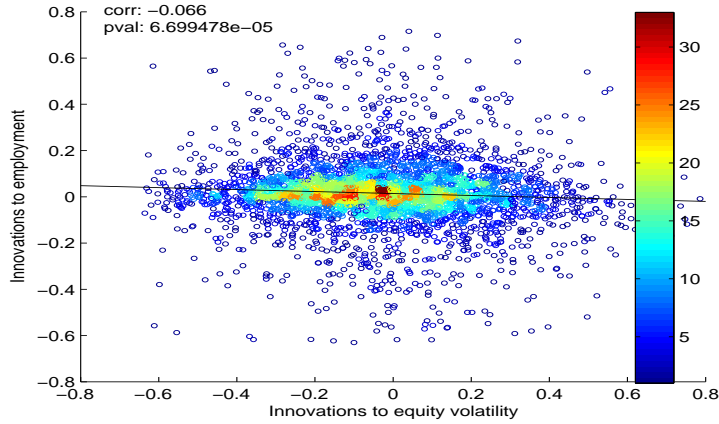
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

though the magnitude of the effect is modest. Figure 1.12 shows the relationship between the year-ahead year-over-year percent change in firm employment against year-over-year change in idiosyncratic equity volatility, for non-recession years. Observations are binned by idiosyncratic equity volatility as there are thousands of observations. Figure 1.13 shows the entire scatter in a heatmap, which confirms the trend. This relationship remains negative and significant (though the magnitude is relatively modest, with a doubling in volatility implying a 2-3% decrease in employment), even after controlling for innovations in firm characteristics, such as profitability, leverage, and Tobin's Q, or year/industry fixed effects. Table 1.2 shows the results of the regression:

$$\Delta emp_t = \beta_0 + \beta_1 \Delta \sigma_{it-1}^{AI} + \gamma X_{i,t-1} \quad (1.10)$$

where $X_{i,t-1}$ represents the controls mentioned above. This illustrates the robustness of the magnitude and the sign of the decrease in employment associated with an increase in firm idiosyncratic risk. To alleviate concerns about measurement error in Compustat employment, Figure 1.14 shows that the relationship between innovations to equity volatility

Figure 1.13: Innovations in firm employment vs. volatility



*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$. The color denotes the number of observations at a given point.

and year-ahead sales growth is quite similar in both direction and magnitude.

1.3.2.4 Relation to Model

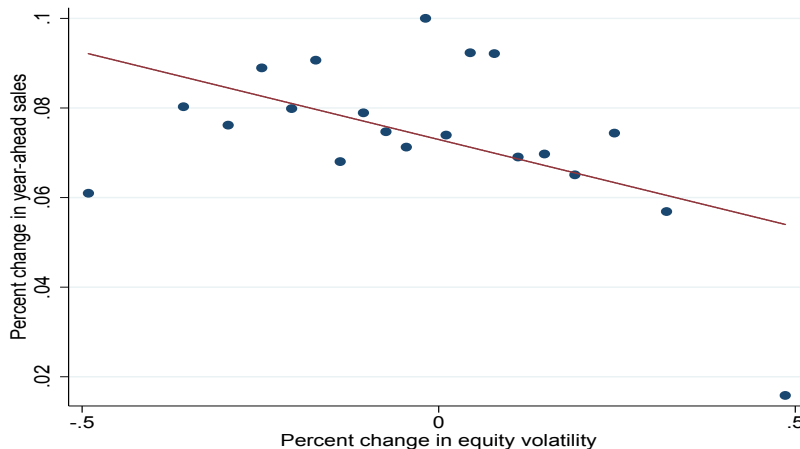
The relationships documented above agree with the qualitative predictions of the model, but comparing them quantitatively is difficult. The true volatility of firm processes are not observed in the data, and the financial market proxies from realized returns are difficult to convincingly characterize in the model.³⁵ Therefore, I turn to a relationship which measures the extent to which default risk affects employment: how predictive innovations in credit spreads are to changes in employment. Figure 1.15 documents this relationship for non-recession years, binning firms by credit spreads; Figure 1.16 displays the entire scatter. There is a significant and robust (to standard controls) negative relationship between innovations to credit spreads and employment.³⁶ This is consistent with the work of Benmelech, Bergman, and Seru (2013), who document that financial constraints play a role in reducing firm (and

³⁵For instance, the variance of realized equity returns may be affected by liquidity considerations or other financial market changes not captured in the model.

³⁶Figure 1.17 documents that this relationship is similar when sales growth is used instead of employment growth.

aggregate) employment.

Figure 1.14: Innovations in firm sales vs. volatility



*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

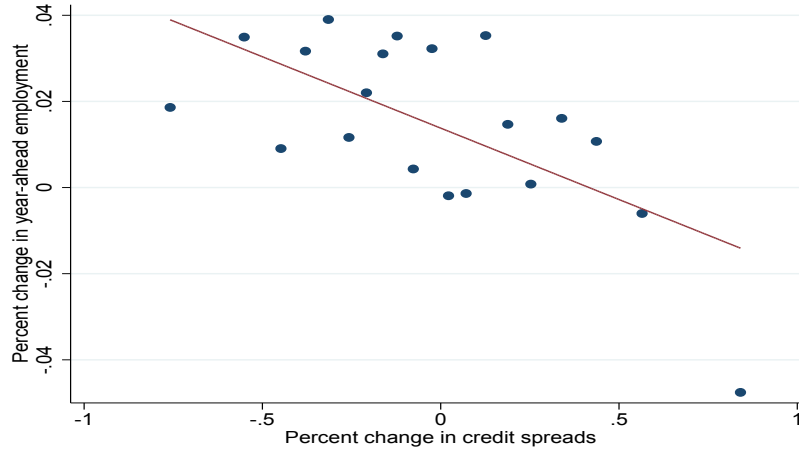
1.4 Model

To evaluate the role of volatility shocks in driving aggregate dynamics, I use a dynamic model with financial frictions and variation in the level of volatility at the firm level. The model is a generalized version of ABK.³⁷

My model differs from ABK in two primary ways. First, I allow for the extent of default costs to vary, controlled by a parameter c . Second, I add heterogeneity in both the level and changes in firm idiosyncratic volatility. Specifically, I let innovations to the volatility of firm specific demand z , vary across firms. This additional heterogeneity allows me to generate cross-sectional implications which can reproduce patterns in the data and be used for calibration.

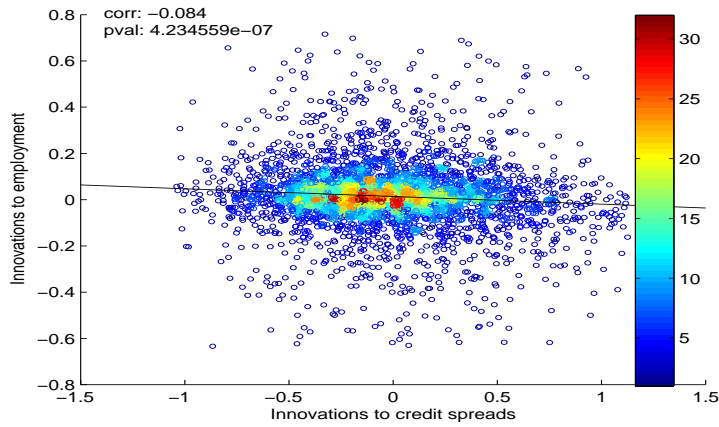
³⁷ABK have, as of October 2015, presented an updated version of their paper, which include some changes to the model. The now use a different agency problem to motivate firms to hold debt and consider the risk-free interest rate to be exogenous. However, the mechanism through which volatility shocks affect employment and the main business cycle results are qualitatively unchanged.

Figure 1.15: Innovations in firm employment vs. credit spreads



*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

Figure 1.16: Innovations in firm employment vs. credit spreads



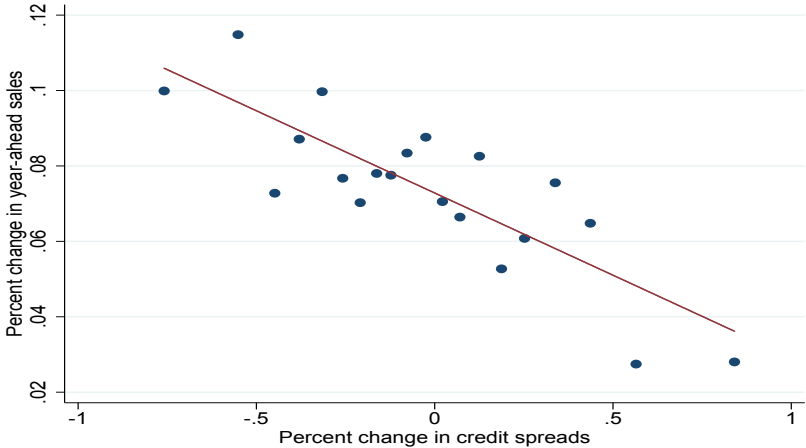
*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$. The color denotes the number of observations at a given point.

The model has a continuum of final good firms, intermediate good firms, financial intermediaries, and households. Intermediate good firms produce differentiated goods using labor which are aggregated into the final good. Final good firms are competitive and produce the final good from intermediate goods. The production technology of final good firms is subject to idiosyncratic shocks augmenting the usage of individual intermediate goods.

These shocks need not be technological: as in ABK they can be interpreted as idiosyncratic *demand* shocks. The volatility of these demand shocks is stochastic. Competitive financial intermediaries lend to the intermediate good firms using standard defaultable debt contracts. Households own all firms and financial intermediaries and pay lump-sum taxes. They make decisions over consumption and leisure, which they both value.

The timing of the model is as follows. First, households make their labor supply decision and firms make their labor demand decisions, and the wage rate is set to clear the labor market. Then, shocks to demand and volatility are realized. Then intermediate good firms produce with the labor choice they made the previous period and the demand shock they received and sell the goods to final good firms, pay workers, choose whether to repay the financial intermediaries, and choose labor for the next period as well as issue additional debt. The final good firms buy the intermediate goods and aggregate them into the final good, which is either consumed or used by entering firms to pay the cost of entry. Potential entrants decide whether to pay the entry cost (in units of the final good) and enter. Households receive all of the net cash flows from intermediaries and firms, and consume the final good.

Figure 1.17: Innovations in firm sales vs. credit spreads



*Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1}-x_t}{\frac{1}{2}(x_{t+1}+x_t)}$. Observations are binned into 5-percentile bins by the x-axis, with a point in the scatter reflecting the mean of those observations in both the x and y axis variables.

1.4.1 Physical Environment

Intermediate good firms produce differentiated goods with technology $y_{it} = l_{it}^\alpha$, where l_{it} indexes the labor stock of firm i at time t . The price that they receive for these goods, as well as how important these goods are for the final good, depend on an idiosyncratic demand shock z_{it} . z_{it} follows a Markov process with transition function $\pi_z(z_{it}|z_{t-1,i}, \sigma_{it})$ and depends on the lagged value of z_{it} as well as the level of stochastic volatility the firm faces, σ_{it} . I do not assume σ_{it} must be equal for all firms — it is a function of an idiosyncratic component, σ_{it}^{id} , and an aggregate component, σ_t^{ag} . Both of these follow Markov processes, with transition functions $\pi_{\sigma^{id}}(\sigma_{it}^{id}|\sigma_{t-1,i}^{id})$ and $\pi_{\sigma^{ag}}(\sigma_t^{ag}|\sigma_{t-1}^{ag})$.

I denote the idiosyncratic state of a firm as $x_{it} = \{z_{it}, l_{it}, b_{it}, \sigma_{it}^{id}\}$, where l_{it} is the employment of the firm and b_{it} its borrowing, and the aggregate state of a firm as $S_t = \{\sigma_t^{ag}, S_{bt}\}$, where S_{bt} is the beginning-of-period aggregate state prior to the realization of shocks. The beginning-of-period aggregate state, $S_{bt} = \{\sigma_{t-1}^{ag}, \Upsilon_t, B_t\}$, is summarized by the aggregate level of volatility before the shock, σ_{t-1}^{ag} , the distribution of firms $\Upsilon_t(x_t)$, and the household's wealth, B_t .

Final good firms buy the intermediate goods and produce the final good with technology:

$$Y_t = \left(\int z(x_{it})y(x_{it})^{\frac{\gamma-1}{\gamma}} d\Upsilon_t(x_{it}) \right)^{\frac{\gamma}{\gamma-1}} \quad (1.11)$$

where $z(x_{it})$ and $y_t(x_{it})$ denote the demand shock and intermediate good production of a firm with state x_{it} .

1.4.2 Final Good Firm's Problem

The final good is used for consumption or to pay the entry cost. Competitive final good firms choose their purchase of intermediate goods to maximize profits, $Y_t - \int p_t(x_{it})y_t(x_{it})d\Upsilon_t(x_{it})$, subject to (1.11). This yields the relative price of the intermediate firm good x_{it} relative to

the aggregate price index:

$$p_t(x_{it}) = z(x_{it}) \left(\frac{Y_t}{y_t(x_{it})} \right)^{\frac{1}{\gamma}}. \quad (1.12)$$

1.4.3 Intermediate Good Firm's Problem

Intermediate good firms make labor, debt issuance, dividend, and entry decisions. They are restricted from paying non-negative dividends, and if they would be compelled to do so they default and any cash flows or remaining firm value goes to the financial intermediaries. It is useful to separately consider the problems of incumbents and entrants.

1.4.3.1 Incumbent Firms

Incumbents enter the period with a stock of labor and debt, l_{it} and b_{it} , and receive their new demand shock and idiosyncratic component of firm-level volatility, z_{it} and σ_{it}^{id} . They choose new amounts of labor and debt issuance, l'_{it} and b'_{it} , as well as dividends d_{it} . The dividend for an incumbent firm can be written as:

$$d_t = p_t(x_{it})l_{it}^\alpha - W(S_{bt})l_{it} - b_{it} + b_{t+1,i}Q(z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t) \quad (1.13)$$

where $W(S_{bt})$ is the wage. The cash flow from debt issuance depends on the firm idiosyncratic shocks, z_{it}, σ_{it}^{id} , as well as firm decisions, $l_{t+1,i}, b_{t+1,i}$, and the aggregate state, S_t . The price of state-uncontingent debt is $Q(z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t)$.

Firms seek to maximize the discounted present value of dividends, modified by a Jensen effect parameter κ . This is motivated by ABK, referring to Jensen (1986), as the following: Managers have incentives to spend built up cash by the firm in ways that benefit themselves at the expense of shareholders, and thus shareholders change management incentives to pay out dividends rather than retain them. The value function of intermediate good firms can

thus be expressed as the following:

$$V(x_{it}, S_t) = \max_{l_{t+1,i}, b_{t+1,i}} \begin{cases} \kappa d_t + (1 - \kappa) E_t [P_{S_t, S_{t+1}} V(x_{t+1,i}, S_{t+1}) | x_{it}, S_t, l_{t+1,i}, b_{t+1,i}] & d_t \geq 0 \\ 0 & d_t < 0 \end{cases}$$

such that (1.13) holds. Here, $P_{S_t, S_{t+1}}$ denotes the stochastic discount factor implied by the consumer's problem. If firms cannot pay a non-negative dividend, they default and are seized by creditors, and the equity holders walk away with no cash.

1.4.3.2 Entrants

Firms can enter by paying an entry cost ξ in terms of the final good. This cost is paid by household in exchange for equity (this is the only equity issuance allowed in the model), and they enter with no debt and idiosyncratic states x_e and σ_e^{id} . Thus the free entry equation can be expressed, modified by the Jensen effect parameter, as the following:

$$0 = \max_{l_{t+1,e}} - \kappa \xi + (1 - \kappa) E_t [P_{S_t, S_{t+1}} V(x_{t+1,i}, S_{t+1}) | S_t, l_{t+1,e}].$$

1.4.3.3 Distribution of firms

The distribution of firms, $\Upsilon_t(x)$, depends on firm leverage, employment, default, and entry decisions. The law of motion for it can be characterized as the sum of several components:

First, incumbent firms which do not default:

$$\Upsilon_t^{ND}(z', l', b', \sigma'^{id}) = (1 - br(z', l', b', \sigma'^{id}, S_t)) \sum_x P(z', \sigma'^{id} | x, S_{t-1}) I(l', b', S_{t-1}, x) \Upsilon_{t-1}(x),$$

where $br(z', l', b', \sigma'^{id}, S_t)$ and $I(l', b', S_{t-1}, x)$ are dummy variables. $br(z', l', b', \sigma'^{id}, S_t)$ is equal to 1 if a firm defaults conditional on realized shocks z', σ'^{id} , choices l', b' and aggregate state S_t . $I(l', b', S_{t-1}, x)$ is equal to 1 if $l' = l^*(x, S_{t-1})$ and $b' = b^*(x, S_{t-1})$.

Second, incumbent firms which do default (but survive default with probability $1 - c$):

$$\Upsilon_t^D(z', l', 0, \sigma'^{id}) = (1 - c) (br(z', l', b', \sigma'^{id}, S_t)) \sum_x P(z', \sigma'^{id} | x, S_{t-1}) I(l', b', S_{t-1}, x) \Upsilon_{t-1}(x).$$

Finally, entering firms:

$$\Upsilon_t^e(z', l', 0, \sigma'^{id}) = e_t P(z', \sigma'^{id} | x_e, S_{t-1}) \mathbb{1}_{\{l' = l_e^*(S_t)\}}.$$

Using these components, the law of motion for the distribution of firms can be written as the following:

$$\Upsilon_t(x) = \Upsilon_t^{ND}(x) + \Upsilon_t^D(x) + \Upsilon_t^e(x)$$

1.4.4 Financial Intermediation

Financial intermediaries lend non-contingent debt to the firms and borrow state-contingent debt from the households. They are perfectly competitive and thus the price of outstanding debt $Q(z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t)$ is pinned down by the following zero profit condition:

$$Q(z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t) = E \left[P_{S_t, S_{t+1}} \left(1 - br(x_{t+1,i}, S_{t+1}) \right. \right. \\ \left. \left. + (1 - \kappa) (br(x_{t+1,i}, S_{t+1})) rr_{t+1}(x_{t+1}, S_{t+1}, b_{t+1,i}) \mid z_{it}, \sigma_{it}^{id}, l_{t+1,i}, b_{t+1,i}, S_t \right) \right].$$

I assume here that if firms default, the resulting cash flows to debtholders are discounted at the discount rate of equity holders, $(1 - \kappa)\beta$. This is micro-founded by specifying that the remaining value of the firm is contracted to be sold to equity holders (or the managers of these firms) in case of default. If I did not make this assumption, then firms would have a perverse incentive to default, as cash flows conditional on defaulting would be discounted at a lower rate than cash flows conditional on surviving.

The recovery rate on debt in the event of default is denoted $rr_{t+1}(x_{t+1}, S_{t+1}, b_{t+1,i})$ and

is defined as follows:

$$rr_{t+1} = \max \left\{ \frac{p_{t+1}(x_{t+1,i})l_{t+1,i}^\alpha - W(S_{bt+1})l_{t+1,i} + (1-c)V_{it}^C(x_{t+1,i}, S_t)}{b_{t+1,i}}, 0 \right\},$$

where $V_{it}^C(x_{t+1,i}, S_t)$ is the continuation value of the unlevered firm after being seized by debtholders, and c is the probability that the firm is destroyed in default. The continuation value of the unlevered firm is expressed as the following:

$$V_{it}^C(x_{t+1,i}, S_t) = \max_{l_{t+2,i}} E \left[P_{S_{t+1}S_{t+2}} V(x_{t+2,i}, S_{t+2}) | x_{t+1,i}, S_{t+1}, l_{t+2,i}, b_{t+2,i} = 0 \right].$$

1.4.5 Households

Households have preferences over consumption of the final good and leisure of the form $U(C_t, L_t)$. They own (and receive) all dividends paid by firms. They also buy state-contingent securities from the financial intermediaries, effectively lending them cash they then extend as credit to intermediate good firms. They discount the future with discount factor β . The problem of households can thus be written as the following maximization problem:

$$V^H(S_{bt}) = \max_{L_t} E \left[\max_{C_t, B_t(S_{t+1})} U(C_t, L_t) + \beta V^H(S_{bt+1}) \right]$$

such that the budget constraint of the household is satisfied:

$$C_t + \sum_{S_{t+1}} B_t(S_{t+1}) P(S_{t+1}|S_{b,t}) = W_t L_t + B_{t-1}(S_t) + \sum d_{it} - \xi M_t(S_t),$$

where M_t is the mass of entry.

1.4.6 Equilibrium

The equilibrium in this model is a sequence of allocations $\{C(S_t), L(S_{bt}), Y(S_{bt})\}$, distribution of firms Υ_t , firm decision functions $\{l(x_t, S_{bt}), b(x_t, S_{bt}), br(x_t, S_{bt})\}$ and prices $\{W(S_{bt}), Q(l', b', x_t, S_{bt}), P(S_{bt})\}$ such that the intermediate good firm's problem is satisfied, the household's problem is satisfied, financial intermediaries earn zero economic profit, final good firms minimize costs, the labor and final good markets clear, the law of motion for the distribution of firms is satisfied, and the free entry condition holds.

1.5 Quantitative Results

This section outlines the parameterization of the model and discusses the solution method. It then documents the cross-sectional implications of the parameterized model and the business cycle implications of a volatility shock.

1.5.1 Parameterization

I parameterize the utility function to be of the form $U(C, L) = \frac{C^{1-\varsigma}}{1-\varsigma} - \frac{L^{1+\nu}}{1+\nu}$, following ABK. I parameterize the process for innovations in z to be the following: $\log(\frac{z_{t+1}}{z_t}) = \mu_z + \sigma_{it}\epsilon_{zit}$, where $\epsilon_{zit} \sim N(0, 1)$ and $\mu_z = -\frac{\sigma_{it}^2}{2}$. Therefore, $E[z]$ does not change in σ_{it} . The process for $\log(\sigma_{it}^{id})$ is set to be an AR(1) process with mean, μ_σ , variance, ϕ_σ , and persistence, ρ_σ . The firm parameters which are chosen are $c, \kappa, \beta, \mu_{\sigma^{id}}, \rho_{\sigma^{id}}, \phi_{\sigma^{id}}$. I allow for a range of parameterizations for c , allowing for both $c = 1$ (default very costly) as in ABK, as well as setting c to the range of values found in the corporate finance literature ($c \in (.084, 0.3)$). The remaining parameters are calibrated in the $c = 0.3$ case, which is on the high end of estimates from the corporate finance literature. Table 1.3 summarizes the parameterization of the remaining parameters in the model. κ is chosen to hit the degree of financial leverage help by non-financial firms in Compustat. I compute the sum of a measure of total liabilities and the sum of market capitalization for all firms for which I have both data series and I

compute the ratio of the sum of liabilities to the sum of market capitalizations across firms. μ_σ , ϕ_σ , and ρ_σ are chosen to hit the behavior of firm idiosyncratic asset volatility, including the mean, persistence, volatility, and how much of the time series variation is represented by the common component of idiosyncratic equity volatility.³⁸ I parameterize aggregate shocks to firm volatility to match dispersion in firm sales growth over the business cycle.

1.5.1.1 Measuring Equity Volatility in the Model

The parameter σ_{it} measures the amount of idiosyncratic volatility of the firm-specific fundamental shock z . However, it does not have a corresponding observable in the data. Thus, I compute a measure of idiosyncratic asset volatility to correspond to the measure I compute from the data. To this end, I compute the expected discounted present value of all cash flows from the firm (both those going to debt and equity holders) for a firm with a given set of state variables. Given the transition probabilities for firm fundamental z and for firm volatility σ_{it}^{id} , I compute the theoretical expected return for the value of firm's assets as well as the standard deviation of returns for the firm's assets. This volatility differs from the measure in the data in that it does not have measurement error and only accounts for changes in business risk or leverage, and not for other variation in asset prices (due to liquidity or other financial market factors). I proxy for these factors by adding noise to the measurement of variance. The process for σ_{it} and for the noise affecting observation of asset volatility is chosen to hit the cross-sectional variation in percent changes in sales growth, the cross-sectional variation in idiosyncratic asset returns, and the persistence and volatility of idiosyncratic firm asset volatility.

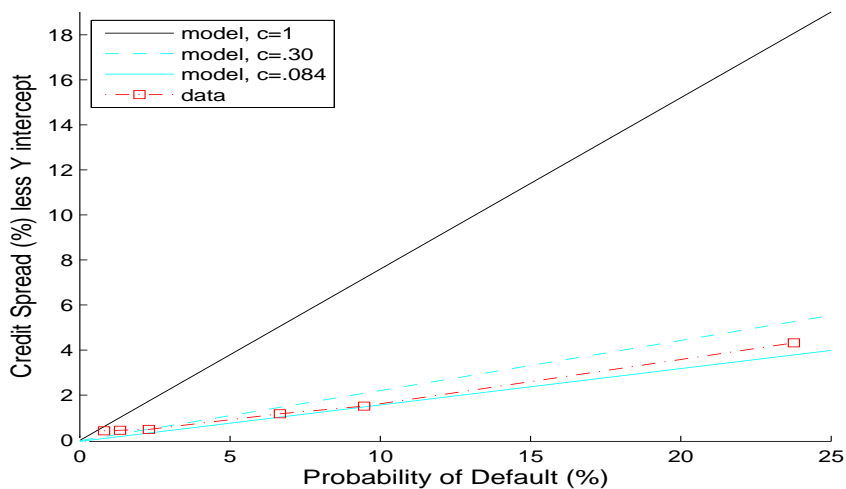
³⁸I find that a common component is responsible for about 25% of within-firm variation in idiosyncratic equity volatility.

1.5.2 Solution Method

The solution method and parameterization I use differs from that used in ABK in a few ways. First, I use very fine grid of productivity shocks, as the quantitative implications of structural models of default can be very sensitive to the grid points. My parameterization for z , which assumes a random walk instead of an AR1, allows me to solve the model for a very fine grid, as I can represent all firm decisions and state variables relative to firm productivity z .³⁹ The cross-sectional implications of my model are generated from the solved steady state. Business cycle implications are generated in the following way: The economy is first assumed to be in steady state, and then a sequence of aggregate shocks are fed in, including volatility shocks calibrated to the micro data for the recent recession. Then firm decision rules are computed via backward value iteration, from which the resulting path of aggregate employment is recovered.

1.5.3 Cross-Sectional Relationships

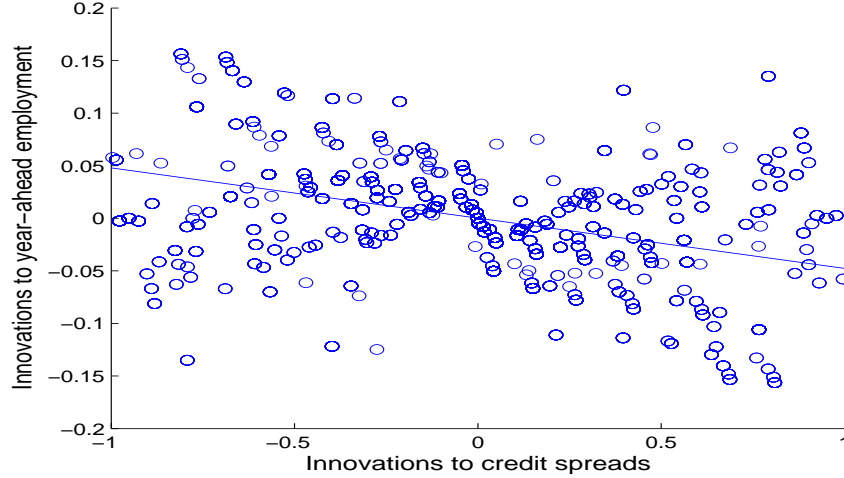
Figure 1.18: Probability of default vs. credit spreads



* The red dashed line reflects data from speculative grade firms, by rating class, for the time period 1984-2012 as reported by Standard and Poor's. I adjust this data to account for risk premia, with the procedure outlined in section 1.2.3.1. The remaining lines are implied by the full model, for varying values of c . All series have the y-intercept from their line of best fit subtracted from them so the slopes can be easily compared.

³⁹Additionally, the literature cited by ABK indicate the demand shocks which are represented by z are quite persistent.

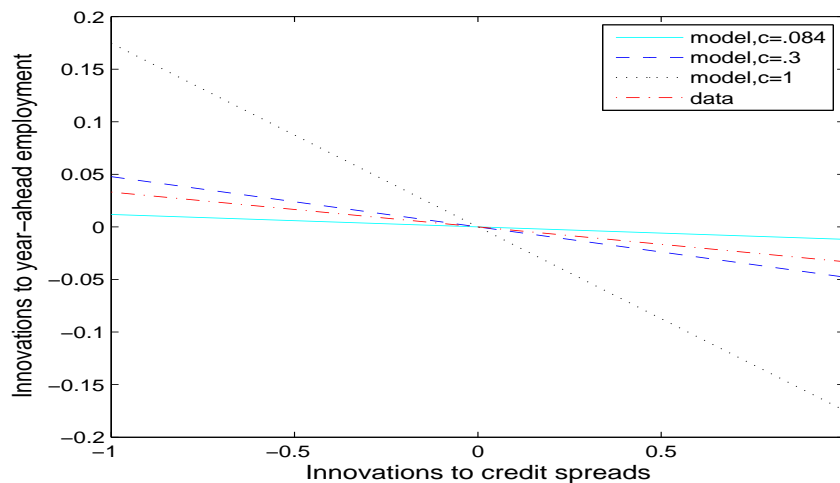
Figure 1.19: Innovations to employment vs. credit spreads, model ($c=.3$)



To analyze the cross-sectional implications of the model, I compute the steady state of the model with idiosyncratic shocks to firm fundamental volatility z . Figure 1.18 shows that the relationship between default rates and credit spreads are entirely inconsistent for the first parameterization ($c = 1$), but generate reasonable results for $c \in [0.084, 0.3]$ (I correct for risk premia in the data using the procedure outlined in section 1.2.3.1). This confirms the results from the simple model. However, it is possible that while default destroys less than 30% of firm value, firms may make decisions as if default were much more costly. For instance, if managers face unemployment in case of default, they have an additional incentive to avoid default. To address this, I consider the relationship between innovations to credit spreads and innovations to year-ahead employment. In the context of the model, this is a measure of how the risk of default (as reflected by credit spreads) affects firm employment choices. If firms made employment decisions as if default were more costly due to higher default costs borne by managers, this relationship should reflect the distortion caused by managerial incentives and correspond to a higher cost of default. Figure 1.19 shows the scatter plot of this relationship in the model in the $c = 0.3$ case. There is a negative relationship between innovations to credit spreads and employment, but it is not perfectly correlated as firms vary in their productivity, volatility, and leverage, and receive shocks to both productivity

and volatility. Figure 1.20 shows the lines of best fit summarizing this relationship in the data and in the model, for a range of values of c . Once again, the relationship in the data lies somewhere between the slopes implied by $c = .084$ and $c = 0.3$. This suggests that firm employment decisions, not just debt pricing, in my model is consistent with firm level data for $c \in [0.084, 0.3]$. If the cost of default were instead parameterized to be $c = 1$, the relationship between these two variables would be much stronger in the model than in firm-level data.

Figure 1.20: Innovations to employment vs. credit spreads

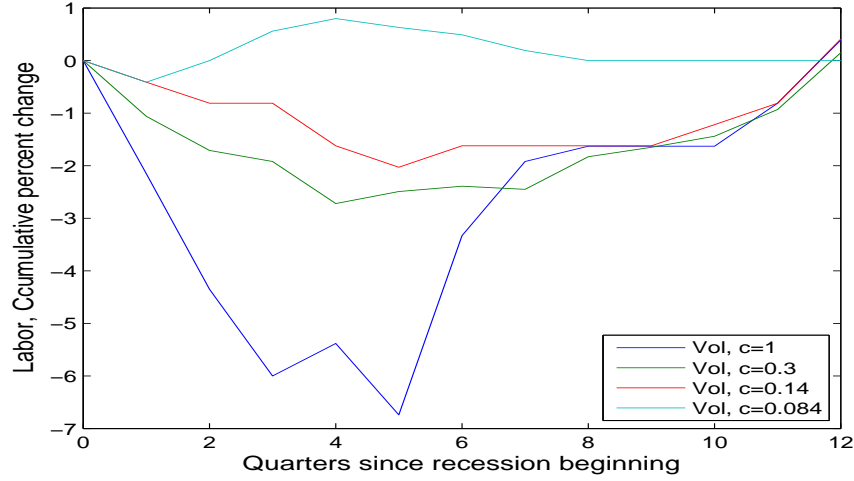


* The red line reflects the line of best fit from an unbalanced panel from 1984-2012, excluding NBER recession years. The remaining lines of best fit are computed using the full model, for varying values of c , in the steady-state.

1.5.4 Business Cycle Implications

As demonstrated in the simple model, the parameterization of c has large effects on the impact of volatility shocks on employment. Figure 1.21 illustrates how the aggregate employment of firms responds to the sequence of volatility shocks implied by the dispersion in firm sales growth seen during the recent 2007-2009 recession. If one follows ABK and assumes default leads to the loss of all firm value ($c = 1$), the model generates large declines in output ($> 6\%$), explaining a great deal of the decline in employment per capita over the recession. However, if c is set to the upper bound of estimates from the corporate finance

Figure 1.21: Aggregate implications of a volatility shock



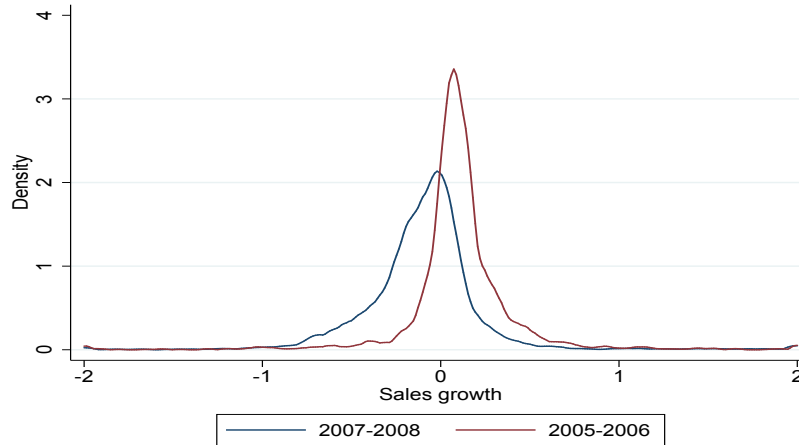
* The lines depict the response of employment, for a variety of calibrations of c , to a sequence of shocks corresponding to the IQR of sales growth during the 2007-2009 recession.

literature, the decline is much more modest, representing at most about a 2.5% decrease. Further reducing c lowers the impact of volatility shocks even more. In the lower bound calibration ($c = .084$), employment only declines slightly before over-correcting. In comparison to these plots, employment per capita fell by around 10%, and the employment of Compustat firms (who did not enter/exit during the recession) fell by just over 5%. Therefore, after calibrating c to match cross-sectional patterns, volatility shocks alone are not sufficient to explain the decline in employment.

1.6 Other Shocks

However, there are other changes which occurred over the business cycle which may affect the decline in employment. I focus on two which are consistent with firm-level data and have a meaningful effect on firms in my model: First, a decline in the skewness of innovations to firm idiosyncratic shocks z ; second, a worsening of the cost of default, c .

Figure 1.22: Distribution of firm sales growth



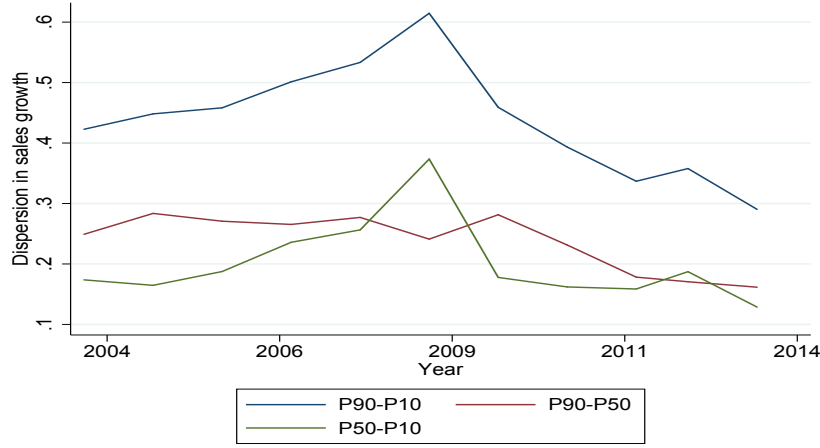
*Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$.

1.6.1 Skewness Shock

Figure 1.24 shows the distribution of year-over-year sales growth during and before the 2007-2009 recession. It is clear that during the recession, firms grew less on average and that the dispersion in growth rates was greater, but the distribution seems to also have become slightly more asymmetric. This becomes much more apparent when looking at measures of outlier-robust dispersion and skewness following Bloom, Guvenen, and Salgado (2015). Figure 1.23 plots three lines: the difference between the 90th and 10th percentile of sales growth, the difference between the 50th and 10th percentile, and the difference between the 90th and 50th percentile. It is apparent that there was a marked increase in the dispersion of sales growth during the recent crisis, as the difference between the 90th and 10th percentile of sales growth widened significantly. However, this was essentially all driven by downside dispersion, the difference between the 50th and 10th percentile, rather than upside dispersion. Bloom, Guvenen, and Salgado (2015) document that this marked decline in skewness is a robust result, and that in general the skewness of firm growth rates in compustat is procyclical. In my model with only volatility shocks, the model underestimates the increase in downside dispersion and overestimates the increase in upside dispersion.

I parameterize a negative skewness shock by assuming that ϵ_{zit} follows a binormal dis-

Figure 1.23: Measures of Dispersion, Upside and Downside



*Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$.

tribution (about the median)⁴⁰. The probability density function of this distribution is the following:

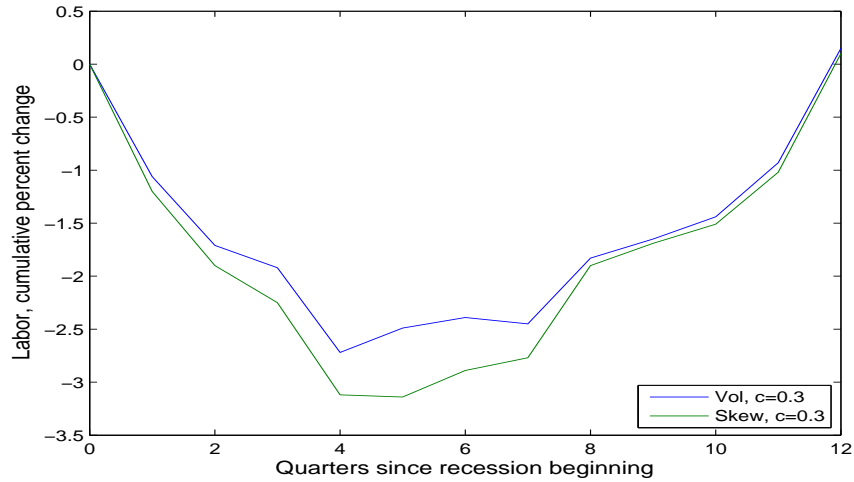
$$f(\epsilon) = \begin{cases} \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{1}{2} \frac{(\epsilon - m)^2}{2\sigma_1^2}} & x \leq m \\ \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{1}{2} \frac{(\epsilon - m)^2}{2\sigma_2^2}} & x > m \end{cases}$$

I can thus parameterize the standard deviations for the upper and lower tails separately, and specify them to hit the numbers documented in Bloom, Guvenen, and Salgado (2015) for upside (90th percentile less 50th percentile) and downside (50th percentile less 10th percentile) of the dispersion of sales growth.

As default is typically a left-tail event, shocks to the skewness of the distribution can increase marginal default probabilities and reduce firm employment further. Figure 1.24 shows the results of an increase in idiosyncratic risk that increased volatility and decreased skewness. This shows that skewness shocks can amplify the decline in employment, but its magnitude is modest (around 1 percentage additional decline in employment).

⁴⁰The binormal distribution has been used to parameterize skewness in financial modeling, see Feunou, Jahan-Parvar, and Tdongap (2014), for example.

Figure 1.24: Aggregate implications of a volatility and skewness shock



* The lines depict the response of employment to a sequence of shocks corresponding to the IQR of sales growth and the observed changes in the difference between upside and downside dispersion (documented by differences in percentiles of sales growth) during the 2007-2009 recession.

1.6.2 Changing Cost of Default

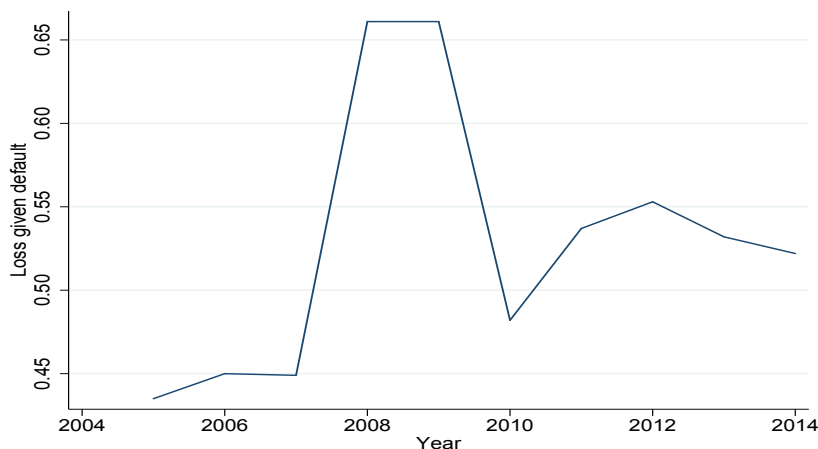
There is considerable evidence consistent with an increased cost of default during recessions. A number of papers document that recovery rates are procyclical (thus losses to debtholders, conditional on default, are countercyclical) and that there is a negative correlation between the default rate and recovery rates conditional on default.⁴¹ This negative relationship is only partially explained by the state of the aggregate economy; this has been interpreted as consistent with the “fire-sale” effect of Shleifer and Vishny (1992).⁴² Figure 1.25 plots the decline in average recovery rates reported by Moody’s during the 2007-2009 recession. Additionally, I argue that a key cross-sectional relationship, the slope of credit spreads vs. the probability of default, changed during the recent crisis in a way consistent with an increased cost of default. As historical default probabilities are highly variable for individual years, I have to use another measure of default risk. I follow Gilchrist and Zakrajsek (2012) and use the probability of default implied by firm micro-data and a Merton “distance-to-

⁴¹Altman (2006) and Altman, Brady, Resti, and Sironi (2005) document key facts and provide a detailed overview of the literature. These results are consistent with Moody’s data and analysis on recovery rates, see Cantor and Varma (2004) and Moody’s (2015).

⁴²See Acharya, Bharath, and Srinivasan (2007).

default” model.⁴³ Figure 1.26 displays this relationship both before and during the great recession. There are two clear changes: first, credit spreads increase substantially given their implied default probability; second, the slope between credit spreads and default probability increases. The level effect, indicating that credit spreads rise more than can be explained by the extent of default risk, is consistent with the finding in Gilchrist and Zakrajsek (2012) that the *excess bond premium*, a measure of credit spreads in excess of what can be explained by default risk, increased substantially in the 2007-2009 recession. The increase in the slope is consistent with a higher cost of default in my model. The slope increases substantially, more than doubling.

Figure 1.25: Procyclical Recovery Rate



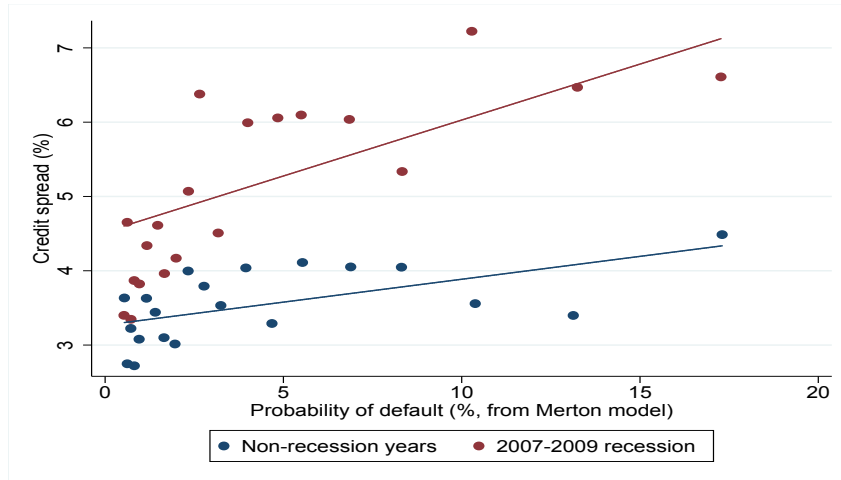
* The data is taken from the average recovery rate as reported by Moody's.

As the cost of default is a first-order driver of the incentive for firms to reduce employment in the face of risk, a shock to the cost of default can reduce employment. I parameterize the shock to the bankruptcy cost c to hit the relative changes in recovery rates during the 2007-2009 recession. Figure 1.27 documents the result of a volatility shock as well as a shock to the level of the default cost.⁴⁴ By itself, a shock to the cost of default does relatively

⁴³The probability of default is a function of the market value of the firm, its liabilities, and the volatility of equity, all taken from firm micro-data.

⁴⁴The cost of default here is 30% before the shock, and increases to just over 50% of firm value during the crisis.

Figure 1.26: Probability of Default vs. Credit Spreads

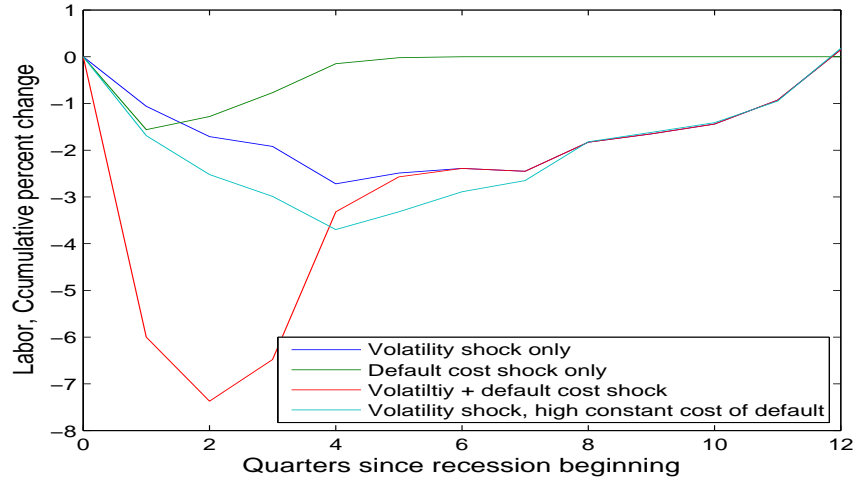


* The x-axis denotes the probability of default implied by a Merton model, computed from the market capitalization of a firm, its liabilities, and realized equity volatility.

little, reducing employment by around 1.5%. However, when fed in together with idiosyncratic volatility shocks, employment declines markedly, generating a roughly 7% decline in employment. The large losses are due to the interaction of the two shocks, not just higher default costs. If c is parametrized as constant and equal to the maximum value of the shock to the cost of default realized in the great recession, the decline in employment is only about half as much (the line labeled “Volatility shock, high constant cost of default” illustrates this in Figure 1.27). The intuition here is similar to the simple model: volatility shocks and shocks to the cost of default interact, as together they raise the incentive for firms to reduce employment in order to avoid losses upon default more than they do separately. In addition, both firm risk-taking and the distortion of firm employment from the costless default case *before the shock* depend substantially on the level of c . If c is high prior to the sequence of shocks, firms choose lower employment and take less risk in terms of leverage and operating leverage than if c was low. Therefore the fact that c is low prior to the sequence of shocks helps amplify the magnitude of the decline in employment.

Figure 1.28 shows the decline in employment implied by the interaction of these shocks for varying mean levels of the cost of default, $c = .084, 0.3$. The shocks are parameterized to hit the *relative* decline in recovery rates; therefore, the maximum level of c parameterized

Figure 1.27: Aggregate Implications of a volatility and default cost shock



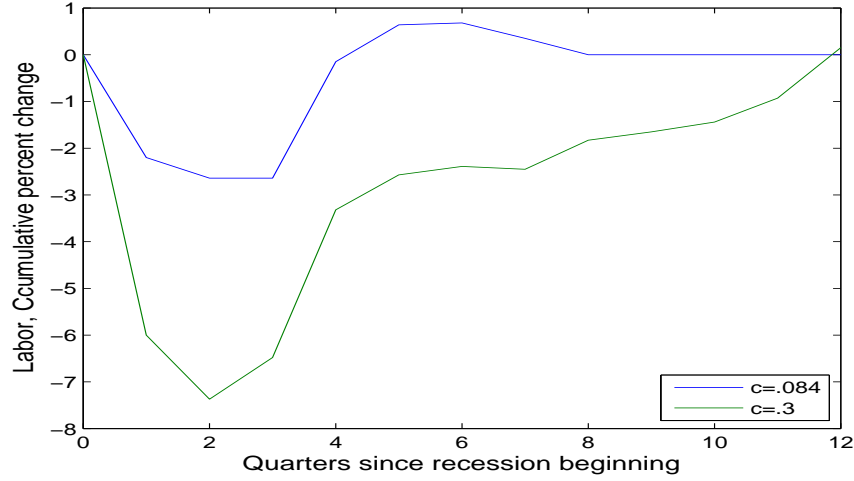
* The lines depict the response of employment to a sequence of shocks corresponding to the IQR of sales growth and calibrated shocks to the cost of default during the 2007-2009 recession. The line denoted "Volatility shock, high constant cost of default" considers the impact of volatility shocks if the cost of default is set to the maximum value of it attained by the calibrated shock during this recession.

is lower if the cost of default is lower ($c = .084$) before the shock. If I instead fed in the same sequence of shocks to the cost of default, the decline in employment would be even greater if the cost of default was lower prior to the recession. The decline in employment by the interaction of these shocks still depends on the typical magnitude of the cost of default. However, given a cost of default, the decline in employment is much larger in response to the combination of these shocks than to volatility shocks alone.

1.7 Conclusion

In this paper I investigate the role of idiosyncratic volatility shocks interacting with financial frictions to drive a decline in employment over the business cycle. I argue that the cost of default has a first-order effect on the size of this decline, and use cross-sectional implications of the model to calibrate the cost of default. The assumption of very high default costs leads to counterfactual implications and are in conflict with an established literature in corporate finance. Parameterizing the cost of default to lower levels, consistent with both the cross-sectional implications and the range of estimates from the literature, greatly reduces the impact of this mechanism over the business cycle. I investigate two additional shocks which

Figure 1.28: Aggregate Implications of a volatility and default cost shock, varying c



* The lines depict the response of employment, for a variety of calibrations of c , to a sequence of shocks corresponding to the IQR of sales growth and calibrated shocks to the cost of default during the 2007-2009 recession.

are consistent with the micro-data for the 2007-2009 recession — a negative idiosyncratic skewness shock and a shock to the cost of default. I find that modeling the risk shock as not only as a volatility shock, but also as a negative skewness shock can amplify employment losses, but not sufficiently to explain the decline in aggregate employment. Shocks to the cost of default, on their own, only have a modest effect on employment. However, when shocks to the cost of default and volatility are fed in together, they can interact to generate a large ($> 7\%$) decline in employment. This suggests that the factors leading to a change in cost of default are key to understanding the effects of higher idiosyncratic volatility during the great recession.

Appendix A: Simple Model

Proof of Proposition 1.1

The default threshold $\bar{z}(l, b)$ is defined as follows:

$$\bar{z}(l, b) l^\alpha - wl - b + V = 0.$$

This can be rearranged:

$$\bar{z}(l, b) = wl^{1-\alpha} + bl^{-\alpha} - Vl^{-\alpha}.$$

Taking the derivative with respect to labor yields:

$$\frac{\partial \bar{z}(l, b)}{\partial l} = (1 - \alpha)wl^{-\alpha} - \alpha bl^{-1-\alpha} + \alpha Vl^{-1-\alpha}.$$

This can be rearranged to find:

$$\frac{\partial \bar{z}(l, b)}{\partial l} = l^{-1-\alpha} (wl - \alpha (wl + b - V)).$$

It immediately follows that (i) and (ii) are equivalent.

The positivity of parameters V, c , and probability density function f immediately imply that (ii) and (iii) are equivalent.

(1.4) immediately implies that (iii) and (iv) are equivalent.

Proof of Proposition 1.2

The normal pdf has the form $f(z) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(z-\mu)^2}{2\sigma^2}}$. Thus the derivative of this with respect to σ is:

$$\frac{\partial f(z)}{\partial \sigma} = -\frac{1}{\sigma^2\sqrt{2\pi}}e^{-\frac{(z-\mu)^2}{2\sigma^2}} + \frac{(z-\mu)^2}{\sigma^3} \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(z-\mu)^2}{2\sigma^2}}.$$

This can be simplified as:

$$\frac{\partial f(z)}{\partial \sigma} = \frac{1}{\sigma^4\sqrt{2\pi}}e^{-\frac{(z-\mu)^2}{2\sigma^2}} ((z-\mu)^2 - \sigma^2).$$

Which implies that $\frac{\partial f(z)}{\partial \sigma} > 0$ if $|\mu - z| > \sigma$, and thus (i) immediately follows. (ii) follows from the fact that $|\mu - z| > \sigma_2$ and $\sigma_2 > \sigma_1$ then $f_2(z) = f_1(z) + \int_{\sigma_1}^{\sigma_2} \frac{\partial f(z)}{\partial \sigma} d\sigma > f_1(z)$.

Derivation of Equity Volatility in the Simple Model

Two Period Case Real equity returns can be computed as follows:

$$\frac{V_{E,2} + div}{\beta V_{E,1}},$$

where

$$V_{E,2} = \beta \begin{cases} 0 & z < \bar{z}(l, b) \\ z l^\alpha - w l - b + V & z \geq \bar{z}(l, b) \end{cases}$$

is the value of equity after z is realized,

$$V_{E,1} = \beta (E[z] l^\alpha - w l + V - F(\bar{z}(l, b)) c V)$$

is the value of equity before debt issuance, and

$$div = \beta \left(b(1 - F(\bar{z}(l, b))) + \int_0^{\bar{z}(l, b)} (z l^\alpha - w l) dF(z) + F(\bar{z}(l, b)) (1 - c) V \right)$$

is the income from debt issuance paid to equity holders. These can be combined to compute the volatility of equity returns in closed form. The expression is omitted here for simplicity, but it is a function of the volatility of z , the amount of leverage, the incidence of default, and the relative size of V to the operating profits. Shocks to the volatility of z will have a first-order effect on the size of equity volatility.

Merton Model The role of the volatility of z on equity volatility can most easily be demonstrated using a Merton model equivalent of this two-period model. The equivalent of “asset value” in the Merton model is $A = z l^\alpha$; this means that if z follows geometric Brownian motion, so does A_t . The equivalent of liabilities are $B = w l + b - V$; the continuation value has to be subtracted here because it does not move proportional in z . Default occurs, as

in a Merton model, if $A < B$. The standard solution of a Merton model then implies that equity volatility, σ_E , can be expressed as the following:

$$\sigma_E = \sigma_z \frac{z_0 l^\alpha \Phi \left(\frac{\log \left(\frac{z(l^\alpha)}{b+wl-V} \right) + \left(r + \frac{\sigma_z^2}{2} \right) (T-t)}{\sigma_z \sqrt{T-t}} \right)}{(1 - F(\bar{z}(l, b))) (E[z_T | z_T > \bar{z}(l, b)] l^\alpha - wl - b + V)}$$

Here it is clear that changes to σ_z will have a large effect on σ_E , though the magnitude of the response also depends on the effect on capital structure considerations.

Appendix B: Dataset Construction

Data Sources

Daily equity prices are extracted from the Center for Research in Security Prices (CRSP). Data from firm accounting statements, at either the annual or quarterly frequency are taken from Compustat. I combine data on credit spreads from the Lehman-Warga and Merrill Lynch databases. The data on credit spreads is limited to the time frame of 1984-2012, thus my empirical analysis is limited to those years.

Computing Asset Volatility

As measures of equity volatility are affected by leverage, I also compute a measure of asset volatility from returns. I follow a procedure consistent with Bharath and Shumway (2008) and Gilchrist and Zakrajsek (2012) in measuring total firm value V_{it}^A and firm asset volatility, σ_{it}^A , whose procedures are all in the spirit of Merton (1974). V_A is the value of assets, V_{it}^B is the value of debt, μ_{it}^A is the mean rate of asset growth, and σ_{it}^A is asset volatility. I recover V_{it}^A and σ_{it}^A from the data closely following the procedure outlined by Gilchrist and Zakrajsek (2012). For each firm, I linearly interpolate the quarterly value of debt from Compustat to a daily frequency. I use daily data on the market value of equity; call this V_{it}^E . I guess a value

of asset volatility, $\sigma_{it}^A = \sigma_{it}^E \frac{V_{it}^B}{V_{it}^E + V_{it}^B}$, where the standard deviation of equity is calculated as the square root of the annualized moving average of squared returns for a firm. To recover the value of assets and the volatility of assets, I follow the procedure outlined in Merton (1974).

Given a guess of σ_{it}^A , I then use the equation

$$V_{it}^E(t) = V_{it}^A(t)\Phi(d_1) - e^{-r(T-t)} * V_{it}^B\Phi(d_2),$$

where $d_1 = \frac{\log\left(\frac{V_{it}^A}{V_{it}^B}\right) + \left(r + \frac{1}{2}(\sigma_{it}^A)^2\right)T}{\sigma_{it}^A\sqrt{T}}$ and $d_2 = d_1 - \sigma_{it}^A\sqrt{T}$, to recover the value of assets. I define r to be the one-year Treasury-constant maturity, which is taken from the Federal Reserve's H.15 report. After converging on V_{it}^A for the given σ_{it}^A , I recompute σ_A from the implied V_{it}^A using the same methodology I use to compute σ_{it}^E . I ultimately converge on σ_{it}^A through a slow-updating procedure.

This is a measure of asset volatility, but not the idiosyncratic component. I use the theory of Merton above to derive a measure of idiosyncratic asset volatility from idiosyncratic equity volatility and asset volatility. Assuming that equity prices follow geometric brownian motion and are a function of asset prices and debt, Ito's lemma allows us to express them as the following: ⁴⁵

$$\sigma_{it}^A = \sigma_{it}^E \frac{V_{it}^E}{V_{it}^A} \frac{\partial V_{it}^A}{\partial V_{it}^E}. \quad (\text{B.1})$$

Rearranging (B.1) and splitting asset and equity volatility into aggregate and idiosyncratic components yields:

$$(\sigma_{it}^{AM})^2 + (\sigma_{it}^{AI})^2 = (\sigma_{it}^{EM})^2 \left(\frac{V_{it}^E}{V_{it}^A} \frac{\partial V_{it}^A}{\partial V_{it}^E} \right)^2 + (\sigma_{it}^{EI})^2 \left(\frac{V_{it}^E}{V_{it}^A} \frac{\partial V_{it}^A}{\partial V_{it}^E} \right)^2,$$

where σ_{it}^{AI} is idiosyncratic asset volatility of firm i and time t , σ_{it}^{AM} is the firm's aggregate

⁴⁵This is equivalent to equation 6 in Gilchrist and Zakrajsek (2012)

(market) asset volatility, and $(\sigma_{it}^A)^2 = (\sigma_{it}^{AM})^2 + (\sigma_{it}^{AI})^2$ (and similarly for equity volatility). Assuming that the idiosyncratic components in this equation correspond to each other and plugging in (B.1) yields idiosyncratic asset volatility as the product of idiosyncratic equity volatility and a de-leveraging factor:

$$\sigma_{it}^{AI} = \sigma_{it}^{EI} \frac{\sigma_{it}^A}{\sigma_{it}^E}. \quad (\text{B.2})$$

Tables for Chapter 1

Table 1.1: Regression of innovations in credit spreads on innovations in volatility

	(1)	(2)	(3)	(4)
	$\% \Delta cs_t$	$\% \Delta cs_t$	$\% \Delta cs_t$	$\% \Delta cs_t$
$\% \Delta \sigma_{Ai,t}$	0.249***	0.031**		
$\% \Delta \sigma_{sale,t}$			0.187***	-0.023
$\% \Delta lvg_t$		-0.024		-0.072
$\% \Delta \Pi_t$		-0.404***		-0.329***
$\% \Delta V_{E,t}$		-0.369***		-0.375***
Year Effects	No	Yes	No	Yes
Bond Characteristic Controls	No	Yes	No	Yes
Bond Rating Effects	No	Yes	No	Yes
N	15474	14700	6865	6631
r ²	0.025	0.438	0.002	0.407

* Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$. Bond characteristics considered include the duration, coupon, and the face value of debt. $\sigma_{Ai,t}$ is idiosyncratic asset volatility, $\sigma_{sale,t}$ is a measure of volatility estimated from a GARCH process on firm sales growth, lvg_t is market leverage, Π_t is profitability (operating profits to book value of assets), and $V_{E,t}$ denotes the market capitalization of the firm.

Table 1.2: Regression of innovations in employment on innovations in volatility

	(1)	(2)	(3)	(4)	(5)	(6)
	$\% \Delta l_{t+1}$	$\% \Delta l_{t+1}$	$\% \Delta l_{t+1}$	$\% \Delta l_{t+1}$	$\% \Delta l_{t+1}$	$\% \Delta l_{t+1}$
$\% \Delta \sigma_{Ai,t}$	-0.024***	-0.021***	-0.028***			
$\% \Delta \sigma_{sale,t}$				-0.029	-0.028	-0.015
$\% \Delta \Pi_t$		0.033***	0.005		0.042*	0.007
$\% \Delta Q_{t-1}$		0.023***	0.025***		0.027***	0.029***
$\% \Delta sale_t$		0.173***	0.126***		0.193***	0.151***
$\% \Delta V_{E,t}$			0.087***			0.074***
$\% \Delta lvg_t$			0.026***			0.016
Year Dummies	No	Yes	Yes	No	Yes	Yes
N	37511	30567	29292	9797	8381	8100
r2	0.001	0.057	0.080	0.000	0.067	0.086

* Data is an unbalanced panel for 1984-2012, excluding NBER recession years. Percent changes are computed as $\frac{x_{t+1} - x_t}{\frac{1}{2}(x_{t+1} + x_t)}$.

Bond characteristics considered include the duration, coupon, and the face value of debt. $\sigma_{Ai,t}$ is idiosyncratic asset volatility, $\sigma_{sale,t}$ is a measure of volatility estimated from a GARCH process on firm sales growth, lvg_t is market leverage, Π_t is profitability (operating profits to book value of assets), Q_t is Tobin's Q, and $V_{E,t}$ denotes the market capitalization of the firm.

Table 1.3: Baseline parameterization

Parameter	Meaning	Value	Notes
<i>Non-firm Parameters:</i>			
ς	CES parameter in utility function	2	Taken from ABK
ν	Controls labor elasticity	0.5	Taken from ABK, implies labor elasticity of 2
γ	Elasticity of substitution	7.7	Taken from ABK, implies 15% markup
<i>Firm parameters:</i>			
α	Returns to scale in labor	0.7	Taken from ABK
κ	Jensen effect parameter	0.05	Chosen to hit ratio of the sum of total liabilities to the sum of market capitalization
β	Discount rate	0.99	
<i>Firm-specific portion of idiosyncratic risk:</i>			
$\log(\mu_{\sigma^{id}})$	Mean	.09	Chosen to hit IQR of sales growth and proportion of idiosyncratic equity volatility not explained by common component
$\rho_{\sigma^{id}}$	Persistence	.92	From autocorrelation of firm idiosyncratic equity volatility
$\phi_{\sigma^{id}}$	Variance	.12	From volatility of innovations to firm idiosyncratic equity volatility

*All variables are at a quarterly frequency.

Chapter 2: Accounting for Productivity Dispersion over the Business Cycle

2.1 Introduction

What drives changes in aggregate productivity? One explanation that has been widely used to explain the variation of aggregate productivity over the business cycle or over time more generally is that frictions to the allocation of labor and capital between firms are time-varying: Greater frictions to the distribution of capital and labor between firms reduce the amount of output produced with a given amount of capital and labor and reduce measures of aggregate productivity.⁴⁶ This paper presents accounting decompositions of changes in aggregate labor, capital, and total factor productivity that addresses this economic mechanism, and can help to quantify the extent to which the changing distribution of labor and capital drive fluctuations in aggregate productivity over time.

The accounting decompositions in this paper rely on the property that aggregate factor productivity ratios can be expressed as the weighted sum of firm-level productivity ratios. Our first decomposition splits changes in measures of aggregate factor productivities into a mean component, changes in the weighted average of log productivities across firms, and a dispersion component, which captures changes in the higher order moments of the distribution of productivities across firms.⁴⁷ The two components add up to the change in a given aggregate factor productivity ratio. We compute the decomposition separately for both aggregate labor and capital productivity. Crucially, for the decomposition of aggregate labor productivity, we require only firm-level panel data on value added and labor, and for the

⁴⁶This economic mechanism plays a role in driving the dynamics of productivity and other macroeconomic aggregates in a number of recent influential papers, including Arellano et al. (2012), Bloom, Floetotto, Jaimovich, Sporta-Eksten, and Terry (2014b), Gilchrist et al. (2014), Khan and Thomas (2013), Midrigan and Xu (2014), and Moll (2014), as examples.

⁴⁷To be precise, the dispersion component can be expressed as a function of the second and higher order cumulants of the distribution of firm productivity measures, while the mean component is only a function of the first cumulant.

decomposition of aggregate capital productivity, we require firm-level panel data on value added and capital.

The allocation of labor and capital may vary across firms not only due to distortions but for technological reasons as well; the second decomposition allows us to group firms (by industry or other categorical groups) to address this point. We implement our first decomposition on each sector, resulting in sectoral mean and dispersion components. We can then weight each sector's mean and dispersion components by sectoral factor shares to obtain aggregate mean and dispersion components. Thus, by an accounting property, the change in aggregate factor productivities can be decomposed into three components: First, an aggregated mean component which captures changes in the weighted average of log factor productivities *within* sectors. Second, an aggregated dispersion component which captures changes in the dispersion of log factor productivities across firms *within* sectors. Third, a sectoral-share component, which captures the changes in the distribution of inputs *between* sectors.

Our decompositions, when applied to aggregate labor or capital productivity, are purely accounting identities. To combine aggregate capital and labor productivity into a measure of total factor productivity, we rely on the standard model assumptions that allow us to compute the Solow residual. We then show that the Solow residual has the nice property that we can express it as the weighted average of the mean, dispersion, and sectoral-share components of capital and labor productivity.

Our decompositions are useful tools for researchers testing whether models where frictions to the allocation of labor or capital across firms play a meaningful role in driving aggregates are consistent with firm level behavior. We present a series of results to demonstrate this point. In the model of Hsieh and Klenow (2009), we demonstrate how our decomposition captures changes in the distribution of the log of marginal revenue factor productivities. We prove that changes in the expected value of the log of marginal revenue factor productivities, as well as changes in production function coefficients, drive changes in the mean component

of our decompositions. We prove that changes in the second central moment and all higher order moments of the log of marginal revenue factor productivities drive changes in the dispersion component of our decompositions.

We then use a more general model of production by heterogeneous firms to demonstrate how distortions to firm capital and labor decisions are captured in our decomposition. We demonstrate analytically that the dispersion component of our decomposition captures changes in productivity due to heterogeneous distortions to firm-level input allocation. The mean component of our decompositions captures changes in technology or common distortions to firm capital or labor choices. We prove that this general model of production has a mapping to a large number of macroeconomic models in the literature that utilize frictions to the allocation of labor or capital across firms to help drive aggregate dynamics.

We compute our decompositions for aggregate labor productivity, capital productivity, and TFP using firm-level data on U.S. nonfinancial public firms. To see if the results are consistent for another large, developed nation, we perform a similar analysis for nonfinancial public firms from Japan. The results for the United States and Japan from the second decomposition applied to labor productivity show that the mean component is highly correlated with movements in aggregate labor productivity and are essentially solely responsible for its cyclical variation. The magnitude of movements in the dispersion component are small, and the dispersion component has a weak negative correlation with changes in aggregate labor productivity. Our results are different for aggregate capital productivity. The dispersion component moves much more closely with changes in aggregate capital productivity, and does play a role in contributing to cyclical variation in aggregate capital productivity. Our decomposition, when applied to TFP, yields the result that the mean component is responsible for the vast majority of its cyclical variation, because much of the cyclical movements in TFP are driven by changes in aggregate labor productivity.

Related Literature The contribution of this paper is to provide accounting decompositions of aggregate labor and capital productivity, which can be implemented without structural estimation, and can guide the specification of firm-level frictions to capital and labor allocation in business cycle models. The fact that our decompositions only require measures of firm-level value added, labor, and capital, and do not require estimation to be computed is an attractive property, as it implies the use of our decomposition not only avoids potential biases from estimation, but also means that our decomposition can be computed in both data and heterogeneous firm models with relative ease. A large number of papers in the literature work with production environments that map into the class of production environments that we rely on to prove how our decomposition maps into models in Section 2.3. The general class of models to which our theoretical results apply include the influential models of Arellano, Bai, and Kehoe (2012), Bloom et al. (2014b), Kehrig (2015), and Khan and Thomas (2013), as only a few recent examples. Thus, the dispersion component of our decomposition reflects changes in the distribution of distortions to firm input allocation in such papers. Hence, the role of frictions to firm labor and capital allocation in a large number of models can be compared to the data through the use of our decomposition. Our empirical results alone can also help to guide model selection in standard, widely-used production environments. In this sense, our decomposition is similar in spirit to Chari, Kehoe, and McGrattan (2007).

Our paper is also related to a number of recent studies which examine the role of reallocation or allocative efficiency in driving aggregate productivity dynamics. One group of papers estimate production function coefficients and firm-level total factor productivities to assess the role of allocative frictions in driving productivity over the business cycle, such as Oberfield (2013), Osotimehin (2013), and Sandleris and Wright (2014). Our approach differs from this set of the literature in that our decompositions are accounting identities requiring only measures of firm-level value added, capital, and labor, and thus we do not require the estimation of production function coefficients. Our method therefore avoids the potential econometric biases in these estimation procedures (which are discussed in Appendix E) and

can be implemented immediately on a wide array of models and data. The magnitude of the dispersion component of our decompositions can be viewed as an approximation to the extent to which allocative efficiency affects aggregate productivity in such models (we show this in Appendix E). Thus, our decomposition, if applied to the respective datasets used in these papers, could be used to complement the paper’s structural approaches and potentially address concerns regarding the assumptions required for estimation. Another group of papers examine the role of resource reallocation through the use of aggregate productivity decompositions, such as Foster, Haltiwanger, and Krizan (2001) and Basu and Fernald (2002). The sectoral share component of our second decomposition also speaks to the role resource reallocation between sectors can play in driving productivity dynamics. Differently from these papers, however, the dispersion component of our decomposition captures the role allocative efficiency plays in driving productivity dynamics.⁴⁸ Additionally, our decomposition does not require the estimation of firm-level TFP.

The rest of the paper proceeds as follows. Section 2.2 defines the components of our decompositions for aggregate labor productivity, aggregate capital productivity, and TFP. Section 2.3 discusses how shocks to firm-level wedges map into the components of our decomposition. Section 2.4 applies our decomposition to data from U.S. and Japanese nonfinancial public firms. Section 2.5 concludes.

2.2 Productivity Decompositions

In this section, we first present our decompositions of changes in aggregate labor and capital productivity, and then we present how to combine these decompositions to perform decompositions of changes in TFP. Decomposition I breaks changes in the log of each aggregate productivity ratio into a mean and a dispersion component to help identify whether it is

⁴⁸Alternatively, adjustment costs could generate a dynamically efficient allocation that observationally is consistent with static misallocation; this point is made in Asker, De Loecker, and Collard-Wexler (2014), e.g.

changes in the mean or dispersion of the log of firm-level productivity ratios that are driving changes in aggregate productivity. Decomposition II allows for groupings of firms (sectors) to each have a mean and a dispersion component, and for the allocation of inputs between each grouping of firms to change over time. In turn, when analyzing changes in aggregate productivity, there is also a sectoral-share component, which reflects how input shares are changing across sectors over time.

2.2.1 Decomposition I: Mean and Dispersion Components

We start with a static decomposition of aggregate labor productivity. We define L as aggregate labor and l as firm-level labor. Aggregate labor is the sum of all firm-level labor. We define K as the aggregate capital stock and k as the firm-level capital stock, where the aggregate capital stock is the sum of all firm capital stocks. The decomposition below holds for capital productivity as well, if we substitute K for L and k for l .

We define Y as aggregate output and v as firm value added, where aggregate output is the sum of all firm-level value added. We have the following identity, which holds at each time t :

$$\frac{L_t}{Y_t} \equiv \sum_i \frac{l_{i,t}}{v_{i,t}} \frac{v_{i,t}}{Y_t}, \quad (2.1)$$

where i indexes the set of firms in the economy.

Building on (2.1), we can now perform a static version of our first decomposition:

$$\log \left(\frac{L_t}{Y_t} \right) = \underbrace{\sum_i \log \left(\frac{l_{i,t}}{v_{i,t}} \right) \frac{v_{i,t}}{Y_t}}_{\text{static mean component}} + \underbrace{\left(\log \left(\sum_i \frac{l_{i,t}}{v_{i,t}} \frac{v_{i,t}}{Y_t} \right) - \sum_i \log \left(\frac{l_{i,t}}{v_{i,t}} \right) \frac{v_{i,t}}{Y_t} \right)}_{\text{static dispersion component}}. \quad (2.2)$$

Aggregate labor to output at each time t is now broken into a “mean component,” which is the weighted average of the log of labor to value added, and a “dispersion component.” If we treat labor to value added as a random variable with a probability density function (reflecting

the number and size of firms with a given productivity ratio), the dispersion component takes the form of the log of the expectation of firm-level labor to value-added ratios less the expectation of the log of firm-level labor to value-added ratios. This term is always non-negative due to Jensen’s inequality. This measure has useful statistical properties related to the measure of entropy in Backus, Chernov, and Zin (2014). Assuming some regularity conditions on the distribution of firm labor to value-added ratios such that the cumulant generating function exists, the dispersion component captures all higher-order cumulants of the distribution of firm-level labor to value-added ratios.⁴⁹ This can be interpreted as the following: The dispersion component captures the effect of all second and higher order moments of the distribution of firm labor productivity on aggregate labor productivity.

We are interested in *changes* in labor productivity. We can recover changes in labor productivity as:

$$\Delta \log \left(\frac{Y_t}{L_t} \right) = \underbrace{-\Delta \sum_i \log \left(\frac{l_{i,t}}{v_{i,t}} \right) \frac{v_{i,t}}{Y_t}}_{\text{mean component}} \underbrace{-\Delta \left(\log \left(\sum_i \frac{l_{i,t}}{v_{i,t}} \frac{v_{i,t}}{Y_t} \right) - \sum_i \log \left(\frac{l_{i,t}}{v_{i,t}} \right) \frac{v_{i,t}}{Y_t} \right)}_{\text{dispersion component}}. \quad (2.3)$$

An increase in dispersion in firm-level labor to value-added ratios decreases aggregate labor productivity. Similarly, an increase in the weighted average of firm-level labor to value-added ratios decreases aggregate labor productivity. Our mean/dispersion decomposition allows us to determine whether it is changes in the mean or the dispersion in the log of firm-level labor to value added which is driving changes in aggregate labor productivity. We present our second decomposition below, which allows each sector to have a mean and dispersion component. Hence, changes in aggregate labor productivity can be driven by

⁴⁹Cumulants summarize the distribution of a random variable, as we explain in more detail in Section 2.3. Backus, Chernov, and Zin (2014) also provide an excellent discussion of why functions of this form capture all higher-order cumulants.

changes in the mean of log firm-level labor to value-added ratios *within* sectors, changes in their dispersion *within* sectors, or changes in the allocation of inputs *between* sectors.

2.2.2 Decomposition II: Mean, Dispersion, and Sectoral Share Components

For a sector (or any given grouping of firms), the identity in (2.1) holds. Hence, if j indexes a given sector, we have the following identity for aggregate labor to added value ratio within that sector at time t :

$$\frac{L_t^j}{Y_t^j} \equiv \sum_i \frac{l_{i,t}^j v_{i,t}^j}{v_{i,t}^j Y_t^j}. \quad (2.4)$$

In turn, for each sector at time t , we can decompose the aggregate labor to added value ratio within a sector into a mean and dispersion component:

$$\log \left(\frac{L_t^j}{Y_t^j} \right) = \underbrace{\sum_i \log \left(\frac{l_{i,t}^j}{v_{i,t}^j} \right) \frac{v_{i,t}^j}{Y_t^j}}_{M_t^j} + \underbrace{\left(\log \left(\sum_i \frac{l_{i,t}^j v_{i,t}^j}{v_{i,t}^j Y_t^j} \right) - \sum_i \log \left(\frac{l_{i,t}^j}{v_{i,t}^j} \right) \frac{v_{i,t}^j}{Y_t^j} \right)}_{D_t^j}, \quad (2.5)$$

where M_t^j is the static mean component in sector j and D_t^j is the static dispersion component in sector j .

By an identity, aggregate labor productivity is equivalent to:

$$\frac{Y_t}{L_t} = \sum_j e^{-M_t^j - D_t^j} \frac{L_t^j}{L_t}. \quad (2.6)$$

This implies that aggregate labor productivity can be expressed as an aggregate of sectoral mean and dispersion components, weighted by the share of labor allocated to each sector. Hence, when we look at *changes* in aggregate labor productivity, we have to account for the fact that input shares of different sectors can be changing over time. In turn, we have a third component, which reflects changes in the input share of a given sector, which we call

the sectoral-share component:

$$\begin{aligned}
\log\left(\frac{Y_t}{L_t}\right) &= \underbrace{\log\left(\frac{\sum_j \left(e^{-M_t^j}\right) \frac{L_{t-1}^j}{L_{t-1}}}{\sum_j \left(e^{-M_{t-1}^j}\right) \frac{L_{t-1}^j}{L_{t-1}}}\right)}_{\text{mean component}} + \underbrace{\log\left(\frac{\sum_j e^{-M_t^j - D_t^j} \frac{L_t^j}{L_t}}{\sum_j e^{-M_t^j - D_t^j} \frac{L_{t-1}^j}{L_{t-1}}}\right)}_{\text{sectoral share}} \\
&+ \underbrace{\log\left(\frac{\sum_j e^{-M_t^j - D_t^j} \frac{L_{t-1}^j}{L_{t-1}}}{\sum_j e^{-M_{t-1}^j - D_{t-1}^j} \frac{L_{t-1}^j}{L_{t-1}}}\right)}_{\text{dispersion component}} - \log\left(\frac{\sum_j \left(e^{-M_t^j}\right) \frac{L_{t-1}^j}{L_{t-1}}}{\sum_j \left(e^{-M_{t-1}^j}\right) \frac{L_{t-1}^j}{L_{t-1}}}\right). \quad (2.7)
\end{aligned}$$

In this decomposition, changes in aggregate labor productivity are broken into three components. First, a mean component which captures changes in an aggregation of sectoral mean log labor productivities. Second, a sectoral share component which captures the effect of the changing allocation of labor between sectors. This second component will be positive if labor is flowing from low labor productivity sectors to high labor productivities ones. Third, a dispersion component, which captures changes in the dispersion of firm log labor productivities *within* sectors.

2.2.3 Decomposing Changes in TFP using Decomposition II

We measure TFP, A_t , as:

$$A_t = \frac{Y_t}{K_t^\alpha L_t^{1-\alpha}}. \quad (2.8)$$

We assume that capital's share of output, α , is positive. We can thus rewrite (2.8) as:

$$\log(A_t) = \alpha \log\left(\frac{Y_t}{K_t}\right) + (1 - \alpha) \log\left(\frac{Y_t}{L_t}\right). \quad (2.9)$$

Taking changes in (2.9),

$$\Delta \log(A_t) = \alpha \Delta \log\left(\frac{Y_t}{K_t}\right) + (1 - \alpha) \Delta \log\left(\frac{Y_t}{L_t}\right). \quad (2.10)$$

In (2.7), we showed that changes in $\log(\frac{Y_t}{L_t})$ can be broken into mean, dispersion, and sectoral-share components. Denote these components for labor as M_t^L, D_t^L , and S_t^L , respectively. Denote these components for capital as M_t^K, D_t^K , and S_t^K , respectively. Hence,

$$\Delta \log\left(\frac{Y_t}{K_t}\right) = M_t^K + D_t^K + S_t^K. \quad (2.11)$$

In turn, we can rewrite changes in \log TFP from (2.10) as changes in the weighted sum of the mean components for capital and labor, the dispersion components for capital and labor, and the sectoral-share components for capital and labor:

$$\Delta \log(A_t) = (\alpha M_t^K + (1 - \alpha) M_t^L) + (\alpha D_t^K + (1 - \alpha) D_t^L) + (\alpha S_t^K + (1 - \alpha) S_t^L). \quad (2.12)$$

2.3 Decomposition Applied to Models

In this section, we demonstrate the economics of our decomposition in standard production environments. First, in the production environment described by Hsieh and Klenow (2009), we demonstrate that changes in the production technology, prices, or the expected value of the log of marginal revenue products of capital will manifest themselves in the mean component of our decomposition. Second, changes in the variance or higher order moments of the log of marginal revenue products of capital will be reflected in the dispersion component of our decomposition.

Building on the above results, we demonstrate in a standard production environment how the components of our decomposition capture changes in the distribution of distortions to firm labor and capital choices. We find that common changes to the frictions to input

choices facing firms are reflected in movements in the mean component. We also derive conditions under which distributional changes in such frictions are reflected in the dispersion component of our decomposition. Such results are derived in a more general framework than that of Hsieh and Klenow (2009), and we identify a number of relevant papers that can be mapped into our environment.

Our results are particularly relevant to the literature that studies the role financial frictions play in amplifying movements in aggregates over the business cycle. We analytically demonstrate how a change in a financial friction in a simple model of production will present itself as a distortion. We then demonstrate that an increase in the extent to which this financial friction affects firms will increase the dispersion in wedges.

2.3.1 Hsieh and Klenow (2009) Production Environment

The model consists of heterogeneous firms that produce differentiated goods. There are S industries, and the outputs of each industry, Y_S , are aggregated into a final good (total output), Y , using Cobb-Douglas technology in a perfectly competitive market. Hence, aggregate output can be defined as:

$$Y = \prod_{s=1}^S Y_s^{\theta_s}, \text{ where } \sum_{s=1}^S \theta_s = 1. \quad (2.13)$$

From standard arguments: $P_S Y_S = \theta_S P Y$, where the price of industry output is P_S and P is the price of the final good, which is set to be the numeraire.

There are M_s firms in a sector s . Industry output, Y_s is produced using CES technology:

$$Y_s = \left(\sum_{i=1}^{M_s} Y_{si}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \quad (2.14)$$

Within an industry, firms are heterogeneous in a few dimensions. First, they vary in aspects of their physical productivity. Second, they vary in the magnitude of frictions to

their labor and capital choices. One can write these two distortions as distortions that affect the marginal products of labor and capital evenly, which one can write as an output distortion τ_Y , and distortions that affect the marginal product of capital relative to labor, which one can write as a capital distortion, τ_K . Firm i within sector s produces output, Y_{si} , from its firm TFP, A_{si} , capital stock K_{si} , and labor L_{si} , using the following Cobb-Douglas technology:

$$Y_{si} = A_{si} K_{si}^{\alpha_s} L_{si}^{1-\alpha_s}. \quad (2.15)$$

Profits of firm i in sector S are thus:

$$\pi_{si} = (1 - \tau_{Y_{si}}) P_{si} Y_{si} - w L_{si} - (1 + \tau_{K_{si}}) R K_{si}. \quad (2.16)$$

From standard arguments, in this setup, the marginal revenue product of capital for a firm, $MRPK_{si} \triangleq \frac{\partial P_{si} Y_{si}}{\partial K_{si}}$, is a function of the rental rate of capital and firm level wedges:

$$MRPK_{si} = R \frac{1 + \tau_{K_{si}}}{1 - \tau_{Y_{si}}}. \quad (2.17)$$

As in Hsieh and Klenow (2009), it is useful to define the marginal product of capital (in total) for a sector as the following:⁵⁰

$$\overline{MRPK}_s \triangleq \frac{R}{\sum_{i=1}^{M_S} \frac{1 - \tau_{Y_{si}}}{1 + \tau_{K_{si}}} \frac{K_{si}}{K_S}}. \quad (2.18)$$

For our decomposition, it is also useful to define the weighted average of the log marginal

⁵⁰There was an error in the specification of this object in the original paper of Hsieh and Klenow (2009). The specification here corresponds to the form in the published corrections.

product of capital for firms in a sector, which is:

$$\overline{LMRPK}_s \triangleq \sum_{i=1}^{M_s} \log\left(R \frac{1 + \tau_{K_{si}}}{1 - \tau_{Y_{si}}}\right) \frac{P_{si} Y_{si}}{P_S Y_S}. \quad (2.19)$$

2.3.1.1 Our Decomposition in this Production Environment

In the environment above, from (2.18) and (2.19) and the definition of the marginal revenue product of capital, our decomposition applied to capital productivity for sector s can be expressed as the following:

$$\begin{aligned} \Delta \log \left(\frac{P_t^s Y_t^s}{K_t^s} \right) = & \underbrace{\Delta \log \left(\frac{1}{\alpha_s} \frac{\sigma}{1 - \sigma} \right) + \Delta \overline{LMRPK}_s}_{\text{mean component}} \\ & + \underbrace{\Delta \log (\overline{MRPK}_s) - \Delta \overline{LMRPK}_s}_{\text{dispersion component}}. \end{aligned} \quad (2.20)$$

We now demonstrate how the changing distribution of marginal productivities are reflected in our decomposition by demonstrating which cumulants of the distribution of log marginal revenue productivities show up in which components of our decomposition. Cumulants are similar to moments; the cumulant-generating function of a random variable is an alternative specification of a probability distribution, similar to a moment-generating function. The first cumulant is the expected value of the variable, the second cumulant is its variance, and the higher order cumulants are polynomial combinations of centralized moments. Consider the distribution of firm log marginal revenue products of capital, reflecting both the mass of firms at a given productivity and their relative output shares (to be precise, the CDF would be written $G_s(X) = \int_{i \in s} \mathbb{1}(MRPK_{si} \leq X) \frac{P_{si} Y_{si}}{P_S Y_S} di$). Denote the cumulants of this distribution as $\kappa_1^s, \kappa_2^s, \dots$. Using properties of the cumulant generating function, our decomposition can be expressed as the following function of the cumulants of the distribution

of log marginal revenue productivities of capital:⁵¹

$$\Delta \log \left(\frac{P_t^s Y_t^s}{K_t^s} \right) = \underbrace{\Delta \log \left(\frac{1}{\alpha_s} \frac{\sigma}{1 - \sigma} \right) + \Delta \kappa_1^s}_{\text{mean component}} + \underbrace{-\frac{\Delta \kappa_2^s}{2!} + \frac{\Delta \kappa_3^s}{3!} - \frac{\Delta \kappa_4^s}{4!} + \dots}_{\text{dispersion component}} \quad (2.21)$$

Note that $\kappa_1^s = \overline{LMRPK_s}$ is the weighted average of the log of firm marginal revenue products of capital, while κ_2^s is the variance of the log of firm marginal revenue products of capital. The mean component captures only changes in technology or $\overline{LMRPK_s}$. Changes in the second cumulant (and thus second central moment), or higher order cumulants (and thus all of the remaining higher order moments) of the distribution of the log of firm marginal revenue products of capital are reflected in the dispersion component of our decomposition.

Note that if firm marginal revenue products of capital are lognormally distributed, then only the first two cumulants are non-zero. In that case, our decomposition is isomorphic to a mean-variance decomposition. Increases in the expected value of the log of firm marginal revenue productivities are reflected in the mean component of our decomposition, while the negative effect of the greater variance of the log of firm marginal revenue products is reflected in the dispersion component of our decomposition. This is apparent if in (2.21) the dispersion component is further broken into variance and higher order terms as below:

$$\Delta \log \left(\frac{P_t^s Y_t^s}{K_t^s} \right) = \underbrace{\Delta \log \left(\frac{1}{\alpha_s} \frac{\sigma}{1 - \sigma} \right) + \Delta \kappa_1^s}_{\text{mean component}} + \underbrace{-\frac{\Delta \kappa_2^s}{2!} + \overbrace{\sum_{n=3}^{\infty} (-1)^{n-1} \frac{\Delta \kappa_n^s}{n!}}^{\text{higher-order terms}}}_{\text{dispersion component}} \quad (2.22)$$

Notice, if we define $\kappa_{L,n}$ as the n 'th cumulant of the log of the marginal revenue product of labor and $\kappa_{K,n}$ as the n 'th cumulant of the log of the marginal revenue product of capital,

⁵¹See Appendix C for details of this derivation.

we can apply our decomposition to changes in TFP using (2.12):

$$\begin{aligned}
\Delta \log(TFPR_s) &= \underbrace{\Delta \log \left(\frac{\sigma}{1-\sigma} \alpha_s^{-\alpha_s} (1-\alpha_s)^{\alpha_s-1} \right) + (1-\alpha_s) \Delta \kappa_{L,1}^s + \alpha_s \Delta \kappa_{K,1}^s}_{\text{mean component}} \\
&\quad \underbrace{- \frac{(1-\alpha_s) \Delta \kappa_{L,2}^s + \alpha_s \Delta \kappa_{K,2}^s}{2!}}_{\text{variance}} + \underbrace{\sum_{n=3}^{\infty} \frac{(1-\alpha_s) \Delta \kappa_{L,n}^s + \alpha_s \Delta \kappa_{K,n}^s}{(-1)^{n-1} n!}}_{\text{higher-order terms}}. \\
&\quad \underbrace{\hspace{10em}}_{\text{dispersion component}}.
\end{aligned} \tag{2.23}$$

2.3.2 Simple Model of Production and Allocation

Given an increase in the dispersion of wedges likely results in a change in value added shares, we demonstrate under what conditions we can analytically demonstrate that an increase in the dispersion of wedges leads to an increase in the dispersion component of our decomposition. Similarly, we demonstrate under what conditions we can analytically demonstrate that an increase in the mean of wedges will increase the mean component of our decomposition, all else equal. We present our results within a similar environment to Hsieh and Klenow (2009), but more general technology.

As in Hsieh and Klenow (2009), the model consists of heterogeneous firms who produce differentiated goods, which are aggregated into a final good (total output), but now with a more general aggregation technology to be described below. Firms are heterogeneous in the following dimensions: They vary in aspects of their physical productivity, z_{it} , and they vary in the magnitude of frictions to their labor and capital choices.

2.3.2.1 Intermediate Good Firm Technology

Firms are indexed by i and time by t . Firm i at time t produces $y_{i,t}$ units of an intermediate good using $l_{i,t}$ units of homogeneous labor and $k_{i,t}$ units of capital with the production function $y_{i,t} = z_{i,t} l_{i,t}^\gamma k_{i,t}^\nu$. Labor and capital are homogeneous, therefore aggregate labor and

capital clearing imply that $\int_i l_{i,t} di = L_t$ and $\int_i k_{i,t} di = K_t$, where L_t and K_t denote aggregate labor and capital.

2.3.2.2 Aggregation Technology and Value Added

Total output, Y_t , is aggregated from firm output with technology $Y_t = (\int_i y_{i,t}^\varphi di)^\phi$. Note that this general form nests the two most common final good technologies considered in the literature as special cases: The CES aggregator and heterogeneous firms producing a single good. The final good sector is competitive and cost minimizing. Standard arguments imply the price of each intermediate good, $p_{i,t}$, is $p_{i,t} = Y_t^{\frac{\phi-1}{\phi}} P_t (y_{i,t})^{\varphi-1}$, where P_t is the price of the final good, which we set to be the numeraire. Therefore value added in real terms, $v_{i,t} = \frac{p_{i,t}}{P_t} y_{i,t}$, can be expressed as a function of prices and firm output:

$$v_{i,t} = Y_t^{\frac{\phi-1}{\phi}} (y_{i,t})^\varphi. \quad (2.24)$$

To compute our decomposition, one requires firm-level productivity ratios and firm-level value-added shares. For a given firm, we can compute a firm-level value-added share as: $\frac{v_{i,t}}{Y_t}$.

2.3.3 The Optimal Allocation of Inputs and the Role of Firm-level Wedges

In this subsection, we solve the optimal allocation of resources in the planner's problem, and we demonstrate how firm-specific wedges can distort the allocation of labor and capital between firms from this allocation.

We show that the optimal allocation of resources in the planner's problem is such that all firms have the same productivity ratios. This choice is unique and can be characterized as a function of the distribution of firm-level TFP, $F_t^z(z)$. We then show that any allocation of capital and labor between firms can be expressed as a function of the optimal input choice and firm-specific wedges. We utilize this final result in the following subsection to evaluate

how shocks to firm-level wedges show up in our decomposition.

2.3.3.1 Optimal Allocation

We now present a proposition that highlights the known result that for any fixed amount of total capital and labor, the optimal allocation of resources (to maximize static output) is such that all firms with identical production function coefficients have the same factor productivity ratios.

Proposition 2.1.

1. *Given a fixed amount of total labor and capital, L_t and K_t , the allocation of capital and labor across firms that maximizes output is such that there are unique optimal labor and capital productivity ratios, $\frac{v_t^*}{l_t^*}$ and $\frac{v_t^*}{k_t^*}$, which are common among all firms and only depend on the CDF of firm productivity, $F_t^z(z)$.*

Proof. See Appendix C. □

A full (static) planner's problem maximizing current welfare could be split into two parts: First, solve for the optimal allocation rule of capital and labor between firms for any fixed amount of both capital and labor; and second, choose the total amount of capital and labor to maximize current period utility. Therefore Proposition 2.1 implies that the allocation which maximizes static utility is one where firms have constant productivity ratios. We do not place any restrictions on the level of statically optimal total labor and capital.

2.3.3.2 Firm-Level Wedges

We then use the optimal labor and capital productivity ratios to define firm-level wedges, defined as the firm productivity ratio, $\frac{v_{i,t}}{l_{i,t}}$ or $\frac{v_{i,t}}{k_{i,t}}$, over the optimal productivity ratio, $\frac{v_t^*}{l_t^*}$ or $\frac{v_t^*}{k_t^*}$. We formally define firm level wedges as:

$$\omega_{l,i,t} \triangleq \frac{v_{i,t}}{l_{i,t}} \frac{l_t^*}{v_t^*}, \tag{2.25}$$

and

$$\omega_{k,i,t} \triangleq \frac{v_{i,t}}{k_{i,t}} \frac{k_t^*}{v_t^*}, \quad (2.26)$$

for labor and capital, respectively.

These wedges capture how far a firm's productivity ratio is from the one that maximizes welfare in the social planner's static optimization problem. They also capture aggregate distortions, which distort every firm's input decision and change aggregate labor or capital, as well as changes in the relative distribution of resources between firms.

In the model of Hsieh and Klenow (2009), which is a special case of this production environment, these wedges can be expressed as functions of the firm-level distortions in their model, τ_{Ysi} and τ_{Ksi} . The firm-level wedges are proportional to these distortions: $\omega_{l,i,t} \propto \frac{1}{1-\tau_{Ysi}}$ and $\omega_{k,i,t} \propto \frac{1+\tau_{Ksi}}{1-\tau_{Ysi}}$.

2.3.4 Shocks to Firm-level Wedges in our Decompositions

In this subsection, we illustrate how changes to the distribution of firm-level wedges are captured in our decomposition and how such changes affect aggregates. The model of production and allocation (from subsection 2.3.2) we consider only has a single sector of production with identical production function coefficients, so we can perform our analysis using Decomposition I. However, Decomposition II first applies Decomposition I individually to each sector and then aggregates up the sectoral mean and dispersion components. Therefore, the way in which our components capture firm-level wedges will be similar for a multi-sector version of our model with production function coefficients varying across sectors. We perform our analysis of shocks to firm-level wedges only for Decomposition I due to the greater tractability and cleaner demonstration of the economics of our decomposition.

We begin *without* making parametric assumptions on the distribution of wedges. Let $\frac{Y_t^*}{K_t^*}$ denote the undistorted (absent any idiosyncratic or common distortions) aggregate capital

factor productivity ratio. Let $F_{\omega,k}$ denote the density function of wedges to firm capital choices, which reflects both the mass of firms and their relative value added shares. Formally, this can be expressed as an integral over firms (denoted by i): $F_{\omega,k}(x) = \int_i 1_{(\omega_{k,i} \leq x)} \frac{v_i}{Y} di$. The cumulants of the log of firm-level wedges are denoted as: $\kappa_{k,1,t}$, $\kappa_{k,2,t}$, $\kappa_{k,3,t}$, et cetera. Cumulants are similar to moments; we discuss their statistical properties in subsection 2.3.1.1. Our decomposition of changes in aggregate capital productivity can be expressed as:⁵²

$$\Delta \log \left(\frac{Y_t}{K_t} \right) = \underbrace{\Delta \log \left(\frac{Y_t^*}{K_t^*} \right) + \Delta \kappa_{k,1,t}}_{\text{mean component}} + \underbrace{-\frac{\Delta \kappa_{k,2,t}}{2!} + \sum_{n=3}^{\infty} (-1)^{n-1} \frac{\Delta \kappa_{k,n,t}}{n!}}_{\text{dispersion component}}. \quad (2.27)$$

Equation (2.27) shows that the mean component captures changes in the undistorted productivity ratio (capturing changes unrelated to distortions) and changes in the first cumulant (which capture common changes in distortions). Changes in the variance of firm log wedges (the second cumulant), or any higher moments of their distribution, are reflected in the dispersion component. Such results easily carry over for labor productivity by replacing capital for labor in (2.27).

We can then use (2.12) to express total factor productivity as a function of undistorted

⁵²See Appendix C for details of this derivation.

TFP, TFP_t^* , and the cumulants of wedges to capital and labor:⁵³

$$\begin{aligned} \Delta \log (TFP_t) = & \underbrace{\Delta \log (TFP_t^*) + \alpha \Delta \kappa_{k,1,t} + (1 - \alpha) \Delta \kappa_{l,1,t}}_{\text{mean component}} \quad (2.28) \\ & \underbrace{\underbrace{\frac{\alpha \Delta \kappa_{k,2,t} + (1 - \alpha) \Delta \kappa_{l,2,t}}{2!}}_{\text{variance}} + \underbrace{\sum_{n=3}^{\infty} \frac{\alpha \Delta \kappa_{k,n,t} + (1 - \alpha) \Delta \kappa_{l,n,t}}{(-1)^{n-1} n!}}_{\text{higher-order terms}}}_{\text{dispersion component}}. \end{aligned}$$

Equation (2.28) shows that the mean component captures changes in undistorted TFP and changes in the first cumulants of log wedges (corresponding to the mean of log wedges). The dispersion component captures changes in the variance or higher order moments of log wedges.

In the remainder of this subsection, we present special cases of the results in (2.27) and (2.28). We show how common changes in wedges and changes in technology are reflected in the mean component of our decomposition. Finally, we show that changes in the variance and higher-order moments of log wedges are captured in the dispersion component our decomposition.⁵⁴

2.3.4.1 Common Shocks to Distortions

We first show that common changes to distortions are reflected *only* in the mean component of our decomposition. First, consider a shock, ξ_{t+1} , that affects firms evenly such that $\omega_{k,t+1} = \xi_{l,t+1} \omega_{k,t}$. This is the sort of shock that would arise, for example, from a distortion to the rental rate of capital faced by all firms. Below, we show the effect of this shock on

⁵³For this exercise, we make the standard assumption that TFP is measured as $TFP_t = \frac{Y_t}{K_t^\alpha L_t^{1-\alpha}}$, where α is constant over time.

⁵⁴These results follow directly from our decomposition and properties of cumulant generating functions; an outline of their derivation are found in Appendix C.

aggregate capital factor productivity and TFP is reflected only in the mean component:

$$\Delta \log\left(\frac{Y_t}{K_t}\right) = \underbrace{\log(\xi_{k,t})}_{\text{mean component}},$$

and

$$\Delta \log(TFP_t) = \underbrace{\alpha \log(\xi_{k,t})}_{\text{mean component}}.$$

Only the mean component will change if the economy is hit by no other shocks. Such results extend to the decomposition of labor productivity when we replace labor for capital.

2.3.4.2 Shocks to the Variance and Higher-order Moments of Distortions

Note that the second cumulant is the variance of log wedges, while all of the higher order cumulants can be expressed as polynomial combinations of the second and higher-order central moments. Therefore (2.27) and (2.28) imply that changes in the variance and any higher-order moments of log wedges are reflected in the dispersion component, without having to make any parametric assumptions.

To provide further intuition, we now demonstrate how shocks to the distribution of wedges are realized in our decomposition under some standard parametric assumptions. For example, if wedges to capital are lognormally distributed with mean $\mu_{\omega,k,t}$ and variance $\sigma_{\omega,k,t}^2$, then changes in aggregate capital productivity can be decomposed as:

$$\Delta \log\left(\frac{Y_t}{K_t}\right) = \underbrace{\Delta \log\left(\frac{Y_t^*}{K_t^*}\right) + \Delta \mu_{\omega,k,t}}_{\text{mean component}} + \underbrace{-\frac{\Delta \sigma_{\omega,k,t}^2}{2}}_{\text{dispersion component}}.$$

With the lognormal assumption, only the first and second cumulant exist. A typical way of adding variation in higher-order central moments (and thus higher-order cumulants) is to create a mixture of lognormals. If wedges to capital are modeled as a mixture of lognormals,

with weights $\lambda_{k,n,t}$ on lognormal distributions with means $\mu_{k,n,t}$ and variances $\sigma_{k,n,t}^2$, then we can express our decomposition as follows:

$$\Delta \log \left(\frac{Y_t}{K_t} \right) = \underbrace{\Delta \log \left(\frac{Y_t^*}{K_t^*} \right) + \Delta \sum_n \lambda_{k,n,t} \mu_{k,n,t}}_{\text{mean component}} - \underbrace{\frac{\Delta \sum_n \lambda_{k,n,t} \sigma_{k,n,t}^2}{2} - \Delta \log \left(\sum_n \lambda_{k,n,t} e^{(\mu_{k,n,t} - \sum_j \lambda_{k,j,t} \mu_{k,j,t})} e^{\frac{1}{2}(\sigma_{k,n,t}^2 - \sum_j \lambda_{k,j,t} \sigma_{k,j,t}^2)} \right)}_{\text{dispersion component}}.$$

The mean component captures changes in the weighted means of the lognormal distributions or in the undistorted factor productivity ratio. All other changes in the distributions will be reflected in the dispersion component. Changes in the variances of the lognormal distributions which make up the mixture will be reflected here, as will higher order moments. For example, a skewed distribution is often parameterized as a mixture of lognormals with different means, which will be reflected, via the term $e^{(\mu_{k,n,t} - \sum_j \lambda_{k,j,t} \mu_{k,j,t})}$, in the dispersion component. Kurtosis is often often parameterized as a mixture of lognormals with different variances, which will be reflected, via the term $e^{\frac{1}{2}(\sigma_{k,n,t}^2 - \sum_j \lambda_{k,j,t} \sigma_{k,j,t}^2)}$, in the dispersion component.

2.3.5 Mapping to Other Models

In this subsection, we show that our model has a mapping to several models of frictions to the allocation of labor and capital between firms. We then demonstrate how frictions in a simple model of financial frictions would be reflected in wedges, and then discuss how our decomposition would capture changes in such frictions.

2.3.5.1 Mapping to Models of Labor or Capital Allocation

Our simple model consists only of a production environment with wedges representing frictions to the allocation of labor and capital. Therefore, there is a mapping to any model

with a production environment consistent with ours. This includes the models of Khan and Thomas (2013), Bloom et al. (2014b), and Arellano et al. (2012), as well as numerous other heterogeneous agent models considered in the macroeconomics literature.

We formally show this correspondence by proving that given the production environment in our model, aggregate output, employment, capital, and the full distribution of output, labor, capital, and technology across firms can be characterized using wedges and either firm-level technology or output shares for any allocation of labor, capital, and technological productivity across firms. We denote $G_t(z, \omega_l, \omega_k)$ as the joint distribution of firm technological productivity and firm-level wedges to labor and capital, $J_t\left(\frac{v}{Y}, \omega_l, \omega_k\right)$ as the joint distribution of firm output shares and firm-level wedges to labor and capital, and $Z_t = \left(\int_i \left(z_{i,t}^{\varphi\left(\frac{1}{1-\nu\varphi-\gamma\varphi}\right)}\right) di\right)^{\phi(1-\nu\varphi-\gamma\varphi)}$ as an index of aggregate productivity. The following proposition states that this representation can map any resulting allocation of resources and aggregates using these firm-level wedges:

Proposition 2.2. *The full distribution of labor, capital, and productivity across firms, $F_t(z, l, k)$, and aggregate output, employment, and capital have a 1-1 mapping with any of the following:*

1. $G_t(z, \omega_l, \omega_k)$.
2. $J_t\left(\frac{v}{Y}, \omega_l, \omega_k\right)$ and a measure of aggregate productivity Z_t .⁵⁵

Proof. See Appendix C. □

2.3.5.2 Mapping to a Model of Financial Frictions

In this subsection, we show that a financial friction in a simple model map can be redefined as a firm-level wedge. We then discuss how a tightening of the friction in the model can generate a greater variance of wedges.

⁵⁵More generally, it can be shown that $J_t\left(\frac{v}{Y}, \omega_l, \omega_k\right)$ together with $\left(\int z^\tau dF^z(z)\right)$, for any $\tau \neq 0$, is sufficient to characterize output, employment, and capital at both the aggregate and firm level.

Consider a simple model where heterogeneous firms produce a homogeneous consumption good with technology $y_{i,t} = z_{i,t} l_{i,t}^b$, where $b < 1$. In this setting, value added is equivalent to firm output, $v_{i,t} = z_{i,t} l_{i,t}^b$. Households have utility function $U(C, L) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{L^{1+\nu}}{1+\nu}$ and discount the future at rate β . Production in this simple model is a special case of the production environment introduced in Subsection 2.3.2; thus, the planner's problem states all firms optimally have the same labor productivity ratios.

The friction in this model is a simple borrowing constraint: Firms have wealth $a_{i,t}$, which we consider exogenous for our analysis. Firms must pay their workers at the beginning of the period but only receive cash flows from production at the end. The borrowing constraint, $l_{i,t}W \leq a_{i,t}\rho$, where W is the wage and ρ is a positive constant, restricts the labor decisions of firms when it binds. Assume firms may also exogenously exit each period with probability δ .

The optimization problem of firms can be expressed as the following Lagrangian:

$$\mathcal{L} = \max_{l_{i,t}} z_{i,t} l_{i,t}^b - l_{i,t}W + \lambda_{i,t} (a_{i,t}\rho - l_{i,t}W_t), \quad (2.29)$$

where $\lambda_{i,t}$ is the Lagrange multiplier on the borrowing constraint. The multiplier is 0 if the borrowing constraint does not bind, and positive otherwise. Taking the first-order conditions of (2.29) and manipulating the labor-leisure condition allows us to express the firm's labor choice as the following:

$$l_{i,t} = z_{i,t}^{\frac{1}{1-b}} b^{\frac{1}{1-b}} Y_t^{-\frac{\sigma}{1-b}} L_t^{-\frac{\nu}{1-b}} (1 + \lambda_{i,t})^{-\frac{1}{1-b}}. \quad (2.30)$$

From (2.30) we can derive firm-level wedges, $\omega_{l,i,t} = \frac{v_{i,t} l_{i,t}^*}{l_{i,t} v_i^*}$, as a function of aggregates and the Lagrange multiplier faced by the firm:

$$\log(\omega_{l,i,t}) = \underbrace{\sigma \log\left(\frac{Y_t}{Y_t^*}\right) + \nu \log\left(\frac{L_t}{L_t^*}\right)}_{\text{Aggregate}} + \underbrace{\log(1 + \lambda_{i,t})}_{\text{Firm Specific}}. \quad (2.31)$$

Note that the wedge can be expressed as a function of the distortion of aggregates from their optimal value (which affect the wage rate) as well as the firm-specific distortion captured by the Lagrange multiplier in the firm's problem. We can express the Lagrange multiplier as the following function of aggregates and each firm's $z_{i,t}$ and $a_{i,t}$:

$$\log(1 + \lambda_{i,t}) = \begin{cases} 0 & z_{i,t} b Y_t^{-b\sigma} L_t^{-b\nu} a_{i,t}^{b-1} \rho^{b-1} \leq 1 \\ \log(b Y_t^{-b\sigma} L_t^{-b\nu} z_{i,t} a_{i,t}^{b-1} \rho^{b-1}) & z_{i,t} b Y_t^{-b\sigma} L_t^{-b\nu} a_{i,t}^{b-1} \rho^{b-1} > 1 \end{cases}. \quad (2.32)$$

Note that the only way that there is no heterogeneity in Lagrangian multipliers is either if (a) the borrowing constraint never binds, or (b) it binds for all firms, but wealth is proportional to productivity ($z_{i,t} = a_{i,t}^{1-b}$). This second condition implies no inefficiencies in the distribution of resources between firms; all inefficiencies arise from the reduced aggregate demand for labor.

Now consider what a shock to borrowing constraints does. Assume that at time $t = 0$, ρ is high enough such that the constraint binds for no firms. Then the mean and variance of $\log(1 + \lambda_{i,0})$ is 0. A decrease in ρ to the point where the constraint binds for some but not all firms leads to a rise in both the mean and variance of $\log(1 + \lambda_{i,t})$. At the same time, the borrowing constraint reduces Y_t and L_t , as some firms cannot hire the amount of labor they would prefer were they unconstrained. Therefore the change to the mean of firm wedges, $\log(\omega_{l,i,t})$, as a result of this shock is ambiguous, as the aggregate component and the mean of the firm-specific component move in opposite directions. However, the direction of the change in variance of the firm-level labor wedge is unambiguous, increasing in response to such a shock.

2.4 Decomposition Applied to Data

In this section, we apply our decompositions of aggregate productivity described in Section 2.2 to data on U.S. public firms and Japanese public firms. In our discussion of the re-

sults applied to U.S. public firms, we also include a comparison of our measures of labor productivity, capital productivity, and TFP to those from the national income and product accounts (NIPA). In Appendix D, we describe how we clean our data on U.S. nonfinancial public firms, and measure the objects of interest.

2.4.1 Discussion of results — Data from the United States

Figures 2.1 and 2.2 display results of year-over-year changes in aggregate labor productivity and its components from Decompositions I and II, respectively. From the eye test alone it should be clear that in the recent recession, aggregate labor productivity and its dispersion component have a negative correlation, and the mean component is highly correlated with aggregate labor productivity. Figures 2.3 and 2.4 further demonstrate this point: Over four recession periods, the mean component moves closely with aggregate labor productivity. In Decomposition II, the dispersion component has very little cumulative change over any of the four episodes in our sample.

Figures 2.5 and 2.6 display results from Decompositions I and II of year-over-year changes in aggregate capital productivity, and tell a different story. The dispersion component is positively correlated with aggregate capital productivity over the past two business-cycle episodes. For previous episodes, the mean component moves more closely with aggregate capital productivity. These results are more starkly apparent in Figures 2.7 and 2.8, which show cumulative changes in aggregate capital productivity and its components from Decompositions I and II. In the recent episodes, for either decomposition, the dispersion component moves much more closely with aggregate capital productivity.

As we describe in the previous subsection on measurement, to compute TFP, by the nature of our assumptions on the production function and values for its coefficients, changes in labor productivity get more weight (65 percent) than changes in capital productivity (35 percent). Hence, as should be expected, we see that the results for TFP are much more qualitatively consistent with the results from what drives changes in labor productivity. This

is apparent in the year-over-year changes charts from Decompositions I and II in Figures 2.9 and 2.10, as well as the cumulative changes charts from Decompositions I and II in Figures 2.11 and 2.12.

Table 2.1 displays correlations between the components of Decomposition II and their respective aggregates. The results from the figures are further codified in this table. The correlation between the dispersion component and sectoral share components and labor productivity are especially striking. Movements in labor productivity are much more correlated with the mean of firm-level log labor to value-added ratios than with their dispersion. These results are dampened when looking at TFP because capital productivity has a positive correlation with its dispersion component. However, the relationship between TFP and its dispersion component is ultimately close to zero.

Our sample represents a significant slice of the U.S. economy; in 2011, it accounted for over 15 percent of GDP and over 17 million employees. To understand the extent to which our sample reflects the mechanisms responsible for driving aggregate productivity changes, we compare the time-series behavior of each aggregate productivity ratio as aggregated from Compustat to that of the respective productivity ratio computed from NIPA. Figures 2.16 and 2.17 show that the time-series properties of TFP computed from both Compustat and NIPA are similar both in their cyclical dynamics and long-term trends. These figures suggest that some of the key forces driving TFP over time are likely present in Compustat data. If there were significant factors driving TFP over the business cycle that existed only in small, private firms, we would expect systematic differences in the behavior of TFP and our measure computed from publicly listed firms over time. However, there are some differences in the measures. TFP from Compustat is more volatile, which is unsurprising given the documented greater volatility of corporate profits measured with generally accepted accounting principles (GAAP) than corporate profits as measured in NIPA.⁵⁶ There are also some slight timing

⁵⁶Hodge (2011) compares the properties of corporate profits computed from the GAAP accounting statements of firms in the S&P 500 index with the corresponding measure from NIPA, finding significantly greater volatility in the S&P measure.

differences, particularly in the timing of the trough (of TFP) of the 2007—2009 recession. These timing differences may be due to the reporting dates of firms in Compustat. However, the measure of TFP for the United States in the Penn World Tables (8.0) has the trough in 2009, so the timing differences may also be due to some technical adjustments made in the NIPA aggregation. In Figures 2.18 and 2.19, we look at changes in each productivity ratio and its NIPA equivalent. We see the timing and volatility issues are present for each productivity ratio separately.

2.4.2 Discussion of results - Data from Japan

In Figures 2.13 and 2.14, we display results from Decompositions I and II of year-over-year changes in aggregate TFP in Japan. The results from the second decomposition are more consistent with those from the United States for the recent recession in that the dispersion component is not correlated with movements in TFP. The results from the first decomposition, however, show the dispersion component to be more highly correlated with aggregate TFP over the recent episode. This result is true for labor productivity as well.

2.5 Conclusion

This paper presents decompositions of changes in aggregate labor productivity, capital productivity, and TFP. We demonstrate how the dispersion component of our decompositions reflects changes in the degree to which frictions affect firms in many heterogeneous firm models that attempt to explain the nature of the business cycle. In turn, computing the components of our decomposition in data and comparing them to the same metrics in a given model of the class we consider will help to assess whether such a model is consistent with firm-level behavior. As we demonstrate in this paper, it is not only useful to compute our decompositions on models that have already been solved; one can also compute our metrics in the data *before* writing down a model to help motivate which mechanisms should be key

in driving patterns over the business cycle.

Appendix C: Proofs and Derivations

Proof for Proposition 2.1

Given the model of production in Subsection 2.3.2, we can define the following Lagrangian for the social planner to solve supposing she gets to allocate a fixed amount of labor and capital across firms, which are indexed by i .⁵⁷

$$\mathcal{L} = \max_{l_i, k_i} \left(\int_i (z_i l_i^\gamma k_i^\nu)^\varphi di \right)^\phi + \lambda_1 \left(K - \int k_i di \right) + \lambda_2 \left(L - \int l_i di \right). \quad (\text{C.1})$$

We want to show that there exists optimal labor and capital productivity ratios that are shared by all firms that share the same production function coefficients. From the first-order conditions of (C.1):

$$\nu \varphi \phi Y^{\frac{1-\phi}{\phi}} \frac{(z_i l_i^\gamma k_i^\nu)^\varphi}{k} = \lambda_1, \quad (\text{C.2})$$

and

$$\gamma \varphi \phi Y^{\frac{1-\phi}{\phi}} \frac{(z_i l_i^\gamma k_i^\nu)^\varphi}{l} = \lambda_2. \quad (\text{C.3})$$

Also, the planner will fully allocate labor and capital to all firms, so:

$$K = \int k_i di, \quad (\text{C.4})$$

⁵⁷To economize on notation, time subscripts are omitted.

and

$$L = \int l_i di. \quad (\text{C.5})$$

With some algebra, it can be shown that:

$$v_i = k_i \frac{\lambda_1}{\nu \varphi \phi}, \quad (\text{C.6})$$

and

$$v_i = l_i \frac{\lambda_2}{\gamma \varphi \phi}. \quad (\text{C.7})$$

Hence, summing over i in (C.6) and (C.7):

$$Y = K \frac{\lambda_1}{\nu \varphi \phi}, \quad (\text{C.8})$$

and

$$Y = L \frac{\lambda_2}{\gamma \varphi \phi}. \quad (\text{C.9})$$

In turn, from (C.6) and (C.8):

$$\frac{Y}{K} = \frac{v_i}{k_i}. \quad (\text{C.10})$$

Also, from (C.7) and (C.9):

$$\frac{Y}{L} = \frac{v_i}{l_i}. \quad (\text{C.11})$$

In turn, all firms will optimally have the same firm-level capital and labor productivity ratios.

We can now express optimal productivity ratios $\frac{v^*}{l^*}$ and $\frac{v^*}{k^*}$ as a function of L , K , and Y^* . From the production technology, (C.10), and (C.11):

$$k_i = Y^{\frac{-1}{\phi}} z_i^\varphi l_i^{\varphi\gamma} k_i^{\varphi\nu} K^* (F^z(z)), \quad (\text{C.12})$$

and

$$l_i = Y^{\frac{-1}{\phi}} z_i^\varphi l_i^{\varphi\gamma} k_i^{\varphi\nu} K^* (F^z(z)). \quad (\text{C.13})$$

Combining the production technology with (C.12) and (C.13), along with some algebra, yields:

$$Y^* = L^{\varphi\phi\gamma} K^{\phi\varphi\nu} \left(\int_i \left(z_i^{\varphi \left(\frac{1}{1-\nu\varphi-\gamma\varphi} \right)} \right) di \right)^{\phi(1-\nu\varphi-\gamma\varphi)}. \quad (\text{C.14})$$

Note that this optimal output is just a function of the distribution of productivity, $F^z(z)$, and total labor and capital.

Thus, we can express the optimal productivity ratios as:

$$\frac{v^*}{l^*} = \frac{Y^*}{L}, \quad (\text{C.15})$$

and

$$\frac{v^*}{k^*} = \frac{Y^*}{K}. \quad (\text{C.16})$$

Proof for Proposition 2.2

This proof is done in the following parts:

1. $F(z, l, k)$ fully characterizes output, employment, and capital.
2. $F(z, l, k)$ has a 1-1 mapping with $G(z, \omega_l, \omega_k)$.

3. $G(z, \omega_l, \omega_k)$ has a 1-1 mapping with $J(\frac{v}{Y}, \omega_l, \omega_k)$ and a measure of aggregate productivity $Z = \left(\int_i \left(z_i^{\varphi \left(\frac{1}{1-\nu\varphi-\gamma\varphi} \right)} \right) di \right)^{\phi(1-\nu\varphi-\gamma\varphi)}$.

Part (i): $F(z, l, k)$ fully characterizes output, employment, and capital.

This must be true, by the definition of production technology and the clearing conditions $L = \int l dF(z, l, k)$ and $K = \int k dF(z, l, k)$. Thus, $F(z, l, k)$ fully characterize aggregate output, employment, and capital.

Part (ii): $F(z, l, k)$ has a 1-1 mapping with $G(z, \omega_l, \omega_k)$.

The portion of the proof has the following parts:

- (a) $F(z, l, k)$ has a unique mapping χ_1 into $G(z, \omega_l, \omega_k)$.
- (b) $G(z, \omega_l, \omega_k)$ has a unique mapping χ_2 into $F(z, l, k)$.
- (c) $\chi_2 = \chi_1^{-1}$.

$F(z, l, k)$ has a unique mapping χ_1 into $G(z, \omega_l, \omega_k)$:

(2.24) combined with the production technology gives us v_i as a function of only z, l, k :

$$v_i = \left(\int z^\varphi l^{\gamma\varphi} k^{\nu\varphi} dF(z, l, k) \right)^{\phi-1} z_i^\varphi l_i^{\gamma\varphi} k_i^{\nu\varphi}. \quad (\text{C.17})$$

Combining (2.25), (2.26), and (C.17) yields:

$$\omega_k(z_i, l_i, k_i, F) = \frac{\left(\int z^\varphi l^{\gamma\varphi} k^{\nu\varphi} dF(z, l, k) \right)^{\phi-1} z_i^\varphi l_i^{\gamma\varphi} k_i^{\nu\varphi} k^*}{k_i v^*}, \quad (\text{C.18})$$

and

$$\omega_l(z_i, l_i, k_i, F) = \frac{\left(\int z^\varphi l^{\gamma\varphi} k^{\nu\varphi} dF(z, l, k) \right)^{\phi-1} z_i^\varphi l_i^{\gamma\varphi} k_i^{\nu\varphi} l^*}{l_i v^*}. \quad (\text{C.19})$$

These equations characterize the wedges implied by a given distribution of capital, labor, and productivity. We can rearrange (C.18) and (C.19) to solve for labor and capital as a function of wedges:

$$k(z_i, \omega_{l,i}, \omega_{k,i}, F) = \left(\frac{\left(\int z^\varphi l^{\gamma\varphi} k^{\nu\varphi} dF(z, l, k) \right)^{\phi-1} z_i^\varphi \left(\frac{l^*}{v^*} \right)^{\gamma\varphi} \left(\frac{k^*}{v^*} \right)^{1-\gamma\varphi}}{(\omega_{l,i})^{\gamma\varphi} (\omega_{k,i})^{1-\gamma\varphi}} \right)^{\frac{1}{1-\gamma\varphi-\nu\varphi}}, \quad (\text{C.20})$$

and

$$l(z_i, \omega_{l,i}, \omega_{k,i}, F) = \left(\frac{\left(\int z^\varphi l^{\gamma\varphi} k^{\nu\varphi} dF(z, l, k) \right)^{\phi-1} z_i^\varphi \left(\frac{l^*}{v^*} \right)^{1-\nu\varphi} \left(\frac{k^*}{v^*} \right)^{\nu\varphi}}{(\omega_{l,i})^{1-\nu\varphi} (\omega_{k,i})^{\nu\varphi}} \right)^{\frac{1}{1-\gamma\varphi-\nu\varphi}}. \quad (\text{C.21})$$

(C.20) and (C.21) allow us to obtain the unique mapping from F to G :

$$G(\bar{z}, \bar{\omega}_l, \bar{\omega}_k) = \int_{z, \omega_l, \omega_k=0}^{\bar{z}, \bar{\omega}_l, \bar{\omega}_k} dF(z, l(z, \omega_l, \omega_k, F), k(z, \omega_l, \omega_k, F)). \quad (\text{C.22})$$

$G(z, \omega_l, \omega_k)$ has a unique mapping χ_2 into $F(z, l, k)$:

Combining (C.18) and (C.19) allows us to express $F(z, l, k)$ as the following:

$$F(\bar{z}, \bar{l}, \bar{k}) = \int_{z, l, k=0}^{\bar{z}, \bar{l}, \bar{k}} dG(z, \omega_l(z, l, k, F), \omega_k(z, l, k, F)). \quad (\text{C.23})$$

This expression is not sufficient to characterize $F(z, l, k)$ as a function of $G(z, \omega_l, \omega_k)$, as the functions $\omega_l()$ and $\omega_k()$ on the right-hand side depend on the term $\left(\int z^\varphi l^{\gamma\varphi} k^{\nu\varphi} dF(z, l, k) \right)^{\frac{\phi-1}{\phi}}$. Note that $\left(\int z^\varphi l^{\gamma\varphi} k^{\nu\varphi} dF(z, l, k) \right)^{\frac{\phi-1}{\phi}} = Y^{\frac{\phi-1}{\phi}}$. All we have to do now is express Y as a function of G . Plugging (C.20) and (C.21) into the aggregate production function yields:

$$Y = \left(\left(\frac{k^*}{v^*} \right)^{\varphi\phi\nu} \left(\frac{l^*}{v^*} \right)^{\varphi\phi\gamma} \left(\int \frac{z^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}}}{\omega_l^{\frac{\gamma\varphi}{1-\gamma\varphi-\nu\varphi}} \omega_k^{\frac{\nu\varphi}{1-\gamma\varphi-\nu\varphi}}} dG(z, \omega_l, \omega_k) \right)^\phi \right)^{\frac{1-\gamma\varphi-\nu\varphi}{1-\phi(\gamma\varphi+\nu\varphi)}}. \quad (\text{C.24})$$

$Y(G(z, \omega_l, \omega_k))$ can thus be defined as a function of z and wedges.

(C.23) and (C.24) can be combined to obtain the functions $\omega_l(z, l, k, G)$ and $\omega_k(z, l, k, G)$.

Thus, we can obtain the unique mapping:

$$F(\bar{z}, \bar{l}, \bar{k}) = \int_{z, l, k=0}^{\bar{z}, \bar{l}, \bar{k}} dG(z, \omega_l(z, l, k, G), \omega_k(z, l, k, G)). \quad (\text{C.25})$$

$\chi_2 = \chi_1^{-1}$:

Combining (C.22) and (C.25) yields the result that $F(z, l, k) = \chi_2(\chi_1(F(z, l, k)))$ for any $F(z, l, k)$. It follows that $\chi_2 = \chi_1^{-1}$.

Part (iii): Claim: $G(z, \omega_l, \omega_k)$ has a 1-1 mapping with $J(\frac{v}{Y}, \omega_l, \omega_k)$ and a measure of aggregate productivity $Z = \left(\int_i \left(z_i^{\frac{\varphi}{1-\nu\varphi-\gamma\varphi}} \right) di \right)^{\phi(1-\nu\varphi-\gamma\varphi)}$.

The portion of the proof has the following parts:

- (a) $G(z, \omega_l, \omega_k)$ has a unique mapping χ_3 into $J(\frac{v}{Y}, \omega_l, \omega_k)$ and pins down Z .
- (b) $J(\frac{v}{Y}, \omega_l, \omega_k)$ and Z has a unique mapping χ_4 into $G(z, \omega_l, \omega_k)$.
- (c) $\chi_3 = \chi_4^{-1}$.

$G(z, \omega_l, \omega_k)$ has a unique mapping χ_3 into $J(\frac{v}{Y}, \omega_l, \omega_k)$ and pins down Z :

(C.17), (C.20), (C.21), and (C.24) can be combined to characterize $\frac{v}{Y}$:

$$\frac{v_i}{Y} = \frac{z_i^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}} (\omega_{l,i})^{\frac{-\gamma\varphi}{1-\gamma\varphi-\nu\varphi}} (\omega_{k,i})^{\frac{-\nu\varphi}{1-\gamma\varphi-\nu\varphi}}}{\int z^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}} (\omega_l)^{\frac{-\gamma\varphi}{1-\gamma\varphi-\nu\varphi}} (\omega_k)^{\frac{-\nu\varphi}{1-\gamma\varphi-\nu\varphi}} dG(z, \omega_l, \omega_k)}, \quad (\text{C.26})$$

and thus express z as a function of $\frac{v}{Y}$, ω_l , ω_k , and G :

$$z\left(\frac{v_i}{Y}, \omega_{l,i}, \omega_{k,i}, G\right) = \omega_{l,i}^{\gamma} \omega_{k,i}^{\nu} \left(\frac{v_i}{Y} \int \frac{z^{\frac{\varphi}{1-\gamma\varphi-\nu\varphi}}}{\omega_l^{\frac{\gamma\varphi}{1-\gamma\varphi-\nu\varphi}} \omega_k^{\frac{\nu\varphi}{1-\gamma\varphi-\nu\varphi}} dG(z, \omega_l, \omega_k)} \right)^{\frac{1-\gamma\varphi-\nu\varphi}{\varphi}}. \quad (\text{C.27})$$

(C.27) can be used to characterize $J(\frac{v}{Y}, \omega_l, \omega_k)$:

$$J\left(\frac{\bar{v}}{\bar{Y}}, \bar{\omega}_l, \bar{\omega}_k\right) = \int_{\frac{v}{Y}, \omega_l, \omega_k=0}^{\bar{v}, \bar{\omega}_l, \bar{\omega}_k} dG\left(z\left(\frac{v}{Y}, \omega_l, \omega_k, G\right), \omega_l, \omega_k\right). \quad (\text{C.28})$$

$G(z, \omega_l, \omega_k)$ trivially maps into a unique Z .

$J(\frac{v}{Y}, \omega_l, \omega_k)$ and Z has a unique mapping χ_4 into $G(z, \omega_l, \omega_k)$:

(C.27), rearranged and integrated, yields:

$$\int \frac{z^{\frac{1-\gamma\varphi}{1-\gamma\varphi-\nu\varphi}}}{\omega_l^{\frac{\gamma\varphi}{1-\gamma\varphi-\nu\varphi}} \omega_k^{\frac{\nu\varphi}{1-\gamma\varphi-\nu\varphi}}} dG(z, \omega_l, \omega_k) = \frac{Z^{\frac{1}{\phi(1-\nu\varphi-\gamma\varphi)}}}{\int\left(\frac{v}{Y}\right) \omega_l^{\frac{\gamma\varphi}{1-\nu\varphi-\gamma\varphi}} \omega_k^{\frac{\nu\varphi}{1-\nu\varphi-\gamma\varphi}} dJ\left(\frac{v}{Y}, \omega_l, \omega_k\right)}. \quad (\text{C.29})$$

Combining (C.27) and (C.29) yields:

$$z\left(\frac{v_i}{Y}, \omega_{l,i}, \omega_{k,i}, J, Z\right) = \omega_{l,i}^\gamma \omega_{k,i}^\nu \left(\frac{\frac{v_i}{Y} Z^{\frac{1}{\phi(1-\nu\varphi-\gamma\varphi)}}}{\int\left(\frac{v}{Y}\right) \omega_l^{\frac{\gamma\varphi}{1-\nu\varphi-\gamma\varphi}} \omega_k^{\frac{\nu\varphi}{1-\nu\varphi-\gamma\varphi}} dJ\left(\frac{v}{Y}, \omega_l, \omega_k\right)} \right)^{\frac{1-\gamma\varphi-\nu\varphi}{\varphi}}. \quad (\text{C.30})$$

(C.30) implies that we can express $G(z, \omega_l, \omega_k)$ as a function of $J(\frac{v}{Y}, \omega_l, \omega_k)$ and Z :

$$G(\bar{z}, \bar{\omega}_l, \bar{\omega}_k) = \int_{z, \omega_l, \omega_k=0}^{\bar{z}, \bar{\omega}_l, \bar{\omega}_k} dJ\left(\frac{v}{Y}(z, \omega_l, \omega_k, J, Z), \omega_l, \omega_k\right). \quad (\text{C.31})$$

$\chi_3 = \chi_4^{-1}$:

These two mappings are trivially inverses of each other.

Other Derivations

Decomposition as a Function of Cumulants

Consider the cumulative density function of firm log capital productivity, weighted by output shares, $G(X) = \int_i \mathbb{1} \left(\log \left(\frac{v_i}{k_i} \right) \leq X \right) \frac{v_i}{Y}$. The mean component expressed as a function of this distribution is: $-\int_x -x dG(x)$, while the dispersion component is:

$$-\left(\int e^{-x} dG(x) - \int -x dG(x) \right).$$

We know, by definition, that the first cumulant can be written as: $\int_x x dG(x)$. A property of the cumulant generating function is that $E[e^{tx}] = \sum_{t=1}^n t^n \frac{\kappa_n}{n!}$, which yields:

$$\int e^{-x} dG(x) = \sum_{t=1}^n (-1)^t \frac{\kappa_t}{t!} = -\kappa_1 + \sum_{t=2}^n (-1)^t \frac{\kappa_t}{t!}.$$

Therefore the mean component can be written as:

$$-\int_x -x dG(x) = \kappa_1.$$

While the dispersion component is:

$$\begin{aligned} -\left(\int e^{-x} dG(x) - \int -x dG(x) \right) &= -\left(-\kappa_1 + \sum_{t=2}^n (-1)^t \frac{\kappa_t}{t!} + \kappa_1 \right) \\ &= -\sum_{t=2}^n (-1)^t \frac{\kappa_t}{t!} = \sum_{t=2}^n (-1)^{t+1} \frac{\kappa_t}{t!}. \end{aligned}$$

This means our decomposition can be expressed as:

$$\log \left(\frac{Y}{K} \right) = \underbrace{\kappa_1}_{\text{Mean Component}} + \underbrace{-\kappa_2 + \sum_{t=3}^n (-1)^{t+1} \frac{\kappa_t}{t!}}_{\text{Dispersion Component}}.$$

Now note that for any variable of the form $z_i = c \frac{v_i}{k_i}$ (such as capital wedges or marginal

products in a Hsieh and Klenow case) yields $\log(z_i) = \log(c) + \log\left(\frac{v_i}{k_i}\right)$. Standard properties of cumulants imply that $\kappa_{1,z} = \kappa_{1,\frac{v}{k}} + \log(c)$, and $\kappa_{n,z} = \kappa_{n,\frac{v}{k}}$ for all $n > 1$. Therefore for such variables, our decomposition implies

$$\log\left(\frac{Y}{K}\right) = \underbrace{\kappa_{1,z} - c}_{\text{Mean Component}} + \underbrace{-\kappa_{2,z} + \sum_{t=3}^n (-1)^{n+1} \frac{\kappa_{n,z}}{n!}}_{\text{Dispersion Component}}.$$

(2.21) and (2.27) follow immediately from this derivation.

Our Decomposition for Different Models and Shocks

Common changes in firm revenue products, whether driven by technology or distortions, are reflected only in the mean component of our decomposition. Consider a change in revenue products such that $\frac{v_{i,t+1}}{k_{i,t+1}} = x \frac{v_{i,t}}{k_{i,t}}$. Then, our decomposition implies that:

$$\begin{aligned} \log\left(\frac{Y_{t+1}}{K_{t+1}}\right) &= \underbrace{-\int \log\left(\frac{k_{i,t}}{v_{i,t}} \frac{1}{x}\right) \frac{v_{i,t}}{Y_t} di}_{\text{mean component}} - \underbrace{\left(\log\left(\int \frac{k_{i,t}}{v_{i,t}} \frac{1}{x} \frac{v_t}{Y_{i,t}} di\right) - \int \log\left(\frac{k_{i,t}}{v_{i,t}} \frac{1}{x}\right) \frac{v_t}{Y_t} di\right)}_{\text{dispersion component}} \\ &= \underbrace{\log(x) - \int \log\left(\frac{k_{i,t}}{v_{i,t}}\right) \frac{v_{i,t}}{Y_t} di}_{\text{mean component}} - \underbrace{\left(\log\left(\int \frac{k_{i,t}}{v_{i,t}} \frac{v_t}{Y_{i,t}} di\right) - \int \log\left(\frac{k_{i,t}}{v_{i,t}}\right) \frac{v_t}{Y_t} di\right)}_{\text{dispersion component}}. \end{aligned}$$

The results in subsections 2.3.4.1 immediately follow from the above. The results in subsection 2.3.4.2 can be derived by using standard formulas for the expectation of log-normally distributed variables.

Specifically, consider the case where the output-share weighted distribution of wedges is a mixture of lognormals. The pdf of wedges is thus $g(\log(\omega)) = \sum_{n=1}^N \lambda_n \phi_n(\log(\omega))$, where ϕ_n , where ϕ_n are normal pdfs and $\sum_n \lambda_n = 1$. Note that lognormal wedges is the special case of this with $N = 1$. Standard formulas for expectations over lognormal distributions imply that $\int \log(\omega) g(\log(\omega)) d\omega = \sum_n \lambda_n \mu_n$ and $\int \omega^t g(\log(\omega)) d\omega = \sum_n \lambda_n e^{t\mu_n + \frac{1}{2}t^2\sigma_n^2}$. The results in subsection 2.3.4.2 immediately follow.

Appendix D: Data and Measurement

Measurement of Objects — Data from the United States

For the empirical analysis in Section 2.4 on U.S. firms, we use annual data on firms that exist in the Compustat database. We take the following steps, in order. First, firms must be headquartered in the United States and have a U.S. currency code. We then keep only firms with December fiscal year-ends. We then drop firms if their employment, property, plant, and equipment — net of depreciation, sales, or our measure of firm-value added — are missing or negative. We then exclude firms with 4-digit SIC codes between 4000 and 4999, between 6000 and 6999, or greater than 9000, as our model is not representative of regulated, financial, or public service firms. We then clean the data by winsorizing each series at the 1st percentile over the entire sample. For our analysis, we lastly only keep data from 1971 to 2011.

Firm-level value added, firm-level capital stock, and firm-level employment are the only firm-level objects we need for our decomposition. When computing year-over-year changes in the components of our decomposition, we also adjust for entry and exit by only keeping data on firms that exist in consecutive years. In the second decomposition, firms are grouped into sectors by two-digit SIC codes.

We measure labor as the number of employees reported in Compustat. We measure capital as the firm's plant, property, and equipment, adjusted for accumulated depreciation. The aggregate capital stock is annual, taken from the Penn World Tables. To adjust for potential changes in the valuation of capital over time, we construct a perpetual inventory measure of the aggregate capital stock and use the ratio of this measure to the value of the aggregate capital stock to deflate the firm-level measure of capital. The investment measure used in the perpetual inventory method is annual gross private domestic investment from the Bureau of Economic Analysis (BEA). To construct our measure of capital using the perpetual inventory method (starting from 1959), we use a depreciation rate of 4.64 percent

and growth rate of technology of 1.6 percent, following Chari, Kehoe, and McGrattan (2007). Our measure is then deflated by the December value of the monthly CPI, which is CPI for All Urban Consumers, seasonally adjusted, from the Bureau of Labor Statistics.

We create a measure of value added in public firms using income accounting. GDP has an income equivalent, GDI, which has similar time-series properties. The major components of this measure have equivalents to income statement measures that are required on 10-K forms for U.S. public firms. In order of magnitude, GDI is made up of the following components: compensation of employees, net operating surplus, consumption of fixed capital (depreciation), and taxes on production and imports less subsidies. While we do not observe the taxes or subsidies on production and imports firms pay in our dataset, we do observe measures of the other three components, all of which make up over 90 percent of GDI for all years in our sample. We observe labor compensation in Compustat annually. If labor compensation is missing, we replace it with selling, general, and administrative expenses. We also observe net operating profits before depreciation, which is the sum of a firm's net operating surplus and its capital consumption. We define a firm's contribution to output as the sum of labor compensation and operating profits before depreciation. In practice, the BEA uses a similar, more detailed approach, where they use firm tax data to aggregate up the components of domestic income and make adjustments for differences between accounting and economic treatment of factors such as capital consumption and inventory valuation.

To compute TFP, following Chari, Kehoe, and McGrattan (2007), we set capital's share of income, $\alpha = .35$ and back it out from (2.9). When we compare our measure against the NIPA-equivalent, we require a NIPA equivalent measure of our value-added measure, a measure of aggregate labor, and a measure of aggregate capital. To compute our NIPA equivalent of our pseudo-GDI measure, we use data from NIPA table 1.12 on National Income by Type of Account. We take compensation of employees (line 2) and subtract government (line 4), then add to this measure corporate profits with inventory valuation adjustment and capital consumption adjustment less taxes on corporate income (line 43). Finally, we

add to this measure consumption of fixed capital, which comes from the BEA. All measures are quarterly, and we only use the fourth-quarter values of these measures. We put this measure in per-capita terms using population including armed forces overseas. This measure is mid-period and monthly. We only keep its December value. We then put this measure in real terms using the CPI measure described in this subsection. Our measure of the real aggregate capital stock was already described in this subsection. This measure is also put in per-capita terms. Our measure of aggregate labor is total non-farm employment and is monthly. We only use the December observation of this variable.

Measurement — Data from Japan

Our data on Japanese public firms comes from the Compustat global database, and our firm-level variables are measured annually. We clean the data as we do for data from the United States, except we only keep firms with currency codes corresponding to the Japanese Yen and country headquarter codes corresponding to Japan. Also, the years of our sample are different: They only cover 2001 to 2011. Consistent with our application to U.S. data, when computing year-over-year changes in the components of our decomposition, we also adjust for entry and exit by only keeping data on firms that exist in consecutive years. In Decomposition II, firms are grouped into sectors by two digit SIC codes

As for the U.S. data, we measure firm-level labor as the number of employees reported and firm-level capital as the firm's plant, property, and equipment, adjusted for accumulated depreciation. We deflate the firm-level Japanese capital stock by the U.S. capital deflator. To put the capital stock in real terms, we deflate it by the OECD's measure of the quarterly CPI in Japan. We only keep the fourth-quarter value of this measure.

In a manner consistent with our application to U.S. data, we create a measure of value added in public firms using income accounting, which is the sum of labor compensation and operating profits before depreciation. As for the U.S. data, if labor compensation is missing, we replace it with selling, general, and administrative expenses. We eventually deflate by

the same Japanese CPI measure as for capital. To compute TFP, we again set $\alpha = .35$ and back it out from (2.9).

Appendix E: Our Decomposition in the Context of Other Methodologies

To apply our decomposition to data, one does not need to estimate firm-level TFP or sectoral production function coefficients. There is already potential for measurement issues biasing the results from our decompositions, as measures of labor, value added, and capital can all be measured incorrectly. Further, we could be incorrectly grouping firms with our sectoral definitions. However, it is easy enough to check different measures of labor, capital, or value added, if available, and see if the results change. Also, one could add measurement error to firm variables and test the extent to which the results change. Similarly, one can check the results from our second decomposition on different definitions of “groupings” or sectors. However, to compute sectoral production function coefficients, as is commonly done in papers assessing the role of labor and capital allocation on productivity over the business cycle, some issues cannot be “checked.” Data from 30 years prior can be crucial in providing “correct” estimates of sectoral production function coefficients. But what if such data are unavailable to the researcher? In addressing the role of resource reallocation in productivity dynamics over the business cycle, the literature has relied on the estimation of these technological measures for all sectors in the economy. In this section, we will demonstrate how some of the econometric biases associated with such an approach can lead one to produce quantitatively and qualitatively different results on the role of allocative efficiency over the business cycle.

We first demonstrate the most difficult-to-correct econometric bias associated with measuring production function coefficients, which is the fact that data for the entire sector over the entire sample are needed to estimate them. We also show that different definitions of factor prices can crucially affect one’s results. Second, we show that our decomposition can help

to assess the role of resource reallocation in productivity dynamics over the business cycle. In particular, in a relatively general setting, we demonstrate that the within-industry component of our decomposition is reflective of within-sector allocative efficiency. Ultimately, this section is meant to demonstrate that our decomposition can, at the very least, be a useful check on such attempts at measuring the role of resource allocation over the business cycle that require estimates of sectoral production function coefficients and firm-level TFP.

Illustrating issues with identification

In Figure 2.15, we demonstrate one possible issue with identification that can severely change the interpretation of the qualitative and quantitative importance of the role of reallocation over the business cycle. We show that our results change substantially when we follow a standard procedure and only slightly vary the estimation procedure for production function coefficients.⁵⁸ Figure 2.15 shows the cumulative change in the contribution of allocative efficiency to TFP over the recent recession for three different standard “versions” of estimating production function coefficients.⁵⁹ We estimate production function coefficients as the average of the ratio of capital expenditures to labor expenditures, $\frac{rk}{wl}$, across firms within a sector over time, where r is the rental rate, k is the capital stock in the firm, w is the wage, and l is labor utilization in the sector. In Version 1, the baseline version, we drop all observations before 1972, use capital and labor utilization from our firm-level data, and estimate the rental rate and wage following Chari, Kehoe, and McGrattan (2007).⁶⁰

We see that in Version 1 of our estimation of production function coefficients, there

⁵⁸Specifically, we implement the approach of Oberfeld (2013) to measure changes in allocative efficiency in our sample of U.S. publicly listed firms. This model of production and aggregation is identical to that in Hsieh and Klenow (2009). Details of our dataset construction and measurement can be found in section 2.4 and Appendix D. As in Oberfeld (2013) and Hsieh and Klenow (2009), we set the elasticity of substitution within sectors to 3.

⁵⁹Positive changes indicate an increase in the extent of allocative efficiency.

⁶⁰Following Chari, Kehoe, and McGrattan (2007) entails setting $r = \alpha * \frac{Y}{K}$ and $w = (1 - \alpha) * \frac{Y}{L}$, where α is defined in the measurement subsection above and come directly from Chari, Kehoe, and McGrattan (2007). Our measures of Y , L , and K are all computed as describe in subsection . These measures are computed differently from how Y , L , and K are computed in Chari, Kehoe, and McGrattan (2007).

seems to be a decrease in allocative efficiency from pre-recession levels to the trough in 2008 to 2009. In Version 2, we take capital, k , and labor expenditures, wl , from the firm-level data, but still estimate r following Chari, Kehoe, and McGrattan (2007). Estimating labor expenditures using firm-level data changes the year-over-year behavior of the contribution of allocative efficiency to TFP. In Version 3, we follow the same procedure as in Version 1 but drop all observations before 1976. In this version, which is only different from Version 1 in that we assume there is slightly less data available decades prior to the recession we are examining, we find an increase in allocative efficiency from 2006 to the trough of the recession. These results demonstrate just one of the potential problems with identification to which the standard model-based approaches are susceptible, a problem that can substantially change the qualitative and quantitative implications of the role of reallocation over the business cycle. Other issues with identification remain; another example is that we find that when we vary the elasticity of substitution across firms between 3 and 10 (standard values used in the literature as noted in Hsieh and Klenow (2009)), the qualitative and quantitative nature of our results change substantially.

Relation to Models of Allocative Efficiency

This part of the appendix demonstrates how the dispersion component of our decomposition relates to the aggregate productivity loss suffered from misallocation in a standard static model of allocative efficiency. We show that the dispersion component directly enters this loss and discuss the relative magnitude of the the social losses and the dispersion components.

We consider the one-sector economy introduced in subsection 2.3.2, and derive two statistics commonly used as measures of productivity loss due to distortions. First, we derive the difference between efficient and observed log TFP. This difference may be driven by misallocation or frictions to the total amount of capital/labor used. Second, we compute the difference between the log TFP implied by the output-maximizing allocation of the observed total labor and capital and observed TFP. This difference will be driven only by misalloca-

tion.

Difference from Optimal Allocation Note that an analogue (in levels instead of differences) to (2.28) yields:

$$\begin{aligned}
\log(TFP_t) &= \underbrace{\log(TFP_t^*) + \alpha\kappa_{k,1,t} + (1-\alpha)\kappa_{l,1,t}}_{\text{static mean component}} \\
&+ \underbrace{\left[-\frac{\alpha\kappa_{k,2,t} + (1-\alpha)\kappa_{l,2,t}}{2!} + \sum_{n=3}^{\infty} (-1)^{n-1} \frac{\alpha\kappa_{k,n,t} + (1-\alpha)\kappa_{l,n,t}}{n!} \right]}_{\text{static dispersion component}},
\end{aligned} \tag{E.1}$$

where $\kappa_{k,n,t}, \kappa_{l,n,t}$ are the cumulants of log wedges in capital and labor to firms productivity ratios, as defined in subsection 2.3.3. This implies that we can express the difference between efficient and realized log TFP as the following:

$$\log(TFP_t^*) - \log(TFP_t) = -\alpha\kappa_{k,1,t} - (1-\alpha)\kappa_{l,1,t} - \alpha D_t^K - (1-\alpha) D_t^L.$$

Where D_t^K and D_t^L are the dispersion components of labor and capital in our decomposition. The first cumulants are the output-share weighted averages of log wedges, and are the only terms other than the dispersion components to enter these losses.

Total Labor and Capital Fixed Consider a firm optimization problem, with fixed stock of total capital and labor within the production environment defined in subsection 2.3.2. Firms take the wage and rental rate as given, and clearing conditions $K_t = \sum_i k_{i,t}$ and $L_t = \sum_i l_{i,t}$ are satisfied. Firm labor and capital choices are distorted from their optimal allocation by $\theta_{i,t}^l$ and $\theta_{i,t}^k$, respectively.

$$\max_{k_{i,t}, l_{i,t}} p_{i,t} y_{i,t} - k_{i,t} \theta_{i,t}^k r_t - l_{i,t} \theta_{i,t}^l w_t. \tag{E.2}$$

The equilibrium decision rules imply that firm marginal revenue products are the following:

$$\frac{v_{i,t}}{k_{i,t}} = \theta_{i,t}^k \frac{r_t}{\varphi\gamma}, \quad (\text{E.3})$$

and

$$\frac{v_{i,t}}{l_{i,t}} = \theta_{i,t}^l \frac{w_t}{\varphi\phi}. \quad (\text{E.4})$$

We can then express the losses due to misallocation in this fixed environment as a function of the rental rate, efficient rental rate, and cumulants of firm-level distortions $\theta_{i,t}^k$ and $\theta_{i,t}^l$.⁶¹

We derive the following expression for $\log(T\hat{F}P_t) - \log(TFP_t)$:

$$\log\left(\frac{T\hat{F}P_t}{TFP_t}\right) = \alpha \left(\log\left(\frac{\hat{r}_t}{r_t}\right) - \kappa_{k,1,t} \right) - D_t^K + (1 - \alpha) \left(\log\left(\frac{\hat{w}_t}{w_t}\right) - \kappa_{l,1,t} - D_t^L \right),$$

where $T\hat{F}P_t$ is TFP if labor and capital are allocated efficiently but total labor and capital are fixed at the observed levels, and \hat{w}_t and \hat{r}_t are the wage and rental rate required to keep total capital and wages fixed if distortions are removed.

⁶¹These distortions differ only from the wedges defined before in that they are distortions from the input choices implied by rental rates and wages instead of the efficient allocation.

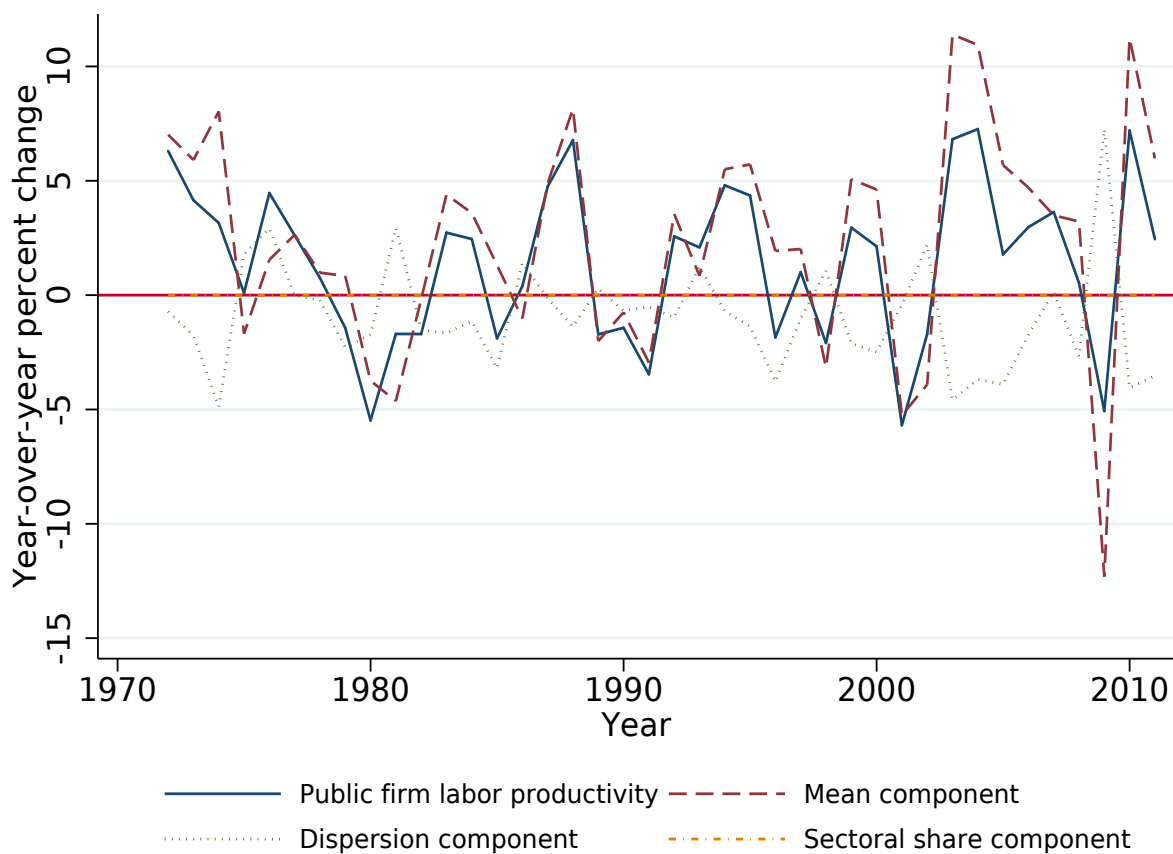
Tables and Figures for Chapter 2

Table 2.1: Correlations between Changes in Aggregates and Changes in their Components from Decomposition II — U.S. Data

	TFP	Labor productivity	Capital productivity
Mean component	0.974	0.955	0.955
Dispersion component	0.042	-0.346	0.399
Sectoral share component	-0.245	-0.329	0.259

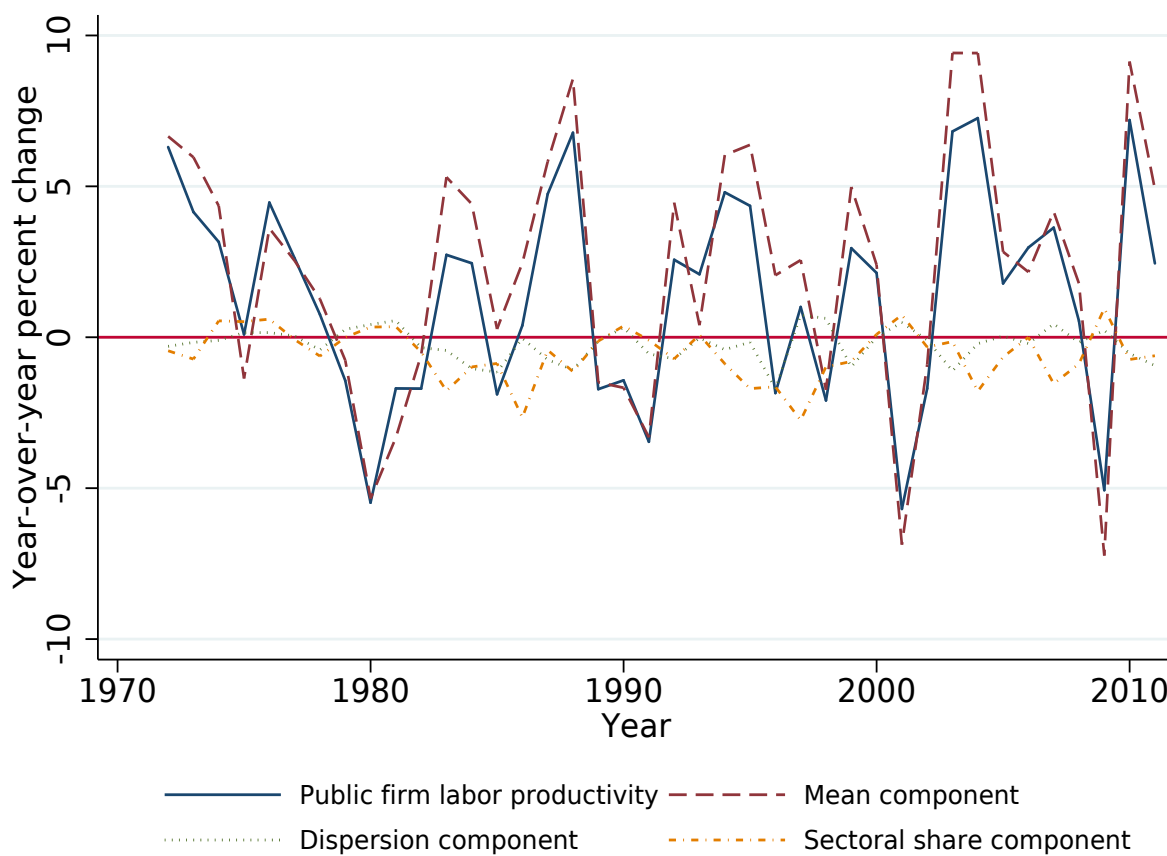
Notes: Sample period is from 1972 to 2011. Data are from U.S. nonfinancial public firms. Firms are grouped by two digit SIC codes. Changes in aggregate measures (TFP, labor productivity, and capital productivity) and components of our decompositions are measured year over year.

Figure 2.1: Decomposition I Applied to Aggregate Labor Productivity: Year-over-Year Changes



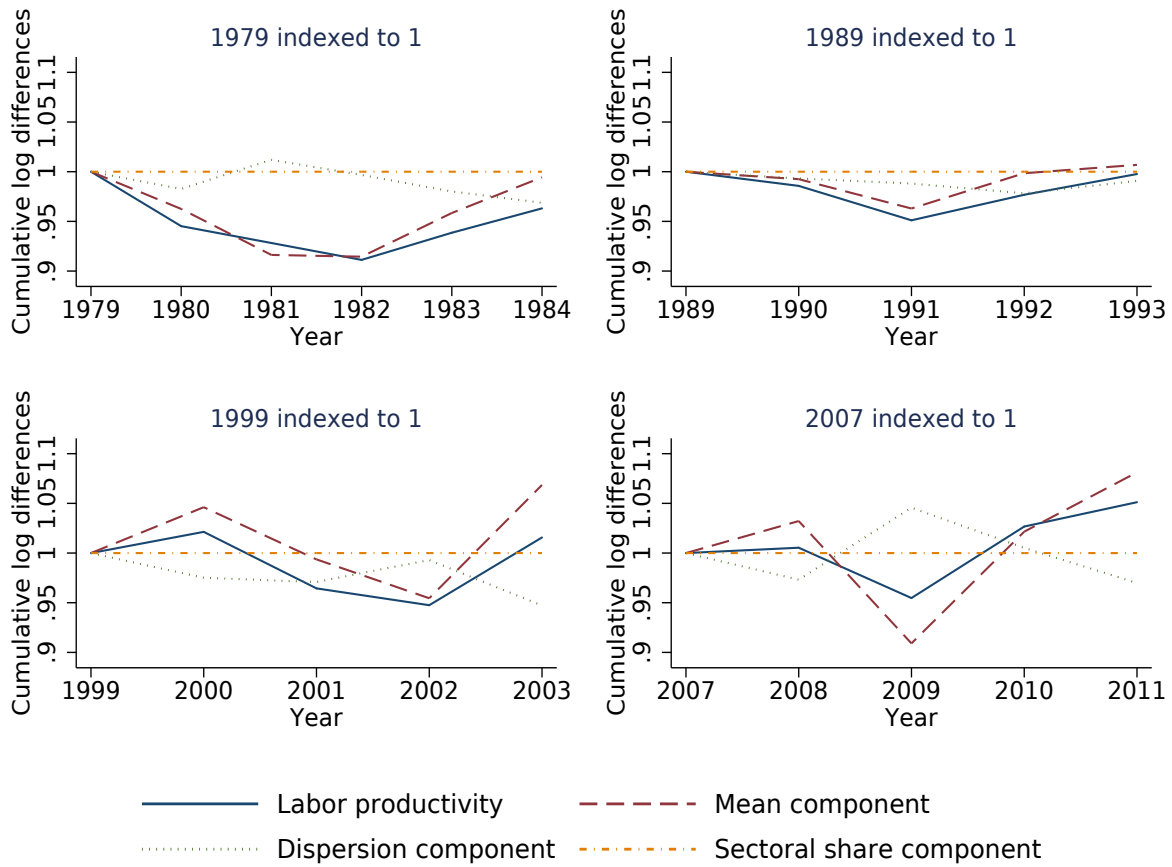
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate labor productivity and its components from Decomposition I using data from U.S. nonfinancial public firms. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 2.2: Decomposition II Applied to Aggregate Labor Productivity: Year-over-Year Changes



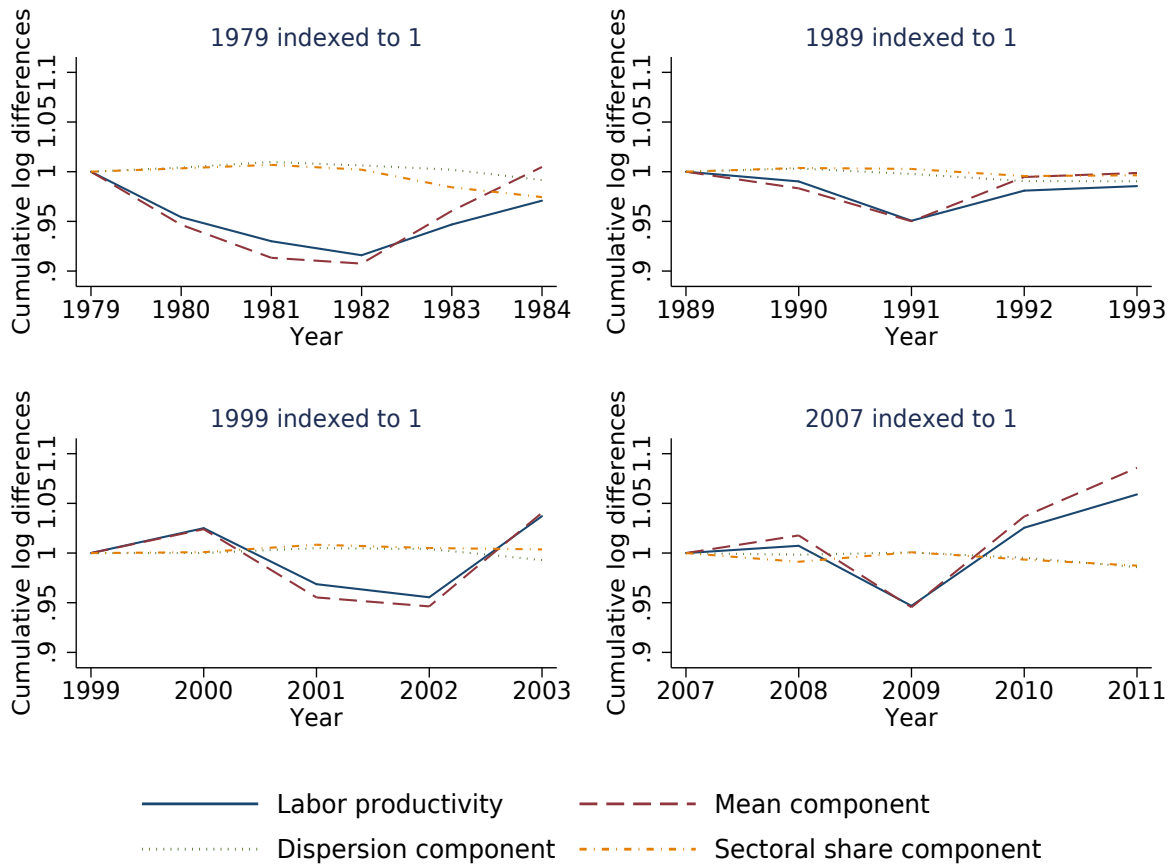
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate labor productivity and its components from Decomposition II using data from U.S. nonfinancial public firms. Firms are grouped by two digit SIC codes.

Figure 2.3: Decomposition I Applied to Aggregate Labor Productivity: Cumulative Changes over Four Business Cycle Episodes



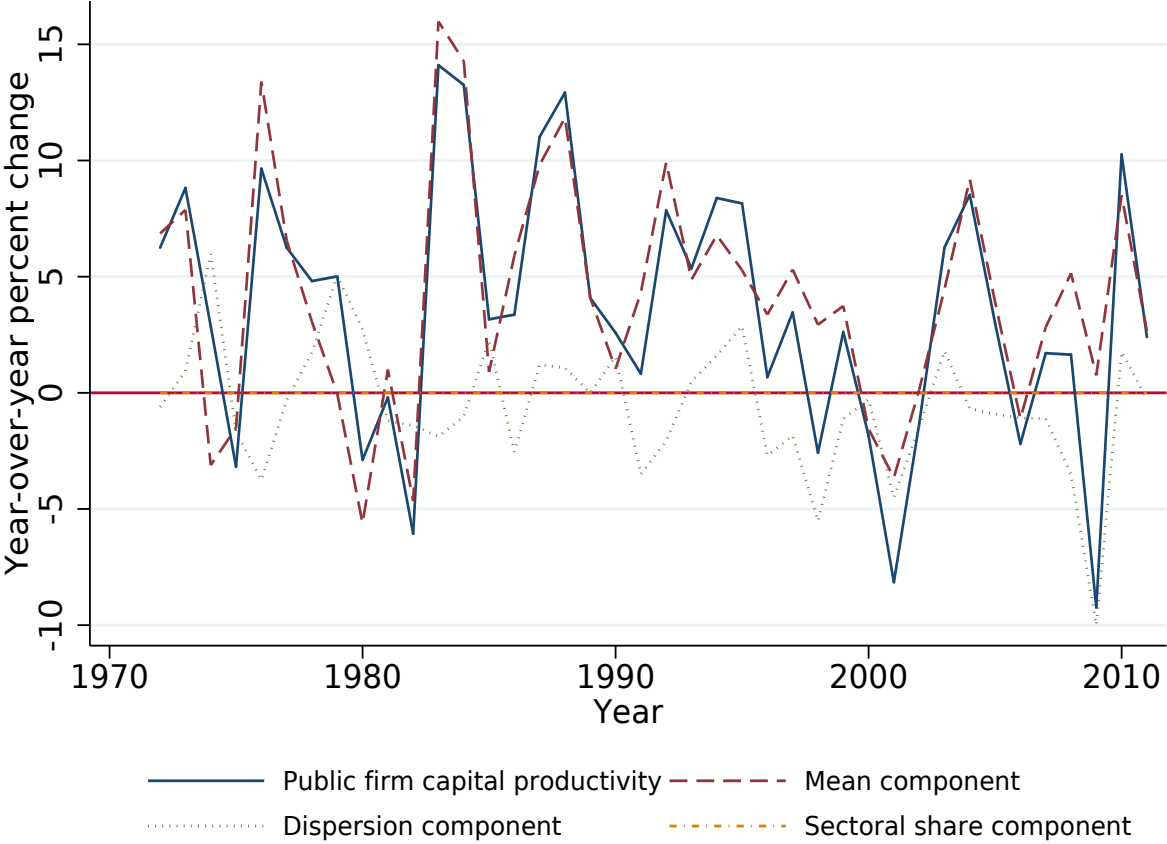
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate labor productivity and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 2.4: Decomposition II Applied to Aggregate Labor Productivity: Cumulative Changes over Four Business Cycle Episodes



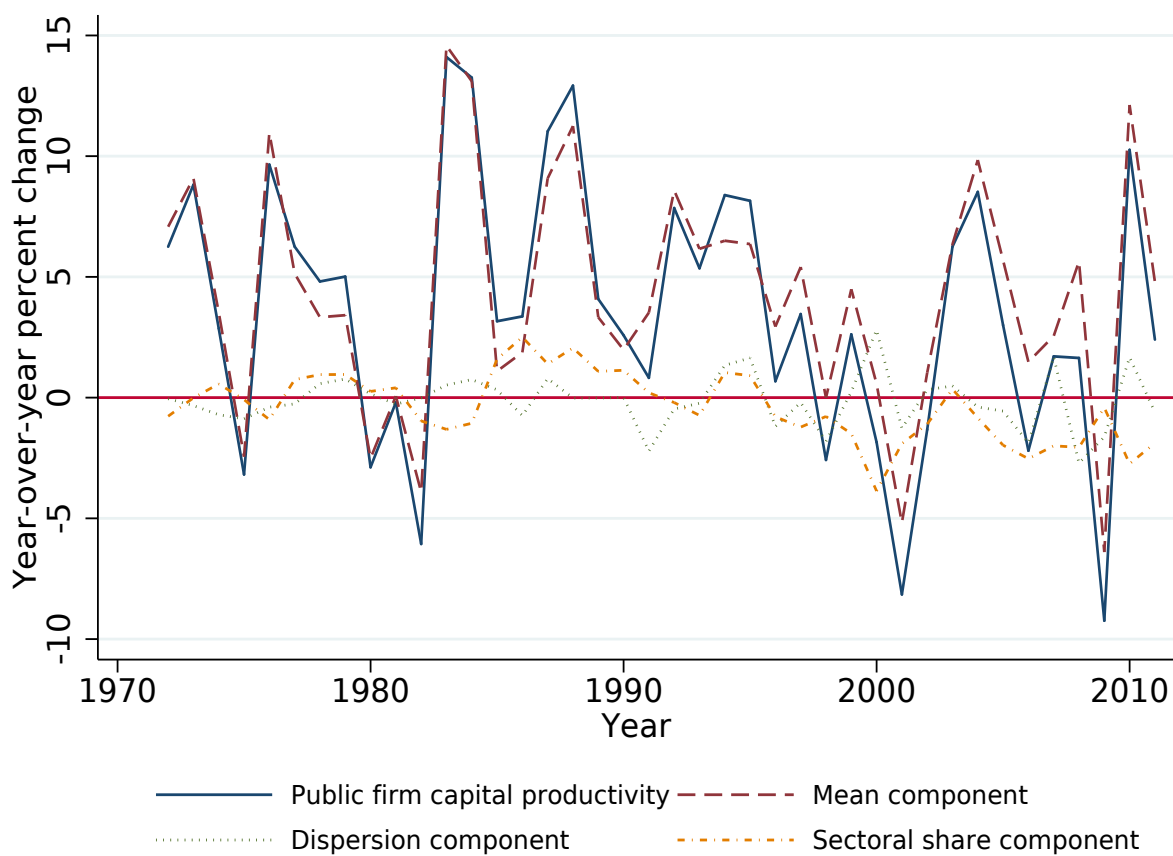
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate labor productivity and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Firms are grouped by two digit SIC codes.

Figure 2.5: Decomposition I Applied to Aggregate Capital Productivity: Year-over-Year Changes



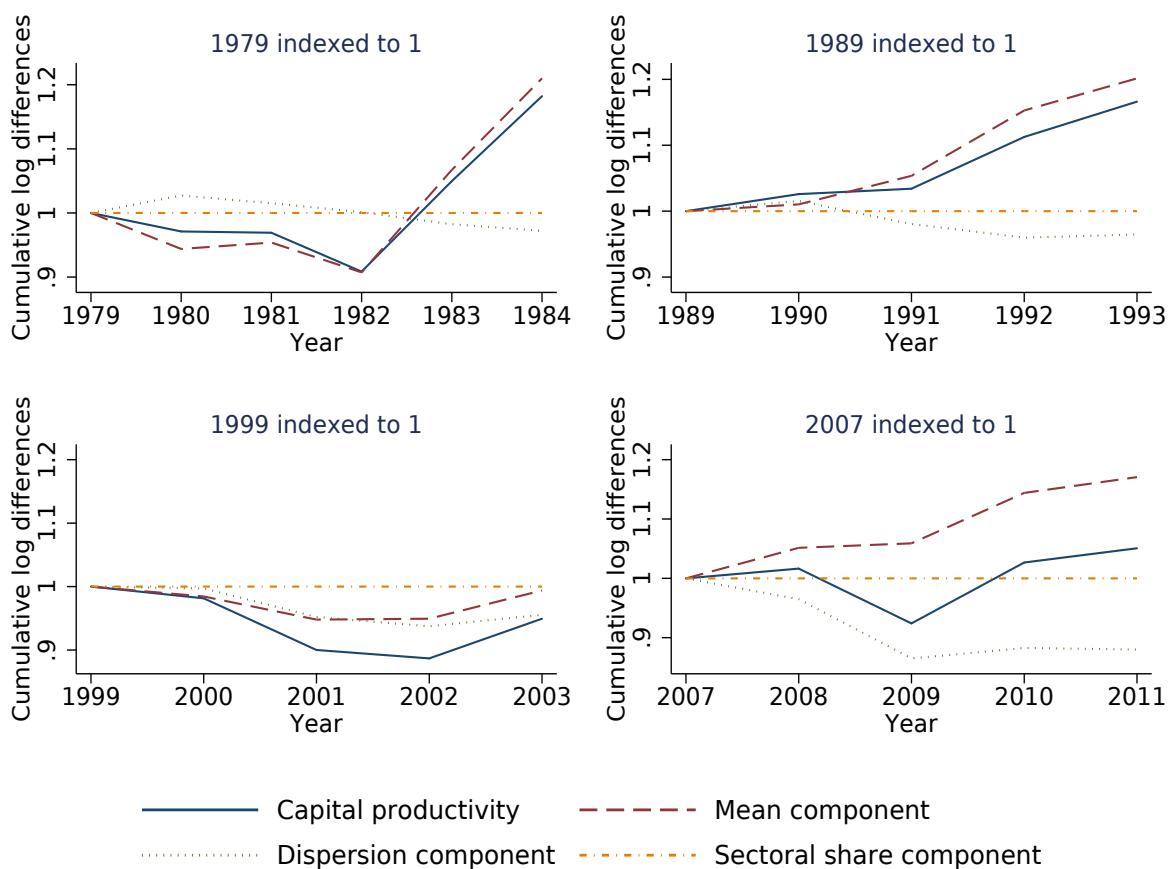
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate capital productivity and its components from Decomposition I using data from U.S. nonfinancial public firms. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 2.6: Decomposition II Applied to Aggregate Capital Productivity: Year-over-Year Changes



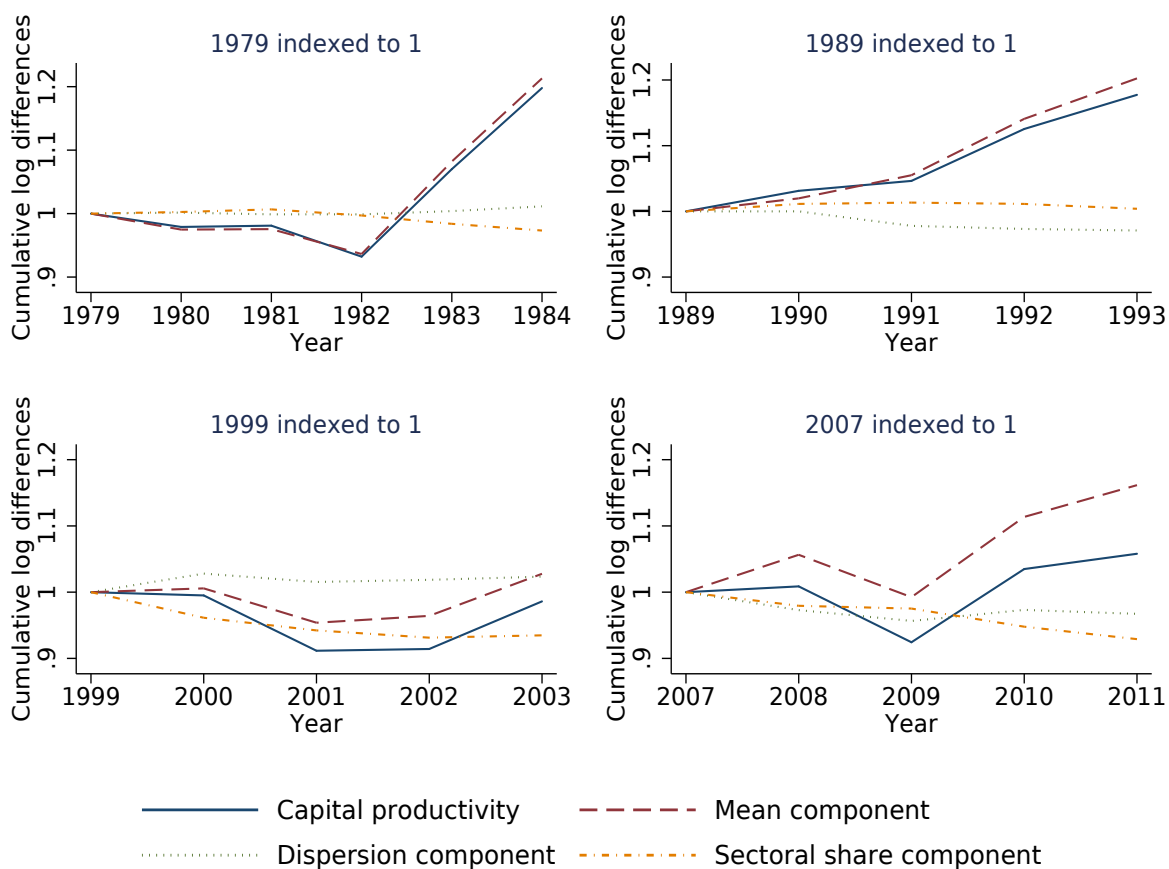
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate capital productivity and its components from Decomposition II using data from U.S. nonfinancial public firms. Firms are grouped by two digit SIC codes.

Figure 2.7: Decomposition I Applied to Aggregate Capital Productivity: Cumulative Changes over Four Business Cycle Episodes



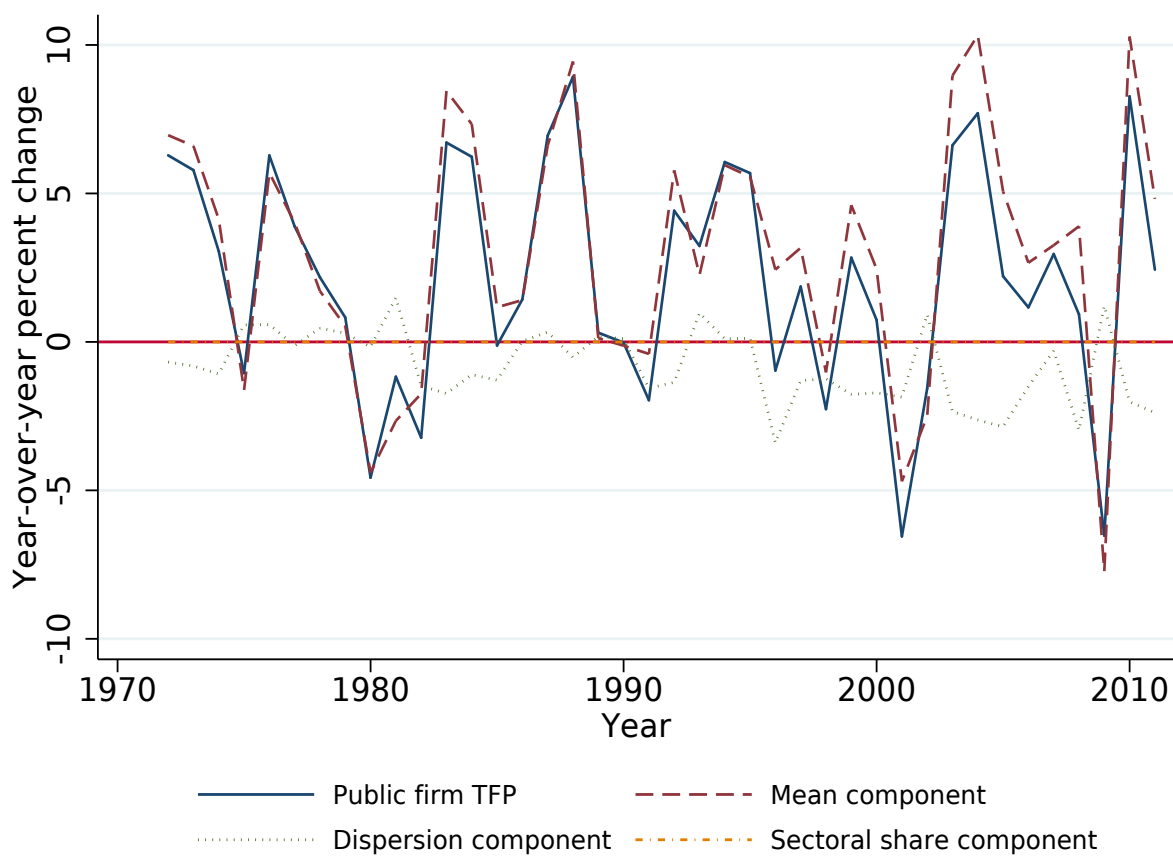
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate capital productivity and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 2.8: Decomposition II Applied to Aggregate Capital Productivity: Cumulative Changes over Four Business Cycle Episodes



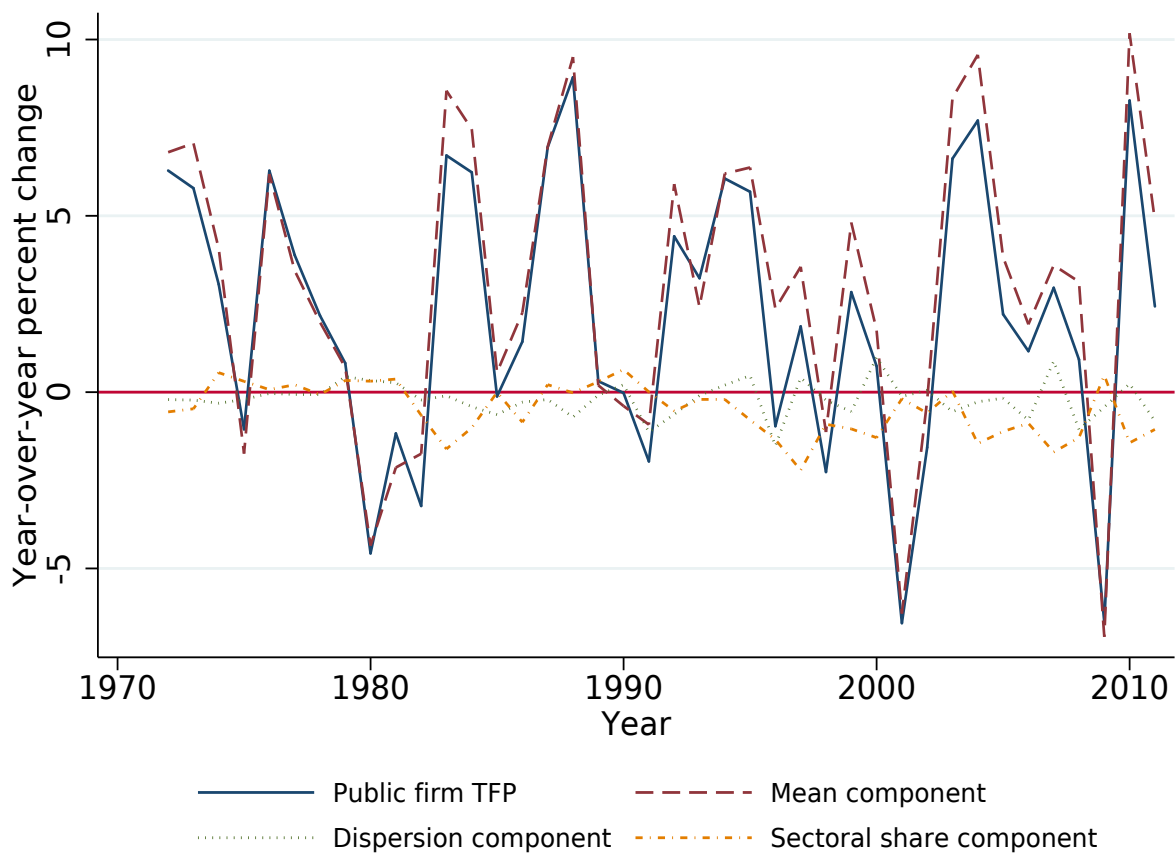
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate capital productivity and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Firms are grouped by two digit SIC codes.

Figure 2.9: Decomposition I Applied to Aggregate TFP: Year-over-Year Changes



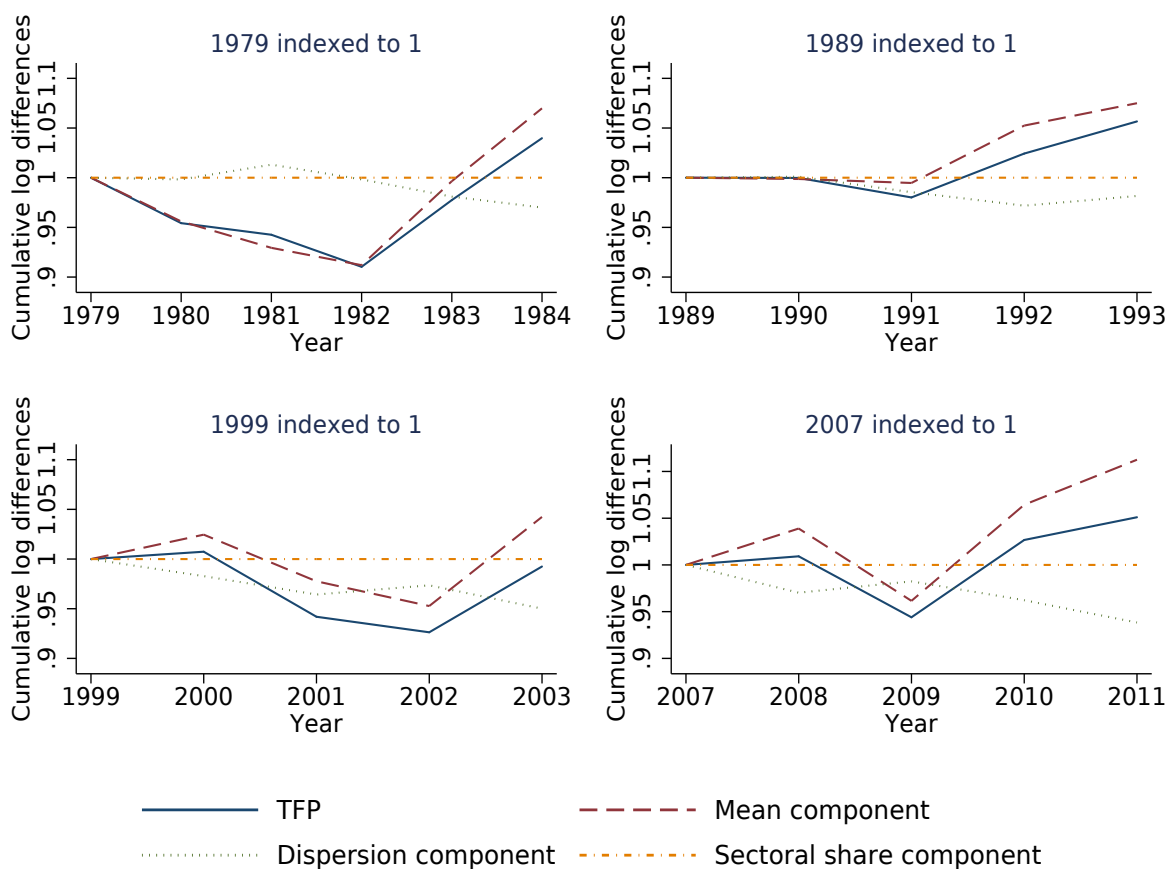
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate TFP and its components from Decomposition I using data from U.S. nonfinancial public firms. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 2.10: Decomposition II Applied to Aggregate TFP: Year-over-Year Changes



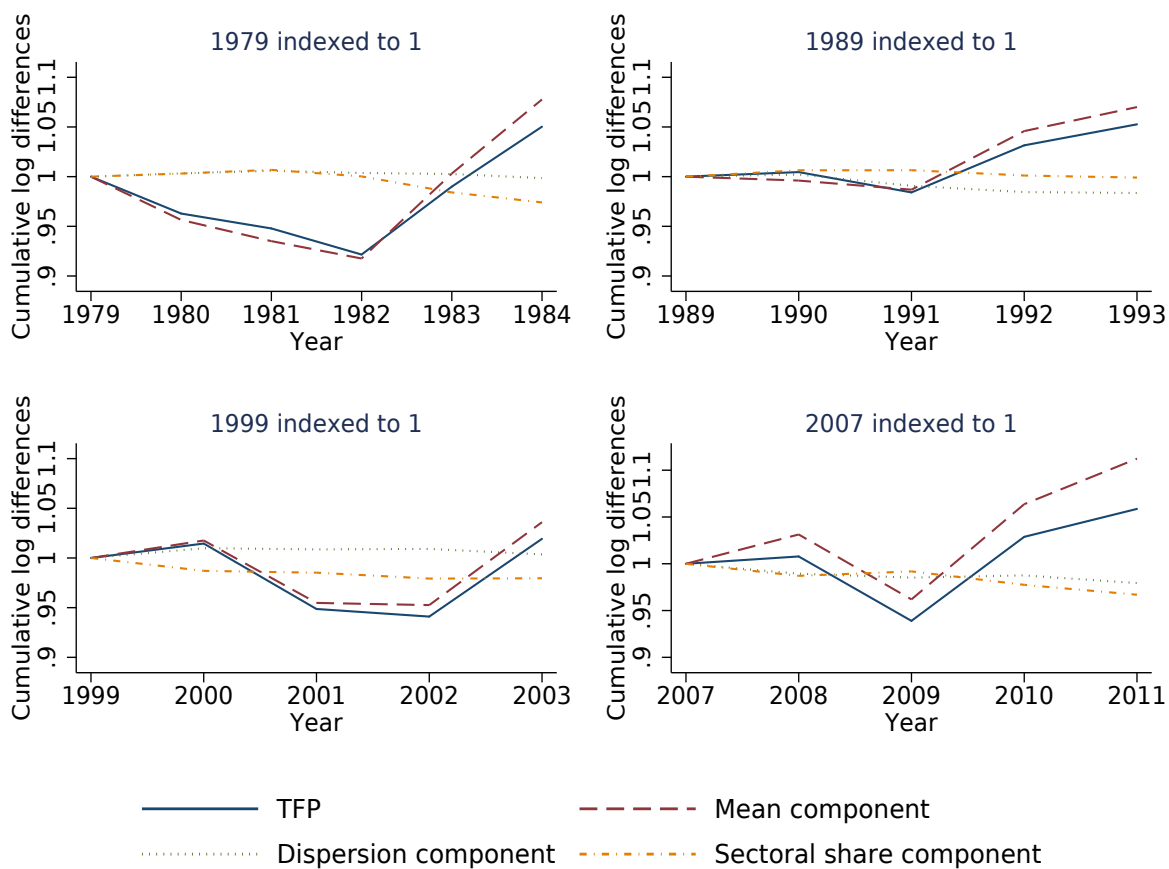
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate TFP and its components from Decomposition II using data from U.S. nonfinancial public firms. Firms are grouped by two digit SIC codes.

Figure 2.11: Decomposition I Applied to Aggregate TFP: Cumulative Changes over Four Business Cycle Episodes



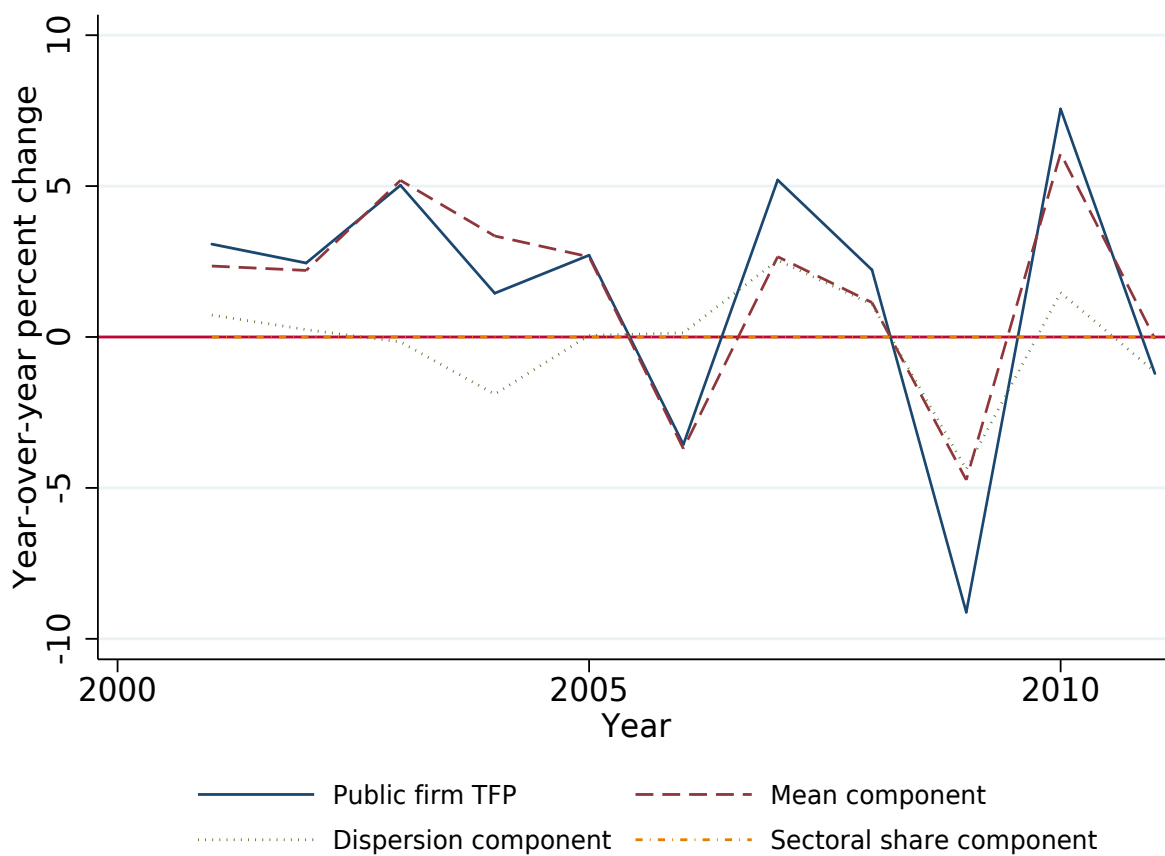
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate TFP and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 2.12: Decomposition II Applied to Aggregate TFP: Cumulative Changes over Four Business Cycle Episodes



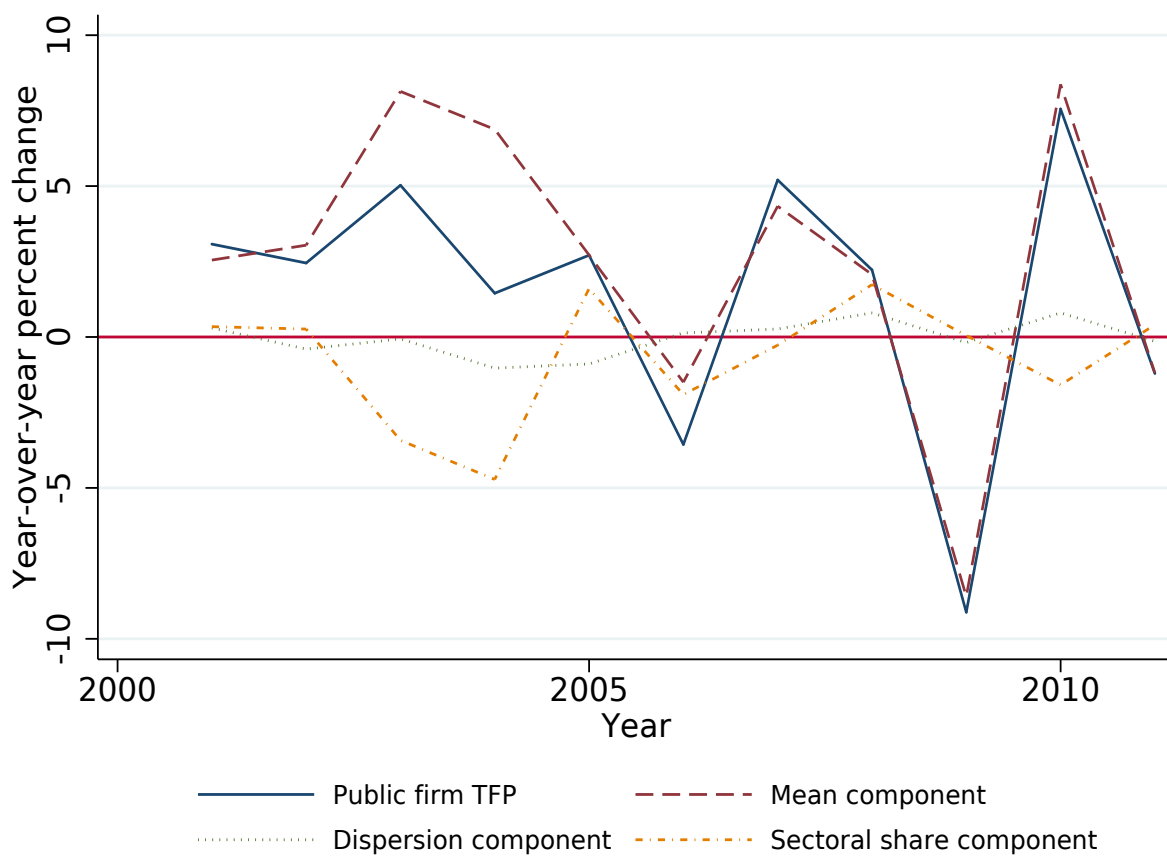
Note: Sample period is from 1972 to 2011. We compute cumulative changes in aggregate TFP and its components from Decomposition I using data from U.S. nonfinancial public firms over four business cycle episodes, with the pre-recession index year in the title of the plot. Firms are grouped by two digit SIC codes.

Figure 2.13: Decomposition I Applied to Aggregate TFP: Year-over-Year Changes



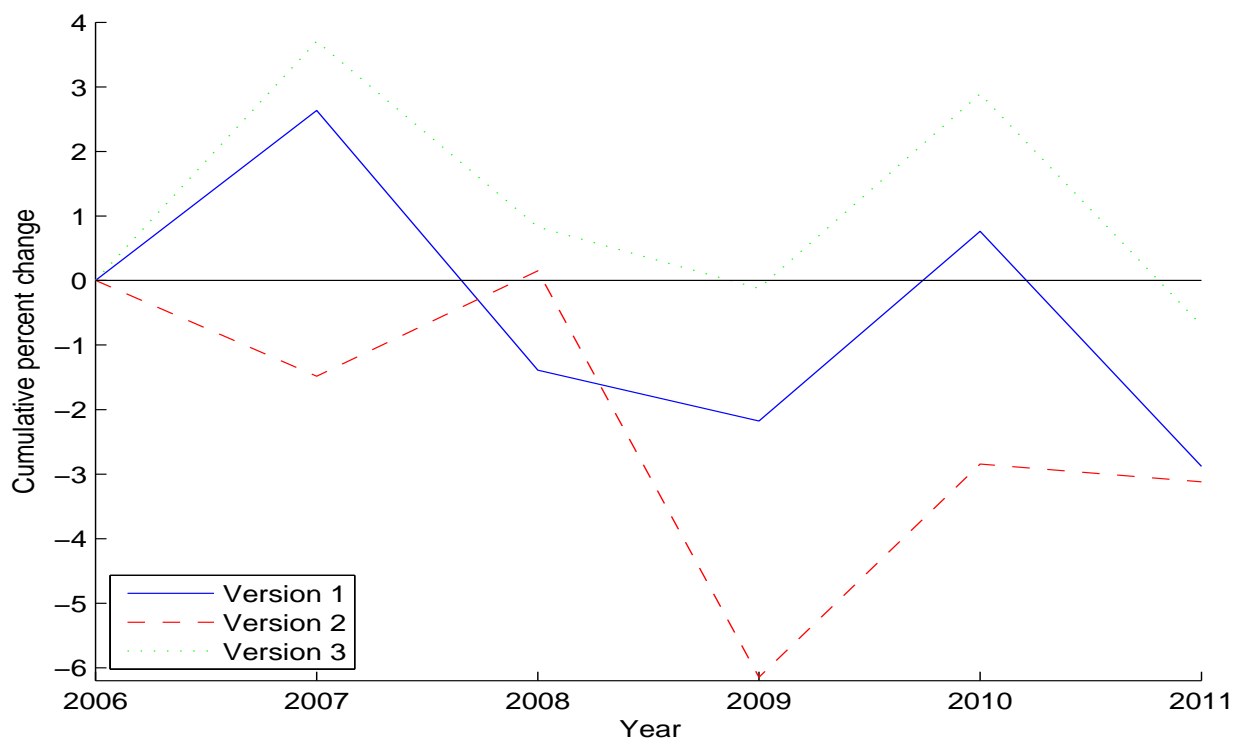
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate TFP and its components from Decomposition I using data from Japanese nonfinancial public firms. Because there is no grouping by sector for Decomposition I, the sectoral share component (the yellow line) is, in turn, flat.

Figure 2.14: Decomposition II Applied to Aggregate TFP: Year-over-Year Changes



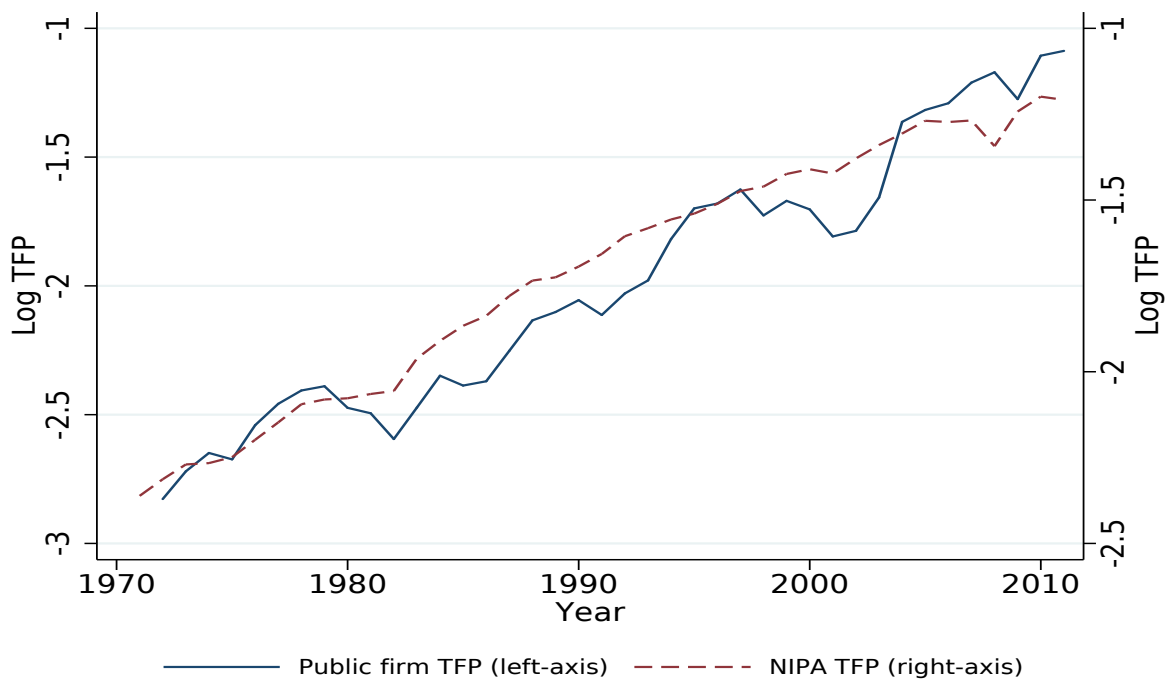
Note: Sample period is from 1972 to 2011. We compute year-over-year changes in aggregate TFP and its components from Decomposition II using data from Japanese nonfinancial public firms. Firms are grouped by two digit SIC codes.

Figure 2.15: Three Estimation Techniques for the Contribution of Allocative Efficiency to TFP



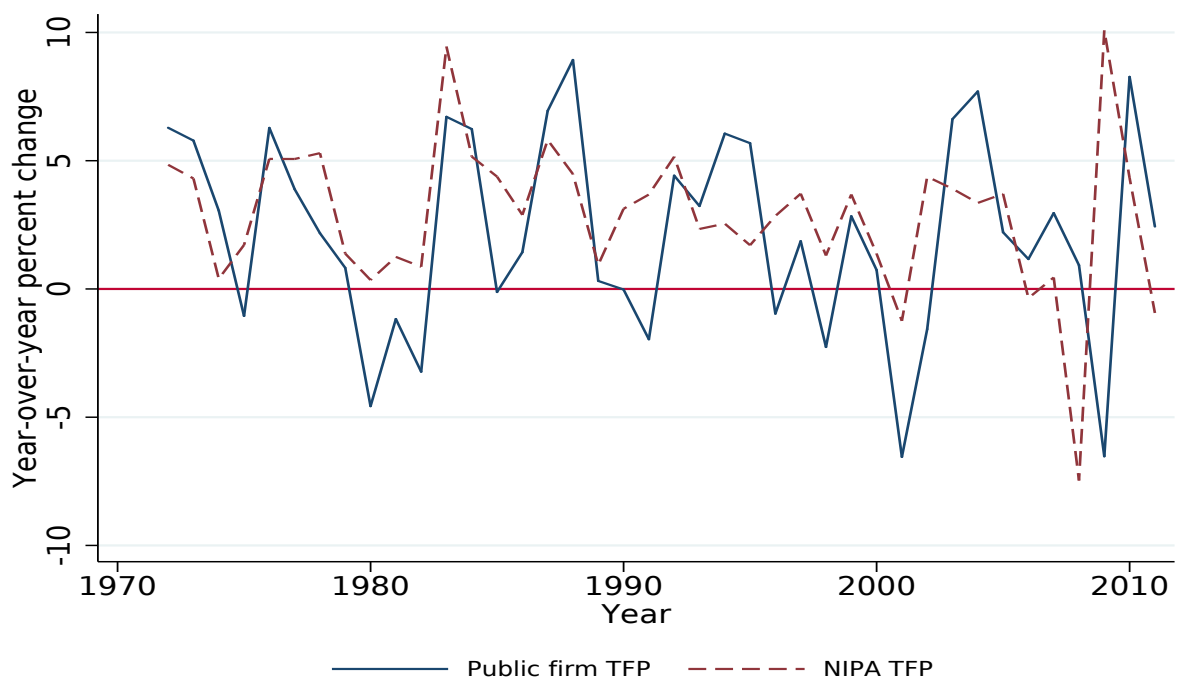
Note: We can express TFP as a function of the hypothetical efficient productivity, TFP_t^{eff} , at time t , and the allocative efficiency of resources a_t such that $TFP_t = a_t TFP_t^{eff}$. The figure shows $\log(a_t) - \log(a_{2006})$. The lines can thus be interpreted as the cumulative percent change in TFP over 2006 levels due to changes in allocative efficiency. The estimation of the different “versions” only differs in the estimation of production function coefficients as follows: (1) Version 1 is the baseline version, (2) Version 2 uses wage data from CompuStat rather than from NIPA, and (3) Version 3 uses data back to 1976 instead of 1972.

Figure 2.16: Trend in Aggregate TFP: NIPA vs. Compustat



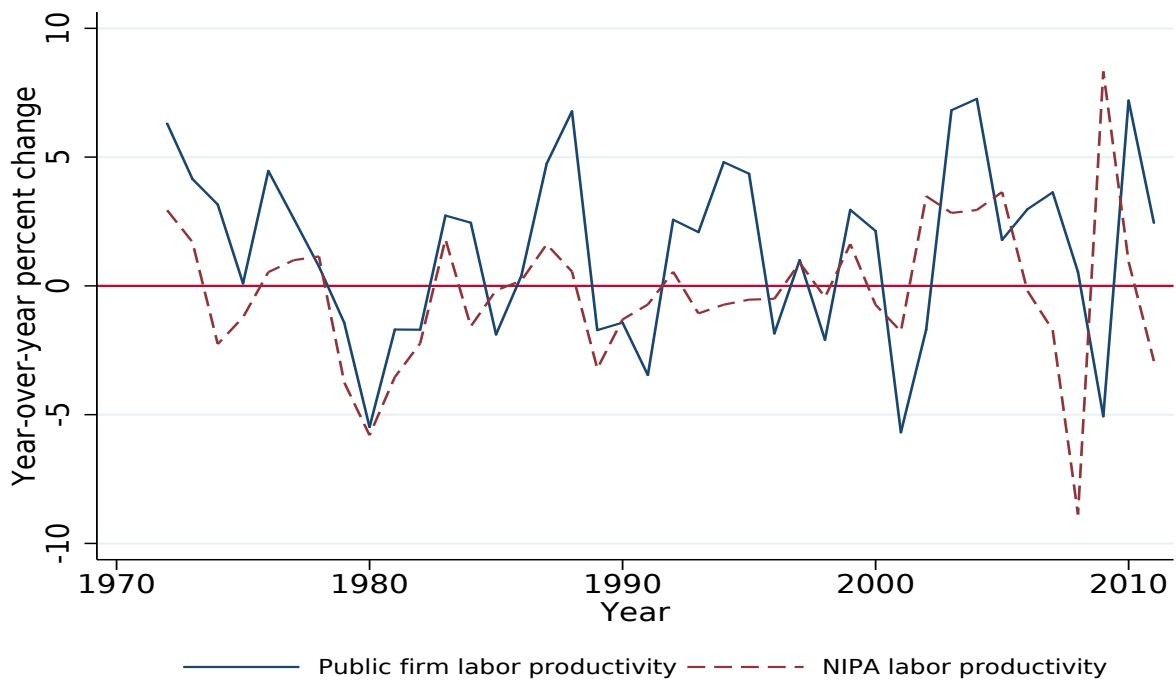
Note: Sample period is from 1972 to 2011. This figure depicts the trend in the natural logarithm of aggregate TFP computed from two different samples. The blue line corresponds to a measure computed from public firm data. The red line corresponds to a measure computed from NIPA.

Figure 2.17: Changes in Aggregate TFP: NIPA vs. Compustat



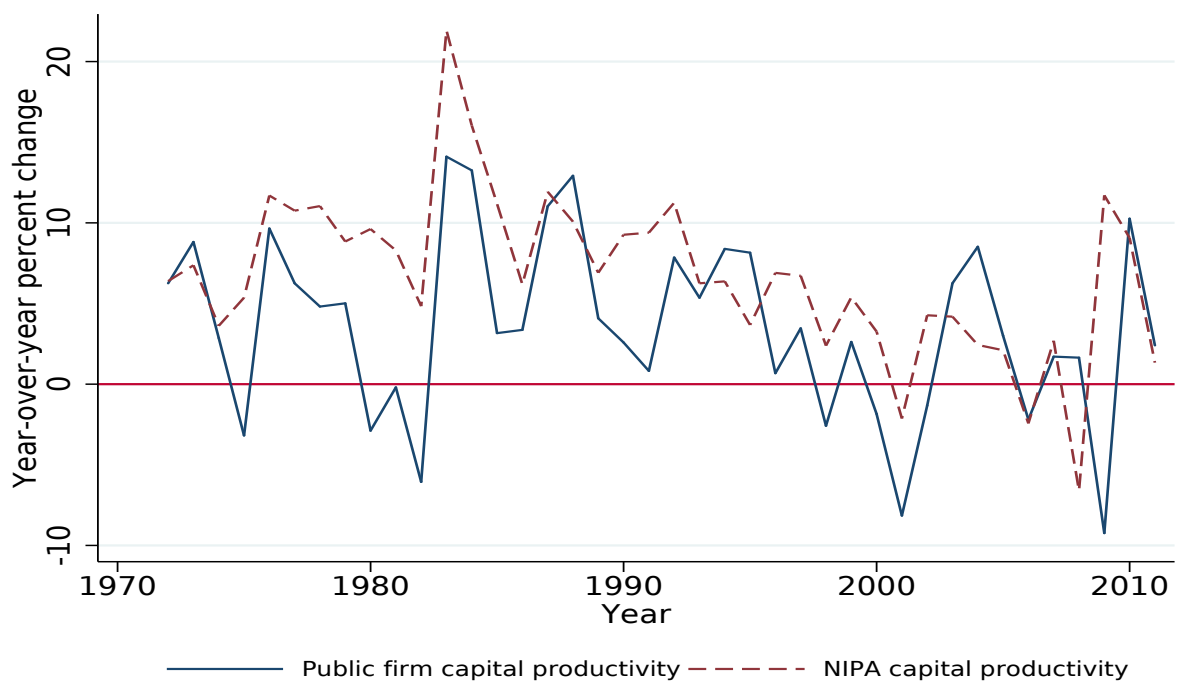
Note: Sample period is from 1972 to 2011. This figure depicts changes in the natural logarithm of aggregate TFP computed from two different samples. The blue line corresponds to a measure computed from public firm data. The red line corresponds to a measure computed from NIPA.

Figure 2.18: Changes in Aggregate Labor Productivity: NIPA vs. Compustat



Note: Sample period is from 1972 to 2011. This figure depicts changes in the natural logarithm of aggregate labor productivity computed from two different samples. The blue line corresponds to a measure computed from public firm data. The red line corresponds to a measure computed from NIPA.

Figure 2.19: Changes in Aggregate Capital Productivity: NIPA vs. Compustat



Note: Sample period is from 1972 to 2011. This figure depicts changes in the natural logarithm of aggregate capital productivity computed from two different samples. The blue line corresponds to a measure computed from public firm data. The red line corresponds to a measure computed from NIPA.

Chapter 3: The Economy-Wide Gains from Resolving Debt Overhang

3.1 Introduction

Myers (1977) first laid out the debt overhang problem; put simply, existing debt can lead equity holders to underinvest, since part of the expected cash flow generated by the investment goes to debt holders, while equity holders bear its costs. Nonfinancial firm debt overhang has been empirically found to affect firm investment and growth in the corporate finance literature, with some relatively large estimates of its effect on firm value.⁶² Nonfinancial firm debt overhang has also been proposed as a potential key driver in slowing aggregate growth and reducing welfare.⁶³ Yet, to our knowledge, the literature has been largely silent in quantifying the aggregate effects of resolving nonfinancial firm debt overhang for productivity and welfare.

This paper directly addresses this gap in the literature. We develop a framework and estimation procedure that lets us ask: What are the gains from resolving debt overhang for nonfinancial firms, both for individual firms and for the aggregate economy in the long run? In our model, firms make endogenous entry and leverage decisions; can potentially default on their debt obligations; are heterogeneous in their investment opportunities; and endogenously innovate but can suffer from debt overhang when making innovation decisions. We estimate key model parameters using data on U.S. nonfinancial public firms, including the parameter that governs the extent to which debt overhang affects firm innovation decisions. We derive bounds on this parameter that account for unobserved heterogeneity in investment

⁶²See work by Lang, Ofek, and Stultz (1996), Giroud, Mueller, Stomper, and Westerkamp (2012), Hennessy (2004), Hennessy, Levy, and Whited (2007) for examples of estimates of the effect of debt overhang on firm investment and growth. See Moyen (2007) for an example of a relatively large estimate of the effect of debt overhang on firm value.

⁶³For example, Lo and Rogoff (2015) highlight debt overhang (corporate, household, and government debt overhang) as a possible cause of sluggish growth in advanced economies.

opportunities potentially driving the patterns between firm growth and firm default risk in the data, and show that our estimated model is well identified and implies estimates of the extent to which debt overhang affects firm growth that are consistent with existing estimates in the corporate finance literature. We find the expected private gains upon entry from resolving debt overhang are modest. The long-run welfare gains from resolving this problem are even smaller than the private gains to a single firm, due to general equilibrium dampening forces. In particular, an endogenous rise in the real cost of innovation and an increase in firm leverage and bankruptcy rates are the primary general equilibrium forces reducing the welfare gains from resolving this problem. However, we find the private gains from resolving debt overhang are nonlinear and rise substantially for firms near default. When the distribution of firm default risk changes to the extent observed during the recent recession, our model implies significant year-ahead employment losses due to debt overhang, and, hence, the expected gains from resolving debt overhang rise substantially.

Our point of departure is a general equilibrium model of firm dynamics similar to the framework of Atkeson and Burstein (2010), which incorporates endogenous process innovation decisions into the model of firm growth described by Luttmer (2007).⁶⁴ Each firm in our model has a specific factor that determines its current opportunities for generating sales and profits. This factor is meant to be general, and can include standard concepts like productivity and manager ability, as well as concepts such as the quality of a firm's product or its brand, to name a few possibilities. In the model, incumbent firms engage in *process innovation*: They have an investment technology through which they can invest resources to lower their marginal cost of producing their differentiated product and, hence, expand profits by expanding sales. Incumbent firms also differ in their *investment opportunities*: They differ in the productivity of their technology for investing to reduce their marginal cost of production so that for some firms it is cheap to invest to lower marginal cost and thus grow

⁶⁴In particular, our model embeds heterogeneity in investment opportunities and endogenous leverage decisions (with the potential for costly default) into a one-country version of Atkeson and Burstein (2010).

sales and profits, while for others it is expensive to do so. At any time, *product innovation* can occur: New firms can pay a fixed cost to enter with a new differentiated product. An innovation of our model is to embed the classical trade-off theory of capital structure into such a framework; here, we build on the model of Leland (1994). Firms take out long-term debt because it has a tax advantage, but do not fully finance themselves with debt because it can lead to costly bankruptcy.

The initial leverage decision of the firm is made to maximize the joint value of equity holders and creditors, but all investment decisions made afterwards are made to maximize only the value of equity holders. When the firm holds debt, marginal benefits from a unit of investment for equity holders vs. equity holders and creditors combined differ. Equity holders thus underinvest relative to the investment decision equity holders and creditors would jointly make. The extent of this underinvestment depends on the firm's investment technology and its default risk. In our model, the cost of innovating is convex, and the convexity of the cost function determines the extent to which the firm innovation decision responds as firms have more default risk. Hence, it is this convexity that governs the extent to which debt overhang affects firms.

We develop a novel estimation procedure to estimate the parameters that govern the firm growth process and the extent to which debt overhang affects firm innovation decisions in our model. To demonstrate why our estimation procedure works in practice, we analytically demonstrate why our moment conditions identify our parameters. We derive a closed-form approximation to the innovation decision of firms. The approximation can be viewed as the innovation choice firms would make in a simplified version of the model where firms make an endogenous innovation decision in the first period, but take their innovation decision as given all periods thereafter. We demonstrate that in practice, under all of the parameter combinations we estimate, the innovation decision in this simple setting provides a tight approximation to the innovation decision in the more general model. Because we can solve the policy function in closed form, we can derive our moment conditions in closed form, and

we show which parameters to a first-order drive which moments. In particular, we show the moments that characterize the nonlinear relationship between firm default risk and firm average subsequent growth are to a first-order driven by the parameter which governs the extent to which debt overhang affects the firm innovation decision.

It is crucial to control for unobserved heterogeneity in our estimation procedure. Firm growth and firm default risk may have a significant relationship because of such unobserved heterogeneity and not debt overhang. If firms with exogenously persistent low growth rates (due to poor investment opportunities) are on average closer to default, then firms near default will grow slower than firms further from default because of differences in their exogenous characteristics, and not just debt overhang. We incorporate such unobserved heterogeneity into our model. To both demonstrate the robustness of our results to a reasonable range of parameter estimates of the extent to which debt overhang affects firms and to overcome potential misspecification issues in accounting for unobserved heterogeneity in firm investment opportunities, we estimate our model under two different sets of moment conditions. First, we use moment conditions which generate a lower bound for the parameter which governs the extent to which debt overhang affects firms. Second, we use moments conditions which generate an upper bound for this parameter. With our closed-form approximation to the firm innovation decision, we demonstrate analytically that the lower and upper bound moment conditions we develop imply lower and upper bounds for the parameter that governs the extent to which debt overhang affects firms. We also show these bounds hold in the more general model with heterogeneity in investment opportunities by generating synthetic data from the model. We obtain moments from the synthetic data, reestimate our model, and find the bounds hold.

We use an indirect inference procedure to locally identify the parameters that govern the mechanisms in our model. The data moments we use to estimate the model come from annual data on an unbalanced panel of U.S. nonfinancial public firms from 1982 to 2012 and a balanced panel of manufacturing firms from 1992 to 1995. We rely on two measures, the

first of which is employment growth. The second measure we use is a measure of firm risk-adjusted leverage, distance-to-default, which is measured as the inverse of the firm's asset volatility times the natural logarithm of the value of a firm's assets relative to the book value of its debt. Distance-to-default is measured in units of the number of standard deviations of annual asset volatility by which the firm's assets must change to equal the firm's book value of debt. A value of 1 implies the firm is one standard deviation from its book value of debt exceeding its assets, 2 implies the firm is two standard deviations from its book value of debt exceeding its assets, and so on.

The moment conditions we specify relate to properties of firm growth and the significant relationship between firm distance-to-default and year-ahead growth we find in the data. Figure 3.1a shows that the average employment growth of U.S. public, nonfinancial firms close to default is almost 7% slower than firms further from default after controlling for year and industry fixed effects, and the relationship is nonlinear. In Figure 3.1b, we show this nonlinear relationship exists for sales and capital growth as well. In Figure 3.1c, we show that this relationship exists and is nonlinear even after controlling for year effects, industry effects, size, age, and access to external finance.

We estimate our model taking the distribution of distance-to-default as given from the data. All moments we use to estimate our model are a function of how a firm will innovate conditional on its distance-to-default, which makes such an approach feasible. In turn, we only need to solve the problem of equity holders in order to estimate our model. Hence, although we specify assumptions on the general equilibrium environment and debt contract, it is only how these functional form assumptions enter the problem of equity holders that affect the identification of parameters in our model. With our estimated model, we then demonstrate that the bounds on the parameter that governs the extent to which debt overhang affects firms imply that our range of estimates of such an effect are consistent with a quasi-natural experiment in the literature, Giroud et al. (2012), and an important structural paper that relies on Q-theory, Hennessy (2004).

Although the distribution of distance-to-default is taken as given for our estimation procedure, the distribution is treated as endogenous for our welfare and firm value counterfactuals.⁶⁵ Firm entry and firm leverage decisions respond to the extent to which debt overhang affects firm investment decisions near default, which in turn affect aggregate welfare. The endogenous distribution of distance-to-default in our estimated model has distributional moments close to those from the data.⁶⁶

We assess the gains from resolving debt overhang were the firm as a whole, rather than equity holders alone, to make the process innovation decision. Although our estimated model implies expected firm value upon entry will only increase modestly for a firm resolving debt overhang, the firm will gain nonlinearly as it gets closer to default. In our estimated model, the shape of the distribution of distance-to-default at any given time can play a significant role in generating the expected future growth rate of firms. When the distribution of distance-to-default changes to the extent observed in the recent recession, absent compensating general equilibrium forces such as prices changing, resolving debt overhang would increase annualized employment growth by between 1.5 – 3%. In our general equilibrium exercise, the long-run welfare gains are more modest and are dampened by the cost of innovation rising and a higher aggregate bankruptcy rate.

Related Literature The literature on debt overhang has continued to develop the theory since Myers (1977), examining the different margins through which this problem can affect firm investment; Diamond and He (2013) and Phillippon and Schnabl (2013) provide two important recent examples of such theoretical work.⁶⁷ Our paper advances the theory as

⁶⁵We do take the distribution of distance-to-default as given for our counterfactuals that assess the gains from resolving debt overhang in terms of expected year-ahead growth in the cross-section and over time.

⁶⁶The distribution of distance-to-default when we estimate the model on our full sample of nonfinancial firms and control for firm heterogeneity in investment opportunities and other firm characteristics (version (6) of our estimation procedure), for example, has mean, standard deviation, skewness, and kurtosis of 5.28, 2.48, 0.47, and 2.41, respectively. The same moments in the data are 4.61, 2.82, 0.41, and 2.23, respectively.

⁶⁷Diamond and He (2013) demonstrate how firm debt maturity interacts with the debt overhang problem, while Phillippon and Schnabl (2013) study a financial sector that suffers from debt overhang and ask under what conditions and how a government should engage in resolving debt overhang for that sector to improve

well: In our model, debt overhang affects the firm innovation decision and firm productivity growth. Our model can match the nonlinear relationship between firm default risk and firm growth that exists in the cross-section. Also, as Atkeson and Burstein (2010) and Luttmer (2007) show, theories of firm growth similar to our own can match a number of additional facts on firm growth and the firm size distribution.

There is an existing set of quantitative studies of the extent to which debt overhang affects firm investment and growth. Two important examples using the Q-theory approach include the work of Hennessy (2004) and Hennessy et al. (2007). There have been a few papers that use quasi-natural experiments to show this problem exists and assess the gains from resolving it, such as Giroud et al. (2012). We demonstrate the extent to which our estimated model is consistent with the estimates of Giroud et al. (2012) and Hennessy (2004) in Section 3.5. Another paper that studies the gains from resolving debt overhang is Moyen (2007). She calibrates a simple model of the firm and studies the gains from resolving this problem for a firm in partial equilibrium. We estimate a model of firm innovation and growth disciplined by the nonlinear relationship between firm default risk and firm growth in the data and find the expected private gains from resolving debt overhang to entering firms to be substantially lower, even in our “upper” bound case of the extent to which debt overhang affects firms.

Our paper is also related to a number of papers which study how debt overhang can affect the economy over the business cycle. Occhino and Pescatori (2012) embed the debt overhang problem in a business cycle model, and find that the debt overhang problem can generate both a labor wedge and investment wedge, and can amplify aggregate shocks. We discipline the extent to which debt overhang affects firm innovation decisions in the cross-section, prior to analyzing its effect over the business cycle. Chen and Manso (2010) demonstrate that the costs of debt overhang can be significantly higher in the presence of macroeconomic risks. Gourio (2014) shows that firm default risk can play a significant role in driving employment losses in a recession. Our results contribute to this literature by demonstrating

welfare.

that heightened firm default risk during recessions can interact with nonlinearities in the firm investment decision (due to debt overhang), and depress firm growth. The role of firm default and insolvency risk in the economy and its measurement is highlighted by Atkeson, Eisfeldt, and Weill (2013).

The rest of the paper follows as such. Section 3.2 presents the model. Section 3.3 defines counterfactuals in the model. Section 3.4 defines our moment conditions and demonstrates analytically why they identify our parameters. Section 3.5 describes the data we use, and the details and results of our estimation procedure. Section 3.6 describes the results from solving the counterfactuals defined in Section 3.3 under our estimates. Section 3.7 concludes.

3.2 The Model

First, we describe a standard production environment under which firms operate. We then define the problem of equity holders. Afterwards, we specify the problem of the debt holders and the general equilibrium environment, and define an equilibrium. We describe our model in this order, because we define some counterfactual objects that only require a solution to the problem of equity holders, some that only require a solution to the problem of the firm, and some that require the solution to the full general equilibrium model.

3.2.1 The Physical Environment

Time is discrete and indexed as $t = 0, 1, 2, \dots$. There is a competitive final good sector and a monopolistically competitive intermediate good sector. The final good is produced from a continuum of differentiated intermediate goods. Intermediate good firm productivities evolve endogenously through process innovation, and the measure of differentiated intermediate goods is determined endogenously through product innovation. All innovation and intermediate good firm production requires labor, which is paid wage w_t . Firms can issue both equity and debt to finance their operations, which are held and priced by households.

Households consume the final good.

Production The final good is produced from intermediate goods with constant elasticity of substitution (CES) production function:

$$Y_t = \left(\int_i y_t^{i^{1-\frac{1}{\rho}}} di \right)^{\frac{\rho}{\rho-1}}, \quad (3.1)$$

where i indexes intermediate good firms, and $\rho > 0$. In our model, there is a standard inefficiency due to the monopoly markup in the production of intermediate goods. To undo this distortion, we allow for a per-unit subsidy, τ^s , on the production of the consumption good.⁶⁸ In equilibrium, standard arguments show prices must satisfy

$$(1 + \tau^s)P_t = \left[\int_i p_t^{i^{1-\rho}} di \right]^{\frac{1}{1-\rho}}, \quad (3.2)$$

where p_t^i is the price set by firm i , and P_t is the price set by final good firms. We choose the price of the final good to be numeraire. Thus, from profit maximization demand for intermediate goods is

$$y_t^i = (1 + \tau_s)^\rho p_t^{i-\rho} Y_t, \quad (3.3)$$

given (3.1) and (3.2).

Firm i produces output, y_t^i , with labor, l_t^i , using the following constant returns to scale production function:

$$y_t^i = \exp(z_t^i)^{\frac{1}{\rho-1}} l_t^i. \quad (3.4)$$

For simplicity, we will refer to e^z as our measure of firm productivity.⁶⁹

⁶⁸The per-unit subsidy on the intermediate good keeps the specification without the tax advantage of debt from being distorted from optimal production, so the tax advantage (and firm debt issuance) does not resolve this entry/production inefficiency.

⁶⁹The physical labor productivity of an intermediate good firm is $e^{z \frac{1}{\rho-1}}$.

3.2.2 The Problem of Equity Holders

At time t , equity holders are indexed by the natural log of their productivity, z_t , the size of their current liabilities, d_t , and the aggregate state, S_t , which contains the distribution Γ_t of firms across (z_t, d_t) .

We will, in turn, summarize the transition function of the aggregate state as

$$S_{t+1} = H(S_t). \quad (3.5)$$

The firm has an investment technology through which it can lower its marginal cost of production. Productivity at the firm level evolves conditional on the investments the firm has made in improving its productivity and on idiosyncratic productivity shocks. We assume that firm productivity follows a binomial process. The amount, Δ_z , that the natural log of firm productivity can move up or down in a period is constant. At time t , with probability q_t the firm's productivity will improve, and with probability $1 - q_t$ its productivity will decrease.

We assume that the cost function is proportional in a firm's size and convex, such that:

$$\phi(q_t, z_t) = \exp(z_t)h \exp(bq_t), \quad (3.6)$$

for all firms, where $b > 0$ and $h > 0$.

We assume that if firms do not default, they have the same value of d as the period before. Equity holders have to pay d to creditors, and d has a tax advantage relative to equity, τ^d , because payments to debt holders are tax deductible. Equity holders exit if their discounted present value of profits falls below 0. When the firm holds debt, marginal benefits from a unit of investment for equity holders vs. equity holders and creditors combined differ, and b governs how much the investment decision responds to this difference. The more levered the firm is relative to business risk, the lower the value of equity. Hence, as b is lower, so that the marginal benefits from a unit of investment for equity holders and the firm as a whole

differ by more, firms will invest less as they are more levered relative to their business risk. In this sense, b controls the extent of the debt overhang distortion.

At every time t , firm i solves

$$\pi(S_t, z_t^i) = \max_{y_t^i} p(S_t, y_t^i) y_t^i - w(S_t) \frac{y_t^i}{\exp(z_t^i)^{\frac{1}{\rho-1}}}, \quad (3.7)$$

subject to (3.3) and (3.4) to maximize profits. With standard arguments, firm i 's p_t^i , y_t^i , and l_t^i can be derived as functions of only z_t^i and the aggregate state.

Given the production environment in Subsection 3.2.1, π_t , from (3.7), of a firm with productivity $\exp(z_t)$ can be written as the firm's productivity multiplied by a scaling factor, $\Pi(S_t)$, which is a function of aggregates and parameters. Formally, $\pi(z_t) = \exp(z_t)\Pi(S_t)$.

Firms have to pay corporate taxes, τ , on their operating profits. Debt has a tax advantage, denoted by τ^d . Innovation requires labor. Equity holders thus receive the following cash flows:

$$CF_E(S_t, z_t, d_t) = (1 - \tau) \left(\pi(S_t, z_t) - w(S_t) \phi(q_t, z_t) \right) - (1 - \tau^d) d_t. \quad (3.8)$$

Hence, the expected discounted present value of profits for equity holders of a firm with state (S_t, z_t, d_t) satisfies the following Bellman equation:

$$V_E(S_t, z_t, d_t) = \max_{q_t} \left\{ 0, CF_E(S_t, z_t, d_t) + E_t[M_{t+1} \left(q_t V_E(S_{t+1}, z_t + \Delta_z, d_t) + (1 - q_t) V_E(S_{t+1}, z_t - \Delta_z, d_t) \right) | S_t] \right\}. \quad (3.9)$$

where M_{t+1} is the pricing kernel that one can derive from the household's problem, and CF_E is defined in (3.8). In a steady state, $M_{t+1} = \beta$. The firm's exit threshold, $\bar{z}_t(S_t, z_t, d_t)$, is a function of the aggregate state and the firm's idiosyncratic state.

The optimal innovation decision of equity holders can thus be written as

$$q_t^*(S_t, z_t, d_t) = \frac{1}{b} \log \left(\frac{E_t[M_{t+1} \left(V_E(S_{t+1}, z_t + \Delta_z, d_t) - V_E(S_{t+1}, z_t - \Delta_z, d_t) \right) | S_t]}{w_t b (1 - \tau) h \exp(z_t)} \right). \quad (3.10)$$

3.2.3 Heterogeneity in Investment Opportunities

One reason that firms grow at different rates is they differ in their investment opportunities. We model firms differing in their investment opportunities following Acemogulu, Akcigit, Bloom, and Kerr (2013); firms differ in the level of their cost function. Define θ_t at time t as the level of investment opportunities for a given firm.

We can amend (3.6), such that the cost function is now

$$\phi(q_t, z_t, \theta_t) = \exp(z_t) \theta_t^{-b} h \exp(bq_t). \quad (3.11)$$

With such a functional form, firms will still differ in their investment decision if $b \rightarrow \infty$, conditional on θ_t . The problem without such heterogeneity has $\theta_t = 1$ for all firms.

Equity holders thus receive the following cash flows:

$$CF_E(S_t, z_t, d_t, \theta_t) = (1 - \tau) \left(\pi(S_t, z_t) - w(S_t) \phi(q_t, z_t, \theta_t) \right) - (1 - \tau^d) d_t. \quad (3.12)$$

Given previous assumptions and parameter restrictions, the expected discounted present value of profits for equity holders of a firm with state $(S_t, z_t, d_t, \theta_t)$ satisfies the following Bellman equation:

$$V_E(S_t, z_t, d_t, \theta_t) = \max_{q_t} \left\{ 0, CF_E(S_t, z_t, d_t, \theta_t) + E_t[M_{t+1} \left(q_t V_E(S_{t+1}, z_t + \Delta_z, d_t, \theta_{t+1}) + (1 - q_t) V_E(S_{t+1}, z_t - \Delta_z, d_t, \theta_{t+1}) \right) | S_t, \theta_t] \right\}. \quad (3.13)$$

Equity holders again have an exit threshold, \bar{z}_t , which is now a function of z_t , d_t , and θ_t .

The optimal innovation decision of equity holders is thus

$$q_t^*(S_t, z_t, d_t, \theta_t) = \log(\theta_t) + \frac{1}{b} \log \left(\frac{E_t[M_{t+1} \left(V_E(S_{t+1}, z_t + \Delta_z, d_t, \theta_{t+1}) - V_E(S_{t+1}, z_t - \Delta_z, d_t, \theta_{t+1}) \right) | S_t, \theta_t]}{b(1 - \tau)w_t h \exp(z_t)} \right). \quad (3.14)$$

(3.14) makes it apparent why as $b \rightarrow \infty$, only θ affects the expected growth rate of firms.

3.2.4 The Debt Contract and Firm Value

In this subsection, we now specify a debt contract consistent with the functional form assumptions in the problem of equity holders and define the problem of the firm.

Perpetuity Debt and Trade-off Theory Following Leland (1994), we will assume that the firm only holds perpetuity debt. The problem of equity holders is the same as in (3.13). Firms hold debt because it has a tax advantage, but do not fully finance themselves with debt because of the possibility of costly bankruptcy.

Timing of Bankruptcy and the Problem of the Firm At the start of each period, t , each incumbent firm has a probability, δ , of exiting, and a probability, $1 - \delta$, of surviving to produce. There is also a discount rate of the firm, r . Notice, then, $e^{-r}(1 - \delta)$ is the discount factor of the firm, which we defined to be β in (3.13).⁷⁰ If the firm survives, equity holders then choose whether to declare bankruptcy or continue to operate. If the firm declares bankruptcy, it loses a fixed proportion, $(1 - \alpha)$, of its productivity, where $\alpha \in (0, 1]$. The existing creditors then gain full equity control of the firm and take out new debt to maximize the joint value of equity holders and new creditors.

⁷⁰If we calibrate the model such that there are multiple periods in a year (say one period is $\frac{1}{\Delta}$ of a year), then the discount factor is $e^{-r\frac{1}{\Delta}}(1 - \delta)$.

If equity holders decide not to go bankrupt, the expected discounted present value of profits for the joint value of equity holders and creditors of a firm with idiosyncratic state variable (z_t, d_t, θ_t) satisfies the following Bellman equation:

$$\begin{aligned}
V_A(S_t, z_t, d_t, \theta_t) = & (1 - \tau) \left(\pi(S_t, z_t) - w(S_t) e^{z_t} \theta_t^{-b} h e^{bq_t} \right) + \tau^d d_t + \\
& E_t[M_{t+1} \left(q_t V_A(S_{t+1}, z_t + \Delta_z, d_t, \theta_{t+1}) + \right. \\
& \left. (1 - q_t) V_A(S_{t+1}, z_t - \Delta_z, d_t, \theta_{t+1}) \right) | S_t, \theta_t \}. \tag{3.15}
\end{aligned}$$

If equity holders decide to go bankrupt, the expected discounted value of the profits of the firm is

$$V_A(S_t, z_t, d_t, \theta_t) = \max_{d_{t+1}} V_A(S_t, z_t + \log(\alpha), d_{t+1}, \theta_t). \tag{3.16}$$

Let $d^*(S_t, z_t, \theta_t)$ be the optimal choice of d_{t+1} that satisfies (3.16). The value of creditors, V_B , is defined as the difference between the value of the firm as a whole, (3.15) and (3.16), and the value of equity holders, (3.9); thus,

$$V_B(S_t, z_t, d_t, \theta_t) = V_A(S_t, z_t, d_t, \theta_t) - V_E(S_t, z_t, d_t, \theta_t).$$

3.2.5 The General Equilibrium Environment

We now fully flesh out a general equilibrium environment consistent with the functional form assumptions that enter the problems of debt holders and equity holders.

Free Entry We assume there is free entry into the economy. New firms are created by purchasing n_e units of labor; a purchase in period t yields a new firm in period $t + 1$ with initial state variables z_t and θ_t drawn from a distribution G . After receiving z_t and θ_t , the firm makes an initial debt decision to maximize the value of equity holders and new creditors.

In any period with a positive mass of entering firms, we have

$$w(S_t)n_e = E_t[M_{t+1} \int_{\theta} \int_z \max_{d_{t+1}} V_{A,t+1}(S_{t+1}, z, d_{t+1}, \theta) G(z, \theta) dz d\theta | S_t]. \quad (3.17)$$

We define $\Gamma_{e,t}$ as the measure of new firms entering the economy at period t that start producing in period $t + 1$.⁷¹

Households Households are endowed with L units of time which they supply inelastically. After all (idiosyncratic and aggregate) shocks are realized, households make a consumption decision, C_t , get paid wages, $w(S_t)$, receive a lump sum transfer of dividends, D_t , and pay a lump-sum tax, T_t .

The recursive problem for households is the following:

$$V^H(S_t) = \max_{C_t} [\log(C_t) + e^{-r} E_t V^H(S_{t+1}) | S_t] \quad (3.18)$$

subject to their budget constraint:

$$C_t = w(S_t)L + D_t - T_t, \quad (3.19)$$

and the aggregate law of motion for S_t , (3.5). The aggregate dividend is the sum of all after-tax profits from intermediate good firms net of entry costs of all newly entering firms.

The Distribution of Firms The distribution of operating firms at time t , $\Gamma_t(z, d, \theta)$, evolves over time as a function of the exogenous exit rate, δ , the choices of q_t by incumbent firms, and the mass of entering firms each period, $\Gamma_{e,t}$. We assume that firm types, θ , are fixed over time for a given firm (but can vary across firms). To simplify the definition of the mass of firms with state $(z_{t+1}, d_{t+1}, \theta_{t+1})$ in period $t + 1$, we break it into four pieces. First,

⁷¹If the mass of entering firms were zero, then (3.17) instead should state that the cost of entering (left hand side) must be greater than or equal to the value of entering (right hand side).

there is a mass of continuing firms who did not go bankrupt who could enter period $t + 1$ with state (z_{t+1}, d_t, θ_t) , which is a function of continuing firms with productivity $z_{t+1} - \Delta_z$ last period that drew positive productivity shocks and continuing firms with productivity $z_{t+1} + \Delta_z$ last period that drew negative productivity shocks:

$$\begin{aligned} \Gamma_{t+1}^C(z_{t+1}, d_{t+1}, \theta_{t+1}) &= (1 - \delta)(1 - q_t(S_t, z_{t+1} + \Delta_z, d_{t+1}, \theta_{t+1}))\Gamma_t(z_{t+1} + \Delta_z, d_{t+1}, \theta_{t+1}) \\ &\quad + (1 - \delta)q_t(S_t, z_{t+1} - \Delta_z, d_{t+1}, \theta_{t+1})\Gamma_t(z_{t+1} - \Delta_z, d_{t+1}, \theta_{t+1}). \end{aligned} \quad (3.20)$$

Second, Γ_{t+1} is also a function of the mass of entering firms who received productivity, z_{t+1} , and investment opportunities, θ_{t+1} , such that they chose coupon payment d_{t+1} :

$$\Gamma_{t+1}^E(z_{t+1}, d_{t+1}, \theta_{t+1}) = \Gamma_{e,t}G(z_{t+1}, \theta_{t+1}). \quad (3.21)$$

Third, Γ_{t+1} is a function of the mass of firms who have productivity $z_{t+1} + \Delta_z - \log(\alpha)$ last period, with type θ_{t+1} and coupon payment d that drew negative productivity shocks, went bankrupt, and chose coupon payment d_{t+1} .⁷²

$$\begin{aligned} \Gamma_{t+1}^B(z_{t+1}, d_{t+1}, \theta_{t+1}) &= (1 - \delta) \int (1 - q_t(S_t, z_{t+1} + \Delta_z - \log(\alpha), d, \theta_{t+1})) * \\ &\quad \Gamma_t(z_{t+1} + \Delta_z - \log(\alpha), d, \theta_{t+1}) dd. \end{aligned} \quad (3.22)$$

Hence, we can define $\Gamma_{t+1}(z_{t+1}, d_{t+1}, \theta_{t+1})$, as the sum of $\Gamma_{t+1}^C(z_{t+1}, d_{t+1}, \theta_{t+1})$, $\Gamma_{t+1}^E(z_{t+1}, d_{t+1}, \theta_{t+1})$, and $\Gamma_{t+1}^B(z_{t+1}, d_{t+1}, \theta_{t+1})$ using (3.20), (3.21), and (3.22).

⁷²It is also possible for firms to have had productivity $z_{t+1} - \Delta_z - \log(\alpha)$ last period, type θ_{t+1} and debt load d , to go bankrupt and choose debt d_{t+1} , although this does not occur in a steady state, so we do not include this case.

Equilibrium In our simple setup, market clearing for the final good requires:

$$C(S_t) = Y(S_t).$$

Market clearing for labor requires

$$\int \int \int l_t(z) \Gamma_t(z, d, \theta) dz d d d \theta + L_{r,t} = L,$$

where $\int \int \int l_t(z) \Gamma_t(z, d, \theta) dz d d d \theta$ is total employment used to produce the intermediate good, whereas $L_{r,t}$ denotes labor spent on process and product innovation.

We can write labor spent on research (process and product innovation) as

$$\Gamma_{e,t} n_e + \int \int \int e^z \theta_t^{-b} h e^{bq_t} \Gamma_t(z, d, \theta) dz d d d \theta = L_{r,t}.$$

A recursive competitive equilibrium in this economy is defined as follows. Given initial distribution, Γ_0 , a recursive equilibrium consists of policy and value functions of equity holders, creditors, and intermediate good firms, $\{l(S_t, z_t), V_E(S_t, s_t), \bar{z}(S_t, x_t), q^*(S_t, s_t), V_B(S_t, s_t), V_A(S_t, s_t), d^*(S_t, z_t, \theta_t)\}$ where $x_t = (d_t, \theta_t)$ and $s_t = (z_t, x_t)$, household policy functions for consumption, $C(S_t)$, aggregate prices, $\{P(S_t), w(S_t)\}$, the mass of new entrants, $\Gamma_e(S_t)$, and the aggregate states including the distribution of firms, S_t , which evolve according to transition function $H(S_t)$ such that for all t : (i) the policy and value functions of intermediate good firms are consistent with the firm's optimization problem, (ii) the representative consumer's policy function is consistent with its maximization problem, (iii) debt and equity holders' value functions and decision rules are priced such that they break even in expected value, (iv) free entry holds (v) labor and final good markets clear, and (vi) the measure of firms evolves in a manner consistent with the policy functions of firms, households, and shocks.

A stationary competitive equilibrium is an equilibrium in which all aggregates, aggregate

prices, and the distribution of firms are constant over time.⁷³ In such an equilibrium, we say these aggregates are in steady-state. We focus only on equilibria with positive entry.

3.3 Definitions of Counterfactual Objects

We use our model to address the following questions: What are the expected private gains from resolving debt overhang for firms in the cross-section and over the business cycle, and for the average firm entering the economy? We also ask: What are the gains for the aggregate economy in the long run from resolving this problem for all firms? In this section, we define the objects that allow us to answer these questions. We discuss our partial and general equilibrium counterfactuals separately.

3.3.1 Partial Equilibrium Counterfactuals

3.3.1.1 Counterfactuals that Only Require the Problem of Equity Holders

With only the cross-sectional distribution of firm distance-to-default from the data and the problem of equity holders, we can compute a counterfactual that assesses the expected private gains from resolving the debt overhang problem. We assess the gains were all firms to make the same innovation decision as the unlevered firm. We are, in turn, comparing the gains were the cost function inelastic where $b = \infty$ to those when the cost function is elastic to the extent that we estimate. We compute the policy function and associated implied annualized growth rate under our estimate for each value of distance-to-default in the data. We then compute the weighted-average value of q and respective implied expected annualized growth rate. We then compare the implied annualized growth rate to the implied annualized growth rate were all firms to make the same decisions as the unlevered firm. Given the fact that there is a distribution of firms in each year, we can also perform this counterfactual year by year.

⁷³We present our model aggregation in a steady-state in Appendix H.

3.3.1.2 Firm Value Counterfactuals

Similar to the counterfactuals above, conditional on firm distance-to-default, we can compare two firms, one that suffers from debt overhang (equity holders make the investment decision) and one that does not (the firm as a whole makes the investment decision), and compare their value functions or expected annualized growth rates assuming prices and the mass of firms do not change.⁷⁴ To recover the gains in terms of firm value upon entry, we hold fixed all general equilibrium effects that could affect firm value (prices, the labor allocation, and the supply of firms), and solve the model again, assuming the firm resolves the debt overhang problem. We then compare the percentage difference between the value function of the average entering firm if the firm does not and does suffer from debt overhang. Following Moyen (2007), we decompose these gains into the gains from operations, the gains from the tax advantage, and the losses from bankruptcy. The value from operations is the expected discounted present value of the firm's production and investment activities. The tax advantage of debt is the expected discounted present value of all interest deductions. The default cost is the expected discounted present value of the deadweight losses from bankruptcy.

3.3.2 General Equilibrium Counterfactuals

There are three distortions in the model we want to focus on: debt overhang, bankruptcy costs, and other equilibrium distortions caused by the tax advantage of debt and bankruptcy costs. We develop a decomposition of the social losses in our baseline model relative to the planner's problem to isolate the effects of these distortions based on counterfactual objects.

Planner's Problem The social planner chooses consumption, product innovation, process innovation, and the labor allocation to maximize her discounted present value of utility such that the final good market clears, the labor market clears, and the law of motion for

⁷⁴We define the policy function were the firm as a whole, rather than equity holders alone, to make the innovation decision in Appendix I.

productivity is satisfied. In our setup, the planner’s problem is the equivalent of setting $\tau^d = 0$ and $\tau = 0$ with a per-unit subsidy, τ^s , on production of the consumption good that undoes the distortion from the efficient allocation from the markup in our model. The subsidy takes value $\tau^s = \frac{\rho}{\rho-1}$. When aggregating our model, we include the subsidy in all counterfactuals and in our base case. We also set $\tau = 0$ to focus on the distortion of interest, which is the social cost due to the tax advantage of debt, and the associated costs of debt overhang and bankruptcy.

Social Loss Decomposition We define the planner’s problem above. Define consumption from the planner’s problem to be C^{EFF} . We define $LOSSES_{EFF}$ as the long-run differences in aggregate consumption between the planner’s problem and our base case with debt overhang:

$$LOSSES_{EFF} = \frac{C^{EFF} - C}{C},$$

where C is consumption from our baseline estimation. We call these losses “social losses,” and moving forward we describe welfare as differences in consumption between steady states. To further decompose these social losses, we create two more consumption measures. To create our first additional consumption measure, we have the firm as a whole, rather than equity holders, make the innovation decision. We solve for a stationary competitive equilibrium given these decision rules and recover a counterfactual object, C^{ND} . We define the losses from debt overhang as

$$LOSSES_{DO} = \frac{C^{ND} - C}{C}.$$

We then create one more object to recover two more counterfactuals objects. We have the firm as a whole make innovation decisions (as in C^{ND}) and also treat α purely as a financial cost. Thus, this cost is a transfer payment, but no productivity is lost in bankruptcy. We

then recover a new consumption measure: $C^{NO\alpha}$. This object gives us the ability to create two counterfactuals, which along with $LOSSES_{DO}$ should add up to $LOSSES_{EFF}$. The first counterfactual object represents the effect of bankruptcy on the total mass of productivity:

$$LOSSES_{NO\alpha} = \frac{C^{NO\alpha} - C^{ND}}{C}.$$

The second is the remaining loss, which can be interpreted as the degree to which α and τ^d distort firm decisions relative to the social planner's choice:

$$LOSSES_{REM} = \frac{C^{EFF} - C^{NO\alpha}}{C}.$$

3.4 Identification of Parameters

As we are interested in estimating both the partial and general equilibrium gains from resolving debt overhang, the parameter that governs the nonlinear relationship between debt overhang and firm default risk will be crucial to our exercise. We infer this parameter from the relationship between a measure of firm default risk and firm growth, accounting for the nonlinearity in the relationship and for unobserved heterogeneity in investment opportunities as a potential driver of this relationship, in a manner explained in this section.⁷⁵ In Section 3.5, we compare moments implied by our estimated parameters to existing estimates on the extent to which debt overhang affects investment and growth from Hennessy (2004) and Giroud et al. (2012), and find our estimates are consistent with those in these papers when

⁷⁵It is important to note that we do impose a functional form on this nonlinearity with our cost function. We have also written down our model with an isoelastic cost function (of which our cost function is a special case) which requires two parameters to govern the nonlinear relationship between debt overhang and default risk rather than one, and allows more flexibility in the functional form of this relationship. Taking our estimates of Δ_z and q_∞ and the moment conditions that first-order drive the identification of these parameters as given, we checked the parameters and associated functional form of the cost function required to match our remaining moment conditions. The functional form was quite close, close enough such that the added benefit of having a clearer economic interpretation from only a single parameter governing the extent to which debt overhang affects firms outweighed the cost of using it, in that the more general cost function implies only a marginal statistic improvement in matching the nonlinearity in the relationship between distance-to-default and growth.

taking their respective sample properties into account. However, we find that our model-implied statistics which correspond to their estimates vary considerably along the margin of distance-to-default. It is this variation that is crucial to generating the results from our counterfactual exercises, and it would be difficult to back out such variation in these statistics from these existing papers alone without imposing assumptions on the nonlinearity in the relationship between default risk and debt overhang one could not confirm without an estimation procedure, such as the one we perform.

In the remainder of this section, we demonstrate analytically in a special case of the general model why our moment conditions identify our parameters. In Section 3.5, we demonstrate that in the full model, our parameters are still sharply locally identified by the same moments. To confirm why this is the case, in Section 3.6, we compare our estimated policy functions and some of the results from our counterfactual exercises to those obtained analytically in the simplified setting, and find their relationship to be extremely tight.

3.4.1 Mappings to Observables

We begin by mapping our state variable and policy function to observables. Here, we assume that investment opportunities are fixed throughout time, and the model is in steady-state. We maintain these assumptions throughout the remainder of this section. We write up how to perform this mapping as a proposition:

Proposition 3.1. *The problem of equity holders in a steady state when firm types are fixed has the following properties:*

- (i) *It can be reduced to two state variables: the firm's investment opportunities, θ , and the number of steps (where the step size is Δ_z) until the firm declares bankruptcy, n .⁷⁶*
- (ii) *Expected annualized employment growth can be fully characterized by the firm's innovation decision, (3.14), and parameters.*

⁷⁶Formally, $n = \frac{z - \bar{z}(d, \theta)}{\Delta_z}$, where $\bar{z}(d, \theta)$ denotes the level of productivity at which a firm with debt d and investment opportunities θ will default.

(iii) The state variable n has a 1-1 mapping with firm distance-to-default defined as: $\frac{\log(V_A(z,\theta)/V_B^*(d,\theta))}{\sigma_A}$, where $V_A(z,\theta)$ is the unlevered value of the firm and $V_B^*(d,\theta)$ is the unlevered value of the firm at its default point.⁷⁷

Proofs. See Appendix F.

Proposition 3.1 implies that the model predictions can easily be compared to well-defined and oft-studied objects in the data. Distance-to-default is a much studied variable in corporate finance with well-established methods for its estimation.⁷⁸

3.4.2 Moment Conditions and Identification in Closed Form

As we showed in Proposition 3.1, we can map the state variable and policy function of the problem of equity holders into observables. We now demonstrate how one can map these observables into moment conditions, and how these moment conditions identify our parameters. To do so, we require a closed-form solution to the policy function of equity holders. To our knowledge, this is not obtainable from the model in Section 3.2. However, we can obtain a closed-form approximation that works well in practice; we demonstrate the approximation works well in practice in Section 3.6. This approximation can be viewed as the policy function of the firm in a special case of the model described in Section 3.2, where we assume the firm innovation decision is made only once in the first period, and in all future periods the firm takes as given the innovation decision to be an exogenous value. We assume that we are using this simple model through the remainder of this subsection.

The closed-form solution to the policy function in the simple model is derived in Appendix F. With the policy function, we demonstrate identification of key parameters for the problem of equity holders, using moments related to the properties of firm employment growth and

⁷⁷For (iii) of Proposition 3.1, we assume that the choice of q of the firm is 0.5. Given we construct DD from daily returns, this is a reasonable assumption. All of our estimates of the innovation decision of the unlevered the firm are close to 0.5 as well.

⁷⁸We discuss how we estimate firm distance-to-default and how our measurement procedure compares to those in the literature in Appendix G.

its relationship to firm distance-to-default. The key model parameters we estimate are b , the convexity of the cost function, Δ_z , the size of steps in the binomial process, and h , the level of the cost function. We define four moments used for local identification of the parameters of the model: the average year-ahead employment growth rate of unlevered firms, the variance of firm employment growth, and regression coefficients from a regression of annualized employment growth on distance-to-default and the square of distance-to-default. The following proposition outlines that, with the closed-form solution to the simple model, we can characterize each of these four moments analytically, and defines key relationships between the parameters and the moment conditions.

Proposition 3.2. *The closed-form solution for the problem of equity holders, as derived in Appendix F, has the following properties:*

- (i) *The derivative of firm expected employment growth to distance-to-default can be characterized in closed form, and its magnitude is proportional in $\frac{1}{b}$.*
- (ii) *The second derivative of firm expected employment growth to distance-to-default can be characterized in closed form, and its magnitude is proportional in $\frac{1}{b}$.*
- (iii) *The expected annualized growth rate of the unlevered firm can be fully characterized as a function of Δ_z , h , and Δ , and is decreasing in h .*
- (iv) *The variance of firm growth rates can be characterized as a function of the expected annualized average growth rate of firms and Δ_z , and is increasing in Δ_z holding the expected average growth rate of firms fixed.*

Proofs. See Appendix F.

The main result is that b has a first-order effect on the first and second derivatives of firm expected growth with respect to firm distance-to-default (DD). This result is crucial for our identification strategy: It says that the parameter which governs the extent to which the debt overhang problem affects firms in our model is to a first-order identified by the regression coefficients of a regression of employment growth on DD and DD^2 . The remainder of the

estimation procedure follows from the next two results. Taking the expected average growth rate of firms as given, Δ_z has a quantitatively first-order effect on the variance of growth rates. Hence, the variance of firm growth rates will be to a first-order driven by Δ_z . Parameters h , Δ_z , and b matter substantially for average growth rates of the unlevered firm, but h has only a second-order effect on the derivatives of firm growth with respect to distance-to-default or the variance of firm growth rates. Hence, taking b and Δ_z as given, h to a first-order pins down the average growth rate of the unlevered firm. Thus, the four moments we define locally identify b , h , and Δ_z . In Section 3.5, we demonstrate that our parameters are well-identified and change in the expected parameters in the more general model described in Section 3.2. We provide further discussion and detail of the local identification of the parameters in Appendix F.

3.4.3 Deriving Bounds on b

The key remaining question necessary for identifying parameters in our model is: How do we control for heterogeneity in firm investment opportunities confounding our estimate of the convexity of the cost function? In particular, firms may be more likely to be near default and not growing if they have worse investment opportunities. Hence, we may overestimate the extent to which debt overhang affects firms if we do not control for such heterogeneity, and thus underestimate the convexity of the cost function. Similarly, if we attribute all of the relationship between firm distance-to-default and firm employment growth to unobserved heterogeneity, we may be underestimating the extent to which debt overhang affects firms. To overcome the difficulties in obtaining accurate point estimates of the extent to which unobserved heterogeneity could be driving the relationships we observe in the data, and to demonstrate the robustness of our results to alternative estimates of our parameters, we define moment conditions that imply upper and lower bound estimates of the extent to which debt overhang affects firms. We prove our bounds hold under some assumptions on the relationship between heterogeneity in firm investment opportunities and firm default

risk.

We first explain the intuition behind why the bounds hold. The estimation procedure finds a lower bound for b by not controlling at all for unobserved heterogeneity. By not controlling for firms having differences in their investment opportunities, we are overstating the role debt overhang plays in driving the relationship between distance-to-default and growth. We argue the estimate we obtain when we demean firm employment growth by its average growth rate implies an upper bound of b . The upper bound argument relies on the fact that debt overhang can affect the firm's average growth rate. Both heterogeneity and debt overhang can generate a positive relationship between distance-to-default and growth. Both have persistence, so many firms with lower average growth rates will also have lower average distance-to-default. By demeaning at the firm level, we are attributing all of the differences in average growth rates to firm heterogeneity. Therefore, in the regression where we demean firm growth rates, we have firms who have low average distance-to-default but thanks to demeaning, not lower average growth. This substantially weakens the relationship between distance-to-default and growth, and thus raises the estimate of b . Hence, for a given panel of firms, we argue we have intuitive reasonable upper and lower bound estimates of b .

We now use the closed-form approximation for firm innovation decisions discussed in Subsection 3.4.2 to demonstrate why the moment conditions above imply bounds on the parameter controlling the extent to which debt overhang affects firm growth, b , in our model. The estimate for b which ignores the presence unobserved heterogeneity will generate a lower bound, b_l , for b , while demeaning firm growth rates will bound b from above.⁷⁹

⁷⁹Instead of writing the propositions in terms of employment growth and DD , as in Proposition 3.2, we write them in terms of q and n . In the proof of Proposition 3.2, we show that one can easily translate the derivative of q with respect to n into a derivative of employment growth with respect to DD , and that this derivative is still proportional in $1/b$. Hence, such results will carry over when we replace q with growth and n with DD in the moment condition.

3.4.3.1 Lower bound

The approach in Subsection 3.4.2 generates firm innovation decisions in closed form. This solution is generalized in Appendix F to allow for firms to be heterogeneous in their investment opportunities as in (3.11). Firm investment opportunities are increasing in θ , which is heterogeneous across firms but fixed over time. Define b_0 as the true value of the parameter b . Assume all parameters besides b are known. Let $\frac{\partial E[q|n_i=n]}{\partial n}$ denote the observed derivative of firm innovation decisions with respect to steps from default, which is a function both of debt overhang and the changing distribution of firm types along steps from default. Define b_L as the value of b such that the derivative of firm innovation decisions with respect to firm steps from default in the model without heterogeneity is set equal to the observed derivative from the data. The following proposition states and proves that if, on average, firm investment opportunities are increasing in firm steps from default, b_L bounds the true value from below:

Proposition 3.3. *If $\frac{\partial E(\theta|n)}{\partial n} \geq 0$, then $b_L \leq b_0$.*

Proof. See Appendix F.

Proposition 3.3 implies that if the average investment opportunities of firms are increasing in their distance from default, then the slope between firm growth and distance to default will be driven not only by debt overhang, but also the changing composition of firm investment opportunities. Therefore we will over-attribute the effect of debt overhang if we do not control for such unobserved heterogeneity in the data and our estimate b_L will be below the true value of b .

3.4.3.2 Upper bound

Assume that we observe a firm over multiple periods. Let $\frac{\partial E[q-\bar{q}|n_i=n]}{\partial n}$ denote the observed derivative of demeaned firm growth rates with respect to firm steps from default, which is a function of debt overhang, the changing distribution of firm types, and a firm's other observed growth rates along firm steps from default. We define b_H as the value of b such that

the derivative of firm growth with respect to firm steps from default in the model without heterogeneity is set equal to the derivative in the data where growth is demeaned at the firm level. The following proposition states and proves that if firm average growth rates conditional on firm types are increasing in n , then b_H bounds the true value from above:

Proposition 3.4. *If $\frac{\partial E(\bar{q}|\theta,n)}{\partial n} \geq 0$, then $b_0 \leq b_H$.*

Proof. See Appendix F.

Proposition 3.4 implies that if firms observed close to default are more likely to also be closer to default in the other periods they are observed, then demeaning a firm's observed growth rates not only removes the variation explained by varying investment opportunities, but also some of the variation explained by its persistent exposure to debt overhang. Therefore we will under-attribute the effect of debt overhang and our estimate b_H is an upper bound for the true value of b .

In Section 3.5, we numerically confirm that our analytical results from the closed-form approximation carry over to the general model. Under a calibration of the process for firm investment opportunities, we show that when we generate synthetic data from the model and obtain the upper and lower bound moment conditions from the synthetic data, after reestimating our model, the conditions needed to prove the propositions above hold and we obtain estimates of b relative to the *true* b used to estimate our model below b with the lower bound moment conditions and above b with the upper bound moment conditions.

3.5 Estimation

In this section, we describe the data we use to estimate our model, the details of our estimation strategy, how we calibrate the remaining parameters in our model, and provide the results from our overidentified indirect inference estimation procedure. At the end of this section, we compare our estimates to those that exist in the literature, and we perform a numerical analysis of our bound argument from Section 3.4.

3.5.1 Data and Measurement

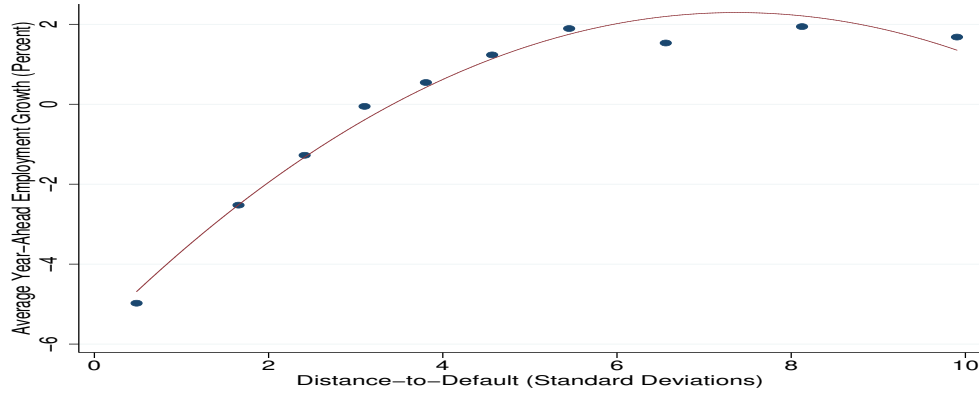
We use nonfinancial public firm data from 1982 to 2012. Equity market data come from CRSP, and annual and quarterly accounting statements come from Compustat. We discuss exact details of how our data is constructed along with some variable definitions in Appendix G. In our estimation procedure, we will rely heavily on properties of employment growth, especially its relationship with firm distance-to-default. If we define V_B as the book value of debt, V_A as the value of assets, and σ_A as the standard deviation of the value of the firm's assets, we can define our measure of firm distance-to-default as $\frac{\ln(\frac{V_A}{V_B})}{\sigma_A}$.⁸⁰

We detail how we construct V_A and σ_A from the data in Appendix G. Distance-to-default is measured in units of the number of standard deviations of annual asset volatility by which the firm's assets must change to equal the firm's book value of its debt. We winsorize the measure to lie between 0 and 10. Employment growth is measured as log differences in employment from year to year. Figures 3.1a and 3.1b plot the relationship between distance-to-default and a detrended measure of average year-ahead growth, where the measure of year-ahead growth is the residuals from a regression of year-ahead employment growth on year and industry dummies. We plot a quadratic fit through the data to demonstrate that the regression, (F.18), discussed in Appendix F, will provide a good fit to the shape of the data. The shape is also similar using a Kernel-smoothing regression. In Figure 3.1c, we plot the residuals from a regression on industry dummies, year dummies, log number of employees at the firm, firm age, and the Whited-Wu index for the firm, which is an index of firms' external financing constraints against firm distance-to-default.⁸¹ The independent variables are all measures that are known to be strongly correlated with growth rates. We see that the relationship looks similar were we to focus on sales growth or capital growth, which should assuage concerns about measurement error in employment in Compustat affecting our results.

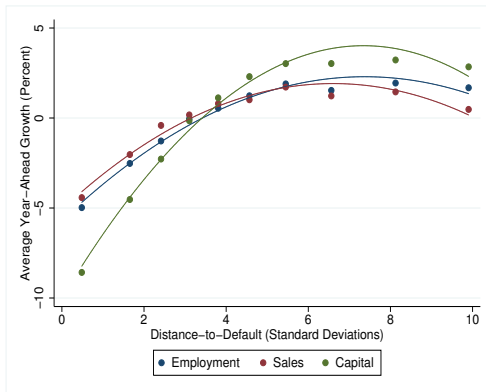
⁸⁰We can also compute distance-to-default as in Merton (1974), and we find very similar data moments.

⁸¹See Whited and Wu (2006) for how to construct the Whited-Wu index.

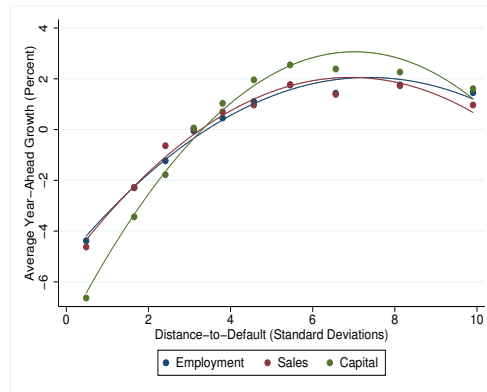
Figure 3.1: Growth vs. Distance-to-Default across U.S. Nonfinancial Public Firms



(a) Employment growth, controlling for year and industry effects



(b) Employment, sales, and capital growth, with year and industry effects

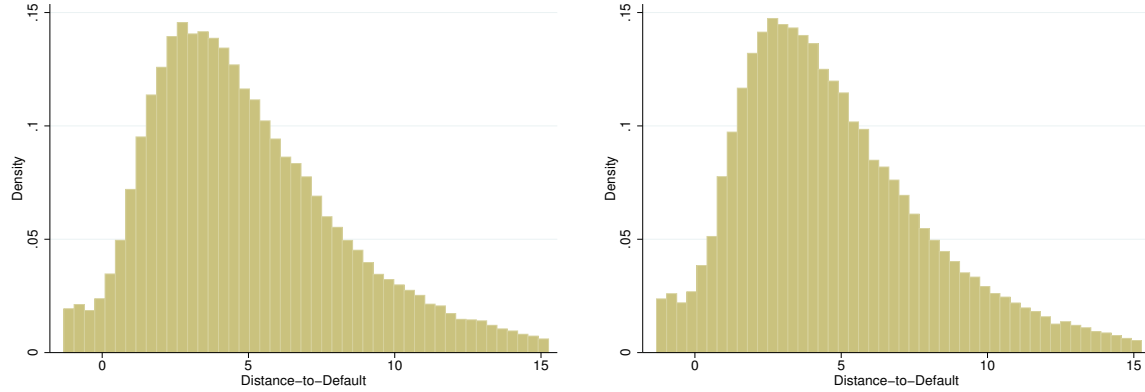


(c) Employment, sales, and capital growth, with additional controls

Sample Period: 1982 to 2012. These figures present binned scatter plots (binned into 10 bins) of a residualized measure of year-ahead growth vs. firm distance-to-default. We also plot a quadratic fit line derived from the underlying data. The y-axis is the residuals from a linear regression on controls, whereas the x-axis is not controlled. Distance-to-default is measured using the methodology described in Appendix G. The additional controls in Figure 3.1c are firm size, firm age, and a measure of firm access to external finance.

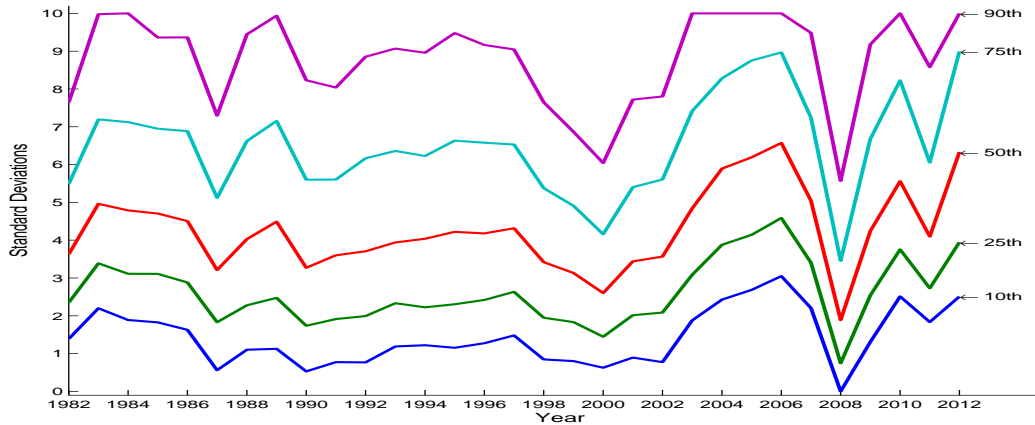
The relationship we establish between firm growth rates and firm distance-to-default will likely exist even absent debt overhang affecting firms. We expect firms that are not growing are on average more likely to have higher leverage relative to their business risk, so, ex ante, we should expect distance-to-default and growth to have a monotonically increasing relationship. The estimation procedure finds reasonable bounds on the extent to which the relationships plotted in Figures 3.1a, 3.1b, and 3.1c could be driven by debt overhang, accounting for such a reverse causality argument.

Figure 3.2: The Distribution of Distance-to-Default across U.S. Nonfinancial Public Firms



(a) The distribution across firm-years excluding NBER recession years

(b) The distribution across firm-years including NBER recession years



(c) The distribution of distance-to-default across firms in a given year

Sample Period: 1982 to 2012. The top left subfigure plots the distribution outside of NBER recession years. The top right subfigure plots the distribution including NBER recession years. Subfigure 3.2c plots the 10th, 25th, 50th, 75th, and 90th percentiles of distance-to-default from 1982-2012. Distance-to-default is measured as described in Appendix G.

3.5.2 Estimation Implementation

We use an indirect inference approach to estimate key model parameters. Our moments are a function of the joint distribution of firm growth rates and firm distance-to-default. We first compute moments in the data, and then compute model-implied moments using a combination of the distribution of firm distance-to-default and firm characteristics in the data and model implied decision rules conditional on firm distance-to-default. This procedure allows us to exactly match the distribution of distance-to-default for each firm over time as

parameters change, thus avoiding some biases associated with the debt contract in the model being misspecified. Further, we inherently correct for important sample characteristics and selection effects in Compustat for which we want to account.

We implement our indirect inference procedure in the following standard way. Say our model moments are the $1 \times n$ vector, $\hat{M}(G)_t$, where G represents the tuple (b, q_∞, Δ_z) , and our data moments are the $1 \times n$ vector, \hat{D}_t . Define $\hat{g} = \hat{M}(G)_t - \hat{D}_t$. We want to minimize $\hat{g}W\hat{g}'$ over G , where W is the weighting matrix, which we choose to be the identity matrix. We then compute standard errors with a nonparameteric block bootstrap procedure with 250 repetitions. To provide more detail, we create 250 new datasets with a bootstrap procedure (resampling firms with replacement). For each dataset, we compute the data moments of interest, while also saving each respective vector of firm distance-to-default. Using each resampled vector of firm distance-to-default as an input into the model, we reestimate the model with respect to the resampled data moments with the indirect inference procedure described above. We then have 250 new estimates of each parameter, and the reported standard deviation for each parameter is the standard deviation reported across those 250 estimates.

3.5.3 Estimation Specifications

Our estimation specifications differ in the data samples used and the methods by which we control for correlates of firm growth and potential firm heterogeneity. Our baseline sample is a large unbalanced panel of nonfinancial public firms that exist at any point in the period 1982 to 2012; this panel corresponds to Versions (1)-(3) of our estimation procedures, cleaned as described in Appendix G. For robustness, we also consider a sample of manufacturing firms that exist in 1992 and survive through 1995; this panel corresponds to Versions (4)-(6) of our estimation procedures.⁸² We choose this panel following Hennessy (2004).⁸³

⁸²Manufacturing firms are defined as those firms with two-digit SIC codes between 20 and 39.

⁸³We have slightly more firms than his panel due to different procedures for cleaning of the data.

Recall, we locally identify model parameters b , Δ_z , and h by comparing moments in the data to moments in the model.⁸⁴ In our estimation procedure, the estimate for h is obtained by directly estimating a related object, the innovation decision of the unlevered firm $q_{\infty, \theta=1}$.⁸⁵ Given the calibrated parameters and estimates for b and Δ_z , $q_{\infty, \theta=1}$ implies an estimate for h . We compare coefficients of a regression of a measure of firm employment growth on distance-to-default and its square. Our other moments are the average growth rate of unlevered firms and the average standard deviation of employment growth. We take the distribution of distance-to-default from the data when we estimate our model; hence, it is only the implied innovation decisions conditional on the distribution that drive our estimates of parameters, not how these estimates would then feed back into changing the distribution of distance-to-default. The distribution for the full sample of nonfinancial firms we consider is plotted in Figure 3.2b.

We run three versions of the estimation procedure for each panel. In the model, we always run the same regression and compute the same moments. The only difference in procedures is how we treat employment growth. The first and fourth specifications demean growth by year and industry averages. The second and fifth specifications control for differences in firm investment opportunities by also demeaning growth at the firm level. The third and sixth specifications are the residuals from regressions where the dependent variable is the variable in the second and fifth specifications, respectively, and the independent variables are age, size, and a measure of firm's access to external finance. As described in Subsection 3.4.3, we can interpret specifications (1) and (4) as specifications which correspond to upper bounds on the effect of debt overhang, while specifications (3) and (6) correspond to lower bounds for the effect of debt overhang.

Table 3.1: Remaining Parameterization

Parameters	Value
<i>Affects Estimation</i>	
Discount factor, β	0.994
Period length, Δ	$\frac{1}{12}$
<i>Does not Affect Estimation</i>	
Tax advantage of debt, τ_d	0.2
Retained value of the firm after bankruptcy, α	0.8
Elasticity of substitution across intermediate goods, ρ	4
Per-period entry cost, $n_e\Delta$	1
Total labor supply, L	1

3.5.4 Remaining Calibration

In Table 3.1, we show our remaining calibration. We set τ_d to 0.2 to match the value chosen in Leland (1998). There exists a range of different estimates in the corporate finance literature. This number will not matter for our estimation procedure, as it will only matter for the choice of debt by the firm. We set the corporate tax rate, τ , to 0 so that when we perform counterfactuals the tax advantage is a pure distortion. Had corporate taxes been positive, the tax advantage will further act as a subsidy to entry; such a policy is of less interest to this paper. The intermediate good firm's problem scales in taxes, so only aggregates will be different (not decision rules) had corporate taxes been positive. Hence, we get the same estimates of parameters no matter the level of τ . We choose α to be 0.8, which is the upper bound of bankruptcy costs found in Bris, Welch, and Zhu (2006). α will also not affect the value of equity holders in our estimation procedure. The overall welfare losses are decreasing in α , since a higher α implies more productivity is lost in bankruptcy. The per-period entry cost and total labor supply are set to one, as these objects' values will not affect our results. We choose ρ to be 4 to match ρ in Atkeson and Burstein (2015). This parameter does not affect firm decisions, only aggregates. Assuming $\rho > 1$, holding all other parameters fixed, the welfare losses from debt overhang are decreasing in ρ . If $\rho \rightarrow \infty$, the CES production function becomes linear, and, in turn, the losses from debt overhang tend toward 0 in the

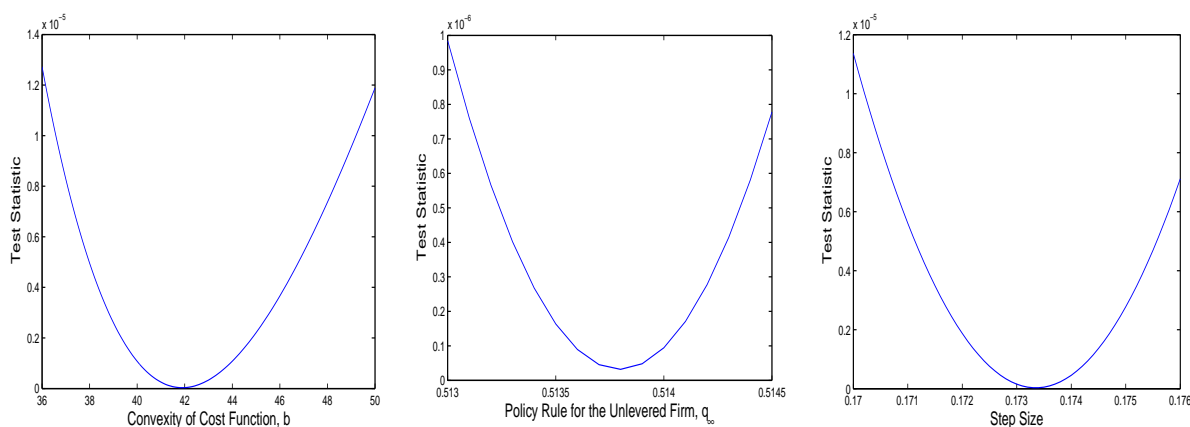
⁸⁴See Appendix F for the derivation of these moments in the model.

⁸⁵This is done to improve numerical accuracy.

limit.⁸⁶ Notice, none of the parameters discussed so far affect the estimates of parameters in our model.

We set the discount factor to 0.994. The discount factor will affect firm decisions and play a role in the estimation procedure. We do not check our results across a range of discount factors; however, our choice fits in the range considered in the literature. The discount factor is $e^{-r\frac{1}{\Delta}}(1 - \delta)$, which given Δ , is a function of a discount rate r and an exogenous exit rate δ . We choose an exogenous exit rate high enough such that our problem admits a stationary equilibrium (so its value is 0.006), and the residual r becomes $\log(1.001)$.

Figure 3.3: Local Identification of Parameters for Version 1 of the Estimation Procedure



(a) Varying \tilde{b} , holding fixed other parameters (b) Varying q_∞ , holding fixed other parameters (c) Varying $\tilde{\Delta}_z$, holding fixed other parameters

In this plot, we hold fixed all parameters and vary one of the parameters, \tilde{b} , q_∞ , and $\tilde{\Delta}_z$ around its local minimum. $\tilde{\Delta}_z$ is equal to $\frac{\Delta_z}{\sqrt{\Delta}}$, and \tilde{b} is $\frac{b}{2\Delta_z\Delta}$. The y-axis is the statistic we minimize over for our estimation procedure.

3.5.5 Estimation Results and Discussion

We show the extent to which our parameters are locally identified in Figures 3.3a, 3.3b, and 3.3c. We hold fixed two of the parameters at their values at which the objective function is minimized and vary the third parameter.⁸⁷ As one can see from the figures, for each

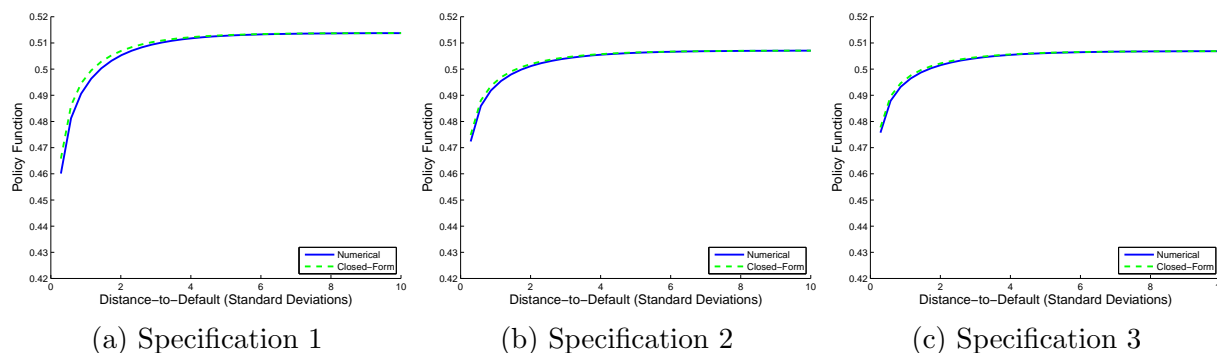
⁸⁶See Acemoglu (2008) Chapter 2 for further discussion of the properties of the CES production function.

⁸⁷When we actually implement our procedure, we use the numerical solution to the model. We search hundreds of thousands of combinations of parameters using a parallelized grid search. Using the closed-form

parameter, we have a clear minimum at our estimate of its value.

Estimation Procedure The results from our estimation procedure are presented in Table 3.2. Across versions of the estimation procedure, the clearest result is that, as expected, when one demeans growth at the firm level, the estimated extent to which debt overhang affects firm innovation decisions decreases (which appears in our results as a higher estimated value of b) in either subsample. The relationship between distance-to-default and growth can be explained to some extent by the fact that the firms that have not been growing are the firms with worse investment opportunities. However, given the functional form of the cost function we assumed and the parameter for b we estimate, the relationship still exists enough such that debt overhang is costly for firms especially as they near default, consistent with findings in the literature. Figure 3.4 presents the effect of debt overhang on firm growth along the margin of distance-to-default implied by our estimates. We discuss the private and public gains from resolving debt overhang implied by our estimates in Section 3.6.

Figure 3.4: Policy Functions Compared across Estimates: Closed-form vs. Numerical



These figures compare the closed-form approximation and the numerically solved policy function for the firm's innovation decision, q , in the model across the estimates in Table 3.2.

3.5.6 Comparing our Estimates to those in the Literature

We focus on two papers in comparing our results to those from the literature: a reduced-form paper with a quasi-natural experiment, Giroud et al. (2012), and a structural paper that

approximation, we can do the same search in a manner of minutes, which helps inform the guess of where to search.

Table 3.2:
Estimation Procedure: Data/Model Moments and Parameter Estimates Across
Specifications

	Larger Core Sample			Manufacturing Balanced Panel		
	Demean by Yr. and Industry (1)	(1)+Demean by Firm (2)	(2)+Addl. Controls (3)	Demean by Yr. and Industry (4)	(4)+Demean by Firm (5)	(5)+Addl. Controls (6)
<i>Data Moments</i>						
β_1 : From Eq. (F.18)	0.0202	0.0121	0.0105	0.0165	0.00902	0.00709
β_2 : From Eq. (F.18)	-0.00136	-0.000856	-0.000735	-0.00123	-0.000722	-0.000614
Avg. Gr. for High DD Firms	0.0166	0.00776	0.00735	-0.0136	-0.00389	-0.00351
Avg. Std. Dev. of Emp. Gr.	0.173	0.159	0.154	0.165	0.152	0.146
<i>Model Moments</i>						
β_1 : From Eq. (F.18)	0.0202	0.0121	0.0105	0.0165	0.00903	0.0071
β_2 : From Eq. (F.18)	-0.00153	-0.000912	-0.000792	-0.0012	-0.000663	-0.000522
Avg. Gr. for High DD Firms	0.0166	0.0078	0.00732	-0.0135	-0.00391	-0.00353
Avg. Std. Dev. of Emp. Gr.	0.173	0.159	0.154	0.165	0.152	0.146
<i>Parameter Ests. (S.E.'s)</i>						
\tilde{b}	41.9 (1.91)	70.6 (4.55)	81.3 (5.72)	45.4	79.9	101
$\tilde{\Delta}_z$	0.173 (0.00106)	0.159 (0.000996)	0.154 (0.000975)	0.165	0.152	0.146
q_∞	0.514 (0.00152)	0.507 (0.00108)	0.507 (0.00114)	0.488	0.496	0.497
<i>Data Sample Properties</i>						
#Firms	5650	5650	5650	894	894	894
Avg. # Employees	5520	5520	5520	5130	5130	5130
Avg. Distance to Default	4.61	4.61	4.61	4.81	4.81	4.81

Notes: Columns (1)-(3) present results from the large unbalanced panel of nonfinancial firms from 1982 to 2012. In Column (1), employment growth is demeaned by year and industry average growth rates. In Column (2), employment growth is demeaned by year, industry, and firm average growth rates. Column (3) regresses employment growth as defined in Column (2) on the Whited-Wu index (a measure of access to external finance), firm age, and the natural logarithm of the number of employees in the firm. The residuals are the new measure of employment growth. Columns (4)-(6) present results from a balanced panel of manufacturing firms that exist between 1992 to 1995. In Column (4), employment growth is demeaned by year and industry average growth rates. In Column (5), employment growth is demeaned by year, industry, and firm average growth rates. In Column (6), employment growth as defined in Column (5) is regressed on the Whited-Wu index, firm age, and the natural logarithm of the number of employees in the firm. The residuals are the new measure of employment growth. High DD is defined as a DD greater than 8. $\tilde{\Delta}_z$ is equal to $\frac{\Delta_z}{\sqrt{\Delta}}$, and \tilde{b} is $\frac{b}{2\Delta_z\Delta}$. The regression coefficient data moments β_1 and β_2 in the larger core sample specifications (1)-(3) are all significant at the 1% significance level. Standard errors for the parameter estimates are computed using a nonparametric block bootstrap, sampling and re-estimating our model 250 times, and are reported only for the larger core sample.

uses Q-theory, Hennessy (2004). Our approach differs in that we do not directly estimate elasticities from the data; instead, we estimate key parameters of a structural model. We compare regression coefficients from the above cited papers to those implied by our estimated model and find, for firms with similar financial soundness to the samples in these papers, that our model generates similar values.

Giroud et al. (2012) use a quasi-natural experiment to estimate the impact of exogenous shocks to firm book leverage on returns, other measures of performance, and sales growth for a sample of highly leveraged firms. We construct a theoretical analog of an estimated regression coefficient that we can derive just from the problem of equity holders, the partial derivative of firm sales growth to exogenous changes in firm book leverage, by computing this derivative for firms with similar book leverage to those in their sample.

Hennessy (2004) uses a Q-theoretic approach to estimate the extent to which debt overhang affects firm investment. He uses the expectation of the market value of lenders' total recovery claim (reflecting both the probability of default and how much lenders recover) scaled by the capital stock as a measure of firm's exposure to debt overhang. He then runs a regression of investment on this measure of debt overhang and controls and finds that this measure has a significant negative relation with investment. The resulting regression coefficient can be interpreted as an estimate of the derivative of investment with respect to this expected recovery claim. We compute, in our model, both the expected recovery claim as well as an analogue of investment, expenditures on innovation, to compute this derivative in our model.

We find that both of these model-implied derivatives change markedly along the margin of distance-to-default in our estimated model, due to the changing intensity of the debt overhang problem. When we account for the sample properties (in terms of leverage or distance-to-default) of these papers, we show that our results are consistent with their estimates.

Comparing our Estimates to Those from a Quasi-Natural Experiment We use the estimate of the derivative of a change in three-year-ahead sales growth to a change in book leverage in Table IV of Giroud et al. (2012) of -0.039. We compute three-year-ahead sales growth in the model.⁸⁸ To compute book leverage in the model, we define a measure of book debt and a measure of book assets. First, we define book assets as $\kappa \exp(z)$ where κ is a free parameter that we calibrate such that the average value of book leverage matches the average value of book leverage in Compustat. Second, we define book debt in the model as short-term debt plus one-half long-term debt, following Gilchrist and Zakrajsek (2012). With some algebra, we can derive book debt for a given value of n as:

$$\frac{\bar{a}}{(1 - \tau^d)} \exp(z) \exp(-\Delta_z n) \frac{1 - \frac{1}{2}\beta^\Delta}{1 - \beta}, \quad (3.23)$$

where $\bar{a} = (1 - \tau^d) \frac{d}{\exp(\bar{z})}$. Notice, $\exp(z)$ in book debt and book assets will cancel. We can then compute the ratio of changes in three-year-ahead sales growth to changes in book leverage at each point on the grid.

Figures 3.5a and 3.5b show our model-implied values of this derivative in the full panel and balanced manufacturing panel, respectively. As this model-implied statistic changes meaningfully along the margin of distance-to-default, it is important to take the leverage of their sample into account. We compare the value of our model-implied derivative at a distance-to-default which corresponds to the median book leverage reported in their paper (1.77) to their estimate of -0.039. In our baseline sample, we find that our estimates imply that this derivative in the lower and upper bound specifications yields derivatives of -0.0553 and -0.0416.⁸⁹ Our estimated model implies a similar, if slightly stronger, derivative at similar levels of book leverage. If we allow ourselves to consider slightly higher levels of distance-to-default (lower book leverage), we can almost exactly hit their estimates in any

⁸⁸We compute three-year-ahead sales growth in a manner similar to how we compute year-ahead employment growth in (F.3) in Appendix F.

⁸⁹This corresponds to specifications (1) and (3), respectively. The distance-to-default values at which we are closest to hitting book leverage of 1.77 are 2.89 and 2.02, respectively.

of our specifications.⁹⁰

Comparing our Estimates to Those from a Structural Paper that Builds on

Q-theory We compare our results to the estimate of the derivative of investment (as a fraction of the capital stock) with respect to the imputed market value of lenders' recovery claim in default normalized by the capital stock (of -0.173) reported in Table III, Column 4 in Hennessy (2004). Our estimate of firm investment is defined as the firm's cost function (which scales in the firm's size, $\exp(z)$). The imputed market value of lenders' recovery claim in default in the model is defined as:

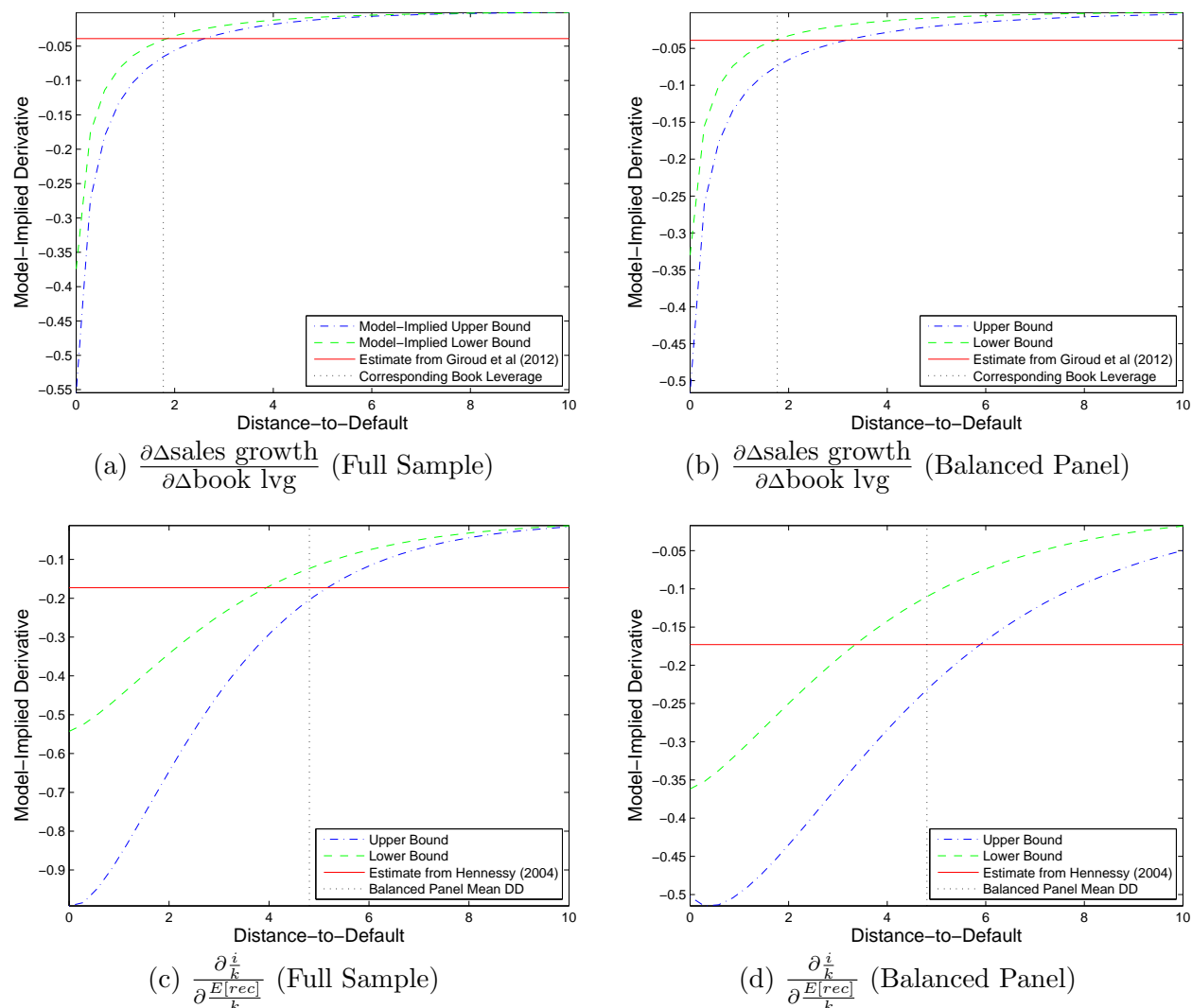
$$(1 - \alpha) \exp(z) \exp(-\Delta_z n) E[\beta^{TD}], \quad (3.24)$$

where $E[\beta^{TD}]$ is the discount rate at the firm's expected time of default, conditional on a firm's n . Because volatility follows a binomial process, we can exactly compute this value given the firm's policy function and the exogenous exit rate. The value of the firm upon entry is also in this term; however, since the cost of entry is one, this value will also be one in equilibrium. Given the capital stock will cancel in both terms, as will $\exp(z)$, we can easily compute this derivative numerically as in the quasi-natural experiment by computing changes in investment relative to changes in the imputed market value of lenders' recovery claim in default.

Figures 3.5c and 3.5d show our model-implied values of this derivative in the full panel and balanced manufacturing panel, respectively. This derivative changes significantly along the margin of distance-to-default, so it is important to take the financial soundness of firms into account when discussing this derivative. The balanced sample of manufacturing firms we use is based off of Hennessy (2004) and has an average distance-to-default of 4.81. In our estimated model specifications based off this sample, we find that our estimates imply that

⁹⁰We find values extremely close to the derivative in Versions (1)-(6) of our estimation procedure of -0.0388, -0.0389, -0.0386, -0.039, -0.0377, -0.0416 at values of distance-to-default of 3.46, 2.31, 2.02, 2.89, 2.31, and 2.02, respectively.

Figure 3.5: Estimation Results compared to those of Giroud et al. (2012) and Hennessy (2004)



The figures above compare model-implied elasticities against existing estimates from the corporate finance literature. Figures 3.5a and 3.5b compare the derivative of 3-year sales growth with respect to book leverage implied by our model at varying levels of distance-to-default against the value estimated by Giroud, Mueller, Stomper, and Westerkamp (2012). Figures 3.5c and 3.5d compare the derivative of scaled investment with respect to the expected discounted value of recoveries by bondholders implied by our model at varying levels of distance-to-default against the value estimated by Hennessy (2004). The horizontal lines correspond to the estimates reported in these papers, while the vertical line corresponds to the summary statistics of their samples. The vertical line in figures 3.5a and 3.5b denote the distance-to-default at which firms' book leverage is equal to the sample median reported in Giroud et al. (2012). The vertical lines in figures 3.5c and 3.5d denote the mean distance-to-default in our balanced manufacturing sample, which is very similar to the sample used in Hennessy (2004). The blue and green lines are model-implied elasticities from the lower and upper bound specifications we estimate. Figures 3.5a and 3.5c are computed using our estimated model from the full, unbalanced panel from 1982 to 2012 and uses specifications (1) and (3) as the upper and lower bounds. Figures 3.5b and 3.5d are computed using our estimated model from the balanced panel of manufacturing firms that exist between 1992 to 1995, with specifications (4) and (6) as the lower and upper bounds.

this derivative in the lower and upper bound specifications yields derivatives of -0.2258 and -0.1067 , which bound the estimate (of -0.173) in Hennessy (2004).⁹¹

3.5.7 Demonstrating the Bounds Hold in the Full Model

We now demonstrate that the upper and lower bounds hold in the full model discussed in Section 3.2. We take values of parameters that come from our estimation procedure, specifically specification (4), where we estimate the model on our core sample of data. Thus, the values of b , Δ_z , and q_∞ used to generate model generated data are 41.9, 0.173, and 0.514, respectively.⁹² We demonstrate that when we simulate data from our model with heterogeneity in investment opportunities, the moments that we use as our lower bound moments generate an estimate of b that is lower than the estimate of b used to simulate model generated data. Similarly, the moments that we use as our upper bound moments generate an estimate of b that is higher than the estimate of b used to simulate model generated data.

We refer to the model with heterogeneity in investment opportunities described in Section 3.2. We assume that firms can be one of two types with values of θ and $1/\theta$. We calibrate θ such that the difference in growth rates between high and low types is equal to the gap in growth rates in the data. In the data, for our core sample, the difference between the 75th and 25th percentile of growth rates is 15.48%, which we can match exactly.⁹³ This value of θ pins down the relative difference in types. We also have to pin down the actual value of the θ of high type firms, which we calibrate such that the implied average mean growth rate of firms is similar to that that we find in our estimation procedure. With such a calibration procedure, we find a value of θ of 0.9343. We assume firm type is the same throughout

⁹¹This corresponds to specifications (4) and (6), respectively. If we use our estimated model specifications using the full unbalanced panel, our bounds become -0.1948 and -0.1184 .

⁹²All references to b and Δ_z in this subsection refer to rescaled values of b and Δ_z . See Table 3.2 for the full description of the transformation.

⁹³If we choose to look at the difference in growth rates of larger percentile gaps, the estimate of b is at least as low in the lower bound case and the estimate of b is at least as high in the upper bound case.

the life of the firm, and firms can enter as either type with equal probability. We simulate 5000 firms for 30 years (with 12 months per year). Firms are drawn from the stationary distribution. For a given firm, if the firm goes bankrupt, we assume we do not observe the firm from that period on. Firms also exit and are not observed thereafter, with exogenous probability $(1 - \delta)$, as in the model. Otherwise, firm innovation decisions determine the probability a firm increases or decreases its productivity each period.

We first find numerically that the proportion of high to low types is increasing in n at each n . Hence, the assumption needed to prove that our lower bound moment condition implies a lower bound for b holds in the full model. We also find that the average mean growth rate of each type is increasing in n at each n , which implies that the assumption needed to prove that our upper bound moment condition implies an upper bound for b holds in the full model. We then reestimate b , Δ_z , and q_∞ . The values of q_∞ we estimate are 0.514 in both the upper bound and lower bound cases. The values of Δ_z we estimate are 0.173 in the upper and lower bound cases, respectively. Hence, the estimates of q_∞ and Δ_z are extremely close to those in Table 3.2. However, in the lower bound case, we find an estimate of b of 17.9, and in the upper bound case, we find an estimate of b of 81.9. Hence, our bounds hold very clearly in the full model.

3.6 Results from Counterfactuals under our Estimates

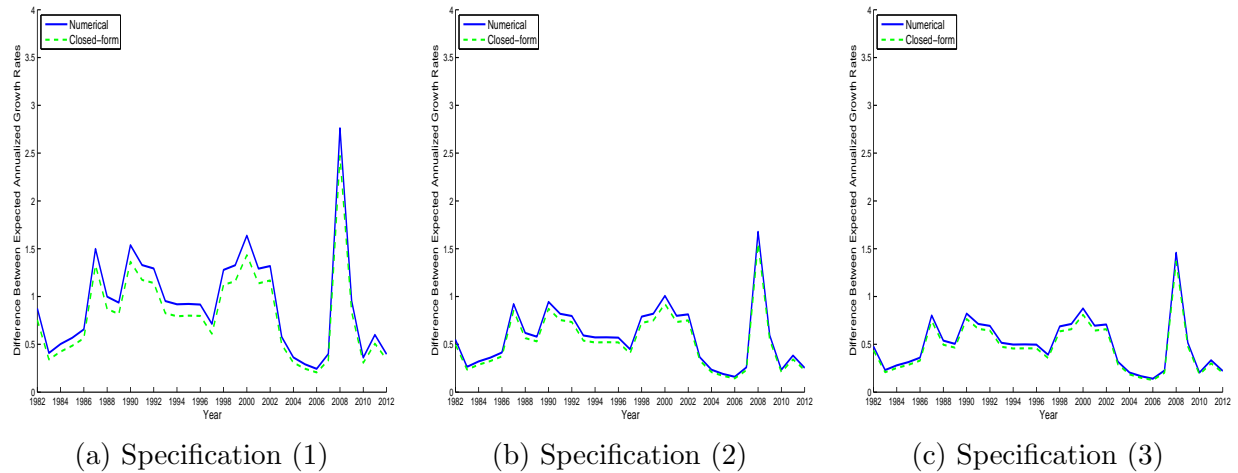
We now use our estimates to provide results from the counterfactuals defined in Section 3.3. Figures 3.4 and 3.6 also compare the results of our closed-form approximation to the full numerical solution of our model.

3.6.1 Partial Equilibrium Counterfactuals

Counterfactuals that only Require the Problem of Equity Holders First, we compare the estimated expected growth rate of firms across the distribution to the expected

growth rate of firms were they to grow at the rate of firms that are unlevered. We find annual gains of 0.952% and 0.515% in the upper and lower bound cases in our baseline sample.⁹⁴

Figure 3.6: Partial Equilibrium Gains from Resolving Debt Overhang by Year



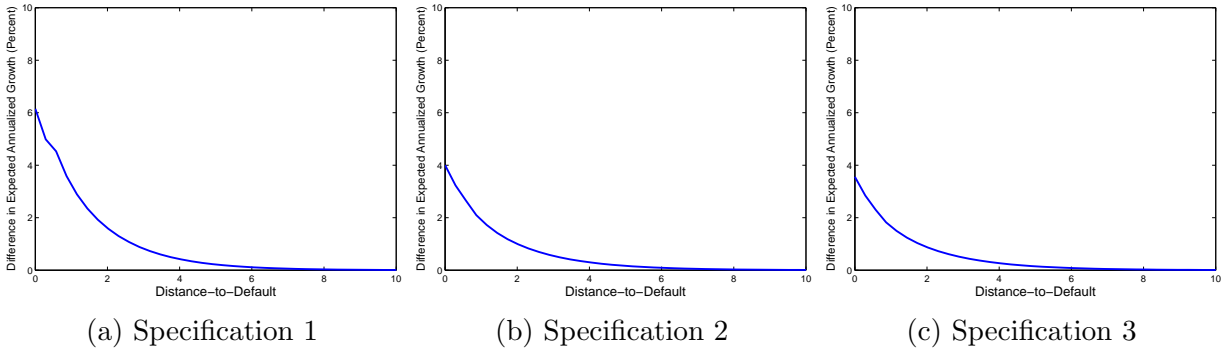
For a given set of estimates in Table 3.2, the lines in a given panel above show the difference between the expected annualized growth rate of firms (in percentage terms) were all firms unlevered and the expected annualized growth rate of firms conditional on the observed distribution of distance-to-default among U.S. nonfinancial public firms in a given year. The green and blue lines respectively show the closed-form approximate and numerical solutions.

We also take the distribution of distance-to-default year by year and perform the same counterfactual as above. Figure 3.6 shows the potential gains from having firms all choosing the investment policy of the unlevered firm year by year. As we expect, these gains increase during times when the distribution of distance-to-default compresses.

Firm Counterfactuals Debt overhang in our model is a highly nonlinear problem. We further demonstrate this in Figure 3.7 across versions of our estimation procedure. We plot the difference in expected annualized growth rates between two firms, one that does not suffer from debt overhang and one that does, for a given value of distance-to-default, assuming prices and the mass of firms do not change. The gains from resolving debt overhang, in terms of annualized employment growth, rise to over 6% for the firms closest to default

⁹⁴These corresponds to specifications (1) and (3), respectively. Our results are similar in the balanced manufacturing panel as well, with specifications (4) and (6) implying annual gains of 0.956% and 0.378%.

Figure 3.7: Percentage Difference in Expected Annualized Growth due to Debt Overhang



The blue line in a given panel is the difference in expected annualized growth in percentage terms between a firm that does not suffer from debt overhang and a firm that does conditional on firm distance-to-default. There exists a kink in some of the panels near default, because the default threshold changes between cases. The counterfactuals above are solved in partial equilibrium (prices and the mass of firms remain constant) and each panel refers to a set of estimates in Table 3.2.

in the upper bound case of our procedure.

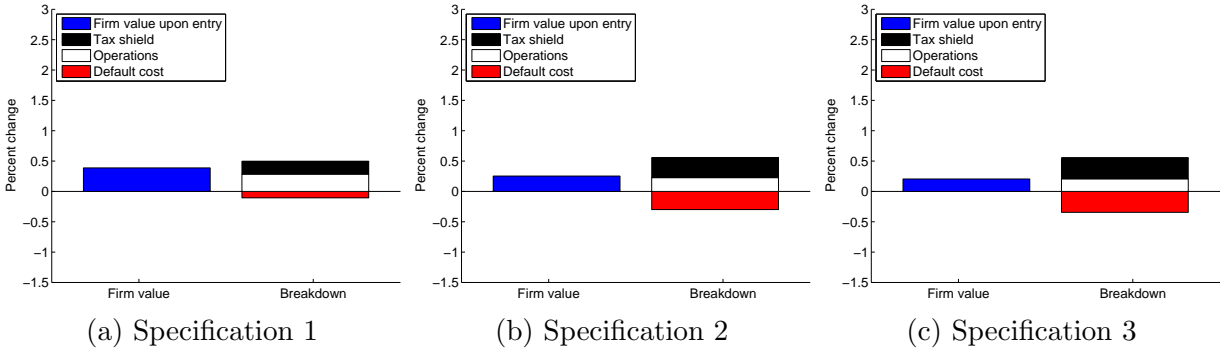
On the other hand, we estimate that the expected gains from resolving debt overhang are modest for an entering firm (in terms of firm value). The blue bar on the left side in a given panel of Figure 3.8 presents the gains as a percent of firm value upon entry. One can find the results from this counterfactual in Table 3.3, as well. Following Moyen (2007), we decompose these into the gains from operations, the gains from the tax advantage, and the losses from bankruptcy. The value from operations is the expected discounted present value of the firm’s production and investment activities. The tax advantage of debt is the expected discounted present value of all interest deductions. The default cost is the expected discounted present value of the deadweight losses from bankruptcy. Most of the gains in partial equilibrium come from gains in terms of the value of operations because, in expectation, the firm makes better investment decisions near default. The firm also anticipates that it will suffer less from debt overhang, so it takes on more debt. In turn, the average entering firm gains more from the tax shield, but also goes bankrupt more often, and these two effects mostly offset.

Table 3.3:
Results from Welfare and Firm Value Counterfactuals under our Estimates

	Larger Core Sample			Manufacturing Balanced Panel		
	Upper Bound Extent of Debt Overhang (1)	Lower Bound Extent of Debt Overhang (2)	Lower Bound Extent of Debt Overhang (v2) (3)	Upper Bound Extent of Debt Overhang (4)	Lower Bound Extent of Debt Overhang (5)	Lower Bound Extent of Debt Overhang (v2) (6)
<i>Parameter Ests.</i>						
<i>(S.E.'s)</i>						
\tilde{b}	41.9 (1.91)	70.6 (4.55)	81.3 (5.72)	45.4	79.9	101
$\tilde{\Delta}_z$	0.173 (0.00106)	0.159 (0.000996)	0.154 (0.000975)	0.165	0.152	0.146
q_∞	0.514 (0.00152)	0.507 (0.00108)	0.507 (0.00114)	0.488	0.496	0.497
<i>Welfare Results</i>						
Social Loss	0.92	1.02	1.04	1.13	1.11	1.11
Debt Overhang Loss	0.22	0.11	0.07	0.17	0.15	0.08
Bankruptcy Loss	0.55	0.76	0.82	0.84	0.82	0.88
Residual Loss	0.15	0.15	0.16	0.12	0.14	0.15
<i>Partial Equilibrium Firm Value Results</i>						
Firm Value Loss	0.39	0.25	0.2	0.69	0.3	0.23
Operations Loss	0.28	0.23	0.21	0.64	0.27	0.23
Bankruptcy Loss	0.21	0.33	0.34	0.73	0.27	0.32
Tax Shield Gain	-0.11	-0.3	-0.35	-0.67	-0.24	-0.32

Notes: This table presents results from counterfactuals computed under our parameter estimates. Specifications (1)-(6) are as described in Table 3.2 and Subsection 3.5.3. We assess and decompose the “Social Loss”, which is the long-run percent change between efficient and baseline consumption. We decompose this loss into three components: “Debt Overhang Loss” is the percent change between consumption in the steady state where debt overhang does not affect firm’s investment decisions and steady-state baseline consumption. “Bankruptcy Loss” is the percentage change between steady-state consumption if debt overhang does not affect firm investment and firms do not lose $1 - \alpha$ of their productivity in bankruptcy and the steady-state consumption if debt overhang does not affect firm investment decisions. “Residual Loss” is the effect of taxes and bankruptcy on firm value, which is the percent change between the social loss and the sum of the two previous counterfactuals. The firm value counterfactuals, “Firm Value Loss”, above assess and decompose the partial equilibrium percent change in firm value upon entry between a firm that does not and a firm that does suffer from debt overhang. We decompose this loss into three components: “Operations Loss” is the expected discounted present value of the firm’s production and investment activities. “Tax Shield Gain” is the expected discounted present value of all interest deductions. “Bankruptcy Loss” is the expected discounted present value of the deadweight losses from bankruptcy. $\tilde{\Delta}_z$ is equal to $\frac{\Delta_z}{\sqrt{\Delta_z}}$, and \tilde{b} is $\frac{b}{2\Delta_z\Delta}$. Standard errors for the parameter estimates are computed using a nonparametric block bootstrap, sampling and re-estimating our model 250 times, and are reported only for the larger core sample.

Figure 3.8: Firm Value Decomposition Across Estimation Specifications

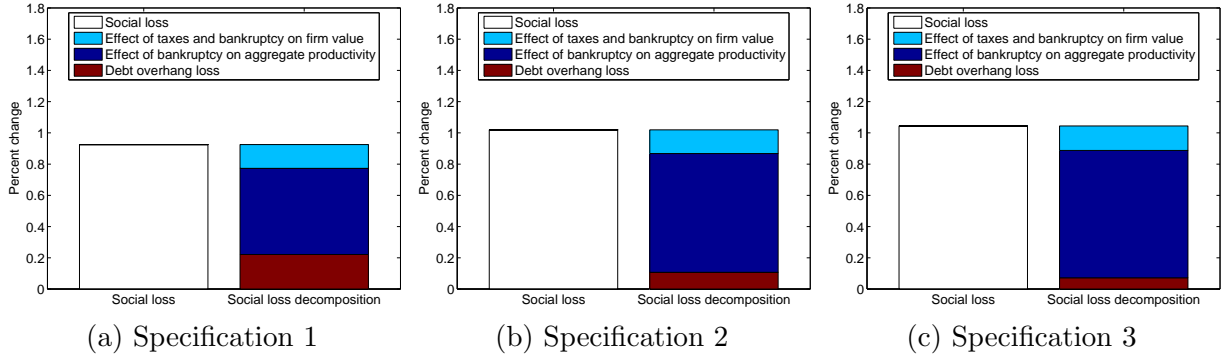


The counterfactuals above assess and decompose the partial equilibrium percent change in firm value upon entry between a firm that does not and a firm that does suffer from debt overhang across the estimates in Table 3.2. The value from operations is the expected discounted present value of the firm’s production and investment activities. The tax shield is the expected discounted present value of all interest deductions. The default cost is the expected discounted present value of the deadweight losses from bankruptcy. The value of operations, the tax shield, and the default cost add up to the gains in firm value upon entry.

3.6.2 General Equilibrium Counterfactuals

Results from our general equilibrium counterfactuals can be found in either Figure 3.9 or Table 3.3. We explain how to compute our general equilibrium counterfactual objects in Subsection 3.3.2. The social losses, which are expressed in terms of baseline consumption, do not vary much with our estimates. We are most interested, in our case, in the decomposition, especially the component that captures the gains from resolving debt overhang. As one can see from Figure 3.9, the gains from resolving debt overhang do not vary substantially with a changing estimate of b . The gains do not vary much as we vary b because in the long run when a large mass of firms increase their innovation decisions, as they do when we resolve the debt overhang problem, they raise the real cost of labor. However, because production, entry, and process innovation all require labor, these activities all become more expensive, thus reducing the incentive of firms to engage in them all else equal. Hence, the rise in the cost of the inputs into innovation (labor) dampen the gains from resolving debt overhang. Further, the aggregate bankruptcy rate will rise, as firms will now have more leverage, on average, as they anticipate they will not suffer from debt overhang near default. Hence, more aggregate productivity will be lost to bankruptcy, dampening the gains from resolving debt overhang.

Figure 3.9: Welfare Analysis Across Estimation Methods



The figures above assess and decompose the long-run percent change between efficient and baseline consumption (social loss) across the estimates in Table 3.2. Debt overhang loss is the difference between consumption in the steady state where debt overhang does not affect firm investment decisions and steady-state baseline consumption. The effect of bankruptcy on aggregate productivity is the difference between steady-state consumption if debt overhang does not affect firm investment and firms do not lose $1 - \alpha$ of their productivity in bankruptcy and the steady-state consumption if debt overhang does not affect firm investment decisions. The effect of taxes and bankruptcy on firm value is the difference between the social loss and the sum of the two previous counterfactuals.

There are two other counterfactuals in this figure which we also explain how to compute in Subsection 3.3.2. The first other object is the effect of bankruptcy on the total mass of productivity. Recall, the interpretation of α in our model is that there is some mass of productivity that is being lost which will be costly to replace. We get rather large losses relative to the losses from debt overhang from this effect. There are a few points to make here given that there are other possible interpretations of α . One could interpret the costs of bankruptcy as not destroying any productivity, but being costly in terms of labor. In this case, we have a smaller, but still significant, blue bar. Another interpretation of α is that the cost of bankruptcy is a direct financial transfer; this will make the dark blue bar zero. In these two cases, the white bar, the social loss, will move close to proportionally with movements in the dark blue bar. Even though our estimation procedure will not change with these different interpretations, the interpretation of bankruptcy is very important in translating the costs of bankruptcy into social losses.

The light blue bar is the effect of taxes and bankruptcy on firm value. Even without debt overhang, there are losses from firm decisions being distorted by the tax advantage and bankruptcy, and these losses are comparable in size to the losses from debt overhang.

3.7 Conclusion

This paper develops and estimates a general equilibrium model of firm dynamics and endogenous innovation in which debt overhang distorts the firm's innovation decision. With our estimated model, we assess the expected private gains and long-run welfare gains from resolving debt overhang. We find that the long-run welfare gains from resolving this problem are small under a range of estimates of the extent to which debt overhang affects firms. The gains are relatively small as well for the average entering firm, especially compared to the private gains for firms close to default. However, our results also suggest that debt overhang may be an important factor affecting firm growth over the business cycle.

Appendix F: Proofs and Value Function Derivations

Proofs to Proposition 3.1

Recall, in Proposition 3.1 we outline properties of the problem of equity holders in a steady state where S_t is constant for all t , and assuming that firm types are constant for a given firm such that $\theta_t^i = \theta^i$ for all t for a given firm i . It is useful to define the Bellman function for equity holders in a steady state where θ , although heterogeneous across firms, is constant for a given firm:

$$V_E(z, d, \theta) = \max_q \left\{ 0, (1 - \tau)(\exp(z)\Pi - w \exp(z)h\theta^{-b} \exp(bq)) - (1 - \tau^d)d + \beta \left(qV_E(z + \Delta_z, d, \theta) + (1 - q)V_E(z - \Delta_z, d, \theta) \right) \right\}. \quad (\text{F.1})$$

First, we will prove that the problem of equity holders can be reduced to two state variables: (1) the firm's investment opportunities, θ , and (2) the number of steps, Δ_z , until the firm declares bankruptcy, n .

Proof. Define $\hat{V}_E = \frac{V_E}{\exp(z)}$. We can thus redefine (F.1) as:

$$\hat{V}_E(n, \theta) = \max_q \left\{ (1 - \tau)(\Pi - wh\theta^{-b} \exp(bq)) - \frac{(1 - \tau^d)d}{\exp(z^*(\theta))} \exp(-n\Delta_z) + \beta \left(q \exp(\Delta_z) \hat{V}_E(n+, \theta) + (1 - q) \exp(-\Delta_z) \hat{V}_E(n - 1, \theta) \right) \right\}, \quad (\text{F.2})$$

where the firm goes bankrupt if $n < 0$ and $\exp(z^*(\theta))$ is the bankruptcy threshold of a given type, θ . Define $\bar{a}(\theta) = \frac{(1 - \tau^d)d}{\exp(z^*(\theta))}$. Given θ and d constant over time for a given firm, $\bar{a}(\theta)$ is constant for a given firm over time. It can be easily verified that $\bar{a}(\theta)$ does not vary in d , because $\exp(z^*(\theta))$ is proportional in d . \square

Next, we prove that expected year-ahead employment growth can be fully characterized by the firm's innovation decision, (3.14), and parameters.

Proof. A firm's expected period-ahead growth rate in the model is $2q(n)\Delta_z - \Delta_z$. We can annualize this growth rate to recover Δy_t^i for firm i between year t and year $t + 1$ as:

$$(2q(n)\Delta_z - \Delta_z + 1)^\Delta - 1 \quad (\text{F.3})$$

\square

Lastly for Proposition 3.1, we prove the state variable n has a 1-1 mapping with firm distance-to-default:

Proof. Note that the unlevered value of the firm can be expressed as $V_A(z, \theta) = e^z \hat{V}_E(\infty, \theta)$. The unlevered value at the default point can be expressed as $V_B^*(d, \theta) = e^{z^*(d, \theta)} \hat{V}_E(\infty, \theta)$. Note that $n(z, d, \theta) = \frac{z - z^*(d, \theta)}{\Delta_z}$. Therefore we can write $DD = \frac{n\Delta_z}{\sigma_A}$. We also know that there is relation between asset volatility and the step size, which reduces to the following relationship under our assumptions: $\Delta_z = \sigma_A \sqrt{\frac{1}{\Delta}}$ ⁹⁵. Therefore, we can write $n = DD\sqrt{\Delta}$, which implies a 1-1 mapping between DD and n .

⁹⁵ Δ_z has a close relationship with asset volatility, σ_A . In particular, $\Delta_z = \sigma_A \sqrt{4q(1 - q)\frac{1}{\Delta}}$, where q is the average q in the economy. If we assume this q is 0.5, which is true if we look at a high enough frequency

□

The Problem of Equity Holders in the Simple Model

To recover the value function and the choice of q in (F.1) and (3.10), respectively, we take the following steps. We first solve for a closed-form solution for the value function with constant aggregates for the problem where firms do not optimize how much process innovation to undergo conditional on their leverage and always choose q as if they were the unlevered firm. We then plug in this solution into the optimal choice of q in the problem with optimally chosen process innovation. As this and the next subsection are solved in steady state, and are used to demonstrate identification and will not be referenced when defining an equilibrium, we drop all time subscripts.

Also, when firm's types are fixed, given h , we can solve for each type's Bellman individually given the choice of q of the unlevered firm. Given a value of h and a value of q_∞ , when types are permanent, a value of θ can then be backed out analytically:

$$\theta = \frac{\Pi\beta(e^{\Delta z} - e^{-\Delta z})}{he^{bq_\infty}(\beta(e^{\Delta z} - e^{-\Delta z}) - b(1 - \beta(e^{\Delta z} - e^{-\Delta z})q_\infty - \beta e^{-\Delta z}))}.$$

The above value of θ implies an associated value of the Bellman of the unlevered firm. Also, note that the bankruptcy threshold may be different conditional on θ . Hence, we can solve for the closed-form approximation over only n , assuming that we are using this solution procedure for each θ and that we have chosen the h to be the h were $\theta = 1$.

From (F.1), the optimal innovation decision of equity holders is

$$q^*(n) = \frac{1}{b} \log \left(\frac{\beta \left(\exp(\Delta z) V_E(n+1) - \exp(-\Delta z) V_E(n-1) \right)}{w(1-\tau)bh} \right). \quad (\text{F.4})$$

The default threshold, $\exp(\bar{z})$, is proportional in debt outstanding. Thus, $(1 - \tau^d) \frac{d}{\exp(\bar{z})}$

of data, then $\Delta_z = \sigma_A \sqrt{\frac{1}{\Delta}}$.

is a constant when aggregates are fixed. Define:

$$\bar{a} = (1 - \tau^d) \frac{d}{\exp(\bar{z})}. \quad (\text{F.5})$$

Now, consider the Bellman in (F.1) except where equity holders always invest as if they were unlevered. Call the innovation decision of the unlevered firm q_∞ . Call the Bellman in this case, \tilde{V}_E .

$$\begin{aligned} \tilde{V}_E(n) = & (1 - \tau) \left(\Pi - w\phi(q_\infty) \right) - \bar{a} \exp(-\Delta_z n) + \beta q_\infty \exp(\Delta_z) \tilde{V}_E(n + 1) \\ & + \beta(1 - q_\infty) \exp(-\Delta_z) \tilde{V}_E(n - 1). \end{aligned} \quad (\text{F.6})$$

We also know

$$\tilde{V}_E(0) = 0. \quad (\text{F.7})$$

We can easily solve for (F.6) with boundary condition (F.7), as this is a linear non-homogeneous second-order recurrence equation with a known solution:⁹⁶

$$\begin{aligned} \tilde{V}_E(n) = & \frac{(1 - \tau) \left(\Pi - w\phi(q_\infty) \right)}{1 - \beta q_\infty (\exp(\Delta_z) - \exp(-\Delta_z)) + \exp(-\Delta_z)} \left(1 \right. \\ & - \frac{\bar{a} \exp(\Delta_z)^{n+1}}{\exp(-\Delta_z) (\beta(1 - q_\infty) - 1 + \frac{1}{2}(1 - \sqrt{1 - 4\beta^2 q_\infty(1 - q_\infty)})} \\ & \left. - \frac{\beta(1 - q_\infty)(1 - \exp(-\Delta_z)) \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty(1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^n}{(\beta(1 - q_\infty) - 1 + \frac{1}{2}(1 - \sqrt{1 - 4\beta^2 q_\infty(1 - q_\infty)})} \right). \end{aligned} \quad (\text{F.8})$$

Now, suppose we have a general cost function as in (3.6). We plug in (F.8), our closed-form solution to the Bellman of equity holders, into the optimal policy function for q for the problem **with** optimally chosen process innovation, (F.4), to recover a closed-form solution

⁹⁶Notice, also as $n \rightarrow \infty$, $V(\infty) = \frac{(1 - \tau)(\Pi - w h e^{b q_\infty})}{1 - \beta q_\infty (\exp(\Delta_z) - \exp(-\Delta_z)) + \beta \exp(-\Delta_z)}$.

to (F.4).

In turn, we can define the choice of q as

$$\tilde{q}^*(n) = \frac{\log\left(\frac{\beta \frac{(\tilde{\Pi} - \phi(q_\infty))}{1 - \beta q_\infty (\exp(\Delta_z) - \exp(-\Delta_z)) + \exp(-\Delta_z)}}{\exp(\Delta_z) - \exp(-\Delta_z) - K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n}{bh}\right)}{b}, \quad (\text{F.9})$$

where

$$K = \frac{\left(\exp(\Delta_z) \left(\frac{2\beta q_\infty \exp(\Delta_z)}{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}\right) - \exp(-\Delta_z) \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)\right) \beta(1 - q_\infty)(1 - \exp(-\Delta_z))}{(\beta(1 - q_\infty) - 1 + \frac{1}{2}(1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}))} \quad (\text{F.10})$$

and $\tilde{\Pi} = \frac{\Pi}{w}$.

Hence, we can recover \tilde{q}^* as a function of n and parameters.

Proofs to Proposition 3.2

Recall, for Proposition 3.2 we will use the closed-form solution for the problem of equity holders from the simple model from Section to define moments in closed form and prove some of their properties.

First, we show the derivative of firm expected employment growth to distance-to-default can be characterized in closed form, and its magnitude is proportional in $\frac{1}{b}$.

Proof. The derivative of q with respect to n is

$$\frac{\partial q}{\partial n} = \frac{1}{b} \frac{\log\left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right) \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n K}{(\exp(-\Delta_z) - \exp(\Delta_z)) + K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n}, \quad (\text{F.11})$$

where K is defined in (F.10). b or n do not enter into K . Hence, as $b \rightarrow \infty$, we find that

$$\frac{\partial q}{\partial n} \rightarrow 0.$$

We can recover this derivative relative to DD by multiplying (F.11) by $\sqrt{\Delta}$. Both objects are proportional in $\frac{1}{b}$. Notice, to calculate the derivative of expected period-ahead growth with respect to n , $\frac{\partial 2q\Delta_z - \Delta_z}{\partial n}$ we just multiply $\frac{\partial q}{\partial n}$ by $2 * \Delta_z$. Further, we can use the relation between $n = DD\sqrt{\Delta}$ to recover:

$$\frac{\partial E \left[\frac{l_{t+1} - l_t}{l_t} \right]}{\partial DD} = \frac{1}{b} \frac{2 \frac{\Delta_z}{\sqrt{\Delta}} \log \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right) \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^{DD\sqrt{\Delta}} K}{(\exp(-\Delta_z) - \exp(\Delta_z)) + K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^{DD\sqrt{\Delta}}}, \quad (\text{F.12})$$

We can see that this derivative goes to 0 as $b \rightarrow \infty$. □

We now prove the second derivative of firm expected employment growth to distance-to-default can be characterized in closed form, and its magnitude is proportional in $\frac{1}{b}$.

Proof. We can find the second derivative of q with respect to n as:

$$\begin{aligned} \frac{\partial^2 q}{\partial n^2} = & \frac{-K}{b} \frac{\log \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right) \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^{2n} K}{\left((\exp(-\Delta_z) - \exp(\Delta_z)) + K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^n \right)^2} \\ & + \frac{1}{b} \frac{\log \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^2 \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^n K}{(\exp(-\Delta_z) - \exp(\Delta_z)) + K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^n}. \end{aligned} \quad (\text{F.13})$$

We can recover this derivative relative to DD by multiplying (F.13) by $\sqrt{\Delta}$. Both objects are proportional in $\frac{1}{b}$. (F.13) is only decreasing in b when the first term is greater than the second. As before, we can find the second derivative of one-period growth with respect to

DD , $\frac{\partial^2 2q\Delta_z - \Delta_z}{\partial D^2}$, by multiplying $\frac{\partial^2 q}{\partial DD^2}$ by $2\Delta_z$, yielding:

$$\begin{aligned} \frac{\partial^2 E \left[\frac{l_{t+1} - l_t}{l_t} \right]}{\partial DD^2} &= \frac{-K}{b} \frac{2 \frac{\Delta_z}{\sqrt{\Delta}} \log \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right) \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^{2DD\sqrt{\Delta}} K}{\left((\exp(-\Delta_z) - \exp(\Delta_z)) + K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^{DD\sqrt{\Delta}} \right)^2} \\ &+ \frac{1}{b} \frac{2 \frac{\Delta_z}{\sqrt{\Delta}} \log \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^2 \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^{DD\sqrt{\Delta}} K}{\left((\exp(-\Delta_z) - \exp(\Delta_z)) + K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)} \right)^{DD\sqrt{\Delta}} \right)^2} \end{aligned} \quad (\text{F.14})$$

If (F.13) is decreasing in b , then (F.14) is decreasing in b , as the first term is greater in absolute value than the second term under the parameter restrictions we introduced. \square

We now prove the expected growth rate of the firm is decreasing in h , and the expected growth rate of the unlevered firm can be fully characterized as a function of Δ_z , h , and Δ .

Proof. The expected growth rate of the unlevered firm is:

$$(2q_\infty \Delta_z - \Delta_z + 1)^\Delta - 1, \quad (\text{F.15})$$

which is clearly increasing q_∞ . Since q_∞ is decreasing in h , expected year-ahead growth for the unlevered firm is thus decreasing in h . \square

We now show the variance of annualized firm growth rates can be characterized as a function of the expected average growth rate of firms across the economy and Δ_z , and is increasing in Δ_z .

Proof. Denote the relative mass of firms at a given state (z, d, θ) as:

$F(z, d, \theta) = \frac{\Gamma(z, d, \theta)}{\int \int \int \Gamma(z, d, \theta) dz d d \theta}$ where $\Gamma(z, d, \theta)$ denotes the mass of firms for a given (z, d, θ) in steady state. We could also write this problem in terms of just two states: n and θ . We can

then write the average q of the economy, \bar{q} , as

$$\bar{q} = \int \int \int F(z, d, \theta) q(z, d, \theta) dz d d d \theta. \quad (\text{F.16})$$

The per-period variance of growth rates is then

$$\begin{aligned} &= \int \int \int F(z, d, \theta) q(z, d, \theta) \left(\Delta_z - \left(2\bar{q}\Delta_z - \Delta_z \right) \right)^2 dz d d d \theta + \\ &\int \int \int F(z, d, \theta) (1 - q(z, d, \theta)) \left(-\Delta_z - \left(2\bar{q}\Delta_z - \Delta_z \right) \right)^2 dz d d d \theta, \end{aligned}$$

from the formula $V(x) = E[x - \bar{x}]^2$.

$$\begin{aligned} &= \int \int \int F(z, d) q(z, d) \left(\Delta_z - \left(2\bar{q}\Delta_z - \Delta_z \right) \right)^2 dz d d d \theta \\ &\quad + \int \int \int F(z, d) (1 - q(z, d)) \left(-2\bar{q}\Delta_z \right)^2 dz d d d \theta. \\ &= 4\Delta_z^2 \left(1 - \bar{q} \right)^2 \int \int \int F(z, d) q(z, d) dz d d d \theta \\ &\quad + 4\Delta_z^2 \bar{q}^2 \int \int \int F(z, d) (1 - q(z, d)) dz d d d \theta. \end{aligned}$$

Now notice that we can use the definition of \bar{q} to simplify it further:

$$= 4\Delta_z^2 \left(1 - \bar{q} \right)^2 \bar{q} + 4\Delta_z^2 \bar{q}^2 \left(1 - \bar{q} \right).$$

$$= 4\Delta_z^2 \bar{q} (1 - \bar{q}) \left((1 - \bar{q}) + \bar{q} \right).$$

$$= 4\Delta_z^2 \bar{q} (1 - \bar{q}). \quad (\text{F.17})$$

$4\Delta_z^2\bar{q}(1 - \bar{q})$ is clearly increasing in Δ_z holding \bar{q} fixed.

□

Discussion of Local Identification

So that we can eventually compare the model to data, we will recover regression coefficients in the model from the following regression:

$$\Delta y_{i,t} = \alpha + \beta_1 DD_{i,t} + \beta_2 DD_{i,t}^2 + \epsilon_{i,t} \quad (\text{F.18})$$

Notice, $\frac{\partial \Delta y_{i,t}}{\partial DD_{i,t}}$ is equal to $\beta_1 + 2\beta_2 DD_{i,t}$, and $\frac{\partial^2 \Delta y_{i,t}}{\partial DD_{i,t}^2}$ is equal to $2\beta_2$, so given these two derivatives and given $y_{i,t}$ and $DD_{i,t}$ are known, we can back out regression coefficients β_1 and β_2 .

We can estimate b , q_∞ (which implies h), and Δ_z with the moments above (the average growth rate of unlevered firms, the coefficients in (F.18), and the variance of employment growth rates), as β_1 and β_2 from (F.18), are proportional in $\frac{1}{b}$, the standard deviation of growth rates across firms outlined in (F.17) is proportional in Δ_z^2 , and expected average growth of zero default risk firms, (F.15), is proportional in q_∞ . We will need to estimate all parameters at once, as b affects the average growth rate in the economy in (F.17), as does q_∞ , q_∞ enters into β_1 and β_2 as does Δ_z , and Δ_z enters into the average growth rate of zero default risk firms (although b does not). When we estimate the model, we will take as given the distribution of firms across distance-to-default from the data. By taking the distribution as given, we can estimate the parameters of the model with the solution to the problem of equity holders and avoid simulation of data. As we show in Section 3.5, our moments are locally identified, and driven by the expected parameters.

Heterogeneity in Firm Types

Suppose that firms are heterogeneous in their investment opportunities with cost function of the type (3.11), as introduced in subsection 3.2.3. It can be verified that the first order condition for q , (3.14), together with the assumption of an exogenous innovation decision q_∞ in the future, results in the following formula for firm expected growth:

$$\tilde{q}^*(\theta, n) = \theta + \frac{\log\left(\frac{\beta \frac{(\bar{\Pi} - \phi(q_\infty))}{1 - \beta q_\infty (\exp(\Delta_z) - \exp(-\Delta_z)) + \exp(-\Delta_z)} \left(\exp(\Delta_z) - \exp(-\Delta_z) - K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n\right)}{bh}\right)}{b}.$$

This can further be simplified as:

$$\tilde{q}^*(\theta, n) = \theta + q_\infty + \frac{1}{b} \log\left(\frac{\exp(\Delta_z) - \exp(-\Delta_z) - K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta_z)}\right)^n}{\exp(\Delta_z) - \exp(-\Delta_z)}\right). \quad (\text{F.19})$$

Proof to Proposition 3.3

Differentiating (F.19) and plugging in for the true value b_0 yields:

$$\frac{\partial E[q|n]}{\partial n} = \int \theta \frac{\partial h(\theta|n)}{\partial n} d\theta + \frac{1}{b_0} \frac{\log\left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty e^{\Delta_z}}\right) \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty e^{\Delta_z}}\right)^n K}{(e^{-\Delta_z} - e^{\Delta_z}) + K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty e^{\Delta_z}}\right)^n}, \quad (\text{F.20})$$

where $h(\theta|n)$ is the probability distribution function of θ conditional on steps from default, n .

Recall that b_L is defined as the value of b such that the derivative of the innovation decision with respect to steps from default in the model without heterogeneity is set equal

to the observed derivative. This condition is the following:

$$\frac{\partial E[q|n]}{\partial n} = \frac{1}{b_L} \frac{\log\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right) \left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right)^n K}{(e^{-\Delta z} - e^{\Delta z}) + K \left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right)^n}. \quad (\text{F.21})$$

(F.20) and (F.21) jointly imply that:

$$b_L = b_0 \left(\frac{\frac{\partial E[q|n]}{\partial n} - \int \theta \frac{\partial h(\theta|n)}{\partial n} d\theta}{\frac{\partial E[q|n]}{\partial n}} \right). \quad (\text{F.22})$$

(F.22) and the assumption that $\frac{\partial E(\theta|n)}{\partial n} = \int \theta \frac{\partial h(\theta|n)}{\partial n} d\theta \geq 0$ yield that $b_L \leq b_0$.

Proof to Proposition 3.4

Recall that b_H is defined as the value of b such that the derivative of the firm innovation decision with respect to steps from default in the model without heterogeneity is set equal to the derivative of the demeaned value of the firm innovation decision to firm steps from default in the data. This condition is the following:

$$\frac{\partial E[q - \bar{q}|n]}{\partial n} = \frac{1}{b_H} \frac{\log\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right) \left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right)^n K}{(e^{-\Delta z} - e^{\Delta z}) + K \left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right)^n}. \quad (\text{F.23})$$

We can express (F.23) in terms of the true value of b_0 as follows:

$$\begin{aligned} \frac{\partial E[q - \bar{q}|n]}{\partial n} &= \frac{1}{b_0} \frac{\log\left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right) \left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right)^n K}{(e^{-\Delta z} - e^{\Delta z}) + K \left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty e^{\Delta z}}\right)^n} \\ &\quad - \frac{1}{b_0} \int_i \log \left(\frac{\left(e^{\Delta z} - e^{-\Delta z} - K \left(\frac{1-\sqrt{1-4\beta^2 q_\infty(1-q_\infty)}}{2\beta q_\infty \exp(\Delta z)} \right)^i \right)}{e^{\Delta z} - e^{-\Delta z}} \right) \frac{\partial L_n(i)}{\partial n} di, \end{aligned} \quad (\text{F.24})$$

where $L_n(i)$ is the probability distribution of steps from default i over the lifetime of all firms, conditional on having observed firms at n steps of default.

(F.20) and (F.23) jointly imply that:

$$b_H = b_0 \left(\frac{\frac{\partial E[q-\bar{q}|n]}{\partial n}}{\frac{\partial E[q-\bar{q}|n]}{\partial n} - \int_i \log \left(\frac{e^{\Delta z} - e^{-\Delta z} - K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta z)} \right)^i}{e^{\Delta z} - e^{-\Delta z}} \right) \frac{\partial L_n(i)}{\partial n} di} \right). \quad (\text{F.25})$$

The assumption that $\frac{\partial E(\bar{q}|\theta, n)}{\partial n} \geq 0$ and (F.19) immediately imply:

$$\int_i \log \left(\frac{e^{\Delta z} - e^{-\Delta z} - K \left(\frac{1 - \sqrt{1 - 4\beta^2 q_\infty (1 - q_\infty)}}{2\beta q_\infty \exp(\Delta z)} \right)^i}{e^{\Delta z} - e^{-\Delta z}} \right) \frac{\partial L_n(i)}{\partial n} di \geq 0. \quad (\text{F.26})$$

Hence, (F.25) and (F.26) yield the result that $b_H \geq b_0$.

Appendix G: Data and Measurement

Data Construction

As described in Section 3.5, our empirical analysis relies on data from U.S. nonfinancial public firms. We take daily stock returns and other equity market data from CRSP and merge them with annual and quarterly accounting data from Compustat. We use the linking table from the CRSP/Compustat merged database to merge the datasets.

For the core sample of firms, we keep only firms with two-digit SIC codes that are not between 60 and 69, are less than 90, and are not equal to 49, following Hennessy and Whited (2007), as our model is not necessarily representative of regulated, financial, or public service

firms. Following Hennesy and Whited (2005), we trim each series at the 2nd percentile except measures that are inherently bounded in nice ranges.

Variable Definitions

Market capitalization is defined as closing price times shares outstanding, and is the data equivalent of the value of equity in the model. To create our measure of distance-to-default, we require the book value of debt, which we define as short-term debt + one-half times long-term debt, where short-term debt is the max of debt in current liabilities (data item 34) and total current liabilities (data item 5), and long-term debt is data item 9. Employment is data item 29. We define book leverage as the book value of debt relative to the book value of assets (data item 6). We ask the reader to consult Whited and Wu (2006) for how to construct the Whited-Wu index.

To create our age measure, we download the entire time series for stock returns for each firm from CRSP. For each date, for each firm, dating back to 1926, we then state the age of a firm is 1 if it is the first date that shows up for the given firm. The age will then be 2 the next year, and so on.

Distance-to-Default

We follow a procedure consistent with Bharath and Shumway (2008) and Gilchrist and Zakrajsek (2012) in measuring firm V_A and σ_A , whose procedures are in the spirit of Merton (1974). V_A is the value of assets, V_B is the value of debt, μ_A is the mean rate of asset growth, and σ_A is asset volatility. We recover V_A and σ_A from the data closely following the procedure outlined by Gilchrist and Zakrajsek (2012). For each firm, we linearly interpolate our quarterly value of debt to a daily frequency. We use daily data on the market value of equity; call this V_E . We guess a value of asset volatility, $\sigma_A = \sigma_E \frac{V_B}{V_E + V_B}$, where the standard deviation of the value of equity is calculated as the square root of the annualized 21-day moving average of squared returns for a firm. Here, we differ from Gilchrist and Zakrajsek

(2012) in that they choose a 252-day horizon for the moving average.

Given our guess of σ_A , we use the following equation from Merton (1974):

$$V_E(t) = V_A(t)\Phi(d_1) - e^{-r(T-t)} * V_B\Phi(d_2)$$

where $d_1 = \frac{\log(\frac{V_A}{V_B}) + (r + \frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}$ and $d_2 = d_1 - \sigma_A\sqrt{T}$ to recover the value of assets. We define r to be the one-year Treasury-constant maturity, which we take from the Federal Reserve's H.15 report. After converging on V_A for the given σ_A , we recompute σ_A from our implied V_A using the same methodology we use to compute σ_E . We ultimately converge on σ_A through a slow-updating procedure.⁹⁷

Appendix H: Aggregation

Our aggregation technology and production environment is equivalent to a one-country version of Atkeson and Burstein (2010) with a per-unit subsidy, τ^s , on the production of the consumption good. We present our model aggregation in steady state below (hence, we remove all time subscripts). We first note that

$$\pi(z) = e^z Y (1 + \tau^s)^\rho w^{-\rho} \frac{1}{\rho^\rho (\rho - 1)^{1-\rho}}.$$

It is then useful to define

$$\Pi = Y (1 + \tau^s)^\rho w^{-\rho} \frac{1}{\rho^\rho (\rho - 1)^{1-\rho}}.$$

The choice of labor by the intermediate good firm is

$$l(z) = e^z Y (1 + \tau^s)^\rho \left(\frac{\rho - 1}{\rho} \right)^\rho w^{-\rho}.$$

⁹⁷We iterate on both σ_A and V_A until they converge to a tolerance of 1e-5. We choose updating parameters for the slow-updating procedure on V_A and σ_A , .25 and .15, respectively, such that 100% of firms converge.

Define the steady-state distribution of firms across states scaled by entry as $\tilde{\Gamma}(z, d, \theta)$. We then find scaled aggregate productivity as

$$\tilde{Z} = \int \int \int e^z \tilde{\Gamma}(z, d, \theta) dz d d d \theta. \quad (\text{H.1})$$

Another useful aggregate to define is average expenditures per entering firm, which we denote by Υ :

$$\Upsilon = n_e + \int \int \int e^z \theta^{-b} e^{bq} \tilde{\Gamma}(z, d, \theta) dz d d d \theta. \quad (\text{H.2})$$

Given Π , \tilde{Z} , and Υ , we can recover the following equilibrium objects:

$$W = (1 + \tau^s) \frac{\rho - 1}{\rho} (\Gamma_e \tilde{Z})^{\frac{1}{\rho-1}}.$$

$$Y = (\Gamma_e \tilde{Z})^{\frac{1}{\rho-1}} (L - L_r).$$

$$L_r = \frac{1}{\rho \xi} L,$$

where $\xi = \frac{\Pi \tilde{Z}}{\Upsilon}$ is the ratio of total variable profits to total expenditures on the research good. Total aggregate productivity is then

$$Z = (\Gamma_e \tilde{Z})^{\frac{1}{\rho-1}}.$$

Appendix I: Resolving Debt Overhang

The Bellman equations can also be solved if the firm as a whole, rather than equity holders alone, were to make the investment decision. It is always equity holders, however, who choose the point at which the firm goes bankrupt. Were the firm as a whole to make the bankruptcy decision, it would never choose to go bankrupt, as bankruptcy entails a deadweight loss. We

define the Bellman equation for equity holders when the firm as a whole makes the investment decision in steady state with fixed types, V_E^{ND} , below:

$$V_E^{ND}(z, d, \theta) = \max \left\{ 0, (1 - \tau) \left(\pi(z) - we^z \theta^{-b} h e^{bq} \right) - d + \tau^d d + e^{-r} (1 - \delta) \left(q V_E^{ND}(z + \Delta_z, d, \theta) + (1 - q) V_E^{ND}(z - \Delta_z, d, \theta) \right) \right\}.$$

We define the Bellman equation for equity holders and creditors combined, V_A^{ND} , below:

$$V_A^{ND}(z, d, \theta) = \max_q \begin{cases} \max_{d'} V_A^{ND}(z + \log(\alpha), d', \theta) & V_E^{ND}(z, d, \theta) < 0 \\ (1 - \tau) \left(\pi(z) - we^z \theta^{-b} h e^{bq} \right) + \tau^d d & \text{else} \\ + e^{-r} (1 - \delta) q V_A^{ND}(z + \Delta_z, d, \theta) \\ + e^{-r} (1 - \delta) (1 - q) V_A^{ND}(z - \Delta_z, d, \theta). \end{cases} \quad (\text{I.1})$$

We then use the first-order condition from (I.1) to find q :

$$q^* = \frac{1}{b} \log \left(\frac{e^{-r} (1 - \delta) (V_A(z + \Delta_z, d, \theta) - V_A(z - \Delta_z, d, \theta))}{b(1 - \tau) h w e^z} \right) + \log(\theta). \quad (\text{I.2})$$

Notice, now, no matter the value of b , the firm does not suffer from debt overhang, as equity holders and creditors are jointly making the investment decision. Because they make the investment decision taking into account the possibility of bankruptcy, if $b < \infty$, the firm will invest more as it is more levered relative to its business risk to avoid bankruptcy.

It is still the case, then, that the value of debt holders, $V_B^{ND}(z, d, \theta)$, is defined as the difference between the value of the firm as a whole and the value of equity; thus,

$$V_B^{ND}(z, d, \theta) = V_A^{ND}(z, d, \theta) - V_E^{ND}(z, d, \theta).$$

Hence, we can compare $V^{ND}(n)$ and $V_A(n)$ given n or across firms to assess the gains from resolving this problem conditional on n or across n . We can also do the same exercise

for the respective policy functions and expected annualized growth rates.

References

- ACEMOGLU, D. (2008): *Introduction to Modern Economic Growth*, Princeton University Press.
- ACEMOGULU, D., U. AKCIGIT, N. BLOOM, AND W. KERR (2013): “Innovation, Reallocation and Growth,” *Working Paper*.
- ACHARYA, V. V., S. T. BHARATH, AND A. SRINIVASAN (2007): “Does Industry-wide Distress Affect Defaulted Firms? - Evidence from Creditor Recoveries,” *Journal of Financial Economics*.
- ALTMAN, E. I. (2006): “Default Recovery Rates and LGD in Credit Risk Modeling and Practice,” *Working Paper*.
- ALTMAN, E. I., B. BRADY, A. RESTI, AND A. SIRONI (2005): “The Link between Default and Recovery Rates: Theory, Empirical Evidence, and Implications,” *Journal of Business*.
- AMAYA, D., P. CHRISTOFFERSEN, K. JACOBS, AND A. VASQUEZ (2015): “Does Realized Skewness Predict the Cross-Section of Equity Returns?” *Journal of Financial Economics*.
- ANDRADE, G. AND S. N. KAPLAN (1998): “How Costly Is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions That Became Distressed,” *Journal of Finance*.
- ARELLANO, C., Y. BAI, AND P. J. KEHOE (2012): “Financial Frictions and Fluctuations in Volatility,” *Working Paper*.
- ASKER, J., J. DE LOECKER, AND A. COLLARD-WEXLER (2014): “Dynamic Inputs and Resource (Mis) Allocation,” *The Journal of Political Economy*, 122, 1013 – 1063.
- ATKESON, A. AND A. BURSTEIN (2010): “Innovation, Firm Dynamics, and International Trade,” *Journal of Political Economy*, 118, 433–484.

- (2015): “Aggregate Implications of Innovation Policy,” *Working Paper*.
- ATKESON, A., A. EISFELDT, AND P.-O. WEILL (2013): “Measuring the Financial Soundness of U.S. Firms: 1926-2012,” *Working Paper*.
- BACHMANN, R. AND C. BAYER (2013): “‘Wait-and-See’ business cycles?” *Journal of Monetary Economics*.
- BACKUS, D., M. CHERNOV, AND S. ZIN (2014): “Sources of Entropy in Representative Agent Models,” *The Journal of Finance*, 69, 51–99.
- BARBERIS, N. AND M. HUANG (2008): “Stocks as lotteries: the implications of probability weighting for security prices,” *American Economic Review*.
- BASU, S. AND J. FERNALD (2002): “Aggregate productivity and aggregate technology,” *European Economic Review*.
- BASU, S., L. PASCALI, F. SCHIANTARELLI, AND L. SERVEN (2009): “Productivity, welfare and reallocation: Theory and firm-level evidence,” *NBER Working Paper No. w15579*.
- BENMELECH, E., N. K. BERGMAN, AND A. SERU (2013): “Financing Labor,” *Working Paper*.
- BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): “The Financial Accelerator in a Quantitative Business Cycle Framework,” in *Handbook of Monetary Economics*, ed. by B. Taylor and M. Woodford, Amsterdam: North-Holland, vol. 1C.
- BHARATH, S. T. AND T. SHUMWAY (2008): “Forecasting Default with the Merton Distance to Default Model,” *Review of Financial Studies*, 21, 1339–1369.
- BLOOM, N. (2009): “The Impact of Uncertainty Shocks,” *Econometrica*, 77, 623–685.
- BLOOM, N., M. FLOETOTTO, N. JAIMOVICH, I. SAPORTA-EKSTEN, AND S. J. TERRY (2014a): “Really Uncertain Business Cycles,” *Working Paper*.

- BLOOM, N., M. FLOETOTTO, N. JAIMOVICH, I. SPORTA-EKSTEN, AND S. J. TERRY (2014b): “Really Uncertain Business Cycles,” *Working Paper*.
- BLOOM, N., F. GUVENEN, AND S. SALGADO (2015): “Skewed Business Cycles,” .
- BRIS, A., I. WELCH, AND N. ZHU (2006): “The Costs of Bankruptcy: Chapter 7 Liquidation Versus Chapter 11 Reorganization,” *Journal of Finance*, 59, 1253–1303.
- BUERA, F. J., J. P. KABOSKI, AND Y. SHIN (2011): “Finance and Development: A Tale of Two Sectors,” *American Economic Review*, 101, 1964–2002.
- CALDARA, D., C. FUENTES-ALBERO, S. GILCHRIST, AND E. ZAKRAJSEK (2014): “The Macroeconomic Impact of Financial and Uncertainty Shocks,” *Working*.
- CANTOR, R. AND P. VARMA (2004): “Determinants of Recovery Rates on Defaulted Bonds and Loans for North American Corporate Issuers: 1983-2003,” *Moody’s Special Comment*.
- CHARI, V., P. J. KEHOE, AND E. R. MCGRATTAN (2007): “Business Cycle Accounting,” *Econometrica*, 75, 781–836.
- CHEN, H. AND G. MANSO (2010): “Macroeconomic Risk and Debt Overhang,” *Working Paper*.
- CHODOROW-REICH, G. (2014): “The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-09 Financial Crisis,” *Quarterly Journal of Economics*, 129 (1), 1–59.
- CHOI, J. AND Y. KIM (2015): “Anomalies and Market (Dis)Integration,” *Working Paper*.
- CHORDIA, T., A. GOYAL, Y. NOZAWA, A. SUBRAHMANYAM, AND Q. TONG (2015): “Are Capital Market Anomalies Common to Equity and Corporate Bond Markets,” *Working Paper*.

- CHRISTIANO, L. J., R. MOTTO, AND M. ROSTAGNO (2013): “Risk Shocks,” *American Economic Review*, forthcoming.
- DAVID, J. M., H. HOPENHAYN, AND V. VENKATESWARAN (2016): “Information, Misallocation and Aggregate Productivity,” *The Quarterly Journal of Economics*.
- DAVYDENKO, S. A., I. A. STREBULAEV, AND X. ZHAO (2012): “A Market-Based Study of the Cost of Default,” *Review of Financial Studies*.
- DIAMOND, D. AND Z. HE (2013): “A Theory of Debt Maturity: The Long and Short of Debt Overhang,” *Journal of Finance*.
- EISFELDT, A. AND T. MUIR (2013): “Aggregate Issuance and Savings Waves,” *Working Paper*.
- EISFELDT, A. AND A. RAMPINI (2006): “Capital Reallocation and Liquidity,” *Journal of Monetary Economics*, 53, 369–399.
- FAMA, E. F. AND K. R. FRENCH (1992): “Cross-Section of Expected Stock Returns,” *The Journal of Finance*, 47, 3247–3265.
- FEUNOU, B., M. R. JAHAN-PARVAR, AND C. OKOU (2015): “Downside Variance Risk Premium,” *Working Paper*.
- FEUNOU, B., M. R. JAHAN-PARVAR, AND R. TDONGAP (2014): “Which Parametric Model for Conditional Skewness?” *The European Journal of Finance*.
- FOSTER, L., J. HALTIWANGER, AND C. J. KRIZAN (2001): *Aggregate Productivity Growth. Lessons from Microeconomic Evidence*, NBER, 303 – 372.
- GILCHRIST, S., J. W. SIM, AND E. ZAKRAJSEK (2013): “Misallocation and Financial Market Frictions: Some Direct Evidence from the Dispersion in Borrowing Costs,” *Review of Economic Dynamics*, 16, 159–176.

- (2014): “Uncertainty, Financial Frictions, and Investment Dynamics,” *Working Paper*.
- GILCHRIST, S. AND E. ZAKRAJSEK (2012): “Credit Spreads and Business Cycle Fluctuations,” *American Economic Review*, 102, 1692–1720.
- GIROUD, X., H. M. MUELLER, A. STOMPER, AND A. WESTERKAMP (2012): “Snow and Leverage,” *The Review of Financial Studies*, 25, 681–710.
- GOMES, J. AND L. SCHMID (2012): “Equilibrium Credit Spreads and the Macroeconomy,” *Working Paper*.
- GOURIO, F. (2014): “Financial Distress and Endogenous Uncertainty,” *Working Paper*.
- HENNESSY, C. (2004): “Tobin’s Q, Debt Overhang, and Investment,” *The Journal of Finance*, 59, 1717–1742.
- HENNESSY, C. AND T. WHITED (2007): “How Costly is External Financing? Evidence from a Structural Estimation,” *The Journal of Finance*, 62, 1705–1745.
- HENNESSY, C. A., A. LEVY, AND T. M. WHITED (2007): “Testing Q Theory with Financing Frictions,” *Journal of Financial Economics*, 83, 691–717.
- HENNESSY, C. AND T. M. WHITED (2005): “Debt Dynamics,” *The Journal of Finance*, 60, 1129–1165.
- HERSKOVIC, B., B. T. KELLY, H. LUSTIG, AND S. VAN NIEUWERBURGH (2014): “The Common Factor in Idiosyncratic Volatility: Quantitative Asset Pricing Implications,” *Working Paper*.
- HODGE, A. W. (2011): “Comparing NIPA Profits With S&P 500 Profits,” *Survey of Current Business*, 91.

- HOPENHAYN, H. (2011): “Firm Microstructure and Aggregate Productivity,” *Journal of Money, Credit and Banking*, 43, 111–145.
- HSIEH, C.-T. AND P. J. KLENOW (2009): “Misallocation and Manufacturing TFP in China and India,” *The Quarterly Journal of Economics*, 74, 1403–1448.
- JENSEN, M. C. (1986): “Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers,” *American Economic Review*, 76, 323–329.
- KAPLAN, S. N. AND G. ANDRADE (1998): “How Costly is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions that Became Distressed,” *The Journal of Finance*, 53, 1443–1493.
- KEHRIG, M. (2015): “The cyclicity of productivity dispersion,” *Working Paper*.
- KHAN, A. AND J. K. THOMAS (2013): “Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity,” *Journal of Political Economy*, 121, 1055–1107.
- KRAUS, A. AND R. LITZENBERGER (1976): “Skewness preference and the valuation of risk assets,” *Journal of Finance*.
- LANG, L., E. OFEK, AND R. M. STULTZ (1996): “Leverage, investment, and firm growth,” *Journal of Financial Economics*, 40, 3–29.
- LELAND, H. E. (1994): “Corporate Debt Value, Bond Covenants, and Optimal Capital Structure,” *The Journal of Finance*, 49, 1213–1252.
- (1998): “Agency Costs, Risk Management, and Capital Structure,” *The Journal of Finance*, LIII, 1213–1243.
- LO, S. AND K. ROGOFF (2015): “Secular stagnation, debt overhang and other rationales for sluggish growth, six years on,” *BIS Working Paper*.

- LUTTMER, E. G. J. (2007): “Selection, Growth, and the Size Distribution of Firms,” *The Quarterly Journal of Economics*, 122, 1103–1144.
- MERTON, R. C. (1974): “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *The Journal of Finance*, 29, 449–470.
- MIDRIGAN, V. AND D. Y. XU (2014): “Finance and Misallocation: Evidence from Plant-Level Data,” *American Economic Review*, 104, 422–458.
- MODIGLIANI, F. AND M. H. MILLER (1958): “The Cost of Capital, Corporation Finance, and the Theory of Investment,” *American Economic Review*, 48, 261–297.
- MOLL, B. (2014): “Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?” *American Economic Review*.
- MOODY’S, I. S. (2015): “Annual Default Study, Corporate Default and Recovery Rates, 1920-2014,” *Moody’s Global Credit Policy Special Comment*.
- MOYEN, N. (2007): “How Big is the Debt Overhang Problem?” *Journal of Economic Dynamics and Control*, 31, 433–472.
- MYERS, S. (1977): “Determinants of Corporate Borrowing,” *Journal of Financial Economics*, 5, 147–175.
- OBERFELD, E. (2013): “Productivity and misallocation during a crisis: Evidence from the Chilean crisis of 1982,” *Review of Economic Dynamics*, 16, 100–119.
- OBERFIELD, E. (2013): “Productivity and misallocation during a crisis: Evidence from the Chilean crisis of 1982,” *Review of Economic Dynamics*, 16, 100–119.
- OCCHINO, F. AND A. PESCATORI (2012): “Debt Overhang in a Business Cycle Model,” *Working Paper*.

- ORLIK, A. AND L. VELDKAMP (2014): “Understanding Uncertainty Shocks and the Role of Black Swans,” *Working Paper*.
- OSOTIMEHIN, S. (2013): “Aggregate Productivity and the Allocation of Resources over the Business Cycle,” *Working Paper*.
- PETRIN, A. AND J. LEVINSOHN (2012): “Measuring aggregate productivity growth using plant-level data,” *RAND Journal of Economics*, 43, 705–725.
- PHILLIPPON, T. AND P. SCHNABL (2013): “Efficient Recapitalization,” *The Journal of Finance*, 68, 1–42.
- RESTUCCIA, D. AND R. ROGERSON (2008): “Policy Distortions and Aggregate Productivity with Heterogeneous Establishments,” *Review of Economic Dynamics*, 11, 707–720.
- SANDLERIS, G. AND M. L. J. WRIGHT (2014): “The Costs of Financial Crises: Resource Misallocation, Productivity and Welfare in the 2001 Argentine Crisis,” *The Scandinavian Journal of Economics*, 116, 87–127.
- SCHAAL, E. (2015): “Uncertainty and Unemployment,” *Working Paper*.
- SHLEIFER, A. AND R. VISHNY (2011): “Fire Sales in Finance and Macroeconomics,” *Journal of Economic Perspectives*.
- SHLEIFER, A. AND R. W. VISHNY (1992): “Liquidation Values and Debt Capacity: A Market Equilibrium Approach,” *The Journal of Finance*, 47, 1343–1366.
- STOCK, J. H. AND M. W. WATSON (2012): “Disentangling the Channels of the 2007-09 Recession,” *Brookings Papers on Economic Activity*, 1, 81–135.
- WELCH, I. (2004): “Capital Structure and Stock Returns,” *Journal of Political Economy*, 112, 106–131.

WHITED, T. AND G. WU (2006): "Financial Constraints Risk," *Review of Financial Studies*, 19, 531–559.