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Macroeconomics and Real Estate Markets

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by

Yongwook Kim

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Macroeconomics and Real Estate Markets

To my wife, Jin kyoung, who has been a constant source of support and encouragement during the challenges of graduate school and life. I am truly thankful for having you in my life. This work is also dedicated to my parents, Sungbok and Jaesik, who have trusted and encouraged me to achieve my dream.

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Curriculum Vitæ

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Abstract

Macroeconomics and Real Estate Markets

by

Yongwook Kim

This dissertation consists of three works that analyze real estate markets from macroeconomic perspectives. In the first essay, a Reverse Mortgage Loan (RML) allows senior homeowners to smooth consumption across time and generations in response to uninsurable shocks. Meanwhile, RML borrowers lose the opportunity to bequeath the whole equity of their home. Borrowing an RML is an intertemporal choice problem, which depends on various factors. This essay studies how intergenerational risk-sharing affects RML origination and intergenerational transfers during the housing boom and bust. I build an overlapping generations model with one-sided altruism to explain how a parent strategically behaves to maximize a dynasty's utility. I find that parents owning a relatively smaller home and scarce liquid assets are the principal borrowers of an RML. As children's income increases, the RML take-up rate initially decreases and then increases. As the size of the bequest motive increases, the RML take-up rate decreases; however, more slowly during the recession.

In the second essay, open enrollment policies provide more public school options by allowing a student to transfer to a public school of her choice regardless of residency. This paper investigates the effect of open enrollment on housing prices and income inequality. I consider school districts in Arizona and North Carolina, as opposite extremes in enrollment policies. I find some evidence on the effect of open enrollment on housing prices and income inequality. In a state with open enrollment, housing prices increase with the number of better schools far from home. The Gini coefficient decreases with the quality

of public education in a state with open enrollment. In an overlapping generations model with altruism, I examine how changes in the quality of public education, private school tuition, and transportation cost affect housing prices and income inequality across states.

The third essay studies how the composition of businesses of different qualities changes between a gentrifying and a non-gentrifying neighborhood in response to higher rent. I investigate how actively commercial gentrification is going on in some neighborhoods in Los Angeles, USA, using Yelp data. Then, I construct a search and matching model with heterogeneous neighborhoods, rents, and search friction. The model predicts a higher proportion of high-quality businesses and rents in a gentrifying neighborhood than a non-gentrifying neighborhood. Depending on whether high and low-quality goods are complements or substitutes, changes in the composition of businesses and rents show different patterns between a gentrifying and a non-gentrifying neighborhood.

Contents

Chapter 1

Reverse Mortgages and Intergenerational Risk-sharing during the Great Recession

1.1 Introduction

A Reverse Mortgage Loan (RML) is a loan secured against home values eligible to homeowners over 62. Unlike conventional home equity loans or home equity lines of credit, RML borrowers can stay in their homes as long as they want without worrying about a monthly repayment. The majority of the RML borrowers consider borrowing an RML for paying various expenses, paying off debts, or helping family members financially (Moulton et al. [16]). That is, retired senior homeowners can smooth their consumption or a family member's consumption through RMLs. Meanwhile, RML borrowers lose the opportunity to bequeath the whole equity of their home.

An RML is a financial instrument to connect the credit and housing markets. Home Equity Conversion Mortgage (HECM) insured by the Federal Housing Administration is the most prevalent RML in the market. Figure 1.1 plots S&P/Case-Shiller home price index and HECM originations over time. Among the interesting features of this figure, I mainly focus on the observation that HECM originations increased during the housing boom, beginning in 2000, and collapsed after the Great Recession. Though the RML take-up rate has been lower than the market expectation, an RML has great potential to grow in the future since most of the U.S. seniors are homeowners. To understand the RML market better, it is crucial to investigate how the RML market is related to the other markets.

This paper studies how intergenerational risk-sharing affects RML origination and intergenerational transfers during the recent housing boom and bust between 2000 and 2009. By studying RML origination, the housing market, and the business cycles, the paper provides insights about the consumption smoothing patterns across time and generations in response to uninsurable shocks. Though I focus on the relationship between the RML and housing markets in this paper, the insights obtained here apply to broader areas where assets owned by different generations are not perfectly correlated.

I build an overlapping generations model with one-sided altruism from a parent to a child.[1] After observing realized shocks in the housing and labor market, a parent chooses her own consumption, transfers to a child, homeownership, and whether to borrow an RML, anticipating a child's optimal decisions on consumption, savings, and homeownership. In contrast to a warm-glow bequest motive, in which a parent earns utility by just leaving a bequest, one-sided altruism allows this paper to investigate how a parent strategically behaves in response to shocks in the housing and labor markets to maximize the dynasty's utility.

To provide some intuition about optimal transfers and RML decisions, I first explore

[1] I do not consider altruism from a child to a parent since transfers from a child to a parent are not common from data relative to the other case, and the main interest of this paper is a parent's transfer and RML decision in response to the child's income and housing risk.

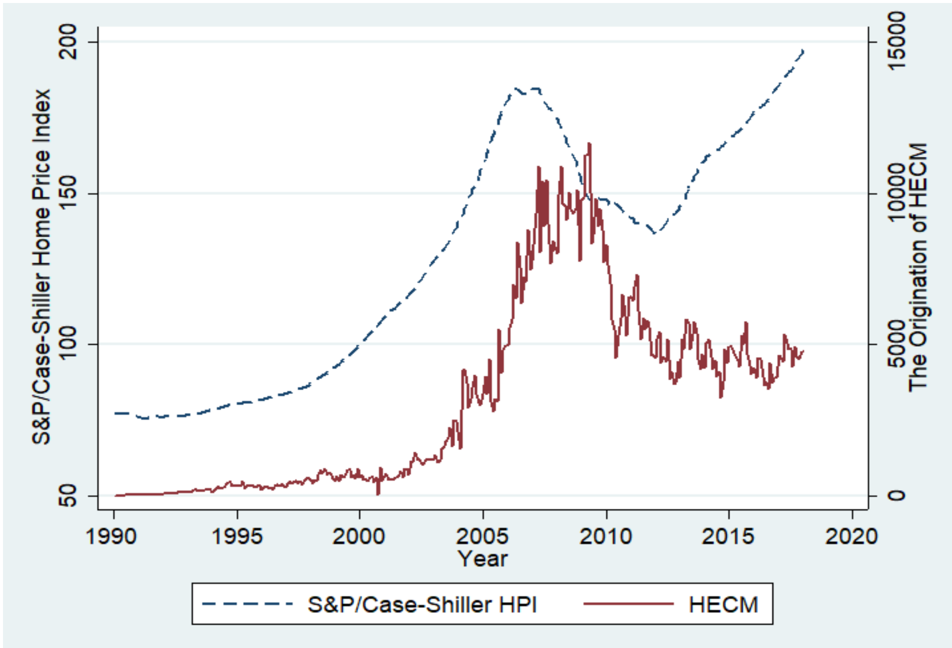


Figure 1.1: Trends of Home Price Index and the Origination of HECM

analytical solutions in a two-period model without uncertainty. A parent always increases transfers as a parent becomes wealthier, or a child becomes poorer. Thus, if a parent borrows an RML, inter-vivos transfers increase. It is, however, ambiguous whether parents transfer more as they become more altruistic. If a parent has large amounts of debt, or a child spends too much relative to her earnings, parents do not increase transfers even if they become more altruistic. In a pooling equilibrium, in which a child always chooses the same homeownership regardless of a parent's RML decision, a parent borrows an RML if it gives higher dynasty wealth than not borrowing an RML. In a separating equilibrium, in which a child chooses different homeownership depending on a parent's RML decision, it is uncertain whether a parent borrows an RML for given housing prices.

For a parent, a home is an instrument for borrowing an RML or bequeathing wealth to a child. For a child, a home can be an investment good. Due to a home's instrumental feature, the most favorable timings to borrow an RML or buy a home do not coincide. A senior homeowner prefers to borrow an RML today to earn higher wealth if the home value decreases in the future. In a separating equilibrium in which the RML leads to homeownership for a child, a senior homeowner borrows an RML today even though an RML does not yield the highest wealth. An altruistic parent has an additional incentive to take out an RML over and above consumption smoothing. A parent also wants to help her children smooth the consumption of housing services. Thus, not only does her own consumption matter but so too does that of her child.

To investigate how optimal transfers and RML decisions respond to different shocks, I consider a richer model and characterize it numerically. I add several sources of uncertainty in an infinite time horizon. I also include housing and RML market's realistic features. I find that seniors owning a relatively smaller home and scarce liquid assets are the principal borrowers of an RML. During the boom, 65% of seniors owning a home of the first half take up an RML, relative to 54% during the recession.[2] As senior homeowners' children become wealthier, the RML take-up rate initially decreases and then increases. During the boom, the take-up rate initially decreases from 37% to 13% and then increases to 65%. During the recession, the take-up rate initially decreases from 41% to 11% and then increases to 67%. These results suggest a decreasing need to help children financially through an RML up to a certain level of children's wealth. However, if children are very wealthy, more parents instead borrow an RML to consume for themselves until the marginal utilities of consumption are equal across generations.

Simulation results reveal that the RML take-up rate decreases as seniors become more altruistic. Unlike bequests, children only receive a fraction of their parents' wealth if parents borrow an RML. Since children prefer more wealth from their parents, they prefer parents leaving them a home rather than borrowing an RML.[3] The take-up rate, however,

[2]These take-up rates are much higher than the data. Since this paper studies the RML origination trend qualitatively, I choose to match the homeownership rate to the data. I do not control all potential factors regarding RML origination since I am interested in the RML origination trend over the size of the bequest motive between the boom and recession.

[3]If there is no altruism, senior homeowners never transfer wealth to children and always borrow an

decreases faster during the boom than during the recession. Since senior homeowners expect a higher home price during the boom, they prefer more bequeathing their home to children to borrowing an RML. The overall take-up rate decreases from 57% to 25% during the boom, whereas it decreases from 59% to 32% during the recession. Due to the decreasing marginal utility of consumption, the decreases in the take-up rate during the boom are prominent if seniors own a small-sized home or children's income is low.

I also find that there is a hump-shaped relationship between transfers by RML borrowers and the size of the bequest motive. Many RML borrowers at a high-level of altruism are the "wealthy hand-to-mouth." That is, the majority of RML borrowers hold scarce liquid assets, owning a relatively small-sized home. They borrow an RML to consume more for themselves rather than transferring more wealth to their children. The changes in transfers over the size of the bequest motive suggest that RML borrowers take up an RML for their children up to a certain level of altruism; however, they borrow an RML for themselves at a higher level of altruism. This pattern of transfers by RML borrowers commonly occurs during the boom and recession. However, during the recession, transfers by RML borrowers at a higher level of altruism decreases moderately than during the boom. During the boom, the average transfer initially increases from 1.23 to 1.427 and then decreases to 1.01. During the recession, it increases from 1.04 to 1.29 and then decreases to 1.25. These results suggest that more senior homeowners take up an RML in response to adverse shocks during the recession.

RML to spend home equity for themselves. As the size of the bequest motive increases, parents increase transfers and less borrow an RML.

1.2 Background

After paying upfront costs, an RML borrower typically receives the loan by a line of credit plan.[4] The loan contract ends, and the borrower (or their heirs, in case of death) has to repay the loan if the borrower dies, moves out permanently, or sells the home. The borrower also has to repay her loan if she fails to pay property taxes, buy homeowner's insurance, or do mandatory home maintenance duty. A feature of an RML is that it is non-recourse. That is, even though the amount of loan balance exceeds the proceeds of home sales at the end of contract, the borrower does not need to repay the excess. A Home Equity Conversion Mortgage (HECM) insured by the Federal Housing Administration (FHA) is the most prevalent RML after 2005. The FHA insures both lenders and borrowers. Lenders are free from the risk that loan balances exceed collateral values when borrowers repay their debts. The FHA also protects borrowers from the risk that lenders go bankrupt before the end of loan contracts. Since there is no monthly repayment unlike a home equity loan, a lender of RMLs does not consider a borrower's ability to pay off monthly repayments. Given the Maximum Claim Amount (MCA),[5] a lender considers a borrower's age and expected future interest rates to determine the available loan amount[6] when it originates RMLs.

1.2.1 Literature review

Despite apparent advantages, the market size for an RML has been smaller than its potential size.[7] Accounting for the fact that the elderly with house-rich and cash-poor

[4] There are a monthly payment (a tenure plan or a term plan), a lump sum payment, a line of credit, or a combination of a lump sum with either tenure plan or term plan as payment plans.

[5] The MCA is the largest loan balance that a lender can claim at the end of a loan contract. The MCA is the lesser of the HECM loan limit and the appraised home value.

[6] This is called the initial principal limit.

[7] There have been debates on the potential size of the market. Rasmussen et al. [26] and Mayer and Simons [13] estimate much larger potential borrowers via RMLs. However, Venti and Wise [23] and

are willing to stay in their home "as long as possible," the take-up rate seems too small. Since an RML was launched in 1989, less than 0.5% of all eligible homeowners originated RMLs per year before 2005. There are differences in the take-up rate of RML across groups. Since the amount of RML balances available depends on the appraised home value, senior homeowners in regions with the fastest rate of increase in housing prices prefer RMLs than in other regions (Mayer and Simons [13], Shan [20], and Haurin et al. [9]). Also, elderly homeowners with higher education are more likely to have RMLs (Chatterjee [5]). Shan [20] and Haurin et al. [9] show different take-up rates of RMLs on geographical, demographical, and credit conditions at the ZIP code level and the state level, respectively. Despite the heterogeneous take-up rate of RMLs at an individual or geographical level, the largest take-up rate in its entire history was only 2.1% of eligible homeowners in 2011 (Nakajima and Telyukova [19]).

The low demand for RMLs is closely related to senior homeowners' attitudes toward dissaving their home equity. According to survey data by RML applicants, the most significant reason seniors do not borrow RMLs is that the elderly liked to own their home completely free of any mortgages (Moulton et al. [16]). Several studies show that the elderly reduce their home equity very slowly or never reduce it unless there are shocks such as the death of a spouse or high medical expenses (Venti and Wise [24], Hurd [10]). Economists argue that senior homeowners rarely dissave their home equity due to a bequest motive and precautionary saving. Though some studies show that bequest motives in the US are not so strong (Hurd [10] and Haider et al. [8]), there is evidence that bequest motives affect the RML demand (Caplin [4] and Moulton et al. [16]). Regarding precautionary savings, many studies show that the elderly are risk-averse against health and mortality shocks (Munnell et al. [17], De Nardi et al. [7], and

Merrill et al. [14] conclude that the RM market will not be popular since it is attractive to old single persons and an only small portion of eligible homeowners satisfy some criteria to initiate the RMLs.

Cocco and Gomes [6]).[8]

On the supply-side, Phillips and Gwin [25] pointed out several risks that lenders face when they originate RMLs. They are longevity risk, interest risk, general home appreciation risk, specific home appreciation risk, and expense risk. The FHA can insure some of these risks. Regulation also seems to limit the supply of RML.

As Figure 1.1 showed, the RML take-up rates of all eligible homeowners increased between 2005 and 2011 from 0.4% to 2.1% per year. The number of HECMs originated per year tripled between 2000 and 2009. As Mian and Sufi [15] showed, the housing market collapse exacerbated the Great Recession via aggregate demand channel. Thus, studying the trend of RMLs is necessary since an RML has the potential to provide liquidity. However, only a few studies have explored why the take-up rate of HECMs increased after 2005. Shan [20] found that a one-year growth rate in house prices negatively affected the take-up rate of RMLs from 1993 to 2007. Haurin et al. [9] found that states where house prices are volatile and currently high, compared to their historical averages, have higher take-up rates. The housing boom and low-interest rates also lead to higher demand for RMLs between 2005 and 2011. Senior homeowners have increased both home equity loans and RMLs during the housing market boom (Shan [20] and Sinai and Souleles [21]). Historically low-interest rates also contributed to higher demand for RMLs. The increasing take-up rates of RMLs between 2005 and 2011, however, have declined by more than 50% after the Great Recession, relative to the peak in 2009. They have not recovered to pre-recession level, even though national home prices have increased and interest rates have remained low.

[8] The elderly may be concerned that some medical expenses are not covered even by Medicaid when some medical emergency happens. They are also concerned that their retirement savings are not sufficient since they are uncertain when they die. High costs, small amounts of loan available, and the complex structure of an RML are also important reasons for the low demand for RMLs (Moulton et al. [16]).

1.2.2 Some Facts

This section reports some relevant facts for thinking about the RML market. I combine HECM loan-level data from the U.S. Department of Housing and Urban Development and the housing price index data from the Federal Housing Finance Agency (FHFA) between 2001 and 2011. HECM data includes all essential features of contracts as well as limited demographic information for all loan applicants. To see how prices affect the origination of RMLs differently between the boom and the recession, I integrate FHFA's monthly purchase-only housing price index data with the zip code level in HECM data. HECM data does not provide information about why senior homeowners borrow RMLs. I use the Health and Retirement Study (HRS) between 2002 and 2012 to fill some missing information. The HRS is a biennial longitudinal survey of Americans over the age of 50.

This section details the family structure, housing, expectations, asset, and income among various topics in the HRS. The HRS data does not have direct information on RMLs. However, the HRS data complements HECM loan-level data in the sense that it shows how seniors dispose of their assets and spend differently during periods of booms and recessions.

Table 1.1 provides descriptive statistics from the HRS for the years between 2002 and 2012. The boom considers 2002, 2004, 2006, and 2012. The Great Recession covers 2008 and 2010. It shows that the boom and Great Recession have discrepancy in some features related to income, wealth, and transfers from parents to children.

- During the boom, 5% of senior homeowners sold their homes. Their average gross return on home sales was 367%. During the recession, only 2% of senior homeowners sold their homes. Their average gross return was 220%. Figure A.1 in the appendix also shows that the distribution of home sales return is more skewed to the right during the recession, relative to during the boom.

- The mortgage delinquency rate for the children of the seniors in the HRS increased from 4% during the boom to 5% during the recession. More young homeowners have either been unable to pay their mortgage on time or deliberately defaulted in situations where the value of their homes rapidly declined.

I examine how likely and how much parents transfer wealth to their children in response to changes in children's income and homeownership over the entire time windows. I also estimate how much the odds of transfer and the amount of transfer change if the elderly's net wealth, how frequently a child contacts a parent, whether a child lives nearby, whether a parent puts a child in the will, or spouse's existence change by one unit.

- Parents are less likely to transfer wealth to a child who earns a higher income or owns a home. The amount of a transfer also decreases with a child's income. For example, a senior homeowner decreases the odds of transferring wealth by 27%[9] if her child's income level changes from "less than $10,000" to "between $10,000 and $35,000." If her child's income level changes from "between $10,000 and $35,000" to "over $35,000," the log odds of transferring wealth decreases further by 57%. When the child's income level changes from "less than $10,000" to "between $10,000 and $35,000," the transfer amount decreases by $2725, and when the child's income level changes from "less than $10,000" to "between $10,000 and $35,000," the transfer amount decreases by $3171.

- Wealthier parents are more likely to transfer wealth to a child, and transfer more. For example, if a senior homeowner's net wealth increases by $100,000, the odds of transferring wealth increase by 1.2%, and the amount of transfer increases by $79.

[9] The log odds decreases by 0.316. Equivalently, the odds of transferring wealth to a child of income between $10,000 and $35,000 is equal to $e^{-0.316} = 0.73$ times of the odds of transferring wealth to a child of income less than $10,000. That is, the odd of transfer decreases by 27%

Table 1.1: Descriptive statistics during the boom and the Great Recession

Boom from the survey in 2002, 2004, 2006, and 2012

	mean	standard dev.	Q_1	Q_2	Q_3
Home return	3.67	6.27	1.17	1.67	3.25
Transfer amount ($)	4462.05	47796.94	0.00	0.00	1000.00
Monthly earnings ($)	3462.41	23328.46	762.50	1775.63	3916.67
Net wealth ($100,000)	3.82	16.68	0.03	0.95	3.26
Net liquid wealth ($100,000)	1.07	10.08	-0.02	0.00	0.30
Net illiquid wealth ($100,000)	2.75	11.32	0.02	1.00	2.80
Homeownership rate	0.75	0.43	1.00	1.00	1.00
Home sales rate	0.05	0.21	0.00	0.00	0.00
Local housing market perception	3.76	1.02	3.00	4.00	5.00
Child mortgage delinquency	0.04	0.19	0.00	0.00	0.00
Observations	62644				

Great Recession from the survey in 2008 and 2010

	mean	standard dev.	Q_1	Q_2	Q_3
Home return	2.20	3.22	1.00	1.43	2.25
Transfer amount ($)	4282.07	26487.76	0.00	0.00	1200.00
Monthly earnings ($)	3583.21	7373.82	750.00	1916.67	4347.00
Net wealth ($100,000)	3.30	10.95	0.00	0.70	2.93
Net liquid wealth ($100,000)	0.76	5.50	-0.07	0.00	0.16
Net illiquid wealth ($100,000)	2.53	7.22	0.00	0.90	2.72
Homeownership rate	0.70	0.46	0.00	1.00	1.00
Home sales rate	0.02	0.15	0.00	0.00	0.00
Local housing market perception	3.91	0.98	3.00	4.00	5.00
Child mortgage delinquency	0.05	0.22	0.00	0.00	0.00
Observations	14819				

Note: Home return is the ratio of home sales proceed over the purchase price. Net liquid wealth includes the total value of stock, bond, checking account, and money market fund net of total value of trusts and debts. The HRS rank the perception of the local housing market from 1(Excellent) to 5(Poor). Child mortgage delinquency rate measures the ratio of children who fell more than 2 months behind on mortgage payments.

If senior homeowner's monthly earnings increase by $100, the amount of transfer increases by $9.

- Due to the absence of a direct measure of bequest motives, I use some alternative measures. If a child contacts a parent more by 100 times per year, the odds of transferring wealth by a senior homeowner increases by 11%. If a child lives within 10 miles of a parent, the odds of transferring wealth increase by 14%. If a parent puts a child in a will, the log odds of transferring wealth increase by 153%.

All these correlations are reported in Tables A.1 and A.2 in the Appendix.[10]

The Great Recession seems to have altered the transfers between parents and children. During the Great Recession, in response to changes in child's income or homeownership, the odds of transfer or the amount of transfer changes in either direction or magnitude, relative to the boom.

- During the Great Recession, senior homeowners were 93% more likely to transfer wealth, though senior renters were 33% less likely to transfer wealth. The odds of transferring wealth by senior homeowners as a child becomes richer decreased less during the Great Recession than the boom. Senior renters are rather more likely to transfer wealth during the Great Recession when a child becomes richer. Senior homeowners and renters were 7% to 32% more likely to transfer wealth to a child who owns a home during the Great Recession. Senior homeowners increase the amount of transfer by $3894 during the Great Recession. Meanwhile, senior renters decrease the amount of transfer by $705.

All these correlations are reported in Tables A.3 and A.4 in the Appendix.[11]

[10]For Table A.1, I use logistic regression of the odds of transfer on child income, child homeownership, net wealth, the number of contact, proximity, a child in a will, and a spouse, with year fixed effect. For Table A.2, I replace the odds of transfer by the amount of transfer and use OLS regression of it with the same independent variables.

[11]For Table A.3, I add a dummy variable of the Great Recession and interaction terms with variables

The previous correlations are based on the HRS and do not show a HECM. In terms of HECM and local housing prices, the share of HECM applicants is different by the group.[12] I also examine how the ratio of loan available to the maximum claim amount changes in response to changes in borrower's age, expected rate, and home price index growth rates.

- Seniors with a lower valued house are the main applicants for HECM. The share of applicants owning houses, which appraised less than two times the median sales price of houses, makes up 77% of HECM applicants on average during the entire history.

- During the boom, the ratio of HECM loan available to the maximum claim amount increases by 0.023% if the one-year growth rate of local housing price index increases by 1%. During the Great Recession, the ratio decreases by 0.006%. These correlations are reported in Table A.5 in the Appendix.

1.3 Model

This section proposes an overlapping generations model with one-sided altruism à la Barro[2]. Through altruism in the model, I investigate how two generations strategically interact for risk-sharing in the housing and labor market. Some structures of the model are based on Nakajima and Telyukova[19], Yao and Zhang[22], and Boar[3].

for a child, in addition to independent variables in Table A.1. For Table A.4, I replace *the Odd of transfer* by *The amount of transfer*.

[12] See Figure A.2 in the Appendix. Senior homeowners in group 1 own a house appraised less than or equal to the median sales price of houses. Senior homeowners in group 2 own a house appraised between the median and two times the median. In group 3, own a house appraised between three times the median and four times the median. Senior homeowners in group 4 own a house appraised greater than four times the median.

1.3.1 Environment

Time is discrete and runs to infinity. An individual of generation t is born in time t. Each lives for two periods, young and old, denoted by a subscript y and o, respectively. Thus, a young individual of generation t and an old individual of generation $t-1$ overlap in time t. I assume that there is one-sided altruism from the old toward the young.

Wages and housing prices are known to both generations at the beginning of time t. All individuals are committed to pay housing expenditures each period. As a renter, she chooses a home size from a set of H_r and has to pay a fraction of housing price each period. As a homeowner, she chooses a home size from a set of H_h and has to pay a home maintenance cost and property tax. To be a homeowner, a young individual can borrow a forward mortgage up to a loan-to-value ratio and has to pay off the mortgage when old.[13] However, an old individual cannot borrow a forward mortgage to buy a home because she cannot pass on her debt to the next generation, by assumption. Anyone who wants to buy a house and has sufficient liquid wealth can buy a house since housing is assumed to be perfectly inelastically supplied at large quantities. Each individual can only own one house.

1.3.2 Endowments

A young individual earns a constant labor income if she is employed. Otherwise, a young individual receives an unemployment insurance. An old individual receives a constant pension income after retirement. A senior homeowner decides whether to borrow an RML in time t. If she does not borrow an RML, the entire home equity will be bequeathed to the old member of generation t at the beginning of time $t+1$, after paying a liquidation cost. On the other hand, if a senior homeowner borrows an RML,

[13] I assume that there is no default on the mortgage.

the remaining home equity will be bequeathed to the old member of generation t at the beginning of time $t+1$. For simplicity, however, I assume that all RML borrowers extract the entire home equity after paying upfront costs.[14] Also, it is not necessary to distinguish an RML payment plans since an RML borrower lives only for one period.

1.3.3 Preferences and Constraints

The old's problem: An old individual of generation $t-1$ chooses the option that provides the highest utility between being a renter, a homeowner and not borrowing an RML, or a homeowner and borrowing an RML. That is,

$$V(S_{o,t}) = \max\{V_{r,n}(S_{o,t}), V_{h,n}(S_{o,t}), V_{h,b}(S_{o,t})\} \tag{1.1}$$

where $V_{r,n}$, $V_{h,n}$,$V_{h,b}$ are the value functions of being a renter, a homeowner and not borrowing an RML, and a homeowner and borrowing an RML, respectively. The first subscript for each value function indicates homeownership (h for a homeowner and r for a renter) and the second subscript indicates for an RML decision (b for a borrower and n for a non-borrower). For each option, given a set of state variables, $S_{o,t}$, the old individual decides current period consumption $c_{o,t}$, the amount of inter-vivos transfer to a young individual $T_{o,t}$, and a home size $h_{o,t}$, to maximize the value functions, which depend on current consumption and a home size, and the young individual's lifetime utility weighted by the strength of bequest motive. The first subscript for each variable indicates the stage of life and the second subscript indicates the time when the variable is determined. The set of state variables, $S_{o,t}$, consists of labor income w_t, the current home price per unit P_t, the amount of liquid asset $a_{y,t-1}$, a home size chosen by the self

[14] For numerical analysis, I relax this assumption and allow senior homeowners with home equity higher than the RML limit cannot extract the entire home equity and the remaining home equity will be bequeathed to their heirs.

in youth $h_{y,t-1}$, a home size and an RML decision $h_{o,t-1}$ and $M_{o,t-1}$, chosen by an old individual of generation $t-2$.

First, the value of being a renter is

$$V_{r,n}(S_{o,t}) = \max_{c_{o,t},h_{o,t},T_{o,t}} \{u(c_{o,t}, h_{o,t}) + \eta v(S_{y,t})\} \tag{1.2}$$

subject to

$$c_{o,t} + T_{o,t} + \alpha P_t h_{o,t} = Q_t + (1-\theta) P_t h_{y,t-1} \mathbb{1}_{h_{y,t-1} \in H_h} \tag{1.3}$$

and

$$c_{o,t}, T_{o,t} \geq 0 \tag{1.4}$$

where η is the strength of the bequest motive, v is the young's value function, α is a rental rate, Q_t is the amount of cash at hand, θ is a fraction of transaction costs of selling a house, and $\mathbb{1}_{h_{y,t-1} \in H_h}$ is an indicator function equal to 1 if $h_{y,t-1} \in H_h$ and 0, otherwise. Equation (1.3) is the budget constraint for a senior renter who rented or owned a house of size $h_{y,t-1}$ when young. If the senior renter was a homeowner when young, she earns the home sale proceeds after paying a transaction cost. Cash at hand consists of a pension, liquid assets, and a bequest. The bequest amount varies depending on whether an old individual of the previous generation was an RML borrower. Thus, cash at hand is equal to

$$Q_t = b + (1+r) a_{y,t-1} + (1-\theta) P_t h_{o,t-1} (1 - M_{o,t-1})$$

where b is a pension income, r is an interest rate on liquid assets, and $M_{o,t-1}$ is a discrete variable equal to 1 if the old individual of generation $t-2$ borrowed an RML and 0, otherwise. The interest rate on liquid assets, r, is defined as

$$r = \begin{cases} r_m \text{ if } a < 0 \\ r_s \text{ if } a \geq 0 \end{cases}$$

where r_m is an interest rate on a forward mortgage and r_s is a risk-free rate on savings. In Equation (1.4), transfer is nonnegative since altruism is one-sided from the old individual to the young individual overlapped in time t.

Second, the value of being a homeowner and not borrowing an RML is

$$V_{h,n}(S_{o,t}) = \max_{c_{o,t},h_{o,t},T_{o,t}} \{u(c_{o,t}, h_{o,t}) + \eta v(S_{y,t})\} \tag{1.5}$$

subject to

$$\begin{aligned} c_{o,t} + T_{o,t} + (\psi + \tau)P_t h_{o,t} &= Q_t, \text{ if } h_{y,t-1} \in H_h \text{ and } h_{y,t-1} = h_{o,t} \\ c_{o,t} + T_{o,t} + (1 + \psi + \tau)P_t h_{o,t} &= Q_t + (1 - \theta)P_t h_{y,t-1}, \text{ if } h_{y,t-1} \in H_h \text{ and } h_{y,t-1} \neq h_{o,t} \\ c_{o,t} + T_{o,t} + (1 + \psi + \tau)P_t h_{o,t} &= Q_t, \text{ if } h_{y,t-1} \in H_r \end{aligned} \tag{1.6}$$

and

$$c_{o,t}, T_{o,t} \geq 0$$

where ψ is a home maintenance cost and τ is a property tax.

Equation (1.6) is the budget constraint for a senior homeowner who rented or owned a house of size $h_{y,t-1}$ when young, and does not borrow an RML. If an individual is a homeowner during the entire life and does not change home sizes between periods, the senior homeowner's disposable income is equal to cash at hand and just need to pay a home maintenance cost and a property tax to remained as a homeowner. However, if the senior homeowner changes a home size, she earns home sales proceed after selling her home of size $h_{y,t-1}$ and has to pay a home price in full in addition to a home maintenance cost and a property tax.[15] If the senior homeowner was a renter when young, her disposable income is equal to cash at hand, and she has to pay a home price, home maintenance cost, and property tax.

Finally, the value of being a homeowner and borrowing an RML is

$$V_{h,b}(S_o) = \max_{c_{o,t},h_{o,t},T_{o,t}} \{u(c_{o,t}, h_{o,t}) + \eta v(S_y)\} \tag{1.7}$$

subject to

$$\begin{aligned}
&c_{o,t} + T_{o,t} + (\psi + \tau)P_t h_{o,t} = Q + (1-\phi)P_t h_{o,t}, \text{ if } h_{y,t-1} \in H_h \text{ and } h_{y,t-1} = h_{o,t} \\
&c_{o,t} + T_{o,t} + (1 + \psi + \tau)P_t h_{o,t} = Q + (1-\theta)P_t h_{y,t-1} + (1-\phi)P_t h_{o,t} \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad , \text{if } h_{y,t-1} \in H_h \text{ and } h_{y,t-1} \neq h_{o,t} \\
&c_{o,t} + T_{o,t} + (1 + \psi + \tau)P_t h_{o,t} = Q + (1-\phi)P_t h_{o,t}, \text{ if } h_{y,t-1} \in H_r
\end{aligned} \tag{1.8}$$

and

$$c_{o,t}, T_{o,t} \geq 0$$

where ϕ are upfront costs of borrowing an RML.

Equation (1.8) is the budget constraint for a senior homeowner who rented or owned a house of size $h_{y,t-1}$ when young, and borrows an RML. If the senior homeowner borrows an RML, she receives additional income of an RML balance. Unlike the timing of liquidation of a bequest home, an RML origination occurs at the same period as homeownership. Thus, the maximum claim amount of an RML is valued at the current home price.

The young's problem: A young individual of generation t chooses between being a renter

[15] There is no qualitative difference in a house except a size. Thus, a senior homeowner who does not change a home size does not need to sell the house she owned when young.

and a homeowner,

$$v(S_{y,t}) = \max\{v_r(S_{y,t}), v_h(S_{y,t})\} \tag{1.9}$$

where v_r and v_h are the value functions of being a renter and a homeowner. Similar to the old individual's value functions, the subscript for each value function indicates homeownership (h for a homeowner and r for a renter). For each value function, given a set of state variables $S_{y,t}$, a young individual decides current consumption $c_{y,t}$, the amount of saving $a_{y,t}$, and a home size $h_{y,t}$, to maximize the value functions. The set of state variables, $S_{y,t}$, consists of a labor income w_t, the current home price per unit P_t, the amount of optimal transfer $T^*_{o,t}$, the optimal home size and RML decision, $h^*_{o,t}$ and $M^*_{o,t}$, chosen by an old individual of generation $t-1$.

First, the value of being a renter is

$$v_r(S_{y,t}) = \max_{c_{y,t},h_{y,t},a_{y,t}} \{u(c_{y,t}, h_{y,t}) + \beta V(S_{o,t+1})\} \tag{1.10}$$

subject to

$$c_{y,t} + a_{y,t} + \alpha P_t h_{y,t} = w_t + T_{o,t} \tag{1.11}$$

and

$$a_{y,t} \geq 0 \tag{1.12}$$

where β is a time discount rate.

Equation (1.11) is the budget constraint for a young individual who rents a house of size $h_{y,t} \in H_r$. Equation (1.12) represents that a renter cannot be a borrower.

Second, the value of being a homeowner is

$$v_h(S_{y,t}) = \max_{c_{y,t},h_{y,t},a_{y,t}} \{u(c_{y,t}, h_{y,t}) + \beta V(S_{o,t+1})\} \tag{1.13}$$

subject to

$$c_{y,t} + a_{y,t} + (1 + \psi + \tau)P_t h_{y,t} = w_t + T_{o,t} \tag{1.14}$$

and

$$a_{y,t} \geq -\delta P_t h_{y,t} \tag{1.15}$$

where δ is a loan-to-value ratio.

Equation (1.14) is the budget constraint for a young individual who buys a house of size $h_{y,t} \in H_h$. Equation (1.15) says that a young homeowner is eligible to borrow a forward mortgage up to a loan-to-value ratio with collateral of the house. Since there is no default on the mortgage, she has to pay off the mortgage $(1 + r_m)a_{y,t}$ when old. If the senior homeowner does not have sufficient liquid wealth to pay off the mortgage, she has to extract her home equity by either decreasing the home size or being a renter after selling her home.

1.3.4 Choices and Equilibrium

I consider a simplified sequential game of two periods. Since the economy starts at time 1, generation 0 (gen-0 henceforth) enters the economy at time 1 as old and dies at the end of time 1. Generation 1 (gen-1 henceforth) enters the economy at time 1 as young and dies at the end of time 2. The initial old are assumed to be homeowners for the entire life. Home sizes of homeowners and renters are normalized to 1. Instead, a homeowner enjoys extra utility from owning a house. Thus, gen-1 just decides whether to buy or rent a house. In this sequential game, gen-0 moves first and gen-1 moves later.[16] I define the following Subgame Perfect Equilibrium (SPE).

Definition: A SPE consists of value functions $\{V^*,\ v^*\}$, gen-0's policy functions $\{c^*_{o,1},\ T^*_{o,1},\ M^*_{o,1}\}$, and gen-1's policy functions $\{c^*_{y,1}, c^*_{o,2}, a^*_{y,1}, D^*_{y,1}, D^*_{o,2}\}$ such that

[16] See the Appendix A.1 for the game tree of this game.

1. By backward induction, given $\{P_1, P_2, w_1\}$ and $\{T_{o,1}, M_{o,1}\}$, gen-1 solves

$$v(D_{y,1}, D_{o,2}) = \max_{c_{y,1}, c_{o,1}, a_{y,1}} \{u(c_{y,1}, D_{y,1}) + \beta u(c_{o,2}, D_{o,2})\}$$

subject to

$$\begin{aligned} c_{y,1} + a_{y,1} + (1 - D_{y,1})\alpha P_1 + D_{y,1}(1 + \psi + \tau)P_1 &= w_1 + T_{o,1} \\ c_{o,2} + (1 - D_{o,2})\alpha P_2 + D_{o,2}\{D_{y,1}(\psi + \tau)P_2 + (1 - D_{y,1})(1 + \psi + \tau)P_2\} &= Q_2 + G_{o,2} \end{aligned} \tag{1.16}$$

and

$$a_{y,1} \geq -\delta P_1 D_{y,1} \tag{1.17}$$

where $Q_2 \equiv b + (1 + r)a_{y,1} + (1 - \theta)P_2(1 - M_{o,1})$, $G_{o,2} \equiv D_{y,1}(1 - D_{o,2})(1 - \theta)P_2 + D_{o,2}(1 - \phi)P_2$, and homeownership $D_{y,1}$ and $D_{o,2}$ are equal to 1 for buying a house and 0 for renting a house, and derives reaction functions of $c_{y,1}(T_{o,1})$, $c_{o,2}(T_{o,1})$, and $a_{y,1}(T_{o,1})$.[17]

2. Given $\{P_1, P_2, w_1\}$ and the reaction functions by gen-1, gen-0 solves

$$\begin{aligned} V_{h,b} &= \max_{c_{o,1}, T_{o,1}} \{u(c_{o,1}, D_{o,1} = 1) + \eta v\} \ \text{if} \ M_{o,1} = 1 \\ V_{h,n} &= \max_{c_{o,1}, T_{o,1}} \{u(c_{o,1}, D_{o,1} = 1) + \eta v\} \ \text{if} \ M_{o,1} = 0 \end{aligned}$$

subject to

$$c_{o,1} + T_{o,1} + (\psi + \tau)P_1 = b + (1 + r)a_{y,0} + (1 - \phi)P_1 M_{o,1} \tag{1.18}$$

[17] Since period 2 is the last period of the economy, all senior homeowners borrow an RML.

and derives $T_{o,1}$ for each value of $M_{o,1}$.

3. After plugging $T_{o,1}$ into $c_{y,1}(T_{o,1})$ and $c_{o,2}(T_{o,1})$, I define v^{RM} and $v^{Bequest}$ such that

$$v^{RM} \equiv \max\{v(1,1), v(1,0), v(0,1), v(0,0)\} \text{ if } M_{o,1} = 1$$

$$v^{Bequest} \equiv \max\{v(1,1), v(1,0), v(0,1), v(0,0)\} \text{ if } M_{o,1} = 0$$

4. Given the optimal v for each value of $M_{o,1}$, gen-0 derives $\{V^*,\ c^*_{o,1},\ T^*_{o,1},\ M^*_{o,1}\}$ such that

$$V^* = \max\{V_{h,b}, V_{h,n}\}$$

5. Given $\{T^*_{o,1}, M^*_{o,1}\}$, gen-1 derives $\{v^*, c^*_{y,1}, c^*_{o,2}, a^*_{y,1}, D^*_{y,1}, D^*_{o,2}\}$ such that

$$v^* = v^{RM} \text{ if } M^*_{o,1} = 1$$

$$v^* = v^{Bequest} \text{ if } M^*_{o,1} = 0$$

Now, I define the following subgame perfect equilibrium for another sequential game in this section only.[18] In this game, gen-1 moves first and gen-0 moves later.[19] In reality, many children make their decisions independently. Altruistic parents help children in financial difficulties after observing them. In a model of two periods, there is no way to reflect this observation for decision making in the next period. To make this game feasible, I assume that the young and old are committed to carry out their decisions simultaneously after first making decisions sequentially.

Definition: A SPE consists of value functions $\{V^\dagger,\ v^\dagger\}$, gen-0's policy functions, $\{c^\dagger_{o,1},\ T^\dagger_{o,1},\ M^\dagger_{o,1}\}$, and gen-1's policy functions $\{c^\dagger_{y,1},\ c^\dagger_{o,2},\ a^\dagger_{y,1},\ D^\dagger_{y,1},\ D^\dagger_{o,2}\}$ such that

[18] With a source of uncertainty, this sequential game can have multiple Markov equilibrium. Thus, it is not feasible to compute equilibrium in a richer model. See Lindbeck and Weibull[11].

[19] See the Appendix A.1 for the game tree of this game.

1. By backward induction, given $\{P_1, P_2, w_1\}$ and $\{c_{y,1}, c_{o,2}, a_{y,1}, D_{y,1}, D_{o,2}\}$, gen-0 solves

$$V_{h,b} = \max_{c_{o,1}, T_{o,1}} \{u(c_{o,1}, D_{o,1} = 1) + \eta v\} \text{ if } M_{o,1} = 1$$

$$V_{h,n} = \max_{c_{o,1}, T_{o,1}} \{u(c_{o,1}, D_{o,1} = 1) + \eta v\} \text{ if } M_{o,1} = 0$$

subject to Equation (1.16) and (1.18), and derives reaction functions $c_{o,1}(a_{y,1})$ and $T_{o,1}(a_{y,1})$.

2. Given $\{P_1, P_2, w_1\}$ and the reaction functions by gen-0, gen-1 solves

$$v(D_{y,1}, D_{o,2}) = \max_{c_{y,1}, c_{o,1}, a_{y,1}} \{u(c_{y,1}, D_{y,1}) + \beta u(c_{o,2}, D_{o,2})\}$$

subject to Equation (1.16) and derives $a_{y,1}$ for each value of $D_{y,1}$ and $D_{o,2}$.

3. After plugging $a_{y,1}$ into $T_{o,1}(a_{y,1})$, I define $V^{B,B}$, $V^{B,R}$, $V^{R,B}$, and $V^{R,R}$ such that

$$V^{B,B} \equiv \max\{V_{h,b}, V_{h,n}\} \text{ if } D_{y,1} = 1, D_{o,2} = 1$$

$$V^{B,R} \equiv \max\{V_{h,b}, V_{h,n}\} \text{ if } D_{y,1} = 1, D_{o,2} = 0$$

$$V^{R,B} \equiv \max\{V_{h,b}, V_{h,n}\} \text{ if } D_{y,1} = 0, D_{o,2} = 1$$

$$V^{R,R} \equiv \max\{V_{h,b}, V_{h,n}\} \text{ if } D_{y,1} = 0, D_{o,2} = 0$$

4. Given v for each value of $D_{y,1}$ and $D_{o,2}$, gen-1 derives $\{v^\dagger, c^\dagger_{y,1}, c^\dagger_{o,2}, a^\dagger_{y,1}, D^\dagger_{y,1}, D^\dagger_{o,2}\}$ such that

$$v^\dagger = \max\{v(1,1), v(1,0), v(0,1), v(0,0)\}$$

5. Given $(a^{\dagger}_{y,1}, D^{\dagger}_{y,1}, D^{\dagger}_{o,1})$, gen-0 derives $\{V^{\dagger}, c^{\dagger}_{o,1}, T^{\dagger}_{o,1}, M^{\dagger}_{o,1}\}$ such that

$$V^{\dagger} = V^{B,B} \text{ if } (D^{\dagger}_{y,1}, D^{\dagger}_{o,1}) = (1,1)$$
$$V^{\dagger} = V^{B,R} \text{ if } (D^{\dagger}_{y,1}, D^{\dagger}_{o,1}) = (1,0)$$
$$V^{\dagger} = V^{R,B} \text{ if } (D^{\dagger}_{y,1}, D^{\dagger}_{o,1}) = (0,1)$$
$$V^{\dagger} = V^{R,R} \text{ if } (D^{\dagger}_{y,1}, D^{\dagger}_{o,1}) = (0,0)$$

Proposition 1. Gen-0 increases transfers to gen-1 if gen-0's wealth increases or gen-1's disposable income in either period decreases. However, it is ambiguous whether gen-0 transfers more wealth when gen-0 becomes more altruistic.

Proof. See the appendix.A.1

In a sequential game which gen-0 moves first, gen-0 transfers wealth to gen-1 until the marginal utilities of consumption are equal across periods and generations. Gen-0 always increases transfers as gen-0 becomes wealthier or gen-1 becomes poorer. However, gen-0 does not always increase transfers as gen-0 becomes more altruistic. If gen-0 transfers "too much" due to high altruism, gen-0 can maximize her utility by decreasing transfers.

Proposition 2. In a pooling equilibrium, gen-0's choice between borrowing an RML and not borrowing an RML is independent of gen-1's homeownership decision.

Proof. See the appendix.A.1

In a pooling equilibrium, gen-1 always chooses the same homeownership regardless of the amount of transfer. That is, gen-1's homeownership decision is not conditional on whether gen-0 borrows an RML or not. Then, gen-0 chooses an option that increases dynasty wealth more, the sum of gen-0 and gen-1 wealth, since both generations consume certain shares. Gen-0 borrows an RML if it makes the dynasty wealth higher than not borrowing an RML. Otherwise, gen-0 does not borrow an RML.

Proposition 3. If gen-1 is liquidity-constrained in both periods, there are cases that gen-1 cannot have sufficient liquidity to buy a home even with transfers from gen-0.

Proof. See the appendix.A.1

If gen-1 is liquidity-constrained in both periods, she cannot always be a homeowner even with transfers from gen-0. If the relative price of P_2 over P_1 is either too high or low, gen-1 cannot have sufficient liquidity to buy a home. If the relative price is too high, the transfer amount from gen-0 is insufficient to buy a home. If the relative price is too low, the saving amount is insuffcient to buy a home.

Proposition 4. In a separating equilibrium, gen-0 decides an RML based on housing prices only if the RML decision leads to the desirable homeownership choice for gen-1. Otherwise, gen-0's RML decision is uncertain.

Proof. See the appendix.A.1

In a separating equilibrium, gen-1's homeownership decision depends on gen-0's RML decision. Gen-0 considers housing prices only if her RML decision leads to the desirable homeownership choice for gen-1. Gen-0 considers more than housing prices if gen-1's utility can reverse the gen-0's RML decision. As a result, gen-0 may borrow an RML in less preferable timing or not borrow an RML in preferable timing regarding housing prices.

Previous propositions do not consider strategic motive in a sequential game between generations. Since the game in this section is non-cooperative game between generations, each generation can achieve what they want by behaving strategically.

Proposition 5. There is a first-mover advantage in a sequential game between two generations. By moving first, gen-1 strategically fails to smooth consumption across periods to induce gen-0 to increase transfers.

Proof. See the appendix.A.1

By moving first, gen-1 fixes the amount of savings. In contrast to the game which gen-0 moves first, the amount of saving does not depend on the bequest motive. Then, gen-1 strategically consumes less and saves more in period 1, relative to the game which gen-0 moves first. As a result, gen-1's consumption in period 1 becomes more valuable to gen-0, making gen-0 increase transfers to gen-1. By the strategic failure of consumption smoothing, gen-1 accomplishes his desirable homeownership and enjoys higher lifetime utility.

1.4 Calibration

To characterize a richer model numerically, I employ several sources of uncertainty and realistic housing and RML markets. Each period a young individual receives a stochastic income out of five levels. Using household income data between 1979 and 2012 from the U.S. Census Bureau, I divide the time window into two episodes in which median household incomes keep increasing or decreasing, and name these episodes the boom and recession.[20] For each episode, I construct a normalized mean household income distribution. During the boom and recession, a young individual receives a stochastic income with an equal probability from a set of $[1.03, 2.59, 4.34, 6.69, 13.67]$ and $[1, 2.53, 4.25, 6.58, 13.46]$, respectively. An old individual receives a constant income of 3.62 and 3.58 during the

[20]The definitions of the boom and recession in this paper do not coincide with the NBER definitions. If I follow the NBER definitions, I cannot construct a reasonable transition matrix of housing prices due to the short duration of recessions.

boom and recession. These values are normalized average household incomes with age of household over 65 from the U.S. Census Bureau.

I use the median home sales data from the U.S. Census Bureau to derive the median sales price trend by the Hodrick-Prescott filter. I define that a housing price is high if the median sales price is over the trend, and low if a median sales price is below the trend. During the boom, on average, housing prices are 10% higher than the trend. During the recession, on average, housing prices are 6% lower than the trend. Given these average deviations relative to the trend, I normalize high (P_H) and low (P_L) housing prices per unit during each episode.

During the boom,

$$P = \begin{pmatrix} P_H \\ P_L \end{pmatrix} = \begin{pmatrix} 3.3 \\ 3 \end{pmatrix}$$

During the recession,

$$P = \begin{pmatrix} P_H \\ P_L \end{pmatrix} = \begin{pmatrix} 3 \\ 2.82 \end{pmatrix}$$

Given housing prices per unit, all individuals choose a home size. Renters choose home sizes from a set of $H_r = [1, 1.5]$ and home buyers choose them from a set of $H_h = [1.75, 2.25, 3, 4.5]$.[21] Housing prices in each period are stochastic and follow the Markov process. Given the average duration of high and low housing prices,[22] I construct the following transition matrices between high and low housing prices matching with average durations.

During the boom,

[21] The elements in H_r are normalized values corresponding to 33rd and 66th percentile of the size distribution of rental houses. The elements in H_h are normalized values corresponding to 33rd, 50th, 75th, and 95th percentiles of the size distributions of owner-occupied houses. (İmrohoroğlu et al. [12])

[22] During the boom, the median sales price is high for two years on average and low for three years on average. During the recession, the median sales price is high for 1.5 years on average and low for 3.25 years on average.

	High price	Low price
High price	1/2	1/2
Low price	1/3	2/3

During the recession,

	High price	Low price
High price	1/3	2/3
Low price	4/13	9/13

Home transaction costs include home preparations, commissions, transfer tax, prorated property tax, utilities, escrow, and title insurance. I use 10% of the sales price as home transaction costs following the Zillow data. Based on the depreciation rate of residential capital, I use 1.7% of home values as home maintenance costs (Nakajima and Telyukova [19]). Among property tax rates over states in the U.S., I use the median property tax rate of 0.91% in Georgia. I set the extra utility of homeownership equal to 6 to ensure that about 72% of seniors own the home during the boom in the benchmark. For a rental rate, I consider financial lenders in a competitive rental market. A lender charges the following rental rate equal to the opportunity costs of offering a house for renting. (İmrohoroğlu et al. [12])

$$\alpha = \psi + \tau + r_m$$

For a HECM up-front cost, using HECM loan-level data between 1989 and 2011, I subtract the net principle limits from the maximum claim amounts and take the average on differences.[23] On average, HECM borrowers take 64% of the maximum principle

[23] I compute net principle limits by subtracting the initial mortgage insurance premiums, origination fees, repair set-asides, and service fee set-asides from the initial principle limits. HECM borrowers pay 2% of the maximum claim amount as the initial mortgage insurance premium. Of the closing costs, origination fees are equal to $2000 or the maximum claim amount, whichever is greater. I ignore third party fees because data on these fees are not available. However, these costs make up very little of the total costs.

limits. HECM borrowers whose home value exceeds the HECM limit will bequeath the excess to their heirs after death. To illustrate this feature, I modify the cash at hand to include the HECM limit as follows

$$Q_t = b+(1+r)a_{y,t-1}+(1-\theta)\{P_t h_{o,t-1}(1-M_{o,t-1})+P_t(h_{o,t-1}-\overline{M})M_{o,t-1}\mathbb{1}_{h_{o,t-1}=4.5}\} \quad (1.19)$$

where $\overline{M}$ is the HECM limit.

Equation (1.19) says that a senior homeowner who does not borrow an RML bequeaths the entire home equity net of liquidation costs. Meanwhile, an old individual who owns a home of 95th percentile in size and borrows an RML will bequeath the excess.

A Young individual who buys a house is eligible to borrow from a lender up to a Loan-to-Value ratio with collateral of their house. I use the LTV ratio of 74% and 70% during the boom and recession from the Freddie Mac loan-level data. Borrowers have to pay the mortgage interest rate of 5.73% and 4.2% during the boom and recession. Savers receive the risk-free rate of 4.09% and 2.30% during the boom and recession.[24]

An old individual receives a renting shock and a health shock. An old individual has to rent a house if a renting shock ϵ_r arrives. This shock captures the unwanted occasion of moving to a nursing home. According to the HRS, between 2002 and 2012, 2.16% to 3.03% of the elderly live in nursing homes. I use the average of 2.5% as a renting shock. With a renting shock, the old's problem, Equation (1.1), is modified by

$$V(S_{o,t}) = \pi_r V_{r,n}(S_{o,t}) + (1-\pi_r)\max\{V_{r,n}(S_{o,t}), V_{h,n}(S_{o,t}), V_{h,b}(S_{o,t})\} \quad (1.20)$$

where π_r is the probability of a renting shock.

If a renting shock happens, the elderly have to be a renter. Otherwise, the elderly choose

[24] For the mortgage interest rate, I use inflation-adjusted 30-year fixed-rate mortgage average. For the risk-free rate, I use inflation-adjusted 10-year Treasury note rate.

Table 1.2: Parameter values for calibration

β	time discount rate	0.96
δ	LTV ratio	0.74/0.7
η	bequest motive	[0.35,0.4,0.45]
θ	home liquidation cost	0.1
ψ	home maintenance	0.017
χ	extra utility of homeownership	1.25/6
τ	property tax	0.0108
ϕ	HECM up-front cost	0.36
r_m	mortgage interest rate	0.0573/0.042
r_s	risk-free interest rate	0.0409/0.023
ϵ_r	a renting shock	0.025
ϵ_h	a health shock	0.2976
γ	consumption aggregator	0.67
σ	coefficient of relative risk aversion	2.006

the best option between $V_{r,n}(S_o)$,$V_{h,n}(S_o)$, and $V_{h,b}(S_o)$. Though I do not model medical expenditures, the elderly are reluctant to decumulate their wealth abruptly if they have to pay for a huge amount of medical expenditure in the future. The old individual with a negative health shock has the following additional utility from not borrowing an RML.

$$\omega(h_{o,t}, M_{o,t}) = \zeta \frac{(P_t h_{o,t}(1 - M_{o,t}) + \xi)^{(1-\sigma)} - 1}{1 - \sigma}$$

where ζ and ξ are adjusting parameters.

I regard that 29.76% of the elderly, who assessed their health status as "fair" or "poor" in the HRS, receive a negative health shock ϵ_h.

Given $S_{y,t} = (w_t, P_t, T^*_{o,t}, h^*_{o,t}, M^*_{o,t})$, the young's value function is replaced by

$$v(S_{y,t}) = \max_{c_{y,t}, h_{y,t}, a_{y,t}} \{u(c_{y,t}, h_{y,t}) + \beta \max\{\mathbb{E}[V_{r,n}(S_{o,t+1}), V_{h,n}(S_{o,t+1}), V_{h,b}(S_{o,t+1})|P_t]\}\}$$

where $V_{r,n}(S_{o,t+1})$, $V_{h,n}(S_{o,t+1})$, and $V_{h,b}(S_{o,t+1})$ are continuation value functions of the old, which depend on $S_{o,t+1} = (a_{y,t}, w_{t+1}, \epsilon_{r,t+1}, \epsilon_{h,t+1}, h_{y,t}, h_{o,t}, M_{o,t})$. The expected value functions are conditional on the current housing price and determined by the probabilities of w_{t+1}, P_{t+1}, $\epsilon_{r,t+1}$, and $\epsilon_{h,t+1}$. For a time discount rate, I use a conventional value of 0.96. The current utility function is defined by

$$u(c_h, h_y) = \frac{(c_y^{\gamma}(\chi(h_y)h_y)^{1-\gamma})^{1-\sigma} - 1}{1-\sigma}$$

where γ is a consumption aggregator, χ is the extra utility of homeownership, and σ is a coefficient of relative risk aversion.

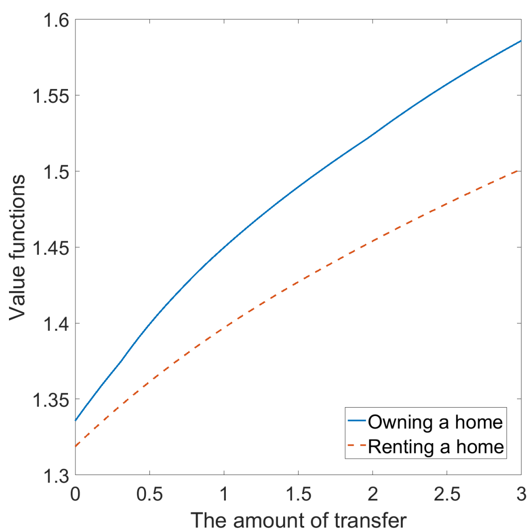
For a consumption aggregator, I use a value of 0.67 as average housing expenditure out of total expenditure from the U.S. Bureau of Labor Statistics. I use a value of 1.2 and 6 for extra utility of homeownership by the young and old to target the homeownership rate of the young and old in the benchmark at 64% and 80%. For a relative risk aversion coefficient, I use a value of 2.006 from Nakajima and Telyukova [19]. For the size of the bequest motive, I use 0.4 as a benchmark and change the value by 0.05 for comparative static analysis. Table 1.2 summarizes all parameter values for calibration.

1.5 Numerical analysis

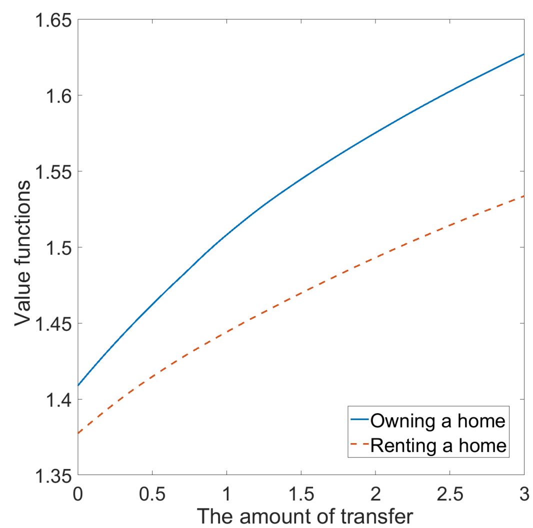
This section proposes the optimal solutions using the richer model with parameter values discussed in the previous section. It then presents simulation results of how likely and how much seniors transfer wealth to young individuals and who borrows an RML, in the steady-state during the boom and recession.

1.5.1 Optimal solution

The fact that a home is a dual good as both consumer and investment goods complicates homeownership decision. Though there are additional costs, such as maintenance costs and property taxes, people want to buy a home to enjoy additional utility. However, people will not buy a home if they have to give up too much consumption. People also consider how home prices will change since they might sell or bequeath the home in the future. For a young individual with sufficient liquidity, Figure 1.2 plots value functions of owning and renting a home over the amount of transfer during the boom. Each panel shows a case in which the elderly borrow an RML or not. As Figure 1.2 shows, if a young individual has sufficient liquidity to buy a home due to a high income or a low housing price, she prefers to own a home regardless of the amount of transfer. Whether an old individual borrows an RML or not does not affect homeownership decision.



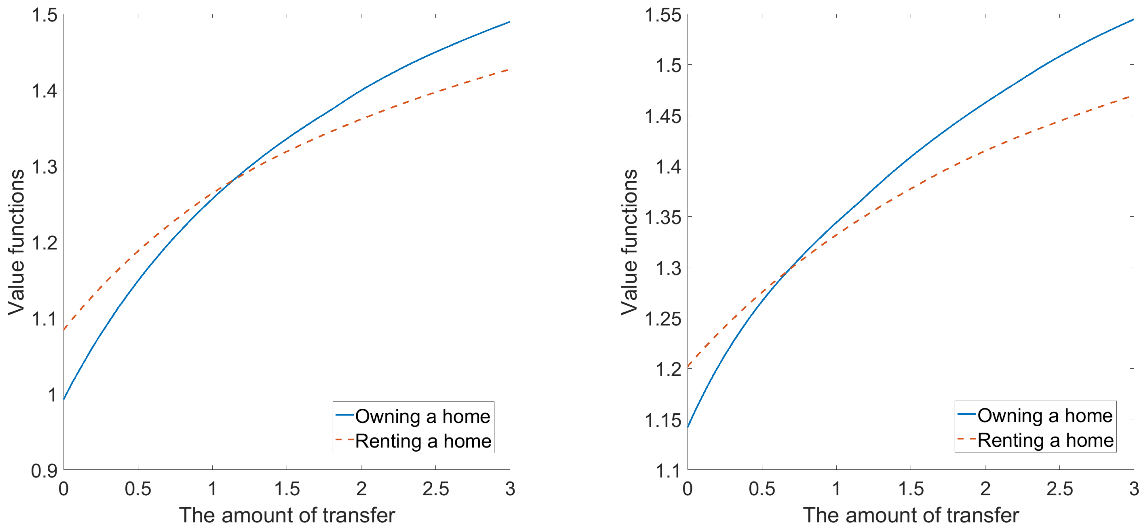
(a) RML



(b) No RML

Figure 1.2: Value functions of the young with sufficient liquidity

However, if a young individual does not have sufficient liquidity due to a low income or a high housing price, whether an old individual borrows an RML can affect a young individual's homeownership decision. For the young with insufficient liquidity, Figure 1.3 plots the young's value functions of owning and renting a home over the amount of transfer during the boom. A young individual needs more transfer to be a homeowner



(a) RML

(b) No RML

Figure 1.3: Value functions of the young with insufficient liquidity

if an old individual borrows an RML since the young will not have a bequest when old. The young extract more resources from the old by being a renter up to the higher transfer amount. The basic structure of the problem is similar during the recession. An old individual's problem is more complicated than a young individual's problem since an old individual considers both homeownership and RML decision to maximize not an individual but dynasty's utility. For an old individual, Figure 1.4 plots value functions of renting a home, owning a home and not borrowing an RML, and owing a home and borrowing an RML over the amount of liquid assets during the boom. As Figure 1.4 shows, an old individual who owns a large amount of debts prefers to rent a home. An old individual who owns a small amount of debts or assets prefers to buy a home and borrow an RML. An old individual who owns a large amount of assets prefers to buy a home and not borrow an RML. The thresholds between these choices vary depending on a set of state variables given to an old individual.

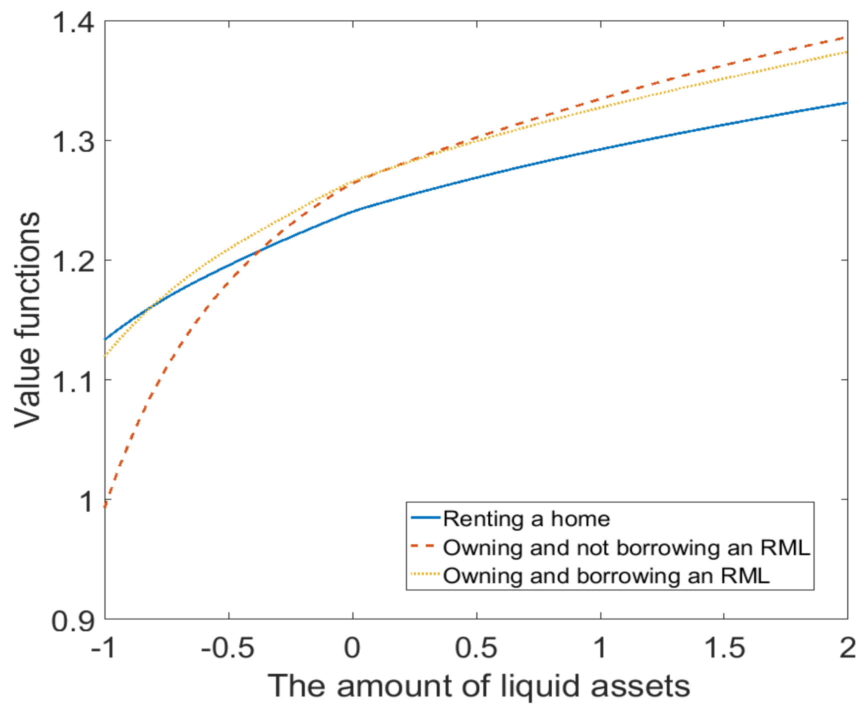


Figure 1.4: Value functions of the old over the amount of liquid assets

1.5.2 Simulation

First, this section reports steady-state results from simulation during the boom and recession in the benchmark. I set the size of the bequest motive at 0.35 in the benchmark. In the benchmark, the homeownership rate of the young and old are 69% and 83%. The RML take-up rate is 41%. With a higher value of the bequest motive, I can match the RML take-up rate with data. The higher value of bequest motive increases homeownership rate of the young, and decreases homeownership rate of the old. Since this paper focuses on RML origination qualitatively, I choose to match the homeownership rates with data rather than the RML take-up rate. Then, I study comparative statics regarding the size of the bequest motive.

Result 1. In the benchmark, the probability of transfer monotonically decreases as young individuals' income increases during the boom and recession. The amount of transfer shows a tendency to decrease. However, it does not monotonically decrease.

During the boom, the probability of transfer monotonically decreases from 1 to 0.13 as young individuals' income increases from the 1st fifth to the 5th fifth. The amount of transfer also decreases from 2.62 to 0.12. However, the amount of transfer to young individuals earning an income of the 4th fifth is higher than the 3rd fifth by 0.07. During the recession, the probability of transfer and the amount of transfer show similar patterns with the boom. The probability of transfer monotonically decreases from 1 to 0.07 as young individuals' income increases. The amount of transfer decreases from 2.3 to 0.05. Compared to the boom, the probability of transfer to young individuals increases except for young individuals earning an income of the 5th fifth. The amount of transfer to young individuals decreases except for young individuals earning an income of the 4th fifth.

Result 2. Overall, senior renters of the 66th percentile are more likely to transfer wealth than senior homeowners during the boom and recession. The senior renters also transfer more wealth than senior homeowners. The probability of transfer and the amount of transfer initially decreases and then increases as the elderly's home size increases.

From Figure 1.4, some old individuals with debts choose to be a renter after selling their home to maximize their utility. The result that senior renters are more likely to transfer or transfer more wealth than senior homeowners seems counterfactual. From the fact that senior renters of the 33rd percentile do not transfer at all, I argue that some senior renters of the 66th percentile can derive higher utility by consuming and transferring more with home sales proceed rather than remaining as a homeowner.

In the benchmark, during the boom, 67% of senior renters transfer wealth, relative to 49% of senior homeowners. As senior homeowners' home size increases from the 33rd to 95th percentile, the probability of transfer increases from 46% to 69%. Among senior homeowners, only senior homeowners owning a home of the 95th percentile are more likely to transfer wealth than senior renters of the 66th percentile. Overall, senior renters of the 66th percentile transfer more wealth than senior homeowners by 42%. Senior homeowners of the 95th percentile transfer wealth more than senior homeowners of the 50th percentile by 131%. Thus, senior homeowners of the 75th and 95th percentile transfer more wealth than senior renters of the 66th percentile by 1% and 24%, respectively.

During the recession, senior homeowners are 2% more likely to transfer than during the boom. Senior renters are 3% less likely to transfer. The amount of transfer by senior homeowners and renters both decreases by 6% and 18%, respectively. Since the recession is less preferable time to sell a home due to lower home prices, the number of senior homeowners who sell their homes to transfer more wealth decreases.

The reason that the probability of transfer and the amount of transfer initially decreases and then increases as the elderly's home size increases is relevant to the RML origination. As Result 5 states below, the majority of RML borrowers are senior homeowners of the 33rd percentile. They have more liquidity to transfer wealth through RML.

Result 3. RML borrowers are more likely to transfer wealth than non-borrower. RML borrowers also transfer more wealth than non-borrowers.

In the benchmark, RML borrowers are not sufficiently less wealthier than non-borrowers for being less likely to transfer or transferring less wealth than non-borrowers. During the boom, 60% of RML borrowers transfer wealth, whereas 49% of non-borrowers transfer wealth. RML borrowers transfer 75% more wealth to young individuals than non-borrowers. During the recession, 58% of RML borrowers transfer wealth, relative to

50% of non-borrowers. RML borrowers transfer 78% more wealth than non-borrowers. However, with a higher value of bequest motive, RML borrowers are not always more likely to transfer or do not always transfer more than non-borrower.

Result 4. The RML take-up rate and the share of RML borrowers initially decrease and then increase as young individuals' income increases.

As young individuals earn a higher income, there is less need to help them financially. Thus, fewer seniors borrow an RML, and they instead bequeath their home to young individuals in the future. However, if young individuals are very wealthy, it is not optimal for old individuals to bequeath their homes to young individuals. Old individuals borrow an RML to consume more for themselves until the marginal utilities of consumption are equal across generations. During the boom, the take-up rate initially decreases from 37% to 13% and then increases to 65% as young individuals' income increases. The share of RML borrowers decreases from 21% to 7% and then increase to 39%. During the recession, the take-up rate and the share of borrowers show similar patterns to the boom. However, the take-up rate and the share of RML borrowers are higher during the recession rather than during the boom except when young individuals earn wages of the 2nd fifth.

Result 5. The RML take-up rate monotonically increases as senior homeowners' home size increases. Seniors owning a relatively smaller house are the principal borrowers for an RML. However, the share of RML borrowers does not monotonically decrease as senior homeowners' home size increases.

During the boom, the RML take-up rate increases from 30% to 69%, as senior homeowners' home size increases. Senior homeowners of the 33rd and 50th percentile make up 65% of borrowers. Especially, the share of RML borrowers owning a home of the 33rd percentile is 42%. During the recession, compared to the boom, the take-up rates by seniors homeowners of the 33rd and 50th percentile decrease by 3% and 2%, and the take-up rates by seniors homeowners of the 75th and 95th increase by 8% and 9%. Then, the share of RML borrowers owning a home of the 33rd and 50th percentile decreases from 65% to 54%.

Since there exists an RML limit, the elderly owning a home larger than the limit can borrow an RML up to the limit and bequeath the excess. That is, some senior homeowners of 95th percentile can be better off by partially liquidating their home equity through RML. Thus, the share of RML borrowers does not monotonically decrease as senior homeowners' home size increases.

Result 6. As seniors become more altruistic, the RML take-up rate always decreases. However, the probability of transfer or the amount of transfer does not monotonically change. The rate of change for these variables is different during the boom and recession.

As seniors become more altruistic, the RML take-up rate decreases since more young individuals prefer senior homeowners not to borrow an RML. Unlike the bequest, seniors do not transfer the entire RML balance to children. However, the take-up rate changes differently during the boom and recession. The overall take-up rate decreases from 57% to 25% during the boom, whereas it decreases from 59% to 32% during the recession. That is, the RML take-up rate decreases more slowly during the recession. The faster decrease in the take-up rate during the boom is mostly caused by senior homeowners owning a home of the 33rd percentile. The RML take-up rate by senior homeowners of the 33rd percentile decreases by 31 percentage points, compared to 22 percentage points during the recession. Since senior homeowners expect that a home price will be more likely to be high in the future during the boom, they less prefer to borrow an RML in terms of housing price expectations. Due to the decreasing marginal utility of consumption, the difference of RML take-up rate over the degrees of bequest motive is salient in the group of senior homeowners of the 33rd percentile. Then, the share of RML borrowers owning a home of the 33rd or 50th percentile decreases from 69% to 59% during the boom, compared to 61% to 50% during the recession.

During the boom, the RML take-up rate decreases faster when young individuals earn an income of the 1st or 2nd fifth than during the recession. The RML take-up rates for them decreases by 37 and 42 percentage points during the boom, relative to 24 and 37 percentage points during the recession. For a similar reason to the rapid decrease in the RML take-up rate by senior homeowners of the 33rd percentile during the boom, senior homeowners can maximize the dynasty's utility by bequeathing their homes to young individuals earning less income in a less preferable time to borrow an RML.

The probability of transfer or the amount of transfer does not monotonically change as seniors become more altruistic. This result is consistent with Proposition 1 in subsection 1.3.4, stating that it is ambiguous whether gen-0 transfers more wealth when gen-0 becomes more altruistic. If the elderly have large amounts of debt or young individuals spend too much relative to earnings, the elderly are less likely to transfer or transfer less wealth, even though they become more altruistic. There are two noticeable changes regarding the transfer. During the boom, as the size of bequest motive increases from 0.3 to 0.4, senior homeowners decrease the amount of transfer by 0.1299. Meanwhile, senior renters increase the amount of transfer by 2.1678. Senior homeowners transfer more than senior renters when the size of the bequest motive is 0.3. However, senior renters transfer more than senior homeowners when the size of the bequest motive is 0.4. This reversion also happens by RML borrowers and non-borrowers during the boom. During the recession, RML borrowers keep transferring more wealth than non-borrowers, as seniors become more altruistic. All the simulation results are reported in Tables A.6, A.7, A.8, and A.9.

1.6 Conclusion

This paper studies how intergenerational risk-sharing affects RML origination during the housing boom and bust. The different frequency between business and housing market cycle makes reconciling RML take-up rates during the boom and bust challenging. Motivated by some facts in the HRS and HECM loan-level data, I introduce an overlapping generations model with one-sided altruism. The model explains how likely and how much seniors transfer wealth to their children. It also answers who borrows an RML during the boom and recession. Most of the results are consistent with the facts in the HRS and HECM data qualitatively. For further research, I suggest the following question. An RML has the potential to break a vicious cycle of the housing market collapse during the recession by providing liquidity. After introducing a production sector, studying how much an RML can alleviate the negative impact of the economy's recession would be an interesting question.

Chapter 2

The Effect of School Choices on Housing Prices and Inequality

2.1 Introduction

Open enrollment allows a student to transfer to a public school of her choice. It gives parents and students school choices by providing more public school options and expanding the educational marketplace geographically. As one of the important factor to determine housing prices, the quality of public education affects where to live and which school to attend. In addition to housing expenses, parents also consider the prospects for their children's future success in their decisions. Open enrollment weakens the connection between housing prices and the quality of public education and makes the problem of residential location and school choice more complicated. Students have an additional option to commute to a school outside of their resident district. This paper examines how open enrollment affects these choice problems faced by parents and students.

According to a 2017 study by the National Association of Realtors, the quality of public schools is an important factor to 26% of home buyers looking for a new home. Though various factors other than schools such as safety, commute times, jobs, and housing inventory also play a part to determine home prices, schools are more important to young home buyers under 36 with at least one kid under the age of 18 at home. Almost half of them consider school districts when they choose where to live. Traditional public school assignments restrict the opportunities from students living in a "bad" school district to attend schools in a "good" school district. Open enrollment has been designed to offer more equitable access to better education opportunities to socioeconomically disadvantaged students. Another potential benefit of open enrollment is to induce public schools to work on continual improvement to recruit and retain students through competition by introducing market mechanisms in educational system.

To motivate the analysis in my model, I explore the effect of open enrollment on housing prices and spatial sorting by income from data. I consider school districts in Arizona and North Carolina, as opposite extremes in enrollment policies. I find fours facts from empirical studies. First, there is a positive correlation between the quality of public education and housing prices in both states. This correlation becomes stronger in a state without open enrollment, relative to a state with open enrollment. Second, housing prices increase with the number of better schools far from home in a state with open enrollment. These observations are consistent with the features of open enrollment to provide more public school options and expand the educational marketplace geographically. Since open enrollment weakens the connection between housing prices and the quality of public education, I explore how parents and students are sorted by income levels depending on open enrollment. I find that there is a negative correlation between the Gini coefficient and the quality of public education in a state with open enrollment. Finally, there is a positive correlation between the Gini coefficient and the existence of a private school in a school district in a state with open enrollment.

I then build an overlapping generations model with altruism and discrete choices for residential location and school choice. In a state without open enrollment, an altruistic parent chooses residential location and her child's school to maximize the sum of her own utility and child's future wage, which depends on her own income, child's ability, and child's education quality. In a state with open enrollment, an altruistic parent has an additional option for her child to commute to a public school outside of her resident district.

Among many comparative statics, I focus on how the Gini coefficient and rent change as education quality, private school tuition, and transportation cost change between states with and without open enrollment. Main results from comparative statics are the following. First, as public education in a "bad" school district improves, income levels are more evenly distributed in a state without open enrollment, while income levels are less evenly distributed in a state with open enrollment. Rents in a "good" school district decrease in both states with and without open enrollment. However, the rent gap across states decreases as public education in a "bad" school district improves. Second, as a private school becomes more expensive to attend, income levels are more evenly distributed in both states with and without open enrollment, though the gap between income distributions get wider across states. Rent in a "good" school district in a state with open enrollment decreases in private school tuition, while that in a state without open enrollment increases in private school tuition. Third, as transportation cost for commute increases in a state with open enrollment, in a "good" school district, income levels become more evenly distributed and rent increases.

This paper is organized as follows. Section 2 introduces relevant studies. Section 3 reports some facts about housing prices and income inequality across states with and without open enrollment. Section 4 develops a model and section 5 exhibits comparative statics based on the model. Section 6 concludes this paper.

2.2 Literature review

Though open enrollment is designed to offer socioeconomically disadvantaged students access to better education opportunities, some evidence shows that take-up rates are small for them due to barrier to full adoption. Higher-income students are more likely to enroll in schools outside their assigned district (Orfiled and Luce (2012), Lavery and Carlson (2015)). Most transfers also take place between districts with relatively high achievement (Carlson et al. (2011)). Lavery and Carlson (2015) find that there is substantial variation in open enrollment participation across demographic groups and grade levels. Unlike Lavery and Carlson (2015) which studied open enrollment in Colorado, Cowen and Creed (2015) find that historically disadvantaged students in Michigan are more likely to participate in open enrollment, however, they are also the most likely to return to their assigned district. In a state with voluntary inter-district open enrollment, school districts consider various factors to adopt open enrollment policies. Brasington et al. (2016) show that demographic characteristics, financial factors, and competitive factors are crucial factors for determining the adoption of inter-district open enrollment policies by school districts in Ohio. Lenhoff (2020) finds that the state school funding formula and the segregated geography of Metro Detroit restrict access to black and economically disadvantaged students in open enrollment.

In survey data, Tenbusch (1993) finds that parents choose a nonresident school for their children due to dissatisfaction with their resident school's educational services and/or administration. He also finds that parent's education level is an influential factor for open enrollment participation. Witte et al. (2008) analyze open enrollment patterns and trends in Minnesota and Colorado and find that students participate in open enrollment based on the socioeconomic characteristics and academic performance of school districts. For example, students in a school district with high percentage of free lunch are more likely to transfer out to a school district with low percentage of free lunch. Fossey (1994) also finds similar results using open enrollment data in Massachusetts. Academic performance also motivates open enrollment participation. Students who did not well academically in the years leading up to their transfer participate in open enrollment in the expectation of academic achievement (Cowen and Creed (2015), Carlson et al. (2018)), though they also quit the program easily. Students who participate in open enrollment as a long-term education option tend to transfer from a disadvantaged district to an advantaged district, relative to students who transfer out temporarily (Carlson et al. (2018)). Students who transfer out temporarily consider family-related issues rather than academic achievement. Ysseldyke et al. (1994) shows that students with disabilities participate in open enrollment looking for special education and more personal attention from the teacher.

The effect of open enrollment on students' academic performance seems ambiguous. Miron et al. (2008) examine and compare nine separate studies on the effect of open enrollment on students' academic performance and find four positive effects, three mixed effects, one slightly negative effect, and one very negative effect. Some studies stress that open enrollment brings the positive effect of competition with another school on students' achievement through systematic changes. (Akey et al. (2009)). Leidwith (2010) finds that increased transfers relevant to open enrollment as well as the wealth of the students' residential neighborhood have a positive effect on the academic performance of students in Los Angeles. Babington and Welsch (2017) find that students transferring out to a better school in Minnesota have higher reading test scores in the subsequent year. Academic performance also depends on the stability of participation in open enrollment. Student who stably participate in open enrollment show small achievement gain, while students who exit the program quickly show small declines in their achievement (Carlson et al. (2018)). On the contrary, several studies report negative or insignificant effect of open enrollment on students' academic performance (Hong and Choi (2015), Lai et al. (2009), Ozek (2009)). One of possible explanations for the negative effect of open enrollment on academic performance is parents' judgment error relevant to parents' education level and less attention to teachers' opinions (Lai et al. (2009)). They find that parents with children in primary schools made more judgment errors. Rothbart (2020) focuses on the change in budget ratio between non-instructional and instructional services within schools. He finds that open enrollment leads to greater expenditures on non-instructional functions rather than instructional expenditure to attract students, which negatively affects students' academic performance.

Though schools are not the only determinant of home prices, home prices are significantly related to test scores and the quality of public schools. Calder (2019) finds that the average zip code associated with the highest quality public elementary school has a 4-fold higher median home price than the average neighborhood associated with the lowest quality public elementary schools. The positive correlation between school quality and housing prices weakens when there are more school choices such as private schools, alternative schools in proximity, and school choice program. (Fack and Grenet (2010), Reback (2005), Schwartz et al. (2014), Calder (2019)). Though parents are willing to pay a premium for school quality when constrained to a specific school based on location, this premium diminishes when the catchment area restrictions are removed (Machin and Salvanes (2010)). The effect of open enrollment on housing prices differs by districts. Generally, residential property values decline in districts that accept transfer students and increases in districts that students transfer out to better districts (Brunner et al. (2012), Reback (2005)).

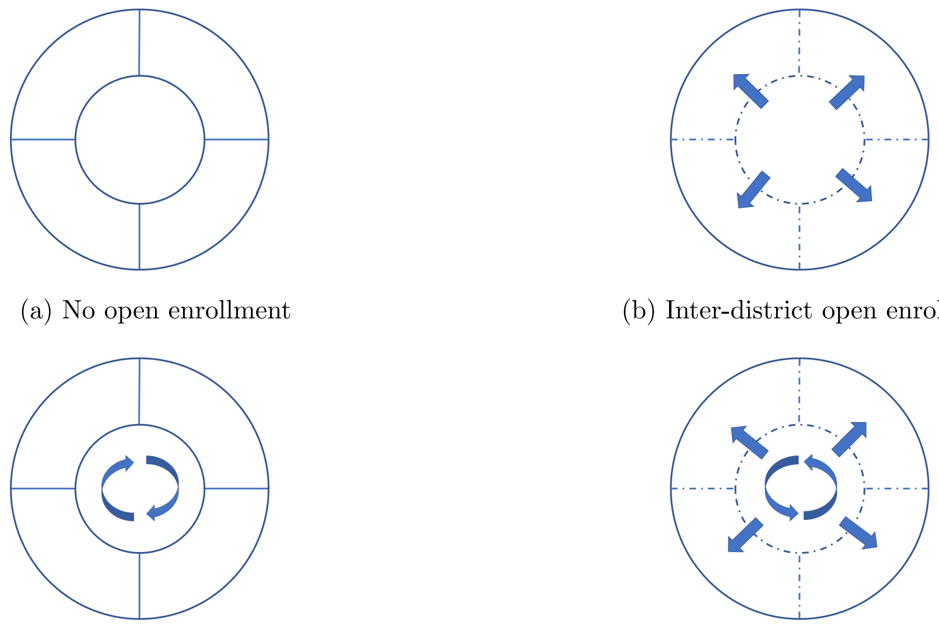
Initiated by Tiebout (1956), many studies have explored residential segregation or sorting associated with public schools and local spillovers (Fogli and Guerrieri (2018), Durlauf (1996), Benabou (1996a, 1996b), Fernandez and Rogerson (1996), Zheng (2017)).

Some studies explicitly focus on segregation or sorting relevant to open enrollment. Epple and Romano (2003) emphasize differences in attributes of students and their households as the result of sorting within district. The combination of a public system with a private school market yields the least residential segregation. The reason is that middle and high-income households whose children attend private school live with lower-income families whose children attend public school (Nechyba (2003)). At higher voucher funding levels, both income segregation and the capitalization of public school quality into housing prices are reduced, relative to a base of no vouchers offered. Ferreyra (2007) extends Nechyba's analysis for economies with multiple public school districts and private schools. Through the simulation of two large-scale voucher programs such as universal voucher and nonsectarian voucher, she finds that both voucher programs affect private school enrollment and housing prices.

2.3 Some facts

There are two types of open enrollment. The first type is intra-district open enrollment, which allows students to transfer to another school *within* their resident school district. The other type is inter-district open enrollment which allows students to transfer to a school *outside* of their resident district. Figure 2.1 describes the flows of student transfers in open enrollment policies. Within either intra-district or inter-district open enrollment, *mandatory* open enrollment requires school districts to participate in the program. *Voluntary* open enrollment allows school districts to choose whether to participate in open enrollment. States with mandatory and voluntary open enrollments usually require mandatory open enrollment in low-performing schools or districts while allowing voluntary open enrollment in the state.

I explore how housing prices and income inequality change with the quality of public



(a) No open enrollment

(b) Inter-district open enrollment

(c) Intra-district open enrollment

(d) Both open enrollments

Figure 2.1: Flow of student transfers in open enrollment

education in Arizona and North Carolina. As Table 2.1 shows, Arizona and North Carolina are in the opposite extreme in terms of school choices in open enrollment. Arizona is one of the states that allow for the most flexible school choices. Arizona has a mandatory inter-district and intra-district open enrollment. It does not even allow voluntary inter-district and intra-district open enrollment. Furthermore, schools accepting students from different districts are responsible for students' transportation to commute to the schools. In another words, public school choices in Arizona are not tied to residential location.

On the other hand, both inter-district and intra-district open enrollment are neither voluntary nor mandatory in North Carolina. Students in North Carolina cannot attend a public school outside of their resident school district.

I collect public school ratings of public high schools from greatschools.org. It offers

Table 2.1: Open enrollment in the US

		Inter-district			
	Voluntary /Mandatory	No/No	Yes/No	Yes/Yes	No/Yes
Intra-district	No/No	AL, MD, NC	KS, NV, NJ NY, ND, OR PA, SC	MS, MO, OK	IA, MN
	Yes/No	DC, VA	ME, MA, NH RI, TN, TX WV, WY	CT, MT	WI
	Yes/Yes		MI	CA, NM	
	No/Yes	AK	GA, KY, WA	IN, LA, OH	AZ, CO, DE FL, ID, NE SD, UT, VT

comprehensive school ratings from a one to ten scale for all public schools in the United States reflecting academic progress, college readiness, equity, and test scores. Since it does not offer school ratings of private schools, I consider the existence or the number of private schools to investigate how private schools affect housing prices and income inequality. Using home sale listings data from Realty Mole Property, I match the latitude and longitude of homes to school attendance boundary data collected by National Center for Education Statistics (NCES). Since school attendance boundary is based on survey data and changes every year, a home could be matched to multiple school attendance boundaries. For homes with multiple matches, I use mean, minimum, and maximum of school ratings within matches to measure the quality of public education. In case of no match, I use all public schools within five miles from homes. I also obtain economic and demographic data by school districts from NCES.

According to descriptive statistics regarding high schools, Arizona and North Carolina have similar percentages of public high schools (Table 2.2). The numbers and ratings of schools come from Greatschool.org as of 2020. In Arizona, the percentage of charter schools is about twice of that of private schools. In North Carolina, the percentage of private schools dominates that of charter schools. On average, North Carolina has better public education than Arizona. However, the disparity in the quality of public education between schools is larger in North Carolina than Arizona.

Table 2.2: High schools in Arizona and North Carolina

	Arizona	North Carolina
Number of school districts	136	116
Number of high schools	939	1061
Number of charter school	257(27%)	66(6%)
Number of private school	142(15%)	399(38%)
Number of public school	540(58%)	596(56%)
Average ratings of charter and public schools	4.92(2.43)	5.19(2.45)
Average ratings of charter schools	5.05(2.97)	5.33(2.3)
Average ratings of public schools	4.82(1.93)	5.17(2.48)

First, I investigate the correlation between housing prices and the quality of public education in Arizona and North Carolina. I estimate the following OLS regression with robust cluster errors in zip codes:

$$\begin{aligned}\log(price) = \gamma_0 + \gamma_1 \textit{Open enrollment} + \gamma_2 \textit{Open enrollment} \times \textit{School quality} \\ + \gamma_3 \textit{Open enrollment} \times \textit{Better school in distance} \\ + \gamma_4 \textit{Near private} + \gamma_5 \textit{Near charter} + X'\beta + \epsilon\end{aligned}$$

where *School quality* is the quality of public school belonging to the school attendance boundary for a home, *Better school far from home* is the number of public schools rated higher than nearby schools within 15 miles, *Near private* is equal to 1 if a private school is within 5 miles from a home, *Near charter* is equal to 1 if a charter school is within 5 miles from a home, and X controls property type, the number of bathrooms and bedrooms, square footage, and metropolitan statistical area.

For column (1), (2), and (3), I use the mean, minimum, and maximum of school ratings for the quality of public education in case there are multiple matches between a house coordinate and a school attendance boundary.

Table 2.3: The effect of open enrollment on housing prices

	Mean	Minimum	Maximum
Arizona	0.498***	0.235***	0.491***
	(0.0950)	(0.0688)	(0.0806)
North Carolina × School quality	0.105***	0.0769***	0.0542***
	(0.00952)	(0.00925)	(0.00734)
Arizona × School quality	0.0568***	0.0543***	0.0208***
	(0.00931)	(0.00727)	(0.00671)
North Carolina × Better school far from home	-0.0104	0.112	-0.00443
	(0.00822)	(0.202)	(0.00382)
Arizona × Better school far from home	0.0137***	0.00647**	0.00502**
	(0.00362)	(0.00274)	(0.00239)
North Carolina × Near private	-0.0851***	-0.0666**	-0.117***
	(0.0316)	(0.0323)	(0.0344)
Arizona × Near private	0.0274	0.0461*	0.0360
	(0.0237)	(0.0250)	(0.0253)
Near charter	-0.0253	-0.00918	-0.0248
	(0.0220)	(0.0244)	(0.0243)
MSA FE	yes	yes	yes
F-value for School quality	13.02(0.0003)	3.78(0.0524)	10.91(0.0010)
F-value for Better school far from home	7.15(0.0077)	0.27(0.6013)	4.35(0.0375)
F-value for Near private	7.96(0.0049)	7.50(0.0063)	12.65(0.0004)
Observations	22648	22648	22648

Standard errors in parentheses
* $p < .10$, ** $p < .05$, *** $p < .01$

Fact 1: There is a positive correlation between the quality of public education and housing prices in both states. The positive correlation is more robust in North Carolina than in Arizona. From Table 2.3 in column (1), when the average public high school rating within a school attendance zone increases by a unit, housing prices increase by 5.8% in Arizona, while it increases by 11.1% in North Carolina. The difference between the effects of the quality of public education on housing prices in Arizona and North Carolina is statistically significant at the 0.05 level. When the minimum and maximum public school rating within a school attendance zone increases by a unit, in Arizona, housing prices increase by 5.6% and 2.1%, respectively. In contrast, they increase by 8% and 5.6%, respectively, in North Carolina. However, in these cases, the differences between Arizona and North Carolina are not statistically significant at the 0.05 level.

Fact 2: There is a positive correlation between better schools far from home and housing prices in Arizona. However, the correlation between them is statistically insignificant in North Carolina. From Table 2.3 in column (1), when the average school ratings of better schools far from home increases by a unit, housing prices increase by 1.4% in Arizona. Housing prices increase by 0.6% and 0.5%, respectively, when the minimum and maximum school rating of better schools far from home increase by a unit in Arizona. However, better public schools far from home are statistically insignificant on housing prices in North Carolina. The differences in the effect of better schools far from home on housing prices in Arizona and North Carolina are statistically significant except when the quality of better schools far from home is measured in the minimum rating of better schools.

Next, I estimate the following regression equation through logit estimation to explore how income inequality relates to the quality of public education in Arizona and North

Carolina. Each variables are measured in a district level. The Gini coefficient is an index to measure the magnitude of income inequality ranging from 0 (Perfect equality) to 1 (Perfect inequality). I estimate the Gini coefficient for grouped data following Tille and Langel (2012). From column (1)-(3) in Table 2.4, I add control variables.

$$\begin{aligned} \textit{Gini index} = \gamma_0 + \gamma_1 \textit{Open enrollment} + \gamma_2 \textit{Open enrollment} \times \textit{Quality of education} \\ + \gamma_3 \textit{Open enrollment} \times \textit{Private schools} + X'\beta + \epsilon \end{aligned}$$

where *Open enrollment* is equal to 1 if state is Arizona, *Quality of education* is the average ratings of public schools within school district, *Private schools* is the number of private schools within school district, and X controls labor participation rate, sex, race, and ethnicity.

Fact 3: There is a negative correlation between the Gini coefficient and the quality of public education in Arizona. However, in North Carolina, the correlation is statistically insignificant. From Table 2.4 in column (3), after controlling sex, race, and ethnicity, when the quality of public education within a school attendance zone increases by a unit, the Gini coefficient decreases by 0.005 and is statistically significant. However, it is statistically insignificant in North Carolina. In Arizona, parents who cannot afford higher housing prices due to better public education are more willing to move to a school district with cheaper houses since their children can still commute to the current school from a new cheaper house. However, in North Carolina, parents are more reluctant to move to a worse school district and willing to sacrifice other than housing expenditure for better education. As a result of open enrollment, the households' income levels become more evenly sorted in a school district in a state with open enrollment.

Table 2.4: The effect of open enrollment on income inequality

	(1)	(2)	(3)
Arizona	-0.0493***	-0.0250	-0.0248
	(0.0177)	(0.0162)	(0.0165)
North Carolina × Quality of education	-0.00869***	-0.00193	-0.00193
	(0.00263)	(0.00255)	(0.00258)
Arizona × Quality of education	-0.00821***	-0.00452**	-0.00452**
	(0.00232)	(0.00214)	(0.00216)
North Carolina × Private school	0.000429	-0.0000127	-0.0000100
	(0.000582)	(0.000525)	(0.000529)
Arizona × Private school	0.00273**	0.00238**	0.00239**
	(0.00114)	(0.00102)	(0.00103)
Sex	yes	yes	yes
Race		yes	yes
Ethnicity			yes
Observations	197	197	197
adjusted R^2	0.251	0.415	0.410

Standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Fact 4: There is a positive correlation between the Gini coefficient and the existence of a private school in a school district in Arizona. However, in North Carolina, the correlation is statistically insignificant. From Table 2.4, districts with private schools in Arizona have a higher Gini index by 0.002, relative to districts with no private school. Like open enrollment, private schools provide more education opportunities. However, due to high tuition, the opportunities are restricted to students whose parents can afford them. That is, private schools attract wealthy parents to a school district. The net effect of private schools and open enrollment in Arizona on the Gini coefficient is positive, weakening spatial sorting by income levels.

2.4 Model

Nechyba (2003) develops a general equilibrium model to study residential segregation between a school district with public schools only and a school district with both private schools and public schools. In his model, parents choose (i) where to live, (ii) whether to send their child to the local public school or a private school, and (iii) how to vote in local or state election on the level of per pupil school spending. He also explored the effect of school voucher program on income and property value across districts. Ferreyra (2007) extends the Nechyba's analysis with multiple public school districts and privates schools including both religious and nonsectarian schools and simulate two large-scale private school voucher program. Based on Nechyba (2003) and Ferreyra (2007), I build an overlapping generations model with one-sided altruism from a parent to a child to investigate how income distributions and housing prices change in the quality of public education, private school tuition, and transportation cost. I abstract from how to vote in local or state election on the level of per pupil school spending. Instead, altruistic parents in a state with open enrollment choose whether to enroll a school outside their residential districts. Before presenting comparative statics, I investigate how a parent's income and her child's ability determine residential location and school choice in the model and whether the model can generate empirical observations in the previous section.

In the model, an individual lives for two periods, child and parent. A parent maximizes the sum of her own utility and her child's future income. The child's future income depends on her parent's income, her ability, and her education quality. I assume that the effect of education quality on income differs by ability. I also assume that commuting to a school far from home reduces the efficiency of education.

I separately consider a state with open enrollment and one without it. Each state has two school districts with "good" public education and "bad" public education. I assume that parents cannot move across states. In a state with open enrollment, a parent in a "good" school district G chooses between enrolling a public school and a private school. A parent in a "bad" school district B has the choice of either enrolling between a public school and a private school or commuting to a public school in district G. A parent in district B in a state without open enrollment cannot commute to a public school in district G. I rank the quality of education increases in the increasing order of a public school in district B, a public school in district G, a private school in district B, and a private school in district G. Finally, housing supply is assumed to be fixed at H in district G and inelastically supplied at a very large quantities in district B. Thus, all residents in district G pay a positive amount of rent until the housing market clears, while residents in district B do not pay rent.

In a state with open enrollment, an altruistic parent in district A chooses between enrolling in a public school and a private school by solving

$$V_G = \max\{V_G^D, V_G^P\}$$

where V_G^D is the value of attending a $\boxed{\text{public school in G}}$ such that

$$V_G^D = \max_c \{u(c) + g(y')\} \tag{2.1}$$

$$\text{subject to} \quad c + R^A \leq y$$
$$y' = \Omega(y, a, e)$$

and V_G^P is the value of attending a $\boxed{\text{private school in G}}$ to maximize (1)

$$\text{subject to} \quad c + R^A + \underbrace{\tau}_{tuition} \leq y$$
$$y' = \Omega(y, a, e)$$

For each value function, a parent chooses consumption c given rent R^G and wage y. The child's future income y' depends on her parent's income y, child's ability a, and child's education quality e.

In a state with open enrollment, an altruistic parent in district B chooses between enrolling in a public school and a private school or commuting to a public school in district G by solving

$$V_B = \max\{V_B^D, V_B^P, V_B^C\}$$

where V_B^D is the value of attending a $\boxed{\text{public school in B}}$ to maximize (1)

$$\text{subject to} \quad c \leq y$$
$$y' = \Omega(y, a, e)$$

,V_B^P is the value of attending a $\boxed{\text{private school in B}}$ to maximize (1)

$$\text{subject to} \quad c + \underbrace{\tau}_{tuition} \leq y$$

$$y' = \Omega(y, a, e)$$

,V_B^C is the value of commuting to a $\boxed{\text{public school in G}}$ to maximize (1)

$$\text{subject to} \quad c + \underbrace{T}_{transportation} \leq y$$

$$y' = \Omega(y, a, e)$$

Parents in a state with open enrollment chooses to reside in school district G if

$$V_G > V_B$$

Otherwise, they chooses to reside in school district B.

In a state without open enrollment, parents decide their residential location and school type by solving

$$V = \max\{V_G, V_B\}$$

where

$$V_G = \max\{V_G^D, V_G^P\}$$

$$V_B = \max\{V_B^D, V_B^P\}$$

for the same value function in a parent's problem in a state with open enrollment.

For utility function, I employ the following logarithmic function.

$$u(c) = log(c + \bar{c})$$

where $\bar{c}$ is the constant autonomous consumption.

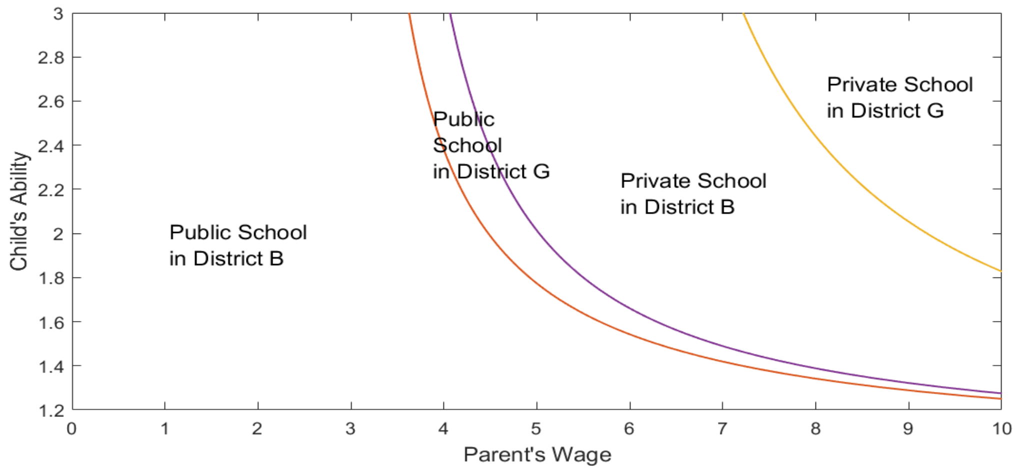
For the child's future income, I assume the following Cobb-Douglas function

$$y' = (a^e)^{1-\alpha} * y^\alpha$$

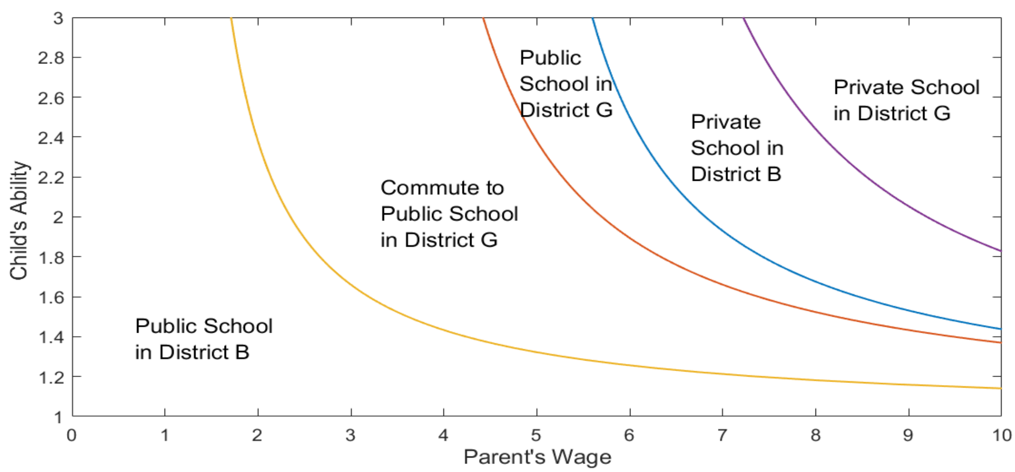
That is, the return to education depends on the child's ability. In Figure 2.2, I characterize a parent's residential location and school choice on the plane of her wage and her child's ability across states with and without open enrollment. I compare the value functions across options that parents can choose. Though a parent's wage is very high, for example, a parent chooses to live in a "bad" school district and enroll in a public school for her child if her child's ability is very low. In a state with open enrollment, a large part of children supposed to attend a public school in a "bad" school district in a state without open enrollment commute to a public school in a "good" district. They are either students with high ability whose parents have medium-income level or students with low ability whose parents have high-income level. In a state without open enrollment, some parents with low-income level choose to reside in a "good" school district if their children have high ability. However, they decide to reside in a "bad" school district in a state with open enrollment, since their talented children still can commute to a public school in a "good" school district. Change in their residential location choices across open enrollment policies lead to difference in income distribution in a "good" school district across states in Figure 2.3.

2.5 Comparative statics

In this section, I investigate how the Gini coefficient and rent in a district with good public education change in response to changes in the quality of public education in a



(a) Residential location and school choices in a state without open enrollment



(b) Residential location and school choices in a state with open enrollment

Figure 2.2

bad school district, private school tuition, and transportation cost.

2.5.1 Quality of public education in a bad district

As Figure 2.4 shows, as the quality of public education in a bad school district increases, the Gini coefficient in a good school district in a state without open enrollment decreases, while the Gini coefficient in a good school district in a state with open enrollment is stable or slightly increases. In a state without open enrollment, higher quality

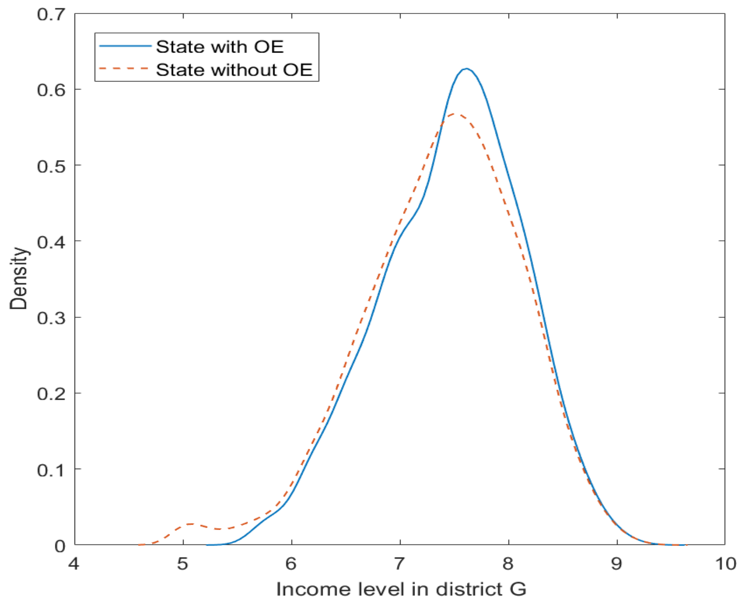
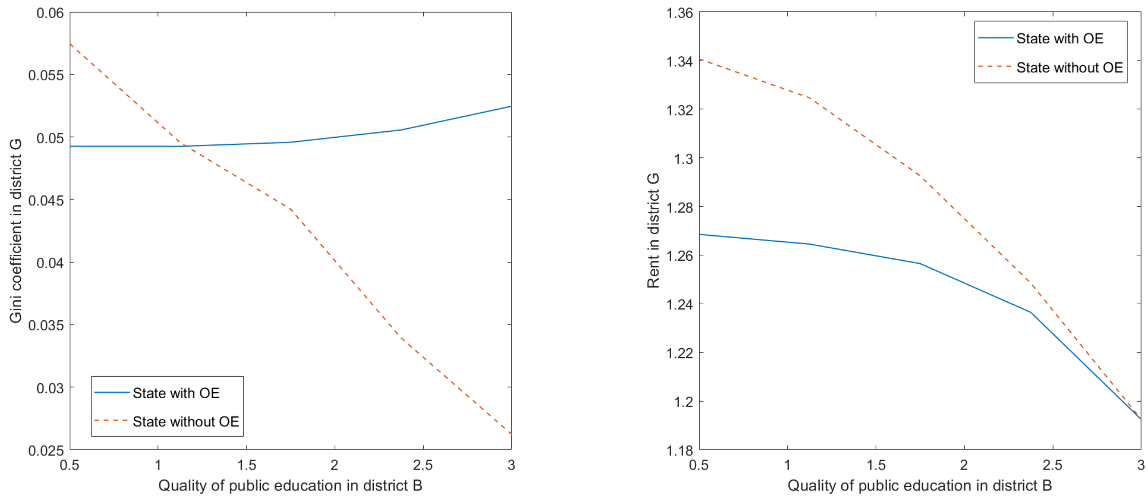


Figure 2.3

of public education in a bad school district means that relatively poor parents with talented children do not need to move out to a good school district to enroll a better school. Thus, income distribution by parents in a good school district becomes narrower, which leads to a smaller Gini coefficient. As the quality of public education in a bad school district increases, a rent in a good school district decreases in both states with and without open enrollment. However, the rent gap across states with and without open enrollment decreases, since the disparity in education qualities between good and bad districts decreases.

2.5.2 Private school tuition

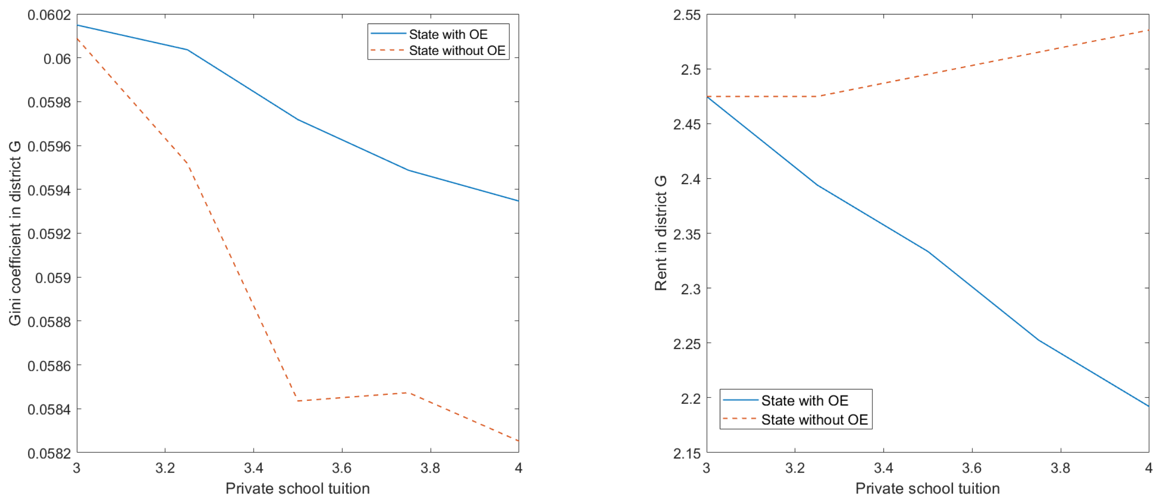
As Figure 2.5 shows, as private school tuition increases, the Gini coefficient in a state with open enrollment and without open enrollment both decrease, though it decreases more rapidly in a state without open enrollment. Private school is an alternative option



(a) (b)

Figure 2.4

to offer high quality of education to students. However, due to higher tuition, it is less approachable than before. Some parents whose children attended a private school in a bad district cannot afford private school tuition anymore. They choose to move to a good school district to attend a good public school. Then, some parents whose children attended a public school in a good school district move to a bad school district, since they cannot afford higher rent due to higher demand of houses in a good school district. As a result, income segregation becomes clearer, and the Gini coefficient decreases in both states. This phenomenon happens more clearly in a state without open enrollment, since there is no option to commute to a public school in a good school district by children who attended a private school in a bad district. Thus, the Gini coefficient decrease more rapidly in a state without open enrollment. Due to higher demand on homes, a rent in a good district in a state without open enrollment increases. Some parents whose children attended a private school in a good district move to a bad district to attend a private school in a bad district, since a private school in a bad district still offers better education than a public school in a good district. Thus, a rent in a good school district decreases



(a) (b)

Figure 2.5

due to lower demand on homes.

2.5.3 Transportation cost

Since transportation cost is relevant only to a state with open enrollment, both Gini coefficient and a rent in a good school district in a state without open enrollment do not change in transportation cost. As Figure 2.6 shows, as transportation cost increases, the Gini coefficient in a good school district decreases in a state with open enrollment. Higher transportation cost implies the discrepancy between transportation cost and a rent in a good school district decreases. Thus, some parents whose children commuted to a public school in a good district choose to move to a good school district and some parents who cannot afford rent in a good school district move to a bad school district. As a result, income segregation becomes clearer, and the Gini coefficient decreases. Due to higher demand on homes in a good school district due to higher transportation cost, a rent in a good school district increases.

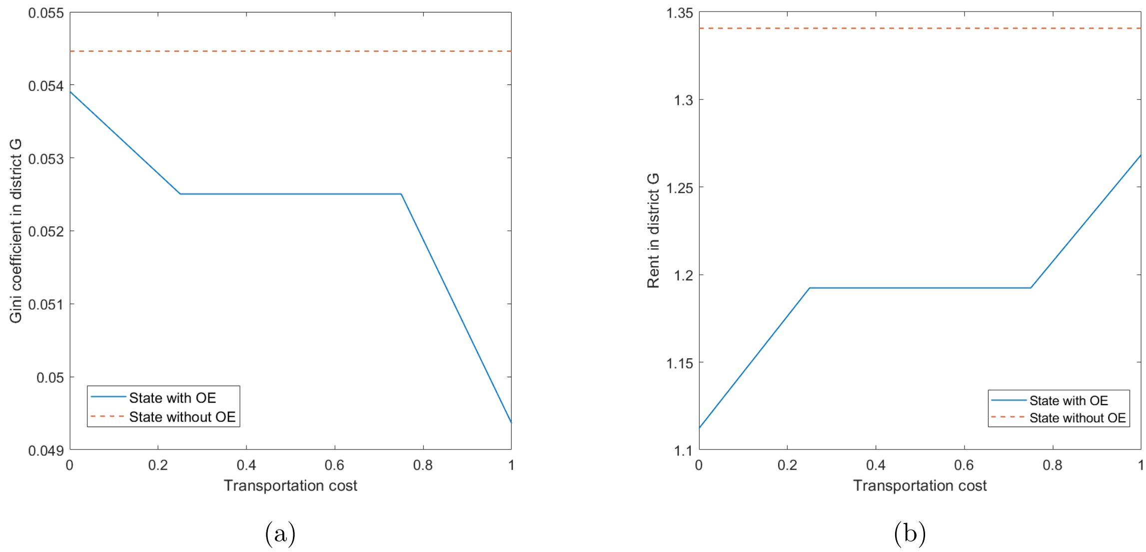


Figure 2.6

2.6 Conclusion

This paper explores the effect of school choices on housing prices and income inequality. Specifically, I investigate how open enrollment affects a parent's residential location and school choice. Based on empirical observations regarding housing prices and income inequality associated with open enrollment, I build an overlapping generations model to mimic the empirical observations. Then, I exhibit rental prices and income inequality over changes in the quality of public education in a bad school district, private school tuition, and transportation cost. This paper contributes to the literature by extending Nechyba (2003)'s model through adding an option to commute to a public school outside of resident district. Future research can consider both local spillover effect as well as open enrollment as determinant of residential location and school choices in a general equilibrium.

Chapter 3

Spatial Sorting: Commercial Gentrification

3.1 Introduction

Commercial gentrification is a process in which more profitable businesses displace small businesses in a disinvested neighborhood. Higher rents caused by various reasons result in the displacement of small businesses in a gentrifying neighborhood. There are two stages in the process of commercial gentrification (Smith, 2006). First, some business owners open their establishments in a disinvested neighborhood, searching for lower rent. Second, if the neighborhood becomes crowded with customers, new businesses enter the neighborhood, searching for a higher profit. After the second stage, rents in a gentrifying neighborhood increases, and some indigenous business owners cannot afford the rent and leave the neighborhood, searching for a lower rent.

This paper studies how the composition of businesses of different qualities changes between a gentrifying and non-gentrifying neighborhood in response to higher rent. I introduce a search and matching framework with two heterogeneous neighborhoods, rents, and search friction building on Acemoglu (2001). To capture the fact that customers head for a gentrifying neighborhood to consume high-quality goods and services, I introduce search friction between customers and businesses in a gentrifying neighborhood. Due to search friction in high-quality retail, customers pay more than a competitive price for goods and services. However, customers pay less in low-quality retail since customers are reluctant to visit low-quality retail in a gentrifying neighborhood.

Most of the literature on retail location decisions relies on either price or non-price competition between firms. In a Hotelling's firm location decisions model, two firms compete with each other, given consumer density, operation costs, and transportation costs in a linear city. Chamorro-Rivas (2000), Karamychev and Van Reeven (2009), and Pal (1998) construct models of firm competition in a game-theoretic framework. Unlike the previous literature, I focus on consumer competition for retail location decisions. Competition between consumers for high-quality goods and services in a gentrifying neighborhood generates positive or negative externalities in terms of prices. As the portion of high-quality retail increases in a gentrifying neighborhood, high-quality retail's total revenue increases due to a positive externality in prices. At the same time, high-quality retails have to pay higher rent by profit-sharing between a retail and a landlord. The tension between higher revenue and higher rent endogenously determines the composition of retail in the steady-state.

Since commercial gentrification exhibits different forms depending on neighborhoods' backgrounds, there is no consensus on how to measure the degree of commercial gentrification. Among empirical commercial gentrification indicators, I employ the rate of displacement for small businesses in a theoretical model. To identify commercial gentrification from real cases, I investigate four neighborhoods in Los Angeles, the United States, in Figure 3.1: Arts District, Little Tokyo, Chinatown, and Boyle Heights. Though they belong to different commercial gentrification types, I apply common indicators like the rate of displacement for small businesses, the share of business entry and exit, and businesses' churn rate over these neighborhoods using Yelp data.

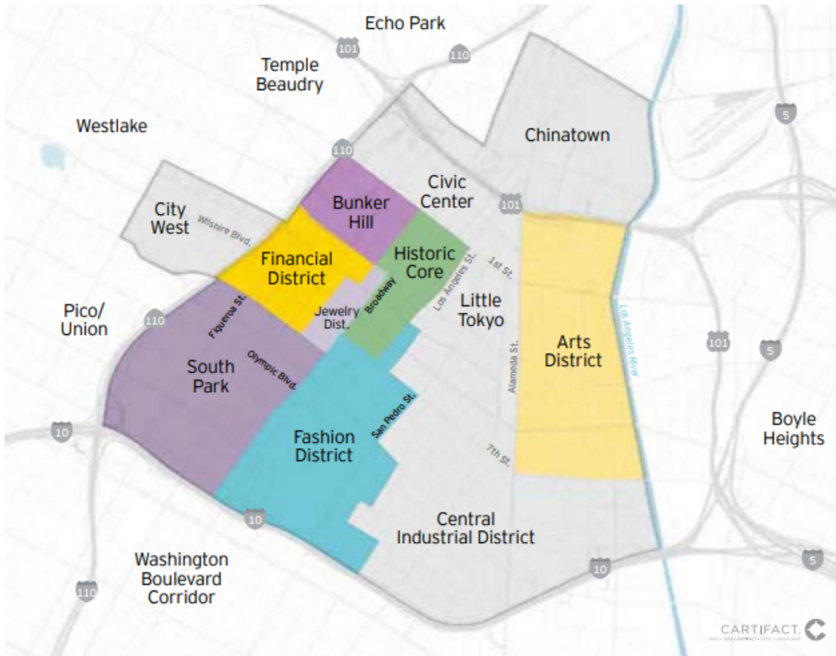


Figure 3.1: Downtown Los Angeles, the United States, Source: Cartifact

The Arts District experiences the most active commercial gentrification among four neighborhoods during the study period between 2005 and 2018. The Arts District has the highest "churn" rate of businesses, which means the portion of entering or exiting businesses over the average number of retails is the highest. Little Tokyo experiences a similar degree of commercial gentrification with the Arts District; however, businesses enter Little Tokyo slowly and exit faster than the Arts District. Compared to the Arts District and Little Tokyo, Chinatown and Boyle Heights experience a moderate commercial gentrification degree. Degrees of commercial gentrification show positive correlations with rental prices in these neighborhoods. The Arts District has the highest median rental prices among four neighborhoods. Median rental prices in Chinatown and Boyle Heights are lower than the others. Neighborhoods of active commercial gentrification also show rapid increases in customers' average evaluation of businesses.

A theoretical model is consistent with the empirical findings. A gentrifying neighborhood has a higher proportion of high-quality businesses than a non-gentrifying neighborhood. On the benchmark, the differences in the proportion of bad businesses in a gentrifying and a non-gentrifying neighborhood are 9.6 percentage points if two types of goods are gross complements and 38.67 percentage points if they are gross substitutes. Changes in rent to the proportion of good businesses show different pattern depending on whether two types of goods are complements or not. Rents in a gentrifying neighborhood increase faster up to around 50% of good businesses and then increase slower than in a non-gentrifying neighborhood if two types of goods are gross complements. This pattern is the opposite to the case of gross substitutes. Combined with the difference in the composition of businesses, changes in rent to the proportion of good businesses can explain differences in rents between a gentrifying and a non-gentrifying neighborhood. Comparative statics show that the differences in the composition of businesses and rents decrease as the magnitude of complementarity increases. However, the differences in the composition of businesses and rents increase as the magnitude of substitutability increases. Similar to the case of gross complements, changes in a firm's bargaining powers over a landlord or a worker's bargaining power over a firm rarely affect the composition of businesses. Changes in rents are more sensitive to a firm's bargaining power over a landlord in the case of gross substitutes. When the bargaining power increases from 0.5 to 0.6, rent differences decrease by 40% and 48% in the case of gross complements and gross substitutes, respectively. However, changes in rents are more sensitive to a worker's bargaining power over a firm in the case of gross complements. When the bargaining power increases from 0.5 to 0.6, rent differences decrease by 112% and 61% in the case of gross complements and gross substitutes, respectively.

3.2 Background

3.2.1 Literature review

Ruth Glass, a British sociologist, first dubbed the term 'gentrification' in 1964 to observe that socio-economically upper-class migrants displace working-class residents in some regions of London, the United Kingdom. Though she focused on residential gentrification, displacement and occupiers' composition changes also apply to commercial gentrification. Until recently, commercial gentrification has received less attention from scholars. However, scholars cannot fully understand gentrification without considering the other since both gentrifications influence each other (Chapple and Jacobus, 2009).

Unlike residential gentrification, commercial gentrification has not been recognized as a serious social problem (Zukin et al., 2009). Some scholars see commercial gentrification as a process of economic process (Cheshire, 2006). Chapple and Jacobus (2009) maintain that commercial gentrification can cause a neighborhood revitalization. Meltzer and Schuetz (2012) found that commercial gentrification improves retail access significantly in low-valued neighborhoods. From interviews with residents in gentrifying neighborhoods, Freeman and Braconi (2004) found that residents welcome diversity and improvement of retails if commercial gentrification does not lead to widespread displacement. Some displaced owner-occupiers do not oppose gentrification if they can settle in another neighborhood, sharing a part of the rent gap (Pratt, 2009; Lee, 2017).

However, owner-occupiers constitute a small portion of displacement. Though commercial gentrification offers new shopping alternatives, not all residents equally access to new retails that usually appeal to higher-income consumers (Sullivan, 2014;Zukin,2009). Though there are some debates about commercial gentrification on local employment, Neumark et al. (2008) find that Wal-Mart stores' opening has adverse effects on local employment and retail earnings. Haltiwanger et al. (2010) find that Big-Box retailing's entry hurts mom-and-pop stores if the Big-Box store of the same industry opens in the adjacent area. Still, who benefits and who loses from commercial gentrification in the long-term is not evident.

There are several types of commercial gentrification, depending on how it started. Chapple et al. (2017) divided commercial gentrification in the literature into four categories: retail upscaling, spaces of commodification, art districts, and transit-oriented districts. First, changes in consumer tastes lead to retail upscaling. The influx of middle or high-income residents in nearby neighborhoods changes the composition of retails. Zukin et al. (2009) explore the role of "boutiques" in changing retail and services composition. Second, business interests or public entities spur commercial gentrification through commodifying spaces (Hackworth, 2002). To boost consumption in a neighborhood, business interests or public entities commercialize ethnic cultures or beautify poverty for cultural tourism (Burnett, 2014; Hackworth and Rekers, 2005). Third, artists can conduct crucial roles for commercial gentrification in art districts. Low rents in a declining industrial area such as the Art District in Los Angeles attract low-income artists who need a spacious atelier. The "artist as a developer" contributes to forming the customer base by attracting other artists, consumers, and tourists (Shkuda, 2015). According to Shkuda (2013), public intervention into the areas promotes art districts' formation. Finally, transit-oriented districts stress the role of transit for the process of commercial gentrification. Cervero and Duncan (2002) and Guthrie and Fan (2013) indirectly suggest the possibility of commercial gentrification depending on rail proximity through commercial property values or commercial building permit activity. Cranor et al. (2015) found different commercial gentrification degrees from six transit-proximate neighborhoods in Los Angeles.

There is no consensus on identifying commercial gentrification. However, there are commonly used indicators for commercial gentrification. Meltzer and Capperis (2016) track whether establishment turnover or "churn" rate increases or business retention rate decreases to determine that a neighborhood experiences commercial gentrification. Ong et al. (2014) use a decline in minority-owned businesses as an indicator of commercial gentrification. After dividing establishments into four types by how necessary an establishment is with NAICS codes, Meltzer and Capperis (2016) use decreasing shares of frequent and necessary establishments or increasing shares of discretionary and infrequent establishments as an indicator. Glaeser et al. (2018) use a similar indicator to quantify neighborhood change with Yelp data. They find the strong correlation between gentrification and increases in grocery stores, cafes, restaurants, and bars. Finally, chain stores' presence and the decreasing number of small businesses can be another indicator of gentrification (Glaeser et al., 2018). Zukin et al. (2009) found that chain stores enter a neighborhood with high population density and contribute to higher rents, which long-term businesses cannot afford. Meltzer (2016) showed that chain stores replace displaced small businesses in a gentrifying neighborhood.

3.2.2 Some facts

To investigate to what extent commercial gentrification is going on in the Arts District, Little Tokyo, Chinatown, and Boyle Heights in Los Angeles, the United States, I track the share of business entry and exit, and "churn" rate of businesses over these neighborhoods. These neighborhoods are appropriate for investigation since they all belong to a typical type of commercial gentrification in the literature and geographically adjacent. Arts District shows a combination of art districts and spaces of commodification by public entities. Commercial gentrification in Chinatown or Little Tokyo is the result of tourism gentrification, ethnic packaging, and commodification of spaces (Lin, 2008; Chapple et al., 2017). Boyle Heights, a Latino enclave in Eastside Los Angeles, is

Table 3.1: Dynamics of business transformation in the four neighborhoods in Los Angeles

	Arts District	Little Tokyo	Chinatown	Boyle Heights
Number of businesses in 2005	7	64	74	118
Number of businesses in 2018	49	132	113	235
Entry	61	157	88	171
Exit	17	88	41	51
The share of entry	2.18	1.60	0.94	0.97
The share of exit	0.61	0.90	0.44	0.29
Churn rate	2.79	2.50	1.38	1.26

a type of transit-oriented districts initiated by the city's light rail transit service in 2009. Geographic proximity has the advantage to control a customer base traveling to these neighborhoods. I also collect customers' local business evaluation as a proxy of retail upscaling.

The main data source I use is Yelp data between 2005 and 2018. Yelp provides basic information about local businesses and publishes customer reviews and evaluations about businesses. If customer reviews and evaluations are not available in Yelp, I refer to customer reviews in Google, Zoomato, Opentable, or Tripadvisor.[1] For opening and closing year of businesses, I collected this information by searching for businesses' websites, newspaper or magazine archive, or social media.[2] The businesses I investigate are restaurants, bars, cafes, and bakeries. I excluded pop-ups, food trucks, street vendors, and food stands since they are less firmly attached to locations.

Table 3.1 shows the dynamics of business transformation in four neighborhoods in Los Angeles, the United States. Following Meltzer and Capperis (2016), I compute the share of business entry and exit, and the churn rate over these neighborhoods. The share of business entry or exit measures the proportion of businesses that enter into or exit a neighborhood over the average number of businesses during the time interval. The

[1] I found business evaluation on Yelp is biased upward than on Google. To get a less biased estimator on customers' business evaluations, I took the sample mean of evaluations from Yelp and Google.

[2] Due to delays or inaccuracy of business registration to the office of finance in Los Angeles, I refer to data from the office of finance only if I cannot find the exact opening or closing years from alternatives.

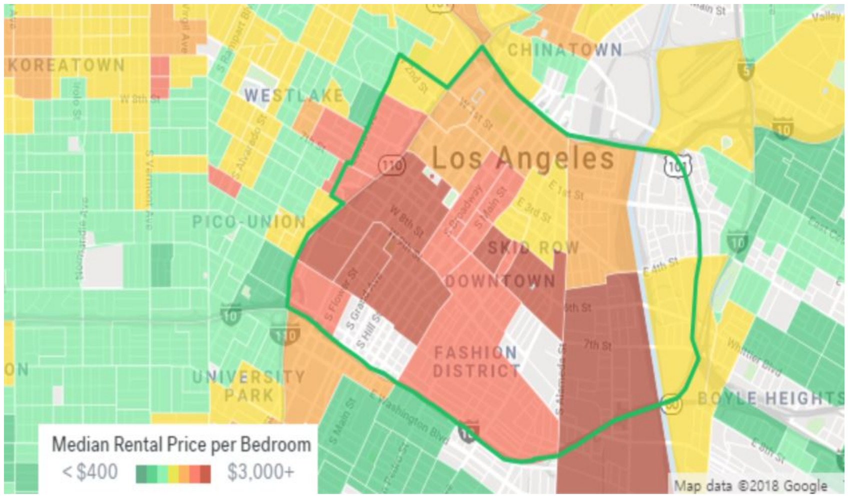


Figure 3.2: Median Rental Price in Los Angeles, Source: Trulia

"churn" rate is defined as the sum of the shares of entry and exit. These measures show how stable the composition of the business is in each neighborhood. According to these measures, the Arts District experiences the most unstable composition of businesses overall among the four neighborhoods, which can be interpreted as commercial gentrification goes on the most actively. Little Tokyo shows a similar degree of commercial gentrification with the Arts District, however, businesses enter Little Tokyo slowly and exit faster than in the Arts District. Chinatown and Boyle Heights show a similar speed of entering a neighborhood, however, the exit of businesses occurs faster in Chinatown than in Boyle Heights. As a result, Chinatown experience more active commercial gentrification than Boyle Heights in overall.

Rental prices show some positive correlations with these measures. Figure 3.2 is a choropleth map of median rental prices in Los Angeles. According to Figure 3.2, the Arts District has the highest median rental prices among the four neighborhoods. Median rental prices in Chinatown and Boyle Heights are relatively lower than the Arts District and Little Tokyo.

Figure 3.3 shows the customers' average evaluation of all businesses in the four neighborhoods over time. According to Figure 3.3, the customers' average evaluation in the Arts District starts to increase sharply from 2008. The other neighborhoods show moderate increases in customers' average evaluation after 2008. Among Boyle Heights, Chinatown, and Little Tokyo, Little Tokyo shows the fastest increase in customers' average evaluation over time. Before 2008, customers' average evaluation was stable or showed slightly decreasing trends in all neighborhoods. To Highlight changes in the composition

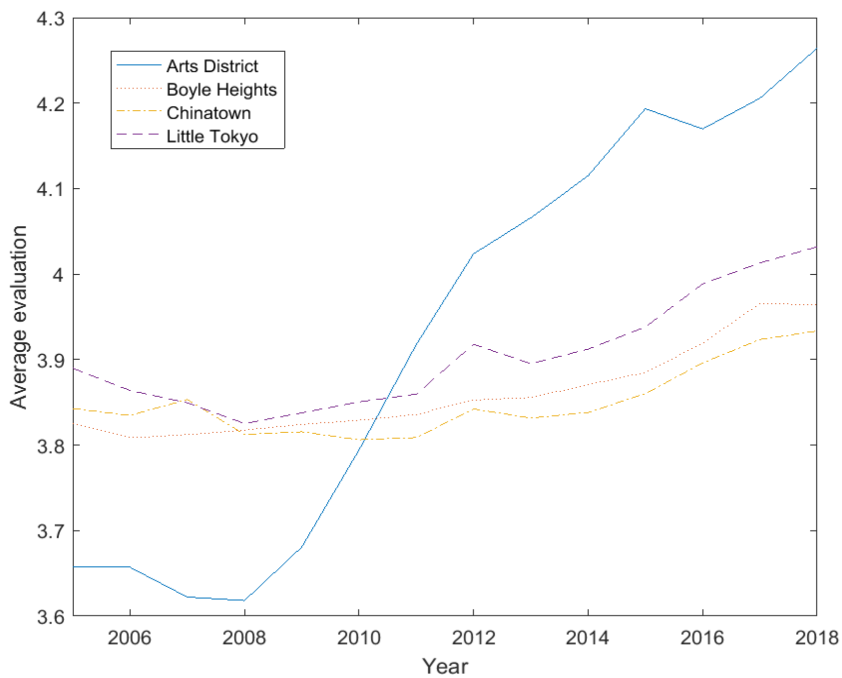
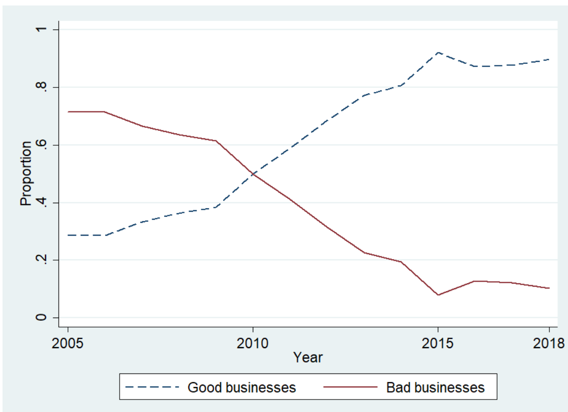


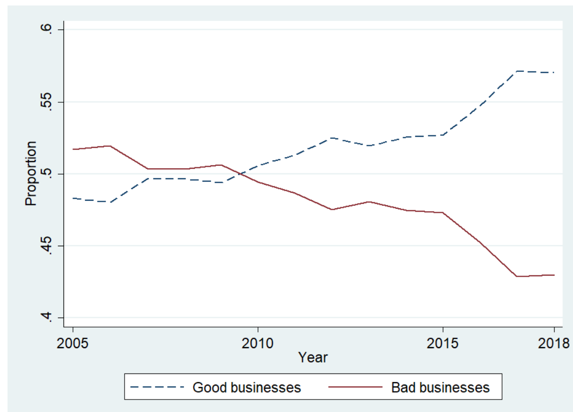
Figure 3.3: Average customers' evaluation from the Yelp

of highly and lowly-evaluated businesses in these neighborhoods, I divide the business into two types, good and bad, and graphed the changes in composition over time. From the Yelp data, I define a good business as a business that exceeds the average evaluation of four and a bad business that is less than four. Figure 3.4 shows the proportion of good and bad businesses in these neighborhood between 2005 and 2018. In the Arts District and Boyle Heights, the proportions of good and bad businesses are reversed around

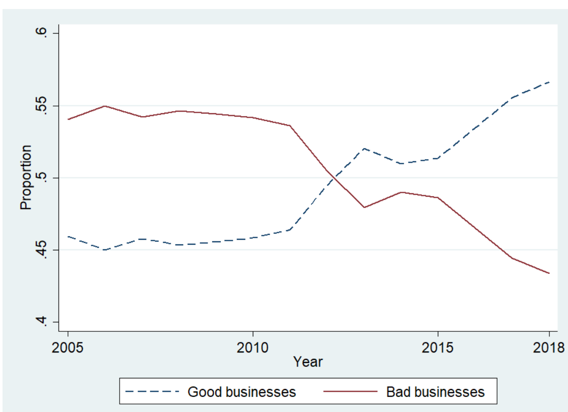
2010. However, in Chinatown and Little Tokyo, the reversion happened later around 2013. Consistent with Figure 3.3, in the Arts District, the proportion of good businesses increased the most rapidly and the ratio of good businesses and bad businesses became about four in 2018.



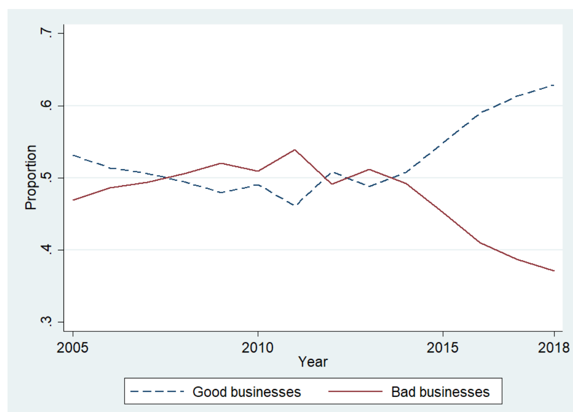
(a) Arts District



(b) Boyle Heights



(c) Chinatown



(d) Little Tokyo

Figure 3.4: Proportion of good and bad businesses

To investigate which businesses contribute most to the trend of average evaluations in four neighborhoods, I decompose the trend of customers' average evaluation into that of entering, exiting, and staying businesses. Customers' average evaluation of exiting businesses is more volatile than the others since the reasons for closing businesses are various such as less profitability, retirement, sick leave, or family issues. Unlike exiting businesses, the trends of average evaluations of entering or staying businesses show clear patterns. Customers' average evaluations of staying businesses in Boyle Heights, Chinatown, and Little Tokyo are fairly stable. The average evaluations of entering and staying businesses in the Arts District all increase the most rapidly over time. Average evaluation of entering businesses in Boyle Heights also increases rapidly over time. The other neighborhoods show moderate increases in average evaluations of entering businesses. These decomposition implies that entering businesses contribute to the increasing trends of average business evaluation in Boyle Heights, Chinatown, and Little Tokyo. In the Arts District, both entering and staying businesses contribute to the increasing trends of average business evaluation.

3.3 Model

This section builds on Acemoglu (2001). I introduce two heterogeneous areas, rents, and search frictions, to explain commercial gentrification and rent differences between a gentrifying and a non-gentrifying neighborhood through a search and matching framework.

3.3.1 Environment

Time is continuous. There are two types of neighborhoods, a gentrifying or a non-gentrifying neighborhood. I assume that only customers freely travel between two neighborhoods. There is a single firm in each neighborhood. A firm opens N vacancies for good or bad businesses. Before filling a vacancy, a firm pays training and preparation costs for each type of business. Then, a firm announces job postings to fill vacancies. At the same time, unemployed workers randomly search for a job. That is, a worker does not distinguish the types of business for a job. A successful match between a worker and a business produces a single good. While a good business produces a high-quality good, a bad business produces a low-quality.

After matches are made, a firm rents spaces for businesses from a sole landlord in each neighborhood and pays rents to the landlord. A firm pays the same rent for each space in the same neighborhood regardless of the type of business. There exists search frictions between customers and businesses to make a demand price different from a competitive price in a gentrifying neighborhood. The search frictions make the demand price of a high-quality good higher than a competitive price. Meanwhile, the search friction makes the demand price of a low-quality good lower than a competitive price. These search frictions capture the fact that consumers heading to a gentrifying neighborhood for a high-quality good are eager to consume in a good business but are more reluctant to consume in a bad business. Some matches end at an exogenous separation rate.

3.3.2 Preference and profits

Customers derive utility from two types of goods. Their utility function is

$$U(Y_b, Y_g) = Q(\alpha Y_b^{\rho} + (1-\alpha) Y_g^{\rho})^{1/\rho}$$

where Y_g and Y_b are the consumptions of a high and a low-quality good, α is a weight on the consumption of low-quality goods, Q is a constant, and $\frac{1}{1-\rho}$ measures substitutability between high and low-quality goods.

Competitive prices of high-quality ($p_{g,n}$) and low-quality ($p_{b,n}$) goods in neighborhood n are equal to the marginal utilities of high and low-quality goods. That is,

$$\begin{aligned} p_{g,n} &= (1-\alpha) Y_{g,n}^{\rho-1} Y_n^{1-\rho} \\ p_{b,n} &= \alpha Y_{b,n}^{\rho-1} Y_n^{1-\rho} \end{aligned} \tag{3.1}$$

where $Y_n = (\alpha Y_{b,n}^{\rho} + (1-\alpha)Y_{g,n}^{\rho})^{1/\rho}$

A firm chooses the proportions of good and bad businesses among N vacancies to maximize profits. A firm earns profits from filled vacancies. In gentrifying neighborhood G, profits from good and bad businesses are

$$\pi_{g,G} = (1+\delta f(\xi_G))p_{g,G} - w_{g,G} - R_G$$
$$\pi_{b,G} = (1-\delta f(\xi_G))p_{b,G} - w_{b,G} - R_G$$

where δ indicates the magnitude of commercial gentrification, ξ_G is a proportion of good businesses in a gentrifying neighborhood, $w_{t,n}$ is a wage received by a worker of a type t business in neighborhood n, and R_n is a rent in neighborhood n. In non-gentrifying neighborhood NG, a profit from a type t business is

$$\pi_{t,NG} = p_{t,NG} - w_{t,NG} - R_{NG}$$

3.3.3 Matching technology

$M(U_n, V_n)$ is the number of matches between unemployed workers and vacancies in neighborhood n, where U_n is the number of unemployed workers and V_n is the number of vacancies, in neighborhood n. I assume that $M(U_n, V_n)$ has a constant return to scale and increases in both arguments. Then, the flow rate of match for a vacancy is

$$\frac{M(U_n, V_n)}{V_n} = q(\theta_n)$$

where $\theta_n \equiv V_n/U_n$ is a job tightness in neighborhood n.

The flow rate of match for an unemployed worker is

$$\frac{M(U_n, V_n)}{U_n} = \theta_n q(\theta_n)$$

$q(\theta_n)$ is a decreasing function of θ_n, and $\theta_n q(\theta_n)$ is a increasing function of θ_n.

For $M(U_n, V_n)$, I assume the following Cobb-Douglas specification

$$M(U_n, V_n) = AU_n^{\sigma} V_n^{1-\sigma}$$

where A is a matching technology and σ is a share for the number of unemployed workers.

3.3.4 Value functions

Depending on the results of search and matching, there exist four value functions. For a worker, there are the value of being unemployed, J_n^U, and the value of being employed in a type t business, $J_{t,n}^E$, in a neighborhood n. For a firm, there are the value of having a type t vacancy, $J_{t,n}^V$, and the value of filling a type t vacancy, $J_{t,n}^F$, in a neighborhood n. The first and second subscript in value functions stand for a type of business and neighborhood.

An unemployed worker has the following flow value of being unemployed in neighborhood n.

$$rJ_n^U = b + \theta_n q(\theta_n)\{\phi_n(J_{b,n}^E - J_n^U) + (1-\phi_n)(J_{g,n}^E - J_n^U)\} \text{ for } n = \{G, NG\} \tag{3.2}$$

where r is a discount rate, b is an unemployment insurance or home production, and ϕ_n is a proportion of bad businesses among total vacancies in neighborhood n.

Equation (3.2) says that an unemployed worker instantaneously receives an unemployment insurance b and will be hired in any type of business by the flow rate of $\theta_n q(\theta_n)$. Given that the worker is hired in any type of business, the worker will be matched to either a bad or good business by the probability of ϕ_n or $1 - \phi_n$. If a worker is matched to either a bad or good business, the worker will gain the value of being employed in a bad or good business.

A worker hired in a type t business in neighborhood n has the following flow value of being employed.

$$rJ^E_{t,n} = w_{t,n} + s(J^U_n - J^E_{t,n}) \text{ for } t = \{g, b\} \text{ and } n = \{G, NG\} \tag{3.3}$$

where $w_{t,n}$ is a wage of worker in type t business in neighborhood n, s is an exogenous separation rate. g and b stand for "good" and "bad" business.

Equation (3.3) says that an employed worker receives a wage of $w_{t,n}$ and will gain the value of being unemployed after losing the value of being employed, with the probability s.

A firm that opens a vacancy has the following flow value of opening a vacancy

$$rJ^V_{t,n} = -k_t + q(\theta_n)(J^F_{t,n} - J^V_{t,n}) \text{ for } t = \{g, b\} \text{ and } n = \{G, NG\} \tag{3.4}$$

where k_t is the vacancy opening cost for type t business. I assume that $k_g > k_b$.

Equation (3.4) says that a firm that opens a vacancy pays the vacancy opening cost k_t such as business interior design and worker training costs and will gain the value of filling a vacancy after losing the value of having a vacancy by the flow rate of $q(\theta_n)$.

To have a constant number of vacancies, a firm should earn a zero net profit from an additional vacancy. By free entry condition,

$$J_{t,n}^{V} = 0$$

Free entry condition implies that

$$J_{t,n}^{F} = \frac{k_t}{q(\theta_n)} \tag{3.5}$$

Thus, the value of filling a vacancy increases in the vacancy opening cost and job tightness.

A business matched with a worker in a gentrifying neighborhood has the following flow value of filling a vacancy

$$\begin{aligned} rJ_{g,G}^{F} &= (1+\delta f(\xi_G))p_{g,G} - w_{g,G} - R_G + s(J_{g,G}^{V} - J_{g,G}^{F}) \\ rJ_{b,G}^{F} &= (1-\delta f(\xi_G))p_{b,G} - w_{b,G} - R_G + s(J_{b,G}^{V} - J_{b,G}^{F}) \end{aligned} \tag{3.6}$$

Equation (3.6) means that a firm that fills a vacancy earns profits, and gains the value of having a vacancy after losing the value of filling a vacancy, with the probability s. Due to search frictions between customers and businesses, a demand price is not equal to a competitive price. A business matched with a worker in a non-gentrifying neighborhood has the following flow value of filling a vacancy

$$rJ_{t,NG}^{F} = p_{t,NG} - w_{t,NG} - R_{NG} + s(J_{t,NG}^{V} - J_{t,NG}^{F}) \text{ for } t = \{g, b\} \tag{3.7}$$

Unlike a gentrifying neighborhood, there is no search frictions between customers and businesses making a demand and a competitive price different in a non-gentrifying neighborhood.

By combining Equation (3.5), (3.6), and free entry condition,

$$\begin{aligned}\frac{(r+s)k_g}{q(\theta_G)} &= (1+\delta f(\xi_G))p_{g,G} - w_{g,G} - R_G \\ \frac{(r+s)k_b}{q(\theta_G)} &= (1-\delta f(\xi_G))p_{b,G} - w_{b,G} - R_G\end{aligned} \tag{3.8}$$

Equation (3.8) is an equation for a vacancy supply curve of a good and a bad businesses in a gentrifying neighborhood. By combining Equation (3.5), (3.7), and free entry condition,

$$\frac{(r+s)k_t}{q(\theta_{NG})} = p_{t,NG} - w_{t,NG} - R_{NG} \text{ for } t = \{g, b\} \tag{3.9}$$

Equation (3.9) is an equation for a vacancy supply curve of a good and a bad businesses in a non-gentrifying neighborhood.

3.3.5 Nash Bargaining

I assume that wages and rents are determined by Nash bargaining. There are two strands of bilateral bargainings. First, wages are determined by asymmetric Nash bargaining between a firm and a worker. A firm and a worker share total net surplus from a match. The bargaining power of a worker is $\beta \in [0, 1]$. A wage is determined by solving

$$w_{t,n} = \arg\max_{w_{t,n}} [J^E_{t,n} - J^U_n]^\beta [J^F_{t,n} - J^V_{t,n}]^{1-\beta}$$

In a gentrifying area, wages are equal to

$$\begin{aligned}w_{g,G} &= \beta\{(1+\delta f(\xi_G))p_{g,G} - R_G + \theta_G(\phi_G k_b + (1-\phi_G)k_g)\} + (1-\beta)b \\ w_{b,G} &= \beta\{(1-\delta f(\xi_G))p_{b,G} - R_G + \theta_G(\phi_G k_b + (1-\phi_G)k_g)\} + (1-\beta)b\end{aligned} \tag{3.10}$$

In a non-gentrifying area, wages are equal to

$$w_{t,NG} = \beta\{p_{t,NG} - R_{NG} + \theta_{NG}(\phi_{NG}k_b + (1-\phi_{NG})k_g)\} + (1-\beta)b \text{ for } t = \{g,b\} \quad (3.11)$$

Proposition 1 Wages are weighted sums of the surplus by a firm with a successful match and the worker's outside option. The surpluses between a gentrifying and a non-gentrifying neighborhood in a successful match are different since a demand price is not equal to a competitive price in a gentrifying neighborhood.
Proof. See the appendix.

Second, rents are determined by Nash bargaining between a firm and a landlord. I assume that all businesses need to pay the same amount of rent for a space, regardless of business type. A firm and a landlord share total net surpluses from a match. Total surpluses depend on how many spaces a firm rents from a landlord. The bargaining power of a firm is $\gamma \in [0,1]$. A rent in neighborhood n is determined by solving

$$R_n = \arg\max_{R_n}[Y_{g,n}J^F_{g,n} + Y_{b,n}J^F_{b,n}]^{\gamma}[(Y_{g,n} + Y_{b,n})R_n]^{1-\gamma}$$

Since each business produces a single good, the number of total matches between unemployed workers and good or bad businesses is equal to the number of high or low quality goods produced, respectively. In a gentrifying neighborhood, rents are equal to

$$R_G = (1-\gamma)\{\xi_{g,G}((1+\delta\xi_{g,G})p_{g,G} - w_{g,G}) + (1-\xi_{g,G})((1-\delta\xi_{g,G})p_{b,G} - w_{b,G})\} \quad (3.12)$$

In a non-gentrifying neighborhood, rents are equal to

$$R_{NG} = (1-\gamma)\{S_{g,NG}(p_{g,NG} - w_{g,NG}) + S_{b,NG}(p_{b,NG} - w_{b,NG})\} \quad (3.13)$$

where $S_{g,NG} \equiv \frac{Y_{g,NG}}{Y_{g,NG}+Y_{b,NG}}$ and $S_{b,NG} \equiv \frac{Y_{b,NG}}{Y_{g,NG}+Y_{b,NG}}$ are the shares of good and bad businesses matched in a non-gentrifying neighborhood, respectively.

Proposition 2 Rents are discounted weighted sums of surpluses by a firm from good businesses and bad businesses-the bargaining power of a firm discounts the rents. The higher the bargaining power of a firm, the lower the rents a firm pays. Surpluses before rents between a gentrifying and a non-gentrifying neighborhood are different since a demand price is not equal to a competitive price in a gentrifying neighborhood.

3.3.6 Steady-state equilibrium

In the steady-state equilibrium, the flows into unemployment and the flows out of unemployment are equal. That is,

$$\underbrace{s(M-U_n)}_{\text{The flow into unemployment}} = \underbrace{\theta_n q(\theta_n) U_n}_{\text{The flow out of unemployment}}$$

where M is the total number of population in a neighborhood n.
Then, the unemployment rate in the steady-state is

$$u_n = \frac{s}{s+\theta_n q(\theta_n)} \tag{3.14}$$

The number of high ($Y_{g,n}$) or low ($Y_{b,n}$) quality goods produced in neighborhood n is equal to the product of employment rate and the number of good or bad business such as

$$\begin{aligned} Y_{g,n} &= (1-u_n)(1-\phi_n)N \\ Y_{b,n} &= (1-u_n)\phi_n N \end{aligned} \tag{3.15}$$

Given the number of high or low quality goods produced in neighborhood n, the competitive prices of high or low quality goods in neighborhood n are determined as

$$\begin{aligned} p_{g,n} &= (1-\alpha)(1-\phi_n)^{\rho-1}[\alpha\phi_n^\rho + (1-\alpha)(1-\phi_n)^\rho]^{\frac{1-\rho}{\rho}} \equiv p_{g,n}(\phi_n) \\ p_{b,n} &= \alpha\phi_n^{\rho-1}[\alpha\phi_n^\rho + (1-\alpha)(1-\phi_n)^\rho]^{\frac{1-\rho}{\rho}} \equiv p_{b,n}(\phi_n) \end{aligned} \tag{3.16}$$

By solving equations for wages, rents, and competitve prices, rents and wages are determined as functions of the portion of bad businesses, ϕ_n, and job tightness, θ_n, in neighborhood n. That is,

$$\begin{aligned} R_n &= R_n(\theta_n, \phi_n) \text{ for } n = \{G, NG\} \\ w_{t,n} &= w_{t,n}(\theta_n, \phi_n) \text{ for } n = \{G, NG\} \end{aligned} \tag{3.17}$$

By plugging the equilibrium prices, wages, and rents into Equation (3.8) and (3.9),

$$\begin{aligned} \frac{(r+s)k_g}{q(\theta_G)} &= (1+\delta\xi_G)p_{g,G}(\theta_G, \phi_G) - w_{g,G}(\theta_G, \phi_G) - R_G(\theta_G, \phi_G) \\ \frac{(r+s)k_b}{q(\theta_G)} &= (1-\delta\xi_G)p_{b,G}(\theta_G, \phi_G) - w_{b,G}(\theta_G, \phi_G) - R_G(\theta_G, \phi_G) \end{aligned} \tag{3.18}$$

$$\begin{aligned} \frac{(r+s)k_g}{q(\theta_{NG})} &= p_{g,NG}(\theta_{NG}, \phi_{NG}) - w_{g,NG}(\theta_{NG}, \phi_{NG}) - R_{NG}(\theta_{NG}, \phi_{NG}) \\ \frac{(r+s)k_b}{q(\theta_{NG})} &= p_{b,NG}(\theta_{NG}, \phi_{NG}) - w_{b,NG}(\theta_{NG}, \phi_{NG}) - R_{NG}(\theta_{NG}, \phi_{NG}) \end{aligned} \tag{3.19}$$

Equation (3.18) and (3.19) represent vacancy supply curves of good and bad businesses in a gentrifying and non-gentrifying neighborhood. The equilibrium job tightness and proportion of bad businesses in a gentrifying neighborhood, (θ_G^*, ϕ_G^*), satisfy Equation (3.18). Similarly, the equilibrium job tightness and proportion of bad businesses in a non-gentrifying neighborhood, $(\theta_{NG}^*, \phi_{NG}^*)$, satisfy Equation (3.19).

Definition: A steady-state equilibrium consists of job tightness and a proportion of bad businesses, $(\theta_n^*, \phi_n^*)_{n=\{G,NG\}}$, unemployment rate, $\{u_n^*\}_{n=\{G,NG\}}$, prices of goods, $\{p_{t,n}^*\}_{t=\{g,b\}\&n=\{G,NG\}}$, number of goods, $\{Y_{t,n}^*\}_{t=\{g,b\}\&n=\{G,NG\}}$, rents, $\{R_n^*\}_{n=\{G,NG\}}$, wages, $\{w_{t,n}^*\}_{t=\{g,b\}\&n=\{G,NG\}}$, and value functions, $\{J_{t,n}^V, J_{t,n}^F, J_{t,n}^E, J_n^U\}_{t=\{g,b\}\&n=\{G,NG\}}$ such that

1. (θ_G^*, ϕ_G^*) and $(\theta_{NG}^*, \phi_{NG}^*)$ satisfy Equation (3.18) and (3.19).
2. $\{p_{t,n}^*\}_{t=\{g,b\}\&n=\{G,NG\}}$ satisfies Equation (3.16) for $\phi_n = \phi_n^*$
3. R_G^*, $w_{g,G}^*$, and $w_{b,G}^*$ satisfy Equation (3.17) for $(\theta_G, \phi_G) = (\theta_G^*, \phi_G^*)$.
   R_{NG}^*, $w_{g,NG}^*$, and $w_{b,NG}^*$ satisfy Equation (3.17) for $(\theta_{NG}, \phi_{NG}) = (\theta_{NG}^*, \phi_{NG}^*)$.
4. u_n^* satisfies Equation (3.14) for $\theta_n = \theta_n^*$.
5. $Y_{g,n}^*$ and $Y_{b,n}^*$ satisfy Equation (3.15) for $u_n = u_n^*$ and $\phi_n = \phi_n^*$.
6. $J_{t,n}^V$ and $J_{t,n}^F$ satisfy Equation (3.4), (3.6), (3.7), and free entry condition.
7. $J_{t,n}^E$ and J_n^U satisfy Equation (3.2) and (3.3).

3.4 Calibration

On the benchmark, I set parameter values to match the portion of good businesses in a gentrifying and a non-gentrifying neighborhood at 78% and 40%, respectively.

Parameter	α	β	δ	γ	ρ	σ	s	N	Q	b	r	A	k_b	k_g
Value	0.5	0.5	0.5	0.5	0.5	0.7	0.2	100	2.5	0.1	0.01	1	0.5	1

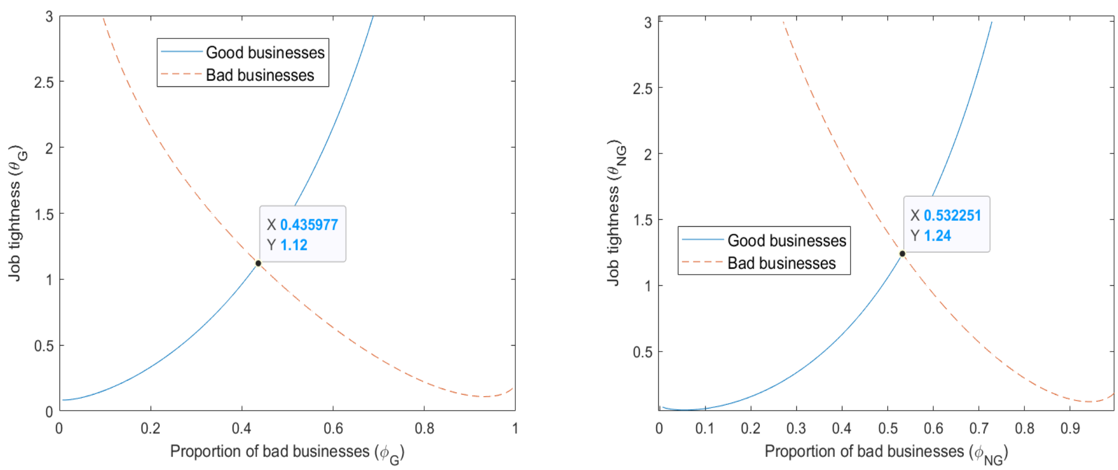
3.5 Comparative statics

For comparative statics, I consider two cases whether two types of goods are gross complements or gross substitutes. I examine how the composition of businesses and rents between a gentrifying and a non-gentrifying neighborhood change in response to changes in complementarity or substitutability, search friction, separation rate, and bargaining powers.

3.5.1 Two types of goods are gross complements. ($\rho < 0$)

Figure 3.5 shows vacancy supply curves of good and bad businesses in a gentrifying and non-gentrifying neighborhood on the benchmark when the two types of goods are gross complements. A firm opens more vacancies of good business in both neighborhoods as the proportion of bad businesses increases, given a fixed vacancy opening cost. In bad businesses, a firm opens fewer vacancies in both neighborhoods as the proportion of bad business increases. As Figure 3.5 shows, on the benchmark, the proportion of bad businesses in a non-gentrifying neighborhood is higher than in a gentrifying neighborhood by 9.6 percentage points in the steady-state. Figure 3.6 shows how rents in both neighborhoods change as the proportion of good businesses increases. Rents increase slightly faster in a gentrifying neighborhood than in a non-gentrifying neighborhood, up to around 50% of good businesses. After that, rents in a gentrifying neighborhood increase more slowly than in a non-gentrifying neighborhood.

Table 3.2 summarizes how much the differences of proportions of bad businesses and rents between a gentrifying and non-gentrifying neighborhood change in response to changes in parameter values. I define the differences as subtracting the value in a gentrifying neighborhood from the value in a non-gentrifying neighborhood. It also shows how much each value changes in percentage relative to the value on the benchmark. As the magnitude of complementarity (ρ) increases, the differences of proportions and rents decrease. The proportions of bad businesses and rents are very sensitive to the changes in the magnitude of search frictions between consumers and businesses. As the magnitude of search friction (δ) increases from 0.5 to 0.7, the differences in the proportion of bad businesses and rents increase by 58.33% and 46.15%. The differences are not sensitive to changes in separation rates. Changes in a firm's bargaining power over a landlord (γ) or that of a worker over a firm (β) do not affect the differences of proportions of bad businesses. However, the higher bargaining power of a firm over a landlord decreases the differences of rents rapidly. In comparison, a worker's higher bargaining power over a firm increases the differences in rents rapidly.



(a) Gentrifying neighborhood (b) Non-gentrifying neighborhood

Figure 3.5: Vacancy supply curves of good and bad businesses ($\rho < 0$)

3.5.2 Two types of goods are gross substitutes. ($\rho > 0$)

Figure 3.7 shows vacancy supply curves of good and bad businesses in a gentrifying and a non-gentrifying neighborhood on the benchmark when the two types of goods are gross substitutes. Compared to the case of gross complements, the curvatures of both curves increase. However, the increases in the curvatures are asymmetric in both neighborhoods. In a gentrifying neighborhood, the curvature change in bad businesses' vacancy supply curve dominates the other. On the contrary, in a non-gentrifying neigh-

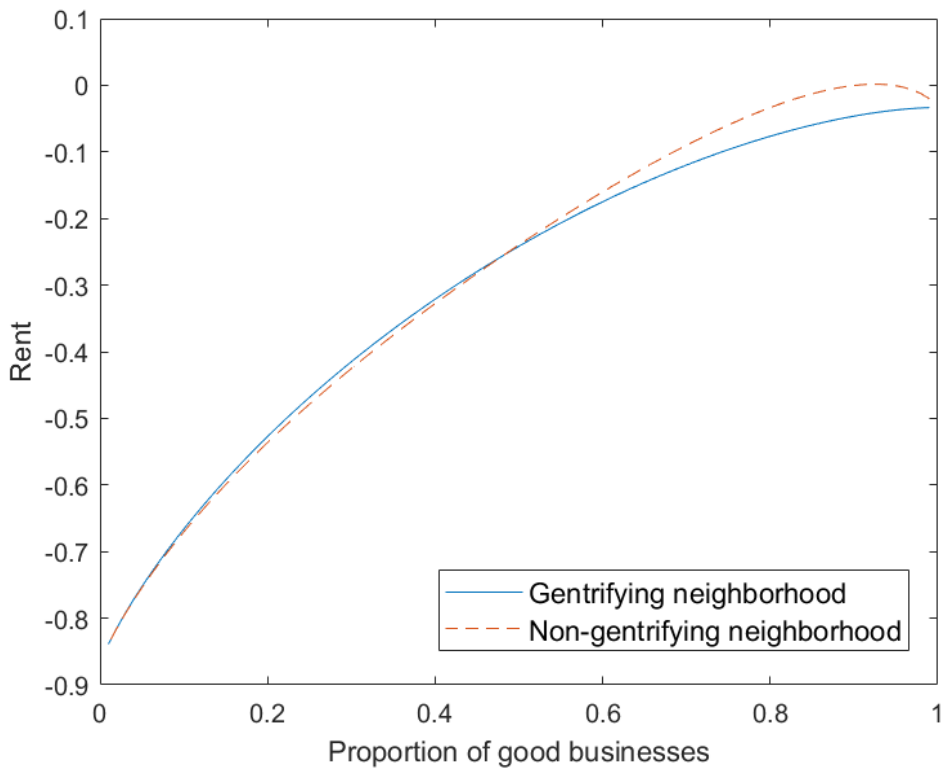
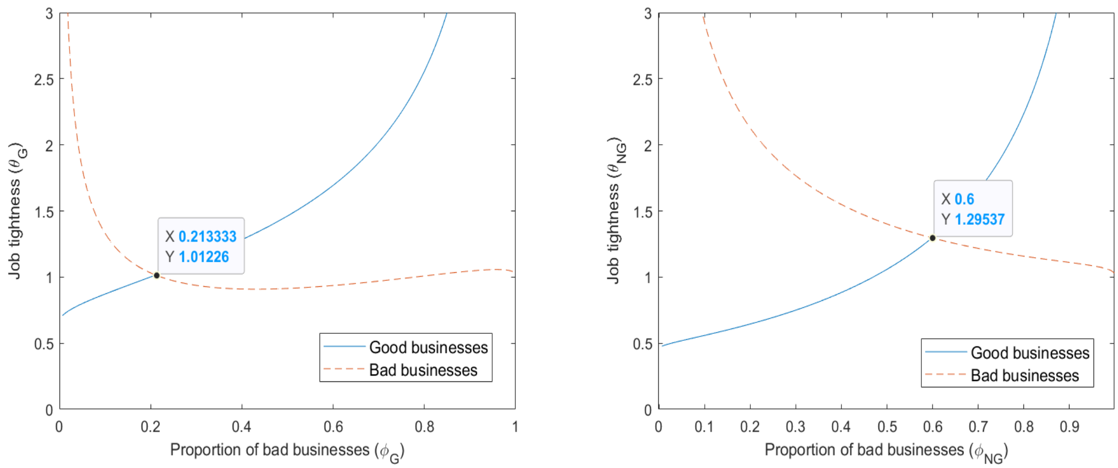


Figure 3.6: Rents over the proportion of good businesses

borhood, the curvature change in good businesses' vacancy supply curve dominates the other. As a result, the proportion of bad businesses in a non-gentrifying neighborhood is much higher than in a gentrifying neighborhood by 38.67%. Rent changes in both neighborhoods also show an opposite pattern to the case of gross complements. Rents in a gentrifying neighborhood increase slightly slower up to around 50% of good businesses and then increase much faster than in a non-gentrifying neighborhood.

Table 3.3 summarizes comparative statics results when the two types of goods are substitutes. The effects of an increase in substitutability are much higher than that of an increase in complementarity. When the magnitude of substitutability increases from 0.5 to 0.6, the differences in proportion and rents increase by 27.39% and 37.66%, respectively. However, the effect of an increase in search frictions between customers and



(a) Gentrifying neighborhood (b) Non-gentrifying neighborhood

Figure 3.7: Vacancy supply curves of good and bad businesses ($\rho < 0$)

businesses is smaller in gross substitutes than in the case of gross complements. Changes in separation rates also do not affect the differences. Similar to the case of gross complements, changes in a firm's bargaining powers over a landlord or a worker's bargaining power over a firm rarely affect the composition of businesses. Changes in rents are more sensitive to a firm's bargaining power over a landlord in the case of gross substitutes. When the bargaining power increases from 0.5 to 0.6, rent differences decrease by 40% and 48% in the case of gross complements and gross substitutes, respectively. However, changes in rents are more sensitive to a worker's bargaining power over a firm in the case of gross complements. When the bargaining power increases from 0.5 to 0.6, rent differences decrease by 112% and 61% in the case of gross complements and gross substitutes, respectively.

3.6 Conclusion

This paper studies spatial sorting in the context of commercial gentrification. Using Yelp data for neighborhoods in Los Angeles in the US, I revisit several measures of

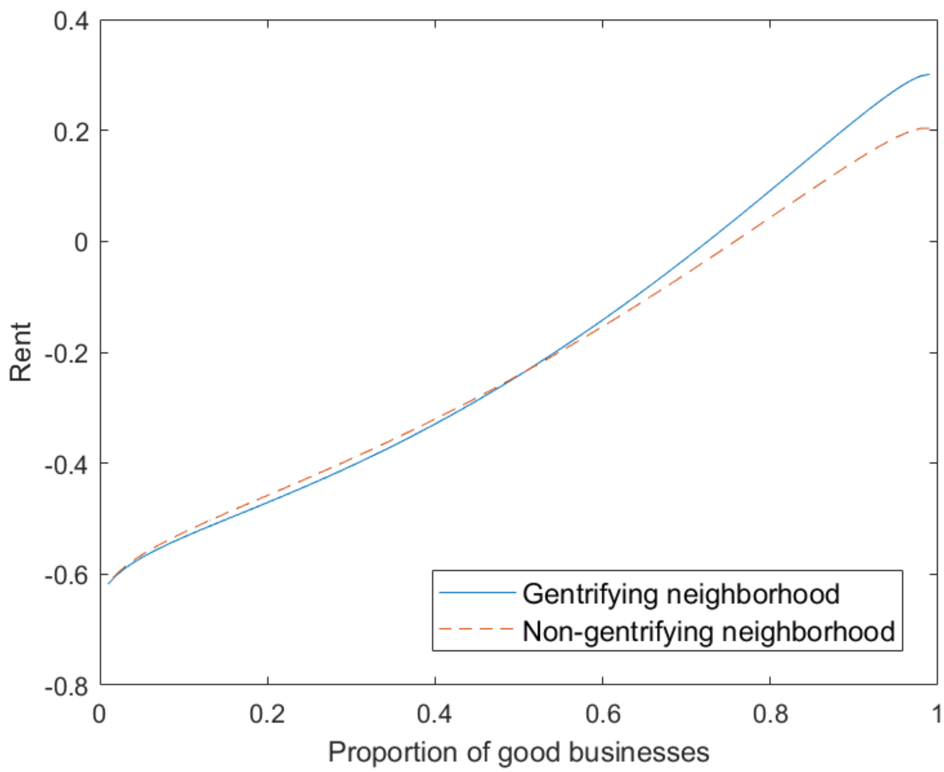


Figure 3.8: Rents over the proportion of good businesses

gentrification. Then, I build a search and match model with the proportion of good business as a measure of commercial gentrification and find that discrepancy between a competitive and a demand price due to search friction in a gentrifying neighborhood is a driving force to make two neighborhoods distinct. Depending on whether high and low-quality goods are gross complements or substitutes, patterns of changes in rents to the proportion of good businesses or sensitivity of the differences to changes in parameter values are different.

This paper is limited to compare the steady-states between a gentrifying and a non-gentrifying neighborhood. As a future research, I think studying the dynamics of the composition of businesses from a non-gentrifying neighborhood to a gentrifying neighborhood is worthwhile. Also, I suggest doing relevant studies by introducing development, taxes, and changes in the magnitude of search frictions through policy intervention.

Table 3.2: The differences between a non-gentrifying and a gentrifying neighborhood. ($\rho < 0$)

	ρ = -0.5	ρ = -0.6	ρ = -0.7
Bad businesses	0.096 (-)	0.09 (-6.25%)	0.081 (-15.625%)
Rent	-0.065 (-)	-0.063 (-3.077%)	-0.055 (-15.385%)
	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.9$
Bad businesses	0.096 (-)	0.152 (58.33%)	0.239 (148.958%)
Rent	-0.065 (-)	-0.095 (46.154%)	-0.131 (101.538%)
	$s = 0.2$	$s = 0.3$	$s = 0.4$
Bad businesses	0.096 (-)	0.098 (2.083%)	0.092 (-4.167%)
Rent	-0.065 (-)	-0.075 (15.385%)	-0.071 (9.231%)
	$\gamma = 0.5$	$\gamma = 0.7$	$\gamma = 0.9$
Bad businesses	0.096 (-)	0.096 (-)	0.094 (-2.083%)
Rent	-0.065 (-)	-0.039 (-40%)	-0.012 (-81.538%)
	$\beta = 0.5$	$\beta = 0.7$	$\beta = 0.9$
Bad businesses	0.096 (-)	0.099 (3.125%)	0.093 (-3.125%)
Rent	-0.065 (-)	-0.138 (112.308%)	-0.212 (226.154%)

Table 3.3: The differences between a non-gentrifying and a gentrifying neighborhood. ($\rho > 0$)

	$\rho = 0.5$	$\rho = 0.6$	$\rho = 0.7$
Bad businesses	0.387 (-)	0.493 (27.39%)	0.613 (58.398%)
Rent	-0.401 (-)	-0.552 (37.656%)	-0.726 (81.047%)
	$\delta = 0.5$	$\delta = 0.7$	$\delta = 0.9$
Bad businesses	0.387 (-)	0.545 (40.827%)	0.6 (55.039%)
Rent	-0.401 (-)	-0.626 (56.11%)	-0.699 (74.314%)
	$s = 0.2$	$s = 0.3$	$s = 0.4$
Bad businesses	0.387 (-)	0.38 (-1.809%)	0.38 (-1.809%)
Rent	-0.401 (-)	-0.379 (-5.486%)	-0.365 (-8.978%)
	$\gamma = 0.5$	$\gamma = 0.7$	$\gamma = 0.9$
Bad businesses	0.387 (-)	0.386 (-0.258%)	0.387 (-)
Rent	-0.401 (-)	-0.209 (-47.88%)	-0.062 (-84.539%)
	$\beta = 0.5$	$\beta = 0.7$	$\beta = 0.9$
Bad businesses	0.387 (-)	0.387 (-)	0.387 (-)
Rent	-0.401 (-)	-0.646 (61.097%)	-0.912 (127.431%)

Appendix A

Appendix: Reverse Mortgages and Intergenerational Risk-sharing during the Great Recession

A.1 Proofs of propositions

For proofs in a deterministic case, I set several parameters equal to zero since it does not change the result qualitatively.[1] For a utility function, I introduce the following CRRA utility function:

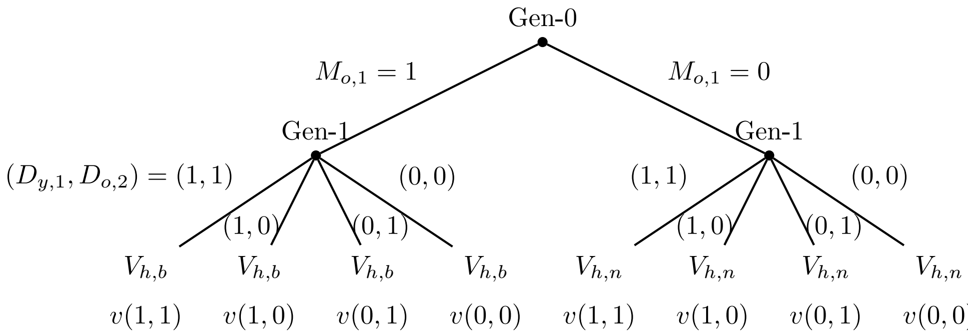
$$u(c, D) = \frac{(c^{\gamma}\chi(D)^{1-\gamma})^{1-\sigma}}{1-\sigma}$$

where $\chi(D) > 1$ if D = 1 and $\chi(D) = 1$ if D = 0. For analytical solutions, I set $\sigma = 1$. Then,

$$u(c, D) = \gamma \ln c + (1-\gamma) \ln \chi(D)$$

[1] $\psi = \tau = \theta = 0$

Sequential Game: Gen-0 moves first



By backward induction, given $\{P_t, w\}_{t=1,2}$ and $\{T_{o,1}, M_{o,1}\}$, gen-1 solves

$$v = \max_{c_{y,1}, c_{o,1}, a_{y,1}} \{\ln(c_{y,1}) + \ln(\chi(D_{y,1})) + \beta\{\ln(c_{o,2}) + \ln(\chi(D_{o,2}))\}\}$$

subject to Equation (1.16) and (1.17).

FOC with respect to $a_{y,1}$:

$$\frac{-1}{w + T_{o,1} - a_{y,1} - E_{y,1}} + \frac{1}{b + a_{y,1} + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2}} = 0$$

where

$$E_{y,1} \equiv (1 - D_{y,1})\alpha P_1 + D_{y,1}(1 + \psi + \tau)P_1$$
$$E_{o,2} \equiv (1 - D_{o,2})\alpha P_2 + D_{o,2}\{D_{y,1}(\psi + \tau)P_2 + (1 - D_{y,1})(1 + \psi + \tau)P_2\}$$

Then,

$$a_{y,1} = \frac{(w + T_{o,1} - E_{y,1}) - \{b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2}\}}{2}$$
$$c_{y,1} = c_{o,2} = \frac{(w + T_{o,1} - E_{y,1}) + \{b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2}\}}{2}$$

Given $\{P_t, w\}_{t=1,2}$ and $\{c_{y,1}, c_{o,2}, a_{y,1}, D_{y,1}, D_{o,2}\}$, gen-0 solves

$$V = \max_{c_{o,1}, T_{o,1}} \{\ln(c_{o,1}) + \ln(\chi(D_{o,1})) + \eta v\}$$

subject to Equation (1.18).

FOC with respect to $T_{o,1}$:

$$\frac{-1}{b + a_{y,0} + (1-\phi)P_1 M_{o,1} - T_{o,1}} + \frac{\eta}{c_{y,1}}\frac{\partial c_{y,1}}{\partial T_{o,1}} + \frac{\eta}{c_{o,2}}\frac{\partial c_{o,2}}{\partial T_{o,1}} \leq 0$$

For an interior solution,

$$T_{o,1} = \frac{2\eta\{b + a_{y,0} + (1-\phi)P_1 M_{o,1}\} - \{(w - E_{y,1}) + (b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2})\}}{2\eta + 1}$$

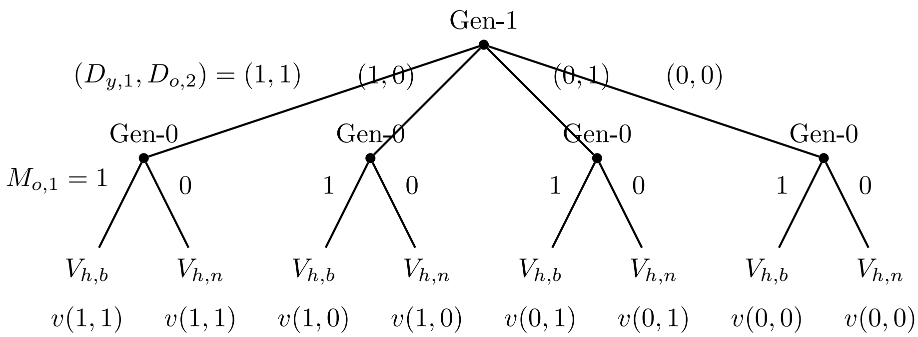
By plugging $T_{o,1}$ into $a_{y,1}(T_{o,1})$,

$$a_{y,1} = \frac{\eta\{(w - E_{y,1}) + (b + a_{y,0} + (1-\phi)P_1 M_{o,1})\} - (\eta+1)\{b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2}\}}{2\eta + 1}$$

Then,

$$c_{o,1} = \frac{\{b + a_{y,0} + (1-\phi)P_1 M_{o,1}\} + \{(w - E_{y,1}) + (b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2})\}}{2\eta + 1}$$
$$c_{y,1} = \frac{\eta\{(b + a_{y,0} + (1-\phi)P_1 M_{o,1})\} + \eta\{(w - E_{y,1}) + (b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2})\}}{2\eta + 1}$$
$$c_{o,2} = \frac{\eta\{(b + a_{y,0} + (1-\phi)P_1 M_{o,1})\} + \eta\{(w - E_{y,1}) + (b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2})\}}{2\eta + 1}$$

Sequential Game: gen-1 moves first



By backward induction, given $\{P_t, w\}_{t=1,2}$ and $\{c_{y,1}, c_{o,2}, a_{y,1}, D_{y,1}, D_{o,2}\}$, gen-0 solves

$$V = \max_{c_{o,1}, T_{o,1}} \{\ln(c_{o,1}) + \ln(\chi(D_{o,1})) + \eta v\}$$

subject to Equations (1.16), (1.17), and (1.18).

FOC with respect to $T_{o,1}$:

$$\frac{-1}{b + a_{y,0} + (1-\phi)P_1 M_{o,1} - T_{o,1}} + \frac{\eta}{w + T_{o,1} - a_{y,1} - E_{y,1}} \leq 0$$

Reaction function by generation 1:

$$T_{o,1}(a_{y,1}) = \frac{\eta(b + a_{y,0} + (1-\phi)P_1 M_{o,1}) - (w - a_{y,1} - E_{y,1})}{\eta + 1}$$

Given the reaction function $T_{o,1}(a_{y,1})$, gen-1 solves

$$v = \max_{c_{y,1}, c_{o,2}, a_{y,1}} \{\ln(c_{y,1}) + \ln(\chi(D_{y,1})) + \beta\{\ln(c_{o,2}) + \ln(\chi(D_{o,2}))\}\}$$

subject to Equations (1.16) and (1.17).

FOC with respect to $a_{y,1}$:

$$\frac{T'_{o,1}(a_{y,1})-1}{w+T_{o,1}(a_{y,1})-a_{y,1}-E_{y,1}}+\frac{1}{b+a_{y,1}+P_2(1-M_{o,1})+G_{o,2}-E_{o,2}}\leq 0$$

For an interior solution,

$$a_{y,1}=\frac{\{b+a_{y,0}+(1-\phi)P_1M_{o,1}\}+(w-E_{y,1})-\{b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2}\}}{2}$$

By plugging $a_{y,1}$ into $T_{o,1}(a_{y,1})$,

$$\mathrm{T}_{o,1}=\frac{(2\eta+1)\{b+a_{y,0}+(1-\phi)P_1M_{o,1}\}-\{(w-E_{y,1})+(b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2})\}}{2(\eta+1)}$$

Then,

$$c_{o,1}=\frac{\{b+a_{y,0}+(1-\phi)P_1M_{o,1}\}+\{(w-E_{y,1})+(b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2})\}}{2(\eta+1)}$$

$$c_{y,1}=\frac{\eta\{(w-E_{y,1})+(b+a_{y,0}+(1-\phi)P_1M_{o,1})\}+\eta\{b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2}\}}{2(\eta+1)}$$

$$c_{o,2}=\frac{\{(w-E_{y,1})+(b+a_{y,0}+(1-\phi)P_1M_{o,1})\}+\{b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2}\}}{2}$$

Proof of Preposition 1

Gen-0 increases transfers to gen-1 if gen-0's wealth increases or gen-1's disposable income in either period decreases. However, it is ambiguous whether gen-0 transfers more wealth when gen-0 becomes more altruistic.

Transfer function by gen-0 is given by

$$T_{o,1}=\frac{2\eta\{b+a_{y,0}+(1-\phi)P_1M_{o,1}\}-\{(w-E_{y,1})+(b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2})\}}{2\eta+1}$$

Then,

$$\frac{\partial T_{o,1}}{\partial(b + a_{y,0} + (1-\phi)P_1 M_{o,1})} = \frac{2\eta}{\eta+1} > 0$$

$$\frac{\partial T_{o,1}}{\partial \eta} = \frac{2(b+a_{y,0}+(1-\phi)P_1 M_{o,1})}{(2\eta+1)^2} + \frac{2\{(w_1-E_{y,1})+(b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2})\}}{(2\eta+1)^2} \lesseqgtr 0$$

$$\frac{\partial T_{o,1}}{\partial(w_1 - E_{y,1})} = \frac{-1}{2\eta+1} < 0$$

$$\frac{\partial T_{o,1}}{\partial(b + P_2(1-M_{o,1}) + G_{o,2} - E_{o,2})} = \frac{-1}{2\eta+1} < 0$$

Proof of Preposition 2

In a pooling equilibrium, gen-0's choice between borrowing an RML and not borrowing an RML is independent of gen-1's homeownership decision.

First, suppose that $v = v(1,1)$.

The value of borrowing an RML ($V_{h,b}$) when $(D_{y,1}, D_{o,2}) = (1,1)$ is

$$\begin{aligned}\ln(c_{o,1}) + \ln(\chi) + \eta v &= (2\eta+1)\ln(c_{y,1}) - \ln(\eta) + (2\eta+1)\ln(\chi) \\ &= (2\eta+1)\ln\left\{\frac{\eta(w_1 + 2b + a_{y,0} + (1-\phi)P_2 - \phi P_1)}{2\eta+1}\right\} + \ln(\chi/\eta) + 2\eta\ln(\chi)\end{aligned}$$

The value of not borrowing an RML ($V_{h,n}$) when $(D_{y,1}, D_{o,2}) = (1,1)$ is

$$\begin{aligned}\ln(c_{o,1}) + \ln(\chi) + \eta v &= (2\eta+1)\ln(c_{y,1}) - \ln(\eta) + (2\eta+1)\ln(\chi) \\ &= (2\eta+1)\ln\left\{\frac{\eta(w_1 + 2b + a_{y,0} + (2-\phi)P_2 - P_1)}{2\eta+1}\right\} + \ln(\chi/\eta) + 2\eta\ln(\chi)\end{aligned}$$

Then,

$$V = \max\{V_{h,b}, V_{h,n}\} = V_{h,b} \text{ if and only if } (1-\phi)P_1 > P_2$$

Next, suppose that $v = v(0,0)$.

The value of borrowing an RML ($V_{h,b}$) when $(D_{y,1}, D_{o,2}) = (0,0)$ is

$$\begin{aligned}\ln(c_{o,1}) + \ln(\chi) + \eta v &= (2\eta+1)\ln(c_{y,1}) - \ln(\eta) + \ln(\chi)\\ &= (2\eta+1)\ln\left\{\frac{\eta(w_1 + 2b + a_{y,0} + (1-\phi-\alpha)P_1 - \alpha P_2)}{2\eta+1}\right\} - \ln(\eta) + \ln(\chi)\end{aligned}$$

The value of not borrowing an RML ($V_{h,n}$) when $(D_{y,1}, D_{o,2}) = (0,0)$ is

$$\begin{aligned}\ln(c_{o,1}) + \ln(\chi) + \eta v &= (2\eta+1)\ln(c_{y,1}) - \ln(\eta) + \ln(\chi)\\ &= (2\eta+1)\ln\left\{\frac{\eta(w_1 + 2b + a_{y,0} + (1-\alpha)P_2 - \alpha P_1)}{2\eta+1}\right\} - \ln(\eta) + \ln(\chi)\end{aligned}$$

Then,

$$V = \max\{V_{h,b}, V_{h,n}\} = V_{h,b} \text{ if and only if } (1-\phi)P_1 > P_2$$

Thus, gen-0 always decides to borrow an RML if $(1-\phi)P_1 > P_2$ regardless of gen-1's homeownership decision.

Proof of Preposition 3

If gen-1 is liquidity-constrained in both periods, there are cases that gen-1 cannot have sufficient liquidity to buy a home even with transfers from gen-0.

Without altruism, gen-1 is liquidity-constrained if

$$w_1 < (1-\delta)P_1$$

or

$$b + a_{y,1} + (1-\phi)P_2 < 0$$

With altruism, gen-0 who borrows an RML transfers her wealth to gen-1 by

$$T = \frac{2\eta(b + a_{y,0} + (1-\phi)P_1) - (w_1 + b + (1-\phi)P_2 - P_1)}{2\eta + 1}$$

and gen-1 saves

$$a_{y,1} = \frac{\eta(w_1 + a_{y,0} - \phi P_1 - (1-\phi)P_2) - (b + (1-\phi)P_2)}{2\eta + 1}$$

Then,

$$w_1 + T = \frac{2\eta(b + w_1 + a_{y,0} + (1-\phi)P_1) - (b + (1-\phi)P_2 - P_1)}{2\eta + 1} > (1-\delta)P_1$$

if and only if

$$(1-\phi)P_2 < (2\eta(\delta - \phi) + \delta)P_1 + 2\eta(b + w_1 + a_{y,0}) - b$$

$$b + a_{y,1} + (1-\phi)P_2 = \frac{\eta(2b + w_1 + a_{y,0} + (1-\phi)P_2 - \phi P_1)}{2\eta + 1} > 0$$

if and only if

$$(1-\phi)P_2 > \phi P_1 - 2b - w_1 - a_{y,0}$$

There exist $(a_{y,0}, w_1, P_1, P_2)$ such that

$$\phi P_1 - 2b - w_1 - a_{y,0} < (1-\phi)P_2 < (2\eta(\delta - \phi) + \delta)P_1 + 2\eta(b + w_1 + a_{y,0}) - b$$

does not hold. Thus, gen-1 cannot be a homeowner even with transfers from gen-0.

Proof of Preposition 4

In a separating equilibrium, gen-0 decides an RML based on housing prices only if the RML decision leads to the desirable homeownership choice for gen-1. Otherwise, gen-0's RML decision is uncertain.

Suppose that $v = v(1,1)$ if $M_{o,1} = 1$ and $v = v(0,0)$, otherwise. $v(1,1) > v(0,0)$

$$\begin{aligned}
V_{h,n} &= \ln(c_{o,1}) + \ln(\chi) + \eta v(0,0) \\
&= \ln(2b + w_1 + a_{y,0} + (1-\alpha)P_2 - \alpha P_1) - \ln(2\eta + 1) + \ln(\chi) + \eta v(0,0) \\
V_{h,b} &= \ln(c_{o,1}) + \ln(\chi) + \eta v(1,1) \\
&= \ln(2b + w_1 + a_{y,0} + (1-\phi)P_2 - \phi P_1) - \ln(2\eta + 1) + \ln(\chi) + \eta v(1,1)
\end{aligned}$$

If $(\alpha - \phi)(P_1 + P_2) > 0$, gen-0 borrows an RML. Otherwise, gen-0's RML decision is uncertain.

Now, suppose that $v = v(1,1)$ if $M_{o,1} = 0$ and $v = v(0,0)$, otherwise.

$$\begin{aligned}
V_{h,n} &= \ln(c_{o,1}) + \ln(\chi) + \eta v(1,1) \\
&= \ln(2b + w_1 + a_{y,0} + (2-\phi)P_2 - P_1) - \ln(2\eta + 1) + \eta v(1,1) \\
V_{h,b} &= \ln(c_{o,1}) + \ln(\chi) + \eta v(0,0) \\
&= \ln(2b + w_1 + a_{y,0} + (1-\alpha-\phi)P_1 - \alpha P_2) - \ln(2\eta + 1) + \eta v(0,0)
\end{aligned}$$

Since $v(1,1) > v(0,0)$, gen-0 never borrows an RML if $(2 - \phi + \alpha)P_2 > (2 - \phi - \alpha)P_1$. Otherwise, gen-0's RML decision is uncertain.

Proof of Proposition 5

There is a first-mover advantage in a sequential game between two generations. By moving first, gen-1 strategically fails to smooth consumption across periods to induce gen-0 to increase transfers. In a sequential game which gen-0 moves first,

$$c_{y,1} = \frac{\eta\{(b + a_{y,0} + (1-\phi)P_1M_{o,1})\} + \eta\{(w - E_{y,1}) + (b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2})\}}{2\eta + 1}$$

$$c_{o,2} = \frac{\eta\{(b + a_{y,0} + (1-\phi)P_1M_{o,1})\} + \eta\{(w - E_{y,1}) + (b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2})\}}{2\eta + 1}$$

$$T_{o,1} = \frac{2\eta\{b + a_{y,0} + (1-\phi)P_1M_{o,1}\} - \{(w - E_{y,1}) + (b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2})\}}{2\eta + 1}$$

$$\mathrm{a}_{y,1} = \frac{\eta\{(w-E_{y,1})+(b+a_{y,0}+(1-\phi)P_1M_{o,1})\}-(\eta+1)\{b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2}\}}{2\eta+1}$$

In a sequential game which gen-1 moves first,

$$c_{y,1} = \frac{\eta\{(w - E_{y,1}) + (b + a_{y,0} + (1-\phi)P_1M_{o,1})\} + \eta\{b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2}\}}{2(\eta + 1)}$$

$$c_{o,2} = \frac{\{(w - E_{y,1}) + (b + a_{y,0} + (1-\phi)P_1M_{o,1})\} + \{b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2}\}}{2}$$

$$\mathrm{T}_{o,1} = \frac{(2\eta+1)\{b+a_{y,0}+(1-\phi)P_1M_{o,1}\}-\{(w-E_{y,1})+(b+P_2(1-M_{o,1})+G_{o,2}-E_{o,2})\}}{2(\eta+1)}$$

$$a_{y,1} = \frac{\{b + a_{y,0} + (1-\phi)P_1M_{o,1}\} + (w - E_{y,1}) - \{b + P_2(1 - M_{o,1}) + G_{o,2} - E_{o,2}\}}{2}$$

That is, gen-1 strategically fails to smooth consumption between periods to exploit gen-0's altruism. In the sequential game which gen-1 moves first, gen-0 transfer more wealth to gen-1 than in the sequential game which gen-0 moves first by $\frac{b+a_{y,0}+(1-\phi)P_1M_{o,1}}{2(\eta+1)}$.

A.2 Regression results

Table A.1: *Odds of transfer*

	(1) Homeowner	(2) Renter	(3) Homeowner	(4) Renter
Child income	-0.369^{***}	-0.0347^{*}	-0.453^{***}	-0.335^{***}
	(0.0148)	(0.0162)	(0.0246)	(0.0696)
Child home	-0.411^{***}	-0.169^{***}	-0.501^{***}	-0.554^{***}
	(0.0231)	(0.0280)	(0.0334)	(0.0969)
Net wealth	0.0164^{***}	0.0176^{**}	0.0122^{***}	0.0299
	(0.00148)	(0.00589)	(0.00147)	(0.0199)
Contact			0.000997^{*}	0.00162
			(0.000490)	(0.00205)
Proximity			0.135^{***}	0.242^{**}
			(0.0295)	(0.0873)
Child in a will			0.934^{***}	1.362^{***}
			(0.0578)	(0.166)
Spouse			0.0782^{**}	0.0743
			(0.0293)	(0.103)
constant	-0.362^{***}	-1.079^{***}	-0.859^{***}	-1.785^{***}
	(0.0351)	(0.0358)	(0.0788)	(0.225)
Year fixed effect	✓	✓	✓	✓
Observations	68290	20634	33697	4523
Adjusted R^2				

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: *Odds of transfer* is the ratio of transferring wealth over not transferring wealth between groups, *Contact* is the number of contacts by a child per year, *Proximity* is equal to 1 if a child lives within 10 miles of a parent or 0, otherwise. *Child income* is an ordinal variable equal to 1 if annual income is less than \$10,000, 2 if annual income is between \$10,000 and \$35,000, 3 if annual income is over \$35,000. *Child home* is equal to 1 for a homeowner and 0 for a renter. *Child in a will* is equal to 1 if a parent puts a child in a will or 0, otherwise. *Spouse* is equal to 1 if a senior has a spouse or 0, otherwise.

Table A.2: *The amount of transfer*

	(1) Homeowner	(2) Renter	(3) Homeowner	(4) Renter
Child income	-1034.3^{***}	-62.91	-1146.2^{***}	-11.54
	(84.75)	(45.06)	(190.1)	(191.6)
Child home	36.20	-70.46	-57.55	-431.8
	(116.5)	(82.98)	(210.0)	(283.3)
Monthly earnings	0.140^{***}	0.00349	0.0942^{**}	-0.0740
	(0.0212)	(0.0114)	(0.0302)	(0.0535)
Net wealth	78.92^{***}	167.3^{*}	79.89^{***}	117.7^{**}
	(16.31)	(66.79)	(19.75)	(42.55)
Contact			3.987	-1.904
			(3.794)	(1.861)
Proximity			-209.0	524.2^{*}
			(188.7)	(247.2)
Child in a will			1056.3^{***}	1029.4^{***}
			(182.5)	(132.9)
Spouse			-636.3^{*}	-372.0
			(254.8)	(210.2)
constant	2300.3^{***}	648.5^{**}	2623.1^{***}	482.0
	(252.7)	(199.5)	(525.7)	(536.9)
Year fixed effect	✓	✓	✓	✓
Observations	68088	20598	33616	4516
Adjusted R^2	0.020	0.038	0.014	0.026

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.3: Odds of transfer

	(1) Homeowner	(2) Renter
Child income=2	-0.312^{***}	-0.0394
	(0.0576)	(0.167)
Child income=3	-0.832^{***}	-0.551^{***}
	(0.0566)	(0.165)
Great Recession	0.660^{**}	-0.404
	(0.216)	(0.428)
Child income=2×*Great Recession*	-0.0610	0.677
	(0.265)	(0.555)
Child income=3×*Great Recession*	-0.160	0.245
	(0.272)	(0.694)
Child home	-0.495^{***}	-0.535^{***}
	(0.0337)	(0.0967)
Child home×*Great Recession*	0.0693	0.281
	(0.211)	(0.654)
Contact	0.000997^{*}	0.00176
	(0.000488)	(0.00203)
Proximity	0.132^{***}	0.241^{**}
	(0.0296)	(0.0873)
Child in a will	0.933^{***}	1.359^{***}
	(0.0575)	(0.166)
Spouse	0.0806^{**}	0.0792
	(0.0292)	(0.103)
Net wealth	0.0122^{***}	0.0303
	(0.00146)	(0.0194)
constant	-1.364^{***}	-2.269^{***}
	(0.0708)	(0.205)
Observations	33697	4523

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: *Great Recession* is equal to 1 if the year of the survey is either 2008 or 2010.

Table A.4: The amount of transfer

	(1) Homeowner	(2) Renter
Child income=2	-2600.2^{***}	-523.4
	(495.3)	(480.7)
Child income=3	-3035.2^{***}	-371.7
	(534.6)	(511.8)
Great Recession	3894.3^{*}	-704.7
	(1681.9)	(612.6)
Child income=2×Great Recession	-1847.8	1162.4
	(2186.7)	(1021.4)
Child income=3×Great Recession	-3352.2	1726.1
	(2193.4)	(1300.2)
Child home	-74.29	-484.8
	(206.7)	(282.5)
Child home×Great Recession	-1091.7	2297.6
	(1359.9)	(3033.0)
Contact	3.937	-2.793
	(3.723)	(1.970)
Proximity	-173.3	514.7^{*}
	(188.2)	(249.2)
Child in a will	1087.7^{***}	1070.6^{***}
	(181.4)	(145.9)
Spouse	-643.3^{*}	-396.1
	(254.5)	(217.1)
Monthly earnings	0.0930^{**}	-0.0761
	(0.0301)	(0.0541)
Net wealth	79.11^{***}	117.2^{**}
	(19.70)	(42.63)
constant	3378.3^{***}	899.1^{*}
	(416.9)	(434.1)
Observations	33616	4516
Adjusted R^2	0.014	0.025

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.5: The ratio of the initial principal limit over the MCA

	(1) Great Recession = Yes	(2) Great Recession = No
Borrower's age	0.00812***	0.00852***
	(808.71)	(548.17)
Expected rate	−0.0421***	−0.0766***
	(−99.97)	(−120.97)
1-year HPI growth	−0.0000627**	0.000229***
	(−2.83)	(7.78)
2-year HPI growth	0.0000206	−0.0000809**
	(0.82)	(−2.72)
3-year HPI growth	0.00000448	−0.0000944**
	(0.18)	(−2.86)
4-year HPI growth	−0.00000665	0.0000832*
	(−0.30)	(2.43)
5-year HPI growth	0.0000129	0.0000209
	(1.05)	(1.09)
constant	0.346***	0.560***
	(154.22)	(89.40)
Year FE	✓	✓
Observations	272178	371540

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note: To determine the initial principal limit, a lender considers the borrower's age and expected rate, including a 10-year Treasury rate and the lender's margin.

Table A.6: The probability and amount of transfer during the boom

The probability of transfer			
	$\eta = 0.3$	$\eta = 0.35$	$\eta = 0.4$
Homeowner	0.4986	0.4916	0.4986
Home of 33rd percentile	0.531	0.4591	0.4513
Home of 50th percentile	0.4004	0.421	0.5016
Home of 75th percentile	0.4708	0.6008	0.5507
Home of 95th percentile	0.619	0.6938	0.6992
Renter	0.28	0.6732	0.8963
RML borrower	0.5778	0.596	0.5403
RML non-borrower	0.3635	0.4876	0.5943
Child's wage of 1st fifth	1	1	1
Child's wage of 2nd fifth	0.6685	0.83	0.927
Child's wage of 3rd fifth	0.309	0.3755	0.6137
Child's wage of 4th fifth	0.2823	0.2776	0.1343
Child's wage of 5th fifth	0.0667	0.1285	0.2231
The amount of transfer			
	$\eta = 0.3$	$\eta = 0.35$	$\eta = 0.4$
Homeowner	0.953	0.9552	0.8231
Home of 33rd percentile	1.0136	0.8047	0.6068
Home of 50th percentile	0.6508	0.727	0.733
Home of 75th percentile	0.9223	1.3677	1.2741
Home of 95th percentile	1.341	1.6795	1.8047
Renter	0.2434	1.3528	2.4112
RML borrower	1.234	1.427	1.0133
RML non-borrower	0.4856	0.8148	1.1869
Child's wage of 1st fifth	2.1328	2.623	2.9769
Child's wage of 2nd fifth	1.141	1.3597	1.69
Child's wage of 3rd fifth	0.3962	0.463	0.6553
Child's wage of 4th fifth	0.5112	0.5369	0.2186
Child's wage of 5th fifth	0.0507	0.1161	0.2212

Table A.7: The probability and amount of transfer during the recession

The probability of transfer			
	$\eta = 0.3$	$\eta = 0.35$	$\eta = 0.4$
Homeowner	0.5492	0.5074	0.5139
Home of 33rd percentile	0.5117	0.5103	0.5028
Home of 50th percentile	0.5199	0.3056	0.3607
Home of 75th percentile	0.5141	0.5381	0.5773
Home of 95th percentile	0.6892	0.6726	0.6749
Renter	0.251	0.6427	0.8588
RML borrower	0.6355	0.5841	0.5627
RML non-borrower	0.3746	0.4971	0.5741
Child's wage of 1st fifth	1	1	1
Child's wage of 2nd fifth	0.6873	0.832	0.8996
Child's wage of 3rd fifth	0.4183	0.381	0.567
Child's wage of 4th fifth	0.3858	0.3633	0.2187
Child's wage of 5th fifth	0.0415	0.0669	0.1708
The amount of transfer			
	$\eta = 0.3$	$\eta = 0.35$	$\eta = 0.4$
Homeowner	0.8549	0.8971	0.8721
Home of 33rd percentile	0.7908	0.759	0.6709
Home of 50th percentile	0.6095	0.5025	0.6365
Home of 75th percentile	0.9379	1.2234	1.0577
Home of 95th percentile	1.2807	1.5557	1.7346
Renter	0.1862	1.1055	1.95
RML borrower	1.035	1.2941	1.2508
RML non-borrower	0.4798	0.7253	0.9778
Child's wage of 1st fifth	1.8526	2.3006	2.7666
Child's wage of 2nd fifth	0.9324	1.2013	1.3164
Child's wage of 3rd fifth	0.4422	0.4381	0.6538
Child's wage of 4th fifth	0.5402	0.6534	0.3846
Child's wage of 5th fifth	0.031	0.0525	0.1312

Table A.8: The take-up rate of RML and the share of RML borrowers during the boom

The take-up rate of RML			
	$\eta = 0.3$	$\eta = 0.35$	$\eta = 0.4$
Overall	0.5708	0.4148	0.2532
Home of 33rd percentile	0.4627	0.302	0.1537
Home of 50th percentile	0.6046	0.4433	0.2638
Home of 75th percentile	0.6531	0.5421	0.3055
Home of 95th percentile	0.7162	0.6897	0.6562
Child's wage of 1st fifth	0.4968	0.3651	0.125
Child's wage of 2nd fifth	0.4679	0.2467	0.0496
Child's wage of 3rd fifth	0.3586	0.133	0.0834
Child's wage of 4th fifth	0.456	0.3263	0.1112
Child's wage of 5th fifth	0.6557	0.6543	0.6352
The share of RML borrowers			
	$\eta = 0.3$	$\eta = 0.35$	$\eta = 0.4$
Home of 33rd percentile	0.4399	0.4239	0.3787
Home of 50th percentile	0.2531	0.2265	0.2066
Home of 75th percentile	0.1618	0.1542	0.095
Home of 95th percentile	0.1452	0.1954	0.3196
Child's wage of 1st fifth	0.204	0.2129	0.1236
Child's wage of 2nd fifth	0.1929	0.1398	0.0466
Child's wage of 3rd fifth	0.1459	0.073	0.0781
Child's wage of 4th fifth	0.1868	0.1871	0.1056
Child's wage of 5th fifth	0.2704	0.3871	0.6461

Table A.9: The take-up rate of RML and the share of RML borrowers during the recession

The take-up rate of RML			
	$\eta = 0.3$	$\eta = 0.35$	$\eta = 0.4$
Overall	0.5892	0.4115	0.3195
Home of 33rd percentile	0.3986	0.2652	0.178
Home of 50th percentile	0.6769	0.4201	0.3378
Home of 75th percentile	0.7683	0.6238	0.3741
Home of 95th percentile	0.7817	0.7781	0.7683
Child's wage of 1st fifth	0.4792	0.412	0.2377
Child's wage of 2nd fifth	0.4473	0.1118	0.0772
Child's wage of 3rd fifth	0.4231	0.1605	0.1482
Child's wage of 4th fifth	0.5009	0.4138	0.2273
Child's wage of 5th fifth	0.6776	0.6686	0.6465
The share of RML borrowers			
	$\eta = 0.3$	$\eta = 0.35$	$\eta = 0.4$
Home of 33rd percentile	0.3217	0.3831	0.3269
Home of 50th percentile	0.2835	0.1569	0.1753
Home of 75th percentile	0.2018	0.1849	0.1042
Home of 95th percentile	0.193	0.275	0.3935
Child's wage of 1st fifth	0.1899	0.2307	0.1786
Child's wage of 2nd fifth	0.1759	0.0619	0.0542
Child's wage of 3rd fifth	0.1672	0.091	0.1112
Child's wage of 4th fifth	0.1979	0.2351	0.1702
Child's wage of 5th fifth	0.269	0.3812	0.4858

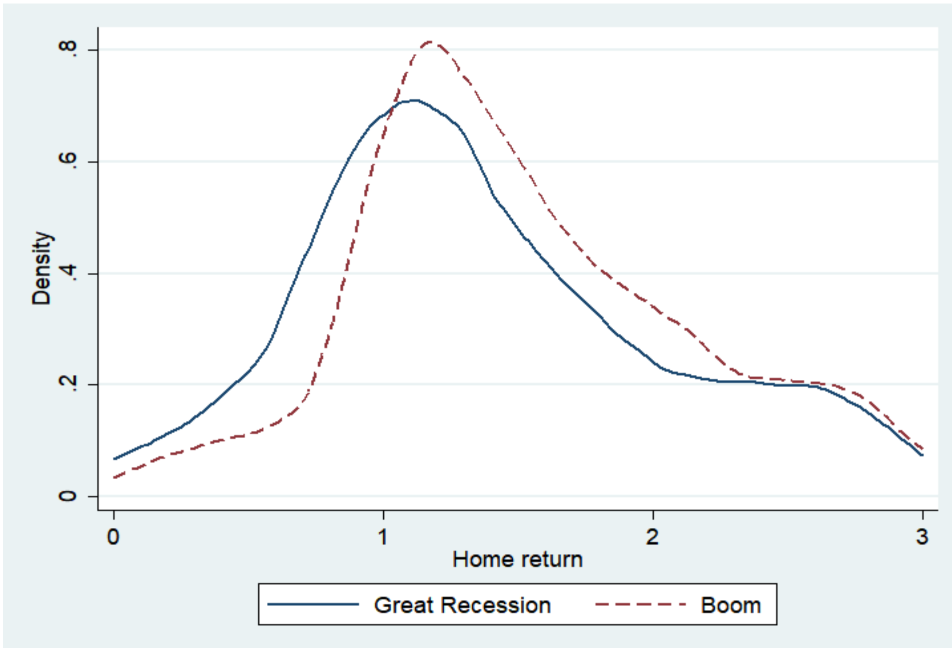


Figure A.1: Density of Home Returns during the boom and the Great Recession

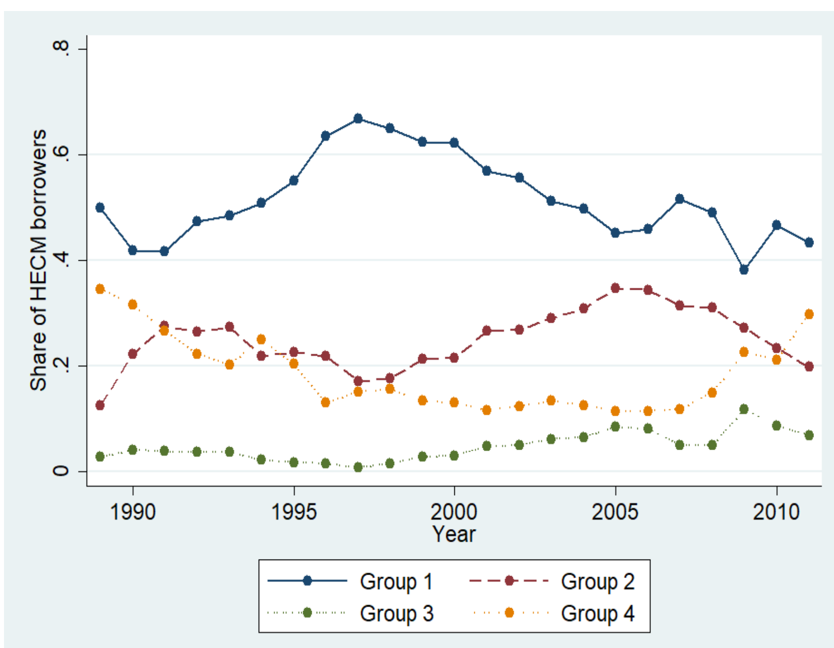


Figure A.2: The shares of HECM borrowers by appraised home values

Appendix B

Appendix: Spatial Sorting: Commercial Gentrification

B.1 Appendix

Proof of the Proposition 1 A wage by the Nash bargaining is determined by solving

$$w_{t,n} = \arg\max_{w_{t,n}} [J^E_{t,n} - J^U_n]^{\beta} [J^F_{t,n} - J^V_{t,n}]^{1-\beta}$$

That is, the wage maximizes the shared rent between a firm and a worker.

The first-order condition with respect to $w_{t,n}$ leads to

$$J^E_{t,n} - J^U_n = \beta(J^E_{t,n} - J^U_n + J^F_{t,n}) \Leftrightarrow (1-\beta)(J^E_{t,n} - J^U_n) = \beta J^F_{t,n} \tag{B.1}$$

From Equation (3),

$$J^E_{t,n} - J^U_n = \frac{w_{t,n} - rJ^U_n}{r+s} \text{ for } t = \{g, b\} \text{ and } n = \{G, NG\} \tag{B.2}$$

From Equation (6), (7), and the free entry condition,

$$\begin{aligned}
J^F_{g,G} &= \frac{(1+\delta\xi_G)p_{g,G} - w_{g,G} - R_G}{r+s} \\
J^F_{b,G} &= \frac{(1-\delta\xi_G)p_{b,G} - w_{b,G} - R_G}{r+s} \\
J^F_{t,NG} &= \frac{p_{t,NG} - w_{t,NG} - R_{NG}}{r+s} \ \text{ for } t=\{g,b\}
\end{aligned} \tag{B.3}$$

By plugging Equation (21) and (22) into Equation (20), the Nash solutions are

$$\begin{aligned}
w_{g,G} &= \beta((1+\delta\xi_G)p_{g,G} - R_G) + (1-\beta)rJ^U_G \\
w_{b,G} &= \beta((1-\delta\xi_G)p_{b,G} - R_G) + (1-\beta)rJ^U_G \\
w_{t,NG} &= \beta(p_{t,NG} - R_{NG}) + (1-\beta)rJ^U_{NG} \ \text{ for } t=\{g,b\}
\end{aligned} \tag{B.4}$$

By combining Equation (3.2) and (B.1), the flow value of being unemployed can be written as

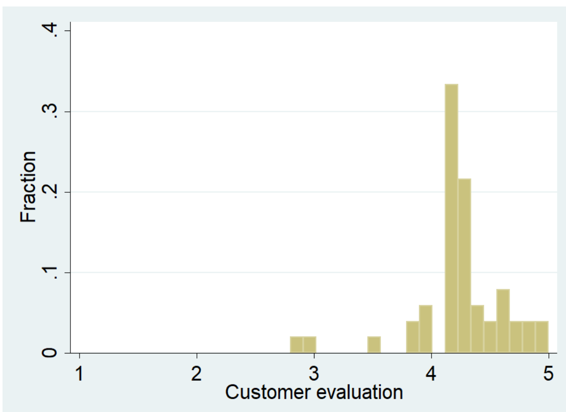
$$\begin{aligned}
rJ^U_n &= b + \theta_n q(\theta_n)\Big\{\frac{\beta\phi_n J^F_{b,n}}{1-\beta} + \frac{\beta(1-\phi_n)J^F_{g,n}}{1-\beta}\Big\} \\
&= b + \frac{\beta}{1-\beta}\theta_n\phi_n k_b + \frac{\beta}{1-\beta}\theta_n(1-\phi_n)k_g
\end{aligned} \tag{B.5}$$

By combining Equation (B.4) and (B.5), wages in a gentrifying area are

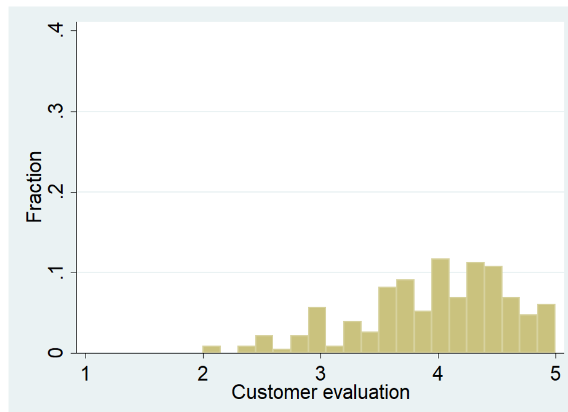
$$\begin{aligned}
w_{g,G} &= \beta\{(1+\delta\xi_G)p_{g,G} - R_G + \theta_G(\phi_G k_b + (1-\phi_G)k_g)\} + (1-\beta)b \\
w_{b,G} &= \beta\{(1-\delta\xi_G)p_{b,G} - R_G + \theta_G(\phi_G k_b + (1-\phi_G)k_g)\} + (1-\beta)b
\end{aligned} \tag{B.6}$$

Wages in a non-gentrifying area are

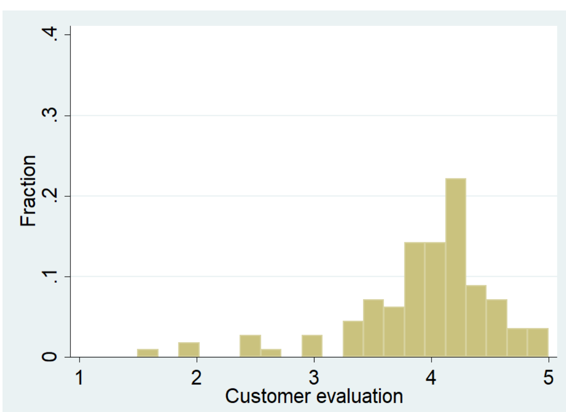
$$w_{t,NG} = \beta\{p_{t,NG} - R_{NG} + \theta_{NG}(\phi_{NG}k_b + (1-\phi_{NG})k_g)\} + (1-\beta)b \ \text{ for } t=\{g,b\} \tag{B.7}$$



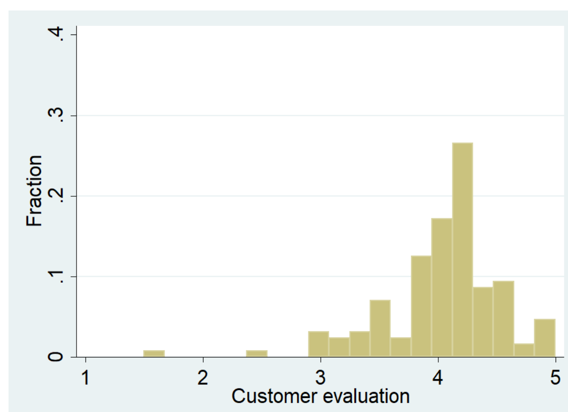
(a) Arts District



(b) Boyle Heights

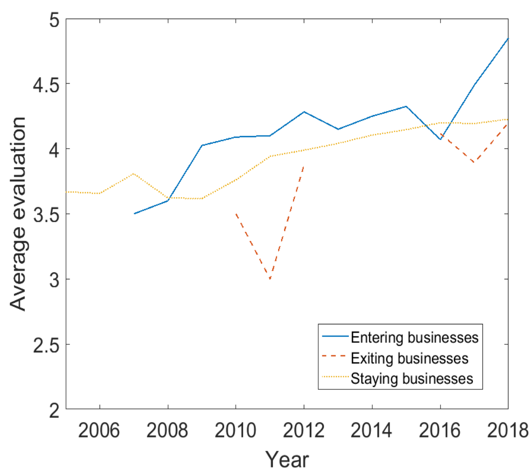


(c) Chinatown

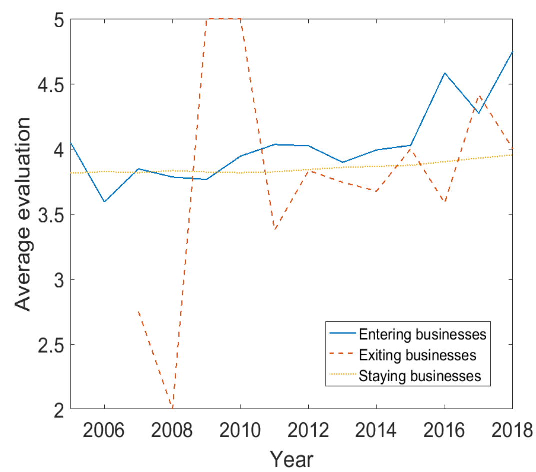


(d) Little Tokyo

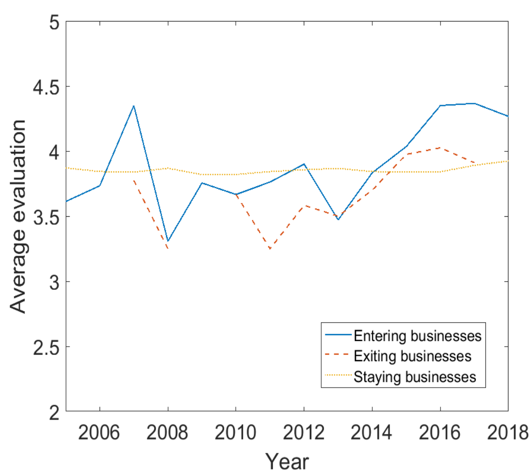
Figure B.1: Histogram of Customers' Evaluation on Active Businesses



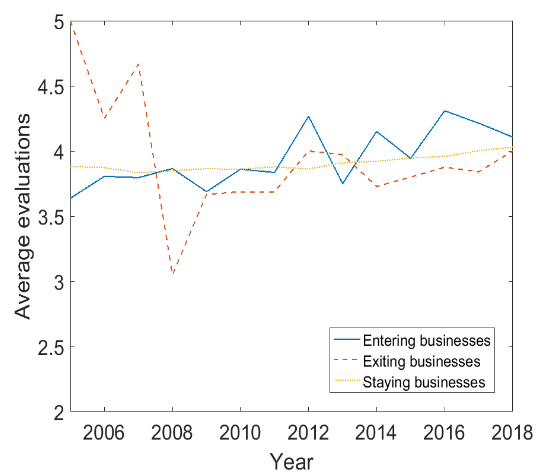
(a) Arts District



(b) Boyle Heights



(c) Chinatown



(d) Little Tokyo

Figure B.2: Customers' average evaluation of entering, exiting, and staying businesses

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