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Jonathan D. Young

January 22, 1960

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Berkeley, California

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ABSTRACT

The purpose of this report is to describe the construction of an optical analog to a symmetric quadrupole by use of an IBM 650 computer program.

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\* This work was done under the auspices of the U. S. Atomic Energy Commission.

## OPTICAL ANALOG FOR A SYMMETRIC QUADRUPOLE

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### INTRODUCTION

This report describes the construction of an optical analog to a symmetric quadrupole by use of an IBM 650 computer program called TRIPOLE.

A quadrupole consists of three magnetic-field sections separated by field-free sections. In a symmetric quadrupole the first and third fields are identical and the two inner field-free sections are likewise identical.

The quadrupole affects the trajectory of a charged particle in much the same way as a ray of light is affected by a system of three lenses. This analogy enables us to describe the effect of the quadrupole in optical terms.

If we consider one field section there is one plane such that any trajectory lying therein experiences the maximum convergent effect. As for trajectories in this plane, the field is analogous to a convex lens of some focal length. However, because of alternation of polarity in the field, there is an orthogonal plane in which the field is analogous to a concave lens of some other focal length. Thus associated with each field section we have two focal lengths. However, we are able to relate both to a single argument. The determination of this argument for each field section is the first step in the development of the optical analog.

For the quadrupole as a whole the optical analog is a convergent-divergent-convergent (CDC) system in one plane, while in an orthogonal plane it is a divergent-convergent-divergent (DCD) system. See Figs. 1 and 2.

By successive application of the simple lens formula and by use of the argument relation mentioned above we are able to derive two transcendental equations for the two arguments. (We note that fields I and III are identical, hence their arguments are likewise identical.) Due to trigonometric terms, we have a multiplicity of solutions. In the program TRIPOLE we seek the least positive values of the arguments which satisfy the two equations. In solving the equations, we must know the object and image distances in both the CDC and DCD planes, and the lengths of field and field-free sections of the quadrupole.

With any point on a trajectory lying in the CDC plane we can associate a vector with two components, respectively the distance from the point to the longitudinal axis of the quadrupole and the slope of the trajectory at the point with respect to that axis. The argument and the length of a field section determine a  $2 \times 2$  matrix which represents the transformation of an entering trajectory by the field. The matrix-vector product is the vector characterizing the trajectory as it leaves the field. A similar but simpler matrix dependent on the length of a field-free section likewise gives the effect on a trajectory as it passes through. The product of all such matrices for all sections gives the total effect of the quadrupole on an entering trajectory lying in the CDC plane.

A similar process used in the DCD plane gives a second matrix representing the total effect of the quadrupole on an entering trajectory which lies in the DCD plane.

From the arguments for the field sections we can compute the focal lengths, and their partial derivatives with respect to object distance. Using the CDC and DCD matrices described above, we can compute the positions of the principal planes and the magnification, and by perturbing the arguments determine the circles of confusion respectively for the CDC and DCD systems.

By using section matrices in the CDC plane successively we can determine the maximum slope for a trajectory originating at a given object distance such that the trajectory lies within all limiting surfaces of the quadrupole and collimator. A similar process gives the maximum slope in the DCD system. For this, the transverse dimensions of the collimator and the radius of the quadrupole orifice must be known.

Complete formulation for the above is given in Appendix 2.

#### DESCRIPTION OF PROGRAM

The basic information must be supplied in IBM 650 machine language floating-point numbers by means of two input data cards. All distances are in inches. See Fig. 3 and Fig. 4

##### First Data Card

First word:  $p$ , object distance in CDC plane measured to centerline of first field section.

Second word:  $p'$ , object distance in DCD plane measured to centerline of first field section.

Third word:  $q$ , image distance in CDC plane measured from centerline of third section.

Fourth word:  $q'$ , image distance in DCD plane measured from centerline of third field section.

Fifth word:  $l_1$ , length of first (and third) field sections.



Sixth word:  $l_2$ , length of second field section.

Seventh word:  $d$ , distance from centerline of a field section to centerline of the next field section.

Eighth word:  $P$ , momentum in Mev/c.

Second Data Card

First word:  $h_c$ , distance from collimator face to longitudinal axis of quadrupole in the CDC plane.

Second word:  $h'_c$ , distance from collimator face to axis of quadrupole in the DCD plane.

Third word:  $k$ , distance from inner face of collimator to centerline of first field section.

Fourth word:  $h$ , distance from field face to axis.

Other words: All zeroes.

OUTPUT FROM TRIPOLE

The output from TRIPOLE is punched cards with IBM 650 machine language floating-point numbers as follows:

First Output Card

Repeats first data card.

Second Output Card

First word:  $\theta_1$  - argument for first (and third) field section

Second word:  $\theta_2$  - argument for second field section

Fifth word:  $(\frac{dB}{dr})_1$  - field derivative for first field in

kG/cm.

Sixth word:  $(\frac{dB}{dr})_2$  - field derivative for second field in kG/cm.

All other words: Zeroes.

Third Output Card

First word:  $|f_1|$  - absolute focal length, first field, CDC plane.

Second word:  $|f'_1|$  - absolute focal length, first field, DCD plane.

Third word:  $|f_2|$  - absolute focal length, second field, CDC plane.

Fourth word  $|f'_2|$  - absolute focal length, second field, DCD

plane.

All other words: Zeroes.

Fourth Output Card

First word:  $\frac{\partial f_1}{\partial p}$  - partial derivative of  $f_1$  with respect to  $p$ .

Second word:  $\frac{\partial f'_1}{\partial p}$  .

Third word:  $\frac{\partial f_2}{\partial p}$  .

Fourth word:  $\frac{\partial f'_2}{\partial p}$  .

Fifth word:  $\frac{\partial g}{\partial p}$  .

Sixth word:  $\frac{\partial g'}{\partial p}$  .

All other words: Zeroes.

Fifth Output Card - A, the CDC Matrix

First word:  $a_{11}$  .

Second word:  $a_{12}$  .

Third word:  $a_{21}$  .

Fourth word:  $a_{22}$  .

All other words: Zeroes.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Sixth Output Card - A', the DCD MatrixFifth word:  $a'_{11}$ Sixth word:  $a'_{12}$ Seventh word:  $a'_{21}$ Eighth word:  $a'_{22}$ 

$$A' = \begin{pmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{pmatrix}$$

All other words: Zeroes.

Seventh Output Card.

First word:  $b(0)$ , distance from centerline of first field section to object principal plane in the CDC system.

Second word:  $b(I)$ , distance from centerline of last field section to image principal plane in the CDC system.

Fourth word:  $M$ , magnification in CDC system.

All other word: Zeroes.

Eighth Output Card

Fifth word:  $b'(0)$  distance from centerline of first field section to object principal plane in DCD system.

Sixth word:  $b'(I)$  distance from centerline of last field section to image principal plane in DCD system.

Eighth word:  $M'$ , magnification in DCD system.

All other words: Zeroes.

Ninth Output Card

First word:  $r$ , radius of confusion in CDC plane.

Fifth word:  $r'$ , radius of confusion in DCD plane.

All other words zeroes.

Tenth Output Card

Repeats second data card.

Eleventh Output Card

First word: maximum slope from object point in CDC plane.

Third word: maximum slope from object point in DCD plane.

## APPENDIX I. OPERATING INSTRUCTIONS

The program deck contains standard drum-clear and load-punch routines.

Place program deck followed by data cards in "read" hopper. Several sets (two data cards each) may be processed successively.

Set

Programmed	Stop		
Control	Run		
Overflow	Stop		
Error	Stop		
Console	70	1951	1951
Display	Program register		

Press

Computer reset  
 Program start  
 Read start  
 Punch start

On completion the program stops with a read command in the program register. About seven minutes are required for each case.

Remove punched cards from punch hopper and print.

## APPENDIX II. FORMULATION

In the CDC plane, successive application of the thin-lens formula gives

$$\alpha f_1^2 f_2 - \beta f_1^2 + \gamma f_1 f_2 + \delta f_1 + \epsilon f_2 - \eta = 0, \quad (1)$$

where

$$\left. \begin{aligned} \alpha &= p + q + 2d, \\ \beta &= (p + d)(q + d), \\ \gamma &= 2(pq + pd + qd), \\ \delta &= d(2pq + pd + qd), \\ \epsilon &= 2pdq, \\ \eta &= qpd^2, \end{aligned} \right\} (2)$$

with  $f_1$ , the focal length of the lens corresponding to the first (and third) magnetic field and  $f_2$ , the focal length for the second field.

For arguments  $\theta_1$  and  $\theta_2$  for the first and second fields respectively, we have

$$f_1 = \frac{l_1}{\theta_1 \sin \theta_1}, \quad f_2 = \frac{-l_2}{\theta_2 \sinh \theta_2}. \quad (3)$$

Substituting these values in (1) and rationalizing yields

$$\begin{aligned} & -\alpha l_1 l_2 - \beta l_1^2 \theta_2 \sinh \theta_2 + \delta l_1 l_2 \theta_1 \sin \theta_1 \\ & + \delta l_1 \theta_1 \theta_2 \sin \theta_1 \sinh \theta_2 - \epsilon l_2 \theta_1^2 \sin^2 \theta_1 - \\ & - \eta (\theta_1^2 \sin^2 \theta_1) (\theta_2 \sinh \theta_2) = 0. \end{aligned} \quad (4)$$

Letting  $x = \theta_2 \sinh \theta_2$  and  $y = \theta_1 \sin \theta_1$ , we write

$$x \left[ -c_2 + c_4 y - c_6 y^2 \right] + \left[ -c_1 + c_3 y - c_5 y^2 \right] = 0, \quad (5)$$

$$\left. \begin{aligned} \text{where } c_1 &= \alpha l_1^2 l_2, & c_4 &= \delta l_1, \\ c_2 &= \beta l_1^2, & c_5 &= \Sigma l_2, \\ c_3 &= \gamma l_1 l_2, & c_6 &= \eta, \end{aligned} \right\} \quad (6)$$

or, more simply,

$$x U_1(y) + V_2(y) = 0, \quad (7)$$

where

$$U_1(y) = -c_2 + c_4 y - c_6 y^2, \quad V_2(y) = -c_1 + c_3 y - c_5 y^2. \quad (8)$$

In the DCD plane (denoted by primes) we have, similarly,

$$\alpha' f_1'^2 f_2' - \beta' f_1' f_2' + \gamma' f_1' f_2' + \delta' f_1' + \epsilon' f_2' - \eta' = 0 \quad (9)$$

$$\left. \begin{aligned} \alpha' &= p' + q' + 2d, & \delta' &= d(2p'q' + p'd + q'd), \\ \beta' &= (p' + d)(q' + d), & \epsilon' &= 2p'q'd, \\ \gamma' &= 2(p'q' + p'd + q'd), & \eta' &= p'q'd^2, \end{aligned} \right\} \quad (10)$$

$$\text{with } f_1' = \frac{-l_1}{\theta_1 \sin \theta_1}, \quad f_2' = \frac{l_2}{\theta_2 \sinh \theta_2}, \quad (11)$$

$$\text{and } t = \theta_1 \sinh \theta_1, \quad s = \theta_2 \sin \theta_2. \quad (12)$$

We have

$$s U_2(t) = V_2(t) = 0 \quad (13)$$

with

$$U_2(t) = -D_2 - D_4 t - D_6 t^2, \quad V_2(t) = D_1 + D_3 t + D_5 t^2, \quad (14)$$

where

$$\left. \begin{aligned} D_1 &= \alpha' l_1^2 l_2 & D_4 &= \delta' l_1 \\ D_2 &= \beta' l_1^2 & D_5 &= \epsilon' l_2 \\ D_3 &= \gamma' l_1 l_2 & D_6 &= \eta' \end{aligned} \right\} \quad (15)$$

Equations (7) and (13) must be solved simultaneously for  $(\theta_1, \theta_2)$  corresponding to the thin-lens system.

The matrix representing the transformation of a trajectory vector by either field-free section is

$$\mathcal{L} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}, \quad \text{where } L = d - 1/2(l_1 + l_2) \quad \text{in either the} \quad (16)$$

CDC or DCD plane.

In the CDC plane the matrix for the first and third fields is



$$\begin{pmatrix} \cos \theta_1 & (l_1 \sin \theta_1)/\theta_1 \\ -(\theta_1 \sin \theta_1)/l_1 & \cos \theta_1 \end{pmatrix} = A \quad (17)$$

while for the second field it is

$$\begin{pmatrix} \cosh \theta_2 & (l_2 \sinh \theta_2)/\theta_2 \\ (\theta_2 \sinh \theta_2)/l_2 & \cosh \theta_2 \end{pmatrix} = B, \quad (18)$$

and the matrix for the whole CDC system is the product

$$A L_B L_A = A. \quad (19)$$

Likewise in the DCD plane we have

$$\begin{pmatrix} \cosh \theta_1 & (l_1 \sinh \theta_1)/\theta_1 \\ (\theta_1 \sinh \theta_1)/l_1 & \cosh \theta_1 \end{pmatrix} = A' \quad (20)$$

$$\begin{pmatrix} \cos \theta_2 & (l_2 \sin \theta_2)/\theta_2 \\ -(\theta_2 \sin \theta_2)/l_2 & \cos \theta_2 \end{pmatrix} = B', \quad (21)$$

and

$$A' L_{B'} L_{A'} = A'. \quad (22)$$

If  $A$  and  $A'$  applied respectively to a trajectory from  $p$  and to a trajectory from  $p'$  give trajectories that will cross the

longitudinal axis near  $q$  and  $q'$  respectively, then our thin-lens formulation is good enough. Otherwise we correct  $(\theta_1, \theta_2)$  by solving

$$\text{Error } (q) = \text{corr } (\theta_1) \frac{\partial q}{\partial \theta_1} + \text{corr } (\theta_2) \frac{\partial q}{\partial \theta_2}, \quad (23)$$

$$\text{Error } (q') = \text{corr } (\theta_1) \frac{\partial q'}{\partial \theta_1} + \text{corr } (\theta_2) \frac{\partial q'}{\partial \theta_2} \quad (24)$$

for  $\text{corr } (\theta_1)$  and  $\text{corr } (\theta_2)$ . The partial derivatives are approximated by difference ratios, using first a small variation of  $\theta_1$  with  $\theta_2$  fixed, then a small variation of  $\theta_2$  with  $\theta_1$  fixed.

The correction process is iterated until desired agreement with  $q$  and  $q'$  is obtained.

From the corrected solution  $(\theta_1, \theta_2)$  we compute

$$\left( \frac{dB}{dr} \right)_1 = \frac{P\theta_1^2}{0.3l_1^2}, \quad \left( \frac{dB}{dr} \right)_2 = \frac{P\theta_2^2}{0.3l_2^2}, \quad (25)$$

and then

$$\left. \begin{aligned} |f_1| &= \frac{l_1}{\theta_1 \sin \theta_1}, & |f_2| &= \frac{l_2}{\theta_2 \sinh \theta_2}, \\ |f'_1| &= \frac{l'_1}{\theta_1 \sinh \theta_1}, & |f'_2| &= \frac{l_2^2}{\theta_2 \sin \theta_2}. \end{aligned} \right\} \quad (26)$$

Next we compute numerators for partial derivatives,

$$N = -\alpha_p f_1^2 f_2 - \beta_p f_1^2 + \gamma_p f_1 f_2 + \delta_p f_1 - \epsilon_p f_2 - \eta_p \quad (27)$$

$$\text{where } \alpha_p = \frac{\partial \alpha}{\partial p}, \text{ etc.,}$$

and

$$N' = \alpha'_{p'} f'_{1^2} f'_{2'} - \beta'_{p'} f'_{1^2} + \gamma'_{p'} f'_{1'} f'_{2'} - \delta'_{p'} f'_{1'} + \epsilon'_{p'} f'_{2'} - \eta'_{p'}. \quad (28)$$

Then we compute

$$\left. \begin{aligned} \frac{\partial f_1}{\partial p} &= \frac{-N}{-2\alpha f_1 f_2 - 2\beta f_1 + \delta f_2 + \delta}, \\ \frac{\partial f_2}{\partial p} &= \frac{-N}{-\alpha f_1 + \delta f_1 - \epsilon}, \\ \frac{\partial q}{\partial p} &= \frac{-N}{-f_1^2 f_2 - f_1^2 \beta_q + 2f_1 f_2 \beta_q + f_1 \delta_q - 2f_2 \epsilon_q - \eta_q} \end{aligned} \right\} \quad (29)$$

$$\left. \begin{aligned} \frac{\partial f'_1}{\partial p'} &= \frac{-N'}{2\alpha' f'_{1'} f'_{2'} - 2\beta' f'_{1'} + \gamma' f'_{2'} - \delta'}, \\ \frac{\partial f'_2}{\partial p'} &= \frac{-N'}{\alpha' f'_{1^2} + \gamma' f'_{1'} + \epsilon'}, \\ \frac{\partial q'}{\partial p'} &= \frac{-N'}{f'_{1^2} f'_{2'} - f'_{1^2} \beta'_q + 2f'_{1'} f'_{2'} \beta'_q - f'_{1'} \delta'_q + 2f'_{2'} \epsilon'_q - \eta'_q} \end{aligned} \right\} \quad (30)$$

Next we compute corrected matrices  $A$  and  $A'$  from the corrected  $(\theta_1, \theta_2)$ , using Eqs. (16) through (21).

Using the vector  $u = (h, 0)$ , characterizing an entering trajectory parallel to the axis, we compute  $Au = v = (x_1, x_2)$  and  $A'u = v' = (y_1, y_2)$ ,

then

$$b(I) = - \left\{ \frac{l_1}{2} + \frac{h - x_1}{x_2} \right\}$$

and

$$b'(I) = - \left\{ \frac{l_1}{2} + \frac{h - y_1}{y_2} \right\}.$$

By symmetry of quadrupole we have

$$b(0) = b(I), \quad b'(0) = b'(I).$$

Next we compute magnifications,

$$M = \frac{s(I)}{s(0)}, \quad M' = \frac{s'(I)}{s'(0)},$$

where

$$\begin{aligned} s(I) &= b(I) + q, & s(0) &= b(0) + p, \\ s'(I) &= b'(I) + q', & s'(0) &= b'(0) + p'. \end{aligned}$$

To determine the radii of confusion we replace  $\theta_1$  by  $\theta_1 - 0.005 \theta_1$  and  $\theta_2$  by  $\theta_2 - 0.005 \theta_2$ , and compute new perturbed matrices  $A$  and  $A'$  by Eqs. (16) - (21), using these values. Using vectors  $u = (h, \frac{h}{p - l_1/2})$  and  $u' = (h, h/p' - l_1/2)$  for trajectories originating at  $p$  and  $p'$  respectively, we compute

$$Au = v = (x_1, x_2), \quad A'u' = v' = (y_1, y_2),$$

then

$$r = x_1 + x_2 (q - l_1/2) \quad \text{and} \quad r' = y_1 + y_2 (q' - l_1/2)$$

give respective radii of confusion.

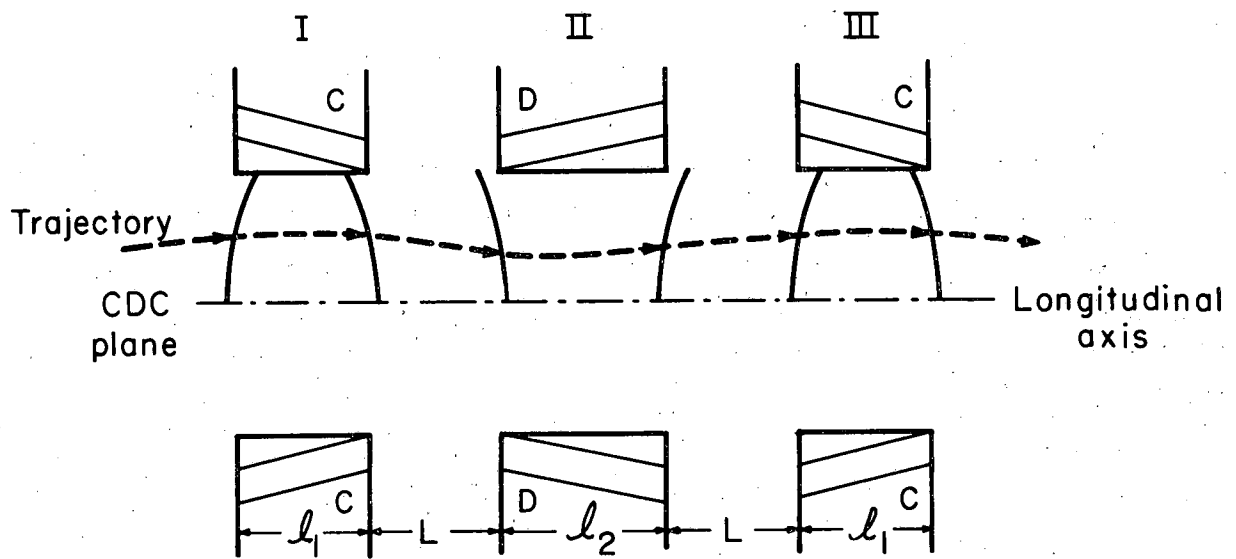
Reverting now to the corrected (not perturbed) solution  $(\theta_1, \theta_2)$  and the associated section matrices,  $A, \mathcal{L}, B$ , we take the vector  $u = \left( \frac{hc(p - \mathcal{L}_1/2)}{p - k}, \frac{hc}{p - k} \right)$  and compute  $Au = u_1 = (x_1, x_2)$

and test for  $x_1 < h$ ; if so then test for  $x_2 > 0$ ; if so the trajectory clears the face of the first field section.

For  $x_2 < 0$ , the trajectory has a maximum height within the field section which can be approximated by finding the intersection of the entering trajectory extended as a straight line, with the exiting trajectory traced back as straight line, for the distance  $h_m < h$  then the trajectory clears the face of the first field.

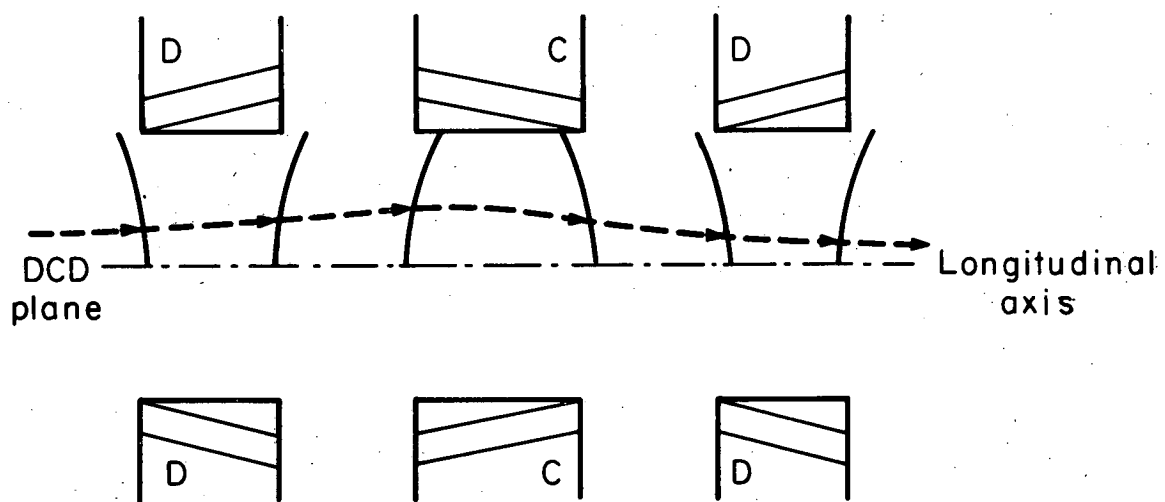
If the trajectory strikes the face of the first field section we reduce the slope of the trajectory from  $p$ .

Similar tests applied to successive sections in the CDC system and to the section in the DCD system finally give the maximum slopes  $\Omega, \Omega'$  for trajectories originating at  $p$  and  $p'$  respectively which will pass through the quadrupole without striking any surfaces.



MU-19115

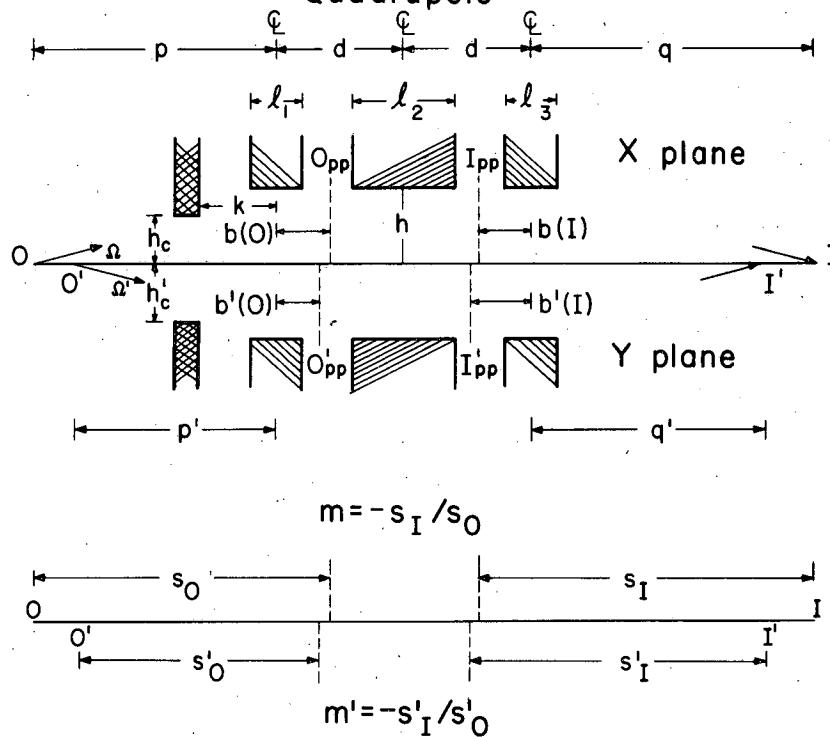
Fig. 1



MU-19116

Fig. 2

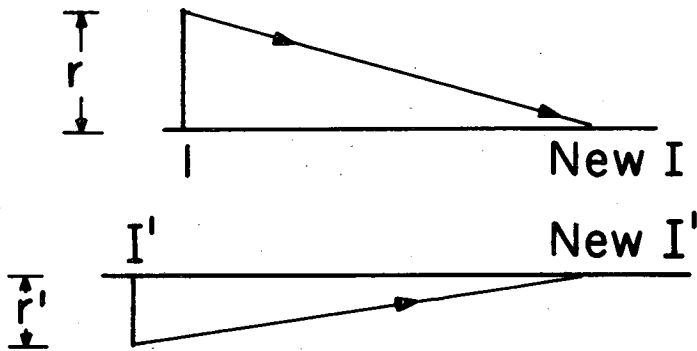
Tripole input and output  
Quadrupole



MU-19117

Fig. 3





$r =$  Radius of Confusion

MU - 19118

Fig. 4

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