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Abstract

This paper develops a simple analytical model of price and frequency competition among freight carriers. In the model, the full price faced by a shipper (a goods producer) includes the actual shipping price plus an inventory holding cost, which is inversely proportional to the frequency of shipments offered by the freight carrier. Taking brand loyalty on the part of shippers into account, competing freight carriers maximize profit by setting prices, frequencies and vehicle carrying capacities. Assuming tractable functional forms, long- and short-run comparative-static results are derived to show how the choice variables are affected by the model's parameters. The paper also provides an efficiency analysis, comparing the equilibrium to the social optimum, and it attempts to explain the phenomenon of excess capacity in the freight industry.

1 Introduction

The adoption of just-in-time (JIT) production processes has changed the face of manufacturing on several dimensions and has particularly affected the demand for freight transportation services. Regan and Garrido (2002) argue that JIT systems have changed the criteria by which freight carriers are selected, increased shipper-carrier communication, reduced the number of carriers used, and increased contractual relationships between producers and carriers. In addition, JIT production has made a particular quality dimension for freight services, the frequency of shipments, increasingly important, with reliability being a related concern.[1] Even beyond the specific JIT context, various authors have shown the importance of freight scheduling to shippers. A principal components analysis by Sheelagh and Gray (1993), which aggregates service characteristics into several main clusters, demonstrates that the component including service frequency is an important determinant of the choices of shippers using sea services as well as those shipping by air. Similarly, Lu (2003) discusses the importance of scheduling in ocean shipping, while providing extensive references to previous work on this topic.

With frequency well-established as a key component of service quality from the demand side, this paper shifts attention to the supply side by analyzing the "supply" of frequency by freight carriers. It does so by developing a model where carriers compete both on price and on this frequency dimension, recognizing its importance to shippers. Though related to several strands of literature on inventory and supply chain management and on logistics services, the current model is among the first to pare away the complexity of freight terminology in order to present the underlying "industrial organization" of freight competition in a stark, simple fashion. Through this simplification, the paper transparently depicts the nature of competition when carriers set price and service frequency, with an associated flexibility in the choice of vehicle size. The analysis is able to present economically intuitive comparative-static results that show the impact of changes in demand and cost parameters on the freight carrier's choice variables. The

model closely follows a previous model of frequency (or "schedule") competition among passenger carriers, developed by Brueckner (2010), and extends it to the freight context.[2]

In the model, producers ship out manufactured final goods to the market. Transportation cost includes, along with the actual per unit transport cost, the cost of storing inventory, which arises from the need to store output until a freight carrier picks it up from the factory. These inventory holding costs, which are analogous to the cost of "schedule delay" in the passenger framework of Brueckner (2010), are modeled very simplistically. Each producer runs a continuous process that generates output uniformly over a period. The time that an average unit of output is held in inventory is inversely proportional to the number f of evenly spread freight dispatches over the period. The "full price" of transportation per unit of output is then $p + \gamma/f$, which is the actual per unit price p charged by a carrier plus the average holding cost per unit.

De Vany and Saving (1977) also use a full-price concept in modeling the capacity and pricing decisions of trucking firms under uncertainty. In their model, stochastic waiting time W gives rise to an inventory holding cost of ηW (parallel to γ/f). W is determined using a complex queuing-theory approach, which was also adopted by a number of subsequent authors.[3] In the present model, by contrast, waiting time depends deterministically on service frequency f in a transparent fashion. A contribution of the paper is thus to provide a more elementary framework for the analysis of frequency competition, which clearly exposes the decisions made by freight carriers.

In a standard competitive set-up, all producers would choose the carrier offering the lowest full price $(p + \gamma/f)$ in the absence of some kind of friction. In the present model, the required friction comes from idiosyncratic "brand loyalty" toward carriers on the part of producers. With brand loyalty, a producer may end up choosing a preferred carrier even when its full price is relatively high.

The nature of brand loyalty has been much debated in economics and in the psychology and marketing literatures. Although development of a model where brand loyalty arises endogenously is beyond the scope of the paper, there are good reasons to think that

such loyalty may nevertheless be an important feature of the freight industry. Most importantly, the evidence shows that relations between freight carriers and shippers often involve long-term contracts (Regan (2004), Hubbard (1999), Hubbard (2001)). Such contracts lead to familiarity, which is likely to breed inertia in future choices (with a change of carrier involving search and other transaction costs). Brand loyalty could also arise from more traditional factors such as locational proximity to a carrier's base of operations or from other idiosyncratic factors. In the model, these considerations generate exogenous brand loyalty, which interacts with price and frequency offerings to yield an allocation of shippers across carriers.

The paper assumes tractable forms for the key functions in the model: the producer cost function, which depends on output; the producer utility function, which depends on profit and brand loyalty to the particular carrier used for shipping; the carrier cost function, which depends on vehicle capacity; the density function for brand loyalties. The analysis solves the carrier's profit-maximization problem and characterizes the resulting competitive equilibrium. It then derives short- and long-run comparative-static results, which show how service frequency, shipment price, and vehicle capacity depend on the parameters of the model. Numerical examples are provided to supplement and generalize the analytical results. The paper also provides an efficiency analysis, comparing the equilibrium to the social optimum, and it attempts to explain the phenomenon of excess capacity in the freight industry.

The paper abstracts away from modeling a specific mode of freight transportation in order to formulate and analyze a very general model of competition between carriers providing freight services. However, it is useful to place the model in some context. The model is easily applicable to the trucking industry, where service frequency and truck sizes are easily adjusted in response to demand. A key distinction in this industry is between truckload (TL) and less-than-truckload (LTL) services, and the model implicitly applies to the LTL case, where the individual shipment volume does not fill an entire truck (requiring the carrier to combine loads). The reason is that the model presumes

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that a shipper uses only a single carrier, which means that its shipment volume cannot exceed the chosen carrier's capacity. This LTL intepretation also applies in the case of container ships, where vehicle size and service frequency can again be easily adapted to meet demand and where a single shipper typically does not produce enough to fill a ship.[4] A similar story can be told about air freight, where the LTL case again seems typical and the fleet deployment strategy is to match appropriate-size aircraft to different routes at frequencies demanded by shippers.

The paper offers a rich analytical framework for studying freight-market competition. The model could be a useful tool for analyzing the impact of freight deregulation, changing tax and toll structures, oil price shocks, and so on.[5] Given the increasing importance of freight services and the dearth of theoretical models of freight competition, the paper provides a useful addition to the transportation economics literature.

2   The Model

2.1   Producer profit maximization

A homogeneous good is manufactured by a continuum of perfectly competitive producers with mass M. The producers rely on the services of n freight carriers to transport their outputs to the market.[6] Using a common technology, the producers each generate Q units of output at cost given by $E(Q)$, an increasing, strictly convex function. Output accumulates as inventory until it is shipped out, with p giving the unit price of shipping the good. With output produced evenly over a time circle of circumference T, the average inventory holding time per unit of output equals $T/2f$, where f is the number of freight dispatches (frequency). Letting μ equal the inventory holding cost per unit of output per unit of time, the average holding cost is then $\mu T/2f$, or, combining the constant terms, γ/f. Thus, the "full cost" of transport per unit of output is $p + \gamma/f$, the actual shipping cost plus the inventory holding cost.[7]

Letting r denote the fixed selling price per unit of output, a producer's profit is then

given by

$$R = \left[r - \left(p + \frac{\gamma}{f}\right)\right] Q - E(Q). \tag{1}$$

The first-order condition for the producer's profit-maximization problem is

$$r - p - \frac{\gamma}{f} = E'(Q). \tag{2}$$

From the first-order condition, it is clear that the solutions for the producer's output and the resultant profits can be expressed as

$$Q\left[r - p - \frac{\gamma}{f}\right] \quad and \quad R\left[r - p - \frac{\gamma}{f}\right], \tag{3}$$

which are both increasing functions of their argument.

2.2 Producer brand loyalty and utility

As discussed earlier, brand loyalty to carriers is an important assumption of the model. Loyalty adds a friction to producer choices that prevents a carrier from losing all its customers when it raises its price or reduces frequency. A producer's brand loyalty for carrier i is captured by the parameter a_i, $i = 1, 2 \ldots, n$, whose size reflects the strength of that loyalty. Note that each producer has a different vector of loyalties $(a_1, a_2, \ldots, a_n)$ to the different carriers, with the density function of loyalties over the mass of producers given by $\Phi(a_1, a_2 \ldots, a_n)$.

A producer's utility (u) depends on the profit earned as well as on the identity of the carrier used for shipping, via the brand loyalty assumption. When a given level of profit is earned using a less preferred carrier, the producer's utility is lower. Conditional on the choice of a particular carrier i, utility is assumed to be multiplicative in profits and the brand loyalty parameter (a_i), and it is written as

$$u(R_i, a_i) = a_i R\left[r - p_i - \frac{\gamma}{f_i}\right] \equiv a_i R_i. \tag{4}$$

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A producer will choose carrier 1 to ship its output when

$$u(R_1, a_1) > u(R_j, a_j), \qquad \forall j = 2, 3 \ldots, n. \tag{5}$$

Substituting from (4), carrier 1 is then chosen when

$$a_j < \frac{a_1 R \left[r - p_1 - \gamma / f_1 \right]}{R \left[r - p_j - \gamma / f_j \right]} \equiv \frac{a_1 R_1}{R_j}, \qquad \forall j = 2, 3 \ldots, n. \tag{6}$$

The inequality in (6) implies that, for a producer to choose carrier 1, its loyalty to all other carriers $(j = 2, 3 \ldots, n)$ must be sufficiently small.

2.3 Carrier shipment volume and profit

The total quantity q_i of shipments transported by a carrier i will depend on the extent of brand loyalty the carrier enjoys among producers and on the prices charged and frequencies provided by all carriers. Carrier 1's shipment volume is found by summing the outputs of producers who choose it:

$$q_1 = \int_{a_1 \, = \, 0}^{\bar{a}_1} \int_{a_2 \, = \, 0}^{\frac{a_1 R_1}{R_2}} \int_{a_n \, = \, 0}^{\frac{a_1 R_1}{R_n}} Q\left(r - p_1 - \frac{\gamma}{f_1}\right) \Phi(a_1, a_2 ..., a_n) \, da_1 da_2 \ldots da_n. \tag{7}$$

To see that the integral adds the outputs of producers using carrier 1, observe that the integrand equals firm output evaluated at p_1 and f_1, which is then weighted by density at the given $(a_1, a_2, \ldots, a_n)$ point, restricting attention to those points in the brand loyalty space where carrier 1 is chosen. This restriction can be seen in the limits of integration, recalling (6) and noting that a_i is assumed to lie in the interval $[0, \bar{a}_i]$, $i = 1, 2, \ldots, n$. Given q_1 from (7), carrier 1's revenue equals $p_1 q_1$.

On the cost side, the cost per departure for carrier 1 is given by $C(s_1)$, where s_1 is the capacity of the transport vehicle. The cost function, which is common across carriers, is increasing in s_1, and $C(s_1)/s_1$ is decreasing in s_1, reflecting economies of tonnage capacity (the cost per ton transported falls as vehicle capacity increases). The total cost for carrier

1 is equal to $f_1 C(s_1)$, frequency times the cost per departure.

Each carrier must have carrying capacity large enough to accommodate its demand, so that $f_1 s_1 \geq q_1$ must hold for carrier 1. As long as vehicle capacity s_1 is freely adjustable, this constraint holds with equality, implying the absence of excess capacity (no partially empty vehicles). Section 8 below assumes instead that there exists a minimum vehicle capacity, in which case excess capacity may emerge as a choice made by carriers in equilibrium (the contraint $f_1 s_1 \geq q_1$ is then nonbinding).

A carrier's objective is to maximize profit, which equals revenue minus cost. Thus, carrier 1's maximization problem can be expressed as

$$\max_{\{f_1, p_1, s_1\}} \pi_1 \; = \; p_1 q_1 \; - \; f_1 C(s_1) \qquad subject\ to\ \ f_1 s_1 = q_1, \tag{8}$$

with q_1 given above in (7). In the maximization problem, the choices of other carriers are viewed as fixed, as seen more clearly below.

3 Functional Form Assumptions

3.1 Producer's cost function

When it chooses carrier 1, producer 1's utility, as expressed in (4), equals

$$u(R_1, a_1) \; = \; a_1 R_1 \; = \; a_1 \left[\left(r - p_1 - \frac{\gamma}{f_1} \right) Q \; - \; E(Q) \right]. \tag{9}$$

The producer's cost function is assumed to take the form

$$E(Q) \; = \; Q^\beta, \quad \beta \; > \; 1, \tag{10}$$

so that the first-order condition (2) for profit (and hence) utility maximization becomes

$$r - p_1 - \frac{\gamma}{f_1} \; = \; \beta Q^{\beta - 1}. \tag{11}$$

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Solving for Q yields

$$Q = \left[\frac{1}{\beta}\left(r - p_1 - \frac{\gamma}{f_1}\right)\right]^{\frac{1}{\beta-1}}. \tag{12}$$

Then, substituting Q into the utility function in (4) gives utility when the producer chooses carrier 1, which equals

$$u(a_1, R_1) = a_1\delta\left[r - p_1 - \frac{\gamma}{f_1}\right]^{\frac{\beta}{\beta-1}}, \tag{13}$$

where $\delta = \left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}}\left(\frac{\beta-1}{\beta}\right)$.

3.2   Distribution of brand loyalty

The next functional form assumption is that the distribution of brand loyalty is symmetric and uniform, which yields[8]

$$\Phi(a_1, a_2 \ldots a_n) = \frac{M}{\alpha^n}, \tag{14}$$

where the upper limit of a_i, the parameter $\bar{a}_i$, equals α for all producers $i = 1, 2 \ldots, M$. Substituting equations (12) and (14) into (7), carrier 1's demand equals

$$q_1 = \int_{a_1=0}^{\alpha} \int_{a_2=0}^{\frac{a_1 R_1}{R_2}} \cdots \int_{a_n=0}^{\frac{a_1 R_1}{R_n}} \left(r - p_1 - \frac{\gamma}{f_1}\right)^{\frac{1}{\beta-1}} \left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}} \frac{M}{\alpha^n} \, da_1 da_2 \ldots da_n$$

$$= (r - p_1 - \gamma/f_1)^{\frac{1}{\beta-1}} \left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}} \frac{M}{\alpha^n} \int_{a_1=0}^{\alpha} \frac{a_1^{n-1} R_1^{n-1}}{\prod_{j\neq 1} R_j} \, da_1$$

$$= (r - p_1 - \gamma/f_1)^{\frac{1}{\beta-1}} \left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}} \frac{M}{n} \frac{\left[\delta \left(r - p_1 - \gamma/f_1\right)^{\frac{\beta}{\beta-1}}\right]^n}{\prod_{j=1}^{n} \delta \left(r - p_j - \gamma/f_j\right)^{\frac{\beta}{\beta-1}}}$$

$$= \left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}} \frac{M(r - p_1 - \gamma/f_1)^{\frac{(n-1)\beta+1}{\beta-1}}}{n \prod_{j\neq 1} \left(r - p_j - \gamma/f_j\right)^{\frac{\beta}{\beta-1}}}$$

$$\tag{15}$$

In the second-to-last line of (15), the term equal to $1/n$ times the final ratio expression equals the share of producers using carrier 1. The ratio expression can be either larger or smaller than unity depending on the price and frequency choices of the carriers, making the share greater than or less than one.[9]

3.3 Carrier's cost function

The cost function for carriers is assumed to be linear in capacity and given by

$$C(s_1) = \theta + \tau s_1, \tag{16}$$

where θ is the fixed cost of operating a vehicle and τ is the marginal cost of additional capacity. Note that (16) reflects economies of capacity, given that $C(s_1)/s_1$ is decreasing in s_1. With this cost function, carrier 1's total cost is

$$f_1 C(s_1) = (\theta + \tau s_1) f_1. \tag{17}$$

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4 The Carrier's Optimization Problem

Since $s_1 = q_1/f_1$ holds from the capacity constraint, carrier 1's total cost in (17) equals $f_1\theta + \tau q_1$. The carrier's profit in (8) can then be written as $(p_1 - \tau)q_1 - \theta f_1$. Substituting for q_1 from (15), profit equals

$$\pi_1 = (p_1 - \tau)\left(\frac{1}{\beta}\right)^{\frac{1}{\beta-1}} \frac{M(r - p_1 - \gamma/f_1)^{\frac{(n-1)\beta+1}{\beta-1}}}{n \prod_{j\neq 1}(r - p_j - \gamma/f_j)^{\frac{\beta}{\beta-1}}} - \theta f_1. \tag{18}$$

Carrier 1 then chooses p_1 and f_1 to maximize profit in Cournot fashion, taking its rivals' choices of price and frequency as given. Note that, while vehicle capacity does not appear as an explicit choice variable, it can be recovered using the relationship $s_1 = q_1/f_1$ after the optimal price and frequency are determined. Also, note that profit is independent of the degree of dispersion of brand loyalty (α), a feature also present in Brueckner's (2010) passenger-based model.

Assuming that the producer's cost function is quadratic ($\beta = 2$) helps to generate tractable analytical solutions. As seen in the numerical examples presented in section 5.6 below, the analytical results derived for this special case also hold for values of β other than 2. When $\beta = 2$, the profit function in (18) reduces to

$$\pi_1 = \frac{M(p_1 - \tau)}{2n} \frac{(r - p_1 - \gamma/f_1)^{2n-1}}{\prod_{j=2}^{n}(r - p_j - \gamma/f_j)^2} - \theta f_1. \tag{19}$$

Carrier 1 chooses f_1 and p_1 to maximize (8), taking f_j and p_j, $j \neq 1$, as given. The first-order condition for f_1 is

$$\frac{\partial \pi_1}{\partial f_1} = \frac{(2n-1)M(p_1 - \tau)}{2n} \frac{(r - p_1 - \gamma/f_1)^{2n-2}}{\prod_{j=2}^{n}(r - p_j - \gamma/f_j)^2} \frac{\gamma}{f_1^2} - \theta = 0, \tag{20}$$

and first-order condition for p_1 is

$$\frac{\partial \pi_1}{\partial p_1} = \frac{M\left[(r - p_1 - \gamma/f_1)^{2n-1} - (2n-1)(p_1 - \tau)(r - p_1 - \gamma/f_1)^{2n-2}\right]}{2n \prod_{j=2}^{n}(r - p_j - \gamma/f_j)^2} = 0. \tag{21}$$

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Given symmetric carriers, the focus is on a symmetric Nash equilibrium, where all carriers charge the same price and provide the same frequency.[10] With p_i and f_i, $i = 1, 2...n$, taking common values denoted by p and f, (21) simplifies (after canceling the multiplicative factor) to

$$r \ - \ p \ - \ \frac{\gamma}{f} \ = \ (2n - 1)(p - \tau). \tag{22}$$

Rearranging, this equation provides a solution for p in terms of f:

$$p \ = \ \frac{1}{2n} \left[r \ + \ (2n - 1)\tau \ - \ \frac{\gamma}{f} \right]. \tag{23}$$

Note, that unlike in a competitive market, where prices are driven down to marginal cost, the equilibrium p given in (23) exceeds the marginal capacity cost τ. This claim follows from (22) given that the left-hand-side expression must be positive.

After imposing symmetry and substituting the solution for p, (20) reduces to

$$\frac{(2n - 1)M\gamma}{2n^2 f^2} \left[r \ - \ \tau \ - \ \frac{\gamma}{f} \right] \ = \ \theta, \tag{24}$$

and rearranging terms yields

$$\frac{4n^2 \theta f^3}{(2n - 1)M} \ - \ \gamma[(r - \tau)f \ - \ \gamma] \ = \ 0. \tag{25}$$

Note that (25) is a cubic equation in f, which (in a manner similar to Brueckner (2004) and Brueckner and Flores-Fillol (2007)) can be used to compute a solution for f in terms of the model parameters. Then, (23) provides a solution for p as a function of f and hence as a function of model parameters.

The solution for f is best illustrated graphically using Figure 1. The first term in (25) corresponds to the cubic curve in the figure while the second term represents the straight line. The points of intersection of the curve and the line satisfy (25) and are

thus candidate solutions for f.[11] Only one of these intersections, however, satisfies the second-order conditions for the optimization problem, which are assumed to hold. These conditions, which require positive definiteness of the Hessian matrix for the problem, imply that the slope of the cubic curve must be greater than the slope of the line at the solution to (25) (see Appendix A.1). Since this condition is satisfied at point B in the figure but not at point A, point B is the relevant solution to (25).

Note that, since prices and frequencies are equal across carriers in a symmetric equilibrium, the distribution of shipments is determined by brand loyalty alone. In a duopoly case $(n = 2)$, producers whose combination of brand loyalties lies below (above) the 45° line in the (a_1, a_2) plane choose carrier 1 (2), a pattern that generalizes to higher n values.

5   Short-Run Comparative Statics

Treating the number of carriers n as an exogenous parameter, short-run comparative-static analysis can be carried out using equations (23) and (25). Long-run analysis, where n is endogenous, is discussed in the next section.

5.1   Effects of parameter changes on frequency

The effects on f of changes in the parameters M, θ, n, τ, and r are directly observable from Figure 1. An increase in θ raises the cubic curve, resulting in a smaller f, while an increase in M lowers the curve, leading to a larger f. An increase in n can be shown to increase the ratio $n^2/(2n-1)$, which raises the cubic curve and reduces f. A rise in r or a fall in τ increases the slope of the line, resulting in a larger f. Thus,

$$\frac{\partial f}{\partial M} > 0, \qquad \frac{\partial f}{\partial \theta} < 0, \qquad \frac{\partial f}{\partial \tau} < 0, \qquad \frac{\partial f}{\partial r} > 0, \qquad \frac{\partial f}{\partial n} < 0. \qquad (26)$$

The effect on f of an increase in γ, which is not directly observable from the figure,

is derived in Appendix A.2, and it is positive:

$$\frac{\partial f}{\partial \gamma} > 0. \tag{27}$$

All these results make sense intuitively. An increase in costs (τ or θ) leads to lower service frequency, while a larger market size (more shippers) increases shipment volumes, inducing carriers to provide more frequent service. The effect of the product price r follows from an indirect effect: profit-maximizing producers increase production when the market price of their product rises, resulting in larger volumes of output to be transported. Carriers, to accommodate this rise, increase frequency. Also, as expected, a higher inventory holding cost γ, which raises the cost of waiting, leads carriers to respond by raising frequency. Finally, a larger number of carriers n competing for a given market size means a smaller share of goods for any particular carrier and, hence, lower frequency.

5.2 Effects of parameter changes on price

To derive the effect of parameter changes on p from (23), both direct effects and indirect effects operating through f must be taken into account. To appraise the indirect effects, note from (23) that f and p are directly related:

$$\frac{\partial p}{\partial f} = \frac{1}{2n}\frac{\gamma}{f^2} > 0, \tag{28}$$

implying that carriers charge a price premium when they increase frequency, improving the quality of service. Since the parameters θ and M do not appear in (23), they affect p only indirectly through f. With (26) showing that a rise in fixed cost θ reduces f while a larger market size M increases f, (28) implies the following impacts on p:

$$\frac{\partial p}{\partial \theta} < 0, \qquad \frac{\partial p}{\partial M} > 0. \tag{29}$$

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Note that economic theory might suggest that a rise in fixed cost should not directly affect price. However, in the current set-up, θ has a negative effect on p via the dependence of price on frequency, which is affected by fixed cost. Also, an increase in the number of producers naturally raises price.

The remaining parameters (τ, r, γ and n) affect price directly as well as indirectly via frequency. In the case of r, the positive direct effect on p is reinforced by a positive indirect effect, given that f rises with r from (26). Therefore, the net effect of an increase in r is to raise p. The derivation of the net price effects for the other three parameters is presented in Appendix A.3. The direct positive effect of marginal cost τ on price dominates the indirect negative effect via frequency, so that p rises with τ. The net effect of a higher holding cost γ is dominated by the direct negative effect, causing p to fall, a result that is not very intuitive. Lastly, although the direct effect is ambiguous, the net effect of a larger number n of competing carriers is to reduce p, a natural conclusion. Thus,

$$\frac{\partial p}{\partial \tau} > 0, \qquad \frac{\partial p}{\partial r} > 0, \qquad \frac{\partial p}{\partial \gamma} < 0, \qquad \frac{\partial p}{\partial n} < 0. \tag{30}$$

5.3   Effects of parameter changes on full price

Having derived comparative-static effects for the basic decision variables f and p, it is useful to analyze the effects of parameter changes on the full price P, which determines the volume of shipments. Using (23), the full price is

$$P = p + \frac{\gamma}{f} = \frac{1}{2n}\left[r - (2n-1)\tau + (2n-1)\frac{\gamma}{f}\right]. \tag{31}$$

As seen in the second expression in (31), the parameters θ and M affect P only through f. An increase in θ reduces f by (26), which raises P, while an increase in M raises f, which reduces P. By contrast, since an increase in τ both increases p and reduces f, the net effect is to raise P, using the first expression in (31). The effects of the remaining parameters are not clear from inspection and are derived in Appendix A.4.

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Following standard economic intuition, the direct positive effect of a change in r in the second expression in (31) dominates the indirect negative effect through f, so that full price rises when product price rises. Similarly, the direct positive effect of γ dominates the negative effect operating through f, leading to an increase in P. Lastly, a rise in the number of competing carriers naturally results in lower a full price. Thus,

$$\frac{\partial P}{\partial M} < 0, \quad \frac{\partial P}{\partial \theta} > 0, \quad \frac{\partial P}{\partial \tau} > 0, \quad \frac{\partial P}{\partial r} > 0, \quad \frac{\partial P}{\partial \gamma} > 0, \quad \frac{\partial P}{\partial n} < 0. \quad (32)$$

5.4 Effects of parameter changes on total shipment volume

Total shipment volume (TS) is the sum of the symmetric equilibrium output produced by M firms or, equivalently, the sum of symmetric equilibrium quantity transported by n carriers (derived from (12) and (15)):

$$TS = MQ = nq = \frac{M}{2}\left(r - p - \frac{\gamma}{f}\right) = \frac{M}{2}(r - P), \quad (33)$$

where the second-to-last equality uses (12) with $\beta = 2$. It is clear that P and TS are inversely related, with $\partial TS/\partial P = -M/2 < 0$. Thus, using (32), it can be concluded that the effects of the parameters θ, τ, γ and n, which affect TS only through P, are

$$\frac{\partial TS}{\partial \theta} < 0, \quad \frac{\partial TS}{\partial \tau} < 0, \quad \frac{\partial TS}{\partial \gamma} < 0, \quad \frac{\partial TS}{\partial n} > 0. \quad (34)$$

Thus, reflecting the increase in P, total shipments fall when any of the cost parameters $(\theta, \tau, \text{ or } \gamma)$ increases, while rising with the number of carriers (in response to a lower P).

The parameters M and r have direct effects on TS along with indirect effects via P. When M increases, both the direct and indirect effects work in the same direction since P falls by (32), resulting in a higher TS. The direct positive effect of r on TS dominates the indirect negative effect via P (see Appendix A.5), so that total shipments again rise with r. This seems like a natural conclusion since it implies that the positive effect of

the product price on shipments dominates the induced effect of more expensive freight services used to transport the output to the market. Thus,

$$\frac{\partial TS}{\partial M} > 0, \qquad \frac{\partial TS}{\partial r} > 0. \tag{35}$$

5.5   Effects of parameter changes on vehicle capacity

Finally, consider comparative-static effects on the vehicle capacity chosen by carriers, which is given by $s = q/f$. To evaluate this ratio, the q expression in (15) is simplified by imposing symmetry and the p solution from (23) is substituted into it. Then, dividing by f yields

$$s = \frac{(2n-1)}{8n} \frac{M}{f} \left[r - \tau - \frac{\gamma}{f} \right]. \tag{36}$$

The comparative-static results for s are derived in Appendix A.6, taking account of the direct parameter effects in (36) along with indirect effects via f. The results are:

$$\frac{\partial s}{\partial \theta} > 0, \quad \frac{\partial s}{\partial \tau} < 0, \quad \frac{\partial s}{\partial M} > 0, \quad \frac{\partial s}{\partial r} > 0, \quad \frac{\partial s}{\partial \gamma} < 0, \quad \frac{\partial s}{\partial n} > 0. \tag{37}$$

The effects of changes in fixed or marginal cost are intuitive: a rise in fixed cost θ provides incentives for carriers to increase vehicle capacity, whereas a higher marginal cost τ encourages the use of smaller vehicles. A larger market and a higher product price both lead to bigger vehicle sizes. Less intuitively, an increase in γ reduces capacity, while an increase in n has the opposite effect. Since a larger γ raises frequency while reducing q through a higher P, $s = q/f$ must fall. Conversely, since a larger n raises q through a lower P while reducing frequency, s must rise.

Table 1 summarizes the short-run comparative-static effects, which are also stated in the following proposition:

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Proposition 1: *The short-run comparative-static properties of the model are as follows:*

(a) *An increase in fixed cost θ reduces frequency and price while raising vehicle capacity. The full price rises and shipment volume falls.*

(b) *An increase in marginal cost τ reduces frequency, raises price, and reduces vehicle capacity. The full price rises and shipment volume falls.*

(c) *An increase in the number of producers M increases frequency, price and vehicle capacity. The full price falls and shipment volume rises.*

(d) *An increase in the market price r of output increases frequency, price, and vehicle capacity. The full price and shipment volume rise.*

(e) *An increase in the inventory cost parameter γ increases frequency and reduces price and vehicle capacity. The full price rises and shipment volume falls.*

(f) *An increase in the number of competing carriers n reduces frequency and price and raises vehicle capacity. The full price falls and shipment volume rises.*

5.6 Numerical results for the short-run equilibrium

Numerical analysis confirms the analytical results derived so far, but its main purpose is to show the same results hold for values of β different from 2. Setting base values for parameters at $M = 1000$, $\theta = 1$, $\tau = 0.01$, $r = 1$, $\gamma = 0.75$, $n = 2$ and $\beta = 2$ leads to an equilibrium with p and f values of 11.4 and 0.241, respectively, as seen in Table 2. As a check on the analytical results presented in equations (26)–(37) and summarized in Table 1, Table 2 shows the effect of changes in the other parameter values for the $\beta = 2$ case. For purposes of illustration, each parameter value is doubled. The results naturally match the analytical findings. For example, increasing the number of producers M from 1000 to 2000 raises f by around 43 percent, to 16.3, and raises price marginally by 2 percent, to 0.246.

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Next, the same exercise is carried out with β values of 1.5 and 2.5, with Tables 3 and 4 showing the results. As seen in these tables, the comparative-static effects are qualitatively identical to those in Table 2. Although the actual magnitudes of the variables are not informative, the percentage changes give a sense of the relative magnitudes of parameter impacts. For example, the effects of parameter changes on f are relatively large in percentage terms compared to the effects on p.

The effects of changes in β on the equilibrium outcomes can also be seen in the numerical results. Borrowing information Table 4, the last row of Table 2 shows the effect of increasing β from 2 to 2.5, holding the other parameters at their base values. It can be seen that an increase in β tends to reduce p, f and P, while raising s.

6   Long-Run Comparative Statics

In the long-run, the number of carriers in the market is endogenous. Carriers freely enter and exit the market, which drives profit down to zero. Evaluating the profit expression (18) at the symmetric equilibrium and setting it equal zero yields

$$\pi \;=\; \frac{M}{8n^3}(2n-1)\left(r-\tau-\frac{\gamma}{f}\right)^2 \;-\; \theta f \;=\; 0. \tag{38}$$

Rearranging terms then yields

$$8n^3\theta f \;-\; M(2n-1)\left(r-\tau-\frac{\gamma}{f}\right)^2 \;=\; 0. \tag{39}$$

Equations (25) and (39) simultaneously determine long-run equilibrium values for the number of carriers ($\bar{n}$) and frequency ($\bar{f}$). These values, substituted into (23), yield a solution for the long-run equilibrium price ($\bar{p}$), and solutions for the full price $\overline{(p+\gamma/f)}$ and vehicle capacity ($\bar{s}$) can then be derived.

Since the equation system consisting of (25) and (39) is analytically intractable, numerical calculations are used to derive the long-run comparative-static effects. Table 5

presents the numerical results, again setting $M = 1000$, $\theta = 1$, $\tau = 0.01$, $r = 1$, $\gamma = 0.75$ and $\beta = 2$ as the base case and now letting n be endogenous. The effects of varying the parameter values on the long-run magnitudes of the endogenous variables are summarized in Table 6. In the table, a $*$ next to a sign indicates that it is different from the short-run sign.

At first glance, the results appear atypical since some of the long-run effects take signs opposite to those of the short-run effects. While the signs for effects on f and s are the same as in the short-run, the effects on p and P change sign between the short- and long-run. In the case of p, this reversal can be explained by noting that parameter changes now have indirect effects through changes in n, in addition to having direct effects on p and indirect effects via f. Thus, the net price effect comprises three partial effects. The indirect effects via n seem to dominate in the long-run comparative-static effects on p and, hence, these effects change signs from the short-run effects.[12]

7 Social Optimum

The next step in the analysis is to compare the equilibrium to the social optimum. Holding n fixed, the social welfare function to be maximized is assumed to equal the sum of producer and carrier profits, given by

$$S = \sum_{m=1}^{M} R_m + \sum_{i=1}^{n} \pi_i = MR + n\pi \qquad (40)$$

Note that producer utility, which includes a brand loyalty term that multiplies profit, is not used in generating the welfare function. The planner's objective could instead include producer utility, but the resulting presence of a multiplicative brand-loyalty factor would prevent a clearcut comparison between the social optimum and the equilibrium. In any case, a welfare function based purely on profit is natural.[13]

Substituting the functional forms for R and π from (13) and (19), the welfare function

can be written as

$$S = \frac{M}{4}\left(r - p - \frac{\gamma}{f}\right)^2 + n\left[\frac{M(p-\tau)}{2n}\left(r - p - \frac{\gamma}{f}\right) - \theta f\right]$$

$$= \frac{M}{2}\left[\frac{1}{2}\left(r - p - \frac{\gamma}{f}\right)^2 + (p-\tau)\left(r - p - \frac{\gamma}{f}\right)\right] - \theta f n. \quad (41)$$

In maximizing welfare, the first-order condition for choice of p gives the standard economic rule of pricing at marginal cost:

$$p_{opt} = \tau. \quad (42)$$

The optimal price p_{opt} given by (42) is then lower than the equilibrium price p_{equi} given by (23):

$$p_{equi} > p_{opt}. \quad (43)$$

This result also holds in the passenger-based model of Brueckner (2010).

The first-order condition for frequency reduces to another cubic equation in f:

$$\frac{4n^2\theta f_{opt}^3}{2M} - \gamma[(r-\tau)f_{opt} - \gamma] = 0. \quad (44)$$

The relation between the optimal and equilibrium frequencies can be derived by comparing (44) and (25). Since $2 < (2n-1)$ for all $n \geq 2$, the cubic curve in equation (44) is higher than the one in (25). The intersection with the line then moves leftward, as seen in Figure 2. Thus,

$$f_{equi} > f_{opt}, \quad (45)$$

indicating that carriers overprovide frequency in equilibrium. Thus, in maximizing their own profits, carriers choose a frequency level higher than the one that would maximize the combined profits of producers and carriers.

This overprovision result is opposite to that derived in the passenger-based model of Brueckner (2010), where carriers underprovide frequency. Thus, while a general conclu-

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sion is that the optimum and equilibrium frequencies differ in models of this type, the exact relation between the two frequencies appears to depend on the details of the model structure. This issue could be further explored by reanalyzing the present model under different functional-form assumptions, looking for a potential reversal of the overprovision result.

The following proposition summarizes the comparison between the equilibrium and the optimum:

Proposition 2:

(a) *The equilibrium price is higher than the optimal price, which equals marginal cost.*

(b) *The equilibrium frequency is higher than the optimal frequency, implying that, in equilibrium, freight carriers overprovide service frequency.*

(c) *The equilibrium full price, shipment volume and vehicle capacity are not clearly comparable to their optimal counterparts.*

8 Excess Capacity

In the analysis above, carriers operate at full capacity (with $fs = q$), so that no vehicle is ever operated partially empty. This outcome, however, can be viewed as unrealistic. Empty miles, which is the distance traveled by a truck without cargo, is an important efficiency measure in the trucking industry. It is claimed that empty miles were substantially reduced following deregulation (Regan 2004). Also, heavy adoption of information technology in the trucking industry has helped reduce empty miles by facilitating better matching of cargoes and available capacity. Despite these improvements, carriers will still not always operate at full capacity.

While various factors (including stochastic demand by shippers) could lead to partially empty freight vehicles, another possibility is that excess capacity arises out of the carrier's

desire to maintain an attractive level of service frequency. This incentive can lead to excess capacity when vehicle capacity is not fully flexible, with carriers constrained by some minimum vehicle capacity $\bar{s}$. In such cases, excess capacity can only be eliminated by reducing frequency, which may be undesirable.

Algebraically, in the presence of a minimum vehicle capacity, carrier 1's profit-maximization problem has two constraints: $f_1 s_1 \geq q_1$ and $s_1 \geq \bar{s}$. While the first constraint held as an equality before, this outcome may no longer be optimal. The optimization is carried out by formulating the following Lagrangian for carrier 1:

$$L = p_1 q_1 - f(\theta + \tau s_1) + \lambda(s_1 - \bar{s}) + \phi(f_1 s_1 - q_1), \qquad (46)$$

where q_1 is given by (15). In this new problem, the first-order condition for frequency is

$$- \tau f + \lambda + \phi f = 0 \qquad (47)$$

If $\lambda = 0$, indicating that the minimum capacity constraint does not bind, then (47) yields $\phi = \tau > 0$, which implies that the capacity constraint binds. In this case, the problem reduces to the one already analyzed. But when $\lambda > 0$, so that the minimum capacity constraint binds, then (47) implies $\phi < \tau$, in which case $\phi = 0$ is admissible, implying excess capacity.

In his passenger-based model, Brueckner (2010) analyzed the parameter changes that cause a reduction in ϕ, arguing that large enough changes would drive ϕ to zero, leading to the emergence of excess capacity. His analysis showed that excess capacity would emerge when the minimum capacity $\bar{s}$ is large, when passenger schedule-delay cost is high (making frequency reductions less desirable), or when the number of carriers is large (limiting shipment volume per carrier). Unfortunately, a parallel analysis in the present model is intractable, and recourse to numerical methods would be required to identify the parameter changes capable of generating excess capacity. While this exercise could be a task for future work, the preceding analysis at least shows the nature of the problem.

9 Conclusion

This paper has presented and analyzed a simple model where freight carriers compete in price and service frequency. The model has captured key industry characteristics, such as the importance of timely deliveries as well as brand loyalty. The paper has used familiar functional-form assumptions to produce a tractable set-up that generates useful comparative-static results showing the short-run effects of the model parameters on frequency, shipment price, vehicle carrying capacity, and other variables. It has derived long-run effects numerically while also providing an efficiency analysis of equilibrium outcomes. Furthermore, the model has provided a possible explanation for the presence of excess capacity in the freight industry as an equilibrium choice made by carriers in the presence of a minimum vehicle capacity constraint.

Freight is shipped by truck, train, aircraft, and ship. All significant economic features, including demand drivers, cost functions, the number of competitors, etc., are vastly different across and even within these freight modes. The model does not capture such differences but rather abstracts away from them in order to focus on a very basic but crucial quality variable: service frequency, and its related choice variable, vehicle size. Despite its abstractions, the model generates clear comparative-static predictions that could be tested with real-world data. Assuming the availability of such data, useful further research might then be empirical in nature, being designed to judge the relevance and accuracy of the model's predictions. Future work could also attempt to generalize the model by eliminating reliance on specific functional forms where possible. Alternatively, the model could be analyzed under different functional-form assumptions, with the goal of gauging the robustness of the results in that fashion.

A Appendix

A.1 Second-order conditions and the relevant f solution

In Figure (1), the cubic curve and the straight line intersect at two points in the positive quadrant, A and B. If the cubic curve is flatter than the line at the solution, as is true at point A, then the second-order conditions are violated, ruling out the solution. The relevant solution is thus point B. The following analysis proves this claim, assuming $n = 2$ so as to make the proof easier to follow.

The second-order derivatives, evaluated at the symmetric equilibrium (when $n = 2$), are

$$\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{3M}{2(r - p - \gamma/f)} \left[-r + 2p + \frac{\gamma}{f} - \tau \right] \tag{48}$$

$$\frac{\partial^2 \pi_1}{\partial f_1^2} = \frac{3M\gamma(p - \tau)}{2f^3(r - p - \gamma/f)} \left[-r + p + 2\frac{\gamma}{f} \right] \tag{49}$$

$$\frac{\partial^2 \pi_1}{\partial f_1 \partial p_1} = \frac{3M\gamma}{4f^2(r - p - \gamma/f)} \left[r - 3p - \frac{\gamma}{f} + 2\tau \right] \tag{50}$$

Using (23), The second-order conditions for maximization require

$$\frac{\partial^2 \pi_1}{\partial p_1^2} \simeq -r + 2p + \frac{\gamma}{f} - \tau = -r + \tau + \frac{\gamma}{f} < 0, \tag{51}$$

where $\simeq$ means 'same sign as', implying

$$r - \tau - \frac{\gamma}{f} > 0. \tag{52}$$

In addition,

$$\frac{\partial^2 \pi_1}{\partial f_1^2} \simeq -r + p + 2\frac{\gamma}{f} = -r + \tau + \frac{7}{3}\frac{\gamma}{f} < 0 \tag{53}$$

implies

$$r - \tau - \frac{7}{3}\frac{\gamma}{f} > 0. \tag{54}$$

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Furthermore,

$$
\begin{aligned}
|H| \; &= \; \left(\frac{\partial^2 \pi_1}{\partial p_1^2}\right)\left(\frac{\partial^2 \pi_1}{\partial f_1^2}\right) \; - \; \left(\frac{\partial^2 \pi_1}{\partial f_1 \partial p_1}\right)^2 \\
&\simeq \; 4(p-\tau)\left[-r \, + \, 2p \, + \, \frac{\gamma}{f} \, - \, \tau\right]\left[-r \, + \, p \, + \, 2\frac{\gamma}{f}\right] \, - \, \frac{\gamma}{f}\left[r \, - \, 3p \, - \, \frac{\gamma}{f} \, + \, 2\tau\right]^2 .
\end{aligned}
$$

After considerable manipulation, it can be shown that the positivity of Hessian reduces to the following requirement:

$$
r \; - \; \tau \; - \; \frac{5}{2}\frac{\gamma}{f} \; > \; 0. \tag{55}
$$

Comparing the above conditions, it is clear that the positivity of the Hessian determinant is the most stringent condition, since satisfaction of (55) implies that (52) and (54) are satisfied.

Now consider the slopes of the cubic curve and the straight line in (25). The slope of the cubic curve is $12\theta n^2 f^2/(2n-1)M = 16\theta f^2/M$ when $n = 2$, and the slope of the straight line is $\gamma(r-\tau)$. But at an f solution, $16\theta f^2/M = (3/f)\gamma((r-\tau)f - \gamma)$ holds, using (25). Therefore, for the cubic curve to be flatter than the line,

$$
\frac{3\gamma}{f}[(r-\tau)f - \gamma] \; < \; \gamma(r-\tau) \tag{56}
$$

must hold. Rearranging, this inequality reduces to

$$
r \; - \; \tau \; - \; \frac{3}{2}\frac{\gamma}{f} \; < \; 0 \tag{57}
$$

If the inequality in (57) holds, then the inequality in (55) cannot be satisfied. Thus, the second-order conditions are violated at point A, where the cubic curve is flatter than the line, ruling out A as the solution. When (55) is satisfied, then the reverse of the inequality in (57) is guaranteed to hold. Thus, if the second-order conditions are satisfied, the cubic curve must be steeper than the line, as at point B. Hence, the claim that B is the relevant solution is established.

A.2 Signing $\partial f/\partial \gamma$

In order to sign $\partial f/\partial \gamma$, recall from the discussion in Appendix A.1 that the cubic curve is steeper than the straight line at the relevant solution. Hence, the inequality

$$12n^2\theta f^2 \;-\; (2n-1)M\gamma(r-\tau) \;>\; 0 \tag{58}$$

must hold. Totally differentiating (25) with respect to f and γ yields

$$\frac{\partial f}{\partial \gamma} \;=\; \frac{(2n-1)M\gamma(r-\tau)f \;-\; 2(2n-1)M\gamma}{12n^2\theta f^2 \;-\; (2n-1)M\gamma(r-\tau)}. \tag{59}$$

The denominator of (59) is positive from (58), while the numerator takes the same sign as $(r-\tau)f - 2\gamma$, which is positive from (55). Hence, $\partial f/\partial \gamma \;>\; 0$.

A.3 Signing effects on p

Accounting for both direct effects p and indirect effects via f, the effect of an increase in γ on p is given by

$$\frac{\partial p}{\partial \gamma} \;=\; p_\gamma \;+\; p_f\frac{\partial f}{\partial \gamma} \;, \tag{60}$$

where p_γ and p_f denote the partial derivatives of the p solution (23) with respect to γ and f, respectively. Substituting from (59) into (60) yields

$$
\begin{aligned}
\frac{\partial p}{\partial \gamma} &= -\frac{1}{2nf} + \frac{\gamma}{2nf^2}\left[\frac{(2n-1)[M(r-\tau)f \;-\; 2M\gamma]}{12n^2\theta f^2 \;-\; (2n-1)M\gamma(r \;-\; \tau)}\right] \\
&= -\frac{1}{nf^2}\left[\frac{n\theta f^3 \;+\; (2n-1)M\gamma^2}{12n^2\theta f^2 \;-\; (2n-1)M\gamma(r-\tau)}\right] \\
&< \;0, \tag{61}
\end{aligned}
$$

using (58). The effects of an increase in τ or n on p can be signed similarly.

A.4 Signing effects on P

The full price P is given by (31). Consider the effect of a change in r:

$$\frac{\partial P}{\partial r} \;=\; P_r \;+\; P_f \frac{\partial f}{\partial r}\,, \tag{62}$$

where P_r and P_f denote the partial derivatives of the P solution (31) with respect to r and f, respectively. Computing $\partial f/\partial r$ and substituting in (62) yields

$$\begin{aligned}
\frac{\partial P}{\partial r} &= \frac{1}{2n} - \frac{(2n-1)}{2n}\frac{\gamma}{f^2}\left[\frac{-(2n-1)M\gamma f}{12n^2\theta f^2 - (2n-1)M\gamma(r-\tau)}\right] \\
&= \frac{1}{2n} + \frac{(2n-1)}{2n}\frac{\gamma}{f^2}\left[\frac{(2n-1)M\gamma f}{12n^2\theta f^2 - (2n-1)M\gamma(r-\tau)}\right] \\
&> 0,
\end{aligned} \tag{63}$$

using (58). The effects of an increase in γ or n on P can be signed similarly.

A.5 Signing partial effects on TS

Given total shipment volume in (33), the effect of a change in r on TS is derived as follows:

$$\begin{aligned}
\frac{\partial TS}{\partial r} &= \frac{M}{2}\left[1 - \frac{\partial P}{\partial r}\right] \\
&= \frac{M}{2}\left[1 - \frac{12n^2\theta f^3 - (2n-1)M\gamma(r-\tau)f - (2n-1)^2 M\gamma^2}{12n^2\theta f^3 - (2n-1)M\gamma(r-\tau)f}\right] \\
&= \frac{M}{2}\left[\frac{(2n-1)^2 M\gamma^2}{12n^2\theta f^3 - (2n-1)M\gamma(r-\tau)f}\right] \\
&> 0,
\end{aligned} \tag{64}$$

using (58). The effect on TS of a change in M can be signed similarly.

A.6 Signing effects on s

The vehicle capacity s is given by (36). Consider the effect of a change in τ:

$$\frac{\partial s}{\partial \tau} \;=\; s_\tau \;+\; s_f \frac{\partial f}{\partial \tau} \, , \tag{65}$$

where s_τ and s_f denote the partial derivatives of the s solution (36) with respect to τ and f, respectively. Computing $\partial f / \partial \tau$ and substituting in (65) yields

$$
\begin{aligned}
\frac{\partial s}{\partial \tau} \;&=\; -\frac{(2n-1)}{2nf} - \frac{(2n-1)M}{8nf^2}\left(r - \tau - \frac{\gamma}{f}\right)\left[\frac{(2n-1)M\gamma f}{12n^2\theta f^2 - (2n-1)M\gamma(r-\tau)}\right] \\
&=\; -\frac{(2n-1)M}{8f}\left[1 + \frac{(2n-1)M\gamma(r-\tau-\gamma/f)}{12n^2\theta f^2 - (2n-1)M\gamma(r-\tau)}\right] \\
&<\; 0,
\end{aligned}
\tag{66}
$$

using (58). All other effects on s can be signed similarly.

B Figures
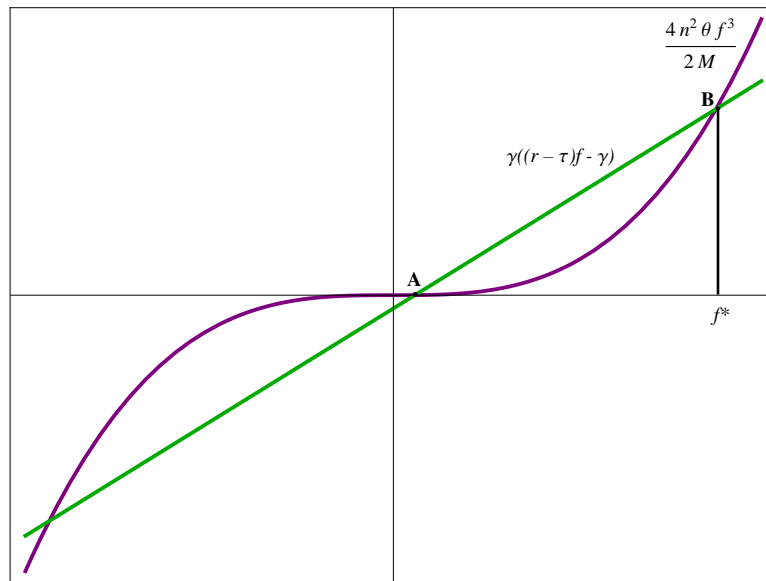

Figure 1: Solution for f
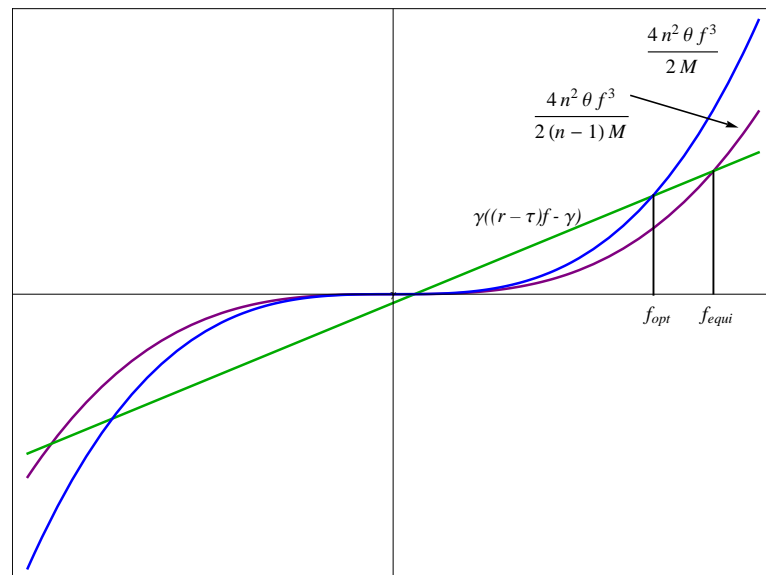
Figure 2: Optimal versus Equilibrium $'f'$

C Tables

Table 1: Short-run Comparative Static Effects

Variable: Parameter:	Frequency (f)	Price (p)	Full Price (P)	Shipment Volume (TS)	Capacity (s)
Market Size (M)	+	+	−	+	+
Fixed Cost (θ)	−	−	+	−	+
Marginal Cost (τ)	−	+	+	−	−
Product Price (r)	+	+	+	+	+
Holding Cost (γ)	+	−	+	−	−
Carriers (n)	−	−	−	+	+

Table 2: Short-run Comparative Static Effects
Numerical Exercise [Case 1: $\beta = 2$]

Parameter Values	Values				Percentage change			
	f	p	P	s	f	p	P	s
Base Case	11.40	0.241	0.307	60.8	−	−	−	−
M = 1000 → 2000	16.29	0.246	0.292	86.9	42.9	2.0	-4.8	42.9
$\theta = 1 \to 2$	7.93	0.234	0.328	84.6	-30.4	-3.0	7.0	39.2
$\tau = 0.01 \to 0.02$	11.34	0.248	0.315	60.5	-0.6	3.1	2.5	-0.6
r = 1 → 2	16.54	0.496	0.542	88.2	45.1	105.8	76.5	45.1
$\gamma = 0.75 \to 1.5$	15.87	0.234	0.328	42.3	39.2	-3.0	7.0	-30.4
n = 2 → 4	8.61	0.123	0.210	91.8	-24.5	-49.0	-31.6	51.0
$\beta = 2 \to 2.5$	12.36	0.289	0.349	56.4	-8.4	-19.8	-13.9	7.3
Base Case: { M = 1000, $\theta = 1$, $\tau = 0.01$, r = 1, $\gamma = 0.75$, n = 2, $\beta = 2$}								

Table 3: Short-run Comparative Static Effects
Numerical Exercise [Case 2: $\beta = 1.5$]

Parameter Values	Values				Percentage change			
	f	p	P	s	f	p	P	s
Base Case	9.83	0.162	0.239	69.7	–	–	–	–
M = 1000 → 2000	14.26	0.166	0.219	98.6	45.1	2.4	-8.3	41.4
$\theta = 1 \to 2$	6.68	0.156	0.269	98.6	-32.1	-3.7	12.6	41.4
$\tau = 0.01 \to 0.02$	9.71	0.170	0.248	69.7	-1.2	5.0	3.8	0.01
r = 1 → 2	21.03	0.336	0.371	69.7	113.9	106.9	55.7	0.01
$\gamma = 0.75 \to 1.5$	13.35	0.156	0.269	49.3	35.8	-3.7	12.6	-29.3
n = 2 → 4	7.44	0.084	0.185	104.6	-24.3	-48.2	-22.5	50.0
Base Case: { M = 1000, $\theta = 1$, $\tau = 0.01$, r = 1, $\gamma = 0.75$, n = 2, $\beta = 1.5$}								

Table 4: Short-run Comparative Static Effects
Numerical Exercise [Case 3: $\beta = 2.5$]

Parameter Values	Values				Percentage change			
	f	p	P	s	f	p	P	s
Base Case	12.36	0.289	0.349	56.4	–	–	–	–
M = 1000 → 2000	17.60	0.294	0.337	80.8	42.3	1.9	-3.6	43.2
$\theta = 1 \to 2$	8.66	0.281	0.368	78.2	-30.0	-2.7	5.2	38.8
$\tau = 0.01 \to 0.02$	12.32	0.296	0.357	56.0	-0.4	2.4	2.0	-0.7
r = 1 → 2	15.81	0.593	0.640	92.2	27.9	105.3	83.2	63.5
$\gamma = 0.75 \to 1.5$	17.32	0.281	0.368	39.1	40.1	-2.7	5.2	-30.6
n = 2 → 4	9.26	0.146	0.227	85.9	-25.1	-49.3	-34.9	52.4
Base Case: { M = 1000, $\theta = 1$, $\tau = 0.01$, r = 1, $\gamma = 0.75$, n = 2, $\beta = 2.5$}								

Table 5: Long Term Equilibrium [Numerical Example]

	Values					Percentage change				
	n	f	p	P	s	n	f	p	P	s
Base Case	4.75	7.95	0.104	0.199	101	–	–	–	–	–
M $= 1000 \rightarrow 2000$	6.17	10.11	0.084	0.158	166	30.0	27.1	-19.3	-20.3	65.2
$\theta = 1 \rightarrow 2$	3.61	6.23	0.130	0.251	120	-23.9	-21.6	24.9	26.2	19.3
$\tau = 0.01 \rightarrow 0.02$	4.69	7.95	0.114	0.209	100	-1.2	-0.05	9.6	5.1	-1.2
r $= 1 \rightarrow 2$	10.27	8.12	0.102	0.195	222	116.2	2.1	-1.8	-1.9	120.7
$\gamma = 0.75 \rightarrow 1.5$	3.61	12.47	0.130	0.251	60	-23.9	56.8	24.9	26.2	-40.3
Base Case: { M $= 1000$, $\theta = 1$, $\tau = 0.01$, r $= 1$, $\gamma = 0.75$, $\beta = 2$}										

Table 6: Long-run Comparative Static Analysis

Variable:	Carriers	Frequency	Price	Full Price	Shipment Volume	Capacity
Parameter:	(n)	(f)	(p)	(P)	(TS)	(s)
Market Size (M)	+	+	$-^*$	–	+	+
Fixed Cost (θ)	–	–	$+^*$	+	–	+
Marginal Cost (τ)	–	–	+	+	–	–
Product Price (r)	+	+	$-^*$	$-^*$	+	+
Holding Cost (γ)	–	+	$+^*$	+	–	–

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Notes

[1]With JIT production, a delay in deliveries or pickups can seriously disrupt production, imposing large costs on the firm. As an example, consider the heavy losses incurred by several West Coast auto assembly plants due to disruption in production caused by the ports strike in September-October 2002. In some cases, a disruption in transportation services may even result in a temporary shut-down of JIT plants.

[2]In work carried out contemporaneously, Kramer and Kramer (2010) also study price and frequency competition between logistics services providers, using a related model. These authors in addition provide a road map to earlier work, which includes a detailed review by Minner (2003) of the inventory-management literature, where the analysis focuses on shipping decisions conditional on carrier prices, service levels and lead times while accounting for inventory holding costs. Several papers (Benjaafar, Elahi, and Donohue (2007), Cachon and Zhang (2006), Jin and Ryan (2009), Yang, Xiao, and Shen (2009)) analyze the impact of carrier competition on choices made by shippers, while a third stream of literature analyzes carrier competition but considers price and frequency competition separately rather than in a single model (Ha, Li, and Ng (2003), Gans (2002), Allon and Federgruen (2007)).

[3]See for example, De Vany (1976), Koenigsberg (1980), Martin (1981), De Vany and Frey (1982), Spiller (1982), De Vany and Saving (1983), Anderson and Ormiston (1983), Masarani and Gokturk (1987) and, more recently, Parra-Frutos (2009).

[4]The flexibility of vehicle size in this case is confirmed by the fact that ships are often custom-built for particular routes.

[5]For background on deregulation in the trucking industry, see Friedlaender and Chiang (1983), McMullen (1987), Beilock and Freeman (1987) and Hubbard (1998).

[6]The producer's optimization problems with shipping of output and input are similar in structure. However, the case where the producer ships the input turns out, surprisingly, to be analytically the much less tractable one. Hence, the paper focuses on the case where the output is shipped.

[7]In reality, inventory holding costs may be firm-specific, leading to different γ values across firms. Such a structure would give rise to differentiated frequency-price equilibria where firms with low holding costs would chose low-frequency carriers while those with high holding costs would choose high-frequency carriers. However, the present model instead introduces firm heterogeneity through brand loyalty, using a symmetry assumption on the loyalty distribution to generate symmetric equilibria.

[8]While brand loyalty is assumed to be exogenous, loyalty could be endogenous in a richer model. In this case, carriers would make efforts to build brand loyalty, providing incentives to producers and incurring costs to improve the chances of signing contracts. Such a model, however, would probably be difficult to analyze.

[9]The share formula would be different under other assumptions on the distribution of brand loyalty. For example, suppose that the natural logs of the individual a_i parameters each follow the extreme value distribution. Then taking logs on both sides of (6), the condition reduces to $log(a_j) + log(R_j) < log(a_1) + log(R_1)$. But with the $log(a_i)$'s having the extreme value distribution, the share of producers chosing carrier 1 is then given by the multinomial logit expression $R_1 / \sum R_j$. The analysis could be carried out under this alternative assumption, and the results would presumably be similar to those derived below.

[10]Asymmetric equilibria arising from cost asymmetries across carriers would be a possible modification of the model. However, analysis of such a framework turns out to be unworkable.

[11]Any intersection in the third quadrant is not relevant since frequency must be positive. Also,

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there might be only one tangency point between the curve and the line in the positive quadrant or no intersection at all. Both these cases are ignored.

[12]To see this conclusion, consider the effect of a rise in a parameter, say, M, on p:

$$\frac{\partial p}{\partial M} = p_M + p_f \frac{\partial f}{\partial M} + p_n \frac{\partial n}{\partial M} , \tag{67}$$

where p_M, p_f and p_n denote the partial derivatives of the p solution (23) with respect to M, f and n, respectively. While M has no direct effect on p, M affects f and, hence, p positively, which results in a positive relationship between p and M in the short-run. However, while the indirect effect via f still exists in the long-run, being captured by the second term in (67) (still positive given Table 6), there exists an additional indirect effect via n. The numerical results indicate that the net effect of a rise in M is a fall in p, and the signs of all other long-run effects on p can be explained in a similar fashion.

[13]Total utility equals total profit times $\Omega = M\alpha/(n+1)$, and this factor would enter the comparison. Note, however, that if social welfare were equal to $1/\Omega$ times producer utility plus carrier profit, the resulting welfare function would reduce to (40).