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ABSTRACT

The Coulomb excitation of ^{235}U has been studied using ^{4}He, ^{16}O, and ^{40}Ar projectiles. Rotational bands at 921 keV and 638 keV (tentative) have been characterized as the K_o+2 and K_o-2 γ-vibrational bands respectively, and a somewhat tentative band at 1053 keV has been assigned as the β-vibrational band based on the ground state.

Two bands at 633 and 822 keV are identified as the 5/2 - [752] and 9/2 - [734] Nilsson states, respectively. The large B(E2) value for exciting these bands indicates that they are strongly Coriolis coupled to the ground state band, 7/2 - [743]. A three-parameter Coriolis calculation, involving all eight components of the $j_{15/2}$ shell-model orbital, can account adequately for all the rotational energies and transition probabilities observed in these three bands.

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KEY WORDS

Coulomb Excitation ^{235}U + ^{40}Ar (182 MeV), ^{16}O (60-78 MeV) and ^{4}He (15-19 MeV). Measured E_γ, I_γ, E_{ce}, I_{ce}. Deduced levels, I, π. Enriched targets.

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1. Introduction

Coulomb excitation is a method ideally suited for studying those low-lying levels of a nucleus which are closely related to the ground state. Thus far, there has been more effort in this area directed toward even-even nuclei than toward other nuclear types. This is because these nuclei are simpler due to the absence at low excitation energies of single-particle states. The levels studied have thus been primarily the collective excitations (rotational and "vibrational") based on the ground states of even-even nuclei. Now, however, it is becoming easier to study the more complicated odd-mass nuclei because instruments of higher resolution—particularly those involving solid-state detectors—are becoming available. In these studies one finds collective states analogous to those in the even-even nuclei, but one also finds other states related differently to the ground state, as the present study of ^{235}U shows.

The work on ^{235}U was begun as an effort to extend a previous study[1] of deformed odd-mass rare-earth nuclei into the heavy-element region. There are two reasons for choosing ^{235}U in particular. The first is simply that it is available as a metallic foil; whereas most heavy odd-mass nuclei are not, due to their short half-lives and intense radioactivity. A more important

reason, however, is that this nucleus should be an exceptionally good one for studying Coriolis effects. The ground state of ^{235}U is a component of the $j_{15/2}$ shell-model orbital, and this orbital provides the only negative-parity levels in the 126-184 shell. Interaction of these components with other negative-parity states is thus reduced due to the large energy spacing. On the other hand, Coriolis interactions among the $j_{15/2}$ components is expected to be large due to the large value of j. Thus we expect the ground state of ^{235}U to be strongly Coriolis coupled to other components of the $j_{15/2}$ orbital, but to be unusually free of couplings to other states.

Newton[2]) has previously studied the Coulomb excitation of ^{235}U and identified the lowest two members of the ground-state band. We have also observed the ground band of ^{235}U; however, our principal aim was to characterize excited bands. Information on the excited bands of ^{235}U also comes from the alpha decay[3,4]) of ^{239}Pu. The ground state of ^{239}Pu corresponds to the Nilsson orbital[5]) 1/2 + [631]; and the alpha decay of this state tends to populate states in ^{235}U similar to itself. Most heavily populated is the 1/2 + [631] state, which has an excitation energy of less than 0.1 keV and a half-life of 26 min[3]). The ground state of ^{235}U corresponds to the Nilsson state 7/2 - [743] which is dissimilar to the 1/2 + [631] state. Thus the Coulomb excitation and the alpha decay could be expected to populate different states and, indeed, very few states are common to both studies. A third method for studying levels in ^{235}U is by means of the d,p and d,t reactions on ^{234}U and ^{236}U and inelastic scattering on ^{235}U. A large amount of information is just now becoming available from such reaction studies but they provide as yet relatively little information on the levels of interest here. However, these three methods together can provide an unusually complete picture of the low-lying excited bands in ^{235}U.

2. Experimental Techniques

In the present paper the results of bombarding ^{235}U with ^{4}He, ^{16}O and ^{40}Ar projectiles are reported. These beams were provided by the Lawrence Radiation Laboratory Hilac. The duty cycle of this accelerator has been increased over the last few years, and varied from about 5% at the beginning of these experiments to 20-30% at the end. The "partial-energy" beams were used so that no degradation of energy was required, and the spread in energy of the beams was about one MeV. The beam energy was measured by scattering the projectiles through 20 deg into a solid-state detector calibrated against the full-energy beams. These energies are expected to be accurate to about 2%.

The ^{235}U used in most of this work was 92% ^{235}U and 8% ^{238}U by mass, and consisted of metallic foils 0.003-0.05 cm thick. A few spectra were taken with foils of higher enrichment when they became available near the end of these studies. The foils were cleaned prior to bombardment by scraping the surface. In most cases, similar ^{238}U foils were bombarded under the same conditions, and lines common to both foils could be ascribed either to impurities or, if the intensities followed the ^{238}U abundance, to ^{238}U.

The electron spectra were taken on a small wedge-gap spectrometer which has been described previously[1]). The thick ^{235}U targets were inclined at 5 or 10 deg to the beam direction, and the electrons were detected at 90 deg to the beam direction. This arrangement minimized the penetration of the projectiles into the foil, and hence also minimized the spread in energy of the detected electrons. The full width at half the height of a peak, $\Delta H\rho/H\rho$ was typically 0.5% although it could be made better (or worse) for special purposes.

The gamma-ray spectra were taken with Ge(Li) detectors whose quality improved during the course of these measurements. The last and best spectra

were taken with detectors about 6 cm^2 in area and 1 cm deep, having a full width at half-maximum height of 2.2 keV for the ^{60}Co 1.33 MeV line. The energies of the transitions were normally determined with these detectors to within 0.5 keV, which was somewhat better than the accuracy of the electron measurements. The agreement between the energies measured using the two methods was normally good. The ground-state rotational band energies could be determined to ±0.2 keV due to the availability of closely-spaced standard lines (^{177m}Lu) and the use of the transitions following ^{235}U alpha decay to insure against energy shifts during the bombardment. The efficiencies of these detectors were measured using the IAEA absolute gamma-ray standards, and are considered to be accurate to 15%.

3. Results

In this section we will describe the measurements and present the spectra and tabulated information. The following measurements were made: (1) 182 MeV ^{40}Ar bombardments to measure the energies of the ground-state rotational band members; (2) 15 MeV ^{4}He and 60 MeV ^{16}O bombardments for B(E2) measurements by comparison with ^{197}Au; (3) gamma-ray studies with 19 MeV ^{4}He; and (4) electron and gamma-ray studies using 78 MeV ^{16}O. These will be discussed in the following paragraphs.

The reason for measuring a number of ground-state rotational band energies in ^{235}U will become apparent in the discussion section. In order to accomplish this we bombarded a 0.01 cm thick ^{235}U foil with 182 MeV ^{40}Ar ions and by multiple Coulomb excitation observed the rotational levels up to 25/2,

Fig. 1. Two problems arose in this measurement. The first was the intense gamma radiation from ^{235}U radioactive decay. Because of this we usually took a spectrum between the Hilac beam bursts (5 msec beam pulses 40 times per sec) with an equivalent gate length and compared it with, or actually subtracted it from, the spectrum taken during the beam burst. This procedure enabled us to identify unambiguously all of the ^{235}U decay gamma rays, but the subtracted spectra were usually poor in the region of subtracted peaks due to 0.1 or 0.2 keV energy shifts between the spectra. However, this method had the advantage of providing an internal standard to measure accurately these energy shifts which were due to the high in-beam count rates. The other problem arose from the presence of ^{238}U in the target. The 159.2 keV $6 \rightarrow 4$ transition in ^{238}U is clearly seen in Fig. 1, and hence one might also expect to see the 211.1 ± 0.5 keV $8 \rightarrow 6$ transition of ^{238}U. However, we have assigned the observed 211.4 keV line mostly to the $21/2 \rightarrow 17/2$ transition in ^{235}U. This assignment is based on a comparison of the present data with spectra taken under similar conditions with a ^{238}U target; the comparison shows that only a small part of the 211.4 keV line can be due to ^{238}U. Table 1 summarizes the energies of the ground-band rotational transitions. Although one could also obtain the rotational B(E2) values from measurements of this type, the experimental refinements and rather extensive mathematical analyses involved are outside the scope of the present study.

The objective of the next set of experiments was to measure the B(E2) values for excitation of the strongest electron and gamma-ray lines Coulomb excited in ^{235}U. We have preferred to do this relative to the well-known 547 keV line in ^{197}Au. It is possible to obtain an accuracy of 10% in

comparing electron or gamma-ray lines with the 547 keV ^{197}Au lines when identical bombarding conditions are used, whereas an absolute accuracy of 10% would be difficult. We feel the ^{197}Au line is known with sufficient accuracy to serve as a reference, and any improvements in this B(E2) value can be applied directly to the present data. The detailed method of calculation for the ^{16}O data has been given in a previous paper[1]), and that for the ^{4}He data is the one outlined in Alder et al.[6]). The use of the comparison method required bombarding energies below that of the Coulomb barrier for gold, and thus lower than optimum for uranium; however, the strongest ^{235}U lines could be accurately measured.

In order to avoid multiple Coulomb excitation, gamma-ray studies were made using 15 MeV ^{4}He projectiles. The results of these B(E2) measurements are given in column 3 of table 2. However, using 60 MeV ^{16}O, we could obtain somewhat improved gamma-ray spectra and much better electron spectra due to the decreased penetration of the target. The ^{16}O gamma-ray results are included in table 2 and the electron results are given in table 3. It is clear that multiple Coulomb excitation could affect the ^{16}O results by as much as 20%, and corrections for this effect have been applied to the ^{16}O data according to the calculations of Lütken and Winther[7]). There is no significant deviation between the $\epsilon_\gamma B(E2)\uparrow$ values from 15 MeV ^{4}He and the corrected 60 MeV ^{16}O values. Comparison of the ^{16}O electron and gamma-ray data gives the conversion coefficients listed in the fourth column of table 3. The multipolarity of the listed lines can be assigned with little ambiguity.

Somewhat better gamma-ray spectra could be obtained using 19 MeV ^{4}He projectiles than were obtained using 15 MeV. However, the improvement was not

great, and the 15 MeV spectrum is shown in fig. 2, together with those for 78 MeV ^{16}O and 182 MeV ^{40}Ar. The relative intensities of the higher-energy gamma-ray lines from ^{4}He and ^{40}Ar bombardments are given in table 4. These values are clearly inferior to those from the 78 MeV ^{16}O data, but were included, with appropriate weighting factors, in the computations of branching ratios. The error in these relative intensities is expected to be around 20% for the stronger lines, and up to 30 or 40% for the weakest lines. The ^{4}He spectra contain a large number of extraneous lines.

The best electron and gamma-ray data for ^{235}U were taken using 78 MeV ^{16}O as projectiles. These spectra are shown in figs. 2 and 3. In table 5, the best energies and relative gamma-ray and electron intensities are listed. A line whose existence is not considered certain is marked by an asterisk. The energies are expected to be accurate to about ±0.5 keV, and the relative intensities to about 15%. Normalization of the electron and gamma-ray spectra was made by assuming the 633 keV transition to be pure M1, as is reasonably clearly indicated in table 3. Conversion coefficients can then be calculated for most of the other transitions, and these are given in column 4 of table 5. The error in these α_K values is expected to be ~ 20% for the strongest lines and up to a factor of two for the weakest. The theoretical conversion coefficients are listed in columns 5, 6, and 7, and the multipolarity assignments for the transitions are given in the last column. For the most part there is little ambiguity in the predominant multipolarity; however, appreciable admixtures of other multipolarities cannot be excluded. There is a reasonably clear trend for the conversion coefficients to be high for the higher energy transitions, and this amounts to 20-30% for the highest energies. We do

not know the cause of this effect. The electron line of the 638 keV transition is particularly poorly measured as it is close to, but much weaker than, the 633 keV line. Nevertheless, the E2 assignment of this line seems reasonable. The 1053 keV photons were not observed, so only a limit for α_k is given in table 5.

In order to determine the multipolarities of the excitations leading to the various lines, the yields of the lines with 60 and 78 MeV ^{16}O were compared. The resulting ratios are given in table 6 where they are compared to the expectations of Coulomb-excitation theory[6]) for E2 and E3 excitations. It is clear that the excitations in all cases are E2. This is particularly significant for the 638 keV transition, whose position in the level scheme is least certain.

4. Discussion

4.1. LEVEL SCHEME

The first step in the interpretation of the data presented in the preceding section is the construction of the level scheme. As mentioned earlier, the levels populated in ^{235}U by ^{239}Pu alpha decay are essentially a completely different set, and hence are of no help in the present situation. Nevertheless, most of the data can be fit rather easily and unambiguously into a level scheme. We will do this in two steps, the first of which involves essentially model-independent arguments (apart from the existence of rotational bands) such as energy sums, transition multipolarities, rotational band energy spacings, and general considerations about transition probabilities. Then in the following

sections a description of the identified bands will be proposed in terms of a more detailed model (vibrational states, Nilsson states, etc.), and a comparison of the observed and expected properties of the bands will be made in as much detail as seems warranted by the data. In such a procedure, no single piece of evidence for a particular band is conclusive, but taken as a whole, we feel the evidence is rather convincing for most of the bands.

The ground-state rotational band of ^{235}U is established unambiguously up to the 25/2 level by energy sums from the ^{40}Ar data (table 1). The absence of the higher states in the ^{16}O data (fig. 1), and especially in the ^{4}He data, strongly support these assignments, as multiple excitation would be expected to be well down with ^{16}O and essentially absent with ^{4}He. In addition, several of these band members were known from previous work[2,3]). This band is shown on the level scheme in fig. 4, and provides a basis for establishing the positions of other bands.

The seven predominantly-M1 lines around 650 keV can be ascribed to a single rotational band having a base level at 633 keV, as indicated in fig. 4. The two pairs of lines separated by energies corresponding to ground state spacings, strongly support this arrangement, and the level spacings, though not entirely regular, are consistent with a K=5/2 band.

The most difficult problem in the level structure has to do with the three or four predominantly E2 transitions around 650 keV. They cannot be fit into the K=5/2 band, and, in addition, their predominant multipolarity is different. The strong 638 keV transition is probably E2, and has been shown (table 6) to result from a single E2 excitation from the ground state. The lack of any other transition separated from this strong one by a ground state

spacing, can most easily be explained if the level is 3/2- and thus can decay by E2 radiation only to the ground state. Two other E2 transitions, together with an E2 component of the 618 keV line (which is seen to be broad in fig. 2) and the 598 keV line of unknown multipolarity, can form two band members based on such a 3/2- level. Although this evidence for the band is rather weak, a 3/2- band is expected in ^{235}U and, as we shall see, the characteristics of this proposed band are in good agreement with those expected.

The M1 lines around 800 keV clearly fall into three pairs which define the 822, 886, and 961 keV levels. The rotational spacings strongly suggest that this is a K=9/2 band. The easiest way to demonstrate this is to set the difference between two successive cascade transitions equal to $2\hbar^2/2\Im$, the lowest order rotational estimate. For this band this gives $\hbar^2/2\Im = 5.7 \pm 0.3$ keV. We can also calculate $\hbar^2/2\Im$ directly from each rotational spacing, and this gives average values of 7.0, 5.8, and 5.0 for K values of 7/2, 9/2, and 11/2, respectively. Only K = 9/2 gives consistent values for $\hbar^2/2\Im$.

The 921, 875, and 818 keV E2 transitions define a level at 921 keV, and the weak lines at 941 and 885 probably define a weakly-populated member of this band at 988 keV. The strong Coulomb excitation of a single level with E2 multipolarity (table 6) almost requires a K value two higher than the ground-state value in a deformed nucleus. Otherwise higher band members would receive sizeable populations. Thus the 922 keV band is likely to have K = 11/2.

The K = 7/2 band at 1053 keV is proposed on the basis only of the strong monopole line(s) at 1053 keV. Such transitions in even-even deformed nuclei strongly suggest the "β-vibrational" band, and in odd-mass nuclei this type of band will have a K value equal to that of the base level (ground state in

this case). The 1053 keV line is presumably a multiplet composed of several $I \rightarrow I$ transitions between the two bands.

Thus the level scheme in fig. 4 accounts for essentially all of the present data. There are several weak gamma rays in the region of 300-450 keV, which probably belong to ^{235}U, but which we have not attempted to identify or interpret in this work. Also a moderately weak transition of 129.1 keV indicates population of the 129.2 keV 5/2 + [622] level identified in the alpha-decay work, but we have not determined if it is directly populated, or fed in the decay of other levels. All the bands identified must have negative parity, like the ground state, due both to the E2 nature of the excitations (table 6), and to the E0, M1, and E2 character of the deexciting radiations. The next sections will comprise a detailed examination of the properties of the bands populated.

4.2. COLLECTIVE BANDS

Of the three bands to which we ascribe predominantly collective character, the best information is available for the K = 11/2- band at 921 keV. Perhaps the strongest indication of the K = 11/2 assignment, as mentioned above, consists of the excitation probabilities of the two observed band members. Table 7 gives the relevant information. Multiple excitation is negligible with ^{4}He projectiles and hence these relative excitation probabilities can be derived from first order perturbation theory and the relative transition probabilities which, in this case, simply depend on the ratio of the squares of the appropriate vector-addition coefficients. When ^{16}O is used as a projectile, multiple excitation is not negligible, and in order to account for this we have used the sudden approximation calculations of Lütken and Winther[7]. These calculations take

into account multiple excitations within the rotational bands, and involve a parameter, q, which is related to the probability of making a rotational excitation. We have taken q = 1.2, which is a rough estimate of that for 78 MeV ^{16}O and thick ^{235}U targets, averaging over projectile angles and target thickness. (We have verified that a q of 1.2 produces about the right effect in the K = 0- band of ^{238}U, where it can be easily tested.) Table 7 shows that the data are in good agreement with the expectations for a K = 11/2 band, and that the only other plausible assignment for the band, K = 9/2, can be excluded.

The separation of the two levels gives a value of $\hbar^2/2\Im$ of 5.1±0.2 keV compared with the ground state value (9/2-7/2 separation) of 5.14±0.03. These are very nearly the same, as is typically found for the γ-vibrational bands in odd-mass nuclei. It is not really clear why they are so nearly equal, as these γ-vibrational bands are not, in general, thought to be very pure. The B(E2) value for exciting this band is: B(E2; 7/2, 7/2 → K = 11/2) = 0.050±0.005 $e^2 \times 10^{-48} cm^4$. This corresponds to 1.1 single-particle units (s.p.u.), using the definition given by Alder et al.[6]), $B(E2)_{sp} = 3 \times 10^{-5} A^{4/3} e^2 \times 10^{-48} cm^4$. This would be rather large for two Nilsson states, and we take it as evidence supporting the predominantly collective nature of the band. An additional argument is the absence of a suitable Nilsson state. The only 11/2- Nilsson state in the 126-184 shell is the 11/2-[725] state. This particular state is probably considerably higher in ^{235}U (see sec. 4.3). Thus we feel it is very likely that the 921 keV band is predominantly the K_0+2 γ-vibrational band based on the ground state.

A comparison of theoretical and experimental relative B(E2) values for de-excitation of the 921 keV band is given in table 8. A difficulty occurs with the 818 keV transition which can arise both from the $11/2 \rightarrow 11/2$ and the $13/2 \rightarrow 13/2$ transitions. We have made the arbitrary assumption that 0.75 of the intensity arises from the former transition, and 0.25 from the latter. One should also keep in mind that the limits of error on our conversion coefficients are not sufficiently small to exclude the possibility that M1 components are having an effect on the relative transition probabilities. The column headed $z = 0$ contains the values derived from the vector-addition coefficients corresponding to E2 decays from a pure $K = 11/2$ band to a pure $K = 7/2$ band. It is well known, however, that in the even-even nuclei, the γ-vibrational band and the ground band mix to a small extent. This mixing affects the branching ratios, and it has been shown[9]) that the corrections to the values given by the vector addition coefficients can be simply expressed in terms of a parameter, z. In a previous paper[1]) we have given the relationships for extending these corrections to the odd-mass nuclei. The last column in table 8 shows the B(E2) values expected if the parameter, z, has a value +0.06. It can be seen that the experimental data are all consistent with this value, whereas they are not consistent with $z = 0$. The only accurately measured ratio, however, is that for B(E2;875)/B(E2;921), from which z is determined to be $+0.063 \pm 0.013$. All z values thus far determined have this sign, and are of this order of magnitude, although in the even-even nuclei near ^{235}U, the average value is less by a factor of two or three. It is not difficult to give a reason for a larger value of z in ^{235}U. We can write z in the form: $z_{IK} = \epsilon_{IK} Q_{grd}/Q_{\gamma\text{-}grd}$ where ϵ_{IK} is the admixed amplitude in the state

of spin I, and Q_{grd} and $Q_{\gamma\text{-}grd}$ represent the E2 transition amplitudes within the ground band and between the ground and γ-vibrational bands, respectively. Whereas Q_{grd} is nearly the same in ^{235}U as in the neighboring even-even nuclei, $Q_{\gamma\text{-}grd}$ is about three times lower. Thus for comparable admixed amplitudes, z would be about three times larger for ^{235}U. A z of 0.06 corresponds to an admixed amplitude in the I = 11/2 state of only about 0.01, so that the mixing is not large.

Although the evidence for a K = 3/2- band at 638 keV is not entirely conclusive, the data on the 638 keV line itself, as mentioned earlier, is difficult to interpret except as the de-excitation of a 3/2- level directly excited at or near 638 keV. Our purpose here will be to show that the other data available are consistent with such a band, even though these data are probably not sufficiently good to justify independently such a detailed analysis.

The transitions in this region which cannot be accounted for by the K = 5/2 band at 633 keV are: (1) definite transitions at 598, 613, 638, 651, 655, and 665 keV which are predominantly or entirely E2; (2) a definite E2 component in the 618 keV line, together with the fact that this line is broad; and (3) a probable line at 646 keV. All these features except the lines at 613, 651, and 646 keV can be accounted for by the proposed K = 3/2- band at 638 keV. The three indicated members of this band are reasonably spaced for a K = 3/2 band and give an average value of $\hbar^2/2\Im$ of 5.1 keV, the same as for the ground band. The B(E2) value for exciting this band is: B(E2; 7/2, 7/2 $\rightarrow$ K = 3/2) = $0.022\pm0.007\ e^2\times10^{-48}\text{cm}^4$. This corresponds to about 0.5 s.p.u. Both of these features of the band are in reasonable accord with the band being the

K_0-2 γ-vibrational band based on the ground state. In fact, this is the only reasonable interpretation of the band as there is no Nilsson K = 3/2- level in the vicinity.

In table 9 we have compared the data for excitation and de-excitation of the 638 keV band with the theoretical expectations for the K_0-2 γ-vibrational band. Several comments should be made about these comparisons. In the excitation of the K = 3/2 band a q value of 1.2 was again used as an average over target thickness and projectile angle. The corrections for multiple excitation (q) and ground-band mixing (z) were applied independently, although they were sufficiently large to make this procedure questionable. The experimental values are not sufficiently good, however, that this constitutes a serious limitation. Also, the ^{40}Ar data suggest that the rotational de-excitations within this band may compete with the interband transitions, and such competition would affect the excitation probabilities in table 9. For the 618 keV transition, we computed the E2 and M1 portions based on the nearby 625 keV transition being pure M1, and used the E2 portion so calculated in table 9. If the 625 keV transition has an E2 admixture, this division can still be nearly correct provided the 618 keV line belonging to the 633 keV band has about the same E2 admixture. Since these two transitions in the 633 keV band are quite analogous, we feel this is probable. The error limit on the 618 keV line in table 9 includes an uncertainty due to this division. The data are probably good enough to indicate a disagreement with the unmixed (z=0) values in table 9, whereas reasonable agreement with the z = +0.07 values is obtained. This agreement indicates to us that the interpretation is reasonable; however, we still must regard this band as somewhat tentative.

As mentioned, there is very little evidence on the β-vibrational band proposed at 1053 keV in ^{235}U, but we feel that the strong electron line at 1053 keV with no corresponding photons is sufficiently unusual to suggest a considerable amount about this band. The limit we could set on the photons was such that $e_K^- / \gamma \geq 0.38$. This effective α_k eliminates all electric multipolarities (apart from E0) and all magnetic ones up to M5. Multipolarities as large as 4 or 5 are very unlikely to arise from Coulomb-excited levels and, although the half-life of the line was not measured, the consistent yield of the line implies a life-time short enough to rule out multipolarity 5 and higher. Thus the only plausible explanation for the electrons is that they are due to one or more E0 transitions. In the nearby even-even nuclei, the lowest excited 0+ state typically decays in part to the ground state via an E0 transition, and is connected with the ground-state band by E2 transitions having B(E2) values of the order of one single-particle unit. This band is called the β-vibrational band, and it is in this sense that we suggest that the band implied by the 1053 keV E0 transition(s) in ^{235}U is a β-vibrational band. If it is similar in nature to the β-vibrational bands in the even-even nuclei, then we probably populate several levels in appreciable intensity, and the 1053 keV transition would be a composite of all the $I_\beta \rightarrow I_{grd}$ E0 transitions. In fact, the line does appear to be somewhat broad, consistent with this interpretation. We have tried to indicate this situation in fig. 4 by dashed lines for two higher band members which also de-excite by the 1053 keV transition.

However, the nature of the β-vibrational band is not very clear in the even-even nuclei, and is even less clear in ^{235}U. If we consider the

2+(β-vib band) → 2+(grd band) transition in ^{238}U, we find[10]) that $e_K^- /\gamma \approx 1$; corresponding to three times fewer photons per K electron than our limit in ^{235}U. Therefore, this ratio could be rather similar for the two bands. The $\epsilon_K B(E2)\Uparrow$ for the 1053 keV line is about $6 \times 10^{-4} e^2 \times 10^{-48} cm^4$, and this is about 15 times lower than that for ^{238}U. Our limit on the 1053 keV photons corresponds to $\epsilon_\gamma B(E2)\Uparrow \lesssim 1.7 \times 10^{-3}\ e^2 \times 10^{-48} cm^4$; however, to allow for the possibility of other unobserved transitions from this (or these) level(s), we can only say that $B(E2;7/2,7/2 \rightarrow 1053) \lesssim 8 \times 10^{-3} e^2 \times 10^{-48} cm^4$. This limit corresponds to about 0.2 s.p.u., almost an order of magnitude less than the observed value in ^{238}U. Thus, while the relative E0 and E2 transition rates from the β-vibrational bands in ^{235}U and ^{238}U may be similar, both absolute rates must be well down in ^{235}U.

It is interesting that the E2 strengths to both the γ- and β- vibrational bands in ^{235}U are down significantly from those for the nearby even-even nucleus, ^{238}U. In ^{235}U, the K_0+2 γ-vibrational band is least affected

$$\left(\frac{B(E2;^{235}U, K = 11/2-)}{B(E2;^{238}U, K = 2+)} \sim \frac{1}{3}\right) ,$$

and this ratio is not much different than was found for the analogous rare-earth nucleus ^{165}Ho. The K_0-2 γ-vibrational band is affected more

$$\left(\frac{B(E2;^{235}U, K = 3/2-)}{B(E2;^{238}U, K = 2+)} \sim \frac{1}{5}\right)$$

and in this case the ratio is significantly smaller than was found for ^{165}Ho

The β-vibrational band is also heavily reduced,

$$\left(\frac{B(E2;{}^{235}U,K = 7/2-)}{B(E2;{}^{238}U,K = 0+)} \lesssim \frac{1}{6}\right)$$

and no analogous state was identified in ^{165}Ho. The reason for these reductions is not really clear; but it may reflect larger and somewhat variable single-particle admixtures in the odd-mass cases. However, since there is also considerable variation among the even-even nuclei, we cannot be too sure of the significance of the effect.

4.3. SINGLE-PARTICLE BANDS

The only plausible interpretation of the K = 5/2- and 9/2- bands at 633 and 822 keV is that they correspond to the Nilsson configurations 5/2-[752] and 9/2-[734]. Both bands are connected with the ground state, 7/2-[743], by E2 transition of around one single-particle unit. These E2 strengths are comparable with those to the proposed collective bands, yet no collective bands based on the ground state and having these K values are expected. Moderate admixtures with the assigned collective bands clearly could not account for this E2 strength. On the other hand, with the above Nilsson assignments, both bands are related to the ground state in that all three are components of the $j_{15/2}$ shell-model orbital. In this case the Coriolis matrix elements between the ground state and these two bands are expected to be very large and can easily admix the bands sufficiently to account for the E2 strengths. In this section, therefore, we will first calculate the expected Coriolis mixings, and then compare in detail the observed properties of these three bands with those calculated.

There are two factors which make this case an exceptionally interesting one for detailed Coriolis calculations. The first one, alluded to above, is that the B(E2) value between the ground and the 5/2- or 9/2- bands determines the admixed amplitude, provided the single-particle E2 component between the bands is negligible. Calculations using the Nilsson wave functions give a B(E2) between the 7/2-[743] and 5/2-[752] bands of ~0.2 s.p.u., and the unfavorable pairing factor $(UU-VV)^2$ might be expected to reduce this to the vicinity of 0.04 s.p.u. The measured value between these bands is ~1.5 s.p.u.; so that most of the strength probably is due to the mixing, although we might expect interference effects from the single-particle E2 components. One might hope that the relative E2 transition probabilities for excitation of these bands could distinguish between the two situations, but for just this case of $\Delta K = \pm 1$ and E2 transitions, one finds identical vector addition coefficients from the admixed and the pure-K components.

The other favorable factor in the ^{235}U case is the presence in the ground and K = 5/2- bands of oscillating terms in the rotational energy spacings. The expression for the rotational energies of a band with a given K value can be written[11]):

$$E_I = E_0 + AI(I+1)\left[1 + \frac{B}{A} I(I+1) + \frac{C}{A} I^2(I+1)^2 + \ldots\right] + (-)^{I+1/2} A_{2K} \frac{(I+K)!}{(I-K)!}\left[1 + \frac{B_{2K}}{A_{2K}} I(I+1) + \ldots\right] \quad (1)$$

where E_0 and A, B, C, ... and A_{2K}, B_{2K}..., are constants for a particular band. The first series in eq. (1) is just the usual I(I+1) expansion, and the

second series gives rise to the oscillating energy spacings referred to above. For the two bands in ^{235}U we can evaluate A_5 and A_7 to be -2.2×10^{-4} and -3.2×10^{-8} keV respectively. Within the framework of our Coriolis calculation, these oscillating terms arise as higher-order coupling of the bands to the K = 1/2- band arising from the $j_{15/2}$ shell-model level—namely 1/2-[770]. The calculated large negative decoupling parameter of this band ($a_{calc} = -7.2$) should produce negative A_{2K} terms in the other $j_{15/2}$ components; but as successively higher K values represent higher-order mixings, we expect these A_{2K} values to decrease sharply with increasing K. The experimental values fulfill both of these qualitative expectations. The problem, then, is to see if we can reproduce the 15 measured rotational spacings and the deduced admixed amplitudes in a Coriolis calculation that starts with plausible and reasonably simple initial conditions. We shall see that a surprisingly good solution can be found.

In the Coriolis calculations there are potentially many parameters, and the first problem one faces is which of these should be varied, and over what region. We can break the problem down into three main areas: (1) how many bands are important, (2) where do we place these bands and their rotational levels, and (3) what Coriolis matrix elements should be used. These three points will be discussed briefly below.

It seems clear that all the components of the $j_{15/2}$ orbital should be included, since the K = 1/2 to K = 11/2 components will surely be important. In addition, low-lying K = 1/2 and possibly K = 3/2 levels could come from the $h_{11/2}$ and $j_{13/2}$ orbitals in the shell above, and from the $p_{1/2}$ orbital coming up from the shell below. Most of our calculations were made with only the

component levels of the $j_{15/2}$ orbital; however, a considerable number were repeated including all the components of the $h_{11/2}$ and $j_{13/2}$ orbitals with no significant difference in the results, even though K = 1/2 and 3/2 components of these orbitals were right in the energy region of interest (1 - 3 MeV). It therefore seems likely that the component from the $p_{1/2}$ orbital, 1/2-[501], will not have much effect on the calculations either, although it may well lie low in ^{235}U. One should realize that what we are saying here is that, given the Nilsson wave functions and the Coriolis force, these other orbitals do not contribute significantly to the observed (K = 5/2, 7/2, 9/2, $j_{15/2}$) bands. It is possible, of course, that they actually may contribute significantly because either a) the Nilsson wave functions are not good enough, or b) other types of interactions are important. However, a good fit to the data can be obtained using only the components of the $j_{15/2}$ orbital.

To locate the components of the $j_{15/2}$ orbital (and, where used, the $h_{11/2}$ and $j_{13/2}$ orbitals) we took the Nilsson eigenvalues, ϵ_ν at $\eta = 4$ and 6 and made a linear interpolation to $\eta = 5.5$, the deformation deduced from the measured ground-band B(E2) value[2]). We then used the simple pairing estimate:

$$E_\nu = \sqrt{(\epsilon_\nu - \lambda)^2 + \Delta^2} \quad , \qquad (2)$$

where E_ν is the energy of the ν component, and the Fermi surface , λ , and energy gap, 2Δ , are adjusted to give the K = 5/2, 7/2, and 9/2 components at about the right energy. We found this procedure gave the Fermi surface to be very near the ground, 7/2, component, and the gap to be ~1.1 MeV. Table 10 gives the calculated position of the components of the $j_{15/2}$ orbital. For

precise fitting of the 5/2 and 9/2 bands, we had to let the initial position of these bandheads vary slightly, but since their final energies are completely specified by the observed bandheads, we do not consider them as parameters in the calculation of the rotational spacings and admixtures. We have also explored the possibility of varying the other bandheads, and in this case they clearly are parameters of the calculation. There is one check on the location of the K = 1/2 and 3/2 components. If we work out in lowest order perturbation theory the expression for the oscillating energy term in the K = 5/2 and 7/2 bands and take their ratio, we get:

$$\frac{A_7(I-5/2)(I+7/2)}{A_5} = \frac{[\bar{E}(1/2,3/2) - E(5/2)]^4}{\bar{E}^4(1/2,3/2)} \epsilon^2(5/2,7/2,I) \qquad (3)$$

where $\bar{E}(1/2,3/2)$ is approximately the mean energy of the K = 1/2 and 3/2 components, $\epsilon(5/2,7/2,I)$ is the admixed amplitude of the K = 5/7 and 7/2 components into each other (these are equal in this limit) for the state of spin I, and E(5/2) is the energy of the K = 5/2 band. For I = 7/2, we know $\epsilon(5/2,7/2,7/2) \sim 8 \times 10^{-2}$ from the B(E2) value for excitation of the 5/2 band, and that E(5/2) = 671 keV; so that we can derive $\bar{E}(1/2,3/2)$ to be about 1750 keV. Our estimate of the mean bandhead energy from the Nilsson eigenvalues with the simplified pairing correction is about 1600 keV; so that we have some added confidence that these energies are reasonable. There is, of course, always the possibility that at ~2 MeV these bands may get badly mixed with nearby bands; but, even if this happens, the present calculation may be sensitive only to a "center of gravity" energy which does not change much from the unmixed bands.

For the rotational energies, we assume all bands to have the same $\hbar^2/2\Im$ initially, and let this be one of the variables in the solution. Nevertheless we know reasonable values for the initial $\hbar^2/2\Im$ must lie between the adjacent even-even value of 7 keV and the average odd-A value in this region of ~6 keV; so that this parameter is confined to rather narrow limits. We take the decoupling parameter of the K = 1/2 component to be the Nilsson value interpolated to $\eta = 5.5$, which is -7.2.

The Coriolis matrix elements for these cases can be written as:

$$H^{cor}(\Omega,\Omega\pm1,I) = -\frac{\hbar^2}{2\Im}\left[(I\mp\Omega)(I\pm\Omega+1)\right]^{1/2}\langle\Omega\pm1|j_{\pm}|\Omega\rangle \qquad (4)$$

where $H^{cor}(\Omega,\Omega\pm1,I)$ is the Coriolis matrix element between states of spin I in bands having Ω and $\Omega\pm1$, and $\langle\Omega\pm1|j_{\pm}|\Omega\rangle$ is the matrix element of the operator $j_{\pm}$ between the two bands. To evaluate them, we calculate $\langle\Omega\pm1|j_{\pm}|\Omega\rangle$ from the Nilsson wave functions for $\eta = 5.5$, which are given in table 10. It is clear, however, that near the Fermi surface these matrix elements should be reduced due to the pairing. In cases of this type (i.e., between components of the "different-parity" orbital in each shell) a factor of two reduction has generally been observed empirically[12], although this is considerably more reduction than the pairing calculations indicate. We can easily see that this will have to be the case in ^{235}U, since we can write from perturbation theory:

$$H^{cor}(5/2,7/2,I) = 671\,\epsilon(5/2,7/2,I) \qquad (5)$$

where $\epsilon(5/2,7/2,I)$ and $H^{cor}(5/2,7/2,I)$ have been previously defined. Again,

for $I = 7/2, \epsilon(5/2,7/2,7/2) \sim 8 \times 10^{-2}$; so that, $H^{cor}(5/2,7/2,7/2) \sim 50$ keV. The full Nilsson value is about 120 keV. The situation is similar for the 9/2 band. For the matrix elements between the upper bands, there is no real previous experience; however, again we can make an estimate from perturbation theory, which gives:

$$\langle 1/2|j_-|3/2\rangle^2\langle 3/2|j_-|5/2\rangle^2 = \frac{A_5[\bar{E}(1/2,3/2) - E(5/2)]^4}{a\left(\frac{\hbar^2}{2\Im}\right)^5} , \qquad (6)$$

where a is the K= 1/2 decoupling parameter, and the rest of the quantities have been defined. If we put in the observed A_5, the $\bar{E}(1/2,3/2)$ from the previous perturbation estimate (eq. (3)), the Nilsson decoupling parameter of -7.2, and $\hbar^2/2\Im = 6.5$, which almost has to be right within 10%, we can calculate a mean value for $\langle 1/2,3/2|j_-|3/2,5/2\rangle$. This mean value turns out to be 7.8 compared with the calculated Nilsson values of 7.25 for $\langle 1/2|j_-|3/2\rangle$ and 7.23 for $\langle 3/2|j_-|5/2\rangle$. Thus for the matrix elements between the upper states this estimate would indicate no reduction from the Nilsson values. We will have more to say about this result later, but for our present purposes it tells us to begin the calculation with the Nilsson value for all the matrix elements except those nearest the Fermi surface.

All of the rotational energy calculations were done using the computer program BETABLE written by T. Clements. This program repeatedly solves the secular determinants for all the I values involved, simultaneously adjusting all the parameters until a least-squares fit to the experimental levels is made. The program also gives the admixed amplitudes for the final fit.

In the first calculation, only one real variable was used, and it is interesting to note that a reasonably good fit to the experimental energies can be obtained. All of the single-particle states from the $j_{15/2}$ orbital were included in this calculation. The Nilsson energies, with a simple pairing correction mentioned earlier, were used for the unobserved states. The Nilsson values for the Coriolis matrix elements at deformation, $\eta = 5.5$, were used except for the two closest to the Fermi surface. These are the very important interactions between the 7/2- ground band and the 5/2- and 9/2- excited bands. The size of these two matrix elements can be derived using perturbation theory from the experimental B(E2) values for exciting the 5/2- and 9/2- bands from the ground 7/2- band. The assumption is made here that the intrinsic B(E2) values are small compared to the admixed collective contribution. This point will be discussed again later. The matrix elements thus obtained are 0.47 and 0.38 of the Nilsson value for the 5/2-7/2 and 9/2-7/2 matrix elements, respectively (these fractions are calculated using $\hbar^2/2\Im = 6.2$ keV). The only variable employed in this first calculation is the rotational constant ($\hbar^2/2\Im$) which affects both the rotational spacings and the size of all the matrix elements apart from the above two.

The results are best shown graphically. Using the usual rotational formula up to terms in $I^2(I+1)^2$, the energy difference between two consecutive states in a rotational band can be written:

$$\frac{E_I - E_{I-1}}{2I} = 2I^2B + \frac{\hbar^2}{2\Im} \qquad (7)$$

where E_I is the energy of the state of spin I, and B is the coefficient of the $I^2(I+1)^2$ term in the power series expansion of the rotational energy in terms of I(I+1). Thus a plot of $E_I - E_{I-1}/2I$ vs. $2I^2$ should give a straight line with slope B and intercept $\hbar^2/2\Im$. The results from the first program are plotted in fig. 5. The 7/2-band is fit fairly well but the slope (B value) is slightly negative instead of positive. This indicates that the matrix elements to the 5/2- and 9/2- bands are too small, since the positive value for B is a direct result of the compression of the ground band by the interaction. The fit to the 5/2- band has the right shape indicating that the influence of the 1/2- band is approximately correct, but the band is too compressed ($\hbar^2/2\Im$ too small) indicating that the matrix element to the 3/2- band should be reduced. The same thing is true of the 9/2- band. This program gives 6.2 keV for the rotational constant ($\hbar^2/2\Im$) prior to the mixing. The rms deviation of all the calculated rotational spacings from the measured ones is 7 keV. A comparison of the experimental and calculated energies is given in table 11.

A vast improvement can be observed in the results of the second program in which the 5/2-3/2 and 9/2-11/2 matrix elements are allowed to vary in addition to $\hbar^2/2\Im$. The matrix elements between the ground band and the 5/2 and 9/2 bands are also allowed to vary but their ratio is held constant. As discussed earlier these two matrix elements can be determined from the experimental B(E2) values assuming the intrinsic B(E2) to be negligible. It can be shown, however, that if there is a significant intrinsic B(E2) contribution, it should interfere destructively with the collective B(E2) causing one to underestimate the amount of mixing and hence the size of

the matrix elements. The ratios of these matrix elements, however, should not be so much affected (the interference is destructive in both cases). This program gives an rms deviation of 0.49 keV compared with the experimental rms uncertainty of 0.27 keV. The rotational constant is a reasonable 6.4 keV, and the matrix elements between the ground band and the K = 5/2 and K = 9/2 bands increase to 0.52 and 0.43 (using $\hbar^2/2\Im$=6.4 keV) of the Nilsson values respectively. The coupling between the K = 5/2 and 3/2 bands decreases slightly to about 0.78 of the Nilsson value and the coupling between the K = 9/2 and 11/2 bands remains at the calculated Nilsson value. The program can thus obviously be run without the 9/2-11/2 matrix element as a variable, making this a three-parameter fit. The calculated energies are shown in table 11 and fig. 6.

Several other programs with more variables were run and a brief discussion of these follows. Allowing the 7/2-9/2 matrix element to vary independently from the 5/2-7/2 one, produced a very slight improvement, with an rms deviation of 0.43 keV. The ratio of these two matrix elements became 0.75 of the value given by the Nilsson wave functions instead of 0.82 as indicated by the B(E2) values and used in the previous calculations. When the band-head energies were allowed to move up or down together in addition to all the above-mentioned variables, an rms deviation of 0.32 keV was achieved by lowering the band heads about 7%. The improvement was due mostly to a better fit for the last two spacings in the K = 5/2 band. The fit to the 7/2 band was essentially unchanged. Additional variation of the 1/2-3/2 interaction did not significantly improve the fit. Allowing the rotational constant of the ground band to vary independently did reduce the rms

deviation to 0.29 keV. It is interesting to note that the best fit from this program gave nearly the same values for the 5/2-7/2 and 7/2-9/2 matrix elements as were obtained from the B(E2) values. In this program the initial K = 7/2 rotational constant dropped to 6.0 keV while the remaining $\hbar^2/2\Im$ rose slightly to 6.6 keV. As mentioned previously, the effect of other single-particle states on the calculation was found to be negligible.

Our conclusion from these calculations is that rather little improvement can be obtained by allowing more variables than the three used in the second program. Although it is tempting to try to fit exactly the admixed amplitudes deduced from the B(E2) values, we expect that the interference from the intrinsic B(E2) values will be destructive and can easily amount to 40%. Thus, calculated amplitudes 20% larger than those given directly by the B(E2) values ($\epsilon^2 \propto$ B(E2)) must be considered quite reasonable, and that is what the second program gives. Also, our amplitudes are based on a Q_0 in ^{235}U of 10 barns, and this number is only known[2]) to about 10%. In the following discussion we will use the results of the second program since it already gives an excellent energy fit, and has simplicity to recommend it over the later programs.

The wave functions derived from the three-parameter calculation described above are listed in table 12. From these we can predict all the relative B(E2) values for exciting the K = 5/2 and K = 9/2 bands and all the relative B(M1) values for de-exciting these bands. The experimental and calculated values are given in tables 13 and 14. We have previously noted that to first order the mixing has no effect on the relative B(E2) values for exciting the various members of either band; it only renormalizes the overall E2 strength. We have again used a q_{eff} value of 1.2 for the fourth column of table 13. The agreement between

the data and the expected values is good, and, being independent of the mixing, lends considerable support to assignments of the levels.

The relative B(M1) values for de-exciting the levels of the K = 5/2 and 9/2 bands are given in table 14. The data are given in column 2, and in columns 3, 4, and 5 are given three calculated values. Column 3 corresponds to the vector addition coefficients and is given mainly for comparison purposes. In column 4 we have given the results from the second program described above using the full Nilsson values for all the M1 strengths. In fact, for the interband M1 transition amplitudes we ought to apply a reduction factor due to the pairing interaction. However, this pairing factor, UU + VV, is the favorable one, and according to most pairing calculations should be ≥ 0.8 for states near the ground state. If the reduction factor is indeed this near unity, it will have little effect on the relative M1 transition probabilities, and column 4 should represent the best theoretical estimate. However, the factor UU + VV should also be the one to apply to the Coriolis matrix elements and we know that in the second program these factors were 0.52 and 0.43 for the 5/2-7/2 and 7/2-9/2 matrix elements, respectively. If we apply these same reduction factors to the interband M1 components, then we get the relative M1 transition probabilities given in the last columns of table 14. Comparing with the experimental data, we find a definite improvement in column 4, over the simple vector addition coefficients. It is harder to decide between columns 4 and 5, and probably here the only significant change is in one of the transitions from the IK = 9/2 9/2 level, which is improved in column 5. We consider the overall agreement between theory and experiment to be reasonably good either for column 4 or column 5.

Our conclusion about these Coriolis calculations is that with as few as three parameters we can account adequately for (1) the 15 observed rotational spacings in the 3 bands, (2) the admixed amplitudes between the 5/2-7/2 and 7/2-9/2 pairs of bands as indicated by the absolute B(E2) values, and (3) all the relative E2 and M1 transition probabilities observed. It should perhaps be mentioned again that if the K = 5/2 and K = 9/2 bandhead energies are to be precisely fit, then two additional parameters are needed in the calculation. We feel the general validity of these calculations is established with very little ambiguity. The only puzzle remaining is why the Coriolis matrix elements near the Fermi surface are reduced so much. Two possibilities occurring to us are: (1) this is just a pairing effect, and the current pairing calculations simply under-estimate this reduction rather seriously, and (2) the Nilsson wave functions are either insufficient, or possibly are shared by several levels, and the state we see only represents the largest single piece left. We cannot really decide between these two alternatives on the basis of the present data.

5. Conclusion

The bands we have identified in the Coulomb excitation of ^{235}U fall into two groups which are: (1) predominantly collective bands based on the 7/2-[743] ground state, and (2) single-particle states which (like the ground state) are components of the $j_{15/2}$ shell-model orbital. Of the collective bands, the information available on the $K_0+2(11/2-)$ band at 921 keV is rather good, but that on the $K_0-2(3/2-)$ band at 638 keV is sufficiently poor to make the assignment somewhat tentative. These two bands comprise the so called "gamma-vibrational" bands based on the ground state. However, it is now recognized that such states in odd-A nuclei can have rather large admixtures

of particular single-particle states. The properties of these two bands have been found to be in good accord with those expected for these assignments. One feature strikes us as somewhat peculiar, although it is perhaps coincidental. That is the very small energy separations (1) between the K_0-2(3/2-) band and the single-particle band 5/2-[752] and (2) between the K_0+2(11/2-) band and the single-particle band 9/2-[734]. Considering states of the same spin, both these separations are less than 35 keV. Not only that, but all four bands have 11/2 states, for instance, within a span of 150 keV (estimating the position in the 3/2- band). We can see no reason for this. The presence of a strong monopole line leads us to propose a third collective band—the so called "beta-vibrational" band—at 1053 keV in ^{235}U. All these collective bands have B(E2) values well below those for the corresponding bands in ^{238}U.

Perhaps the more interesting bands in ^{235}U are the K = 5/2-[752] and 9/2-[734] single-particle bands at 633 and 822 keV, respectively. These two bands, together with the ground 7/2-[743] band are all components of the $j_{15/2}$ orbital, which provides all the negative parity levels in the 126-184 shell. One thus expects that this group of levels will be relatively free of mixtures from other levels, but due to the large j value will have very large Coriolis interactions with one another. A Coriolis calculation using as few as three parameters can account adequately for all the information on these three bands. The three parameters all have reasonable values, but the Coriolis matrix elements between the ground band and both the K = 5/2 and K = 9/2 bands are somewhat lower than expected. These must be around half of the Nilsson values according to the data, but cannot be brought lower than about 0.8 of the

Nilsson value according to current pairing calculations. We do not understand whether this is an inadequacy of the present pairing calculations or an indication that the wave functions are different from the Nilsson values. Apart from this point, these bands seem to be adequately understood.

REFERENCES

1) R. M. Diamond, B. Elbek, and F. S. Stephens, Nucl. Phys. 43 (1963) 560

2) J. O. Newton, Nucl. Phys. 3 (1957) 345

3) C. M. Lederer, J. M. Hollander, and I. Perlman, Table of Isotopes (1967) John Wiley and Sons, and references therein

4) B. R. Mottelson and S. G. Nilsson, Mat. Fys. Skr. Dan. Vid. Selsk. 1 No. 8 (1959); F. S. Stephens, Frank Asaro, and I. Perlman, Phys. Rev. 113 (1959) 212, and references contained in these compilations

5) S. G. Nilsson, Mat. Fys. Medd. Dan. Vid. Selsk. 29 No. 16 (1955)

6) K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Mod. Phys. 28 (1956) 432

7) A. Lütken and A. Winther, Mat. Fys. Skr. Dan. Vid. Selsk. 2 No. 6 (1964)

8) L. A. Sliv and I. M. Band, Coefficients of Internal Conversion of Gamma Radiations, Academy of Sciences of the USSR, Moscow-Leningrad, Part I, K-shell (1956); Part II, L-shell (1958)

9) O. B. Nielsen, Proc. Rutherford Jubilee Int. Conf. (Heywood, London, 1961) p. 317; P. G. Hansen, O. B. Nielsen, and R. K. Sheline, Nucl. Phys. 12 (1959) 389

10) R. M. Diamond, B. Elbek, and F. S. Stephens, unpublished data

11) A. Bohr and B. R. Mottelson, monograph, to be published

12) E. K. Hyde, I. Perlman, and G. T. Seaborg, The Nuclear Properties of the Heavy Elements II, pp. 732-733 and 953-954; I. Perlman, Proc. Int. Conf. on Nucl. Struct. Kingston, Canada, ed. D. A. Bromley and E. W. Vogt, Univ. of Toronto Press, Toronto, North Holland Publishing Co., Amsterdam (1960) 556; Irshad Ahmad, Ph.D. Thesis, University of California Radiation Laboratory Report UCRL-16888, 1966, unpublished

Table 1

Ground-state rotational band transitions[a])

I	I → I-1	I → I-2
9/2	46.3	
11/2	56.8	103.1
13/2	67.6	124.4
15/2	78.4	146.1
17/2	89.6	168.0
19/2	[b])	189.4
21/2	[b])	211.7
23/2	120.5	232.4
25/2	134.0	254.5

[a]) All these transitions are expected to be accurate to ±0.2 keV.

[b]) These lines were masked by the uranium K x rays.

Table 2

$B(E2)^{a}$) values for the strongest lines in ^{235}U Coulomb excitation

E_γ	$E_{excit.}$	ϵ_γ $B(E2)\uparrow$ 15 MeV ^4He	ϵ_γ $B(E2)\uparrow$ 60 MeV ^{16}O	ϵ_γ $B(E2)\uparrow$ 60 MeV ^{16}O corr. to q=0[b])	Adopted[c]) $\epsilon_\gamma(BE2)\uparrow_{q=0}$
625	671	1.59±0.24	1.23±0.18	1.33±0.19	1.50±0.15
633	633	1.81±0.27	1.50±0.21	1.74±0.24	1.79±0.18
638	638	1.31±0.19	1.25±0.19	1.47±0.22	1.36±0.14
822	822	1.29±0.19	1.1±0.3	1.3±0.3	1.29±0.18
921	921	3.12±0.45	2.4±0.4	2.7±0.5	2.98±0.30

[a]) All B(E2) values are for excitation, are in units of $e^2 \times 10^{-50}$ cm^4, and are relative to the 547 keV line in ^{197}Au, which was taken[6]) as 43.0.

[b]) We have used $q_{eff} \approx 0.8$ (see ref. 7) for 60 MeV ^{16}O, corrected this to q=0.

[c]) We have weighted the ^4He data twice as heavily as the ^{16}O data.

Table 3

Measured conversion coefficients for the prominent lines[a]) in ^{235}U

E_γ	$\epsilon_\gamma B(E2)\Uparrow$ [b]) 60 MeV ^{16}O	$\epsilon_K B(E2)\Uparrow$ 60 MeV ^{16}O	Exp. $\alpha_K \times 10^2$	Theo.[c]) $\alpha_K \times 10^2$ M1	E2	E1	Mult. Assign.
618	0.84±0.17[d])	0.065±0.009	7.7±1.9	11.0	2.0	0.7	M1-E2
625	1.39±0.14	0.158±0.016	11.4±1.7	11.0	1.9	0.7	M1
633	1.54±0.15	0.202±0.020	13.1±2.0	11.0	1.9	0.7	M1
638	1.16±0.12	~0.016	~1.4	10.0	1.9	0.7	(E2)
822	1.12±0.11	0.073±0.007	6.5±1.0	5.3	1.2	0.4	M1
840	0.84±0.17[d])	0.046±0.005	5.5±1.4	5.0	1.1	0.4	M1
921	2.62±0.30	0.030±0.005	1.15±0.23	3.9	1.0	0.4	E2
1053	< 0.2[d])	0.061±0.006	> 30	2.7	0.8	0.3	E0

[a]) All B(E2) values are for excitation, are in units of $e^2 \times 10^{-50}$ cm^4, and are relative to the 547 keV line in ^{197}Au, which was taken[6]) as 43.0 for gamma rays, and 0.68 for electrons.

[b]) Small corrections to the 60 MeV ^{16}O gamma-ray data have been made on the basis of table 2.

[c]) The theoretical conversion coefficients are taken from Sliv and Band[8])

[d]) These gamma rays were not directly measured at 60 MeV, but were obtained by correcting the 78 MeV ^{16}O data to 60 MeV ($q_{eff} \approx 1.2 \rightarrow q_{eff} \approx 0.8$).

Table 4

Gamma-ray relative intensities from ^{4}He and ^{40}Ar bombardments

E_γ	Rel. Inten.		Ave. ^{4}He Rel. Inten.	^{40}Ar Rel. Inten.
	15 MeV ^{4}He	19 MeV ^{4}He		
618	49	59	54	68
624	99	108	103	93
633	131	131	131	92
638	91	94	92	131
665	14		14	14
671	41	48	44	24
674	15		15	26
776	13	11	12	15[b])
784	15	14	15	21[b])
818	16		16	54
821	52	56	54	40
840	45[a])	40[a])	43[a])	28
875	40	59	49	54
920	(100)	(100)	(100)	(100)
1053		< 13	< 13	

[a]) There are impurity lines very near this line in the ^{4}He spectra.

[b]) There are probably contributions to these lines from extraneous peaks.

Table 5

Data from ^{235}U + 78 MeV ^{16}O; multipolarity assignments

E_γ	Rel. Inten.	Nor. Rel. Inten. e_K	Exp. $\alpha_K \times 10^2$	Theo.[a] $\alpha_K \times 10^2$ M1	E2	E1	Predom. Mult.
598.2	7						
601.4	4	0.3	8.0	12.2	2.1	0.8	M1
607.0	14	1.2	9.0	11.8	2.1	0.8	M1
612.8	13						
617.6	70	4.5	6.5	11.4	2.0	0.7	M1-E2
624.8	110	9.7	8.8	10.9	1.9	0.7	M1
633.1	120	[12.6][b]	[10.7]	10.7	1.9	0.7	M1
637.9	105	1.3	1.3	10.5	1.9	0.7	E2
646.3*	4						
651.4	11	0.5	2.6	9.9	1.8	0.7	E2
654.7	8						
664.6	13	0.5	3.8	9.3	1.8	0.6	E2(+M1)
671.0	38	2.9	7.7	9.1	1.7	0.6	M1
674.4	26	2.6	10.0	9.0	1.7	0.6	M1
719.0*	~9						
775.3	18	1.8	10.0	6.3	1.3	0.5	M1
782.9	18	1.1	6.3	6.1	1.3	0.5	M1
790.7	7	0.5	6.6	5.9	1.3	0.5	M1
818.1	25	3.1 (0.4)					
821.6	46	3.1 (2.7)	5.9	5.3	1.2	0.4	M1
839.5	34	2.0	5.8	5.1	1.2	0.4	M1

Table 5 (continued)

E_γ	Rel. Inten. γ	Nor. Rel. Inten. e_K	Exp. $\alpha_K \times 10^2$	Theo.[a] $\alpha_K \times 10^2$ M1	E2	E1	Predom. Mult.
858.2	9						
874.5	47	0.7	1.5	4.5	1.1	0.4	E2
884.7*	14						
890.8*	12						
920.6	100	1.3	1.3	3.9	1.0	0.4	E2
936.8*	16						
941.0	10						
1053.0	<5	1.9	> 38	2.7	0.8	0.3	E0

*The existence of these lines is considered questionable.

[a]) The theoretical conversion coefficients are from Sliv and Band[8]).

[b]) This electron intensity is normalized to the value for an M1 transition (see table 3).

Table 6

Relative yields from ^{16}O bombardments at 78 and 60 MeV

E_γ	$E_{excit.}$	Exp. Y_{78}/Y_{60}	Theo. Y_{78}/Y_{60}	
			E2	E3
633	633	3.4±0.4	3.5	4.6
638	638	3.3±0.4		
625	671	3.8±0.4	3.6	4.6
822	822	4.3±0.6	4.3	5.4
921	921	4.7±0.7	4.8	5.9

Table 7

Excitation of the 921 keV band

$I_iK_i \rightarrow I_fK_f$; ΔE	Exp.		Theo. if K_f=11/2		Theo. if K_f=9/2	
	^{4}He	^{16}O	q=0	q=1.2[a])	q=0	q=1.2
7/2 7/2 $\rightarrow K_f$ K_f;921	(1.0)	(1.0)	(1.0)	(1.0)	(1.0)	(1.0)
7/2 7/2 $\rightarrow K_f+1$ K_f;988	< 0.07	0.18±0.06	0	0.16	0.57	0.73

[a]) From calculations of Lütken and Winther [7]).

Table 8

Relative B(E2) values for de-excitation of the 921 keV band

Transition	Exp.	Theo. z=0	Theo. z=+0.06
11/2 11/2 → 7/2 7/2;921	(1.0)	(1.0)	(1.0)
11/2 11/2 → 9/2 7/2;875	0.65±0.07	0.39	0.64
11/2 11/2 → 11/2 7/2;818	0.31±0.11[a])	0.10	0.27
13/2 11/2 → 9/2 7/2;941	(1.0)	(1.0)	(1.0)
13/2 11/2 → 11/2 7/2;885	1.9±0.9	0.71	1.4
13/2 11/2 → 13/2 7/2;818	(1.2)[b])	0.26	0.9

[a]) This represents an arbitrary division of the 818 keV gamma ray and the error limits have been adjusted to take account of the uncertainty so introduced.

[b]) This is a small fraction of the total 818 keV transition, and hence is completely uncertain.

Table 9

Relative B(E2) values for excitation and de-excitation of the 638 keV band

$I_i,K_i \rightarrow I_f K_f$; ΔE	Exp.	Theo. z=0 (q=1.2)	Theo. z=+0.07 (q=1.2)
7/2,7/2 → 3/2,3/2;638	(1.0)	(1.0)	(1.0)
7/2,7/2 → 7/2,3/2;665	0.38±0.20	0.76	0.53
7/2,7/2 → 7/2,3/2;701	0.16±0.06	0.47	0.17
7/2,7/2 → 9/2,3/2;(745)	[a]	0.26	0.03
5/2,3/2 → 7/2,7/2;665	(1.0)	(1.0)	(1.0)
5/2,3/2 → 9/2,7/2;618	2.7±1.0[b]	1.2	2.3
7/2,3/2 → 7/2,7/2;(701)	< 0.6[a]	0.28	0.13
7/2,3/2 → 9/2,7/2;655	(1.0)	(1.0)	(1.0)
7/2,3/2 → 11/2,7/2;598	1.4±0.5	0.79	1.5

[a]) Not observed.

[b]) This intensity results from a division of the 618 keV line (see text).

Table 10

Initial values[a]) for bandhead energies and $\langle \Omega \pm 1 | j_{\pm} | \Omega \rangle$

Ω	Band head energies	$\langle \Omega \pm 1 \| j_{\pm} \| \Omega \rangle$
1/2	1810	
		7.25
3/2	1370	
		7.23
5/2	630	
		7.06
7/2	0	
		6.70
9/2	820	
		6.12
11/2	2220	
		5.24
13/2	3770	
		3.86
15/2	5510	

[a]) Values for $\eta = 5.5$.

Table 11

Calculated and measured rotational spacings

Transition	First Calc. (1 para.)	Second Calc. (3 para.)	Measured Value
K=7/2			
9/2 → 7/2	47.7	46.6	46.3±0.1
11/2 → 9/2	58.2	57.1	56.8±0.1
13/2 → 11/2	68.7	67.7	67.6±0.1
15/2 → 13/2	79.1	78.4	78.4±0.2
17/2 → 15/2	89.7	89.2	89.6±0.2
19/2 → 17/2	99.6	99.8	99.8±0.2
21/2 → 19/2	110.7	111.3	111.9±0.2
23/2 → 21/2	119.2	121.1	120.5±0.2
25/2 → 23/2	132.4	134.2	133.9±0.3
K=5/2			
7/2 → 5/2	28.6	38.6	38.0±0.2
9/2 → 7/2	37.9	49.6	49.6±0.3
11/2 → 9/2	43.7	57.5	57.0±0.4
13/2 → 11/2	58.7	71.7	72.8±0.6
K=9/2			
11/2 → 9/2	60.8	63.6	64.2±0.2
13/2 → 11/2	72.3	76.1	75.6±0.3

Table 12

Mixing amplitudes from the second (three-parameter) calculation

Assigned state K	I	Ω=1/2	Ω=3/2	Ω=5/2	Ω=7/2	Ω=9/2	Ω=11/2	Ω=13/2	Ω=15/2
5/2	5/2	0.011	0.106	0.994					
	7/2	0.030	0.164	0.982	-0.092				
	9/2	0.034	0.213	0.965	-0.132	-0.075			
	11/2	0.078	0.266	0.937	-0.162	-0.135	-0.013		
	13/2	0.062	0.297	0.913	-0.183	-0.200	-0.027	-0.001	
7/2	7/2	0.001	0.008	0.092	0.996				
	9/2	0.002	0.016	0.138	0.988	0.071			
	11/2	0.004	0.025	0.176	0.979	0.104	0.007		
	13/2	0.005	0.035	0.211	0.968	0.131	0.012	—	
	15/2	0.011	0.049	0.243	0.956	0.155	0.018	0.001	—
	17/2	0.010	0.059	0.272	0.944	0.176	0.025	0.002	—
	19/2	0.023	0.075	0.300	0.930	0.195	0.031	0.002	—
	21/2	0.018	0.086	0.324	0.917	0.212	0.038	0.003	—
	23/2	0.041	0.107	0.349	0.906	0.227	0.045	0.004	—
	25/2	0.026	0.115	0.368	0.888	0.242	0.052	0.005	—
9/2	9/2	0.003	0.016	0.063	-0.080	0.995			
	11/2	0.012	0.037	0.109	-0.125	0.980	0.099		
	13/2	0.014	0.062	0.160	-0.169	0.960	0.142	0.006	

Table 13

Relative[a]) B(E2) values for excitation of the 633 and 822 keV bands

$I_iK_i \rightarrow I_fK_f;\Delta E$	Exp	VAC[b])	VAC (q_{eff}=1.2)
7/2 7/2 → 5/2 5/2;633	(1.0)	(1.0)	(1.0)
7/2 7/2 → 7/2 5/2;671	1.30±0.15	0.96	1.09
7/2 7/2 → 9/2 5/2;721	0.62±0.15	0.38	0.66
7/2 7/2 → 11/2 5/2;778	0.22±0.09	0.06	0.29
7/2 7/2 → 13/2 5/2;851	~0.09	--	0.10
7/2 7/2 → 9/2 9/2;822	(1.0)	(1.0)	(1.0)
7/2 7/2 → 11/2 9/2;886	0.91±0.14	0.57	0.73
7/2 7/2 → 13/2 9/2;961	0.31±0.14	--	0.19

[a]) The absolute B(E2) values obtained from the second calculation are uniformly ~40% larger than those measured, as discussed in the text.

[b]) To first order the mixing does not affect these relative B(E2) values.

Table 14

Relative B(M1) values for de-excitation of the 633 and 822 keV bands

$I_iK_i \rightarrow I_fK_f;\Delta E$	Exp.	VAC	Second Calc. (no pairing corr.)	Second Calc. (pairing corr.)
7/2 5/2 → 7/2 7/2;671	0.27±0.04	0.29	0.26	0.24
7/2 5/2 → 9/2 7/2;625	(1.0)	(1.0)	(1.0)	(1.0)
9/2 5/2 → 7/2 7/2;(721)	<0.1	0.04	0.06	0.05
9/2 5/2 → 9/2 7/2;674	0.32±0.11[a])	0.50	0.39	0.37
9/2 5/2 → 11/2 7/2;618	(1.0)	(1.0)	(1.0)	(1.0)
11/2 5/2→ 9/2 7/2;(731)	<0.2	0.08	0.15	0.11
11/2 5/2→ 11/2 7/2;(674)	(0.3)[b])	0.66	0.44	0.44
11/2 5/2→ 13/2 7/2;607	(1.0)	(1.0)	(1.0)	(1.0)
9/2 9/2 → 7/2 7/2;822	(1.0)	(1.0)	(1.0)	(1.0)
9/2 9/2 → 9/2 7/2;775	0.48±0.09	0.23	0.27	0.39
9/2 9/2 → 11/2 7/2;719	~0.3[c])	0.02	0.01	0.01
11/2 9/2→ 9/2 7/2;840	(1.0)	(1.0)	(1.0)	(1.0)
11/2 9/2→ 11/2 7/2;783	0.62±0.13	0.41	0.56	0.62
11/2 9/2→ 13/2 9/2;(715)	<0.2	0.06	0.02	0.01
13/2 9/2→ 11/2 7/2;858	(1.0)	(1.0)	(1.0)	(1.0)
13/2 9/2→ 13/2 7/2;791	0.8±0.3	0.56	0.90	0.94
13/2 9/2→ 15/2 7/2;(712)	<1	0.10	0.02	0.01

[a]) A fraction of both the 618 and 674 keV gamma ray have been assigned elsewhere, and the limit of error includes the uncertainty so introduced.

[b]) This number is uncertain as most of the 674 keV radiation is assigned elsewhere. We include it here only for completeness.

[c]) The existence of this line is questionable.

FIGURE CAPTIONS

Fig. 1. Gamma-ray spectra of ^{235}U bombarded with ^{16}O and ^{40}Ar projectiles. This is the energy region where the rotational crossover transitions in the ground band occur and these are indicated on the figure, as are a few other lines of interest.

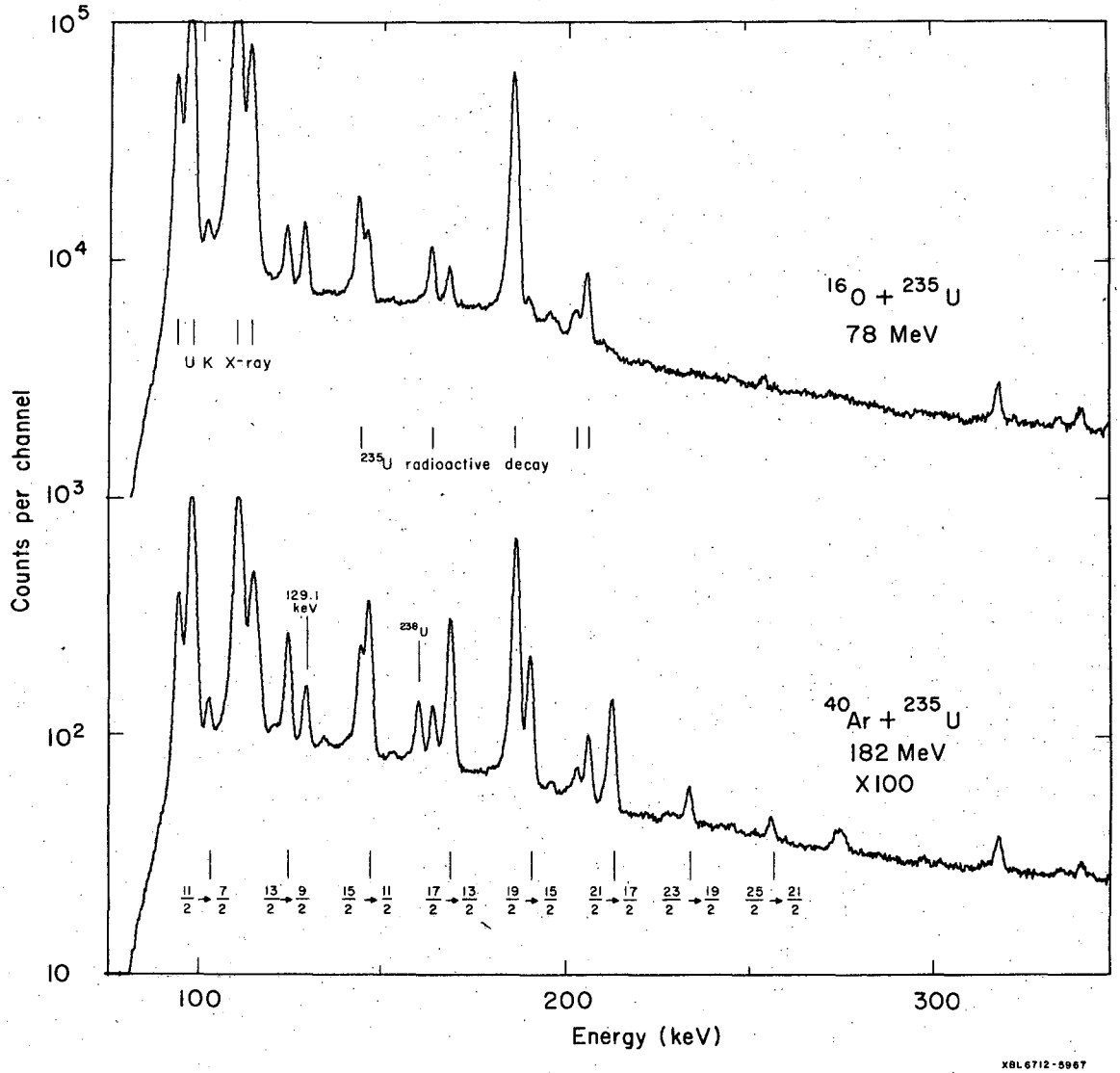
Fig. 2. Gamma-ray spectra of ^{235}U in the region 400 to 1000 keV. The three spectra correspond to Coulomb excitation by ^{4}He, ^{16}O, and ^{40}Ar projectiles, as indicated on the figure. The ^{4}He spectrum was taken with a slightly different amplifier gain than the other two. Each individual spectrum contains some lines not belonging to ^{235}U Coulomb excitation.

Fig. 3. Electron spectrum of ^{235}U Coulomb excited by 78 MeV ^{16}O projectiles. Most of the conversion lines identified are listed on the figure.

Fig. 4. Levels Coulomb excited in ^{235}U.

Fig. 5. Rotational spacings of bands in ^{235}U. The points are the experimental data, with the height of a point covering the error limits, and the lines correspond to the spacings obtained from the one-parameter Coriolis calculation.

Fig. 6. This plot is like Fig. 5, except (1) the lines correspond to the three-parameter Coriolis calculation, and (2) the ordinate scale has been doubled.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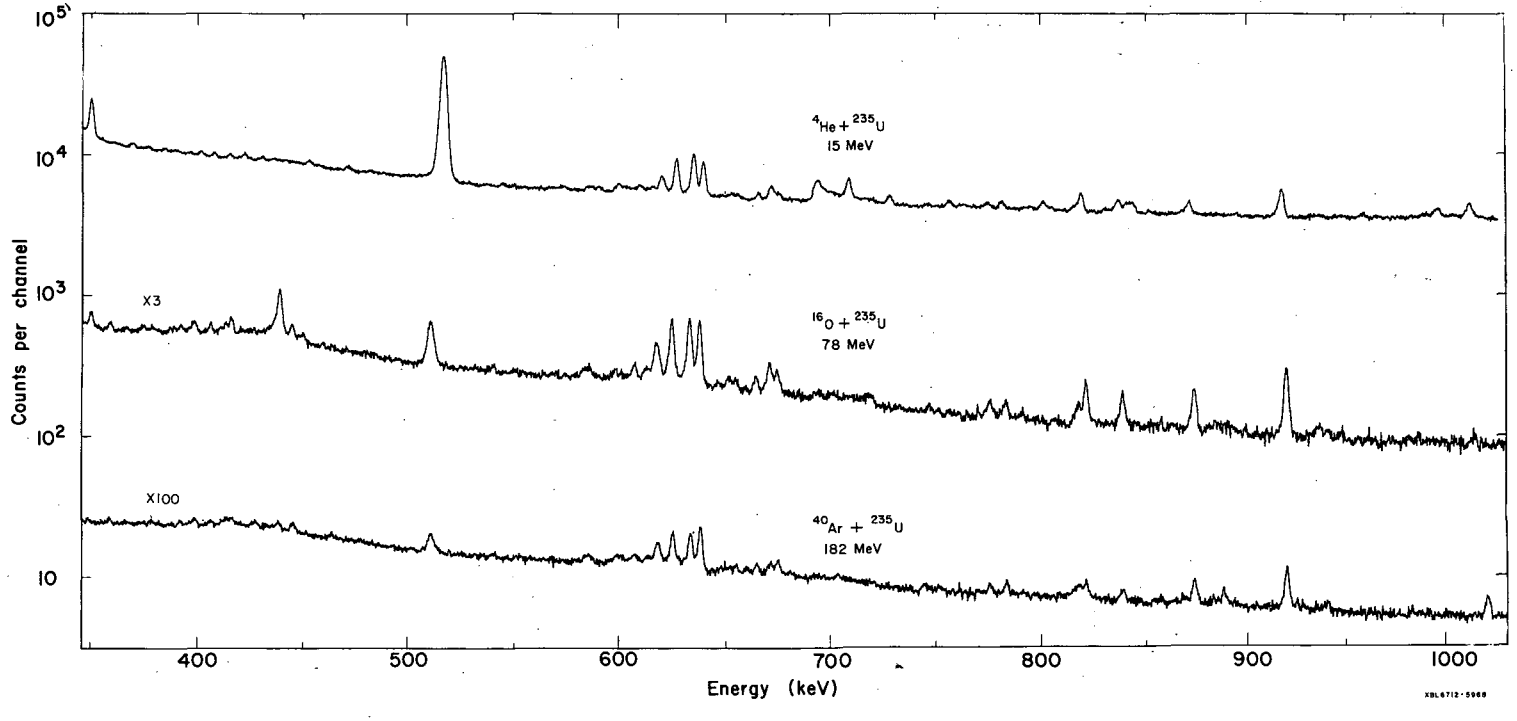
10^3
10^2
10
Counts per channel
U K X-ray
$^{16}O + ^{235}U$
78 MeV
^{235}U radioactive decay
129.1 keV
^{238}U
$^{40}Ar + ^{235}U$
182 MeV
X100
$\frac{11}{2} \rightarrow \frac{7}{2}$
$\frac{13}{2} \rightarrow \frac{9}{2}$
$\frac{15}{2} \rightarrow \frac{11}{2}$
$\frac{17}{2} \rightarrow \frac{13}{2}$
$\frac{19}{2} \rightarrow \frac{15}{2}$
$\frac{21}{2} \rightarrow \frac{17}{2}$
$\frac{23}{2} \rightarrow \frac{19}{2}$
$\frac{25}{2} \rightarrow \frac{21}{2}$
100
200
300
Energy (keV)
XBL 6712-5967

Fig. 1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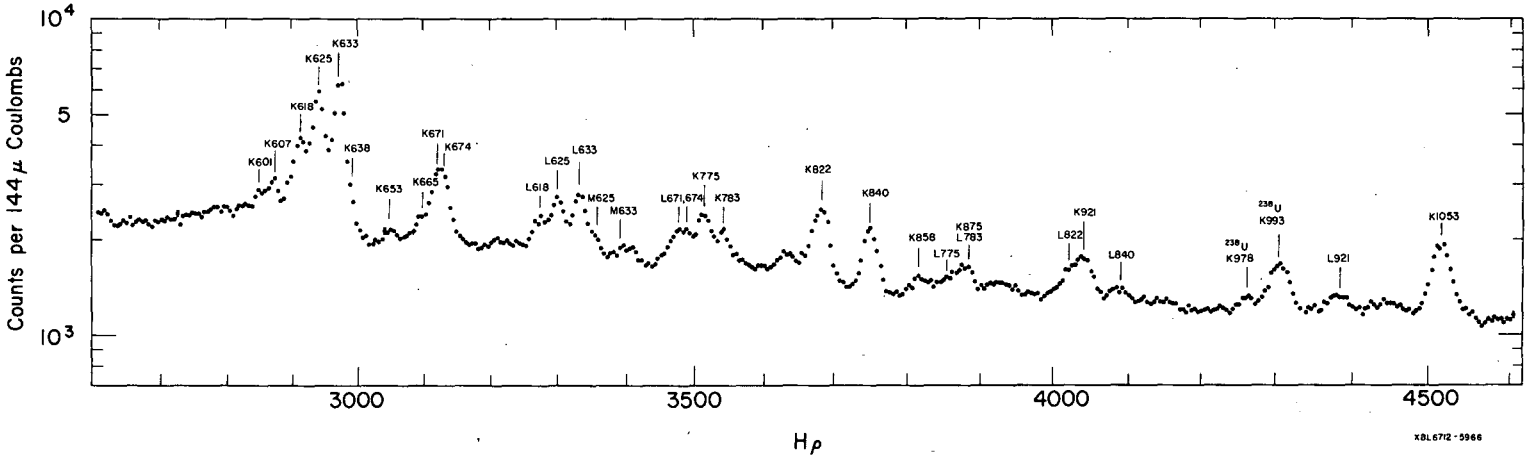
10^4
10^3
10^2
10
X3
X100
$^4He + ^{235}U$
15 MeV
$^{16}O + ^{235}U$
78 MeV
$^{40}Ar + ^{235}U$
182 MeV
400
500
600
700
800
900
1000
Energy (keV)

Fig. 2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10³
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K625
K618
K607
K601
K638
K653
K671
K674
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L625
L633
L618
M625
M633
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K775
K783
K822
K840
K858
K875
L783
L775
K921
L822
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4000
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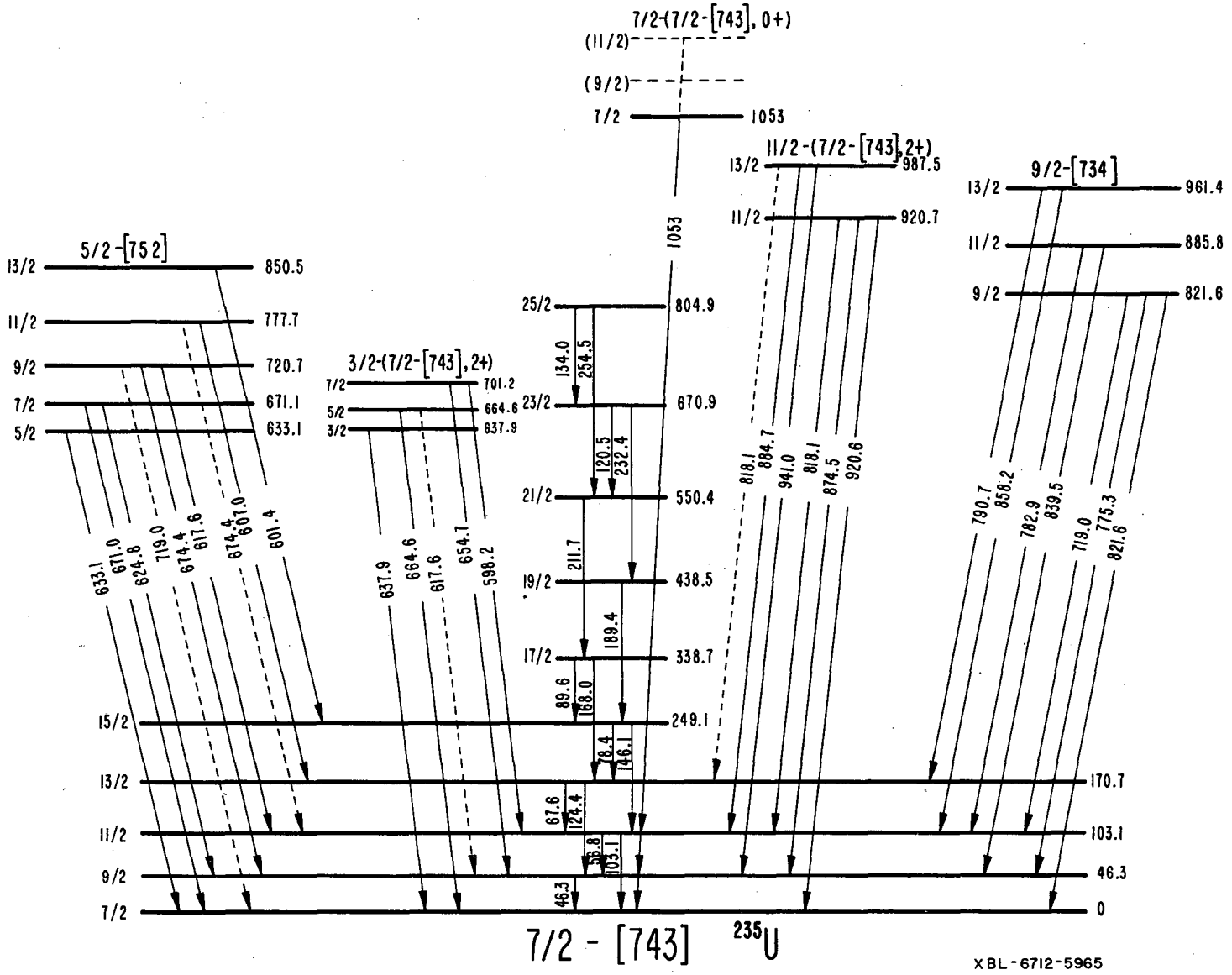
Hρ
XBL6712-5966

Fig. 3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(9/2)
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1053
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11/2
920.7
9/2-[734]
13/2
961.4
11/2
885.8
9/2
821.6
5/2-[752]
13/2
850.5
11/2
777.7
9/2
720.7
7/2
671.1
5/2
633.1
3/2-(7/2-[743], 2+)
7/2
701.2
5/2
664.6
3/2
637.9
25/2
804.9
23/2
670.9
21/2
550.4
19/2
438.5
17/2
338.7
15/2
249.1
13/2
170.7
11/2
103.1
9/2
46.3
7/2
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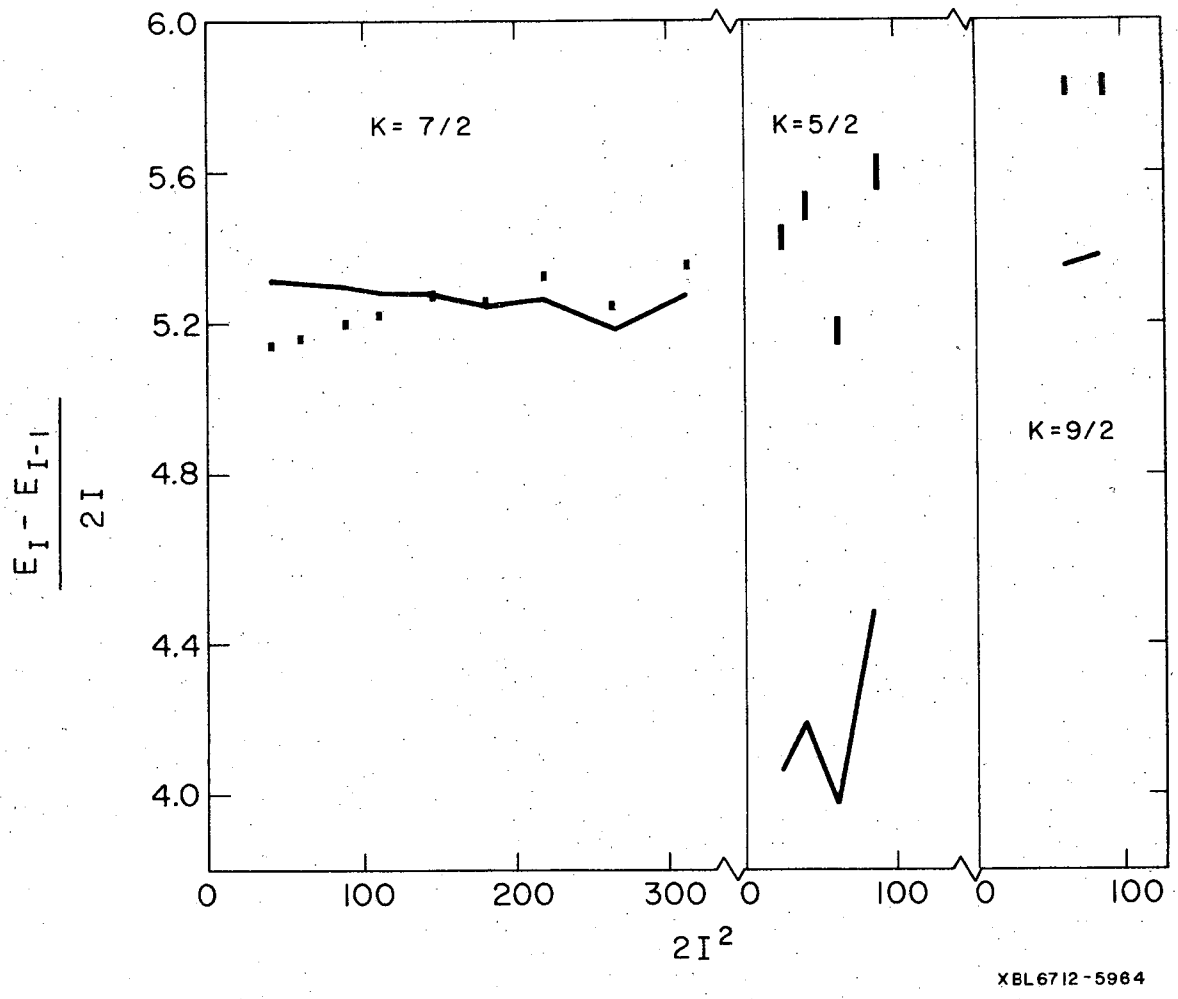
254.5
120.5
232.4
211.7
189.4
89.6
168.0
78.4
146.1
67.6
124.4
56.8
103.1
46.3
818.1
884.7
941.0
818.1
874.5
920.6
790.7
858.2
782.9
839.5
719.0
775.3
821.6
633.1
671.0
624.8
719.0
674.4
617.6
674.4
601.0
601.4
637.9
664.6
617.6
654.1
598.2
7/2 - [743] 235U
XBL-6712-5965

Fig. 4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5.6
5.2
4.8
4.4
4.0
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K=5/2
K=9/2
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100
200
300
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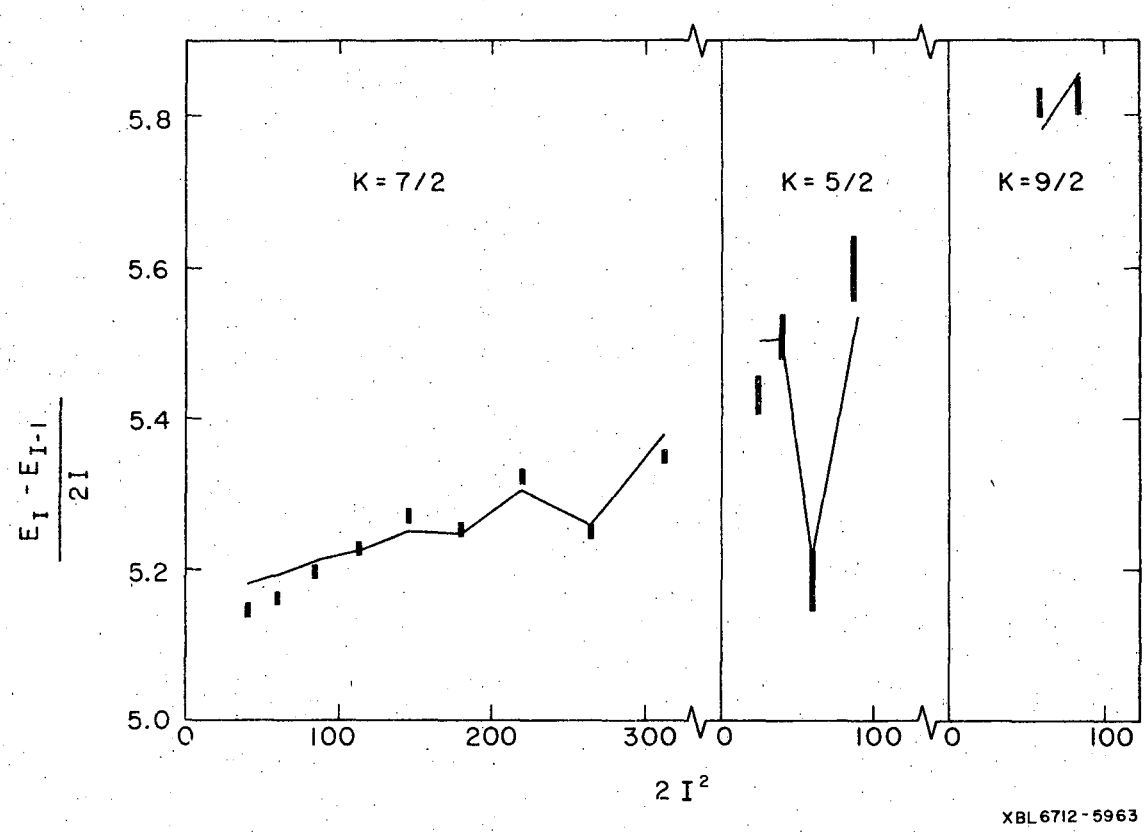
100
0
100
$\frac{E_I - E_{I-1}}{2I}$
$2I^2$
XBL6712-5964

Fig. 5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K = 5/2
K = 9/2
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5.4
5.2
5.0
0
100
200
300
0
100
0
100
$\frac{E_I - E_{I-1}}{2I}$
$2I^2$
XBL6712-5963

Fig. 6