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Authors
Baker, James A.
Stubbins, Warren Fenton.

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ABSTRACT

A computational procedure has been devised to give the shim changes required to correct the magnetic field of the 184-inch cyclotron, thus avoiding the empirical shimming formerly used. A mathematical model of the shim element suggested by Lambertson's study of the effect of small volumes of iron on the pole faces of magnets was used. The magnetic field is described by stating its value on the median plane above the 240 shim-element centers. A mapping function, calculated from the induced dipoles and their images, which gives the effect on the magnetic field in the median plane for a unit change in the height of each of 240 shim elements was computed. The effect of each shim-element change in the 23,000-gauss field is determined for all centers. Two hundred forty simultaneous equations are solved by an iterative process to find the necessary shim-element changes. Those calculated for a large measured field change corresponded to the ones producing the change in the 184-inch model magnet. The application of this method to shimming the 184-inch cyclotron will be reported.

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I. INTRODUCTION

Experimental and analytical studies of particle trajectories in high energy accelerators indicate that, for circular machines, efficient acceleration is achieved by the creation of a uniform, well tailored, magnetic field.

Necessary changes in the iron shim elements on the pole faces of a large cyclotron to achieve the desired magnetic field are determined by computation. A mathematical model of a shim element is assumed. The effect on the magnetic field produced by a unit change in the thickness of each shim element is computed for discrete points in the cyclotron. The resulting relationship is called the mapping function. The field change resulting at discrete points from the change of any or all shim elements may be calculated from the mapping function. Conversely, the necessary changes in shim elements to achieve a desired magnetic-field change may be computed. The latter computation is the problem of most interest.

It is necessary to avoid empirical shimming that requires repeated additions to and subtractions from the pole tips. In our very highly saturated magnetic field, 23,000 gauss, each shim element has a measurable effect at every place in the gap; a change of one shim literally requires compensation everywhere. For a field uniformity of ± 0.01%, about 2 gauss, an empirical process to produce the desired magnetic field that converges slowly is not satisfactory. The difficulty is emphasized by the inconvenience of repeatedly changing 490 shim elements in an 11- to 15-inch gap, 20 feet in diameter.

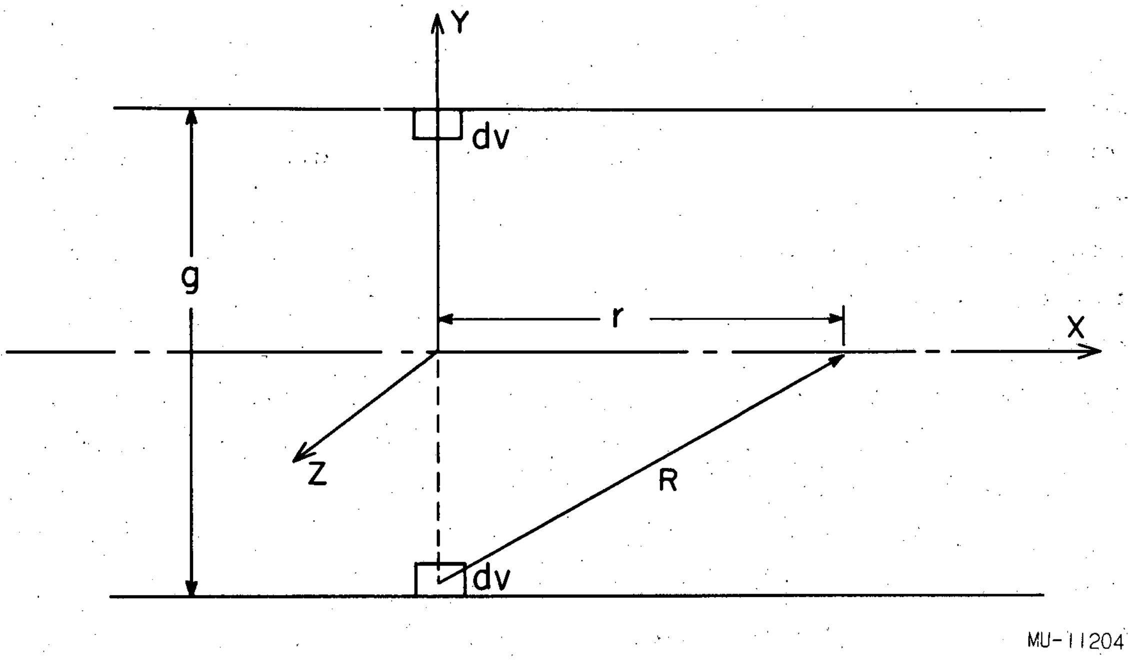

Fig. 1. Geometrical arrangement of small volumes of iron placed upon infinite pole tips separated by a gap g.

II. MATHEMATICAL MODEL OF THE SHIM ELEMENT

We extend a method due to Lambertson,[1] who computed the effect of placing symmetrically two small volumes of iron adjacent to the pole faces. The change in the magnetic field results from the magnetic dipoles induced in the volumes of iron by the main magnetic field. From Lambertson's calculations and experiments, it was apparent that his procedure could be extended to the shimming of a cyclotron.

In Fig. 1 we show two pole tips of infinite extent in the x, z planes, with a small volume of iron on each pole tip. The magnetic field on the median plane resulting from the induced dipoles may be obtained from the vector potential, $\vec{A}$, by

$$\vec{B} = \nabla \times \vec{A}. \tag{1}$$

We shall be concerned with the y component of this field. The vector potential of a dipole is given by

$$\vec{A} = \vec{M} \times \nabla\left(\frac{1}{R}\right) , \tag{2}$$

where $R = (x^2 + y^2 + z^2)^{\frac{1}{2}}$. The dipole density is

$$m = \frac{B_s}{4\pi} , \tag{3}$$

where B_s is the saturated magnetic inductance. For the magnetic field component oriented along the y axis, we choose

$$\vec{M} = m\,(0, 1, 0). \tag{4}$$

In the x, y plane for a small volume, dv, on the lower pole, we obtain on the median plane a field change

$$dB_y = 2m\,dv\left[\frac{y^2 - \frac{1}{2}x^2}{(x^2 + y^2)^{5/2}}\right] \tag{5}$$

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[1] Glen Lambertson, UCRL Engineering Note 4121-12, MT-1, Sept. 1954 (unpublished).

Owing to the symmetry and orientation of the dipoles, this component may be doubled to include the effect of the other small volume. If we let $y = \frac{g}{2}$ and $x = r$, then we have for both volumes

$$dB_y = \frac{B_s}{\pi}\left(\frac{2}{g}\right)^3 \left[\frac{1 - \frac{1}{2}\left(\frac{2r}{g}\right)^2}{\left[1 + \left(\frac{2r}{g}\right)^2\right]^{5/2}}\right] dv \qquad (6)$$

We now consider the reflections of these dipoles in the iron pole tips. In the high magnetic field the permeability of the iron is quite small. The first image of a dipole in the adjacent pole has a strength $\frac{\mu-1}{\mu+1}$ times the strength of the dipole. Each successive image has its strength further diminished by this factor. Because $\frac{\mu-1}{\mu+1}$ is considerably smaller than unity for our cyclotron, it is only necessary to consider the first image. The dipole and its first image increase the strength of the field by the factor

$$1 + \frac{\mu-1}{\mu+1} = \frac{2\mu}{\mu+1} . \qquad (7)$$

Thus

$$dB_y = \frac{B_s}{\pi}\left(\frac{2}{g}\right)^3 \frac{2\mu}{\mu+1} \left[\frac{1 - \frac{1}{2}\left(\frac{2r}{g}\right)^2}{\left[1 + \left(\frac{2r}{g}\right)^2\right]^{5/2}}\right] dv \qquad (8)$$

We shall obtain the total field change from a shim element by integrating over the volume change of the element.

III. THE 184-INCH SHIMS

Model measurements of the magnetic field of the 184-in. cyclotron on a 1/16-size model showed that a central field value of about 23,000 gauss could be obtained. This was accomplished by reducing the magnetic gap and adding additional ampere-turns near the gap in the original cyclotron.

The proper magnetic-field contour is then achieved by the arrangement of iron shims on both pole faces. Figure 2 shows the mechanical arrangement of the shim elements. The shim-element dimensions were selected in the belief that they would provide sufficient flexibility in modifying the field and that their boundaries would not be "seen" on the median plane.

The shim elements are of substantial thickness, from 1.371 to 2.795 in. and fastened to a base plate. The shimming procedure involves removing the shim element and either reducing its thickness by machining or increasing it by inserting a piece of thin shim stock between the shim element and the base plate. The shim stock is cut to the boundary dimensions of the shim element. The shim element clamps the shim stock firmly in place as the bolts are tightened.

In the calculation, the shim elements were taken to match those mechanically processed. However the elements in the outermost ring of shims were grouped in pairs and the innermost rings were further subdivided. The number of elements involved in the calculation then became 240 instead of 245 as mechanically arranged. The computation was based on identical changes top and bottom, so for the 240 units of computation 490 actual shim elements are represented, half on the upper pole face and half on the lower.

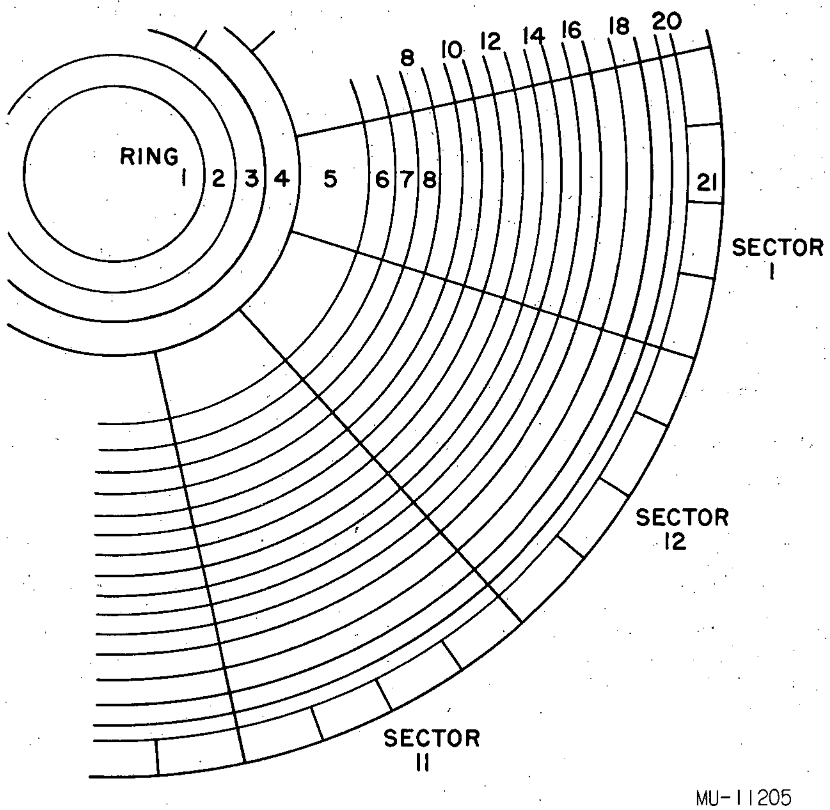

Fig. 2. Arrangement for dividing the pole surface into shim elements.

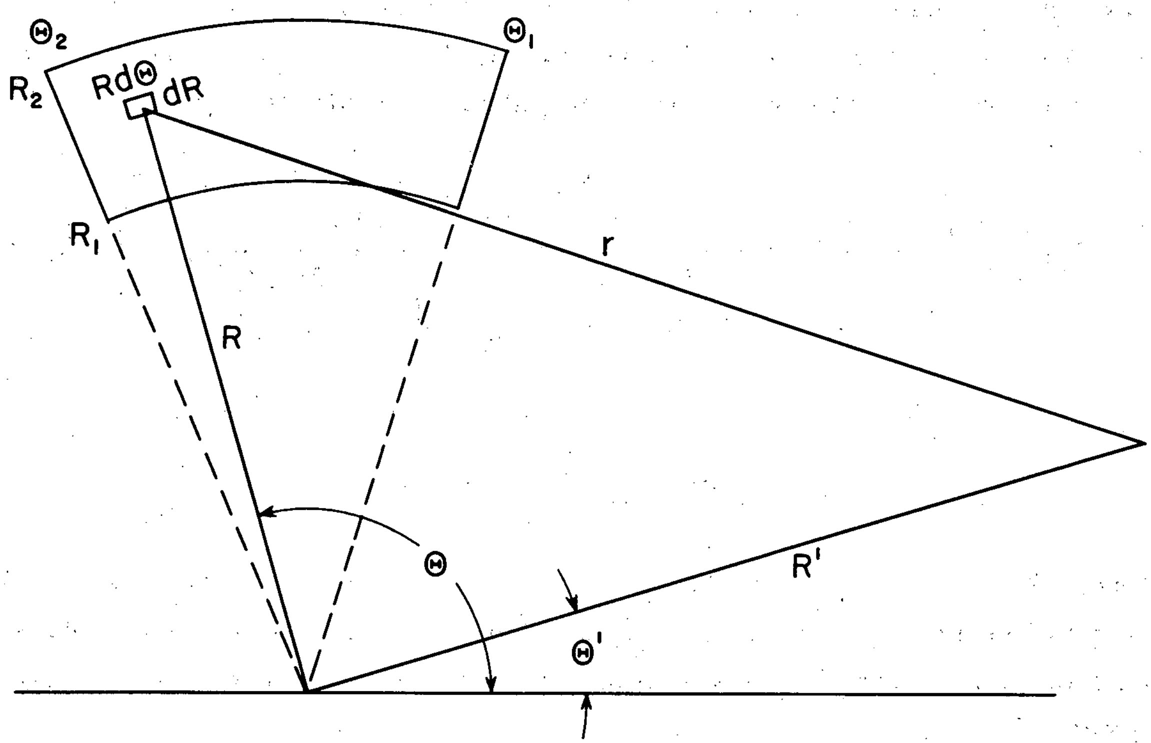

Fig. 3. Area of a typical shim element to be integrated in evaluating the integral I at the point R', θ'.

IV. THE MAPPING FUNCTION

By using equation (8), we may obtain the change in field resulting from a change in the thickness of a shim element. The volume element in cylindrical coordinates is given by

$$dv = h\, R\, dR\, d\theta\,, \tag{9}$$

where h is the uniform change in height over an entire shim element. Figure 3 shows the arrangement considered in the calculation.

Let A_{ij} be the change in the magnetic field at the center of shim element i resulting from a unit change in the height of shim element j.

Then

$$A_{ij} = \frac{B_s}{\pi} \left(\frac{2}{g}\right)^3 \frac{2\mu}{\mu + 1} \int_{R_1}^{R_2} \int_{\theta_1}^{\theta_2} \left[\frac{1 - \frac{1}{2}\left(\frac{2r}{g}\right)^2}{\left[1 + \left(\frac{2r}{g}\right)^2\right]^{5/2}} \right] R\, dR\, d\theta, \tag{10}$$

and

$$r = \sqrt{R^2 + R'^2 - 2R\,R' \cos(\theta - \theta')}\ , \tag{11}$$

where R_1, R_2, θ_1 θ_2 bound shim element j and R', θ' are the coordinates of the center of shim element i. The calculation applies only to the midplane. The integral in equation (10) is evaluated for each shim element. For convenience, we set

$$G = \left[\frac{1 - \frac{1}{2}\left(\frac{2r}{g}\right)^2}{\left[1 + \left(\frac{2r}{g}\right)^2\right]^{5/2}} \right], \tag{12}$$

and

$$I = \iint G R\, dR\, d\theta\,. \tag{13}$$

The computation of I was performed by two methods. At first a particular shim element was selected and the function I was calculated in both radial and azimuthal directions. Calculation was accomplished by means of a planimeter and an overlay of concentric circles. The area of the shim element lying between two circles was determined and multiplied by a mean value of G corresponding to the average distance from the center. The center of the concentric circles was placed at the point where the field was being evaluated.

This initial calculation served two purposes:

(a) it permitted an evaluation of the range and form of I for a given shim, and

(b) it gave a means of checking the calculations performed on the digital computer.

Later the calculation of the integrals I for all typical shim elements at the necessary points through the cyclotron was programed and performed with the IBM-650 digital computer. All possible symmetries that shortened the calculation were used, and the calculation of the effect of change of a single radial row of shim elements sufficed. The mapping function A, was found by azimuthal transposition of the values for the single radial row.

Figures 4 and 5 show the function I for a shim element in the 15th ring. In Fig. 4 the marked points on the curve indicate the value of I at the center of the shim element in the ring of the given number in the same radial row. Figure 5 shows how sharply the aximuthal influence of this shim element changes. The shim element in the 15th ring has approximately 156 square inches of surface. A 1/8-inch change in the height of each of a pair of these shim elements results in an approximately 100-gauss change on the midplane above its center. At the most remote shim-element center the resulting change is 0.025 gauss.

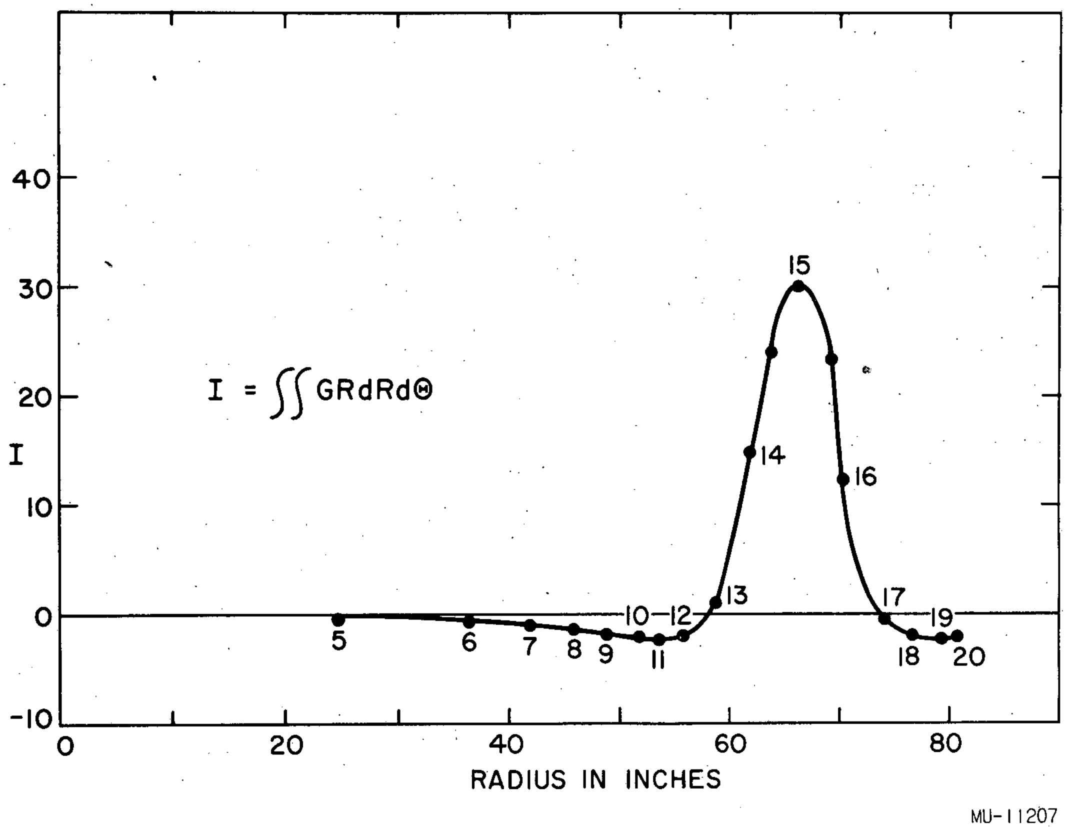

Fig. 4. Radial effect in a sector of increasing the shim element thickness in ring 15 of the same sector.

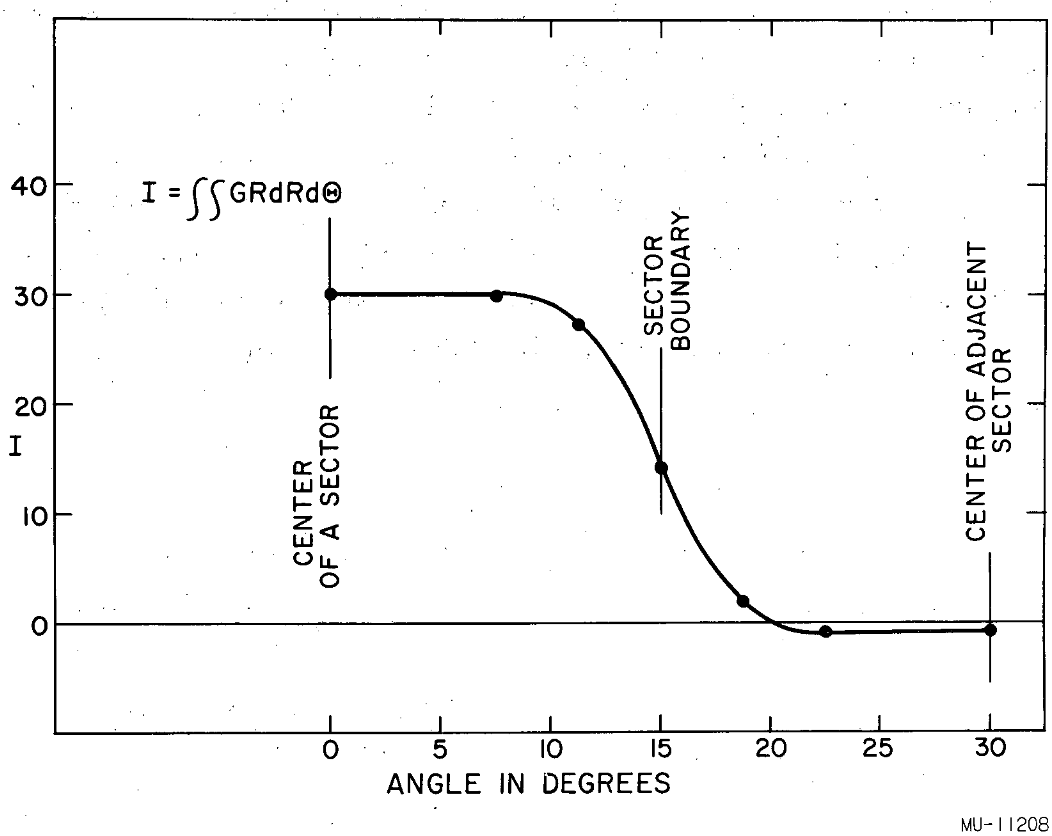

Fig. 5. Azimuthal effect of increasing the thickness of one shim element in ring 15.

V. DEFINITION OF THE MAGNETIC FIELD

The two questions of whether the magnetic field is determined to a sufficient accuracy everywhere by its value at the 240 distributed points and whether it can be modified to the required accuracy by using 240 shim elements were resolved by two independent arguments.

A sampling theorem[2] for continuous functions that have bounded rates of variation is applied to the magnetic field whose rate of change is determined by the gap. This shows that no separation between any of the 240 points exceeds the spacing required to completely determine the field.

A comparison was made between the fields resulting from normally placed shim elements and from a smoothly varying iron surface. The difference in the magnetic field was a maximum variation on the median plane of 0.02% occuring at the discrete boundaries of shim elements of different thicknesses. Local variations of this amplitude are acceptable, and the mean field values are obtainable within the desired accuracy with 240 shim-element pairs.

[2]Stanford Goldman, Information Theory (Prentice-Hall New York, 1953), p. 67.

VI. CALCULATION OF SHIM CORRECTION

Initially the magnetic field will differ from the desired magnetic field owing to radial and azimuthal variations. The shimming problem consists of modifying the pole-face contour to provide the required final field. The amount of change required in each shim element may be calculated using the mapping function.

We define the error between the measured field and the desired field at the center of the i^{th} shim element as B_i. If we recall that the mapping function, A_{ij}, is the effect at the center of shim element i of a unit change in the height shim element j then we see that the total magnetic field change at the center of the shim element i resulting from changing all the shim elements is

$$\sum_{j=1}^{240} A_{ij} h_j = b_i \tag{14}$$

where h_j is the change in the height of shim element j in terms of the unit change. For the proper field correction it is required that $B_j = b_j$; then the error will be corrected as the shim elements are changed to correspond to h_j. Therefore we have the problem of solving 240 simultaneous equations of the form

$$\sum_{j=1}^{240} A_{ij} h_j - B_i = 0, \quad \text{where } i = 1, \ldots , 240 . \tag{15}$$

This may be written as the matrix-vector equation

$$A \vec{h} = \vec{B}, \tag{15'}$$

which may be used to obtain the following:

(a) The shim element changes required to reduce the field error to as small a value as desired.

(b) The best arrangement to minimize the field error everywhere, if only some shim elements are changed.

(c) The field everywhere resulting from prescribed shim element changes.

VII. SOLUTION OF THE MATRIX EQUATION

Because A is a large (240 by 240) matrix, it was decided to use an iterative technique in the solution of the equations (15'). Moreover, when an iterative procedure is used one avoids most of the very severe round-off errors that accompany "finite" methods.

Since the matrix element A_{ij} is the effect wrought in the field at shim i by changing shim j, it is seen from Fig. 4 and 5 that

$$\left| A_{jj} \right| \gg \left| A_{ij} \right|, \quad \text{where } i \neq j. \tag{16}$$

That is to say, when shim j is changed, the change in field is far greater above shim j than it is anywhere else.

With this fact in mind we substitute for A the normalized matrix, A', obtained from A by dividing through each column by the diagonal element it contains:

$$A'_{ij} = \frac{A_{ij}}{A_{jj}} \quad . \tag{17}$$

We notice that

$$A'_{jj} = 1,$$

and that

$$\left| A'_{ij} \right| \ll 1, \quad \text{with } i \neq j .$$

Thus we see that A' is very close to the unit matrix. This leads us to choose the Gauss-Siedel iterative technique. This technique, as applied to the solution of

$$A'\vec{h} = \vec{B} \tag{18}$$

consists of making an initial guess, $X^{(0)}$, at the solution and then obtaining an improved "solution," $X^{(1)}$, as follows:

$$X_1^{(1)} = B_1 - \sum_{j=2}^{240} A'_{ij} X_j^{(0)}$$

$$\vdots$$

$$X_K^{(1)} = B_K - \sum_{j=1}^{K-1} A'_{Kj} X_j^{(1)} - \sum_{j=K+1}^{240} A'_{Kj} X_j^{(0)} . \tag{19}$$

In general, when one has found the mth iterate, $X^{(m)}$, one obtains $X^{(m+1)}$ as follows:

$$X_K^{(m+1)} = B_K - \sum_{j=1}^{K-1} A'_{Kj} X_j^{(m+1)} - \sum_{j=K+1}^{240} A'_{Kj} X_j^{(m)} . \qquad (20)$$

We notice that this method would converge in one iteration if A' were equal to the unit matrix.

For an initial guess at the solution, we choose $X^{(0)} = B$. This amounts to assuming that in changing a given shim we affect the field only above that shim.

To test the convergence of this process, we compute after the Kth iteration

$$r^{(K)} = \max_i (AX^{(K)} - B)_i .$$

This procedure has been coded for the Univac at the Livermore site of UCRL. On a sample problem, the process converged to two significant figures after six iterations. Each iteration required approximately 10 min.

VIII. TEST PROBLEM

The 1/16-size scale model of the 184-in. magnet was modified by the addition of a ring of iron, and the change in the magnetic field was measured. (The ring of iron in the model did not correspond to the shim elements later processed for the 184-in. magnet and chosen in our computation.) The experimental confirmation of our shimming calculation uses these measured results. The error field was defined at each shim-element center with the same radial distribution used for all azimuths.

In Fig. 6 the shim change on the model and the resulting magnetic-field change are plotted to the scale of the full-size machine. Figure 7 shows the calculated shim-element changes along a radius superimposed on Fig. 6. These were calculated by the process described above and represent the result of nine iterations.

An examination of the calculated changes and comparison with the experimental change confirms the principle and the method of solving this problem. The experimental magnetic-field change is very large and nonlinearities, which are not accounted for in the calculation, do not appear serious. The computational procedure is also checked by the uniformity of shim-element changes azmuthally. An error in handling the matrix elements would certainly be seen here. Figure 8 shows the azimuthal shim-element height variations for the most unfavorable ring.

The application of this procedure in shimming of the 184-inch cyclotron will appear in a later report.

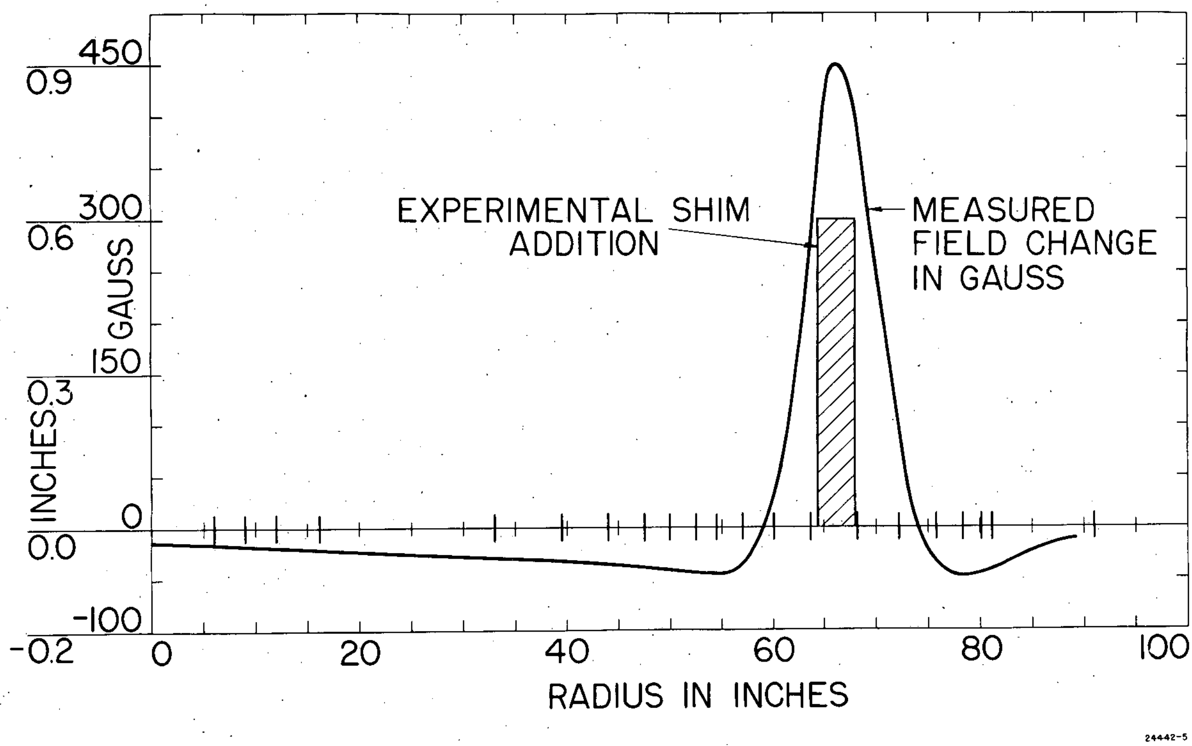

Fig. 6. Radial profile of the iron ring added to the 1/16-size scale magnetic model of the 184-inch cyclotron and the resultant radial magnetic-field change. This is plotted to correspond to the full-scale machine.

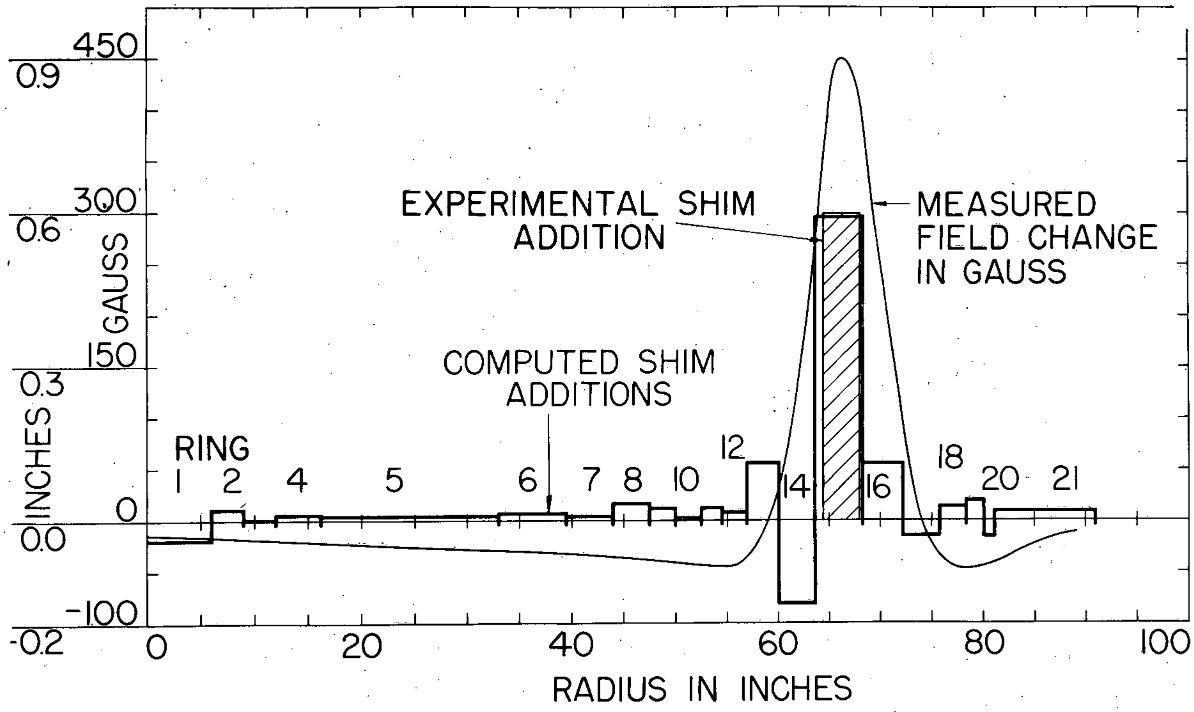

Fig. 7. Radial profile of the computed shim-element changes to achieve the magnetic-field change caused by the experimental iron ring. These data are superimposed upon Fig. 6.

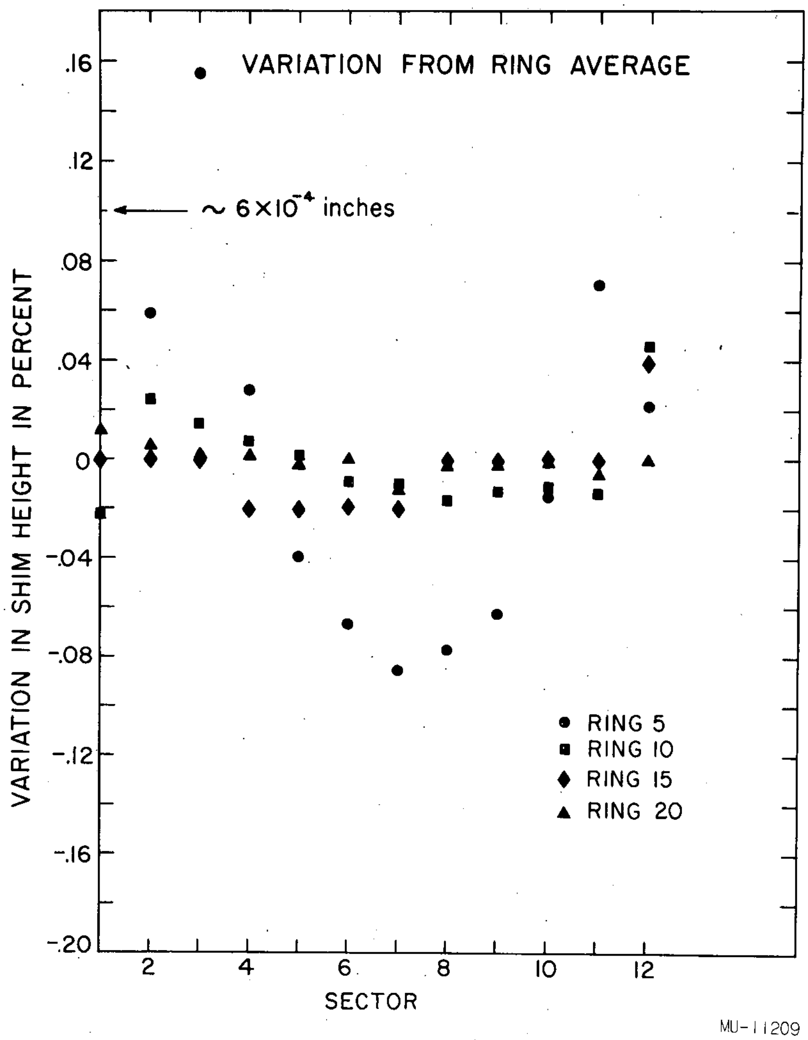

Fig. 8. Percentage variation from the average of the height of shim-element changes calculated for several rings.

ACKNOWLEDGMENT

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