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# Mode Excitation From Sources in Two-Dimensional EBG Waveguides Using the Array Scanning Method

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**Abstract**—An efficient semianalytical algorithm for the evaluation of the field and modal excitation by a line source in a two-dimensional electromagnetic bandgap (EBG) waveguide is presented. The method allows for an accurate and efficient calculation of the near field from the source inside the EBG waveguide, as well as the amplitude of the EBG waveguide mode that is excited. The same method can be applied to a wide variety of structures, as well as other types of sources and discontinuities.

**Index Terms**—Electromagnetic bandgap (EBG) materials, Green's function, periodic structures, waveguides.

## I. INTRODUCTION

THE DESIRE to achieve low-loss propagation in the millimeter-wave and optical ranges has recently motivated research into new ways of guiding electromagnetic waves. An electromagnetic bandgap (EBG) material with a row of defects (missing elements) constitutes a waveguiding structure that provides an attractive alternative to conventional waveguides [1], [2].

Here, we demonstrate a new method of moments (MoM)-based semianalytical method to efficiently model the fields and the mode excitation inside a two-dimensional (2-D) EBG waveguide. To the knowledge of the authors, this is the first time that an algorithm specifically tailored to the efficient calculation of the fields and mode excitation from a source inside of an EBG waveguide has been presented. The method also allows for physical insight into the physics of the field excited by the source and the modal excitation from the source. The method is explained for 2-D EBG waveguides, though the algorithm can easily be generalized to other EBG structures. The EBG waveguide under study consists of a periodic array of either metallic or dielectric posts, or holes in a dielectric slab, periodically spaced in the  $x$ - $z$  plane. (For simplicity, the case of metallic posts is considered here.) The method uses the one-dimensional (1-D) periodicity properties of the waveguide along the  $z$  direction in order to calculate the field of the line source by means of an array scanning method (ASM) [3], [4]. To improve the computational efficiency of the method, a 2-D Ewald acceleration scheme [5], [6] is used to improve the convergence of the periodic free-space Green's function that is used in the

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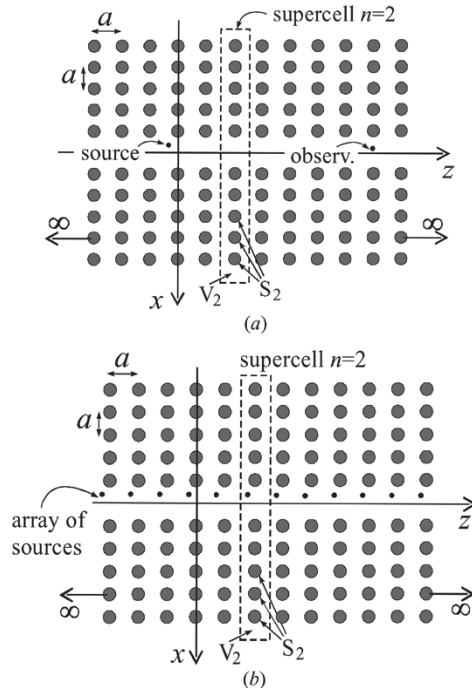


Fig. 1. (a) Geometry of the EBG waveguide and its excitation. The waveguide is an infinite periodic structure with period  $a$  along  $z$ , and truncated along  $x$ . In the figure, the periodic supercell  $n = 2$  is shown. The line source is located in the  $n = 0$  supercell.  $S_n$  denotes the surfaces of all conductors in the  $n$ th supercell. The volumetric region of the  $n$ th supercell is denoted by  $V_n$ . (b) An infinite array of line sources located in the waveguide. The original source is that in the  $n = 0$  supercell. In the ASM, the infinite periodic array is used to synthesize the field due to the single impressed source.

unit-supercell analysis. The near field from the source, as well as the amplitudes of the guided modes that are launched by the source, can be directly determined.

## II. EXCITATION OF THE EBG WAVEGUIDE

The geometry of the EBG waveguide is shown in Fig. 1. It is a periodic structure with period  $a$  along  $z$ , and is truncated along  $x$  with a finite number of rows of metallic posts of radius  $r_p$  on either side of the channel, embedded in a homogeneous background material. A periodic "supercell," defined by the dashed line, consists of a single column of posts along  $x$  with a missing central element. The region of the  $n$ th supercell is denoted by  $V_n$ , while the surface of the posts in this supercell is denoted by  $S_n$ . We assume that the electric field is polarized along the  $y$  axis, and since there is no variation along the  $y$  axis in this 2-D problem, propagation is both  $TE_z$  and  $TE_x$ . We denote by  $J_S(\mathbf{r})$  and  $E(\mathbf{r})$  the surface current and electric field, respectively, directed along  $y$ . An impressed electric line source  $J^i(\mathbf{r}')$  is located in the  $n = 0$  supercell at  $\mathbf{r}' = \mathbf{r}_0 = (x_0, z_0)$ , and the

electric field is evaluated at an observation point  $\mathbf{r} = (x, z)$  located in the waveguide. In the following, the fields of a single line source (and its modal excitation) in the waveguide are determined by using the ASM discussed below.

### A. Array Scanning Method (ASM)

A convenient calculation of the aperiodic excitation of a periodic structure can be obtained using the ASM, as the method was called in [3], though it had seen previous use, e.g., in [4]. The first step is to note the following relation between an infinite periodic array of impressed linearly phased line sources  $J^{i,\infty}(\mathbf{r}', k_z)$  with currents directed along  $y$ , see Fig. 1(b), and a single line source  $J^i(\mathbf{r}')$ :

$$J^{i,\infty}(\mathbf{r}', k_z) = \sum_{m=-\infty}^{\infty} \delta(z' - z_0 - ma) \delta(x' - x_0) e^{-jk_z ma}$$

$$J^i(\mathbf{r}') = \frac{a}{2\pi} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} J^{i,\infty}(\mathbf{r}', k_z) dk_z \quad (1)$$

where  $k_z$  is an impressed wavenumber along  $z$ . The single line source  $J^i(\mathbf{r}')$  is thus synthesized from the phased array of line sources along the  $z$  axis by integrating in the phase-shift variable  $k_z$  over the Brillouin zone. The electric field at any point  $\mathbf{r}$ , produced by the periodic array of line sources in free space is denoted as  $G^\infty(\mathbf{r}, \mathbf{r}_0, k_z)$ . The field produced by the periodic array of line sources in the EBG waveguide environment is denoted by  $G_{\text{EBG}}^\infty(\mathbf{r}, \mathbf{r}_0, k_z)$ . By the same weighted superposition used in (1), the electric field produced by the single source  $J^i(\mathbf{r}')$  in the waveguide is then given by

$$G_{\text{EBG}}(\mathbf{r}, \mathbf{r}_0) = \frac{a}{2\pi} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} G_{\text{EBG}}^\infty(\mathbf{r}, \mathbf{r}_0, k_z) dk_z. \quad (2)$$

Calculation of  $G_{\text{EBG}}^\infty(\mathbf{r}, \mathbf{r}_0, k_z)$  is discussed in the next section.

### B. MoM Solution Using the ASM

The method of moments (MoM) is used to solve the electric field integral equation (EFIE)

$$\int_{S_0} J_{S,\text{post}}^\infty(\mathbf{r}', k_z) G^\infty(\mathbf{r}, \mathbf{r}', k_z) d\mathbf{r}' = -G^\infty(\mathbf{r}, \mathbf{r}_0, k_z), \quad \mathbf{r} = S_0 \quad (3)$$

in order to obtain the surface current  $J_{S,\text{post}}^\infty(\mathbf{r}, k_z)$  (periodic along  $z$  with period  $a$ , except for a progressive phase shift) on the various metallic posts within the supercell, excited by the impressed periodic line-source array current  $J^{i,\infty}(\mathbf{r}', k_z)$ . The EFIE equation(3) implies that the field  $E_{\text{post}}^\infty$  produced by  $J_{S,\text{post}}^\infty$  plus the incident field  $G^\infty(\mathbf{r}, \mathbf{r}_0, k_z)$  produced by the array of sources  $J^{i,\infty}(\mathbf{r}', k_z)$ , must vanish on the surface of the posts within the supercell  $S_0$ . Note that in (3) the numerical integration is carried out only on the posts in the supercell  $n = 0$  (see Fig. 1) since we use the periodic Green's function  $G^\infty(\mathbf{r}, \mathbf{r}', k_z)$  for the field produced by a periodic array of line sources in free space successively phased as  $\exp(-jk_z na)$ . The EFIE (3) is then discretized according to the MoM procedure. The mutual coupling between the various basis-function

elements (to construct the MoM matrix) is then evaluated efficiently using the periodic Green's function  $G^\infty$ , whose series representation is accelerated using the Ewald method modified for 2-D geometries [5], [6]. Typically only two or three terms in each of the Ewald sums are sufficient to achieve good accuracy.

The periodic electric field at any point  $\mathbf{r}$ , produced by the periodic current on the posts comprising the EBG structure, is evaluated by integrating over the post currents comprising the single supercell  $S_0$ , as

$$E_{\text{post}}^\infty(\mathbf{r}, k_z) = \int_{S_0} J_{S,\text{post}}^\infty(\mathbf{r}', k_z) G^\infty(\mathbf{r}, \mathbf{r}', k_z) d\mathbf{r}'. \quad (4)$$

(On the posts, this field must cancel the field  $G^\infty(\mathbf{r}, \mathbf{r}', k_z)$  due to the periodic array of line sources in free space.) The periodic Green's function for the array of phased line sources in the EBG environment is given as

$$G_{\text{EBG}}^\infty(\mathbf{r}, \mathbf{r}_0, k_z) = G^\infty(\mathbf{r}, \mathbf{r}_0, k_z) + E_{\text{post}}^\infty(\mathbf{r}, k_z). \quad (5)$$

Equation (2) is then used to obtain the field of the single line source inside the EBG waveguide. The post current and the electric field in an arbitrary  $n$ th cell, induced by the single impressed line source in the supercell  $n = 0$ , are evaluated using ASM as

$$J_{S,\text{post}}(\mathbf{r} + na \hat{\mathbf{z}}) = \frac{a}{2\pi} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} J_{S,\text{post}}^\infty(\mathbf{r}, k_z) e^{-jk_z na} dk_z$$

$$G_{\text{EBG}}(\mathbf{r} + na \hat{\mathbf{z}}, \mathbf{r}_0) = \frac{a}{2\pi} \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} G_{\text{EBG}}^\infty(\mathbf{r}, \mathbf{r}_0, k_z) e^{-jk_z na} dk_z \quad (6)$$

with  $\mathbf{r} \in S_0$  for the post currents and  $\mathbf{r} \in V_0$  for the  $G_{\text{EBG}}$  field. Note that the total electric field  $G_{\text{EBG}}(\mathbf{r} + na \hat{\mathbf{z}}, \mathbf{r}_0)$  can also be obtained by applying the integration (6) to the scattered field  $E_{\text{post}}^\infty(\mathbf{r}, k_z)$ , and then adding the cylindrical field radiated by the line source at  $\mathbf{r}_0$ .

### C. Mode Excitation

Assuming a mode with a propagation wavenumber  $k_z = k_{z0}$  exists in the EBG waveguide, the terms  $J_{S,\text{post}}^\infty(\mathbf{r}, k_z)$  and  $G_{\text{EBG}}^\infty(\mathbf{r}, \mathbf{r}_0, k_z)$  have a pole at  $k_z = \pm k_{z0}$ . (The poles corresponding to the mode are periodically spaced in the  $k_z$  plane, and it is assumed here that  $k_{z0}$  is the pole that lies within the Brillouin zone of integration, with a positive real part.) For a lossless dielectric background material, the pole  $k_{z0}$  is still slightly below the real  $k_z$  axis for a propagating waveguide mode, corresponding to leakage loss through the finite number of rows of posts. The pole  $k_{z0}$  for such a mode is on the lower sheet of the branch point at  $k_z = k_0$ , where  $k_0$  is the free space wavenumber, when the guided mode is a fast wave ( $\text{Re}(k_{z0}) < k_0$ ). The mode is therefore an improper mode (the modal fields exponentially increase in the air region beyond the confines of the EBG structure). The residue evaluation at  $k_z = k_{z0}$  or  $k_z = -k_{z0}$ , for  $z > z_0$  or  $z < z_0$ , respectively, furnishes the modal current and electric field in the waveguide. (The residue of the pole at  $-k_{z0}$  is the negative of the residue of the pole at  $k_{z0}$ .) The modal field at any point in the EBG

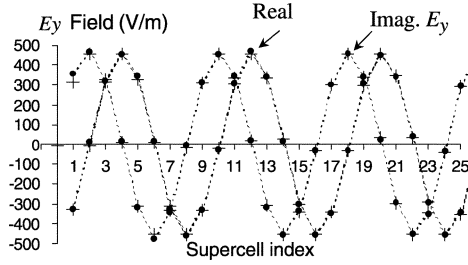


Fig. 2. Real and imaginary parts of the electric field  $E_y$ . The numerically determined full-wave field (dots) and the extracted field of a single dominant waveguide mode (crosses) are calculated at the center of each supercell. Dashed curves connect the data points for convenience.

waveguide may be written in terms of the modal field at the corresponding shifted observation point within zeroth supercell

$$G_{\text{EBG}}^{\text{mode}}(\mathbf{r} + na\hat{\mathbf{z}}, \mathbf{r}_0) = -ja\varepsilon_{\pm} R_G(\mathbf{r}, \mathbf{r}_0, \varepsilon_{\pm} k_{z0}) e^{-jk_{z0}|n|a} \quad (7)$$

where  $\varepsilon_{\pm} = \text{sgn}(z + na - z_0)$  and  $\mathbf{r} = (x, z) \in V_0$ . (Note that the  $\text{sgn}$  function is always positive for  $n > 0$ , and always negative for  $n < 0$ .) In this expression, the residue is calculated numerically as  $R_G(\mathbf{r}, \mathbf{r}_0, k_{z0}) = \lim_{k_z \rightarrow k_{z0}} [(k_z - k_{z0}) G_{\text{EBG}}^{\infty}(\mathbf{r}, \mathbf{r}_0, k_z)] = \lim_{k_z \rightarrow k_{z0}} [(k_z - k_{z0}) E_{\text{post}}^{\infty}(\mathbf{r}, k_z)]$  where we note that the field  $G^{\infty}(\mathbf{r}, \mathbf{r}_0, k_z)$  in (5) has no pole. Similar concepts apply to post currents  $J_{S, \text{post}}(\mathbf{r} + na\hat{\mathbf{z}})$ .

The calculation of the modal amplitude in this manner is very efficient because it involves only a periodic MoM solution, which in turn requires an analysis of only a single unit supercell (and this is made even more efficient by using the Ewald method to accelerate the periodic Green's function [6]). For example, in the specific structure considered in Section II-D, a supercell consists of eight posts. Using 16-pulse basis functions on each post gives a total of 128 basis functions. Such a calculation is therefore much more efficient than discretizing the entire EBG waveguide structure.

#### D. Numerical Results

As a simple example we analyze a waveguide as in Fig. 1(a) (with metallic rods of radius  $r_p = 0.2a$  in free space, where  $a$  is the period. The zeroth bandgap for the EBG structure that surrounds the channel (the first complete stop band for the  $\text{TE}_z$  polarization) exists when  $0 < a/\lambda_0 < 0.48$  ( $\lambda_0$  is the free-space wavelength), while the first bandgap exists for  $0.72 < a/\lambda_0 < 0.83$  [7]. Since we would have multimode propagation in the waveguide (i.e., higher order waveguide modes) for frequencies in the first bandgap, we analyze here the field propagation in the zeroth bandgap. The waveguide is modeled as in Section II-B, assuming four rows of posts parallel to the  $z$  axis on each side of the defect-channel. An electric line current is placed within the waveguide at  $(x, z) = (0, 0)$  with normalized frequency  $a/\lambda_0 = 0.3$ .

Fig. 2 plots the real and imaginary parts of the electric field  $E_y$  on the axis of the waveguide, sampled at locations  $z_n = na$ , the center of the  $n$ th supercell (see Fig. 1). The field is evaluated using (6) with (4) and (5) and also by using the modal solution (7), corresponding to a single dominant waveguide mode. It is clear that the field is essentially that of a single propagating mode, with guided wavelength  $\lambda_g = 8.0a$ . The two solutions are superimposed on the plot, showing that for this particular case the higher order waveguide modes are negligible even at

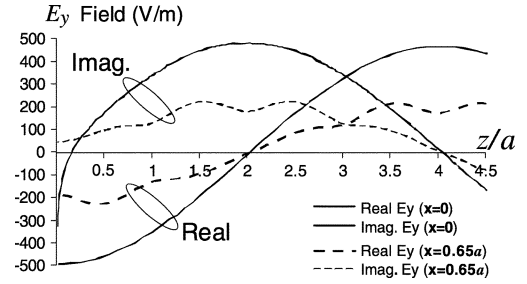


Fig. 3. Real and imaginary parts of the electric field  $E_y$  versus the continuous coordinate  $z/a$ , evaluated at two distinct locations:  $x = 0$  (along the center of the waveguide, continuous lines) and  $x = 0.65a$  (dashed lines). Off the waveguide center, higher order Floquet waves result in oscillations in the field.

the  $n = 1$  supercell. Hence, the variation from cell to cell is  $\exp(-jk_{z0}na)$  with  $k_{z0} = 2\pi/\lambda_g$ . The field variation plotted versus  $z$  within a unit cell would not be a pure sinusoid, in general, since higher order Floquet waves would be present.

Fig. 3 shows the field  $E_y$  plotted versus  $z$ , where  $z$  now changes continuously from near zero to a distance of 4.5 cells away from the source ( $z = 1.35\lambda_0$ ). The field is plotted along the center of the waveguide at  $x = 0$  (continuous line) and closer to one of the walls of the waveguide, at  $x = 0.65a$  (dashed line). The imaginary part of the field at  $x = 0$  exhibits an expected singularity near the source, whereas at  $x = 0.65a$  the field, which is still mainly that of a single waveguide mode, is more oscillatory with changing  $z$  due to the higher order Floquet waves that comprise the mode.

### III. CONCLUSION

The results of this letter can be extended to numerically model and evaluate the reflected and transmitted modes when discontinuities are present in an EBG waveguide, and this will be the subject of future studies. In the present summary, the algorithm is specialized to a 2-D EBG waveguide made of metallic posts in order to simplify the discussion and the formulation, but the algorithm can also be applied to 2-D and three-dimensional EBG waveguides made of dielectric material.

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