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APPLICATION AND OPERATION OF THE UCRL DIFFERENTIAL ANALYZER

John Killeen

June 8, 1953

Berkeley, California

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APPLICATION AND OPERATION OF THE UCRL DIFFERENTIAL ANALYZER

John Killeen

Radiation Laboratory, University of California Berkeley, California

June 8, 1953

ABSTRACT

The purpose of this report is to explain how the UCRL differential analyzer solves differential equations. It includes a description of the machine from a functional point of view and an explanation of the method of setting up equations for analyzer solution. It also includes operating procedures peculiar to this instrument.

9

TABLE OF CONTENTS

1

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1.	Introduction	2			
2.	The Purpose of the Differential Analyzer.	2			
3.	The Idea of the Differential Analyzer.				
4.	The Principle of the UCRL Differential Analyzer.	4			
5.	The Structure of the UCRL Differential Analyzer.	6			
6.	Method of Setting Up Differential Equations for Analyzer				
	Solution with Examples.	14			
7.	Scaling a Problem for the UCRL Differential Analyzer.	22			
8.	Procedure in Wiring and Operating the Differential Analyzer.	29			
	References	33			

-2-

APPLICATION AND OPERATION OF THE UCRL DIFFERENTIAL ANALYZER

1. Introduction

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The differential analyzer described in this report has been in operation in the theoretical group of the Radiation Laboratory for eighteen months. It was constructed in 1951 by the Radiation Laboratory under the supervision of Earl Sorensen and Carroll Gordon. The construction details are described in reference 1. For eight months during 1951 the theoretical group operated a differential analyzer, smaller but similar in design, which was leased from the University of Illinois. It was built by Professor Arnold Nordsieck of that university and it proved the need for a similar machine at this laboratory. The availability of these instruments has been a valuable addition to the computing facilities of the theoretical group.

I would like to thank Mr. William Gerkin for his valuable suggestions, interesting discussions, and continued interest in the operation of the analyzer.

2. The Purpose of the Differential Analyzer.

The differential analyzer is an instrument for evaluating the solution of ordinary differential equations. It solves linear equations with variable coefficients and non-linear equations. It can be used to solve systems of ordinary differential equations with a single independent variable.

On the UCRL differential analyzer graphical information can be fed into the machine as the problem is being solved. This enables the machine to solve equations which contain empirical functions. -3-

The applications of the instrument are many since ordinary differential equations appear in all branches of science and engineering. Typical examples of problems well suited to analyzer solution are equations of motion of charged particles in electromagnetic fields, one dimensional wave equations, radioactive decay chains, mechanical and electrical oscillations, deflection of beams, fluid flow, etc.

3. The Idea of the Differential Analyzer.

The idea of the differential analyzer is well known^{2,3}. It consists of a number of integrators which can be connected together so as to solve an ordinary differential equation directly for given boundary conditions. The output of one integrator may be used continuously as the integrand or variable of integration of one or more other integrators and this system of interconnected integrators generates the solution of the equation.

As an example of this method consider the harmonic oscillator differential equation:

$$\frac{d^2 y}{dx^2} = -y$$

Performing an integration with respect to x this equation can be written:

 $\frac{dy}{dx} = -\int y \, dx \qquad \text{Integrator 1}$

The solution is obtained by another integration:

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$$y = \int \frac{dy}{dx} dx$$
. Integrator 2

The solution of this equation is then obtained using two integrators connected such that the output of integrator 2 is the integrand of integrator 1 and the output of integrator 1 is connected to the integrand of integrator 2 with a change of sign. The variable of integration of both integrators is the independent variable of the machine.

-4-

4. The Principles of the UCRL Differential Analyzer.

The UCRL machine is a mechanical differential analyzer. The independent variable of the differential equation and all the functions generated in solving the equation are represented by rotations. The numerical value of the function is directly proportional to the total number of rotations of the shaft which represents it. The proportionality factor is a calculated scale factor for that function for a particular problem.

An integrator is a continuously variable gear. They are of the ball and disc type. A horizontal disc mounted on a vertical shaft and free to rotate in either direction is the driving component. Driven by this component is a vertical wheel which rolls on the disc. This wheel has a spherically ground edge so it is actually a spherical section which maintains a point contact with the disc. The wheel and disc are steel with precision ground surfaces and the friction between them on contact is sufficient to drive the wheel without slipping. The vertical axis of the disc is supported in a movable carriage so the point of contact of wheel and disc may be continuously varied. This variable displacement of the point of contact from the center of the disc supplies a variable gear ratio. The vertical wheel is mounted on a horizontal shaft in such a way as to allow it to tip in and out slightly as it rotates permitting the wheel to roll into new positions always maintaining only a point contact.



Denoting a rotation of the disc by Δx , the displacement of the point of contact from the center of the disc by y, the radius of the spherical section by c, and a rotation of this wheel by Δz , the equation of the gear can be written:

$$c \Delta z = y \Delta x$$

then the total number of rotations of the driven wheel can be written:

$$z = \frac{1}{c} \int y \, dx \, .$$

Consequently, if the variable of integration of an integral is made proportional to the rotation of the disc and the integrand proportional to the displacement of the point of contact from the center of the disc, then the total rotation of the driven wheel will be proportional to the integral.

The connections between units on the UCRL differential analyzer are made by an electro-mechanical system. In order to transmit the mechanical rotations representing functions from one unit to another self-synchronous

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-6-

electric motors (selsyns) are used. So for each unit there are selsyns used as outputs (generators) and selsyns used as inputs (motors). In electric motor drives the independent variable selsyn which drives all the other selsyns by means of the connections made between units. The actual connections between selsyns are made by patch cords on a plug board (Fig. 1) where there is a socket for each selsyn properly labeled by its mathematical use. An integrator, for example, has two input selsyns which are the integrand and variable of integration and one output which is the integral (Fig. 2). This system affords a great saving in space and in the time needed to ter up a problem. To change completely from one problem to another reduires less than twenty minutes.

5. The Structure of the UCRL Differential Analyzer

The UCRL differential analyzer consists of two complete units which can be used separately to solve different problems or can be wired for dual operation which results in an analyzer of double the tatacity for the solution of more complicated problems

Each unit has six integrators and a facility for connecting a seventh integrator. Under dual operation this gives a total capacity of "fourteen integrators which is sufficient to bandle a wave corner, if problems. The two extra integrators are mounted apart from the regular machine but are connected with other units on the regular plug board.

Each analyzer unit has two input-output tables with the becessary hand cranks for input operation (Fig. 1). There are two activitional inputoutput tables mounted separately along with the two estra operations. These are also connected with the main plug board. When the two enalyzer units are wired for dual operation there are trem the theorem is input-cut ut tables available for a problem

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Fig. 3 Front view of a single analyzer unit.

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These units consist of a horizontal rectangular table which holds a piece of 7 x 10 inch graph paper. On the table a movable index carriage rests on horizontal and vertical beams. These beams which traverse the length and width of the table are fastened to blocks which engage the ordinate and abscissa lead screws. The lead screws are driven by selsyns through a friction coupling. The beam blocks may be disengaged from the lead screws and moved along the guide way for initial setting. The friction coupling the selsyn shaft.

When using the tables as input tables, a known functional relationship is previously plotted on the regular graph paper. The independent variable of the function may be one of the dependent variables of the differential equation, i.e., a known function of the solution, f(y), is fed into the machine as the problem is being solved. The abscissa selsyn is driven by the selsyn representing the variable y which is being generated in the machine. The ordinate selsyn is connected to a crank driven selsyn. An operator then keeps the index on the plotted curve by turning the crank which drives the ordinate as the machine drives the abscissa. The rotations being fed into the machine by the operator are then proportional to f(y). To illustrate this method consider the differential equation

$$\frac{d^2 y}{dt^2} = f(y)$$

y

where f(y) is some known function of the dependent variable, such as the force on a particle where the force is given as a function of position. The equation can be written

$$'(t) = \int f(y(t))dt$$
. Integrator 1

Another integration with respect to t gives

$$y(t) = \int y'(t)dt$$
. Integrator 2

So two integrators and an input table are needed to solve this equation. The abscissa of the input table is driven by the output of integrator 2. The ordinate of the input table is driven by a hand crank which also drives the integrand of integrator 1. The output of integrator 1 drives the integrand of integrator 2. The variables of integration of both integrators are connected to the independent variable selsyn which is driven by the power supply. These are all the connections that are necessary to solve the equation. The tables are used as outputs to record the solutions as they are being generated. In this case both the ordinate and abscissa of the table are driven by functions which appear in the machine explicitly as rotations. In this case the sighting index is replaced by a pen which draws the functional relationship desired. The solution may be plotted against the independent variable or in the case of coupled equations the dependent variables may be plotted against each other to give trajectories, for example; or the first derivative may be plotted against the solution to give a phase plot.

On each analyzer unit there are six scaling multipliers (Fig. 4). These multiply a function by a constant. The constants on each multiplier vary from 0.1 to 0.9 in 0.1 intervals. They are actually just cone shaped gears and to set the constant factor the appropriate gears are meshed. The gears are coupled to selsyn shafts so there are two selsyns to each multiplier--an input and an output.

Adding two functions is done electrically on the UCRL differential analyzer. There is a type of selsyn motor called a "differential selsyn"

-11-



ZN 630

Fig. 4 Multipliers

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which algebraically adds the output signals of two selsyns. Consequently, the rotations of this type selsyn represent the sum of the two input functions. An adder then consists of a differential selsyn, with its two inputs, coupled mechanically to an ordinary selsyn which then furnishes the output signal. On each analyzer unit there are four such adders. In addition the integrand selsyns of six integrators, three on each unit, are also differential selsyns. This means that these integrators will perform integrals of the type

$$\int \left[f(x) + g(x) \right] dx .$$

Sec.

This, of course, is very helpful in that it saves the use of an adder providing the sum f(x) + g(x) is not needed explicitly for some other operation.

On all integrators there is a counter which records the rotations of the integrand lead screw and is a measure of the integrator displacement and thus is proportional to the value of the function being integrated. These counters (and consequently the displacements) are all set at the beginning of each analyzer run by the initial conditions of the problem. There are also two extra counters on each analyzer unit which may be used to measure any other functions in the problem. These counters have their own selsyns and are connected to the functions in the usual way.

The differential analyzer has been constructed to give an accuracy of 0.1% for the operation of its components. The counters can be read accurately for three figures, consequently the solutions can be recorded to three figures. The actual accuracy of a solution depends largely on the problem, i.e., the number of integrators used, number of input functions, running time, etc. For most problems the solutions are good to within 1.0%.

-13-

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6. Method of Setting Up Differential Equations for Analyzer Solution with Examples.

In setting up a differential equation for analyzer solution we operate on the equation directly, i.e. expressing the highest order derivative explicitly we then integrate both sides of the equation repeatedly until the solution is expressed in terms of a number of integrals. Consider the general equation of order n.

$$y^{(n)} = f(x, y, y', y'', ..., y^{(n-1)})$$

$$y^{(n-1)} = \int f(x, y, y', ..., y^{(n-1)}) dx$$

$$y^{(n-2)} = \int y^{(n-1)} dx$$

$$...$$

$$y = \int y' dx$$

Ordinarily such integrals cannot be performed since the integrands contain unknown functions but with the use of a system of interconnected integrators of the differential analyzer these integrals can be performed simultaneously.

Although an individual integrator can be used to solve a definite integral as a planimeter does this is a very inefficient use of the analyzer. Frequently persons with skill in solving differential equations are able to express the solution as a rather complicated definite integral and then desire to use an analyzer to evaluate this expression. This does not make use of the real advantage of the analyzer which is the creation of a mechanical system which obeys the given differential equation. To illustrate the method of setting up equations for analyzer solution we shall consider several examples.

Consider the equation

$$\frac{d^{2}y}{dx^{2}} + (a + \cos x)y = f(x)y^{2} + g(x)y \frac{dy}{dx} + h(x) \left(\frac{dy}{dx}\right)^{2}$$
with initial conditions
$$\begin{cases} y(0) = y_{0} \\ y'(0) = 0 \end{cases}$$

where a is a constant and f(x), g(x), h(x) are known functions of x and are given in tabular form. We can write the equation as

$$y' = -\int (a + \cos x)y \, dx + \int f(x)y^2 \, dx + \int g(x)y \, y' \, dx + \int h(x)y'^2 \, dx$$

We can express the above integrals as Stieltje's integrals:

$$y' = -\int (a + \cos x) d \left[\int y dx \right] + \int f(x) d \left[\int y^2 dx \right] + \int g(x) d \left(\frac{y^2}{2} \right) + \int h(x) d \left[\int y' dy \right].$$

We shall write this as

$$y' = -\int (a + \cos x) \, du + \int f \, dv + \int \underline{g} \, dy^2 + \int h \, dw$$

where
$$u = \int y \, dx$$
$$v = \int y^2 \, dx$$
$$y^2 = \int 2 \, y \, dy$$
$$w = \int y' \, dy$$

-15-

The $\cos x$ can be generated on the analyzer with two integrators

$$\cos x = -\int \sin x \, dx$$
 $\sin x' = \int \cos x \, dx$

and the solution is then obtained with one more integrator

$$y = \int y' dx$$

This problem requires eleven integrators as is seen and three input tables for the known functions f(x), g(x), h(x). It also requires three adders to combine the terms which make y'. As outputs we would plot y vs x, y' vs x, y' vs y. This would then use the three remaining plotting tables. A wiring diagram for this set up is seen in Fig. (5). In an actual set up all the functions are prefixed by scale factors so that the value of the function is then expressed in number of turns of a selsyn shaft. The choosing of appropriate scale factors will be discussed in the next section.

A particular solution of a differential equation is started on the analyzer by setting the initial displacements of all the integrators used. Consequently, the initial values of all functions which are integrands must be specified. If all of these values are known initially then the problem is called a one-point boundary value problem. The preceeding problem is an example of this type.

If, however, the initial values of functions which are integrands are not all known, but other conditions are given then the unknown function values must be estimated and trial solutions run until the given boundary conditions are satisfied. An example of this type of problem will now be considered.









Consider the system of equations

$$\frac{df}{dx} = -g\left[f^2 + g^2 + (1+\beta)\right]$$

$$\frac{dg}{dx} = -\frac{2g}{x} + f\left[f^2 + g^2 - (1-\beta)\right]$$

with the conditions that f and g vanish as x goes to infinity. We also specify that g(0) = 0. The problem is to find the discrete values of f(0) which lead to solutions with no modes, one mode, two modes, etc., and that are asymptotic to zero.

It is observed that x = 0 is a singular point of the second equation. Consequently, the system cannot be solved by an analyzer at the origin. The system can be solved in the small finite interval, $0 \le x \le \epsilon$, by method of series since g(0) = 0, so the analyzer solution can begin at $x = \epsilon$, which is a regular point. Solutions exist only for a certain range of values of the numerical constant, β . We shall assume that this range has been determined from other considerations.

We may now write the equations in a form suitable for analyzer set-up.

 $f = -\int \left[f^{2} + g^{2} + (1 + \beta) \right] g \, dx$ $g = -\int \frac{2g}{x} \, dx + \int \left[f^{2} + g^{2} - (1 - \beta) \right] f \, dx \, .$

We then express the integrals as the following Stieltjes integrals:

$$f = -\int \left[f^2 + g^2 + (1 + \beta) \right] d \left[\int g dx \right]$$

$$g = -\int 2g d \ln x + \int \left[f^2 + g^2 - (1 - \beta) \right] d \left[\int f dx \right]$$

So the analyzer set-up is

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$$f = -\int \left[f^{2} + g^{2} + (1 + \beta) \right] du$$

$$g = -\int 2g d \ln x + \int \left[f^{2} + g^{2} - (1 - \beta) \right] dv$$

$$u = \int g dx$$

$$v = \int f dx$$

$$f^{2} = \int 2f df$$

$$g^{2} = \int 2g dg$$

The function $\ln x$ is generated as follows

$$ln x = \int \frac{1}{x} dx$$
 $\frac{1}{x} = -\int \frac{1}{x} d ln x$

This problem then requires nine integrators and two adders. There are no input functions in this problem; consequently, the operator needs only to set the initial displacements and start the machine and the solutions are generated automatically. As outputs we plot f vs x, g vs x, and f vs g.

The wiring diagram for this problem is shown in Fig. 6. The function which goes to the integrands of the first two integrators is the sum $f^2 + g^2$, however their initial displacements are different. The first is initially displaced by an amount proportional to $\begin{bmatrix} f_0^2 + g_0^2 + (1 + \beta) \end{bmatrix}$ and the second by an amount proportional to $\begin{bmatrix} f_0^2 + g_0^2 - (1 - \beta) \end{bmatrix}$. That the system cannot be solved on the analyzer at the point x = 0 is obvious from the last two integrators.

In these examples we have made use of some standard connections for generating elementary functions. A list of some of these standard connections follows.



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for $x \neq 0$

for $x \neq 0$

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1.
$$e^{ax} = a \int e^{ax} dx , a \text{ feal}$$
2.
$$\sin x = \int \cos x dx$$

$$\cos x = -\int \sin x dx$$
3.
$$\sinh x = \int \cosh x dx$$

$$\cosh x = \int \sinh x dx$$
4.
$$x^{2} = \int 2 x dx$$
5.
$$x^{3} = \int 3 x^{2} dx$$

$$x^{2} = \int 2 x dx$$
6.
$$x^{4} = \int 2 x^{2} dx^{2}$$

$$x^{2} = \int 2 x dx$$
7.
$$\ln x = \int \frac{1}{x} dx$$

$$\frac{1}{x} = -\int \frac{1}{x} d \ln x$$
8.
$$\sqrt{x} = \frac{1}{2} \int \frac{1}{\sqrt{x}} dx$$

$$\frac{1}{\sqrt{x}} = -\frac{1}{2} \int \frac{1}{\sqrt{x}} dx$$

$$\frac{1}{\sqrt{x}} = 2 \int \frac{1}{\sqrt{x}} d \ln x$$

. • .

We also make use of the following relations:

1.
$$ku = \int k \, du$$
, $k = \text{real constant}$
2. $\int f(x) g(x) \, dx = \int f(x) \, d\left[\int g(x) \, dx\right] = \int g(x) d\left[\int f(x) dx\right]$
3. $uv = \int u \, dv + \int v \, du$.

7. Scaling a Problem for the UCRL Differential Analyzer.

The independent variable and all the functions generated in solving a system of differential equations are measured in the differential analyzer by selsyn shaft rotations. Consequently, we must select suitable proportionality factors between the numerical value of the function and the number of rotations of the selsyn shaft which represents it. In doing this we are governed by certain rules imposed by the design of the integrators and plotting tables.

We should recall from section 4 in the description of an integrator that the radius of the vertical integral wheel is a constant, c, which appears in the integrator equation as

$$z = \frac{1}{c} \int y dx$$
.

Since all the variables in a system are expressed in rotations; the constant, c, must also be given in these units, and furthermore in the same units as the integrand displacement, y, as is evident from the above equation in moment form $c \Delta z = y \Delta x$. For the integrators on the UCRL differential analyzer this constant is equal to 50, since a displacement equal to the radius of the integral wheel requires 50 turns of the integrand lead screw.

Consequently, in a problem set-up the integrals are written in the following form:

UCRL-2239

$$(\alpha z) = \frac{1}{50} \int (\beta y) d(\gamma x)$$

where $(\mathfrak{T} z)$, (βy) , $(\mathfrak{T} x)$ are the variables expressed in selsyn shaft rotations and \mathfrak{T} , β , \mathfrak{T} are scale factors assigned to the variables z, y, x for a particular integrator. An expression $(\mathfrak{T} x)$ means that a change in the variable x by one unit requires \mathfrak{T} turns of the selsyn shaft representing x. The scale factors must of course satisfy the relation:

$$\Upsilon = \frac{\beta \Upsilon}{50} .$$

We are, however, further restricted in choosing these factors by the maximum allowable displacement of the integrators. On the UCRL analyzer the total displacement of the integrators is equivalent to 100 turns of the integrand lead screw--50 turns for positive displacements and 50 turns for negative displacements. Consequently, when selecting scale factors for functions which are integrands the positive and negative variation of the product must not exceed 50. That is, if y is an integrand function and β is its scale factor for an integrator then $|\beta y| \leq 50$. On the other hand it is desirable to choose integrand scale factors such that the maximum displacement is attained, for purposes of increased accuracy and ultimate saving in running time.

The next consideration in choosing scale factors is the size of the plotting tables. The total variation of the abscissa is equivalent to 200 turns of the abscissa lead screw and the total variation of the ordinate is equivalent to 140 turns of the ordinate lead screw. These lead screws have twenty threads per inch so the graph paper which is used is

UCRL-2239

7" x 10" with twenty lines per inch. This means that the coordinate axes may be conveniently labeled so that function values may be read off the plots in their original units. When assigning a scale factor for a variable that is used as an ordinate or abscissa of a table it must be selected so as to keep the total variation of the variable on the paper.

On the integrators and plotting tables there are limit switches which automatically stop the machine if any of the variables exceed the design limits. This prevents damage to the instrument and is a signal to reconsider and probably rescale the problem.

Although a scaling set-up may be successful insofar as the problem is contained within the limits of the machine it is usually desirable to re-scale it to make optimum use of the integrators and plotting tables. Consequently, the first set-up should generously allow for the variations in the unknown functions.

We shall now consider an example of the method of scaling for the UCRL differential analyzer.

Consider the equation

$$m\ddot{z} = eE_{z}(z, t) = eE_{0}(z) \cos(\omega t + \delta_{0})$$
with initial conditions
$$\begin{cases} z(0) = 0 \\ \dot{z}(0) = v_{0} \end{cases}$$

We shall find it convenient to transform independent variable by $\theta = \omega$ t then the equation becomes

$$\frac{d^2 z}{d\theta^2} = \frac{e}{m \omega^2} E_0(z) \cos \left(\theta + \delta_0\right).$$

So for analyzer solution we shall consider an equation of the form

-24-

-25-

 $\begin{cases} z(0) = 0 \\ z'(0) = v_0 / \omega \end{cases}$

$$z'' = f(z) \cos (\theta + \delta_0)$$

with initial conditions

and where f(z) is a known function of position and can be tabulated and plotted for the range of z which is required. This function f(z) will be fed continuously into the analyzer from an input table. The integrals to be considered are the following:

$$z' = \int f(z) \cos (\theta + \hat{S}_{0}) d\theta = \int f d \sin (\theta + \hat{S}_{0})$$

$$z = \int z' d\theta$$

$$\sin (\theta + \hat{S}) = \int \cos (\theta + \hat{S}) d\theta$$

$$\cos (\theta + \hat{S}) = -\int \sin (\theta + \hat{S}) d\theta$$

The ranges of the variables can be stated and are tabulated below.

Region I:

$0 \leq z \leq 4$	
$0 \leq f(z) \leq 0.4$	
0 ≤ z' ≤ 1.0	(estimated)
$0 \leq \theta \leq 4\pi.$	(estimated)

Region II:

 $4 \leq z \leq 14$ $0 \leq f(z) \leq 0.4$ $0 \leq z' \leq 5.0$ (estimated) $4\pi \leq \theta \leq 10\pi$ (estimated)

-26-

We have selected an example that makes use of the technique of splitting the range of solution up into regions so as to make better use of the integrators and for ease in following the given input functions. A scaling set-up for Region I would be as follows:

Integrators

$$100 \ z' = \frac{1}{50} \int 125 \ f \ d \ 40 \ \sin (\theta + \delta)$$

$$100 \ z = \frac{1}{50} \int 100 \ (z' - .5) \ d \ 50 \ \theta + 50 \ \theta$$

$$40 \ \sin (\theta + \delta) = \frac{1}{50} \int 40 \ \cos (\theta + \delta) \ d \ 50 \ \theta$$

$$40 \ \cos (\theta + \delta) = -\frac{1}{50} \int 40 \ \sin (\theta + \delta) \ d \ 50 \ \theta$$

Plotting Tables

250 f vs 50 z 100 z' vs 50 z 10 θ vs 50 z.

Multipliers

$$.2(50 \theta) = 10 \theta$$

 $.5(250 f) = 125 f$
 $.5(100 z) = 50 z$.

A set-up for Region II would be:

Integrators

$$100 z' = \frac{1}{50} \int 125 f d 40 \sin (\theta + \delta)$$

20 z = $\frac{1}{50} \int 20(z' - 2.5) d 50 \theta + 50 \theta$

۴

40 sin
$$(\theta + \delta) = \frac{1}{50} \int 40 \cos (\theta + \delta) d 50 \theta$$

40 cos $(\theta + \delta) = -\frac{1}{50} \int 40 \sin (\theta + \delta) d 50 \theta$.

-27-

Plotting Tables

250 f vs 20 z 20 z' vs 20 z 5 θ vs 20 z.

Multipliers

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 $.1(50 \theta) = 5 \theta$.5(250 f) = 125 f.2(100z') = 20 z'.

A wiring diagram for Region I is on Fig. 7. It should be noticed that a function may have one scale factor for use on a plotting table and another on an integrator. When such functions are generated in the machine the largest scale factor is generated and the others are obtained from it by means of the multipliers which only multiply by tenths.

In this example we have used on integrator 2 the technique of displacing an integrand function so as to make use of all the displacement for integrating instead of just the positive side. This is particularly effective when the integrand function remains near zero for some time. In employing this technique we of course must make use of an adder for each such displacement.

A sample initial values sheet for the preceding problem is given below.







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		REGION I		REGION II		
		<u>Start</u>	Finish	Start	Finish_	
	8.	30 ⁰				
	θ.	0	θ _{z=4}	$\Theta _{\mathbf{z}=4}$	θ z=14	
	θ+δ.	30 ⁰	θ _{z=4} +30°	θ z=4 +30°	e _{z=14} +30°	
	Z	0	4	4	14	
	z'	0.2	z' _{z=4}	z' z=4	z' z=14	
	f	0	.1	.1	•05	
 1:	125 f	0	12.5	12.5	6.25	
2:	100(z'5)	-30.0				
3:	40 cos (θ + δ)	34.66				
4:	40 sin (0 + S)	20.0				

8. Procedure in Wiring and Operating the Differential Analyzer.

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After an analyzer set-up has been carefully planned and written out we are then ready to put this set up on the machine. A typical procedure for doing this will be discussed. A detailed description of the various controls as well as the plug board is given on pp. 34-48 of reference 1.

If the set-up requires more components than can be supplied by a single analyzer unit, then both analyzer units should be connected for dual operation. This should be done first and with the power off. If the set-up

-30-

requires only one unit then they should be disconnected and both units used separately.

The <u>power</u> should be turned <u>on</u> while patching the plug board according to the wiring diagram for a particular problem. During this wiring the wheels should be up and independent variable motor off. When wiring the board after each connection is made the lamp board underneath the analyzer should be observed to make sure all the lamps are out. This means that the selsyns connected are in phase. If a bulb is on the patch cord connecting the selsyn indicated should be disconnected and connected again repeatedly until the light goes out.

Each patch cord is wired to retain the proper polarization. The cords should be plugged so that the cable drops down from the plug. To <u>introduce a negative sign</u> in a connection, the plug on one end of the cord making the connection should be rotated 180° so that the plug is upside down.

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After the problem is wired up the multipliers should be set and the necessary input and output sheets placed on the plotting tables. We are then ready to set initial conditions. The integrator <u>wheels should be up</u> during all of this preliminary work. The initial displacements of the integrators should be set first. By holding over the switch opposite each integrator the independent variable drives the integrand selsyn of that integrator. Consequently, the displacement is easily changed and is measured by the counter mounted on the shaft of the integrand selsyn. Positive displacements are measured in tenths of turns from 0.1 to 50.0 and negative displacements are indicated on the counters by readings from 50.0 to 99.9. Care must be taken when near the edge of the disc to change the displacement in the proper direction. -31-

When more than one integrand has the same function, setting one will change the other so the others should be disconnected and set separately and then connected again. Integrators that can be set to the same number when connected must be chosen. (Not all integrand selsyns can be set accurately together.) The best choice is a regular integrand selsyn along with an integrand with a differential selsyn. Then one input to the differential selsyn is locked by a crank which can adjust the differential selsyn to the same setting as the regular selsyn.

It frequently happens, when an integrand function is an output of a multiplier, that the setting of the integrand displacement is impossible because of the reverse in mechanical advantage. The integrand plug should be disconnected momentarily until the setting is made. After connecting the plug again a finer adjustment can be made by disengaging the multiplier, turning the output gear slightly until the setting is reached and then engaging the gear carefully at the proper multiplier ratio.

After the integrator displacements are set the independent counters are set in the same way and the settings recorded. The plotting tables are then set according to the initial conditions of the functions plotted and pens placed in the holders on the output tables.

At this point the variac of the independent variable motor should be set at zero. The variac <u>must</u> always be set to zero when the wheels are lowered. To start the analyzer solution after all is set--lower the integrator wheels. This automatically turns on the independent variable motor. The speed of the motor is then gradually increased from zero to proper running speed by slowly turning up the variac. When the solution is completed turn down the variac, raise the wheels, and remove the pens from the output tables.