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Mathematics, Language, and Learning: A Longitudinal Study of Elementary Teachers and Their Mathematics Teaching Practices

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Education

by

Cathery Yeh

Dissertation Committee:
Associate Professor Rossella Santagata, Chair
Professor Raul A. Fernandez
Assistant Professor Tesha Sengupta-Irving

2016

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DEDICATION

This dissertation is dedicated to my husband and children. Thank you for embarking on this journey with me. We made it!

> To teach in a manner that respects and cares for the souls of students is essential if we are to provide conditions where learning can most deeply and intimately begin.
>
> bell hooks

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ACKNOWLEDGMENTS

I would like to thank many people for their continued support and encouragement throughout this process. This journey was not one that I completed alone.

I am deeply grateful for the support and guidance of my dissertation committee. First, I want to acknowledge the wonderful guidance and tutelage of my advisor, Professor Rossella Santagata. Rossella, where should I begin – there are not enough words to describe how grateful I am. Thank you for showing me the beauty of research and scholarship and its potential for impact, for pushing to be my best but reminding me to also be kind to myself, and for serving as a role model of the type of advisor/scholar/mentor I hope to become. Without your guidance and persistent support, this dissertation would not have been possible. Professor Tesha Sengupta-Irving, thank you for letting me in your world. I appreciate the hard questions asked and the reminder of where I am right now in my professional journey and where I hope to go. Professors Raul and Nancy Fernandez, muchismas gracias! UCI and, then, CSUF became my home when I met you both. Thank you for the advice and guidance during this journey and for showing me that the best type of mentoring is through example and from the heart. Professor Mark Ellis, I wouldn't be here without you. Thank you for encouraging me to get a doctorate and for believing in me even before I believed any of this was possible.

I would not have been able to complete this journey without the community, "the village," that has helped me juggle the role of parent, student, wife, and educator. Thank you moms and dads for working all day and still watching the girls in the evening. There is no love like the love of grandparents. Thank you Jeff, Katy, Vero, Wendee, Jinny, Bing, Ken, Lyn, Keith, Teresa, Remy, Catherine, and everyone else who helped the family with babysitting, carpooling, meals, and your friendship, love and understanding. We are so fortunate to have friends that have become family and that love Em and El as their own.

A very big thank you to my UCI community for helping me survive this process: Hana, Edelina, Bri, Alma, Analicia, Jess, Huy, Alfredo, Sonja, Leona, Sara, Tina, Tara, Marcela, Janet, Priyanka, Jody, Val, and Jeanne. Thank you for reminding me that one of the best parts of graduate school is the people you meet along the way. Each one of you is such an inspiration.

To my Titan family – Mark, Fernando, Nick, Antoinette, Tara (yes, girl, you show up in two places), Tonja, and so many more -- thank you! This year has been such a learning experience, but I cannot have asked for a better start in the academy. Thank you for keeping me and the FLM program afloat and for your encouragement, words of wisdom, and your friendship. No matter where I am, I know this is just the start of our academic workship/friendship.

Thank you to the four teachers in this study. Thank you for opening up your classroom, your teaching practice, and your lives.

To my own students at Hillside, Arnold, UC Irvine, and CSUF, my scholarship is and will always be grounded in you. Thank you for allowing me to learn with and from you.

Finally, to my family, you guys are amazing. Thank you for taking part in this journey with me, for encouraging me to take risks and for serving as a daily reminder of how blessed I truly am. And to my better-half, David, this is all possible because of you. You have sacrificed so much- your work and your own sleep to support me through it all. We made it!! This degree is as much yours as it is mine!

Em and El, this degree is for you… as a reminder that we never stop learning and that there is no limit to what we can achieve.

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ABSTRACT OF THE DISSERTATION

Mathematics, Language, and Learning:
A Longitudinal Study of Elementary Teachers and Their Mathematics Teaching Practices

By

Cathery Yeh

Doctor of Philosophy in Education

University of California, Irvine, 2016

Associate Professor Rossella Santagata, Chair

Elementary school mathematics has gained increased attention in the last few decades. A growing field of research has studied the programmatic design and development of elementary mathematics teaching in teacher education; however, few studies have examined longitudinally the mathematics teaching of novice elementary teachers. Existing longitudinal studies on elementary mathematics teaching have generally focused on the effects of teacher preparation on their beginning practices and have examined novice teachers as a homogenous group. This dissertation consists of two studies that examined longitudinally novice elementary teachers and their mathematics teaching practices during their first two years of professional teaching. The first paper examined how three novice bilingual teachers organized mathematics learning for their emergent bilinguals. Data are drawn from three longitudinal case studies and include videotaped classroom observations and interviews of their mathematics instruction. Specifically, the study examined the type of supports the teachers provided to develop student learning of the language of mathematics, who holds the authority of knowledge the classroom community, and how and what student repertoires of practice were utilized. The study findings highlight the complexity of bilingual teaching in the context of supporting students in learning the language of

mathematics. The second paper examines the role of reflection as a vehicle for teacher change. Reflection has been identified in teacher education as a vehicle for professional growth and development; however, there are few studies that specifically look at the relation between reflection and teaching. This pair of case studies details the mathematics teaching practices and the lesson reflections of two novice teachers over a two-year period, examining their relationship, and how it may contribute to their development over time. Findings from this study highlight the potential for teacher learning in attending to student thinking in teaching and lesson reflection. In light of the two study findings, specific recommendations for teacher preparation are provided. A model of teacher knowledge is proposed.

INTRODUCTION

School mathematics plays a central role in students' sense of identity and their academic achievement. And teachers are critical players in how they shape the classroom learning environment to promote engagement in school mathematics responsive to students' identities, experiences, and understanding of mathematics. Just as classroom teachers must learn about their students to develop mathematics instruction responsive to their students, mathematics teacher-educator researchers must also learn about novice teachers' experiences to design learning opportunities responsive to teachers' professional experiences in teacher preparation and the induction period. Currently, there are few studies that longitudinally examine the mathematics teaching practices of novice elementary teachers.

This dissertation takes a situated lens and view mathematics teaching practice neither as static nor fixed, rather as an evolving dynamic process in which teachers learn to participate in and across communities (Wenger, 1998). Two studies are shared that longitudinally examine novice elementary mathematics teachers and their mathematics teaching practices during their first two years of professional teaching. The first paper explores the teaching practices of three bilingual teachers and how they organize mathematics learning for their emergent bilinguals. The second paper examines the interrelation of instruction and reflection and its potential to promote teacher professional growth over time.

There is shared consensus in the field of mathematics education that the best learning opportunities are ones situated in practice and made relevant to the daily experiences of students (K-12 school-age students and preservice teachers who are students of teaching) (Ball & Cohen, 1999; Civil, 2002; 2007; Foote, 2009; Grossman, Hammerness, & McDonald, 2009; National Council of Teachers of Mathematics [NCTM], 2000; 2014). While exploratory in nature, the

goal of this dissertation is to better understand the teaching experiences of novice teachers and to use what is learned to better support their learning and, in return, their own students' learning of mathematics.

Background

Our lived experiences shape how we make sense of the world around us. I began my teaching career sixteen years ago in downtown Los Angeles, working with students of color and language minorities like me. As a classroom teacher, I visited over 300 hundred student homes to get to know my students and their families to determine how I could leverage their informal and formal mathematics knowledge, their language practices, and their daily experiences as resources for learning. This desire to better create student-centered instruction inspired me to go back to school to begin my doctoral studies.

For the past six years, I have worked with the elementary mathematics pre-service teachers at the University of California, Irvine as a graduate researcher for a longitudinal, mixed-method study investigating the impact of an elementary mathematics methods course that integrates a research-based curriculum designed to develop pre-service teachers' abilities to analyze teaching and teach in ways that promote generative learning for students and teachers (for more information about the multi-method longitudinal study, see Santagata & Yeh, 2014; Yeh & Santagata, 2015). The multi-method study investigates the effects of the course both short term- pre-service teachers' experiences and learning during teacher preparation - and long term - on their professional experience in the classroom and with their teaching communities during their first three years of professional teaching. As the project's graduate researcher, I was actively involved in both phases of the study. For the last four years, I have taken part in the case

study component where I visited the classrooms of the study participants, observing and debriefing their lessons.

Though similar in goal to the larger NSF project in which teacher learning and teachers' mathematics practices are examined, the two studies described here only followed teachers who taught in similar settings to my own as a classroom teacher. Distinct from the larger project, the study focus is on bilingual teachers and teachers who are working in culturally and linguistically diverse settings. For this reason, four of the 10 case study participants were selected for the dissertation. Three of the four teachers are discussed in the first study, and the second study includes two of the four participants.

Literature Review

Mathematics-Achievement-Access

Mathematics carries particular power in our society. The consequences of student participation or denied opportunities for participation in school mathematics are well-documented. Students' sense of belonging and identity as mathematics learners start early on in schooling (Boaler, 2015). Their early mathematics achievement and conceptions of self set the way for course-taking pathways (e.g. taking part in rigorous middle and high school mathematics and science courses) with significant implications for academic and career trajectories (Stinson, 2009). More than any other subject, mathematics impacts college acceptance, college choice, and later success in the labor force (Lee, 2012; 2013).

However, it is widely documented that success in mathematics is not random but rather falls into distinct patterns based on racial, ethnic, linguistic, and gender divisions (Gutiérrez, 2002, 2008; Gutstein, 2008; Stinson, 2009; Tate, 1995). Analyses of school achievement, course-taking patterns, and standardized-test data reveal prevalent patterns of inequity. Much of

prior literature on equity in mathematics education has summarized achievement gaps and proposed solutions to help remedy the gap (e.g. Peressini, 1997; Silver, Smith, & Nelson, 1995). Rochelle Gutiérrez (2007) refers to the emphasis on the "gap" as problematic as "such an approach implies that the people being served by the programs need to improve but not the mathematics" (p. 37). Gutiérrez argues, and I share in this perspective, that students' mathematics performance is not a reflection of their innate ability or dispositions/soft skills but a product of the organization of students' mathematics education. As such, there needs to be more attention paid to mathematics as it is realized in classrooms and experienced by students.

The focus of study in this dissertation is the mathematics classroom. In line with Wenger's (1998) notion of "learning as becoming," learning is viewed here as localized, dynamic, and co-constructed, and in relation to classrooms as communities of practice. Taking a situated lens, the two studies presented here examine how the classroom ecology, co-constructed by the teacher and students, provide opportunities to engage in classroom mathematics that supports both students' and teachers' generative learning.

Centering the Teaching of Mathematics on Students

This dissertation is guided by a vision of mathematics teaching that is grounded in two related pieces of literature. The first is the vision proposed by reform-based movements as *ambitious*. During the last few decades, the mathematics education research community has developed a robust knowledge base about the forms of mathematics teaching practices that support the development of students' understanding of central mathematical ideas (Hiebert & Grouws, 2007; Kilpatrick, Martin, & Schifter, 2003; Stein, Remillard, & Smith, 2007). The National Council of Teachers of Mathematics (2000, 2006, 2015) and the new state standards (CCSSM, 2010) describe a set of learning goals encompassing conceptual understanding,

procedural fluency, and habits or practices of reasoning for students to form connections between rules, procedures, and concepts. The instructional vision proposed in the standards has been called "ambitious" (Kazemi, 2008; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010), as it is a shift from the rote-based, teacher-driven culture of mathematics teaching that has and is still pervasive in U.S. schools (Gallimore, 1996; Stigler & Hiebert, 2009).

In this dissertation, a central component of ambitious instruction and its focus is the role of students in the teaching and learning process. Kang and Anderson (2015) define "ambitious instruction" as "practices of deliberate and ongoing attention and actions that move student thinking forward" (p. 865). Therefore, the center of the teaching of mathematics is the students. This is bounded by the conception that mathematical knowledge and understanding cannot be transferred directly from teacher to students (Franke & Kazemi, 2001; Freire, 1968). Students must be agents of their own learning, be provided opportunities to engage in rich mathematical tasks in collaboration with others in the learning community, and to construct their own knowledge and understanding in ways that reflect, refine, and extend their prior knowledge, skills, and experiences (Ball, Thames, & Phelps, 2008; Franke et al., 2009; Hufferd-Ackles, Fuson, & Sherin, 2004; Stein & Smith, 2011).

Centering the teaching of mathematics on students brings me to the next related piece of literature on teaching mathematics grounded in *equity*. Equity is a broad all-encompassing term that is frequently heard and used. Often, equity is blurred with equality (Gutiérrez, 2007, 2013). For example, equality in mathematics education often equates to *sameness*, meaning that all students have access to the *same* instruction, *same* quality of teachers, *same* form of instruction, and *same* supports for learning. However, learning does not take place in a vacuum and must

account for past social, political, and historical injustices, student identities, and other contextual factors (Gutiérrez, 2013, Powell & Frankenstein, 1997).

The focus on equity, here, considers how classrooms as learning spaces can promote broader and more meaningful participation. This vision of equity requires mathematics teaching be responsive to students' backgrounds, experiences, and knowledge and particularly to students' linguistic resources, the language practices student bring into the classroom. It recognizes that school mathematics has historically served some groups of students, privileging some over others, based on a Western frame of reference (D'Ambrosio, 1985; Gutiérrez, 2007, Powell & Frankenstein, 1997). As such, equitable classroom learning requires moving away from "sameness" to personalizing mathematics instruction to capitalize on and leverage students' informal and formal mathematics, hybrid-language practices, and experiential knowledge as resources for schooling (Gutiérrez et al., 1996; Moschkovich, 2011; Planas & Civil, 2013). This requires, again, attention to the students, and looking closely at their culture-based strengths as resources for learning.

Elementary Mathematics Teacher Education

> Persistent and unacceptable gaps narrow and ultimately disappear when all students have access to rigorous, high-quality mathematics, taught by teachers who not only understand mathematics but also understand and appreciate learners' social and cultural contexts in meaningful ways.
>
> (NCTM, 2015, p. 65)

The vision of the ambitious and equitable mathematics teaching proposed above is advocated by policy makers and the education research community (Kazemi, 2008; NCTM, 2000; 2014; Lampert, et al., 2010). There is general consensus that effective mathematics teaching for ALL learners must be responsive to students' mathematical, linguistic, and experiential knowledge. The question, then, is how to prepare pre-service teachers to enact these practices, and make them part of their beginning instructional repertoire. The teacher-education

community has centered on two approaches.

One is the focus on preparing teachers to learn to engage in high-leverage practices (Grossman, Hammerness, & McDonald, 2009; Lambert et al., 2010), with the idea that teaching involves a set of routines that occur with high frequency. Through structured activities that approximate the work of teaching, teachers can learn to enact specific sets of practices that place them on a trajectory for achieving the visions of ambitious and equitable teaching described above (Kazemi, Franke, Lambert, 2009; Lampert et al., 2010; Thompson et al., 2013; Aguirre et al., 2012; Aguirre & del Rosario Zavala, 2010).

Another is the approach focused on preparing teachers to learn in, and from, practice through systematic analysis of teaching (Hiebert & Morris, 2012; Hiebert, Morris, Berk, & Jansen, 2007; Santagata & Guarino, 2011). This approach involves the design and enactment of instruction that provides teachers opportunities to gain insight into student thinking and to use student-based evidence of learning to determine the effectiveness of instruction and next steps.

Both approaches to teacher education have demonstrated the potential to deepen teachers' attention on students during instruction and reflection, to improve teachers' mathematics teaching practices, and to promote generative learning for students AND teachers (e.g. Franke & Kazemi, 2001; Jacob, Franke, Carpenter, Levi, & Battey, 2007; Kazemi, et al., 2009; Santagata, Zannoni, & Stigler, 2007; Santagata & Yeh, 2014; Sun & van Es, 2015). While this work has been invaluable in deepening our understanding of pre-service teachers' learning, we know very little about what happens to novice teachers when they graduate from teacher preparation and begin teaching mathematics in their own classrooms.

Novice Elementary Teachers and Their Mathematics Teaching Practices

The first few years of professional teaching has been identified as a critical period where students of teaching become teachers of students (Beauchamp & Thomas, 2011; McCormack, Gore, & Thomas, 2006). It is a period that has often been described as" dramatic and traumatic" in nature, prompting Veenman (1984) to use the term "reality shock" to describe the collapse of novice teachers' ideals of teaching with the harsh and challenging realities of everyday classroom life (p. 143). Much of the research on novice teachers has shown that most leave teacher programs with idealistic and reform-oriented views of teaching (e.g. student-centered, constructivist), but adopt "traditional" instructional methods (e.g. teacher-centered, didactic) once they begin professional teaching (Cohen, 1988; Costingan, 2004; Flores, 2006; Lortie, 1975). Many factors have been blamed; among them: teachers' individualized factors (lack of mathematical and pedagogical knowledge and beliefs about mathematics, teaching, and their students (Aguirre & Speer, 2000; Ball, 1991; Hodgen & Askew, 2007; Ma, 1999; NRC, 2001; Speer 2008; van Es & Conroy, 2009)), or contextual factors (culture of the school, institutional and political pressures) that contend with teaching responsive to students (Anderson & Stillman, 2011; Diamond & Spillane, 2004; Flores & Day, 2006; Lubinski, Otto, & Rich, 1996; Steele, 2001; Vacc & Bright, 1999).

The literature on novice teachers has enhanced our understanding of the experiences and challenges of teaching ambitiously and equitably during the transition period. However, there are limitations to the existing studies; for example, most have examined only the first year of professional teaching (e.g. Anderson & Stillman, 2011; Diamond & Spillane, 2004; Flores & Day, 2006). As a result, we know very little about what happens to novice teachers after the first year when or if the "reality shock" wears off.

Second, the literature on novice elementary teachers and their mathematics teaching practices is scarce. What has been assumed about novice elementary teachers' mathematics teaching practices are often extrapolations from other subject areas, or, broadly, teaching in general (Wang, Odell, & Schwille, 2008). The studies that do exist have examined the "effects" of teacher preparation, often from an input-output model (e.g. a teacher learns at the university, then transfers that learning into action), describing whether novice teachers "can" or "can not" put into practice what has been taught from their university preparation (e.g. Cady, Meier, & Lubinsky, 2006; Tower, 2010; Steele, 2001). Learning to teach mathematics is seen as a fixed set of principles and methods that teachers take up whole from teacher preparation and put into action into the classroom. What is needed are studies that move beyond dualistic measures of mathematics teaching, as something teachers simply "can" or "can not" do well, but as complex, composing of multiple components, and developmental, a point along a learning progression. Longitudinal studies are needed that examine the opportunities for learning as teachers engage in the daily work of teaching.

Lastly, most studies have examined novice teachers as a homogenous group. Given our growing, linguistically-diverse student population, there is an emphasis on diversifying our teaching workforce (e.g., Sleeter, 2015). Teachers who share cultural and linguistic backgrounds with their students can disrupt assumptions and stereotypes about language-minority student groups and develop instruction that builds and leverages students' cultural and linguistic resources (Achinstein & Aguirre, 2008; Galinda &Olguin, 1996; Sleeter, 2015). However, given the language politics in the U.S., bilingual teachers have received the majority, if not all, of their own schooling and their preparation in English with an acquisition model of "acquiring English" rather than the maintenance of bilingualism and have had little to no university coursework on

learning methods to support emergent bilinguals (Sutterby, Ayala, & Murrillo, 2005). Currently, we know very little about novice elementary bilingual teachers and their mathematics teaching practices. Just as classroom teachers must take on the role of "teacher-researchers to learn firsthand about the lived realities of students… and use this knowledge as the basis for curricular units" (Civil, 2007, p. 106), mathematics teacher-educator researchers must also learn about our novice teachers so we can design instructional modules based on the knowledge learned from their lived experiences.

Study Contribution

Shulman (1987) contends "teaching is, essentially, a learned profession" (p. 9). It therefore follows that learning to teach is a lifelong developmental process in which teachers are learners on their own professional journeys. This dissertation takes a step forward and longitudinally examines novice elementary mathematics teachers and their mathematics teaching practices.

This study contributes to current literature in the following ways:

- A focus on elementary school mathematics;
- A focus on bilingual teachers and how they organize mathematics learning for their emergent bilinguals (paper 1);
- A longitudinal approach that examines the interrelation of instruction and reflection, and its potential to promote teacher professional growth over time (paper 2).

The overarching research questions of this dissertation are: What are the mathematics teaching trajectories of elementary school teachers during their first two years of professional teaching? What can we learn from their teaching to inform our work as mathematics teacher- educator researchers?

Below are the research questions for each of the studies:

- How do three novice bilingual teachers organize mathematics learning for their emergent bilinguals? (paper 1);
- How does each teacher's mathematics teaching and teaching reflection change over time? What is the relationship between their mathematics teaching and their lesson reflections, and how might that contribute their development over time? (paper 2);

Study Summaries

The first paper takes a sociocultural lens and examines the classroom ecology for learning language and mathematics, examining how three novice bilingual teachers organize mathematics learning for their emergent bilinguals. Specifically, I look at the opportunities for learning provided by each teacher in relation to: (a) the types of support provided to develop students' learning of the language of mathematics; (b) who holds the authority of knowledge in the classroom community; and (c) how, and what, students' repertories of practices are utilized. Data sources consist of videotaped classroom observations of mathematics instruction over their first two years of teaching. Study findings highlight the complexity of bilingual teaching in the context of supporting students in learning the language of mathematics. Given the growing linguistic background of U.S. students and the little preparation currently available to support teachers working with emergent bilinguals, specific recommendations for teacher preparation are provided based on study findings.

The second paper shifts the focus from students' learning opportunities to mathematics teachers' learning opportunities. This study examines the learning-to-teach trajectory of two novice elementary teachers over a two-year period who were comparable in terms of their teaching backgrounds, with similar mathematics teaching practices at the end of teacher

preparation. However, their mathematics teaching by the end of their second year looked quite different. Given their different trajectories, this study seeks to make sense of each teacher's learning-to-teach trajectories. Specifically, I examine the relation between their mathematics teaching and their reflections on teaching and how it may contribute to their professional growth over time.

Chapter 2

Math is More Than Numbers

Introduction

"Ethnic identity is twin skin to linguistic identity – I am my language. Until I can take pride in my language, I cannot take pride in myself." (Anzaldua, 1987, p.253).

As the population of emergent bilinguals [1]increases in U.S. public schools, so do concerns about the growing dissonance between research on the education of emergent bilinguals and how they are educated. The miseducation of emergent bilinguals in the United States is well-documented (García, Kleifgen, & Falchi, 2008; Téllez K. & Moschkovich, J., & Civil, M, 2011). Classroom practices reflect U.S. educational language politics of assimilation and deficit framing, and curricula, assessment, and instruction have continually failed to reflect their voices and experiences (Civil, 2002, 2007; Turner, Varley, Gutièrrez, Simic-Muller, & Díez- Palomar, 2009; Valenzuela, 2002). In the area of mathematics, limited linguistic support, low expectations, and rote-based teaching have plagued their learning experiences (Chapa, & De La Rosa, 2006; Chval & Pinnow, 2010; Cuevas, 1984; García, et al., 2008; Moschkovich, 2012).

A viable solution is a focus on diversifying the teacher workforce to better reflect the linguistic and cultural diversity of our student population (Banks, 1995; Cochran-Smith, 2004; Sleeter, Neal, & Kumashiro, 2015). While a growing body of research highlights the linguistic and academic benefits of bilingual classrooms, we currently know very little about the teaching

[1] Definitions of bilingualism vary based on the epistemological stance on language. This study takes a sociocultural lens; I use the definition of bilinguals provided by Valdés-Fallis as "the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations" (p. 4). This definition characterizes bilingualism not only as an individual trait but also as a social and cultural phenomenon as one participates in the language practices of a community.

practices of beginning bilingual teachers (Sleeter et al., 2015; Quiocho & Rios, 2000; Villegas & Irvine, 2010; Villegas & Lucas, 2004); as such, there is a need for studies that examine the early instructional experiences of novice bilingual teachers. This paper draws on sociocultural theory and research to examine how three novice bilingual elementary teachers organize mathematics learning for their emergent bilinguals; specifically, I examine the type of supports the teachers provided to develop student learning of the language of mathematics, how learners are positioned within the classroom community, and how, and what, student repertoires of practice are utilized.

This paper makes two significant contributions. First, it highlights the complexity of bilingual teaching in the context of supporting students in learning the language of mathematics. Second, in light of the study findings, specific recommendations are provided as to how teacher preparation programs can better support pre-service teachers to engage in mathematics teaching practices that build on students' linguistic and cultural identities. I begin with a review of literature on bilingualism and language in mathematics, and the teachers' role in organizing student learning.

Bilingualism and Bilingual Learners

I begin the literature review with an analysis of the language and labels currently used to describe students who speak languages other than English. In the United States, policymakers refer to these students as English language learners (ELLs), or as limited English proficient students (LEPs), as federal legislators did in the original No Child Let Behind Act. These labels have often served as a proxy for other demographic labels (for example, in place of "Latin@") [2]

[2] I use the term Latin@ to reference a person (or persons) who's geographic heritage is Latin American. This usage is similar to the U.S Census Bureau's usage, wherein the terms Hispanic and Latino synonymously refer to "those who classify themselves in one of the specific Hispanic or Latino categories: 'Mexican,' 'Puerto Rican,' or 'Cuban.' However, the term "Hispanic" carries negative connotations for some, who argue that the term refers to either the country of Spain or the Spanish language. Few persons of Latin American origin have any connection with Spain and many who share this ethnic heritage may not speak Spanish. Therefore, I use the term Latin@s. I use the @ sign to indicate both an "a" and "o" ending (Latina and Latino). Traditionally, it is customary for groups of males to be

rather than an accurate portrayal of students who speak languages other than English (Gándara & Contreras, 2009). An important point is how the use of ELLs and LEPs as labels signal the omission of an idea critical in considering issues of equity in teaching (García, et al., 2008).

English language learners (ELLs) or limited English proficient students (LEPs) are, in fact, emergent bilinguals. In ignoring the bilingualism that these students develop through schooling in the United States, schools perpetuate inequities in their education by discounting the cultural and linguistic resources emergent bilinguals bring, and assume their educational needs are the same as monolingual students (Gándara & Contreras, 2009 García, et al., 2008; Moschkovich, 2012). The deficit framing of bilingual learners and bilingualism has historical roots that have been pervasive in the research and teaching of emergent bilinguals.

Bilingualism, Language, and Mathematics Learning

Research on language and learning started with a focus on bilingualism and bilingual learners from a deficiency perspective. The majority of the studies undertaken before 1980 argued that bilingualism impaired learners' linguistic, cognitive, and educational development (Reynold, 1928; Saer, 1923, both cited in Saunders, 1988). Bilingualism was considered unnatural, and the widespread view was that the cognitive effort required to learn two languages diminished children's ability to learn other content (Reynold, 1928; Saer, 1923, both cited in Saunders, 1988). These findings influenced the focus of language in mathematics education for emergent bilinguals.

Over the last three decades, the focus of language in mathematics education for bilingual and multilingual learners has focused on the challenges the English vocabulary, oral fluency, and

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written as Latinos, females as Latinas, and a group of females and males to be written in the form that denotes only males with Latinos. The term is written Latin@ with the "a" and "o" intertwined, as opposed to Latina/Latino, to show a sign of solidarity with individuals who identify as lesbian, gay, bisexual, transgender, queer, and questioning (LGBTQ).

comprehension present (Spanos & Crandall, 1990; Spanos, Rhodes, Dale, & Crandall, 1988). The problems of emergent bilingual mathematics learners were brought to the fore by Halliday (1975). Halliday (1978) highlights the discipline-specific language of mathematics, noting that learning mathematics includes constructing multiple meaning for words. Mathematics is viewed as a sign system that includes language aspects unique to the mathematics register, which Halliday defines as:

> A register is a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings. We can refer to the "mathematics register," in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is being used for mathematical purposes. (p. 195)

Halliday's (1978) notion of a mathematical register establishes an understanding of the unique ways language constructs mathematical knowledge. The kind of mathematics students need to develop through schooling uses language subtly different in meaning than its everyday use (e.g., difference, factor, table). Within the language of mathematics, there even exist different meanings for mathematical words (e.g., base of a triangle vs. base of a power, or median of a data set vs. median of a triangle), and different words to describe a single sign (e.g. + as plus, increased). As such, Halliday (1978) has argued that learning the language of mathematics is part of the process of learning school mathematics.

Recent research on emergent bilinguals has continually fallen into two camps: a focus on an assimilationist, nativist perspectives (e.g. the obstacles of vocabulary, oral fluency, and comprehension in comparison to monolingual speakers), and the repertoire of practices, the resources (Civil, 2002; 2007; Gutiérrez & Rogoff, 2003) emergent bilinguals bring to learning mathematics. Studies from a sociocultural perspective, the lens through which this study is framed, have examined the construction of mathematical knowledge in social context, focusing

not only on the challenges emergent bilinguals face, but also on the strengths of their linguistic and cultural resources (Banks, 1996; Gutiérrez & Rogoff, 2003; Moschkovich, 2002; Téllez, Moschkovich, Civil, 2011).

Mathematics as Language Socialization

A sociocultural perspective on learning mathematic builds on the understanding that learning is situated in context (Brown, Collins, & Duguid, 1989; Greeno, 1994; Lave & Wenger, 1991) and the notion of Discourses (Gee, 2005). An important aspect of mathematics learning is that it is language socialization, a discourse process, and a situated activity (Cobb, Wood, & Yackel, 1993; Lemke, 1990; Moschkovich, 2002). This perspective implies that mathematics learning is a cultural practice, and language serves as a mediational tool for learning. Therefore, the focus is not just on the language of mathematics, its register and terminology, but its socio-cultural and mathematical norms (Cobb et al, 1993; Lave & Wenger, 1991; Nasir, 2002, Sullivan et al., 2002), the type of valuing, acting, and thinking associated with learning mathematics in the classroom (Gee, 2005), moving the focus away from the individual to the dynamics of social interactions, and its impact on students' identity development.

For this reason, language, particularly one's first language, cannot be isolated from its social and cultural dimensions (see the survey in Civil, 2010). Language is more than simply words; it is key to one's identity and validation of that identity (Anzaldua, 1987; Nieto, 2002). Language extends to a person's definition of self, his or her relationship to the community, and his or her status in the wider sociocultural and political milieu (Anzaldua, 1987; Cummins, 2000; Gutiérrez, 2002). As students move through school, they come to learn who they are as mathematics learners through their experiences in mathematics classrooms, in their interactions with teachers, parents, and peers, and in relation to their anticipated futures. The switch from

one language to another (e.g. from the first language to the language of instruction) is not only a matter of proficiency level, but an interplay between the different cultures embedded in a complex network of symbols, norms, sociocultural practices, and identities. In a study of student interactions within a reform-based mathematics Calculus class, Gutiérrez (2002) found bilingual students fluent in English chose to use Spanish during group work not as a necessity, but as a way of bonding with others in class. Moschkovich's (2007) study of code-switching in mathematical conversations builds on this argument of language hybridity. Her analysis of transcripts of student mathematical discussions found that code-switching between English and Spanish, everyday colloquialism and school-based discourse was the most prevalent when students were negotiating ideas and engaging in mathematical arguments. These studies show that language, and choice of language, are more than a reflection of language proficiency. Language is a reflection of who we are, where we come from, and who we are trying to become.

Validating and maintaining one's linguistic identity is intricately linked to academic performance. Bilinguals who are able to read, write, and communicate in their primary/home language do better in school even when English is the medium of instruction, are more likely to enroll in advanced mathematics, to complete school, and enter higher education (Cardenas, Robledo, & Waggoner, 1998; Khisty, 2004). Instruction in a student's home language, and even the explicit valuing of it in classrooms, plays a significant role in language-identity achievement for bilingual learners (Khisty, 2004; Moschkovich, 2002). In other words, language, identity, and mathematics are intricately linked, and play a significant role in whether emergent bilinguals have access to education, and whether schooling is oppressive or liberating. And teachers play a critical role in shaping the learning ecology.

Linguistic Diversity of Teaching Force

In the U.S., where linguistic and cultural diversity is growing rapidly, the teaching work force remains predominantly monolingual and white. While students of linguistic diversity comprise 47% of the nation's K-12 students, 83% of the teaching force is white and monolingual (National Center for Education Statistics, 2014). Such a skewed racial representation means most public school teachers come into teaching with limited cross-cultural backgrounds, knowledge, and experiences (Sleeter, 2001; Sleeter, Neal, & Kumashiro, 2015). As a group, they have enjoyed unacknowledged privilege denied to many of their students, and may not see language and identity as central components to learning (Sleeter, 2015).

Nieto (2002) believes how teachers and schools view the language of students is what matters most. Emergent bilinguals have been normalized into oppression by language. Spanish, the language spoken by the largest ethnic group in the U.S., is associated with poverty, marginalization, and an obstacle to academic success (Gonzalez, 2001; Gutiérrez, 2003; Sleeter, 2015). As such, there is continual movement toward monolingualism rather than bilingualism or multilingualism, and issues of power and equity permeate our society and our classrooms through language.

For this reason, a linguistically-diverse teaching force must be cultivated in our American public schools. "Teachers who share cultural, linguistic, racial, and class backgrounds with their students provide a source of connection and examples of successful adults for the youth" (Achinstein & Aguirre, 2008; p. 1507). Such teachers are critical for student success, and to disrupting the assumptions and stereotypes of underserved student groups (Achinstein & Aguirre, 2008; Quiocho & Rios, 2000; Villegas & Irvine, 2010; Villegas & Lucas, 2004).

Study Purpose and Research Question

The need for well-prepared bilingual teachers is critical, which helps to explain why a growing body of research has focused on teacher recruitment, and the practices of exemplar teachers (e.g., Gutiérrez, 2003; Khisty, & Chval, 2002; Khisty & Morales, 2004; Moschkovich, 2002; Razfar, Licón, Khisty, & Chval, 2011). Despite this, there is a dearth of studies on the teaching practices of novice bilingual teachers. Previous research reports that bilingual teachers may be better at organizing instruction that supports students' learning of languages, leverages students' repertoires of practice, and positions bilingual learners as able, capable contributors of knowledge (Achinstein & Aguirre, 2008; Galinda & Olguin, 1996; Sleeter, 2015). However, given the language politics in the U.S., bilingual teachers received the majority, if not all, of their own schooling, and their preparation in English, with an acquisition model, of "acquiring English" rather than maintaining bilingualism, and have had little to no university coursework on methods supporting emergent bilinguals (Sutterby, Ayala, & Murrillo, 2005). Given the assimilationist approach to language acquisition in the U.S., one has to wonder if being bilingual and bicultural is enough?

Sleeter (2001) contends that the teacher-education research community has concentrated on examining teacher preparation when more attention should be placed on what actually happens when graduates of teacher- preparation programs begin to teach. The induction period, that unique phase during the transition from a student of teaching to a teacher of students, is considered a critical period of identity development (Beauchamp & Thomas, 2011; McCormack, Gore, & Thomas, 2006). A close examination of novice teaching is critical for school leaders, and teacher educators to consider in our work to better support our teacher workforce and our students.

This study examines the mathematics teaching practices of three novice bilingual teachers working with Latino emergent bilinguals, with the focus on elementary mathematics teaching. Spanish is the language spoken by the vast majority of U.S. bilingual students.. Close to half (45%) of all Latin@ children going to U.S. schools are emergent bilinguals; although it is important to note that Latin@ students are not a linguistically homogenous group and reflect a range of language practices (Lazarín, 2006; Téllez, Moschkovich, & Civil, 2011). The focus of this study is specifically on their opportunities to learn mathematics. In elementary schools, mathematics is often the first content area for transition in English (Fabelo, 2008; Remillard & Cahnmann, 2005). It is also a subject area in which our language minorities, and particularly the Latino-emergent bilinguals, have been greatly underserved (Gándara & Contreras, 2009; García, et al., 2008; Gutiérrez, 2002; Valenzuela, 2002). Given the mis-education of our emergent bilinguals and the gatekeeper status of mathematics, greater attention must be placed on their mathematics learning experiences.

This study answers the following research question: How do three novice bilingual teachers organize mathematics learning for their emergent bilinguals? To provide insight into this question, I examine: (a) the types of supports provided to develop student learning of the language of mathematics; (b) who holds the authority of knowledge in the classroom community; and (c) how and what students' repertoires of practice are utilized.

Methods

Study Context

The present study is based on observations conducted in three elementary classrooms. All three teachers attended the same teacher credential program, a one-year post-baccalaureate elementary program at a West Coast public university in the United States. The credential

program focused on developing pre-service teachers' competencies in four specific areas: 1) developing an inquiry stance; 2) supporting second-language learners; 3) collaborating with faculty, peers, and mentor teachers to continually improve practice; and 4) appreciating the unique resources students bring to the classroom. All three teachers were situated in linguistically-diverse field placement settings, and had two 10-week quarters of mathematics methods courses and separate courses on child development, educational equity, theories and methods in English language development, and methods of instruction for special populations.

Teacher Portraits

The teachers in this study are part of a larger project examining the impact of an elementary mathematics methods course on pre-service teachers' learning in the context of their teacher preparation, and after graduation. A short portrait of each teacher as well as a description of their classroom and their students is provided. Pseudonyms are used throughout.

Laura. Laura, in her mid-twenties, identified as white Latina. Laura grew up in a bilingual, bicultural household with a stepmother who intentionally only spoke Spanish at home. Laura took Spanish in middle school, high school, and majored in Spanish in college. At graduation, Laura did not plan to pursue a career in teaching, but struggled for a year to find work. With both parents as teachers, Laura saw teaching, particularly a career as a bilingual teacher, as a stable career opportunity.

By the second year of hire at her school, Laura had developed a reputation for her strength in mathematics teaching, and was selected by the district to develop curricula, frequently being observed by teachers from other schools and districts interested in bilingual mathematics instruction responsive to students' thinking.

School/*Students*. Laura taught first grade (n=24) in a dual-language immersion program.

During Laura's first year at Valadez, the school was in its third year as a two-way language school. The dual language program at Valdez started as a community grassroots movement. The goal of the two-language program was to support students to develop bilingual and biliteracy skills in Spanish and English. Half of the children in school were native Spanish-speakers, and the other half were native English-speakers. Most were born in the United States, with parents who were first or second generation immigrants from Mexico, Costa Rica, and El Salvador. Instruction at her grade level was a 90/10 model, with curriculum instructed 90% in Spanish and 10% in English. At the time of Laura's hire, mathematics instruction at Valdez was considered quite traditional--textbook-driven, and procedural-focused. When I began my visits, the school had requested additional support in math professional development, and a district math coach came weekly for a year to work with the teachers.

Kassandra. Kassandra, in her late twenties, identified as Vietnamese American. Kassandra spoke Vietnamese, Spanish, and English. Though born in the U.S., Kassandra did not learn English until she started Kindergarten, and had strong memories of being in an English-only school context, and in an English as a second language (ESL) pull-out program where she left her homeroom to work with an English language teacher on vocabulary and syntax. Kassandra frequently used Spanish in her own classroom and with students and their parents. She described her Spanish as acquired from language classes in middle and high school, and from daily interactions growing up in her community, as well as now in her daily interactions with students and families. By Kassandra's second year of teaching, she was the lead teacher for her Kindergarten grade level team, and led the planning of the instructional curriculum.

School/Students. Kassandra taught kindergarten (n = 24) at Excel Academy, a public charter school in an urban area with grounded pedagogical roots in culturally responsive

teaching. Kassandra taught in an English-immersion classroom setting where English was used as the language of instruction. At the time of hire, Excel had just opened, and the student population consisted only of students of color, with three-fourths of the students Latin@, and a fourth African American. All of Kassandra's students were born in the United States. Her students represented the full spectrum of proficiencies in Spanish and English; some students came into school speaking only Spanish, some were bilingual, and some spoke only English. Ninety-five percent of the students enrolled at Excel qualified for free and reduced lunch. Kassandra's charter management organization housed their own online professional development program where teachers were required to utilize their online video-based modules based on feedback from their principal and curriculum support staff. By the second year, Kassandra's school was recognized as one of the highest-performing elementary schools in their area.

Elise. Elise, in her mid-twenties, identified as Mexican American. Elise had wanted to be a teacher since kindergarten. She, herself, had been in a dual-language program until the passage of proposition 227 in California. The passage of the proposition led to the elimination of bilingual classes and Elise's transition to English-only settings in second grade. Elise entered teaching with the intent to share critical knowledge with her Latino students. Deeply committed to supporting students' self-identity, Elise was actively involved in a community mentoring program in which she mentored Latina youths. Elise taught at the same school where she student taught, and her mentor teacher during her field experience became her grade-level colleague.

School/*Students*. Elise taught second grade (n = 28) at the same school as Laura. Her students were at the intermediate level of language development. The majority of Elise's students started at Valadez in kindergarten and received instruction in Spanish and English for the last three years. During Elise's second year, her district underwent a textbook adoption period during

which the second grade classrooms were selected to pilot the two new textbooks under consideration for adoption. Elise spent her second year piloting two different mathematics textbooks.

Data Sources

This study primarily draws on videotaped observations of the classroom interactions during mathematics instruction, however, multiple data sources were used to provide background on the teachers' perceptions of their lessons, their own schooling experiences, professional development opportunities, and the school culture in which they taught. Here, I describe the primary data source (videotaped mathematics lessons) and secondary sources (videotaped interviews, field note observations, and member check interview).

Primary Data Source – Videotaped Mathematics Lessons

After graduation, each teacher was visited at her school site three times a year: once at the start of the school year, once midway through the year, and once during the last month of the school year. At each visit, the mathematics lesson was observed and videotaped (three visits a year, six visits across two years). The camera focused on capturing the teacher-student interaction and student-student interactions during whole class instruction and group work. The mathematics lesson ranged from 40 to 70 minutes in length. All the videotaped lessons were transcribed.

Secondary Data Sources

Post-lesson reflection: A post-lesson interview (three visits a year, six interviews across two years) was conducted after the lesson observation. Interviews conducted were semi-structured; four questions were posed during each interview: 1) What was the main learning goal of this lesson?; 2) How did it go? What was surprising? What worked as planned? What didn't?;

3) What would you do differently if you were to teach the lesson again?; And 4) What did you learn from teaching this lesson? Interviews ranged from 20 minutes to two hours. All interviews were recorded and transcribed verbatim. The lesson reflections were used to provide background on planning and decision-making in their lessons.

All day shadowing field notes: For the last visit made each school year, I shadowed the teacher all day. Participant-observation field notes were taken which included contextual and interactional details and observations, as well as practical and methodological reactions to the events in a memo format. In addition to the field notes, a longer interview was conducted and videotaped at the end of the day during the last visit of each school year. This last interview asked teachers to share their overall perspective of the year, what they learned, how their teaching connected back to their experience during the credential program, and their experience in the school community. These date sources were used to provide background on the teachers' perceptions of their lessons, their own schooling experiences, professional development opportunities, and the school culture in which they taught.

Member check interview: To increase the credibility of the study, additional one-on-one interviews were scheduled (Anfara, Brown, & Mangione, 2002; Lincoln & Guba, 1985). Teacher portraits and descriptions of classroom practices were shared with each teacher to ensure they accurately captured their teaching practices and their experiences.

Data Analysis

The method of analysis chosen for this study was a qualitative method of thematic analysis (Maxwell, 2004). The analysis incorporated both the inductive data-driven approach and the deductive approach (Boyatzis, 1998; Fereday & Muir-Cochrane, 2006). This approach complemented the research questions by allowing the three broad dimensions (i.e., language

supports; students' positioning; and the students' repertoires of practices) under study to guide the deductive thematic analysis process while capturing the nuances of specific practices for each dimension to emerge directly from the data inductively (Boyatzis, 1998; Fereday & Muir-Cochrane, 2006).

All 18 transcribed mathematics lessons were entered into the Dedoose data management program. The process of coding included the use of both the lesson transcripts and the videotaped lessons. During data analysis, I would observe the videos while tracking my analysis of the transcriptions on Dedoose. Memos of the nonverbal aspects of the classroom interactions that were not captured in the transcriptions, but were relevant to the three broad dimensions were written on Dedoose.

Following, I provide a detailed description of the analytic process to demonstrate the credibility or trustworthiness (Koch, 1994) of the interpretive research. The step-by-step process of analysis is outlined to demonstrate transparency in how I formulated the overarching themes from the data. The process presented here is a linear, step-by-step procedure; however, the research analysis process was iterative and reflexive.

The first stage of data analysis was to operationalize each of the three broad dimensions: language supports, authority of knowledge, and students' repertoires of practices. This first stage was deductive in nature as I drew upon existing literature in relation to the three dimensions. For the dimension of language supports, I drew upon the literature that examined language strategies that focused on students' languages as resources and addressed more than just vocabulary acquisition but students' participation in mathematical discussions. Specifically, I drew upon the guiding principles suggested by Ramirez and Celdón-Pattichis (2012) and Aguirre & Bunch (2012) that discussed resources to promote active participation in mathematics classrooms for the

five language modalities: reading, listening, speaking, writing, and representing. For the dimension of authority of knowledge, I drew upon Chval's prior work (Pinnow & Chval, 2015; Razfar, Khisty, & Chval, 2010) to examine how the teacher organized learning in which the authority of knowledge resided with the teacher or was distributed between teacher and student. Specifically, I examined who was positioned as the authority of knowledge (i.e., the teacher or student(s)), and how. Last, for the dimension of students' repertoire of strategies, I drew upon the concept of students' multiple funds of mathematical, linguistic, and experiential knowledge (Aguirre et al., 2011; Foote, 2009; Gonzalez, Moll, & Amanti, 2005) and repertoires of practices (Gutiérrez, Baquedano-Lopez, & Tejada, 1999). Specifically, I examined teachers' use of possible out-of-school (e.g. sharing with siblings) and classroom based experiences (e.g. school garden), student strategies and sense making, and students' language practices as resources for learning.

Guided by the literature described above, my first goal was to operationalize each of the dimensions in relation to the data. To do so, I first selected two random lessons and read and re-read the lesson, highlighting aspects of the lesson that referred to language supports, the authority of knowledge, and students' repertories of practices. Following the analysis process of the two lessons, I created definitions and examples of each of the three dimensions. Another researcher was invited to identify aspects of the lesson as it related to each dimension in two more lessons. Our results were compared, and we modified how the dimensions were operationalized. The definitions and descriptions of the three broad dimensions are provided in Table 1.

Table 1

Dimension Definition and Description

Dimension 1	
Label	Language support
Description	Teacher strategies and resources to support students' development of language to actively participate in the mathematics lesson across the five modalities: reading, listening, speaking, writing, and representing
Example	Teacher shows a picture of ginger bread man linking the Spanish word "hombre de pan de jengibre" to the picture. (Connect language with visual supports)
Dimension 2	
Label	Authority of knowledge
Description	Who is positioned as the authority of knowledge (teacher or student(s) and how (e.g. teacher select student strategy or idea to share)
Example	Student gives the answer of 8 instead of the correct answer of 9, and the teacher leads the class to choral count one-by-one the total amount of apples in the problem (Teacher as authority of knowledge and teacher-led strategy)
Dimension 3	
Label	Students' repertoire of practices
Definition	Students' repertoires of practices utilized to promote learning: students' own math strategies and sense-making, experiences, and language practices: everyday ways of talking, English, Spanish, code-switching, and developing school-based mathematics discourse
Example	Student public share of student-invented strategy and teacher documents student problem solving method on the board (Students' own math strategies and sense-making)

In the next stage, I went back and analyzed all 18 lessons to highlight aspects of the lesson that referred to language supports, authority of knowledge, and students' repertoires of practice. At this time, I also identified specific teaching practices that were representative of each dimension as they emerged from the data, with the goal of capturing variances across lessons and teachers. For example, Table 2 provides the teaching practices found in the data as they relate to the dimension of repertoire of student practices. The goal of this stage of analysis was not to

produce a frequency count of the practices but to develop an understanding of which teaching practices were used and its development throughout the lesson. As well, it is important to note that the dimensions are not independent from each other. Often, a classroom practice was highlighted as representative of multiple dimensions. For example, allowing students to code-switch between English and Spanish would be considered as representative of all three dimensions.

Table 2

Teaching Practices for the Dimension of Repertoire of Student Practices

Students' Repertoire of Practice
Subcategory 1: Verbal Resources for Communication
Language of Instruction
Everyday Ways of Talking
Developing School-based Mathematics Discourse
Hybridity of Spanish and English
Subcategory 2: Nonverbal Resources for Communication
Gestures
Student work
Subcategory3: Student Strategy
Student Strategy
Student Explanation
Teacher Strategy and Explanation

To help identify themes and patterns for each teacher and across time, time-ordered matrixes (Miles & Huberman, 1994) were developed for each teacher with each column representing a lesson and rows for each of the three dimensions (e.g. language strategies, student positioning, and repertoires of practice). Each cell highlighted the teaching practices seen for that specific dimension and how patterns of classroom interaction unfolded during the lesson. Table 3 provides an example of the matrix for Laura's lesson for visit 1, lesson 1.

Last, vignette compositions were written to document similarities and dissimilarities in teaching practices over time and across participants. At this time, illustrative moments were selected, such as the ones highlighted in the findings to display instances that represent the patterns and themes that emerged for each teacher.

Limitations

Limitations occur for all studies. Specifically, I will discuss the limitations within my data and my own role as a researcher. First of all, I realize that my findings are bound to the unique context in which the study was conducted (Corbin & Strauss, 2009). My study examined the mathematics teaching practices of elementary school teachers who were graduates from the same credential program. They are all bilingual/multilingual and had received the majority of their own K-12 schooling and teacher education training with instruction in English, and they taught in schools where their students were also linguistically diverse with varying degrees of proficiency in English and Spanish. Therefore, I cannot generalize the study findings to the general population of teachers or even to all bilingual teachers (Corbin & Strauss, 2009; Erickson, 1986). However, to ensure transferability, I illuminated patterns and themes through "rich descriptions" of the case study participants, vignettes from their classrooms, and interview quotes. As stated by Erickson:

> *The "thick description" that has been generated, however, enables observers of other contexts to make tentative judgments about applicability of certain observations for their contexts and to form "working hypotheses" to guide empirical inquiry into these context (1993, p. 33).*

Second, much of the data were coded and themes identified by one person. To establish credibility in my findings, I tried to "live in" the data (Corbin & Strauss, 2009), engaging in

rigorous and multiple rounds of analyses. During my data analysis and in the writing of my dissertation, I enlisted the assistance of a second researcher to help create the coding framework and to discuss themes present in the data. I shared regularly my coding and analysis with colleagues knowledgeable about qualitative research and met with a colleague fluent in Spanish to discuss my translations and interpretations. As well, I conducted member checks to ensure I accurately represented their personal and classroom narratives.

As well, I realize the influence of my own positionality. I am an Asian-American woman, who served as an elementary school teacher for over a decade working in culturally and linguistically rich settings. I, myself, am multi-lingual and taught in Spanish, English, and Chinese. My scholarship centers on diversifying the teaching profession and achieving educational equity for non-dominant student groups. By listing my sociocultural and professional identifications, I acknowledge the complexities and limitations that my position creates in researching the lives of teachers and students.

I am also aware of my own possible influence during the classroom visits. I was in class taking field notes as the teachers were taking their mathematics methods courses, and I served as a teacher's assistant and taught two of the four teachers in a summer credential course also focused on mathematics teaching. I realize that my regular visits may have impacted how they taught or how they thought I wanted them to teach.

At the same time, I believe being with the teachers during their teacher preparation and initial years of teaching and my familiarity and personal experience as a teacher have also provided a rapport and hopefully a richness of data that might not otherwise been possible. Furthermore, my own classroom experience provided me with a better framework for understanding what questions might elicit the information that I was seeking. My familiarity with

the language and jargon of teaching (e.g. IEP, PLC, CGI) has been invaluable in teasing out innuendos of meaning that are specific to the context of teaching and working in linguistically rich settings.

Findings

In this section, I describe how Laura, Kassandra, and Elise organized mathematics learning for their emergent bilinguals. The findings reveal the complexity of teaching language and mathematics. Findings will be shared in relation to the three dimensions: (a) the types of supports provided to develop students' learning of the language of mathematics; (b) who is authority of knowledge in the classroom community; and (c) how and what students' repertoires of practices were utilized.

Types of Supports to Develop Students' Learning of the Language of Mathematics

The findings show that all three teachers demonstrated a commitment to support students' learning of the language of mathematics. Laura, Kassandra, and Elise used practices that aligned with research on language acquisition and bilingual education that built upon students' linguistic, cultural, and experiential knowledge. Specifically, these are the practices identified in almost every lesson observed by all three teachers:

- Connect mathematics lessons with students' life experiences (Barwell, 2003; Civil, 2002; 2007).
- Encourage students to use multimodal approaches (e.g. pictures, words, numbers, gestures) to communicate (Chval & Khisty, 2009; Moschkovich, 2012).
- Use visual supports, such as concrete objects, illustrations, and gestures in classroom conversations (Moschkovich, 2002; 2005; Setati, 2008).

- Connect language with visual representations (e.g. manipulative materials, tables, graphs, and equations) (Moschkovich, 2002; Chval & Khisty, 2009; Morales, Khisty, & Chval, 2003).
- Have essential ideas, concepts, representations, and words on the board so that students can refer back to throughout the lesson (Begolli & Richland, 2013; Stigler et al., 1996).
- Promote the use of students' native language, English or Spanish, during instruction (Cummins, 2000; Moschkovich, 2002, 2007).

While all three teachers used similar strategies to support learning, there were clear variances in how the strategies were used. I will highlight the differences with an analysis of Laura's interactions with her first grade students and compare her classroom interactions with those of Kassandra and Elise.

Laura. Laura's mathematics instruction followed a consistent routine. The class began with a math talk activity in which students would engage in mental math – the solving of problems mentally, or choral counting, skip counting of numbers—to explore mathematical concepts and number patterns and relationships. Then, the class engaged in a problem-solving activity.

Laura typically led a classroom discussion about the problem before the students began working on it. During the problem launch, Laura usually asked the students the following questions: "What do you understand about the situation of the problem?; " "What do you know about the answer without solving the problem?" Students were encouraged to solve the problem using manipulatives, through multiple representations (e.g. pictures, words, numbers, and tables), and strategies. As students worked independently or in pairs to solve the problem, Laura circulated around the room observing, asking probing questions, and annotating students'

approaches to the problem. This was then followed by a public sharing of student solutions with a discussion focused on the connection between student strategies or representations. During the second year, after the public solution sharing of strategies, Laura asked students to write down and explain one student presenter's strategies in their own words. Laura discussed how this addition was made to "hold students accountable for explaining and critiquing each others' thinking" (November, Year 2).

In the following sections, I provide analysis of Laura's interactions with her students to illustrate how Laura created a classroom in which language was the heart of the social context in which mathematics learning occurred. Students engaged in the analysis of mathematics concepts and strategies while developing the language.

What follows are excerpts from a lesson taught during a November visit in Laura's second year to show features typical of the problems, practices, and norms in her classroom throughout the two years. During this visit, her first grade students were introduced to their first join change-unknown word problem (a problem combining two amounts where one addend is unknown; e.g., 3 + __ = 10):

Nostros tenemos __ fotos en el altar. Los estudiantes ponen algunas fotos más. Ahora hay ___ fotos in el altar. ¿Cuántas fotos ponen los estudiantes? [We had ____ photos on the altar. Now, we have ___ photos in the altar. How many photos did the students put on?] 3/10 13/20 83/100

What follows is an excerpt from the public solution strategy share for the word problem above:

Excerpt 1[3] Laura teaches [November, Year 2]

[3] The following transcription symbols are used in all transcript excerpts
/ English translation follows
(( )) Description of nonverbal communication such as gestures, gaze, movement, etc.

T: Ahora va venir Sammy. Y Sammy también va ha explicar su idea de los números 3 y 10. / Now Sammy is going to explain his ideas about the number 3 and 10.

((*Sammy walks up and stands next to Laura. Laura writes Sammy's name down on the white chart paper below the word problem*.))

T: Sammy, explícanos esta idea. ¿Como contaste? / Sammy, explain to us the idea. How did you count?

S1: Yo conté en mi mente ((*points to his forehead*)) que 3 y 1, 2, 3, 4, 5, 6, 7 mas y 3, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. ((*counts with his fingers*)) / I counted in my mind 3 and 1, 2, 3, 4, 5, 6, 7 plus 3, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

T: ¿Oh Sammy, entonces tu empezaste con el numero TRES? / Sammy, so you started with the number TRES?

S: Si / Yes

((*Laura writes the number three below Sammy's name on the poster sheet*.))

T:Por qué empezaste con el numero TRES? / Why did you start with the number THREE?

S: Porque nosotros tiene TRES (*points to the number three written in the word problem displayed on the white chart poster*) y los estudiantes pone algunos mas (*points to the word problem again*). / Because we have 3 and the students put some more.

T: ¿Oh tenemos 3 fotos (*finger circling the three in the word problem*) en el altar y los estudiantes ponen alguna mas? / Oh we have 3 photos in the altar and the students put some more?

T: Dígale a su amigo lo que dijo Sammy ahora. / Tell your friend what Sammy just said.

This episode demonstrates how Laura fostered an integrated approach to language and content through problem solving and meaning making, not problem answering. The focus of the discourse was on understanding why Sammy counted on from three in relation to the problem asked. By orienting students towards Sammy's reasoning, Laura directed students to analyze the words in the problem and its meaning. In this brief turn after the partner discussion of Sammy's strategy, Laura called on another student to build on to Sammy's strategy:

Capital letter to show emphasis in speech.

T: Entonces, Sammy tu empezaste con el numero 3. ¿Por qué Sammy empezó con el número 3? ¿Quién nos puede decir? / Sammy, you began with the number 3. Why do you think Sammy started with the number 3? Who can tell us?

S1: Porque los otros tienen 3 fotos y los estudiantes PONEN ALGUNOS MAS, don't know how much ((*Lea points to the words "ponen alguno mas" in the word problem*)). But al final es 10 FOTOS / Because we have 3 photos and the students PUT SOME MORE, don't know how much more ((*Lea points to the words "put some more" in the word problem*)). But at the end, it's 10 PHOTOS.

T: ¿OH AL FINAL ES 10? Okay, entonces voy a escribir 10 ((*Laura writes ten on the same line as the three on the chart paper)*). Y sabemos que ahora hay 10 fotos. ¿Cómo llegaste de 3 a 10? ¿Que hiciste? / So at the end is 10? Okay. I'm going to write 10 ((*writes ten on the same line as the three on the chart paper*)). We know that now there are 10 photos. How did you get from 3 to 10?

S2: Sammy pone 3, 3 fotos, en su mente después 4, 5, 6, 7, 8, 9, 10 -- 10 fotos. ((*counts with her finger*)) / Sammy put 3, 3 photos, in his mind then 4, 5, 6, 7, 8, 9, and 10 – 10 photos ((counts with fingers)).

T: Entonces tenias 3 en tu mente y contaste 3 contamos juntos 3, 4, 5, 6, 7, 9, 10. ¿Entonces habían 3 y pusimos cuantos mas? / So you had 3 in your mind and you counted, lets all count together, 3, 4, 5, 6, 7, 8, 9, 10. ((*writes the numbers between the 3 and the 10 already written on the chart pape*r)).

T: Y aquí entonces 1, 2, 3, 4, 5, 6, 7. (represents the jumps on chart paper. See figure 1). Siete brincos para llegar al diez. / So now 1, 2, 3, 4, 5, 6, 7. Seven jumps to get to ten.

T: ¿Por qué? ¿Porque 7 brincos? / Why 7 jumps?

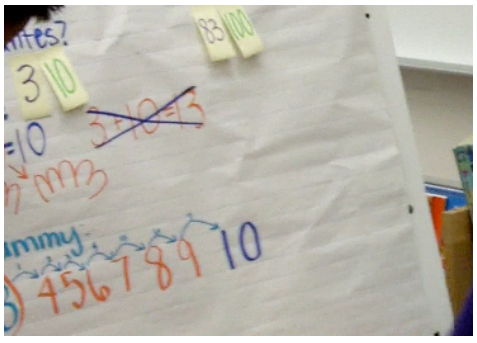

Figure 1. Documentation of Sammy's Strategy

The short vignette above highlights the start of the classroom discussion of Sammy's strategy. The public sharing of Sammy's strategy lasted for over seven minutes. Multiple students, like Leah and Analicia, offered an explanation with the focus on not just making sense

of Sammy's strategy but how his strategy connected to the problem asked. By doing so, students were required to explain Sammy's sense making of the problem's context and to then analyze and explain the underlying characteristics of a change-unknown problem type.

Students developed language through engaging in the mathematics. As in the scenario discussed above, students arrived at meanings and definitions as they engaged in the collective analysis of each other's reasoning and problem solving. The classroom interactions of Kassandra and Elise followed a different pattern.

Kassandra and Elise. Kassandra and Elise set a specific time before the math instruction to teach language. Dutro and Moran (2003) call the preteaching of vocabulary prior to content instruction "frontloading vocabulary". Frontloading vocabulary is a commonly suggested strategy to support students' comprehension and acquisition of new vocabulary. I provide an example of frontloading from an observed lesson in Kassandra's classroom.

Kassandra did not follow an established district textbook but created lessons based on her students' learning. While lessons were created based on her knowledge of students' development, her school required instruction to follow a gradual release model, sometimes referred to as "I do, we do, you do." Kassandra began each lesson with a think aloud in which she explicitly modeled her thought process for reading and solving problems ("I do"). After, the class would practice the strategy as a group ("We do"). Then, the remaining instructional period would be devoted to individual student practice of worksheets while she worked with a small group of students ("You do").

The following description of a lesson, observed in February of Kassandra's second year, illustrates her classroom interactions and the explicit teaching of terminology typically seen in both Kassandra and Elise's mathematics lessons. In this 30-minute lesson, the class worked on

direct modeling of addition problems. A group of 24 kindergarten students sat in five rows facing the board and the teacher. The lesson began with an introduction of the key mathematical terms in the lesson:

T: Can you show me what we do in an addition problem?

Ss: ((*Students lift up their hands and bring their hands together to clap))* Put them together. ((*Choral Response*))

T: We put them together. That means we put two groups of things together. ((*Kassandra lifts up her hands and brings her hands together to clap))* Yesterday, we were telling stories about our friends and putting them together. Remember, we had three girls sitting at the turquoise table ((*Kassandra draws three circles on a paper projected by the document camera))* and then two more girls ((*Kassandra draws two more circles on the paper*)) came to join them. And then, how many girls do we have ALL TOGETHER?

Ss: 4/5 ((*Choral response*))

T: Let's count them together.

Ss: 1,2,3,4,5 ((*Students choral count as Kassandra points to each circle on the board.*))

T: Five is our answer. ((*Kassandra writes 3+2 = 5 below the five circles.))* We are going to call our answer THE SUM ((*Points at the 5 in the equation*)). Can we say that?

Ss: The sum. ((*Choral response*))

T: The sum is what? Can someone with a quiet hand tell me what the sum is? ((*No one raises their hand.*)) Juan?

S1: I don't know.

T: Can a friend help Juan and tell us what the sum is? Brian?

T: The answer to our addition problem. Can we repeat that? The sum

Ss: the sum ((*choral response))*

T: is the answer

Ss: is the answer ((*choral response*))

T: to our addition problem.

Ss: to our addition problem. ((*choral response*))

T: Brian, can you repeat the answer? The sum…

S2: The sum is the answer to the addition problem.

T: Juan, can you repeat what Brian said.

S1: The sum is

T: our addition answer.

S1: our addition answer.

Right after, Kassandra moved on to the math lesson in which the class learned how to direct model addition problems with counters. The scenario above illustrates how language was treated as a self-contained entity separate from the rest of the mathematics lesson. Krashen (1989) uses the term "target language" to describe key vocabulary explicitly taught wherein the acquisition of the word becomes the focal point of the instructional activity. This process of frontloading language and rehearsing technical mathematical terms were regularly found in Kassandra and Elise's mathematics lessons.

The Authority of Knowledge Within the Classroom

For this dimension, I examined who was positioned as the authority of knowledge in the classroom– the student(s) or teacher. Laura distributed the authority of knowledge between herself and the students. For Kassandra and Elise, they felt a strong sense of responsibility to serve as the language and mathematics experts. I use an example from Laura's classroom and another from Elise's to illustrate the type of interactions that took place.

Laura. Laura used various strategies to distribute the authority of knowledge in her classroom. One of the key strategies Laura invoked was to orient students as the knowledge and content experts. The Sammy strategy share scenario above highlights the stance Laura assumed

for herself in relation to the mathematics problems and her students. In the classroom, Laura positioned herself as the facilitator as well as a learner within the classroom community rather than the authority of the mathematics and linguistic content. As shown in the two scenarios above, Laura often adopted a posture of uncertainty ("Three? Why did you start with the number three?") to open up space for students to take on the expert role ("Tell a friend Sammy's strategy. Why 7 jumps?").

In a later component of the public solution share, Laura called on a third student, Jesus, to explain his strategy. Jesus was a quiet student. In Laura's class, the class had established an expected norm for students to initiate explanations and commentaries on each others' strategies; students were not waiting for permission from Laura to speak. Prior to this visit, Jesus had not spoken out during the discussions.

It is important to note that a student like Jesus who is reticent, but growing in confidence, may often not be asked to share in most classes. As well, teachers often select strategies based on a perceived order of sophistication of strategies. The strategy Jesus used to solve the photo problem was at a direct modeling stage where he was explicitly representing each and every object in the problem. Students sharing based on the level of sophistication of the strategy is commonly seen (Kazemi & Hintz, 2014; Smith & Stein, 2009). In such a setting, Jesus possibly would go first as he "modeled all."

During Jesus's public sharing, he was not audible and pointed to his work instead of giving a verbal response. Laura allowed silence but encouraged Jesus to use multimodal approaches to communicate. Laura held his paper out for the class to show how Jesus had solved the problem. Jesus first drew twenty dots and crossed out thirteen. Laura drew his strategy on the white poster (see Figure 2). Even without his verbal speech, Laura positioned Jesus as an expert

as he displayed his work visually and the class unpacked his strategy together. What follows is an excerpt exemplifying how Laura deliberately distributes the knowledge authority by positioning Jesus and the class as the experts instead of herself:

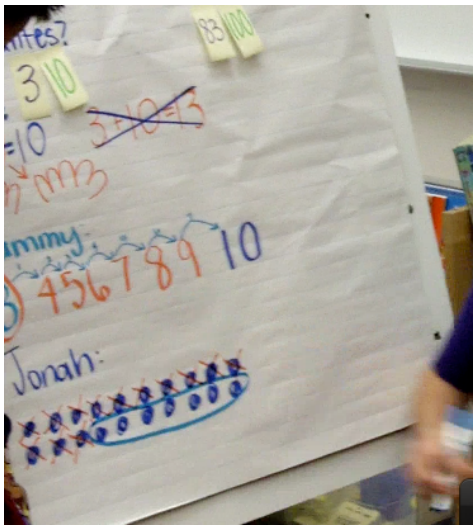

Figure 2. Documentation of Jesus's Strategy

T: Jesus, adelante. Escuchando a Jesus, Jesus ¿Que hiciste?/ Jesus, come forward. Let's all listen to Jesus. Jesus, what did you do?

((*Jesus's talk is unaudible.))*

T: ¿Jesus primero dibujo veinte que? Veinte. / Jesus first drew twenty what? Twenty.

S1: Veinte puntos/ Twenty dots.

T: Veinte puntos, cuenten conmigo. El primero puso veinte puntos. Tres, dos, uno. / Twenty dots, let's all count together. He first put twenty dots. Three, two, one.

Ss: Uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, diez, once, doce, trece, catorce, quince, dieciséis, diecisiete, dieciocho, diecinueve, veinte./ One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty. ((*Students chorally count the dots as Laura draws them on the chart paper.))*

T: El dibujo veinte puntos, ¿Por que dibujastes veinte Jesus?/ He drew twenty dots. Jesus why did you draw twenty?

S1: No me acuerdo/ I don't remember.

T: Él no se acuerda porque dibujo veinte. ¿Sophie tu tienes una idea?/ He doesn't remember why he drew twenty. Sophie, do you have an idea?

S2: ((*Student walks up to the chart poster and stands next to Jesus.))* El dibujo veinte porque ahí, veinte fotos *(points to the space for "twenty" in the word problem written on the chart paper)/* He drew twenty because of that, twenty photos ((*points to the space for "twenty" in the word problem written on the chart paper))*.

T: ¿Porque hay veinte ahi? Ahora hay veinte fotos en el altar.¿Jesus luego que hicistes? / Because there's twenty? Now there are twenty photos on the altar. Then what did you do?

((*Jesus holds out his paper for Laura to see.))*

T: Dibujastes veinte. El quito trece. Hacemos eso, cuenten conmigo, vamos a quitar trece puntos, tu cuenta tambien okay? Cuenten./ You drew twenty. He took away thirteen. Let's do that, counting with me. We are going to take away thirteen dots. You count too, okay?

Ss: ((*Students chorally count the numbers as Laura crosses out thirteen circles from the 20 circles drawn on the chart paper.))* Uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, diez, once, doce, trece./ One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen ((*choral response*))

T: Okay Jesus quito trece. Y luego que paso? Hay siete que se quedan. ((*Circles seven dots on the paper*)) Jesus puedes explicar que significan esos siete? Que son? / Okay Jesus took away thirteen. And then what happened? There are seven left. ((*Circles the seven dots on the paper*)) Jesus can you please explain what does the seven mean? What are they?

((*Jesus's response is unaudible.))*

T: Explícale a su amigo lo que hizo Jesus. / Explain to your friend what Jesus did.

((*The students talk amongst themselves and then Laura clapped to catch their attention.))*

T*:* Okay. Alguien puede explicar porque Jesus empezó con veinte y terminó con trece? Por que hizo esto? Jesus, escoge a otra persona que pueda explicar lo que hiciste. / Who can explain why Jesus started with 20 and ended with 13? Why did he do this? Jesus, pick another person that can explain what you did.

((*Jesus points to a student sitting in near the back The student walks up an stands next to Jesus an the chart poster.))*

S3: ((*Student walks up to the poster sheet and stands next to Jesus.))* Habían trece al principio (*uses finger to circle the thirteen crossed out dots*) y luego habían veinte y estos son los que ponen (*uses finger to circle the seven circled dots)* / There were thirteen (*uses finger to circle the thirteen crossed out dots*) in the beginning and then there were twenty and those are the ones they put there (*uses finger to circle the seven circled dots*).

T: Hmm, okay. ((*puts finger on check to show a pensive look.*)) Alguien. / Hmm, okay. Anyone…

S4: ..Porque Jesus did it backwards a Sammy… so it's twenty take away thirteen is seven. / But Jesus did it backwards a Sammy… so it's twenty take away thirteen is seven.

T: Veinte quita trece es igual a siete y tú dijiste él lo hizo al revés a Sammy. Qué quieres decir?/ Twenty minus thirteen is equal to seven. ((*Writes 20-13=7 under the dot representation*)) You said that he did the reverse to Sammy. What did you mean?

S4: Jesus hizo…/ Jesus did…

S5: Sammy counted up. Sustracción / es opposite. / Sammy counted up. Subtraction is opposite.

S6: Sammy lo hizo una suma. Sustracción es el revés. Es opposites. / Sammy lo hizo una suma. Sustracción es el revés. They are opposites.

In the scenario above, Laura demonstrated some of the ways she distributed the knowledge authority between herself and the students. One way is her consistent questioning, displaying a sense of wondering ("Alguien puede explicar porque Jesus empezó con veinte y terminó con trece? Por que hizo esto?/ Who can explain why Jesus started with 20 and end with 13? Why did he do this?"). Laura intentionally paused during certain parts of a student share and asked the rest of the students to explain the reasoning behind that strategy. This was a common move to open up the floor for peer mediation. It was a normative expectation for students to provide explanations and commentaries on the strategy shared. As students shared their thinking, they presented their thoughts in whatever language and modality that made sense to them. Choice of language and nonverbal communication provided more access for students to contribute to the conversation.

The first student Jesus called on (notice how Jesus was the one who called on students to explain his thinking) used his finger to circle the thirteen crossed-out dots, and then pointed to the words "thirteen photos" in the problem to explain the relationship between his strategy and

the problem's context. This opened up the floor to a total of six students to explain his strategy. The explaining and clarifying of Jesus's strategy led students to arrive at the discovery that Sammy's counting-up strategy was "opposite," or "the reverse" (an inverse relationship) of Jesus's subtraction strategy. Seeing counting up as a viable strategy, or the relationship between addition and subtraction, allows students more versatility when solving addition or subtraction problem types (Carpenter, et al., 2001; 2015).

This sequence typifies the discourse arrangements within the classroom and is a good example of the collective sense making fostered and organized by Laura. Throughout the discussion, Laura facilitated the problem solving, putting the onus of explanation and analysis on the students who were expected to publicly share their thinking in multiple ways, and at multiple times. Students' share-outs were not always fully articulated ideas, partial explanations were often heard, and students built upon each others'. The focus of the discussion was not on the answer but the making sense and solving of the problem, allowing more voices and ideas to be heard. Although Laura positioned the students as the authorities of knowledge, she was an active participant in organizing the dialogic discourse that took place. Throughout the classroom visits, Laura took notes of student strategies during their independent work time to determine who should share and in which order. During her lesson reflections, Laura regularly talked about the intentional selection and sequencing of specific solution strategies as well as strategies to increase participation.

Kassandra and Elise. Laura engaged in various strategies to distribute the knowledge authority by focusing on collective sense-making of student strategies, whereas Kassandra and Elise felt a strong sense of responsibility to serve as the language and mathematics experts in the

classroom. I use an example from Elise's class to illustrate the type of interactions that took place.

Elise's mathematics lessons were rich in verbal and nonverbal language practices. Both Spanish and English were valued and frequently heard in the classroom. As in Kassandra's classroom, Elise explicitly taught key vocabulary using visuals and hands-on experiences to link concepts and build mathematics language. Manipulatives, total physical responses, and chants as mnemonic techniques were used to help students remember procedures and concepts. Relevant content vocabulary was presented at the beginning of the lesson, modeled by Elise, and sentence stems provided to support students' appropriation in their own speech. While students engaged regularly in mathematics across modalities (e.g. visual, kinestic, auditory), Elise felt the responsibility of serving as the content and language expert fell upon her.

Elise organized the mathematics instruction into two parts. Instruction began with students engaging in a mathematics activity using total physical response (e.g. students used their hands to represent the hands of a clock, created a human number line, searched for geometric shapes in classroom scavenger hunts). After the whole class activity, students worked independently on worksheet problems or at learning centers reinforcing the day's lesson.

I highlight a lesson that took place during a similar period as the other vignettes. This lesson took place during March of Elise's second year. By this time, Elise had voluntarily attended a series of district-funded professional workshops on understanding children's development of numerical fluency and algebraic reasoning and children's mathematics learning trajectories.

In this scenario, the class was on their second day working on regrouping in subtraction with place-value models. Elise followed a developmental approach to teaching math

concepts/skills through a concrete-to-representational-to-abstract sequence of instruction (CCSI, 2010; Van de Walle et al., 2011) The day prior, the class used models (concrete objects) to represent the process of regrouping: working with place-value blocks and place value charts to connect the procedural skill of regrouping (the decomposing of a ten unit for ten ones) to its conceptual basis (each place value position is related to the next by a constant multiplier of ten as we are based on a base-ten system). Each student had their own set of base-ten blocks as place-value models and a place-value chart to organize the objects (see Figure 3). The dialogue that follows is from the start of the mathematics lesson during which Elise demonstrated the process of regrouping from concrete to representational - connecting the written recording of each step of the procedure to the modeling with the base-ten blocks.

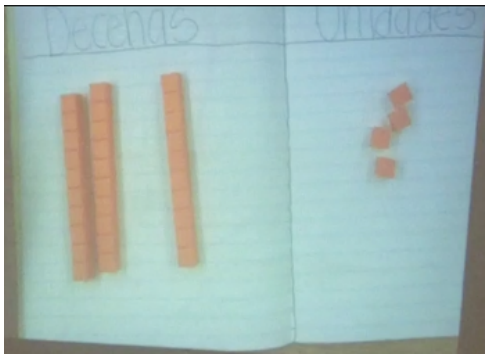

Figure 3. Teacher display of subtraction model

T: Con los bloques de diez, yo les voy a dar un cuento sobre algo, así que tienen que escuchar calladitos. ¿Por ejemplo, si digo Diego tiene 34 pájaros ((*Elise places the three base ten rods in the tens place and four units in the ones place of the place value chart. See Figure 3*.)), cuantas decenas voy a poner? / With the blocks of 10, I'm going to give you a story. So you all must listen quietly. For example, if I say Diego has 34 birds, how many tens am I going to have?

((*Elise points to the three base ten rods on the tens side of the place value chart.*))

Ss: Tres/ Three. (*(A few students respond chorally.))*

T: ¿Cuantas me faltan para ser treinta cuatro? / How many more do I need to make thirty-four?

((*Elise points to the four ones units on the ones side of the place value chart.*))

Ss: Cuatro/ Four ((*A few students responde chorally.))*

T: No es necesario tocar nada en este momento. ((*Elise looks around the room and waits for students to put down the base ten blocks.*)) Pero su hermanito, Allen, le abrió la jaula, y se le fueron veinticinco. Se fueron veinticinco pájaros ¿Que voy hacer? Se fueron veinticinco pájaros. ¿ Que vamos hacer?/ You don't need to touch anything right now. But his little brother, Allen, opened the cage, and twenty-five left. Twenty-five birds left. What are we going to do?

Ss: Vamos a quitar una docena. / We are going to take away a ten. *((A few students respond chorally.))*

T: No es necesario tocar nada en este momento. / You don't need to touch anything right now.

((*Elise waits for students to put down the base ten blocks in their hands*.))

T: ¿Si, vamos a quitar una docena y que se va hacer? / Yes, we are going to take away a ten and what is that going to make?

Ss: Pone una unidades/ Put ones. *((A few students respond. Elise takes away a ten rod from the ten place-value column.*))

T: ¿Las pongo en unidades? / I put them with the ones? ((*Elise places the ten rod into the ones place value column.*))

Ss: No. (*Choral response*.)

T: ¿las voy a poner aquí? ¿Qué va a pasar? / Am I going to put them here? What is going to happen? (((*Lifts the placed ten rod from the ones place-value column and holds it up for students to see.))*

S1: Vas a quitar dos decenas/ You are going to remove two tens

T: ¿DOS decenas? / TWO tens? ((*Elise holds up the ten rod in hand and shakes it.))*

S1: No

Ss: Una docena/ One ten.

T: Una docena. (*Elise holds up the ten rod in hand and shakes it.))* ¿Donde se va a ir esta docena? / One ten. Where is this ten going to go?

Ss: En las unidades/ In the ones. ((*Elise places ten individual units in the ones column*.))

T: ¿Y vas a cambiarlas porque? / And why are you going to change them

Ss: por unidades, se fueron / for ones, they left

T: Veinticinco se fueron entonces ya puedo quitar veinticinco? / Twenty-five left and now can I take twenty-five?

Ss: Si / Yes ((*Choral response.))*

T: ¿si se fueron veinticinco cuantas quito? ¿Dos decenas? / If twenty-five left, how many do I take out? Two tens?

S2: No, una / no, one

((*Elise removes the remaining two ten rods from the tens place*.))

T: y que mas?/ and what else?

Ss: cinco unidades/ five ones

T: Cinco unidades? / Five ones?

((*Elise removes five one units from the ones column. Nine ones unit remain.))*

T: Y cuantas son mi respuesta? / And what is my answer?

Ss: Nueve/ Nine.

In the dialogue above, we see how Elise took a developmental approach to linking procedural skills to its conceptual basis. Elise was building on from the concrete experience the day prior to the representational stage – linking each step of regrouping to the written recording. She first represented the minuend with the place value blocks and made trades first (i.e., exchanging 1 for 10 in the position to the right) before subtracting. This process of "trading all" before subtracting has been shown to help prevent errors in subtraction (Van de Walle, et al., 2007) and demonstrated Elise's attention and understanding of students' mathematical development.

Physical and visual models were regularly used by Kassandra and Elise to connect procedures and concepts; however, the models used as well as the process for problem solving were determined by them, not their students. At one point, a student appeared to suggest a

different way to subtract ("Vas a quitar dos decenas/ You remove two tens."). It's possible that the student may have wanted to subtract by place value – taking the two tens of 25 from 34 first; however, Elise didn't ask him to explain his thinking but directed the students' attention back to the ten rod in hand as she wanted students to regroup before subtracting.

By prescribing a solution path (even when linked to a conceptual basis), Laura and Kassandra limited students' opportunity to engage in their own sense-making and be agents of their own learning. In doing so, Kassandra and Elise remained as the authority of knowledge in the classroom.

How and Which Students' Repertoires of Practice are Utilized

The last category is related to the prior and examined which students' repertoires of practice were utilized. Specifically, I examined how students' repertories of practice (Gutiérrez & Rogoff, 2003)— language practices, verbal and non-verbal gestures, and student strategies – were used during mathematics instruction. In the analysis, I identified the teachers utilizing the following students' repertoires of practices: students' mathematics strategies, lived experiences, everyday talk, school-based mathematics discourse, English, Spanish, the hybridity of English and Spanish, and gestures.

All three teachers created lessons relevant to students' lived experiences, were multi-modal, and encouraged hybrid-language practices. With a focus on addition and subtraction algorithms in K-2 grades, all three teachers created addition and subtraction problems related to how mathematics could be used to solve problems in students' daily life. Lessons were built around common events where mathematics may be used at home (e.g. making brownies, sharing with a sibling), and shared school experiences (e.g. field trips, ordering school supplies, school garden). In addition, every math lesson asked students to use multimodal approaches (e.g.

pictures, words, numbers, and gestures) to show their thinking. Hybrid-language practices (e.g. every day, native language, second language, and school-based discourse) were heard as students interacted with each other in class and during recesses. However, the way in which the teachers conceptualized language and mathematics impacted how students' repertories of practice were leveraged for learning. I use examples from Laura's classroom and contrast it against Elise and Kassandra to illustrate these differences.

Laura. Laura engaged in teaching practices that built on students' knowledge, skills, experiences, and language practices: students' home language, everyday ways of talking, and developing mathematical discourse as resources for learning (Civil, 2007; Foote, 2009; Gonzalez, Moll, & Amanti, 2005; Gutiérrez, et al., 1999). Here, I'm going to highlight specifically how Laura built upon the hybridity of experiences, languages, and meaning making in her classroom.

In the episode of the photo problem shown earlier, Jesus was the third student to present his strategy during the lesson. In all three student presentations, the students used different representations and approaches to solving similar problems. Note how Jesus solved the same word problem as Sammy (3/10) but with a different number set (13/20). Laura always provided word problems with three sets of number choices to allow students to wrestle with similar mathematical ideas but using numbers students felt comfortable working with.

Laura also utilized students' linguistic repertories and their hybrid and multimodal practices as resources for communication and sense making in her classroom. Students' home language, everyday ways of talking, and emergent mathematics discourse were encouraged in class (Civil, 2007; Foote, 2009; Gonzalez, Moll, & Amanti, 2005; Gutiérrez, Baquedano-Lopez, & Tejada, 1999).

The following scenario illustrates how Laura built upon students' repertoire of practice over time. I discuss a visit made two months after the photo-problem lesson described earlier. The class regularly engaged in student-strategy share. In prior lessons, solution strategies were labeled with the students' names (e.g. Rosa's strategy). In this visit, the lesson focus was ""estrategias para juntar /strategies for adding" to link students' addition strategies to its mathematical term. However, the defintion was not given by Laura to the students but unpacked by the students as they analyzed each strategy's features.

The following episode highlights how Laura organized a classroom community that recognized and honored the various repertoires of practice students' brought from their own experiences as well as the repertoire of practices established within their classroom community.

T: Cada vez que hacemos, Cada vez que hacemos un problema ay personas que comparten su idea, verdad? / Whenever we do a problem, there is other people who share the same idea as you, right?

Ss: Si. / Yes

T: si, cada vez que lo hacemos. Pero yo estaba pensando, estaba pensando que sus ideas no tienen nombre. Entonces si yo le digo, as la idea de Caleb, no sé de qué estoy hablando. Entonces hoy quería hacer una lista de las ideas que tenemos. Entonces yo llame esta hoja estrategias para juntar. Que es una estrategia? Que es una estrategia? Quien ha escuchado esta palabra? / Yes, whenever we do it. But what I was thinking (pause), I was thinking that your ideas do not have a name. Therefore, if I tell you all to do Caleb's idea, I don't know what I am talking about. Today, I would like to do a list of the ideas that we have. Therefore, I call this page strategies for adding. ((*Laura points to the words "Estrategias para juntar/strategies for adding" on the chart paper*.))

((*The chart paper is covered by another sheet of chart paper. Laura lowers the top paper to reveal the first strategy "Modelar de uno en uno (Modeling one by one)." The strategy title and the visual example are shown.))*

T: la primera estrategia es de Ella, es la estrategia de Ella. Su estrategia el nombre de su estrategia es modelar de uno en uno. Esta es una manera que muchos de ustedes están usando modelar de uno en uno. Y que están observando de las estrategia de Ella? Como es SU ESTRATEGIA? Que ha hecho. Hmmm. / The first strategy is Ella's. This is Ella's strategy. Her strategy's name is modeling one by one. This is one strategy that many of you use. What do you observe about Ella's strategy? What is her strategy? What has she done? Hmmm.

((*Laura puts her hand to her face to show a pensive look. Many students in the class follow and model the same poise.))*

T: Piensa todos pensando observando su estrategia. Que están observando? Que están observando de la idea de Ella, Alex? / Everyone think about her strategy. What do you observe? What do you observe about Ella's idea, Alex?

S1: ((*Alex walks up to the poster and points at Ella's strategy*)) um, tiene quince y catorce-trece the same one as this ((*Alex walks over to the class math wall and points to strategy on another poster where the student is also modeling all (Figure 4 and 5*)) because quince y trece and then right there he put quince y trece mas, but then she counts each one – one, two, three, four, five, six, seven, eight, nine, ten eleven, twelve, thirteen, fourteen, fifteen, (touches each drawn unit and counts out loud) and then one, two, three, four, five, six, seven, eight, nine, ten eleven, twelve, thirteen, fourteen, fifteen, and then you and then you also could do this. /um, there's 15, and 14, 13, the same one as this because 15 y 13 and then right there he put 15 and 13 more, but then she counts each one, two, three, four, five, six, seven, eight, nine, ten eleven, twelve, thirteen, fourteen, fifteen, and then one, two, three, four, five, six, seven, eight, nine, ten eleven, twelve, thirteen, fourteen, fifteen.

T: Oh Entonces tu estas diciendo, el esta diciendo que en la idea de Ela cuenta todas las cosas. Entonces Alex dijo que cuenta cada palo, verdad? Cuenta de uno en uno voy a escribir eso. / Okay, what you are saying, he is saying that Ella's idea is to count.. Alex said to count each of the lines, right? Count one by one, I am going to write this.

((*Laura writes "cuenta todas las cosas / counts one by one" under the modeling one by one representation on the chart paper.))*

T: Algo más que observan de su idea de Ella? Ana? / His idea is to count one by one. Is there anything else you observe about Ella's idea? Ana?

S2: una manera fácil también is to take quince first and add uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve los demás más los demás le va hacer fácil solo hacer números. / An easy way to also do this is to first take the first fifteen plus the other one, two, three, four, five, six, seven, eight, nine and then the rest of them plus the rest of them. It will be easier to just use those numbers.

T**:** tu estas diciendo que sería más fácil usar números. / You are saying that it would easier to use numbers?

S2: Si porque te vas a hacer cansada de escribir muchos muchas líneas. / Yes because you will get tired of drawing so many lines.

T: En esa tienes que escribir muchas cosas. Voy a escribir esto también, entonces en esta idea escribes, ((*Laura writes "escribes muchas cosas / writes down a lot" on the chart paper.))*

. Addy algo más? / In this one, you have to write many things. I am going to write this one too. ((*Laura writes "escribes muchas cosas / writes down a lot" on the chart paper))* Addy, anything else?

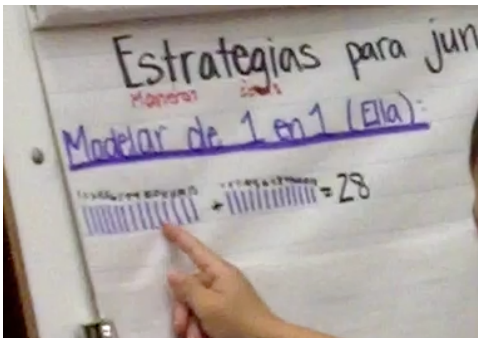

Figure 4. Ella's strategy

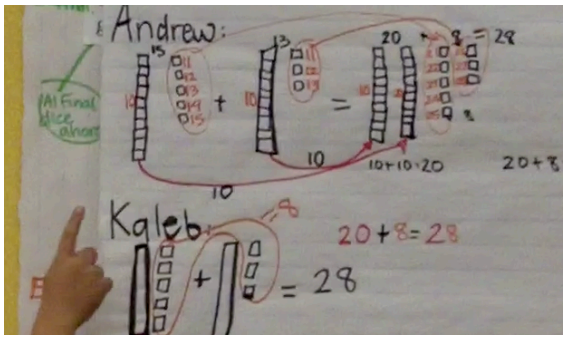

Figure 5. Student gesturing to one by one model

The interaction above shows some of the ways Laura drew on students' repertoires of practice to engage the class in collaborative problem solving. First, Laura used students' ways of thinking and reasoning about mathematics as the focus of discussion. Laura displayed student strategies on a math wall filled with posters of strategies gathered from lessons past. The posters were often referenced during group discussion, as seen in the vignette above, and students often went up to the posters to explore other strategies. Varied strategies and representations (e.g. drawings, words, and equations) were included on the posters.

The students spoke in Spanish, English, and code switching between Spanish and English during every visit. As seen in the scenario above, students moved in and out of a range of language practices across the participation structure: pairs, small group, and whole class. The flow of language practices, students moving between English and Spanish, informal and school-based discourses, were most observed when students were trying to communicate their reasoning and sense making. It appeared Laura's students often articulated the academic language of mathematical concepts and numbers (school-based discourse commonly heard in class) in Spanish and would use their native language to provide illustrations ("una manera fácil también is to take quince first and add uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve los demás más los demás le va hacer fácil solo hacer números"). Laura accepted Alex's response, but rephrased

a more complete response in Spanish so other students could hear the discourse. ("El esta diciendo que en la idea de Ela cuenta todas las cosas. Entonces Alex dijo que cuenta cada palo, verdad? Cuenta de uno en uno voy a escribir eso. / Okay, what you are saying, he is saying that Ella's idea is to count.. Alex said to count each of the lines, right? Count one by one, I am going to write this.). As in the example above, Laura acknowledged students' own words, gestures, and meanings while also seizing these opportunities to have students hear from her how the discourse should be and to reinforce their understanding through mathematical meanings.

As well, semiotic resources other than spoken or written language, including gestures, were often used. In the example above, Alex pointed to another student example of a modeling one-to-one strategy as a way to explain how the strategy required students to "count one by one." By revoicing his explanation in Spanish using the mathematical terms and writing down his observation on the poster sheet, Laura showed that all forms of communication – verbal or nonverbal, English or Spanish – were valued. This is especially important, as research examining emergent bilinguals shows that they often rely upon semiotic resources other than spoken or written language to participate in classroom interactions (Cekaite, 2009; Setati, 2008). Gutiérrez uses the term "hybrid spaces" to describe classrooms that create or sustain the resources and identities students bring from their personal and prior experiences (Gutierrez, et al., 1999).

Kassandra and Elise. In the classrooms of Kassandra and Elise, I saw a relationship between the student strategies used and the hybridity of language practices. Students moved between language practices in their student-to-student interactions and during recesses; however, student strategies and hybrid and multimodal practices were seen much less frequently during classroom discussions than in Laura's. To provide contrast as to how Laura utilized students'

repertoires of practice during strategy shares, I offer a vignette of Elise's class discussion on a strategy for subtraction with regrouping. The strategy is not the traditional U.S. subtraction wherein the algorithm subtracts from right to left, one place-value column at a time, regrouping as necessary. In this strategy, the students are subtracting numbers in expanded form, the subtrahend and minuend are written in expanded form in vertical order with the same place value arranged in columns and differences found for each place value.

T: ((*points to the number "427" written in the problem 427 – 182.)*) ¿Ok, que son las escenas de 427? / Ok, what is the expanded form for 427?

S1: Cuatro / Four

T: ¿Cuatro, y como vamos hacerlo?/ Four, how are we going to do it?

S2: cuatrocientos /400

T: Cuatrocientos/ 400. ((*writes down 400, 20, 7 on the worksheet projected on the overhead, then points to the 20 and 7))*.. y que es esto? veinte y siete y ciento ochenta y dos /And what is this? 20 and 7. And 182 ((*points to the 182*))

SS: cien/100 ((*Elise writes down 100 below the 400 in the expanded form of 427*))

SS: 80 and 2 ((*Elise writes down 100, 80, 2 in columns by place value below the 400, 20, and 7)*)

T: ochenta y dos. ¿Que es siete menos dos? / eighty and two. What is seven minus two?

Ss: Cinco/ five ((*writes 5 down and points to "20-80"))*

S3: sesenta/ sixty

T: espérese ¿Cómo se puede restar 20 de 80? ¿Que le vas a decir?/Hold on, how can you subtract twenty from eighty? What are you going to say?

S4: Puedes prestar de los 400 / You can borrow from the 400.

T:)/ Le prestamos 400 y ese se convierte / We borrow from 400 and what does this one convert into? ((*points at the 400*))

S4: Este se convierte en 300 / That one turns into 300 ((*Elise crosses out the 400 and writes 300 above)*).

T: ¿Por que? / Why?

S4: Porque le presaste uno prestado uno al 20./ Because you lend one to the 20.

T: ¿Por que le prestas?/ Why do you lend it?

S6: Porque 80 es mas que 20/ 80 is more than 20.

In the interaction above, the class was using verbal and nonverbal communication (gestures and the expanded form) to make sense of the regrouping process in subtraction. However, the mathematics representation and how to regroup were determined by Elise. What follows is a brief episode of the discussion for regrouping in the next problem:

((The class already decomposed 639 and 256 into place value in expanded form together and this is shown on the paper projected on the overhead.))

T: ¿Entonces que estamos haciendo con nueve menos seis ?/ so what are we doing with 9-6?

S1: Tres/ three

T: ¿Jay, que vamos hacer con treinta y cincuenta?/ Jay, what are we doing with thirty and fifty?

S6: Puedes prestar de seiscientos y poner quinientos/You borrow from six hundred and you have five hundred

T: quinientos? / Five hundred? ((*Elise crosses out the 600 and writes 500 above*))

S6: quinientos / five hundred

T: ¿pero porque le vas a dar una centena al treinta? / But why are you giving one hundred to the thirty?

S6: Porque cincuenta es mas que treinta /Because 50 is more than 30.

The vignettes above provide a window into the classroom discourse around regrouping for two problems, however, the same discursive pattern continued for the remaining 30 minutes. As seen in the two vignettes above, there was very little language hybridity during strategy shares in Elise's classroom. The class discussion was only in Spanish. This was common and can be expected. The students in Elise's classroom were a grade above than Laura's and had more

opportunities to develop their Spanish proficiency. However, there was less variance in what was said. Note how student responses for both problems for the regrouping process ("Porque le prestaste uno al 20/ Because you lend one to the 20." "Puedes prestar de seiscientos /You borrow from 600."), and the reason why regrouping was needed ("Porque 80 es mas que 20/ 80 is more than 20/ 80 is more than 20." "Porque cincuenta es mas que treinta /Because 50 is more than 30.") were very similar. Throughout the remaining lesson, students talked about "borrowing" or "lending" to the next place value, and the need to regroup because the subtrahend was "more than" the minuend.

There was a clear relationship between the variance in strategy shared to variances in language practices seen and heard. In both Elise and Kassandra's classes, Spanish, mathematics representations, and gestures were frequently used, but the teacher modeled the representations and gestures first. By doing so, student responses were limited to a repetition of certain words and phrases ("you borrow one" "more than").

Setati and colleagues (Setati, Adler, Reed, & Bapoo; 2008), in their observation of a multilingual classroom, found that there were limited variance in language practices when students responded to steps to procedures that was already taught than questions focused on students' meaning making. Setati and colleagues (Setati, et al., 2008) attributed this finding to the fact that the two types of questions in fact require two forms of discourse that have different sets of linguistic and mathematical knowledge demands. While the former can be constructed through simple memorization of set terms or the language to describe the sequence of actions taught, the latter requires the learner to understand and convey the reasoning beyond the actions.

Discussion

This study examined how three novice bilingual teachers, Laura, Elise, and Kassandra, organized mathematics learning for their emergent bilinguals. To provide insight into this question, I examined (a) the types of supports provided to develop students' learning of the language of mathematics; (b) who was positioned as the knowledge authority; and (c) how and what students' repertoires of practices (Gutiérrez & Rogoff, 2003) are utilized.

First of all, the study findings confirm prior research on the importance in cultivating a linguistically diverse teaching force (Achinstein & Aguirre, 2008; Quiocho & Rios, 2000; Villegas & Irvine, 2010; Villegas & Lucas, 2004). All three teachers provided a classroom ecology that valued and honored students' linguistic, cultural, and experiential knowledge. They showed an understanding that learning took place in cultural context and worked to connect students' prior knowledge and experiences – both individual and cultural – in their lessons. Their mathematics lessons often connected to students' lived experiences, used multimodal approaches to communicate mathematically, and encouraged students to use their native language, English or Spanish, in class.

While all three teachers used strategies that valued students' linguistic and experiential knowledge, how strategies were used in the three classrooms revealed the complexity of the construct of learning language and mathematics. Specifically, I will discuss the findings in relation to two prevailing conceptual lenses for understanding learning – learning as acquisition and learning as participation – and argue for the danger of exclusive reliance on one particular conceptual lens.

A widely employed conceptual lens for understanding the learning of language and mathematics is the acquisition model. The acquisition model prevails within research and in

education settings because of its utility. This model views learning as the acquisition of specific knowledge and concepts. This is commonly seen in all classrooms including the three classrooms observed; all three teachers led each lesson with the goal of supporting students to "acquire" knowledge of specific concepts (e.g. part-whole relations) and skills (e.g. subtraction regrouping). Within the acquisition model of learning, new knowledge is acquired and accumulates through a developmental progression, building from one concept to the next, through teacher facilitation and scaffolding. Conceiving learning as a developmental trajectory provides a useful model for structuring learning (e.g. building students' Cognitive Academic Language Proficiency (CALP), or formal academic language, from their Basic Interpersonal Communication Skills (BICS), or day-to-day language, or the use of concrete-representational-abstract stages of developing understanding of mathematical concepts). The work of Cognitively Guided Instruction have shown the educational benefit in professional developments that deepen teachers' understanding of students' learning trajectories and pedagogical considerations responsive to students' development.

However, as Sfard (1989) has argued, an overemphasis on one metaphor for learning can be dangerous. Conceiving learning only from an acquisition model has often led to the treatment of concepts as its own object of learning and separate from the context in which it is learnt. A widely endorsed teaching practice, and seen in the classrooms of Kassandra and Elise, is the frontloading of the vocabulary before the rest of the mathematics lesson -where language is taught independent from its application. As well, the idea of concepts building upon each other have led to viewing development of language and mathematics learning as linear, to notions that certain vocabulary or mathematics concepts must be taught before students can engage in rich mathematical talk or more challenging mathematical tasks, and that knowledge must transfer

from the teacher to the students. This linear transfer metaphor is widespread within formal education systems, shapes curriculum and pedagogy in the U.S, and was seen in both the classrooms of Kassandra and Elise. Their lesson structure and the progression of lessons followed a gradual release of responsibility in which they slowly decreased the amount of "scaffolding" provided until students were able to acquire the learned action on their own (Duke & Pearson, 2002; Graves & Pearson, 2003; Pearson & Gallagher, 1983). Though Kassandra and Elise taught concepts based on knowledge of students' development, the onus of teaching fell only on them and, therefore, limited the students' repertoires of practices (e.g. linguistic, mathematical, and experiential) utilized in the classroom.

However, just as dangerous is to view learning only as participation. While the acquisition model is strongly entrenched in what happens in individual minds, the participation model views learning as situational, indexically bound to the social context (e.g. Brown et al., 1989; Lave & Wenger, 1991). Cognition and knowing, then, are distributed over the environment, and learning is located in these networks of distributed activities of participation. However, the participation metaphor has been charged with embedding the learning so completely within the given context that the individual is lost, and it is unclear how learning transfers from context to context (e.g. Sfard, 1998; Elkjaer, 2003).

Therefore, both conceptions of learning are needed and a balanced approach can be seen in Laura's classroom. A balanced model of learning that considers both acquisition and participation would view learning as an ecology between the individual and the environment. For example, Laura strategically designed mathematical problems and selected student strategies to help students acquire knowledge of specific concepts and skills based on her own knowledge of students' development in language and mathematics. As well, Laura established a learning

ecology with constructed socio-cultural norms that privileged participation and distributed knowing over both individuals AND the community. The focus of classroom interactions was to develop student's individual understanding through collective sense-making. By doing so, Laura and her students embraced, and built upon, the repertoire of practices available in the classroom. Mathematics lessons were driven by students' own ways of problem solving, and multimodality and language crossing were encouraged and utilized to support meaning making. By valuing diverse voices and perspectives, Laura and her students created a more egalitarian space for learning where students saw themselves in what was being taught. Students who might have been dismissed as less capable in more traditional settings (e.g. developing in school-based discourse, solving problems using direct modeling, and having only partial answers) were recognized by others and saw themselves as an integral part of the learning community (Boaler, 2000; Lampert, et al. 2010; Wenger, 1998).

It is important to connect what was seen in these classrooms to the broader education community. While both models, learning as participation and learning as acquisition, are recognized in the field of research and practice, the acquisition model is the dominant framework for learning within our current research and teaching community. Formal school-based learning often follow the assumption that students should receive and learn what was clearly communicated and explicitly taught by the teacher (Cardenas, et al., 1988; Freire, 1970/2000; Valenzuela, 2002; Weissglass, 2002). And students who cannot and/or refuse to defer their agency and passively receive knowledge are attributed individual blame, labeled as "not a math person," "slow," and "lazy," often marginalizing students of color and emergent bilinguals (Setati, 2008; Gutièrrez, 2013). A participation models reminds us that students' learning as well as teachers' learning to teach is in relation to the environment and the opportunities for learning

provided within such a context. This brings me to the next section to consider the context of teachers' learning in teacher preparation. In the next section, I discuss the implications of this study's findings on teacher preparation.

Implications for Teacher Education

> Because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming – to become a certain person or, conversely, to avoid becoming a certain person. Even the learning that we do entirely by ourselves contributes to making us into a specific kind of person. We accumulate skills and information, not in the abstract as ends in themselves, but in the service of an identity.
>
> (Wenger, 1998, p. 215)

In current U.S. schools, the majority of classroom teachers are not prepared to work with emergent bilinguals (Ballantyne et al., 2008; Chval & Pinnow, 2010; Menken & Antuñez, 2001). With few exceptions, such as in Florida and California, most states are only beginning to require specific mandates for teacher preparation programs (Menken & Antuñez, 2001). When available, preparation is limited to one course devoted to "English language acquisition," learning methods of instruction from an acquisition model focused on linguistic assimilation.

Specially designed academic instruction for English (SDAIE) strategies, grounded in acquisition models of learning, drives preparation courses. These language methodologies incorporate teacher's active modeling and teaching of conversational and academic English through a sheltered English approach (Krashen, 1989). A teacher skilled in sheltered instruction utilizes techniques such as visual aids, modeling, demonstrations, graphic organizing, native language support, and adapted text to make content comprehensible (Echevarraia & Short, 2004). However, the focus is on language acquisition. Focusing solely on supporting students to acquire a second language is a phenomenon Valenzuela (1999) has called "subtractive bilingualism." In the U.S, subtractive bilingualism has permeated most of the programs dealing with bilingual communities (Brown & Souto-Manning, 2007, Crawford, 2004; Cummins, 2000;

Nieto, 2002; Perez, 2004). The acquisition model dominated Kassandra and Elise's teaching, as this was what they experienced as students during their schooling and what was taught in their preparation courses.

I argue that schools and teacher preparation models must move away from a language-acquisition model of learning towards a more balanced framework that considers both learning as acquisition and participation. So, what would it look like for teacher preparation programs to embrace a balanced conception of learning?

Complicating Conceptions of Language–and-Mathematics

First, a dual model complicates views of how bilingual students learn and expands what counts as competence in communicating mathematically. From a pure acquisition perspective, a central developmental assumption for psycholinguistics is that learners proceed from the simple to the complex, from Basic Interpersonal Communication Skills (BICS), or day-to-day language, to Cognitive Academic Language Proficiency (CALP), or formal academic language. Students who do not speak according to a pre-conceived standard form of language are then seen as less competent (García & Gonzalez, 1995; Mosckovich, 2002; 2010). This conception of the development of language and mathematics learning as linear leads to notions that certain vocabulary or mathematics concepts must be taught before students can engage in rich mathematical talk or more challenging mathematical tasks. This has often led to classroom instruction for emergent bilinguals to be watered-down with limited opportunity to communicate mathematically (Fabelo, 2008; Gándara & Contreras, 2009; García, Kleifgen, & Falchi, 2008).

How can teacher educators promote a change in learning the language of mathematics for emergent bilinguals using a dual model? First, this requires an understanding of students' development but to move away from viewing development in learning mathematics (e.g. teacher

to student, concrete to abstract, basic to advanced) and language as linear (e.g. BICS to CALP, Spanish to English, school to home). Mathematics discourse is a system that includes ranges of language practices (school-based, home, playground), and multiple forms (verbal, gestures, concrete objects, and drawings) to communicate and support meaning making. Students' choices of languages (note how I use "languages," as almost all communication requires a hybridity of languages), and selection of mathematics practices is often not a reflection of ability but linked to identity, community, power, and status (Anzaldua, 1986; Barwell, 2003; Setati, 2008). This leads to the next recommendation for teacher preparation.

Reimagining Teacher Preparation

The balanced metaphor to learning requires a reevaluation of the typical model of mathematics teacher preparation that focus solely on teachers' acquisition of knowledge and skills for ambitious teaching. Currently, the mathematics teacher education community has developed a robust knowledge base about the forms of instruction, often called high-leverage practices, that teachers should learn to support students' development of central mathematical ideas (Hiebert & Grouws, 2007; Kilpatric et al, 2003; Stein et al, 2007; Lampert et al, 2010).. However, there is an underlying assumption that these practices are the "right" teaching methods and strategies for all students. This one-size-fits-all model do not consider the sociocultural reality and the Western, assimilationist perspective of learning that shape classroom practice. Without considering participation, power, and identity, instruction that is *ambitious* will not be *equitable* for all learners, particularly for students of color and emergent bilinguals that have historically and are continually underserved by traditional schooling practices.

To support teachers to engage in mathematics instruction that is both *ambitious* and *equitable*, mathematics methods courses cannot solely focus on the development of math content

and pedagogy without consideration of the broader sociopolitical context in which learning takes place. As well, the teacher preparation programming must change. This would mean that one isolated teacher-preparation course (such as "multicultural education," or "English language acquisition)" is insufficient; opportunities for pre-service teachers to analyze, reflect upon, and develop effective mathematics teaching for emergent bilinguals must be purposefully integrated across curricula. Teacher preparation programs must include opportunities for pre-service teachers to learn how to establish and maintain productive learning environments for their emergent bilinguals within the mathematics context.

Pre-service teachers' learning must be situated in practice (Ball & Cohen, 1999; Aguirre et al., 2013; Dale & Cuevas 1992). Pre-service teachers will need opportunities to view and experience effective mathematics learning environments for all students, particularly emergent bilinguals. They will need opportunities to see how rich mathematical tasks can be enhanced to grant access to all students, how teachers can productively support students' dual development of language and mathematics through active participation in mathematical communication, and how emergent bilinguals can flourish when challenged mathematically in and through productive interactions as members of learning community. A dual model of learning would, and should, allow teacher educators, like me, to support teachers like Kassandra, Elise, and Laura to:

- Embed language learning in mathematics rich context;
- Balance opportunities to develop literacy skills within mathematics meaning making and communicative capabilities;
- Scaffold students' understandings and production of academic language in content specific ways;

- Provide unscripted spaces where students can make meaning on their own terms and draw more openly on their full linguistic resources.

Analyzing the Socio-Political Issues in Multilingual Classrooms

Given the political nature of language, pre-service teachers must have opportunities to reflect upon and analyze the dynamics of power in classrooms. To move away from the perception of language as a neutral object, it is necessary to question the various visible and invisible messages that are conveyed to students (who are all language users) through differing representations and the valorizations of certain languages (and language uses). Pre-service teachers need to understand the interplay of mathematics education and the linguistic-social-and-political issues that affect students' academic and identity development. Khisty (2006) reminds us that:

> Education for subordinated groups can mean self-determination, and this is intertwined with empowerment, self-respect, respect for one's history and community. From this perspective, understanding development in mathematics is to understand the relationship of a constellation of sociocontextual factors. Within this constellation is the nature of language use, the resultant discourse community in mathematics classrooms, and students' participation in this discourse community, especially when there is more than one cultural language (p. 438).

Monolingual learners (or classrooms) have continued to serve as the norm. To view cultural and linguistic diversity as resources towards learning, bilinguals need to be described and understood on their own terms and not in comparison to monolingual students (Moscкhovich, 2010).

Classroom interactions are never neutral spaces, communicating as they do to students the value of their language, culture, and identity. As seen in Laura's classroom, her interactions were organized around problem solving, collective sense making, and dialogic communication that built upon student agency and identity. However, Laura's classroom is not the norm. Given the growing linguistic diversity of our student population and their miseducation, much more

attention in teacher education and mathematics education research must be placed on the learning opportunities of bilingual students and bilingual teachers.

Chapter 2

The Interrelation of Reflection and Action

Introduction

Attending and responding to students' mathematical thinking is at the heart of the pedagogical reforms in mathematics education (Common Core Standards Initiative [CCSI], 2011; National Council of Teachers of Mathematics [NCTM], 2000; 2014; National Research Council, 2001, 2007). This vision of instruction has been described as "responsive (Stylianides & Stylianides, 2014), "responsible" (Ball & Forzani, 2011), "inquiry-based" (Alton-Lee, Hunter, Sinnema, & Pulegato-Diggins, 2011), and "ambitious" (Jackson & Cobb, 2010; Lampert, et al., 2010; Thompson, Windschitl, & Braatan, 2013), as it requires a paradigm shift from the teacher-centered, cultural norms that dominate U.S. schooling (Gallimore, 1996; Stigler & Hiebert, 2009).

Instruction that values and respects the mathematical thinking of all students and places students' mathematical thinking and reasoning at the heart of decision-making is both ambitious in agenda and enactment. I, along with others (e.g. Ball & Forzani, 2011; Santagata, 2013; Sherin, Jacobs, & Phillip, 2011), argue that the deliberate and ongoing attention and action teachers must take to move student thinking forward necessitates complex knowledge, skills, and dispositions. As Burton (2004) noted:

> It is not easy to organize a classroom where the mathematics is not prescribed but is generated through the activities of the students and where it is the responsibility of the teacher to help draw the mathematics out of the activities, help the students to interrogate the many different forms of it which they offer, and expect student involvement in the process of questioning, justifying, challenging, and reflecting (p. 372).

Therefore, ambitious teaching carries a unique challenge in that it demands commitment and continual development. The importance of reflection as a mechanism for learning in teacher education is well-documented.

Reflection

Dewey (1910), who was foundational in the exploration of reflection, characterized reflection as a disciplined, conscious, explicit and critical thought process that contributes to the intellectual and moral development of a person (cited in Bailey, 2006). Reflection has found a solid ground in teacher education as a vehicle for professional growth and development (Davis, 2006; Loughran, 2002; Wilson, 2009; Schön, 1983, 1987). Extending from the work of Dewey, Schön (1987) characterizes reflection as reflection–in-action and reflection–on-action. Reflection-in-action guides teachers' in the moment of decision-making, while reflection-on-action includes the planning and reflecting back on one's practice. Schön saw reflection as a vehicle for teachers to build their own knowledge from their teaching. According to Schön (1987), teachers can improve their teaching by analyzing, adapting, and challenging their assumptions through a self-sustaining cycle of reflecting on one's theory and practice, and learning from one situation to inform the next.

Building on Schön's work is the construct of teacher noticing. Classrooms are complex settings. During a lesson, teachers are bombarded with a plethora of information, events, and interactions; teachers need to learn to "see" what matters in a classroom interaction. Recent research advocates teachers explicitly focus on the disciplinary substance of student thinking (e.g., Coffey, Hammer, Levin & Gant, 2011; Franke & Kazemi, 2001; Stein, et al., 1996). Teachers need to be able to determine noteworthy ideas in student work and student talk, and reflect on decisions in order to proceed with their lesson and future lessons (Jacobs, et al., 2010;

Star & Strickland, 2008; van Es & Sherin, 2008). This aspect of "seeing" what is noteworthy during teaching is defined as teacher noticing (Mason, 1994; 2002; Sherin et al., 2011; van Es & Sherin,2009).

Teacher noticing involves two core processes: (1) attending to important elements of instruction; and 2) making sense of what has been noticed (Mason, 2002; Sherin & van Es, 2009). More recently, this work has been extended to noticing and reflection after instruction (Santagata, et al., 2007; Santagata & Angelici, 2010; Shern & van Es, 2009). Building from the construct of teacher noticing, a series of projects have utilized a specific framework, the Lesson Analysis Framework, for teachers to reflect on teaching in systematic ways (Hiebert et al., 2003; Santagata & Angelici, 2010). The framework includes four steps in which the teacher: 1) considers the learning goal(s) of the lesson; 2) examines evidence of student progress towards the learning goal; 3) reflects on the strategies and activities the teacher utilized that helped students to make progress towards the learning goals and those that were not as effective; and 3) lastly, considers alternative strategies to use and/or next steps in the lesson. This four-step process provides a systematic, integrated, and evidence-based analysis of instruction, characteristics identified as central to productive reflections (Dewey, 1933; Davis, 2003; Hiebert et al., 2007; Santagata, et al., 2007).

Teacher Reflection and Teaching Practice

Given the significance placed on reflection in teacher education, what is the role of reflection on practice? What is the relationship between reflection and teaching? Initial studies on the relation between teaching and reflection have focused on expert teachers.

The literature on expert-novice teaching shows that expert teachers have a more complex view of teaching than novice teachers; they see, attend to, and analyze the connections and

relationships in a classroom in more integrated ways (Berliner, 2001; Brown, Collins, & Duguid, 1989; Chi, 2011; Schon, 1983). In comparison, novice teachers find it difficult to veer from a planned lesson to notice and take up student ideas (Berliner, 2001; Franke, Webb, Ing, Freund, & Bailey, 2009; Jacobs, et al., 2010).

As well, there is some evidence of the specific relationship between analysis of teaching and teachers' mathematics instructional practices. Van Es and Sherin (2008) examined teachers' thinking through participation in a video club designed to help experienced elementary teachers learn to notice and interpret students' mathematical thinking. Van Es and Sherin (2008) argue that noticing and interpreting are important skills to use in the midst of instruction to respond to student learning and within the process of productive reflections. Their study found that participation in the video club led to the teachers' increased focus on interpreting students' mathematical thinking. In addition, some of the teachers in the study appeared to slow down their instruction, and ask more questions of their students during instruction.

Santagata and Yeh (2014) examined the impact of a teacher preparation course, the Learning to Learn from Teaching (LLfT), to develop elementary pre-service teachers' ability to learn to systematically analyze teaching practice through video analysis. The LLfT course included a series of activities, using the Lesson Analysis Framework described earlier, for preservice teachers to collectively plan and analyze classroom teaching in relation to evidence of students' learning. Santagata and Yeh examined the relationship between the pre-service teachers' ability to teach in ways that made student thinking visible to their ability to use student-based evidence to assess lesson effectiveness. Findings revealed that elementary pre-service teachers who attended to student thinking during instruction were also more likely to cite mathematics-specific evidence of student learning to comment on their teaching. Additionally,

the teachers who did not draw on student thinking and learning in their evaluation of teaching also did not create opportunities in their lessons for students to express their thinking.

Sun and van Es (2015) also found similar results for secondary pre-service teachers. Sun and van Es compared the mathematics teaching practices of teachers in the LLfT project to a cohort that did not participate in the project. Analysis of the teaching practices between both groups showed the LLfT group that learned to systematically analyze teaching created more opportunities to see, notice, and pursue student thinking during instruction.

There has been an increased attention to teachers' analysis skills to reflect on teaching and its relation to classroom practice. Kersting and colleagues (2012) found that teachers' performance on their classroom video analysis (CVA) survey, an instrument examining teachers' analysis abilities, was positively related to students' learning. Santagata and Yeh's (2016) longitudinal study of three novice teachers' processes to attend, elaborate, and propose improvements in teaching through analysis of their CVA responses and interview data revealed that revolving these processes around student thinking and learning led to moving their students' and the teachers' own learning forward. As well, ZDM Mathematics Education (2016) recently published a special issue on the perception, interpretation, and decision-making process in relation to classroom teaching.

These studies highlight the role of noticing and interpreting as central elements of instruction and reflection. They offer a variety of ways to think about and conduct interventions that facilitate teachers' development of productive reflection, analysis of teaching, and mathematics teaching responsive to students, and specifically the possible relation between reflective practices that systematically analyze teaching and pre-service teachers' mathematics instructional practices. However, the majority of these studies examined development in

reflection and teaching as teachers participated in some form of professional development, the field of teacher education lack information about the role of systematic reflection and teaching when novice teachers engage in the daily work of teaching. Given the high focus in teacher education on reflection, particularly as a vehicle for teacher learning, it is important to examine the relation between teaching and reflection in novice teachers after graduation from teacher preparation. Specifically, what is the relation between reflection and teaching, and its role in teacher learning over time?

Study Purpose

This study examines the relation between reflection and teaching in two novice teachers, and how it may contribute to their teacher development over time. Specifically, I examine, in detail, the mathematics teaching practices and the teaching reflections of two novice teachers, Faith and Elise, over a two-year period. Both Faith and Elise were part of a larger project that examined the short- and long-term impact of a mathematics method course on the teaching practices of beginning elementary teachers (Santagata & Yeh, 2014; Yeh & Santagata, 2015; Santagata, Yeh, & Mercado, under review). In the initial analysis of their classroom practices, I found both Faith and Elise used similar mathematics teaching practices near the end of their teacher preparation. However, their teaching by the end of their second years looked quite different. Given their different trajectories, they provided an ideal situation to examine the relation between teaching and reflection, and its possible contribution to their professional development over the two-year period. This study seeks to answer the following questions:

1) How does each teacher's mathematics teaching and lesson reflection change over time?
2) What is the relationship between their mathematics teaching and their lesson reflections, and how might that contribute their development over time?

The Intersection of Systematic Reflection and Ambitious Instruction

I used a common set of dimensions to characterize Elise and Faith's mathematics teaching and lesson reflections: 1) rigor of mathematical tasks; 2) student thinking and understanding; and 3) mathematical discourse. Although the three dimensions alone do not capture the complexity and multidimensionality of teachers' decision-making during mathematics instruction and reflection, they represent important aspects of mathematics instruction as recommended by the National Council of Teachers of Mathematics (NCTM, 2000; 2006; 2014), and the National Research Council (NCR, 2001; 2007; Kilpatrick, Swafford, & Findell, 2001). All three dimensions are associated with student achievement but could differ from teacher to teacher, even given the same curriculum (Hiebert et al., 1997; Stein, Smith, & Henningsen, 2009). What follows is a brief description of the rationale for the selection of each of the three instructional dimensions.

Rigor of mathematics tasks. Rigor of mathematics tasks has been shown to have implications for students' learning, as well as students' motivation, their perceptions of what mathematics is, and what it means to be successful (Hiebert & Wearne, 1993; Middleton & Spanias, 1999; Mueller, Yankelewitz, & Maher, 2011; Sengupta-Irving & Enyedy, 2015; Stein et al, 2009). The study focus on the rigor the task encompasses means the degree to which the task is cognitively demanding and requires students to problem solve and to draw upon their prior knowledge. These facets of mathematics tasks have been deemed "worthwhile" by NCTM and other mathematics education researchers as the rigor of math tasks chosen and posed to students lays the foundation for students' learning opportunities and teachers' opportunities to gauge

students' mathematics understanding (Boaler & Staples, 2008; Hiebert & Wearne, 1993; Stein, Grover, & Henningsen, 1996).

Student thinking. The study focus on student thinking encompasses the degree to which student thinking is made visible, how the teacher attends to, pursues, and builds on students' thinking to inform instruction, and how student thinking is used to reason about the effectiveness of instruction. The teachers' attention to and reasoning about students' thinking to inform instructional decision are central components of reform-based instruction (CCSI, 2011; NCTM, 2000; 2014; NRC, 2001, 2007; Franke, et al., 2007; Silver & Stein, 1996), as well as characteristics of productive reflection and analysis of teaching (Hiebert & Morris, 2012; Santagata, et al., 2007).

Mathematical discourse. Research indicates that engaging students in mathematical discourse can lead to increased student knowledge and understanding (Franke et al, 2001; Michaels, O'Connor, & Resnick 2008; Moschkovich, 2007; Yackel & Cobb, 1996). This dimension examines the quality of classroom discourse: students' opportunity to engage in mathematics communication (verbal and non-verbal) and discussions that deepen their conceptual understanding through explanation and critique of each others' reasoning.

Participants

In this set of case studies, I examine the teaching practices and the lesson reflections of two novice elementary mathematics teachers, Elise and Faith, both graduates of the same teacher preparation program. When I examined their prior experiences and teaching contexts, the two were comparable. Elise and Faith received their undergraduate degrees in the social sciences and entered a teacher preparation program right after completion of their undergraduate degrees. Both had years of experience before entering the teacher preparation program working in

classroom settings as a tutor and teacher's assistant. Their student teaching during and professional teaching after teacher preparation occurred in schools serving linguistically- and culturally- rich student populations. At the time of hire, the mathematics instruction at both sites was governed by district pacing plans, quarterly assessments, and district-mandated curriculum, and both teachers used mathematics textbooks from the same publishing company.

Specifically, Elise and Faith were selected for the study as their mathematics teaching practices looked very similar near the end of their teacher preparation. Observation of their videos from the Performance Assessment for California Teachers (PACT) Teaching Event, a portfolio assessment that measures pre-service teachers' ability to plan, enact, assess, and reflect on a lesson sequence, revealed similar teaching practices. Elise taught a lesson on creating a story problem from a multiplication equation, while Faith taught a lesson on finding the perimeter of common geometric shapes.

Both teachers utilized the same instructional sequence. They first modeled the procedures on the overhead projector, guided students in practice, then followed with students' independent practice of mathematical tasks. The classroom discourse observed in the videos consisted of the traditional three-part I.R.E. classroom discourse sequence (Cazden, 1988): the teacher would initiate a closed question, a student would be called on to respond using "equity sticks" (a student name is written on each popsicle stick, and selection of students is based on the stick drawn), and the response is evaluated by the teacher. Student responses were limited to final answers, and, at times, the teacher-determined steps to arrive at an answer (e.g. the steps to create a word problem from a multiplication equation or steps to find the perimeter for a common geometric shape).

While both teachers used similar teaching practices, they varied in their ability to attend

to student's mathematical thinking in videos of math instruction. As part of the larger project they participated in, they completed the classroom video analysis (CVA) survey. The CVA assessment examined teachers' ability to attend to and elaborate on students' mathematical thinking and learning as made evident in the video clips observed (Kersting, 2008). Participants were asked to watch ten 1-3 minute video clips of classroom instruction on the topic of whole number operations and rational numbers. At the end of teacher preparation, Faith's CVA responses displayed greater attention to and analysis of students' mathematics thinking than Elise's. Of the ten CVA responses, half of Faith's responses made direct reference to students' thinking and learning and analyzed in depth the mathematics at the basis of the lesson. In comparison, Elise's responses only described the students' mathematics thinking made evident in the clip, and she made no reference to student thinking in three of her CVA responses.

Data Sources

The analysis presented for this study relies on videotaped mathematics lessons and lesson reflections gathered during their first and second years of teaching after graduation from teacher preparation. I visited Elise and Faith at six time points during a two-year period: the beginning, middle, and the last month of each school year. During each visit, I observed and videotaped their mathematics lessons. Lessons ranged from 50 to 75 minutes in length.

After each observed mathematics lesson, the teachers were asked to reflect on the lesson just observed. Videotaped interview reflections ranged from 30 to 80 minutes in length. The interviews were semi-structured; four questions were posed during each interview: 1) What was the main learning goal of this lesson?; 2) How did it go? What was surprising? What worked as planned? What didn't?; 3) What would you do differently if you were to teach the lesson again?; and 4) What did you learn from teaching this lesson?

Data Analysis

Data analysis occurred in two phases related to the two research questions. The first phase examined each teacher's mathematics teaching and lesson reflection over time. The second phase examined the relationship between teaching and reflection, and how it might contribute to their development over time.

Phase One

The first question sought to examine changes in mathematics teaching and lesson reflections over time. I used the set of dimensions described earlier to characterize Elise and Faith's mathematics teaching and lesson reflections: 1) rigor of mathematical tasks; 2) student thinking and understanding; and 3) mathematical discourse.

The data analysis for this question occurred in three phases, and the process was both deductive and inductive in nature (Boyatzis, 1998; Fereday & Muir-Cochrane, 2006). This first stage was deductive in nature as I drew upon existing literature in relation to the three dimensions. To examine the rigor of mathematics tasks, I drew on the taxonomy of mathematics tasks developed by Stein and colleagues (Stein, et al., 1996; Stein & Lane, 1996; Stein & Smith, 1998) to develop the coding scheme. For the student thinking dimension, I built upon a prior coding scheme used in an earlier study (Santagata & Yeh, 2014) that examined the teachers' attention to student thinking in their lesson reflections and how student thinking was made visible and pursued during instruction. For the dimension of mathematical discourse, I drew upon the Math Talk Community framework developed by Hufferd-Ackles and colleagues (2004).

The first phase included analysis of the mathematics lessons, beginning with a random selection of two lessons from each teacher. First, I watched each taped lesson and read the

transcripts to identify evidence of each of the dimensions (cognitive demand, student thinking, and mathematical discourse) in the data. Focusing on one dimension at a time, each lesson was read to identify places demonstrating evidence of that dimension, which I annotated the instances with emergent codes and preliminary comments. Then, each instance of the dimension was reread, the data coded, previous categories refined, and other sub-categories identified (Strauss, 1987). The same process was repeated for each of the three dimensions. Using this recursive process, each dimension was operationalized.

Then, a second researcher used the coding scheme to identify places in the data discussing the dimension in the lesson, and annotated each instance in the data with preliminary comments. We came together after analysis of each data source to check for consistency and to refine codes. This process was repeated for one fourth of the data to operationalize each dimension first for the videotaped lesson. A coding framework to analyze the mathematics lessons was developed consisting of the three dimensions – rigor of task, student thinking, and mathematical discourse – and connected to a 3-point rubric scale (see Table 1).

Once a coding framework was developed, I returned to the entire data set and coded all 12 videotaped lessons holistically. Each lesson received a score of 1 through 3 in each of the three categories: rigor of task, student thinking, and mathematical discourse. A sample of 25% of the lessons were randomly selected and coded by another researcher. Each researcher reviewed the data independently and met after to discuss the scores. Percent agreement for the three categories – rigor of task, student thinking, and mathematical discourse--were 83%, 100%, and 100%, respectively. The rating scale captured both the frequency (e.g. how often the teacher enacted these high-leverage practices in their teaching) and reflection to reformed-based teaching (see Table 1).

Table 1

Coding Framework for Videotaped Lessons

Category	1	2	3
Rigor of Math Task	Students primarily receive, recite, or perform routine procedures without analysis or connection to underlying concepts of mathematical structure.	Knowledge is treated unevenly during instruction. Deep understanding of some mathematical concepts is countered by superficial understanding of some other ideas. At least one idea may be presented in depth, but in general the focus is not sustained.	At least half of the lesson includes task(s) that require close analysis of procedures and concepts, involves complex mathematical thinking, utilizes multiple representations AND demands explanation/justification
Student Thinking	Student thinking is only minimally visible. The focus of student thinking is on the correctness of answer. Students may provide short answer-focused responses. The teacher rarely solicits student explanations, and these are limited to procedural steps.	Student thinking is made visible. The teacher elicits students' thinking beyond the answer, and begins to ask how or why they arrived at the answer.	The teacher elicits students' thinking and builds on their responses. The teacher may press for additional explanation beyond the initial how and why, pose alternate examples/questions for students to think about, or draw other students' attention to students' ideas.
Mathematical Discourse	Mathematical discourse and communication (can be verbal or non-verbal) are limited. Only the teacher is talking, or discourse generally focus only on the Initiate-Response-Evaluate (IRE) sequence.	Students are given some opportunities to talk to a partner, show their thinking (verbally or nonverbally) (e.g. whiteboard, base-ten blocks, worksheet with problem-solving strategies, physical response), but student talk of mathematics in not in rigorous ways (debate, explanation, communication, reasoning, and making generalizations).	There are multiple opportunities for students to engage in mathematical discourse and communicate in rigorous ways (debate, explanation, communication, reasoning, and making generalizations) and show their significance to mathematical understanding.

The second phase included an analysis of the lesson reflection interviews. The mathematics lesson coding framework was used as the starting block in developing the reflection coding framework to provide alignment between the two coding frameworks. The same process – of identifying segments of the interview data that referenced the three categories and creating a coding framework that both draws on prior research and captures the variances seen in the data – described in the lesson analysis was applied to the lesson reflections. A coding framework to analyze the lesson reflection interview transcript was developed consisting of the same three categories – rigor of math task, student thinking, and mathematical discourse – and also connected to a 3-point rubric scale. The rating scale captures both the frequency (e.g. how often the teacher attended to this category during the lesson reflection) and reflection of ambitious mathematics teaching (see Table 2).

Once the reflection coding framework was developed, I returned to the entire data set and coded all 12 transcribed reflection interviews. A sample of 25% of the reflection interviews were randomly selected and coded by another researcher. Each lesson reflection received a score of 1 through 3 in each of the three dimensions: cognitive demand, student thinking, and mathematical discourse. Percent agreement for the three categories – rigor of task, student thinking, and mathematical discourse--were 83%, 83%, and 91%, respectively. For the third phase, matrices were constructed to examine patterns between teachers' lesson reflections and mathematics lessons and across time.

Table 2

Coding Framework for Lesson Reflections

Category	1	2	3
Rigor of Math Task	Teacher's discussion of the lesson goal, task design, and its implementation is on students receiving, reciting, or memorizing procedures and definitions. There is no talk of conceptual understanding, or opportunity for mathematical analysis or exploration.	Teachers' discussion of the lesson goal, task design, and its implementation contains some mention of conceptual understanding or opportunity for mathematical analysis/exploration, but there is substantial discussion of students receiving, reciting, or memorizing facts, procedures, and definitions.	The majority of the discussion of the lesson goal, task design, and its implementation focus on conceptual understanding and opportunities for students to analyze procedures and concepts, engage in complex mathematical thinking, utilize representations, and demands explanation/justification.
Student Thinking	Little to no mention of student thinking/ understanding, or evidence used does not accurately gauge student thinking/understanding (e.g. speed, completion of problems assigned, being quiet).	Mentions student thinking/understanding, but the justification/evidence is inconsistent. Some comments are grounded in evidence and others are not.	Talks in depth about student thinking and understanding generally with justification and evidence. The eliciting and building from students' thinking is explicitly stated. Teacher talks about next steps, or changes to future lessons based on what was learned from student-based evidence.
Mathematical Discourse	Discussion of mathematical discourse and communication (verbal or nonverbal) generally focus only on the answer, or the correctness of the solution method.	Discussion of mathematical discourse and communication (verbal or nonverbal) is discussed as being important. The teacher talks about getting students to partner talk and show their thinking (verbally or nonverbally); however, there is no discussion about talking about mathematics in rigorous ways (e.g. debate, explanation, communication, reasoning, and making generalization), and their significance to mathematical understanding	Discussion (verbal and non-verbal) of strategic opportunities for students to engage in mathematical discourse and communication are linked to the mathematics of focus. The role of mathematical discourse and communication is central to mathematics learning. Student-to-student discourse is discussed as central (e.g. encouraging students to ask questions to each other, listen closely to responses of peers).

Phase Two

The second research question sought to understand the relation between each teacher's teaching practice and lesson reflections and to consider how they might contribute to their development over time. While the rubric scores helped to capture the teachers' development over time, they did not provide insight into the relationship between teaching and reflection and how they might contribute to teacher learning. Thus, I explored the nature and substance of each teacher's lesson reflections (what aspect of the classroom lesson did teachers notice) and then referenced back to the videotaped mathematics lessons to identify what aspects of the lesson they noticed (and did not notice). To do this, each of the data sources was subjected to a vertical analysis (Miles & Huberman, 1994); in other words, they were examined independently and across time. A second phase was then carried out through a comparative analysis (Miles & Huberman, 1984) between the lesson reflection and videotaped mathematics lessons. Third, a horizontal analysis (Miles & Huberman, 1994) was carried out in which the two cases were compared

In the first phase, four steps were followed. Drawing on prior literature (Yeh & Santagata, 2014; van Es & Sherin; 2008), I reread each interview, and memos were created to summarize what the teachers noticed (i.e., math thinking, pedagogy, climate, and classroom management), and how they talked about what they noticed (descriptive, evaluative, and interpretative). Second, units of analysis were created based on topic segmentation (i.e., discussion of lesson goal, effectiveness of lesson, changes to be made in future lessons, lesson learned) for each lesson reflection. Third, each unit segment was analyzed in terms of what the teacher noticed (e.g. students' thinking, pedagogy, classroom climate, classroom management), and how they discussed each topic in relation to their interpretation of the lesson (what

interpretation and decision-making are discussed). Throughout the analysis process, constant comparative analysis (Glaser & Straus, 1968) was used, coded segments were compared to each other, and coding revisited several times to account for the richness of the data and my own understanding of what was emerging in the analysis. Then, summaries were written for each data across each interview – highlighting what they noticed, and their interpretation and decision-making in relation to each lesson.

In the second phase, I went back to identify what aspects of the lesson the teacher noticed in their lesson reflection (also noting what they did not notice). This was done for each mathematics lesson. Summaries were also written for each data –highlighting what they noticed and what they didn't, and pulling illustrative lesson vignettes and reflection quotes.

In the last phase, a comparative or horizontal analysis (Miles & Huberman, 1994) was carried out in which the individual case themes were compared across the two cases. For this part of the analysis, much of the data were coded and themes identified by one person, and the analysis then discussed with colleagues. This process allowed for consistency in the methods but failed to provide multiple perspectives from a variety of people with varying expertise. To ensure the trustworthiness of my study (Anfara, Brown, & Mangione, 2002), I shared regularly my coding and analysis with colleagues knowledgeable about qualitative research. As well, I conducted member checks to ensure I accurately represented their personal and classroom narratives.

Findings and Discussions

In this section, the results of the analysis for the two case study participants, Elise and Faith, are presented in two sections. The first section presents the scores each teacher obtained from my analysis in the dimensions of rigor of math tasks, student thinking, and mathematical

discourse for their mathematics teaching and lesson reflection trajectory over time. The second section uses vignettes of their lessons and excerpts from their reflections to illustrate the relation between teaching and reflection, and how that might contribute to their development over time.

Examining Change in Teaching and Reflection Over Time

How does each teacher's mathematics teaching and lesson reflection change over time? Findings from the analysis of the mathematics teaching and lesson reflections using the three-point rubrics were used to examine patterns in their development over the two-year period. Both Elise and Faith showed some improvement in their teaching over time, yet had very different patterns of development. I first present findings on Elise's mathematics teaching and lesson reflection trajectories in the dimensions of rigor of the task, student thinking, and mathematical discourse. Then, findings for Faith's mathematics teaching and lesson reflection trajectories, using the same dimensions, are shared.

Elise

Rigor of mathematics. Figure 1 captures the rigor of the mathematics in Elise's teaching and lesson reflections over the six time points. Elise showed improvement from year two, visit one to year two, visit two in the rigor of the mathematics tasks implemented during her lessons, but the score dropped again at the last visit. This means that throughout the six visits, five of the six lessons observed asked students to primarily recite or perform routine procedures prescribed by Elise without opportunity for students to engage in their own analysis of the underlying reasoning behind the procedure. The rigor of mathematics score for Elise's lesson reflections began higher than her teaching, but aligned at year two, visit two. In five of the six lesson reflections, knowledge was treated unevenly, meaning Elise discussed the importance of teaching mathematics for understanding, but her descriptions of the lesson activities included

both high and low tasks that were often contradictory. Elise talked about posing activities that use tools and representations to make connections that promote understanding, but Elise decided which tools, representations, and pathways for students to use. In her reflections, there was never mention of engaging students in analysis of tasks, in cognitive struggle, or exploration of problem solving and invented strategies.

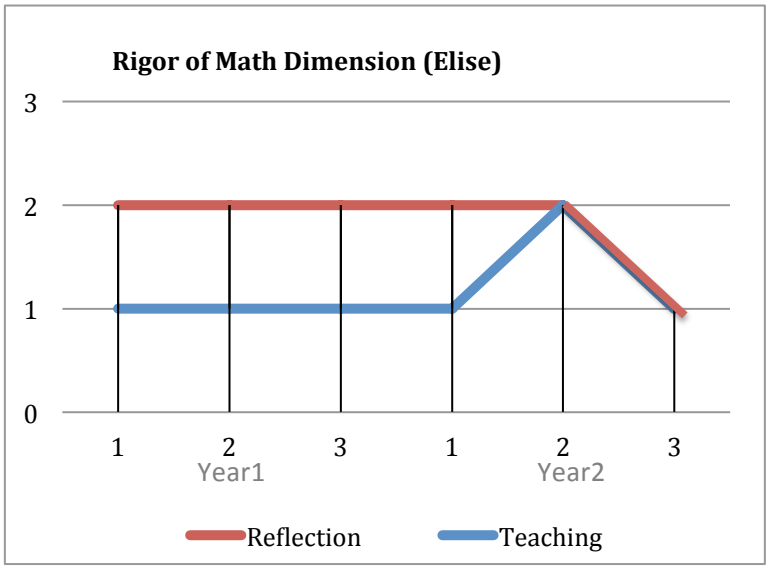

Figure 1. Elise's scores on the rigor of math dimension

Student thinking. Elise received the same score in the student thinking dimension for the mathematics lessons and teaching reflections in her first year, meaning Elise rarely elicited student thinking beyond just the answer in the three visits made during the first year, and her lesson reflections consisted of very little mention of student thinking. In year two, there was a shift first in her teaching when she began to elicit student thinking beyond just the answer, asking how students arrived at an answer, or why. In year two, visit two, her lesson reflection score caught up to her teaching when she began to talk about her students' thinking/understanding, at times grounding her analysis of their understanding with descriptions of how students solved problems, and at other times citing evidence that would not be revealing

of student learning (e.g. speed, completion of problems assigned, being quiet). Figure 2 captures Elise's scores in the student thinking dimension for her mathematics teaching and lesson reflections over time.

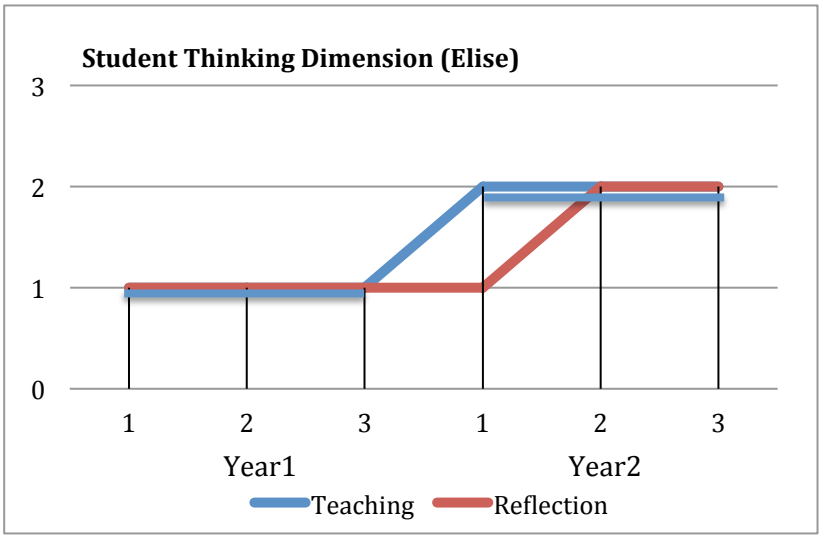

Figure 2. Elise's scores on the student thinking dimension

Mathematical discourse. Elise received the same score in the mathematical discourse dimension for her teaching and lesson reflection throughout the six visits, meaning that students were given opportunities to talk with a partner or show their work (verbally or nonverbally) using math manipulatives or tools during instruction. As well, mathematical discourse and communication were discussed as important in her lesson reflections. However, student talk of the mathematics observed during instruction and discussed during the reflection focused on sharing answers. Discursive practices to foster shared understanding of mathematical ideas through analysis and comparison of student approaches and arguments were not seen in her teaching or lesson reflections. Figure 3 captures Elise's scores for the math discourse dimension for her mathematics teaching and lesson reflections over time.

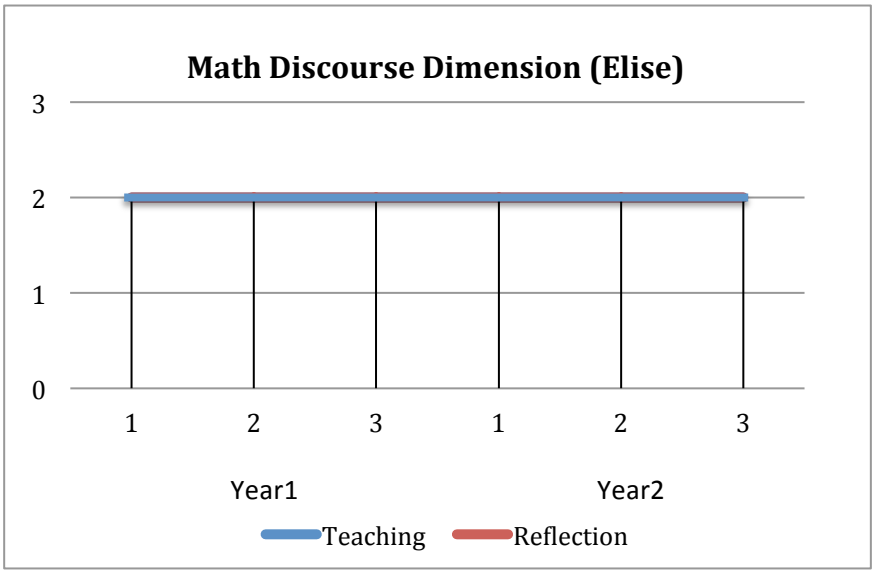

Figure 3. Elise's scores on the math discourse dimension.

Rigor of mathematics. Figure 4 captures the rigor of the mathematics in Faith's teaching and lesson reflections over the six time points. Faith's mathematics teaching received scores of 1 during her first year, and the rigor of the mathematics increased in year two when the instruction shifts from the students performing routine procedures without analysis to students engaging in tasks that require analysis in year two. Faith began with higher scores in her lesson reflections than her teaching. During year one, there was little focus on conceptual understanding in her teaching while understanding of mathematics concepts was consistently discussed as important in her reflections. During year one, Faith, like Elise, described lesson activities that included both high and low tasks; Faith talked about posing activities that use tools and representation to make connections to promote understanding, but Faith determined which tools, representations, and pathways for students to use.

Like her teaching, Faith's lesson reflection scores increased in year two, meaning she talked explicitly about the importance of allowing students opportunities to engage in more activities requiring more complex mathematical thinking: analyzing procedures and concepts,

coming up with students' own representations and strategies, and demanding explanations and justifications of their reasoning.

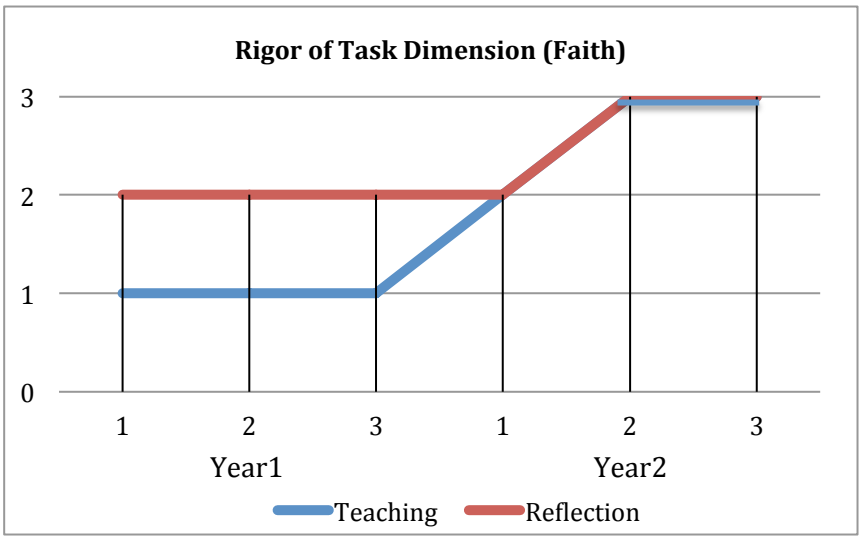

Figure 4. Faith's scores on the rigor of math task dimension.

Student thinking. Faith's score in her mathematics instruction remains the same until an increase at year two, visit two, when her instruction shifted from an elicitation of student thinking (prior to year two, visit two) to building on from student responses where she pressed students for additional explanation beyond the initial how and why or drew other students' attention to the sharer's ideas. For her lesson reflections, Faith received high scores for five of the six visits. In those five lesson reflections, Faith based her analysis of the effectiveness of her instructional strategies on students' understanding of the lesson goals, and provided specific examples of student actions that served as evidence of student understanding or struggle. As well, her lesson reflections consistently discussed next steps or changes to future lessons based on her analysis of student progress. Figure 5 captures Faith's mathematics teaching and lesson reflections over time.

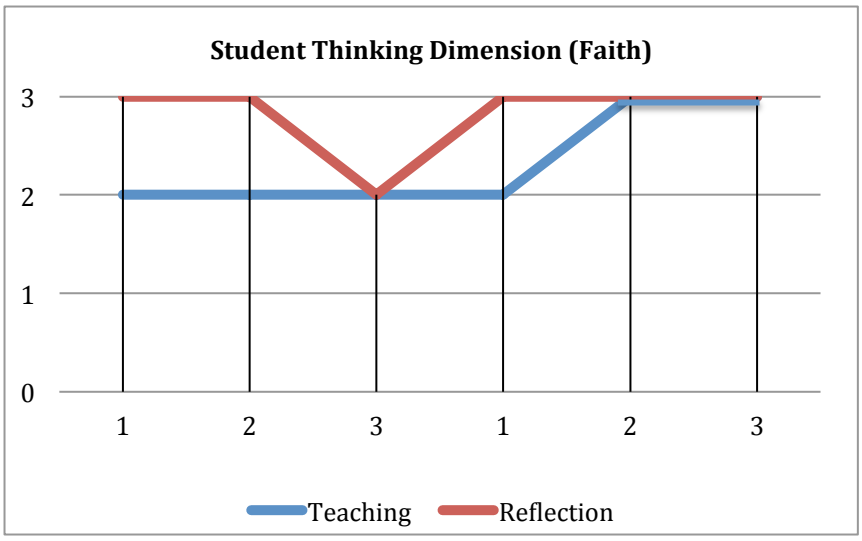

Figure 5. Faith's scores on the student thinking dimension.

Mathematical discourse. Faith received the same score on the mathematical discourse dimension for her mathematics teaching and lesson reflections in her first year (see Figure 6). This means that students were given opportunities to talk with a partner or show their work (verbally or nonverbally) using math manipulatives or tools during instruction, as well as mathematical discourse and communication was discussed as important in her lesson reflections. However, student mathematical talk observed during instruction and discussed during the reflection were answer-driven. In year two, there was a shift, first in her lesson reflection in which discursive practices of analysis and comparison of student approaches and arguments were discussed as central to mathematics learning. In year two, visit two, her teaching score caught up to her reflection score when her instruction also provided opportunities for students to engage in debate, explanation, communication of reasoning, and generalizations of strategies and concepts.

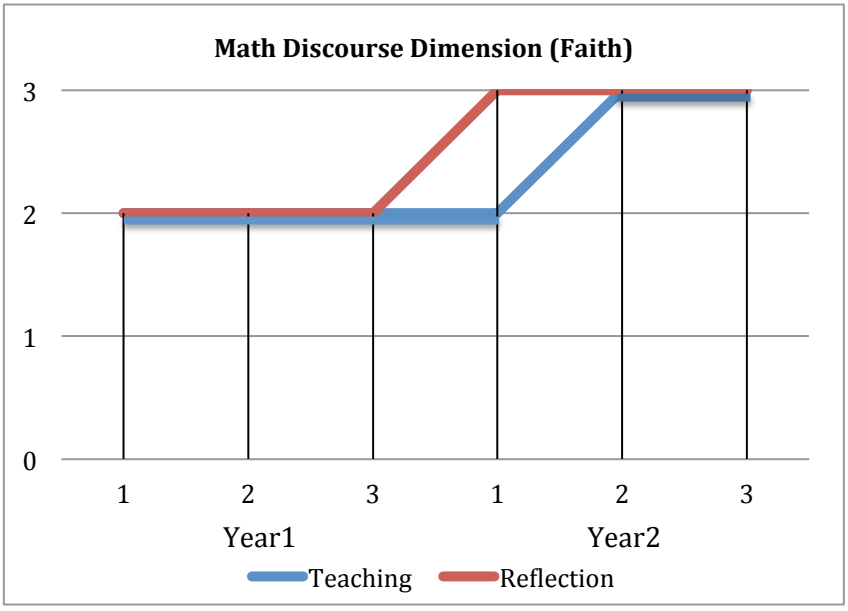

Figure 6. Faith's scores on the math discourse dimension.

Comparison of the learning-to-teach trajectory of Elise and Faith. Elise and Faith's instructional practices and lesson reflections during the first year were representative of what has been identified as typical of the culture of schooling in the U.S. (Barrettt et al., 2002; Stigler & Hiebert, 1999). Both teachers expressed a commitment to teaching mathematics with understanding but taught in ways where the source of problem solving and mathematical ideas came from them. The teacher and her mathematical ideas were the currency of classroom activity while the students' role was to rehearse and memorize the series of steps prescribed. While both Elise and Faith engaged students in mathematical communication, the talk remained surface level, focused on procedural accuracy or the correctness of answers.

The only difference between Elise and Faith during year one was their attention to student thinking in their teaching and lesson reflections. Neither teachers positioned student strategies and ideas as the objects of inquiry; however, there were distinct differences in their attention to student thinking. While Elise provided little opportunity for student thinking to

emerge during classroom activities or used interpretations of student thinking to reflect upon instructional effectiveness, Faith consistently made space to elicit student thinking in her instruction (though she never took up the student ideas) and used information about her students to determine what worked in her lesson and what needed to be improved.

In year two, both teachers showed some growth and greater alignment between their teaching and reflection. However, Faith showed much greater improvement. By the end of year two, Faith's teaching and reflection echoed a vision of teaching aligned with reformed-based initiatives: cognitively rigorous, discursively rich, and focused on students' reasoning and sense making.

Relationship Between Teaching and Reflection, and Contributions to Development Over Time

As discussed in the previous section, Elise and Faith ended up in different places in their teaching by the ends of their second year. What was the relationship between their mathematics teaching practices and their lesson reflections, and how might it contribute to their development over time? To answer the second research question, I use vignettes and interview quotes to illustrate their different trajectories and highlight how this may have occurred. I begin with a description of the relations between Elise's teaching and reflection over time and follow with a description of Faith's trajectory for comparison.

Elise

The first year. After graduation, Elise was hired to teach second grade at the same school where she had student taught. Her master teacher during the field component of teacher preparation became her grade-level colleague. Throughout the six visits, Elise's lessons followed a two-part structure. At the start of the lesson, the class convened in a meeting area for a warm-

up activity during which students engaged kinesthetically or visually with the mathematics content (e.g. finding geometric shapes around the class, choral counting with the hundreds chart, calendar work). The warm-up activity was followed by the "student work period."

The "student work period" was different in structure than the directed instruction seen during her student teaching. Elise did not begin the work period modeling a procedure or skill for students to practice. Instead, Elise passed out the math worksheets so the students could work on problems independently. In a lesson reflection, Elise stated that she strategically provided time for "*students to solve the worksheet problems on their own*." She didn't want students to "*just rehearse previously taught methods*," as she wanted students "*to be able to wrestle with the mathematics on their own first.*" . After a few minutes of individual work, the class convened to discuss the solution, and Elise modeled step-by-step the correct way to solve the problem on the overhead. Then, students worked independently on the remaining packet problems for the rest of the lesson.

To illustrate the type of classroom interactions and lesson reflections typical during Elise's first year, I use vignettes and reflection quotes from a lesson taught four months into her first year. This lesson focused on comparing two or three-digit numbers using place-value understanding. In her reflection, Elise wrote that the instructional goal was to address a common struggle she had seen in her class:

> *My learning goal was for [the students] to work with greater than and lesser than … I wanted them to know the place value of the numbers because there are some students that still feel like one number is bigger … confusing the hundreds, tens, and ones. That's where they are having trouble.*

Elise had planned to start the lesson with a human number-line activity to help students "*know the place for the numbers*" as well as provide opportunities to practice the academic language: "*I*

try to incorporate [the academic language] in the number line [activity] 'the smaller, bigger, between and after' because those are the words that the kids don't understand."

In the activity, Elise gave each student an index card with a number written, and the students were asked to place themselves in order from least to greatest in relation to the magnitude of the other students' numbers. What follows is a brief episode of the twenty-minute activity:

T: (Elise hands an index card to Krystal.): *What number do you have?*

S1: (Krystal stands up and faces the lined up students.*): 518*

T*: Where are you going to go? (*Krystal still stands in her spot. 5 second silence.) *Between Bella and Delanie or between Alicia and Bella?*

S2*: Bella and Delanie.*

T*: It's Krystal's turn. Where are you going to go, between whom will you go?*

S1*: Next to Delanie.*

T*: Why? Because your number is bigger or smaller than Delanie's?*

S1*: My number is bigger than Delanie's.*

T*: Bigger than Delanie's or smaller?*

S1*: Smaller.*

T*: Smaller, very good. Go more over there.* (Elise hands a card to another student.) *Chase, what number is this?*

S1*: 909*

T*: Where are you going? Is it bigger or smaller than Delanie's?*

S1*: Smaller*

The above scenario is representative of the type of activities and discourse practices typical during Elise's first year of teaching. Elise's mathematics lessons showed her

commitment to creating learning experiences that were engaging, hands-on, and grounded in students' mathematical understanding. Lessons observed included a substantial amount of kinesthetic activity with physical response as well as student talk. Elise's own talk consisted mostly of questions, often beginning with an open-ended prompt. As seen in the scenario above, students were asked to explain where they would go on the number line and why. The initial prompt of asking students to determine their position along a number line had the potential to engage students in reasoning about magnitude, place value, and relational thinking.

Each one of Elise's teaching moves was in relation to her students' thinking and acting. However, examination of Elise's sequence of questioning revealed that the classroom discourse patterns were quite representative of what is typically seen in U.S. schooling (Franke & Kazemi, 2001; Stigler & Hiebert, 1999). Her question sequence often funneled student responses to a specific procedure or answer. While she often pressed students for a justification of "*Why? Why there?,"* Elise would reduce the complexity of the problem to two choices, "*It's bigger or smaller?*" to alleviate student discomfort which in turn lowered the mathematical struggle and rigor and limited space for student reasoning to surface.

This, in turn, limited Elise's own access to whether, or how, students were making sense of mathematics. Elise's instruction during the first year provided little space to observe student thinking. In general, Elise gave very little evidence of student ideas or understanding to guide her own thinking, either in the moment of teaching or retrospectively in her reflections.

Elise's lesson reflections during the first year revealed her struggle to manage the complex setting of classroom teaching. She discussed the multiple demands, goals, and concerns – classroom management, student motivation and interest, developing conceptual understanding, and completion of student work –that influenced how she planned and implemented her lessons.

In each reflection, Elise demonstrated a commitment to teaching mathematics with understanding *("I wanted them to know the place value..")*, to creating instruction that built upon their current understanding ("*some students that still feel like one number is bigger ... confusing the hundreds, tens, and ones")*, and to developing students' language for mathematical communication ("*try to incorporate [the academic language] in the number line [activity] 'the smaller, bigger, between and after")*.

While Elise attended to the range of features that define ambitious instruction, her reflections were absent of descriptions of the students' problem-solving methods, thinking, or reasoning as it unfolded during lessons. When asked to reflect on the success of her lessons, without reference to students' engagement with, and learning of, the mathematics to guide her thinking, Elise's analysis was limited to her ability to complete the lesson as planned. As an example, Elise's response to the effectiveness of the comparing numbers lesson described earlier, Elise stated:

> *"It went well. I had people finishing but others didn't, but they are getting a little bit better. It's just practice. A lot of them were able to finish the packets."*

Absent of descriptions of the students' thinking and reasoning, Elise's gauge of student learning often relied on students' ability to complete the assigned task. In all three lesson reflections during the first year, Elise talked about completing assignments as a measure of lesson effectiveness, and when asked about what she would do differently based on what was learned, Elise stated:

> *"I think one of my things is giving them more of, I think this is kind of boring for them. They get tired. So I would want to give more fun activities for them to do but with learning. Last time, I did like a little bingo game with comparing numbers and they had fun, and after they did that they finished their (worksheet) really quickly."*

Dewey (1933) described reflective thought as "active, persistent, and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends" (p. 9); therefore, reflection is a process that informs future action and is grounded by evidence. Elise's reflections showed clear respect and attention for her students as learners; however, Elise's analysis of her teaching was often not based on the most accurate gauge of students' learning (e.g. classroom behavior, student interest, completion of math packets). A possible reason for the evidence used could relate back to her own classroom instruction. Elise's sequence of questioning reduced student discomfort and student response choices and limited her ability to gather evidence of her students' understanding. Without evidence of students' mathematics understanding as a reference point, Elise's analysis of instruction and decisions for possible next steps were limited to other aspects of the classroom interaction. Elise's suggestions for lesson improvement were always based on students' engagement with the math content. However, without understanding of students' engagement in the mathematics, providing "fun activities" for students was a continual focus throughout her first two years.

The second year. Elise's journey throughout her second year was marked by attempts to create a more student-centered learning environment. With new state standards, Elise explained during the interview that the school had adopted a new textbook with a heavier focus on context-based mathematics problems, math talks, and visual math models. Her local district offered a

series of professional developments, and Elise eagerly attended workshops on CGI[4] instruction, FactsWise [5], and math talk activities (i.e., choral counting, number strings, counting collections).

In her second year, Elise's mathematics consisted of more partnership work, math projects, and use of visual and kinesthetic mathematics models. In the lessons observed during the second year, Elise incorporated many of the mathematics activities learned from her math professional developments. Her students engaged in choral counting activities, did "mental math" composing and decomposing numbers in different ways, and rotated between math centers. While the mathematics activities were more hands-on, involved more student talk, and shifted away from the worksheet packets, the mathematical tasks were often limited to building procedural fluency rather than conceptual understanding. Choral counting activities and number talks discussed the procedures in nature with very little discussion of the reasoning behind it. To illustrate, I explain a lesson observed three months into the second year. Part of a two-day activity, the class was split up into groups where they rotated to one of the four centers every 30 minutes. Elise described the lesson goals as:

> *"Each group had a different goal: basic facts with the iPads, counting numbers from twenty to two hundred, and then the other [center] works on forming tens. That's where a lot of kids were missing the foundation of how to form tens in different ways. So, last week they did the counters. So, this time, they're going to use the abacus. So each day and each time, a different tool. With me, the comparing [word] problems. The different comparing problems have the different missing parts."*

Elise's center worked on comparing word problems with missing unknowns, and the other three centers on counting activities and games to develop basic facts fluency. What follows is a brief scenario highlighting the interactions that took place in Elise's group:

[4] Cognitively Guided Instruction (CGI) is a professional development program based on an integrated program of research focused on (a) the development of students' mathematical thinking; and (b) instruction that influences that development (Carpenter et al., 1996; 2015).
[5] FactsWise is a research-based approach to teaching basic facts fluency that prioritizes 5s, 10s, and part-whole thinking (Common Core Level 3 thinking) (FactsWise, http://ellipsismath.blogspot.com/p/factswise.html)

(The student worksheet is projected on the overhead.)
T: *Ok, now you are going to do number 4.*

(Elise points to the worksheet projected on the overhead to the word problem "There are 16 birds on the tree. Nine red birds fly away. Then, four crows come to the tree. How many birds are on the tree now.")

T: *Okay, show me 16.*

*(*Elise models and connects together 16 unifix cubes, then, waits for the students in the group to do the same.)

T: *What happens after? 9 red birds leave. What happens here?*

S1: *Take 9 away.*

T: *Okay, take 9 away*. (Elise takes away 9 unifix cubes from the 16 and waits for the students to do the same.) *Now, 4 crows come to the tree. What do we do?*

S1: *We add 4 more.*

T: *We add 4 more because they come to the tree. How many birds do we have in the tree now?*

S2: *14*

T: *How many do you have? (*points to a student)

S3: *11*

T: *How do you know that it's 11?*

S3: *I counted. There's 7, and I added four more: 8, 9, 10, 11* (counting as she stacks up four more unifix cubes from the original stack of 7).

T*: okay. Yes.*

During the small group interaction, Elise broke down the word problem into discrete parts for analysis. Each student physically modeled the steps of the word problem using their own set of manipulatives. However, questions were limited to a particular way of problem solving instead of the mathematical reasoning. At times, students were called on to explain their thinking. However, Elise, like many U.S. teachers, did not know how to approach wrong answers

(Santagata, 2004; Santagata & Bray, 2015). Wrong answers were often ignored, and the correct answers pursued. As seen above, Elise did not ask the student with the answer of 11 to explain her thinking but asked the student with the correct response to articulate how she arrived at that answer. While classroom interactions focused on procedural skills, the smaller group structure and Elise's elicitation of "hows" provided more opportunity for students to show their thinking and problem solving process, verbally and nonverbally.

Aligned with more space during instruction for Elise to see and hear student thinking, Elise's reflections also showed greater attention to students' engagement with the mathematics content. For example, in discussing the student who had given the response of 11 above, Elise stated:

> *"With Maya [who stated 11], I think that she was getting there, but she's still struggling with when to add or subtract, without saying what the problem needs her to know. So, I think, she's still struggling with some ideas but she's getting better at modeling."*

During the visits made in the second year, Elise's reflections continued to attend to features of instruction that new teachers tend to notice (Sherin et al., 2011; Star & Strickland, 2008; van E & Sherin, 2008), including speed ("*most completed the front and back*"), behavior ("*they didn't throw the blocks today*"), and making math fun ("*They really enjoyed it. They don't see it's math related but it is*)" However, her reflection also included more attention tp student thinking than the year prior. Student strategies and their implications for understanding began appearing in her lesson reflections. Elise talked more explicitly about how students were solving problems and used these descriptions to justify her assessment of the lesson along with the other considerations.

Faith

The first year. Faith taught fourth grade in a large urban school district in Southern California. Faith's lessons also followed a consistent structure throughout her first year teaching. The lesson observation we discuss here took place also in the fourth month into Faith's first year of teaching. In this lesson, students were learning to "*convert a mixed number into an improper fraction*" through an understanding of their equivalence. The mathematics lesson began with a word problem on fraction equivalence displayed on the document camera. Students were given five minutes to solve and show their work independently on individual whiteboards. As students worked, Faith circulated the room, repeatedly emphasizing: "*Solve the problem in multiple ways. Show your thinking*." Then, the class would convene for a discussion. What follows is an excerpt of the discussion:

(Faith is sitting in the front of the room, next to the document camera, facing the students. The mathematics problem is projected on the screen.)

T: *Alright, let me see. Let's see your work. I'm seeing a lot of visuals.* (Students lift their white boards to their chest or over their heads.) *I see numbers, great. And I really appreciate students who are showing me their thinking in more ways than one. Excellent. Very nice. Okay, my friends at table 1, I need to see yours. Let me see your board again. Come on, let's go. Even if you're not done, that's okay. It's alright, Juan, even if you're not done. It's okay. I want to see your thinking. Aright, great. Boards down.*

T: *Alright, let's talk about this. Melissa and Jennifer went apple picking. Melissa picked ½ of the bushel. Jennifer picked 4/8 of a bushel. Melissa says that they picked the same amount, but Jennifer says that she picked more. Which girl is correct? And explain how we know.*

(A few students raise their hands. Faith, places a blank white sheet of paper under the document camera.)

T*: Alright, let's take a look at this. I'm going to say Melissa is going to be blue, and for me, I like to use bars. They're just a little easier for me. I don't draw perfect circles.* (Faith draws a blue rectangle on the paper.) *Melissa will be blue, Jennifer will be black.* (Faith draws another rectangle of the same size in black next to the blue rectangle.) *Okay, so I notice that my first fraction says that she has half of a bushel, 1 over 2,* (Faith writes ½ under the blue rectangle*) 4 over 8* (Faith write 4/8 under the black rectangle). *Alright, well, for me, when I look at my fractions, I am going to first look at the denominator I want to know how*

many total pieces it takes to make 1 whole here. (Points to the blue rectangle with her marker and then partitions the rectangle into two equal parts vertically.) *So, 2 whole pieces, I know my numerator tells me I'm just taking about one of them* (colors one of the two partitioned parts blue*). So, for the second one, how many total pieces do I need to divide it into? How many total equal pieces?* (Faith pulls a popsicle stick from a jar.) *Lanette*?

S1: *Eight*.

T: *Eight. And what's our vocabulary word for our bottom here?* (Pulls another popsicle stick from the jar.) *Raul?*

S2: *Eight?*

T: *Eight. Our vocabulary word.*

S2: *Ohh, no.*

T: *Do you need a second to think about it?*

S2: *Yes.*

T: *Alright. Call on someone to help you out.*

S2: *Julian.*

S3: *Eighths?*

T: *Eighths. I do like that. Julian is telling us that these are separated into eighths. You got the "th" at the end, great! What else is it called? Sally?*

S4: *Ummm.*

T: *Look at your math dictionary, see Samantha is using her tool right there. Go ahead Samantha.*

S5: *Denominator.*

T: *Denominator. Right, my denominator here says eight*. (Points to the 8 in the 4/8 displayed on the screen.) *So, I'm going to separate it into eight equal pieces. Let's do that.* (Partitions the black rectangle quickly into eight equal pieces first by cutting vertically into fourths and the fourths in half vertically again). *I know that my numerator here says 4* (shades in a partitioned eighths one at a time as she counts out loud) *– 1,2,3, and 4.*

The above episode is representative of whole-class discussions in Faith's classroom during the first year. Faith began the lesson displaying a word problem on the projected screen.

Students would be given independent time to solve the problem on their own and encouraged to show their thinking and to solve in multiple ways. As students worked independently, Faith walked around asking probing questions of individual students, indicating a focus on student thinking. Then, when the whole class discussion began, the agency of the learning and teaching shifted from the students to Faith. Faith would demonstrate a specific procedure to solve the problem step-by-step. As in Elise's class, Faith gave much of the explanation and reasoning while students' verbal responses were limited to closed questions (e.g. answers to the next step of a procedure "*eight,*" and academic terms "*denominator*"). The whole class discussion would then be followed by students' independent practice of a few problems assigned from the district-assigned textbook. (Both Faith and Elise used mathematics textbooks from the same publishing company.).

During their first years, both Elise and Faith implemented lessons in teacher-centered ways where the discussion focused on teacher-prescribed procedures and through directive, leading questions. What was different between the two teachers was Faith's elicitation of student thinking. In each of Faith's mathematics lessons, Faith repeatedly asked students to show their thinking on paper or on the white board before starting the class discussion. While Faith made room for student thinking to be made visible during instruction, her class discussions never followed up and built on from student strategies. The strategy explicitly taught was Faith's own problem solving and reasoning. While Faith was not able to build on and extend student thinking in her teaching, Faith's lesson reflections showed great attention to her students' thinking and reasoning.

I use Faith's reflection from the same lesson described above to highlight some typical features. Here, Faith was asked to reflect on the effectiveness of the lesson:

"Um, not so well. Only because the students didn't reach the goal. They weren't getting [it]. Even yesterday when we were doing equivalent fractions, they were still a little iffy on the conceptual understanding. Because I know that when I brought in the procedural thinking, of, whatever, you know, times 2 times 2, divide 5 divide 5. You know, they understood that in the way that I was directly telling them, this is the strategy, or this is the way to do it. But, when asking to explain or even talk about showing it in different ways, they had a bit of trouble."

While Faith's lesson reflections were peppered with concerns that plague many first year teachers (e.g. classroom management, feeling overwhelmed, learning to navigate new standards and district-assigned curricula), Faith's analysis of her teaching effectiveness was based on the students and their progression towards the lesson's mathematical goal. In this lesson, the goal was for students to understand that mixed numbers and improper fractions are equivalent fractions. In reflecting on the lesson, Faith described specific instances during which she assessed students' thinking during the instruction:

> "[I was] just walking and monitoring; I saw a lot of them were trying and some of them were showing their thinking, it wasn't quite there yet. Because when I was asking questions, like Randy and a few other students, they were very hesitant. They couldn't tell me how the two were related….I was trying to bring back that prior knowledge of – mixed numbers are equal to improper numbers, you know, to see that relationship. They couldn't make that connection with equivalence. "

Right after this comment, Faith went on to describe the instructional decisions that may have impacted her students' inability to see the two fraction notations as equivalent: the way in which she represented the fraction visually ("*with the whole in black, and the part in blue"*) that led to student confusion. She again cited specific aspects of student talk as evidence:

> *"They said, 'Alex – they're not equal. Because Alex ate four and Randel ate 1.' And I would say 'Okay, I noticed that you're comparing Randel and Alex and what they ate, but that's not what we're doing, we're combining it together. We counted them in different ways.' So I think that was the disconnect, is that they say that – they weren't sure what to separate and what to combine. And because I was making two comparisons in a way… I think that was where they got confused. "*

The excerpt above exemplifies Faith's attention to student thinking and the use of evidence of her students to analyze her own teaching. This attention to student thinking was

present in every lesson reflection. In addition, Faith consistently used her students' understanding of the math concept to reflect upon her lesson effectiveness and to consider changes in her instruction. During the first year, every suggestion for lesson improvement was based on what she could have done to better explain and model mathematical concepts for her students. In the second year, Faith shifted focus from the importance of her explanations of mathematical ideas to her students' explanations.

The second year. In the second year, Faith, like Elise, tried to shift to "*a new way of doing math*" that aligned with the demands of the new state standards. Faith described the type of teaching required in the new standards as "*more student-centered and having the teacher be more of a facilitator… stepping back from the directed teaching part and having students defend their answers, reason their answers, and look at strategies in multiple ways using different tools.*" Faith's district also offered a series of professional developments, and Faith took part in a monthly CGI workshop for four months.

Faith's teaching also shifted during the second year. During the first visit, Faith continued to teach in a similar manner to the year prior. The students were given a double-digit multiplication word problem to solve using multiple strategies. Faith called on different students to explain their strategies and would then demonstrate step-by-step the procedures to solve the problem. However, near the end of the class discussion, a student shared a strategy Faith had not thought of. In visits prior, Faith only built upon her own strategies. Here, Faith asked the student to explain his strategy as she tried to represent it on the overhead:

(Faith is sitting in the front of the room, next to the document camera, facing the students. The mathematics problem is projected on the screen.)

S1: *I have a different strategy.*

T: *Christian? What strategy did you use?*

S1: *I used um, I used tens and ones*.

T: *The tens and ones, so, which, like, can you give me some of the steps you did?*

S1: *Twenty-seven, I put two tens and seven ones; for sixty I put six tens and (*Faith interrupts*.)*

T: *Okay, so six tens for sixty,* (Faith writes "6 x 10 = 60" on the paper projected by the document camera) *and then for twenty-seven what did you do*?

S1: I *put two tens – seven ones.*

T: (Faith writes 2 x 10 + 7 = 27 on the paper). *Okay. Two tens and seven ones, okay. So did you put it in, in boxes, or did you put it more like, regrouping? Which one? … Can I have a little help because I don't understand what you - I need a little help guiding me through it.* (A student has his hand raised). *I think some of your classmates are understanding what you're saying, can we ask them? (*Faith looks at Christian, and Christian nods*.) Yeah? Okay. Alan?*

T: Can you try to explain what Christian is saying?

S2: Oh, he's trying to say like you break apart the numbers into tens blocks and ones units.

T: Okay so units and tens. So I know that here are my tens blocks, tens here, and seven, okay. (Faith begins to draw base-ten blocks of ten rods on the paper.) Like this, Christian? Okay, and I have six tens blocks, right? (Faith draws six ten rods.) So what do I do with them? What do I do with them? Christian?

The short exchange above illustrates the first time during the visits when Faith actively pursued a student strategy, and her intent was to make sense of and accurately represent the student's thinking. Faith even asked the student if her interpretation was accurate, ("*Like this, Christian? I have six tens blocks right?*). This instructional move then invited Christian to explain his strategy further. During the lesson reflection, Faith made reference to this exchange with Christian and Alan as what she had learned from her lesson:

> *"Christopher's strategy, I totally didn't expect that at all. I mean I was improvising up there, hoping like, he could, he could lead me. I know that at the beginning of the year, had I heard that I might have said, okay that's a great one, but we haven't learned that yet or we're not learning that today, but I wanted to see where he was going. And I love that Alan, he was there, he goes, 'I think I know what he's saying, like, let me try.' I loved that collaboration even when they weren't partners. I could tell he was listening, and he was also*

trying to help us figure out. Okay we're using tools to use a new strategy that actually we haven't even talked about, because we already know these other strategies, we might take those same, that same skill or same knowledge to apply them to a new one and just see it differently. So I really, I really love that, I thought that was something that was a good takeaway."

By the next visit, Faith had made distinct shifts in how her lesson was structured and how students came to understand concepts. Unlike the lessons observed prior, the students were not sitting at their individual desks but together on the carpet facing Faith as she sat by an easel with chart paper. Faith began with a class choral read of the word problem: "*Luke Skywalker rode his land speeder 0.75 miles in the morning and 1.10 miles in the afternoon. Use a model to show how many miles Luke rode all together.*" Faith then led a discussion about the contextual and mathematical ideas in the word problem (e.g., Faith showed the class a photo of a land speeder and asked students to give their own meaning of the word "model" in context of the problem), then, discussing with the class the norms and expectations for partnership work ("*Show your partner that you're listening. When you're looking at your partner and you're listening, what are some words that you can say when they're talking*? *Or when you have a question? A disagreement?")*

During partnership work, Faith walked around asking students specific questions in relation to how they represented the problem and taking notes on student approaches (the annotation of student strategies started the second year). After about 10 minutes, the class convened. Faith spent the remaining 35 minutes strategically calling on student pairs to share as she documented their strategies on a poster sheet.

T: *Okay, so, I'd like to hear some from partnerships on what you guys did using models and to explain your thinking. Explain your thinking. Jace?*

S1: *Sam and I draw a number line.*

T: *Drew a number line. Okay. Hold on*. (Faith spoke out loud as she wrote " Jace and Claud's work. Drew a number line" on the chart paper.)

S2*: Yes, we drew a number line.*

T*: You drew a number line, which is absolutely a model to show your thinking.* (Faith draws a line next to "Jace and Claud's work" written on the chart paper.) *Okay, now what?*

S2*: Um, we drew a line on it.*

S1*: We drew a line um. Right there.* (Points to the number line.)

T*: Ok where's right there? So tell me when to stop.* (Faith points her marker at the start of the number line and slowly moves over the line until the student tells her when to stop and draw the hash mark.)

S1*: Stop.*

T*: Ok here*? (Faith draws a small vertical line, a hash mark, at the half way mark of the number line.)

S1&2*: Yeah.*

T*: Ok.* (Faith continues to move to the right over the number line.)

S2*: And then right there.*

T*: Ok*. (Faith draws another hash mark on the line.)

S2*: And then I wrote a 1.10.*
T: I think we may be skipping a step. Where did you get that from (points to the first part of the segmented number line)? *What do I do with this model?*

In the vignette above, the two students were representing the two distances traveled (0.75 miles and 1.10 miles) on the number line. Faith had the students explain where to place the indicated mark on the number line. She explained during the interview that she strategically asked students to tell her "*where to stop on the line*" to gauge their understanding of the magnitude of each distance on the number line. Faith intentionally asked the students ("*Where did you get that from? What do I do with this model?*) to press the class to articulate how features of the visual model connected back to the problem asked. For the remaining class session, three

more student pairs shared their strategies, some with partial answers and estimated responses. The increased focus on student reasoning was evident in Faith's lesson reflections as well.

During Faith's second year, she consistently focused on student thinking and used student evidence to consider the lesson effectiveness and next steps. However, the focus shifted from an analysis of how students solved problems to how they were able to provide evidence of their reasoning. In response to the effectiveness of the lesson shown above, Faith stated:

> *"I think it went well. I saw, the evidence from what I heard, their body language, their work in itself, the poster…. They were able to talk about what they were doing, and I think there was a lot of evidence on evidence. They were able to really tell me, I got this number from this sentence or this word. Before, they would just pick numbers and do something with it…And now to make that connection for models and visuals."*

Another shift in the lesson reflections from year one was the focus of Faith's suggestions for lesson improvement. In the first year, Faith discussed specific pedagogical strategies to better explain and model concepts for her students. In the second year, her suggestions shifted from how she could better explain concepts to how she could encourage students to better explain their thinking. For example, Faith's lesson suggestion for the land speeder problem was:

> *"[I would add to the problem] to explain more. Explain with models and words… put it in words so if your partner is reading it, they'll be able to see everything that you've done. You can really explain what your numbers and pictures mean."*

By the end of year two, Faith's inquiry stance towards student thinking in her reflection was seen in her instruction. At the start of the year, as in the vignette with Christian above, Faith began to take up students' unsolicited ideas. By the second visit, the focus was on students' cognitive work instead of her own guided instruction. Faith not only elicited student ideas but pressed students to elaborate on their thinking with justification and explanation.

Accounting for Differences Between Faith and Elise's Teaching Practice and Reflection Over Time

Mathematics education reform calls for mathematics instruction that is flexible, adapting one's teaching in the midst of instruction based on attention to student ideas that arise in the midst of instruction (NCTM, 2000, 2014). Faith and Elise began their professional teaching careers with mathematics teaching practices typical of what is seen in new teachers and representative of the culture of teaching commonly seen in U.S. schools (Barrett et al., 2002; Britzman, 1991; Raymond, 1997; Stigler & Hiebert, 1999). At the start of their professional teaching, Faith and Elise made few adaptations in their instruction as the lessons unfolded and generally taught their lesson as planned. Students' sense making and problem solving were not the driving force of instruction; the teachers, their strategies and reasoning, were the currency of classroom activity. However, by the end of the second year, there were clear differences in their learning-to-teach trajectories. What led to these changes?

There are many possible factors that can lead to teacher change: the school context, professional development opportunities, and change in new curriculum and tests (Cooney, 2001; Fennema & Nelson, 1997; Tirosh & Graeber, 2003). Faith and Elise experienced all three. They just started their professional careers, were adapting to new standards, and took part in CGI professional development workshops. While both grew in their practice through time, Faith made a more significant change in her practice. What may have been the motivation for Faith's change? What opportunities facilitated teacher learning and led to a change in teaching over time?

While Faith and Elise's teaching looked similar at the onset of their professional teaching, there were clear differences between their lesson reflections. Elise's reflections were

often descriptive rather than analytical and revealed the wide range of features of classroom interactions (e.g. classroom management, student engagement, student learning) Elise noticed to reason about her teaching. Faith, even from the start of her professional teaching, centered her inquiry on the students. In every lesson reflection, Faith analyzed her teaching systemically. Analysis of classroom interactions was in relation to evidence of students' learning of the lesson goals and then considered the teacher's actions contributing to them. Decisions for instructional change and next steps were grounded in her own sense making and reasoning of student thinking. Her students' mathematical understanding influenced her analysis of her teaching and considerations for next steps. Even before graduation, Faith showed a stronger inclination to attend to and reason about student thinking than Elise (as seen in their CVA scores), and this attention to student thinking in her own teaching was seen after graduation.

Dewey (1933) described reflection as a meaning-making process that moves a learner from one experience into the next with deeper understanding through systematic, disciplined, scientific inquiry into practice. Specifically, for mathematics teaching, Stockero (2008) defined productive reflection as "analyzing classroom events… to identify often subtle differences in students' mathematical understandings and the ways… the teachers' actions contributed to them" (p. 374-375). Faith's lesson reflections and later Lisa's embodied the characteristics of productive reflection: systematic, disciplined attention to student thinking, and analysis of instruction based on evidenced-based reasoning. In relation to the Lesson Analysis Framework (Hiebert et al, 2003; 2007, Santagata, et al, 2007), and Sherin's noticing framework (Sherin, 2011; van Es & Sherin, 2009), Faith's reflections from the onset of professional teaching identified what was important in her lessons – students' engagement with the mathematics-- which she then used to determine students' understanding, the effectiveness of instruction, and

considerations for next steps. This process of collecting data from instruction to evaluate students' learning to inform instruction became an ongoing cycle for assessing, revising, and making change in instruction.

However, reflection alone seemed not enough. Schön (1987) talked about the connections between reflection–in-action (the decisions made in the midst of teaching) and reflection–on- action (the looking back on practice after the event). The noticing of students' thinking in both reflection AND teaching appear to play a critical part in teacher learning. On the surface, Faith and Elise's teaching practices looked similar at the onset. A closer examination of the nature of classroom interactions revealed differences in Elise and Faith's attention to student thinking, even from the first visit. Faith repeatedly provided "check points" at the beginning of her lesson and during independent student work to assess students' progress, though she did not build upon student ideas in her class discussion until the second year. In comparison, during the first year, Elise often began with an open-ended question but often intervened, limiting student response choices and her access to whether or how students were making sense of the mathematics. Without evidence of her students' thinking as a reference point, Elise could only ground her lesson analysis on superficial aspects of students' learning—following the lesson as planned, student interest, motivation, and speed in completion of work. In year two, Elise's increased attention to student thinking during instruction was followed with teaching reflections that also leveraged student thinking to inform her own learning.

Sun and van Es (2015) and Santagata & Yeh (2013) also found that teachers who attended to student thinking in their analysis of teaching also enacted teaching practices that made space to assess student thinking. The relation between teaching and reflection can be seen in Faith's development through time and later Elise's development in year two. Given Faith's

development during the two years, attention to student thinking in teaching and reflection seems to be a powerful mechanism for teacher learning. However, it is important to note that it took almost two years of elicitation before Faith was able to build on and respond to her students' thinking in ways that allowed student ideas to guide instruction. Faith's struggle to build on students' responses is a common one. Franke and colleagues (2009) have found that even notice teachers readily elicited an initial student explanation but found it difficult to follow up and pursue student thinking in ways that supported students' learning (Franke, Fennema, Carpenter, Ansell, & Behrend, 1998; Franke, Webb, Chan, Ing, Freund, & Battey, 2009).

These in-the-moment decisions about how to respond to students' verbal or written explanations have been described by Philipp and colleagues (2013) as professional noticing of children's mathematical thinking, which extends beyond Sherin's (2011) definition of noticing. Phillip and colleagues extend Sherin's construct of noticing to consist of three distinct but related subskills: (a) attending to children's strategies, (b) interpreting children's understanding, and (c) deciding how to respond on the basis of children's understanding (Jacobs, et al., 2010; Jacobs et al., 2011). It seemed for Faith during her first year and Elise during her second that they were able to attend to and perhaps interpret student thinking but struggled on how to respond.

Research suggests that the sort of impromptu response to student thinking requires deep and flexible knowledge of the discipline, close attention to students' ideas, and an understanding of the terrain of students' development in order to engage in decision-making around student's thinking (Ball & Cohen, 1999; Hill et al., 2008; Franke & Kazemi, 2001). It is very likely that Elise and Faith had very little opportunity to listen to students' mathematical explanations prior to their teaching to develop enough of an understanding about students' thinking and the content to know what to do with what they heard. An important contribution from the study findings is

how teachers' attention to students' thinking during instruction and reflection can serve as a vehicle for teachers to deepen their knowledge of the mathematics, the pedagogy, and of their students. Just as Faith stated:

> *... it was a lot of experimenting. I think that's the best way I can describe it. Because there were times when I would try something new and it would work so well and I would feel so confident and like, yes I'm doing something right, you know, I can see it, my evidence is right in front of me, it's in my kids. And then there would be days where I'll try something and it's just, a complete disaster. So it was a lot of experimenting. And um, a lot of learning from them [the students]* (Faith, year 1, visit 3)

Conclusion

"A great deal of learning would be required for most teachers to be able to do the kind of teaching and produce the kind of student learning that reformers envision for none of it is simple. This kind of teaching and learning would require that teachers become series learners in and around their practice, rather than amassing strategies and activities."

(Ball & Cohen, 1999, p. 5)

Recent research in teacher education has examined ways to best support teachers to develop ambitious teaching practices. There is shared consensus that this work is "ambitious," and that mastery of ambitious instruction by graduation isn't feasible (Ball & Cohen, 1999; Grossman, Hammerness, & McDonald, 2009; Santagata & Guarino, 2011). Some have focused on the work of enactment – the development of beginning teachers' competencies in high-leverage practices (Grossman, et al., 2009; Lampert et al., 2010) that occur with high frequency in lessons, while others have focused on learning from teaching, the analytic skills to observe and reflect on teaching that could lead to generative growth (Hiebert & Morris, 2012; Santagata, et al., 2007).

The study findings highlight the significance of the relationship between the enactment (of high leverage practices) and the reflection (through systematic analysis) of ambitious teaching. Both approaches deconstruct the complexity of teaching to important features of instruction. However, it is not the discrete skills and routines of practice that are important, but

how they come together to support student learning and teachers' learning to teach. As Jansen, Grossman, and Westbroek (2015) stated, "recomposition of practices" is critical (p. 142).

The study findings suggest that centering teaching and reflection on students' mathematical thinking had the potential to serve as a vehicle to support teachers' development of pedagogy, mathematics, and student understanding. Therefore, noticing of student thinking is at the core of the enactment of ambitious teaching and productive reflections (Lampert, et al., 2010; Santagata, et al., 2007; Sun & van Es, 2015;Thompson, et al, 2013). An important next step would be to identify the principles of designing teacher preparation to develop pre-service teachers' noticing skills in their enactment and reflection of ambitious teaching.

As well, our findings highlight the importance of longitudinal studies that follow pre-service teachers from teacher preparation to their school context to understand their learning-to-teach process. There are currently too few studies investigating how our novice teachers navigate the complex task of teaching after graduation. I argue that studies like this that require researchers to look and listen closely to the experiences and voices of our novice teachers are important so teacher preparation can be more responsive to our prospective teachers' learning.

Chapter 3

Conclusion

Goals and Results of this Study

This dissertation focuses on teacher practice, and builds on the conception that classrooms can serve as an important place of learning for students and teachers. The central goal of this dissertation was two-fold: to better understand the teaching experiences of novice elementary mathematics teachers and to use what was learned to consider how to better support pre-service teachers' learning during teacher preparation. As such, this dissertation longitudinally examined novice elementary teachers and their mathematics teaching practices during their first two years of professional teaching. Specifically, the two studies examined how the classroom ecology, co-constructed by the teacher and students, provided opportunities to engage in classroom mathematics that support students' learning (study 1) and teachers' learning (study 2). Broadly, the overall research questions for this dissertation were:

- What are the mathematics teaching trajectories of elementary school teachers during their first two years of professional teaching?
- What can we learn from their teaching to inform our work as mathematics teacher-educator researchers?

For the remaining part of the conclusion, I discuss the findings as they relate to the two broader research questions.

Research Question 1: Mathematics Teaching Trajectories of Novice Elementary Teachers

What are the mathematics teaching trajectories of elementary school teachers during their first two years of professional teaching? Both studies closely examined the mathematics

teaching practices of the four elementary school teachers. Below I provide a summary of each study in relation to this research question.

The first paper: Math is More Than Numbers. This first study, Math is More Than Numbers, examined how three novice bilingual teachers, Laura, Elise, and Kassandra organized mathematics learning for their emergent bilinguals. Specifically, I examined the opportunities for learning provided by each teacher in relation to: (a) the types of support provided to develop students' learning of the language of mathematics; (b) who was the knowledge authority in the classroom community; and (c) how, and what, students' repertories of practices were utilized.

First of all, the study findings confirm prior research advocating for a diversified teacher workforce; that teachers sharing linguistic and cultural backgrounds with their students may be better at building cultural bridges to learning (e.g., Achinstein & Ogawa, 2008; Achinstein, Ogawa, Sexton, & Freitas, 2010; Villegas & Lucas, 2002). All three teachers created a learning ecology that value students' linguistic, cultural, and experiential knowledge. Their mathematics lessons often connected to students' lived experiences, used multimodal approaches for communication, and encouraged students to use their native language, English or Spanish, in class. While similar language teaching practices were used, their classroom interactions around these similar strategies revealed differing orientations towards learning language and mathematics.

Laura created a learning ecology in which students developed language through individual and collective engagement in mathematics. Therefore, learning the language of mathematics was not the end product, but part-and-parcel with the mathematics learning. Laura's students developed their mathematics reasoning, in, through, and with language. As the class engaged in collective sense making, Laura's classroom privileged participation and

distributed knowing by building upon the repertories of practices (e.g. ways of knowing and problem-solving, everyday language, Spanish, English, school-based discourse, and everyday experiences) available in the classroom. During visits, mathematics lessons were driven by students' own ways of problem solving, and "half-baked ideas," multimodality, and language crossing were encouraged and leveraged to support meaning making. Laura's classroom ecology embodied a balanced conception of learning as both acquisition and participation in which students' individual understanding and the collective groups' meaning making were valued and utilized to promote students' learning of the language of mathematics. .

In contrast, the classroom interactions of Kassandra and Elise embodied an acquisition model of learning. In their classrooms, discrete knowledge (e.g. learning specific mathematical terms) and skills (e.g. subtraction regrouping) were explicitly taught by the teacher to the students. The lesson structure and teaching of concepts flowed from the concrete to abstract, from teacher-supported to student-appropriated, following a linear progression of development. As concepts and the mathematics language were taught to students, the students had little opportunity to develop their own strategies and meaning for solving mathematical problems. Therefore, students' opportunities to communicate mathematically were limited to final answers, use of prescribed vocabulary and terms, and an appropriation of the school-based discourse modeled by the teacher.

The study findings illustrate the complexity of bilingual teaching in the context of supporting students in learning the language of mathematics. Teachers' conceptions of learning affect their pedagogy and what is learned in their classrooms. As well, the findings point to the cautionary tale of the assumption of the cultural match (Achinstein & Aguirre, 2008; Achinstein & Ogawa, 2011). A collection of studies highlighted by Achinstein and Ogawa (2011) reported

that linguistically and culturally diverse teachers, when compared with white, monolingual teachers, positively impacted the achievement, attendance, retention, and college-going rate of students of linguistic and cultural diversity. However, Achinstein and Ogawa (2011) also found that the reality of the cultural match was far from ideal. The assumption of cultural match often downplays the significance of prior school experiences, and leads to the assumption that linguistically-diverse teachers are immune to dominant discourses of mathematics and language.

Educational language politics of assimilation, as well as rote-based teaching, have historically plagued the learning experiences of emergent bilinguals and are most likely the type of schooling bilingual teachers have experienced themselves as students (Chapa, & De La Rosa, 2006; Chval & Pinnow, 2010; Cuevas, 1984; García, et al., 2008; Moschkovich, 2012). Given this, is it fair to expect bilingual teachers to teach language-and-mathematics meaningfully if they have never experienced it themselves as students and have received very little teacher preparation to do this type of work (Chapa, & De La Rosa, 2006; Chval & Pinnow, 2010; 2008; Menken & Antuñez, 2001)? This study's findings and others show that while recruitment of bilingual teachers is important, preparation and support during and after teacher preparation is equally and significantly vital.

The second paper: The Interrelation of Reflection and Action. The second paper shifts the focus from the students' opportunities to learn to teachers' opportunities to learn to teach in and from their mathematics teaching. This study examined the learning-to- teach trajectory of two novice elementary teachers, Elise and Faith, over a two-year period. Both teachers were comparable in terms of their teaching backgrounds with similar mathematics teaching practices at the end of teacher preparation. However, their mathematics teaching by the end of the second year looked quite different. Because of their different trajectories, this study

examined the relationship between their teaching and reflection, and its possible contribution to their professional growth over the two-year period. Specifically, this study sought to answer the following questions:

3) How did each teacher's mathematics teaching and teaching reflection change over time?
4) What was the relationship between their mathematics teaching and their lesson reflections, and how might it contribute to their development over time?

How did each teacher's mathematics teaching and teaching reflection change over time? The study findings determined that, on the surface, Faith and Lisa's teaching practices looked very similar at the onset of their professional teaching. However, a closer examination of the nature of the classroom interactions revealed differences in how Lisa and Faith attended to student thinking. While both classroom lessons were teacher-driven, Faith repeatedly provided "check points" at the start and near the end of her lessons to assess student thinking, while Lisa's sequence of questioning limited student response choices and her access to students' thinking and reasoning. Their differing attention to student thinking was also found in their lesson reflections. Lisa's reflections were often descriptive rather than analytical, describing the multiple features of classroom instruction (e.g. classroom behavior, student engagement, her own lesson plans) noticed and used to reason about her instruction. Faith, even in her reflection of the first lesson observed, centered her inquiry into practice on the students. Her students' mathematical understanding drove her analysis of her teaching and her considerations for next steps. In year two, both Faith and Elise showed growth and greater alignment between their teaching and reflection as they grew in their attention to student thinking in their instruction and lesson reflection.

What was the relationship between Faith and Lisa in their mathematics teaching practices and their lesson reflections, and how might that contribute to their development over time? At the onset of Faith's visits, her lesson reflections embodied the characteristics of what literature would define as "productive reflections": systematic, disciplined, and analytical based on evidence of students' understanding and reasoning (Dewey, 1933; Hiebert et al, 2003; 2007; Santagata et al., 2007). In every lesson reflection, Faith identified what was important in her lessons – students' engagement with the mathematics--then used the information about her students to determine students' understanding and the effectiveness of instruction, and then considered next steps and alternatives in relation to what was learned. This process of collecting data from instruction to evaluate students' learning and then to inform instruction became an ongoing cycle of assessment, revision, and experimentation of changes in instruction.

The noticing of students' thinking in both the reflection AND teaching seemed to play a critical part in Faith's learning. The "check points" during instruction afforded Faith the opportunity to assess student progress and gave her the necessary data to analyze her teaching. In comparison, Lisa's questioning sequence limited student response choices and her access to whether or how students were making sense of the mathematics. Without evidence of her students' thinking as a reference point, Lisa could only ground the analysis of her lesson on other aspects of students' learning—following the lesson as planned, student interest, motivation, and speed in completion of work. The relation between Lisa's attention to students' thinking in teaching and reflection was also seen in the latter half of year two. As Lisa attended more to student thinking in her instruction, Lisa also showed greater attention to student thinking in her reflection.

The study findings highlight the relationship between teaching and reflection and point to the potential for learning in teachers' attention to student thinking. Teaching responsive to students' understanding requires a complexity of interwoven knowledge of mathematics, teaching, and students. It seems, by attending to students' mathematical thinking, Faith and Lisa made sense of what they noticed in relation to their own existing knowledge of the content, students' thinking, and their pedagogy, allowing them to become better and better at using what was heard from their students to make decisions. As Faith described when asked to reflect back on her first year of teaching, she stated:

> *... it was a lot of experimenting. I think that's the best way I can describe it. Because there were times when I would try something new and it would work so well and I would feel so confident and like, yes I'm doing something right, you know, I can see it, my evidence is right in front of me, it's in my kids. And then there would be days where I'll try something and it's just, a complete disaster. So it was a lot of experimenting. And um, a lot of learning from them [the students]* (Faith, year 1, visit 3)

Research Question 2: Informing our Work as Mathematics Teacher-Educator Researchers

This dissertation longitudinally documented the classroom mathematics teaching practices of four elementary teachers working in linguistically-rich communities. What can we learn from their teaching to inform our work as mathematics teacher-educator researchers? The study's findings confirm the complex and situated nature of teaching and learning. "Teaching occurs in particulars- particular students interacting with particular teachers over particular ideas, in particular circumstances" (Ball and Cohen, 1999, p. 10). This means that the classroom as a learning ecology is influenced by and interacts with both the teacher's and the students' experiences, identities, and their own cultural, historical, and economic context (Darling-Hammond, 2002, Weissglass, 2002). There is no particular form of scripted teaching that works for all teachers and students.

My engagement in the dissertation work has led me to closely reflect upon the type of learning goals for mathematics methods courses that attend to both ambitious and equitable teaching. Teachers need to be able to:

1. Understand that students have a wealth of knowledge (e.g. mathematical, cultural, community, linguistic, experiential) that can be utilized and built upon in instruction to support mathematics learning.
2. Recognize the influence and interaction teachers' own dispositions, beliefs, attitudes, experiences, and identities have on their own mathematics teaching.
3. Plan, enact, and reflect on one's teaching practice in ways that generate knowledge for professional growth.

These set of goals for the mathematics methods course articulate a more balanced conceptualization of learning. Mathematics methods courses cannot solely focus on the development of math content and pedagogy without consideration of the broader social context in which learning takes place. For the remaining part of the conclusion, I introduce and build on Rochelle Gutiérrez's proposed model of knowledge for mathematics teaching as a possible framework to organize mathematics methods courses centered on ambitious and equitable teaching.

Conocimiento: Rethinking "Knowledge"

Gutiérrez (2013) proposed a model of the types of knowledge teachers need to teach mathematics called "conocimiento." Gutiérrez (2013) proposed that the knowledge teachers need to teach mathematics involves four types: content knowledge, pedagogical knowledge, knowledge of students, and political knowledge (Figure 1). I will describe each component of Gutiérrez's knowledge framework and then discuss an addition.

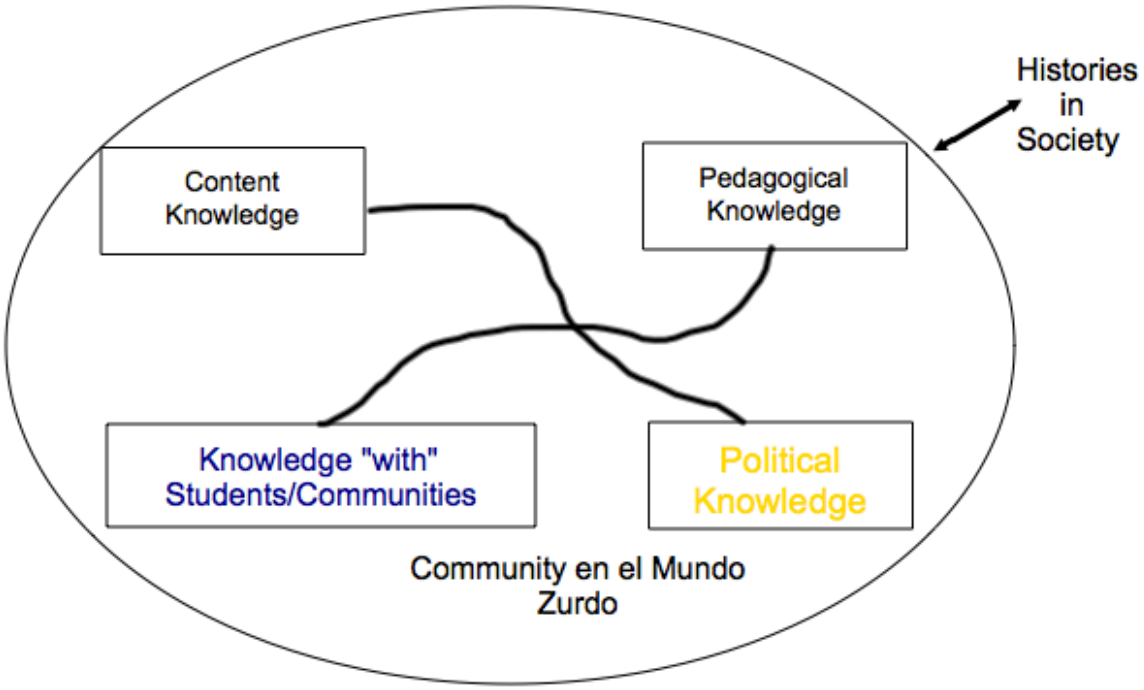

Figure 1. Gutierrez's proposed model of knowledge

The mathematics education community has focused on three specific areas of teacher knowledge: 1) content knowledge, 2) pedagogical knowledge, and 3) knowledge of students. Deborah Ball and colleagues (Ball, 2000; Ball, Thames, & Phelps, 2008; Hill, Sleep, Lewis, & Ball, 2007), building off the seminal work of Lee Shuman (1986), have argued that the knowledge for teaching only begins with content knowledge (e.g., knowing the meanings and connections, not just procedures, for double-digit multiplication) and requires knowledge of the pedagogy and of students (e.g. selecting a context and number choices for a multiplication problem to build on to students' experiential and math knowledge. Most preparation programs have centered their focus specifically on these three areas of teacher knowledge (e.g. see Ball et al, 2008; Hill, Sleep, Lewis, & Ball, 2007). Taking a practice-based approach, a growing number of mathematics methods courses engage pre-service teachers in designing settings,

representations of teaching, and activities that approximate practice and decompose it into parts that are more manageable for learning (e.g., Ball et al, 2008; Grossman, et al., 2009). While learning and knowledge is situated within the classroom, most courses have often fostered the perspective that good teaching is just good teaching for all (Gutiérrez, 2013; Weissglass, 2002).

Consideration of identity politics and issues of power are often not included in math methods courses (Aguirre, et al., 2013; Barwell, 2003; Gutiérrez, 2013). When the attention is placed on the learning experiences of non-dominant student groups (e.g. emergent bilinguals, lower performers, students of color, working-class students), the focus is on a few differentiation strategies "to meet students' needs" in a lesson designed from a mainstream, monolingual, Westernized perspective (Ellis, 2009; Gutiérrez, 2013). What is often missing are critiques of the role of structural inequities of power and privilege and how they impact status, participation, and competence in mathematic classrooms. Gutiérrez (2013) describes this teacher awareness as political knowledge.

Gutiérrez's framework of conocimiento offers an important set of lenses for mathematics teacher educators to consider in our work with pre-service teachers. The framework can serve as an analytical framework to examine teacher-preparation courses, classroom practices, and deficit narratives written about education in general. It is important to note Gutiérrez's definition of a teacher's conocimiento is not a set of static knowledge but a process of knowing as one works and learns with students. The framework itself is about reflexivity, the development of awareness and change in oneself and with others, but there is no clear indication in the framework for reflexivity.

Aligned with a process of change is Freire's theory of praxis. Freire (1970/2007) defines praxis as "reflection and action upon the world in order to change it" (p. 51). As such, praxis is

not the end in itself, but the action – the process of reflecting on and putting theoretical knowledge into practice – that leads to pedagogical transformation. Praxis is a three-step process whereby teachers: (1) reflect on theories and understandings gained from readings, observation, and discussion; (2) incorporate these perspectives into view of equitable mathematics teaching and learning; and (3) determine actions that help achieve equitable practice (Wager & Foote, 2013).

Rethinking Knowledge: The Role of Reflection and Action for Generative Knowledge

An important addition to Gutiérrez's framework then is the explicit inclusion of the process of reflection and its connection to action. Specifically, teaching for equity requires continual reflection, action, and learning as a teacher (Dewey, 1933; Freire, 2007; Chao, Murray, & Gutiérrez, 2014). I see a bridging between the four knowledge bases where reflection and action with a focused inquiry on students (their ways of knowing, their language practices, and experiences) is central in the framework, serving as a vehicle for generative, ongoing learning for the other knowledge bases (see Figure 2). I argue that the knowledge of reflecting and acting with focused inquiry provides a vehicle for teachers to generate agency for their students as well as serve as an agent of their own growth across all knowledge bases (Battey & Chan, 2010; Franke et al., 1988; Chao, et al., 2014).

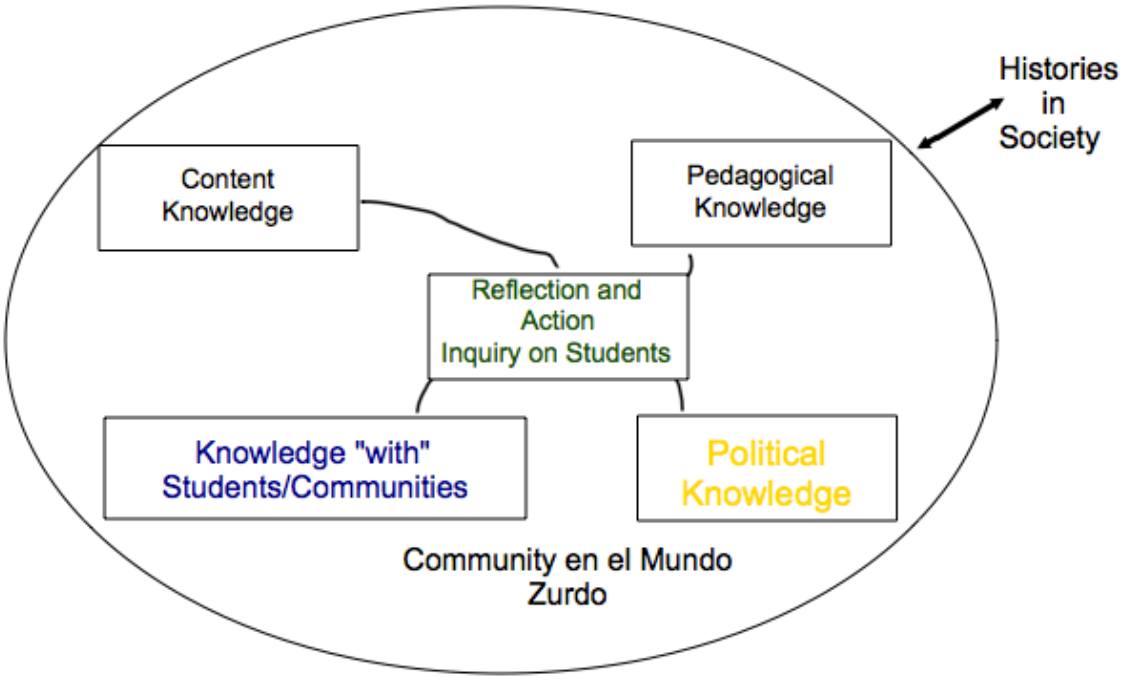

Figure 2. Revised knowledge framework

In the remaining section, I discuss how reflection and action focused on students can serve as vehicle for generative growth across all knowledge bases. As well, I will explain instructional activities and their progression within a methods course using the revised knowledge framework with reflection and action at its core. I have loosely organized the knowledge of reflection and action focused on students into three interrelated categories: inquiry into individual students, classrooms, and the broader community.

Inquiry into individual students. To start, reflection and action with a focused inquiry on students means teachers become learners of their students (Carpenter, et al., 2015; Santagata, et al., 2007; Jacobs, et al, 2010). The study findings and other studies on teachers' noticing of student thinking have shown that teacher attention to student thinking and reasoning has the generative potential to improve teacher knowledge of the mathematics, pedagogy, and of students (Carpenter et al, 2015; Santagata & Yeh, 2014, Sun & van Es, 2015).

What instructional activities can support pre-service teachers' inquiry into individual student thinking and problem solving? A close examination of individual student's problem solving through videos and paired with opportunities for pre-service teachers to conduct their own interviews with students can be used to challenge the idea that there is only one single right algorithm for solving math problems and highlight students amazing potential and capability to invent their own productive strategies (e.g., Jacobs, Lamb, & Philipp, 2010; Santagata,et al., 2007; Sherin & van Es, 2009; Stockero, 2008).

Method courses may include pre-service teachers watching videos of emergent bilinguals solving problems using their own strategies and in hybrid languages. Then, having pre-service teachers focus on productive aspects of students' mathematical ideas and language use, what students are able to do, and rooting claims about students in evidence instead of assumptions. Opportunities to attend closely to students, using their own problem solving processes and in their own language practices, can challenge narrow conceptions of student competence as well as deficit narratives that failure for marginalized students is normative (Battey & Chan, 2010; Ghousseini, Franke, & Chan, 2014; Hand, 2012; Turner, Dominquez, Maldonado, & Empson, 2013).

Inquiry into classrooms. To understand that student competencies are in relation to the opportunities for learning provided, it is important to then shift the focus of analysis to the classroom. Using artifacts of practice – lesson plans, classroom cases, and videos of other classroom interactions and the pre-service teachers' own classrooms--they can analyze the learning that took place. First, they can begin with an explicit analysis of the goals underlying the activities and then explore how particular aspects of the activity likely helped or hindered students' learning (Hiebert et al., 2007; Santagata, et al. 2007, van Es & Sherin, 2009).

Taking a step forward, shift the lens to consider who was noticed and who was not, thereby explicitly exploring aspects of power, participation, and positioning in the classroom (Aguirre, et al, 2013; Esmonde & Langer-Osuna, 2013; Turner et al, 2013). Are there differences in how students are participating? Which students are heard? Which ones are not? What pedagogical choices can be made to ensure more students have a voice in the construction of mathematical knowledge in the classroom?

Attention to all students in the classroom, particularly those who are often silent or silenced can lead to an interrogation of cultural norms (e.g. teacher as the authority figure) and how particular aspects of the instructional design (e.g. the context of the math problem and how it is presented) and responses to student ideas (e.g. only building on students with the correct answer) impact student positioning and their opportunities to learn. Then, consider pedagogical changes that can give voice and position all students competently (e.g. valuing student invented strategies, students' use of hybrid and multimodal language practices, and developing mathematics communities that support collective sense making).

Inquiry into the relation between the classroom and beyond. And then, teachers need to look beyond the classroom. Mathematics teaching responsive to students requires detailed knowledge of who students are. Metanarratives passed on through media, research, and in daily conversations within the school walls often perpetuate powerful and narrow ways of framing students based on race, class, gender, and language (Banks, 1993; Chao, et al., 2014; García, et al., 2008; Gutiérrez, 2002; 2013). To move beyond an essentialized notion of students, it is important for teachers to become familiar with the local context, to survey the community to access students out-of-school mathematical practices (deAbreu & Cline, 2008; Foote, 2013; Nasir, 2007), and funds of knowledge (Civil & Andrade, 2002; Civil, Planas, & Quintos, 2005).

A deep commitment and attention to students means acknowledging students' hybrid identities (Gutiérrez, et al., 1990; Setati, 2006; Gutiérrez, 2013) and embracing classrooms as hybrid spaces (Gutiérrez, 2013).

Mathematics teacher educators' own reflection and action focused. Lastly, to encourage pre-service teachers to engage in the process of reflection and action focused on students, I, as a teacher educator, should also explore my own process of reflection and action, and consider how I can honor the diverse ways of knowing my pre-service teachers bring to the method courses (Kalinec-Craig, 2013; Berk & Hiebert, 2009). To encourage classroom practices that are dialogic, where expertise is shared, and students' own linguistic, cultural, and experiential knowledge is honored and utilized, I must design classroom seminars reflecting such practices. And this begins with an examination of my experiences, assumptions, and knowledge about mathematics-language-learning-identity with respect to preparing new teachers for the diverse classroom.

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Appendix A

Matrix of Identified Practices and Developments Laura Visit 1.1

Laura Visit 1, Year 1	
Dimension	
Language Supports What strategies supported students' development of language of mathematics for active participation? Multimodal and multisemiotic Kinesthetic Visual supports Connect language with visual representations Essential ideas, concepts, and rep, and words on board for reference Promote use of students' native language during instruction	- Choral counting activity from 22-121 with two hundreds chart (one made into 101-200). Students count forward and backward and use fingers to represent the digits in tens and ones. -numerical fluency- FactsWise- 5 and some more class-made chart of five and some more with part-whole grid, dots on ten frame, and sentence begins with student noticing talk connect ten frame to part-whole grid to number sentence) as C say number sentence, Lauren makes equal sign with both arms every time as points to equal sign and class choral count is equal to in number sentence. Students model gesture. - Communicative property talk T says, "let's count in reverse way." A student says the answer is the same: "S: Because, because they are the same if you change it to five and five it is still the same." T. first focus on understanding meaning (what does it mean?) and then ask to say the word with partner, whole class, partner Breaks apart word in syllable breaks on board, has individual students say the word, and whole group breaking apart into syllable to emphasize the pronunciation. Tens relationship from one place value to next Then, teen number activity where objective is to break apart numbers 0-15 into tens and ones (e.g. 7 is 0 tens and 7 ones) using individual base ten units (once with ten goes in cup – van del walle activity on identifying tens) Activity shifts to rug talk (One what? What represents one? What does it look like? What is ten? What would represent ten? Focus is on students explain ten individual units make up one ten. Lauren by poster paper ask writing down student observation/explanation with words, numbers, and models (student choice) "T: Does someone have an observation about units and tenths? ( ) pay attention?"

Student Positioning Who is positioned as the authority of knowledge and how? Choral count- student – Spanish, lead student gestures Numerical fluency- teacher, student, Spanish, gestures Communicative property- student, school and everyday, English and Spanish, student gestures to number sentence, student explain (6) Tens relationship- student, English and Spanish, student explanation (6) noticing, relationship tens to hundred (8 student explanation)	Choral count Students are called up to lead class on daily choral counting; student points to number with stick as the others read out loud. Lauren stands or sits to side as student lead. Spanish only Numerical Fluency: Fives and some more T: Hmmm. (puts index finger on chin- thinking face) Which numbers, which numbers are the ones that we are studying, what do they have in common? Tell your friends." Student share- tell students that idea belong to group when shared Calls on multiple students (6) to share what they notice without stating right or wrong in between. Idanil speak in English and Spanish. Boy in blue walks to board to point to L. representation of Idanil's explanation in English and Spanish. Three raise hand but most just add on. T. repeatedly rephrasing what is said by one student and asking other students for their perspective. Often rephrasing student response and asking "what else? What do you think?" Documents on poster sheet student response. Pauses in between student responses and seem to be contemplating (hand on face) what they're saying. This seems to be intentional. Communicative property talk T says, "let's count in reverse way." (intentional = says in interview) A student states the answer is the same: "S: Because, because they are the same if you change it to five and five it is still the same." 3 more make comment about how number sentence have same numbers but different order. T. ask students for meaning(what does it mean?) and writes their meanings down and then practice pronunciation and repeat of word A student says "it's reverse" in English and Lauren rephrases in Spanish Class discussion on shared understanding of what ten unit mean. Lauren writes on the top of the white poster ten in black ink and one in red ink. A student states one unit and Lauren draws the unit cube. Asks students what that one unit signifies? 5 Ss response. Some says one. Another states one unit. Lauren asks students to explain what each unit cube represent in relation to the number line. A student state that one unit cube is equal to 1 in English and Spanish. Students start to shout out what a ten represents. Lauren asks students "what can I draw to demonstrate one ten?" Questions are often broad, focus rather than funneling. Does someone have an observation about units and tens? 7 student give observation and T. charts. Then, asking students to tell each other relationship between one ten and ten ones and then what they notice about tens to one hundred (student speak to each other in Spanish and English). Ask for relationship. Pair share again to each other (Spanish and English). At the end, multiple students (5) respond with 3 raising hand but most building on from each other. All in Spanish. Each time T. asking why. Two times pause during student share show thinking face and ask class to explain "What did Joseph say? What does he mean?" Does not state right or wrong but writes ideas on poster sheet Focus of question is not just on how one arrives at answer but what it means. Particularly the whole class discussion at end of class on noticing the base of ten in place value system.