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Publication Date
1961-04-10

UCRL -9644

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UCRL-9644

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Contract No. W-7405-eng-48

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Experimental evidence for the occurrence of an $I = 1$, Λ-π resonant state (Y^*) as an intermediate step in the reaction

$$K^- + p \rightarrow \left\{ \begin{array}{c} Y^{*-} + \pi^+ \\ Y^{*+} + \pi^- \end{array} \right\} \rightarrow \Lambda + \pi^+ + \pi^- \qquad (1)$$

has been presented by Alston et al.[1] for K^- (lab) momentum 1150 Mev/c. The same reaction has also been studied recently by Berge et al.[2], and the related K_2^0-p reactions have been analyzed by Martin et al.[3] for K_2^0 momentum 975 Mev/c. The interpretation of these data in terms of the mass (M^*), half-width $(\Gamma/2)$, spin (J), and parity of the Y^* state has been confused by the evidence that the Y^* decay in these reaction sequences (1) cannot be regarded as the decay of a free particle. In this Letter, we show that this evidence can be largely understood as due to interference effects arising from the requirement of Bose statistics for the final pions, and we discuss the extent to which these Y^* parameters may ultimately be determined from data in this momentum range.

From the reported estimates (10 to 30 Mev) for $\Gamma/2$, the mean distance traveled by the primary pion in one Y^* mean lifetime is more than 4 fermis.

† Work done under the auspices of the U.S. Atomic Energy Commission.

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It is therefore a plausible assumption that the successive pion-emission processes do not interfere dynamically to any marked degree. In this case we are led to an amplitude $M(1,2)$ for the reaction sequence $\overline{K} + N \rightarrow Y^* + \pi$, $Y^* \rightarrow \Lambda + \pi_2$, of the general form[4]

$$M(1,2) = \mathcal{O}(\sigma, q, p_1, P_2)\, A(P_2) , \tag{2}$$

where q and p_1 denote the c.m. momenta of the K^- meson and the primary pion, and P_2 denotes the momentum of the secondary pion in the π_2-Λ rest frame. The final configuration $\Lambda + \pi_2 + \pi_1$ may also be specified by giving ℓ, the orbital angular momentum of π_1, and L, the orbital angular momentum in the π_2-Λ system which corresponds to the Y^* spin and parity; the form of $\mathcal{O}$ is then determined for total angular momentum j by the angular momentum coupling and the $(K\Lambda)$ parity and by centrifugal barrier considerations. The role of the resonant state is represented by the second factor,

$$A(P) = \exp(i\delta(P)) \sin \delta(P)/P^{2L+1} . \tag{3}$$

The amplitudes for the two sequences (1) must be added coherently, and their sum must correspond to a final state with correct symmetry for interchange of the two pions. Thus, for total isotopic spin I = 0 and 1, the amplitudes M_0 and M_1 are given by

$$M_0 = M'(1,2) + M'(2,1) , \tag{4a}$$

$$M_1 = M''(1,2) - M''(2,1) . \tag{4b}$$

For comparison with experiment, it is convenient to consider the probability distribution $P(E_+,E_-)$ on the (E_+,E_-) phase-space diagram. Since the distribution $P(E_+,E_-)$ sums over all orientations of the $\Lambda\pi^+\pi^-$

plane, the contributions to $P(E_+,E_-)$ from initial states of different angular momentum and parity add incoherently. Since $P(E_+,E_-) = \frac{1}{2}(M_0 + M_1)^2$ and $P(E_-,E_+) = \frac{1}{2}(M_0 - M_1)^2$, the symmetrized probability distribution $\{P(E_+,E_-) + P(E_-,E_+)\}$ is simply a superposition of the separate $I = 0$ and $I = 1$ distributions, without interference between them. The observation that the experimental distributions $P(E_+,E_-)$ and $P(E_-,E_+)$ do not differ markedly from each other at 760 and 850 Mev/c suggests that, for these two momenta, the reaction may be dominated by a single isotopic-spin channel.

Two $\Lambda\pi^+\pi^-$ configurations are of particular interest:

(A) $E_+ = E_-$, and maximum Λ recoil energy. For this configuration, the two pions have the same c.m. momentum, and $M(1,2)$ and $M(2,1)$ are necessarily equal. For 850 Mev/c and below, the Y^{*+} and Y^{*-} bands overlap strongly and marked interference is to be expected at A (see Fig. 1 of preceding Letter[2])--constructive for $I = 0$, destructive for $I = 1$. The experimental distributions at 760 and 850 Mev/c show an especially low density of events in the region near A, which suggests that it is the $I = 1$ channel which is dominant at these momenta.[5] The density of events expected in this region for the models discussed below is compared with the data in Table I.

(B) $E_+ = E_-$, and the Λ hyperon at rest. For this configuration, the two pions have equal and opposite momenta in the c.m. system. With $I = 1$ $(I = 0)$, $M(1,2)$ and $M(2,1)$ interfere constructively or destructively according as $\ell + L$ is odd (even) or even (odd). The observed distributions at 760 and 850 Mev/c indicate strong constructive interference in the region near B, leading to the conclusion that, with $I = 1$, $\ell + L$ is odd.[6] An $S_{1/2}$ Λ-π resonance therefore requires a p-wave primary pion at these production energies, a $P_{1/2}$ or $P_{3/2}$ Λ-π resonance requires an s- (or d-) wave primary pion.

Table I. The number of $\Lambda + \pi^+ + \pi^-$ events expected in the region near A, with both E_+ and E_- less than the limit E_m stated, for the K^--p interaction data presently available.[a]

	Configuration $(\ell L_J)_j^I$					
	$(sS_{1/2})_{1/2}^0$	$(pP_{3/2})_{1/2}^0$	$(pP_{3/2})_{3/2}^0$	$(pS_{1/2})_{1/2}^1$	$(sP_{3/2})_{3/2}^1$	Events Observed
850 Mev/c; E_m = 150 Mev	35.8	26.0	8.5	12.3	11.0	11/262
760 Mev/c; E_m = 125 Mev	29.8	32.8	8.6	9.7	10.5	18/252
760 Mev/c; E_m = 120 Mev	17.4	21.1	4.2	3.1	2.2	8/252

a. From Reference 2.

We now discuss in detail the calculations of the symmetrized (E_+,E_-) distributions for some cases of particular interest, to illustrate the nature of the interference effects resulting from Bose statistics. These cases represent only the simplest possibilities for fitting the available data, and are neither unique nor exhaustive examples.

$S_{1/2}$ Λ-π Resonance

With the assumption of p-wave excitation, we have

$$M(1,2) = (a\underset{\sim}{q} \cdot \underset{\sim}{p}_1 + ib\underset{\sim}{\sigma} \cdot \underset{\sim}{q} \times \underset{\sim}{p}_1)(M_2 - M^* + i\Gamma/2)^{-1} , \qquad (5)$$

where M_2 denotes the total energy in the Λ-π_2 rest frame. Figure 1a compares the calculated distribution of $M(\Lambda$-$\pi)$ for $M^* = 1385$ Mev and several values of $\Gamma/2$ with the 850 Mev/c data (obtained by projecting the symmetrized (E_+,E_-) plot onto one axis). These curves display an insensitivity to $\Gamma/2$, for an adequate fit[7] to the data is obtained with any value of $\Gamma/2$ between 20 and 30 Mev. As shown in Table I, the destructive interference obtained for the $I = 1$ $pS_{1/2}$ case is in reasonable accord with the data: $I = 0$ $sS_{1/2}$ production, on the other hand, gives too high a density of events near A, as well as a poor fit to the mass distribution.[8]

Figure 2 compares the distributions obtained for 1150 Mev/c with this model, for the same M^* and $\Gamma/2 = 25$ Mev, with the experimental data.[1] For $I = 1$, the fit obtained is quite adequate: $I = 0$ $pS_{1/2}$ production can also fit these data but, the interference at B being destructive, only with a larger width $\Gamma/2$. Since this K^- energy lies just above the strong $I = 0$ $\bar{K}$-N resonance (at 1100 Mev/c K^-(lab) momentum), there is, of course, little reason to believe that this Y^* production should necessarily involve the same states as are effective at 850 Mev/c. In fact, these data show appreciable asymmetry between Y^{*+} and Y^{*-} production, an indication that both $I = 0$ and $I = 1$ production must be occurring from the same angular-momentum states.

The low density of events with small E_+ or E_- at 510 and 620 Mev/c can also be understood in terms of $pS_{1/2}$ production, as a result of the centrifugal barrier faced by the outgoing p-wave primary pion.

$P_{3/2}$ Λ-π Resonance

For $I = 1$ production, we consider s-wave excitation

$$M(1,2) = (2q \cdot P_2 - i\sigma \cdot q \times P_2)\, A(P_2) , \qquad (6)$$

where the phase shift $\delta(P_2)$ has been taken to have the form known for the (3,3) resonance,[9] with suitable parameters M^* and Γ. The fit to the 850-Mev/c mass distribution is quite adequate with $\Gamma/2 = 25$ Mev, as shown in Fig. 1a; the same parameters again give an adequate fit to 1150-Mev/c distribution. Figure 1b shows the calculated distributions for $I = 0$ production in the $j = 1/2$ and $3/2$ states $(pP_{3/2})$. Neither of these configurations reproduces the data. Table I shows that for $(pP_{3/2})_{1/2}$, the constructive interference near A is too strong; $(pP_{3/2})_{3/2}$ does give small intensity in this region because the Y^*-decay distribution (relative to production direction) happens to have approx $(7-6 \cos^2 \theta)$ form for this case, but this angular distribution is incompatible with the data (see Fig. 3). Although a rough fit may be obtained by superposing these two configurations, it is clear that this comparison adds support to our assignment of $I = 1$ production.

changed from τ to Λ per D. Miller 4/25/61

The distributions at 510 and 620 Mev/c are poorly fitted by the $I = 1$ $(sP_{3/2})$ configuration. Within the limits of statistics, however, we find that the low-energy data are adequately described by the $I = 0$ $(pP_{3/2})_{3/2}$ configuration.[10]

$P_{1/2}$ Λ-π Resonance

For s-wave excitation, we have

$$M(1,2) = \sigma \cdot P_2 \, A(P_2) , \tag{7}$$

where $A(P_2)$ is taken to have the form appropriate to a P-wave resonance, that used already with Expression (6). The symmetries required for fitting the (E_+,E_-) plots are similar to those for the $P_{3/2}$ possibility just discussed.

The highly anisotropic distribution observed for Y^* decay relative to its direction of motion may also be understood in terms of Bose statistics. For $I = 1$ and $\ell + L$ = odd, the configuration with the secondary pion emitted forwards gives constructive interference, so that the angular distribution for Λ emission in the Y^* decay is peaked backwards. The decay angular distributions (averaged over all production directions) calculated for $(sP_{3/2})$ and $(pS_{1/2})_j$ for 850 Mev/c are shown in Fig. 3 and compared with the experimental data.

The decay angular distribution for Y^* production at 0 and 180 deg is of particular interest, in view of Adair's general result for its dependence on J. Again, the angular distributions characteristic of isolated decay are strongly modified by the requirements of Bose statistics. The distortion of the idealized $(1 + 3 \cos^2 \theta)$ distribution for $J = 3/2$ is shown in Fig. 4a. The difficulty in distinguishing conclusively between the cases $J = 1/2$ and $J = 3/2$ at 850 Mev/c is apparent.[11] The character of these distortions is closely linked with the requirement of constructive interference at B.

As the K^- momentum is increased further, the primary pion energy will exceed the energy of the secondary pion and Bose interference effects will become less and less important. In the calculations for 1150 Mev/c, this is already apparent.[12] As shown in Fig. 4b, the distortion of the Adair distribution for $J = 3/2$ is much less severe; unfortunately the paucity of data at this momentum does not yet allow a decision on the Y^* spin. It is clear that, in further experiments at higher K^- momenta, the effects of Bose statistics will be sufficiently reduced to allow the possibility of a clearcut distinction between $J = 1/2$ and $J = 3/2$ for the Y^* state.

For Y^* production from a K^--p state of angular momentum j and isotopic spin I, it is important to note that the geometric limit on the absorption cross section is $(2j + 1)\pi\lambda\!\!\!^{-2}/4$, which takes the value $2.85(j + \frac{1}{2})$mb at 850 Mev/c. At this momentum, the cross section observed for Reaction (1) is 3.2 ± 0.3 mb, comparable with the geometric limit for $j = 1/2$. This limitation provides a severe condition[13] which must be satisfied by any simple model of the production process. For $I = 1$, this condition is satisfied by $(sP_{3/2})$ production, but not by $(sP_{1/2})$ production. The possibility $(pS_{1/2})_{3/2}$ [together, perhaps, with some $(pS_{1/2})_{1/2}$ production] also satisfies this condition. At this point we note that the angular distributions for Y^* production are isotropic at 760 and 850 Mev/c. This is consistent with $(sP_{3/2})$ production, of course. For the $(pS_{1/2})_{3/2}$ case,[14] a calculation of the angular distribution (including Bose effects) shows that a strong $\cos^2\theta$ term is to be expected at these energies; this discrepancy could be reduced by a large admixture of $(pS_{1/2})_{1/2}$ excitation of suitable phase. The large value observed for the $(\Lambda + \pi^+ + \pi^-)$ production cross section is most probably an indication that the distribution receives contributions from a number of partial waves and configurations, of which only the dominant terms have the symmetry specified in our discussion.

In conclusion, we are glad to express our deep indebtedness to Mr. Joseph Schwartz for his enthusiastic computational assistance in this work.

FOOTNOTES AND REFERENCES

1. M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, N. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters 5, 520 (1960).

2. J. Berge, P. Bastien, O. Dahl, M. Ferro-Luzzi, J. Kirz, D. Miller, J. Murray, A. Rosenfeld, R. ~~Tripp, and M. Watson, Phys.~~ Rev. Letters, Pion-Lambda Resonance (Y^*), (preceding letter). See also M. H. Alston and M. Ferro-Luzzi, Revs. Modern Phys., Pion-Hyperon Resonances, (to be published, 1961). per D. Miller 4/25/61

3. H. J. Martin, L. B. Leipuner, W. Chinowsky, F. T. Shively, and R. K. Adair, Phys. Rev. Letters 6, 283 (1961).

4. A matrix element of this form has also been used by B. Sakita (Angular Correlations in $K^- + p \rightarrow \Lambda + \pi^+ + \pi^-$, submitted to Nuovo Cimento, 1961) in a discussion of some Λ polarization effects which can occur in this process in consequence of the effect of Bose-statistics interference.

5. This conclusion may be checked directly by measurement of the Y^* production cross section in K^--n or K_2^0-p collisions, where only the $I = 1$ interaction can be effective. If Y^* production is dominantly from the $I = 1$ channel, then we expect $\sigma(K^- + n \rightarrow Y^* + \pi) = 2\sigma(K_2^0 + p \rightarrow Y^* + \pi) \approx 2\sigma(K^- + p \rightarrow Y^* + \pi)$. It is of interest that Martin et al. (ref. 3) have reported strong Y^* production in 975-Mev/c K_2^0-p collisions, although a precise value for the cross section is not yet available.

6. That this evidence is significant for $I = 1$ is shown by calculations based on specific models with $\ell + L$ even. The case $\ell = L = 0$ is strongly excluded, as it requires the density to vanish along the line $E_+ = E_-$. If $I = 0$ held, this evidence would require $\ell + L$

even for the dominant production states. The simplest possibilities, ($sS_{1/2}$, $pP_{1/2}$ or $pP_{3/2}$) can be excluded by the argument given aobve (see also Table I), or by the comparison in Fig. 3. An exceptional situation which could be consistent with $I = 0$ production is mentioned in Ref. 8. It is possible, of course, that at these energies both $I = 0$ and $I = 1$ production occurs dominantly in the states with the symmetry specified here; it should be emphasized that these come from initial states of opposite parity and that, since they do not interfere, they would contribute no asymmetry to the $P(E_+, E_-)$ plot.

7. In this fitting, no question arises concerning the identification of "background" to be subtracted: all events are included. As explained above, the weakly contributing states do not interfere with the resonant contributions, and their influence is thereby minimized.

8. The assumption of $I = 0$ production in the $sS_{1/2}$ configuration gives too high a density of events in the region near A (see Table I). However, it should be remarked here that an s-wave Λ-π resonance associated with a $\bar{K}$-N bound state is not necessarily well-fitted by a Breit-Wigner amplitude, but may be long-tailed on the low-energy side (cf. R. H. Dalitz, Revs. Modern Phys., On the Strong Interactions of the Strange Particles, to be published (1961)). An asymmetric resonance amplitude of reasonable total width, which falls rapidly on the high-energy side, would be small in the region of A for K^- momenta 760 and 850 Mev/c; and the density of events would be low in this region, despite the constructive interference. We

remark also that at 510 Mev/c, rather too many events are found in the neighborhood of B: a resonance amplitude with a long tail on the low-energy side would give a stronger constructive interference in this region, in accord with this observation.

9. M. Gell-Mann and K. M. Watson, Ann. Rev. Nuclear Sci. 4, 219 (1954). Eq. 11. (added per D. Miller)

10. This state has been considered because the K^--p data at 400 Mev/c (see L. Alvarez, The Interactions of Strange Particles, Lawrence Radiation Laboratory Report UCRL-9354, Aug. 1960) shows evidence for strong interaction in the $p_{3/2}$ state. If the $K\Lambda$ parity is odd and the π-Λ resonance is $P_{3/2}$, then the $I = 0$ configuration $(pP_{3/2})_{3/2}$ may be expected to play a major role in the $\Lambda\pi\pi$ state in this energy region.

11. We note here that, in the Adair distributions, there is generally interference between amplitudes from different initial angular momentum states, so that these distributions are more sensitive to small admixtures of configurations of different symmetry.

12. Note that the interference effects are model-dependent in that they depend on the form assumed for the function $A(P_2)$ at energies well above the π-Λ resonance energy. Somewhat stronger interference terms could be obtained by modifying the phase shift $\delta(P_2)$ in this region, or by including additional momentum-dependent factors.

13. For $I = 0$ absorption, we would have

$$\sigma(\Lambda + \pi^0 + \pi^0) = \frac{1}{2}\,\sigma(\Lambda + \pi^+ + \pi^-),$$

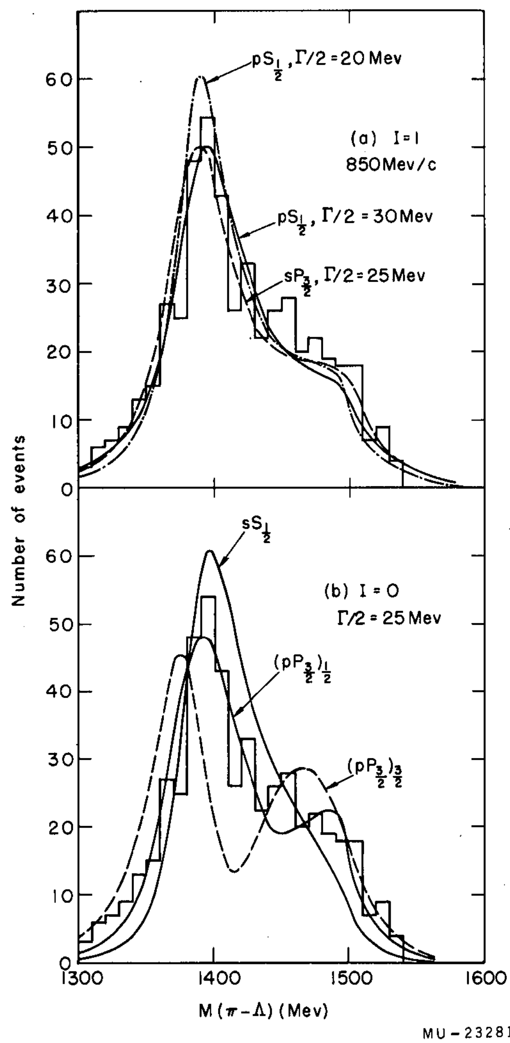
so that the Y^* production cross section must be increased to 4.8 ± 0.5 mb to allow for these unobserved events.

14. R. H. Dalitz (Phys. Rev. Letters 6, 239 (1961)) has suggested that

enhanced $I = 1$ production would be expected as a consequence of the (3,3) resonant interaction between the primary pion and the nucleon of a $J = \frac{1}{2}$ $\bar{K}$-N bound state. This interpretation suggests naturally that the primary pion be emitted in the p wave with $j = \frac{3}{2}$. Although the configuration $(pS_{1/2})_{3/2}$ fits the decay angular correlations and the (E_+, E_-) plot rather well, it appears excluded by the production angular distributions observed.

FIGURE CAPTIONS

1. The $M(\pi - \Lambda)$ distribution observed at 850 Mev/c is compared with (a) the calculated distributions for the $I = 1$ $(pS_{1/2})$ and $I = 1$ $(sP_{3/2})$ configurations with $M^* = 1385$ Mev and various values of $\Gamma/2$, and (b) the calculated distributions for the $I = 0$ configurations $sS_{1/2}$, $(pP_{3/2})_{1/2}$, and $(pP_{3/2})_{3/2}$ with $\Gamma/2 = 25$ Mev.
2. The $M(\pi - \Lambda)$ distribution observed at 1150 Mev/c is compared with calculated curves for the $pS_{1/2}$ and $sP_{3/2}$ configurations. An adequate fit is obtained for the $I = 1$ configuration with $\Gamma/2 = 25$ Mev, but a fit for the $I = 0$ configuration requires a larger width.
3. The angular distribution of the pion from Y^* decay, relative to the Y^* direction of motion, for all events at 850 Mev/c with $M(\pi - \Lambda)$ between 1360 and 1410 Mev. The calculated curves for $I = 1$ $sP_{3/2}$ and $I = 1$ $pS_{1/2}$ production reproduce the general trend rather well. The curve for $I = 0$ $(pP_{3/2})_{3/2}$ production is plotted to show that the data strongly exclude the possibility that this is the dominant production state.
4. Adair distributions are shown for forward- and backward-produced Y^* decay (integrated over the mass range $1360 \leqslant M(\Lambda - \pi) \leqslant 1410$ Mev) at K^- momenta 850 Mev/c and 1150 Mev/c, where the angle θ_Λ of the Λ particle in the Y^* rest frame is measured from the direction of motion of the Y^*. The distortion from the expected $(1 + 3\cos^2\theta_\Lambda)$ distribution for $J = \frac{3}{2}$ is much weaker at 1150 Mev/c than at 850 Mev/c. However, the present statistics at 1150 Mev/c do not yet allow the cases $J = \frac{1}{2}$ and $J = \frac{3}{2}$ to be distinguished.



$pS_{\frac{1}{2}}$, Γ/2 = 20 Mev
(a) I = 1
850 Mev/c
$pS_{\frac{1}{2}}$, Γ/2 = 30 Mev
$sP_{\frac{3}{2}}$, Γ/2 = 25 Mev
$sS_{\frac{1}{2}}$
(b) I = 0
Γ/2 = 25 Mev
$(pP_{\frac{3}{2}})_{\frac{1}{2}}$
$(pP_{\frac{3}{2}})_{\frac{3}{2}}$
Number of events
M (π−Λ) (Mev)
MU-23281

Fig. 1

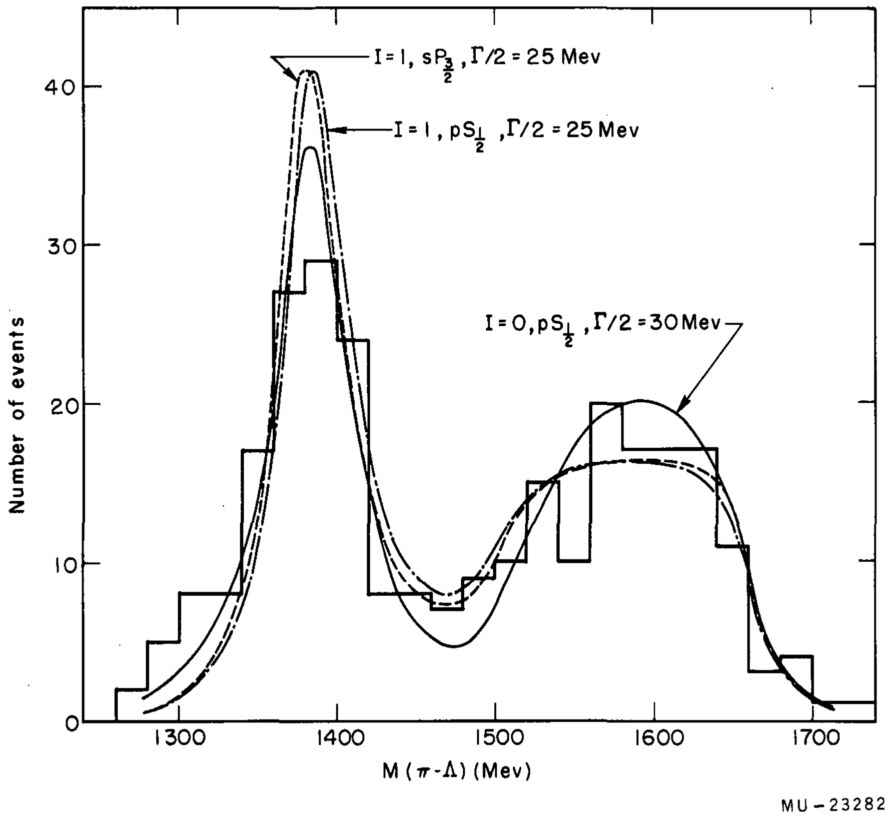

I = 1, sP$_{\frac{3}{2}}$, Γ/2 = 25 Mev
I = 1, pS$_{\frac{1}{2}}$, Γ/2 = 25 Mev
I = 0, pS$_{\frac{1}{2}}$, Γ/2 = 30 Mev
Number of events
M (π-Λ) (Mev)
0
10
20
30
40
1300
1400
1500
1600
1700
MU-23282

Fig. 2

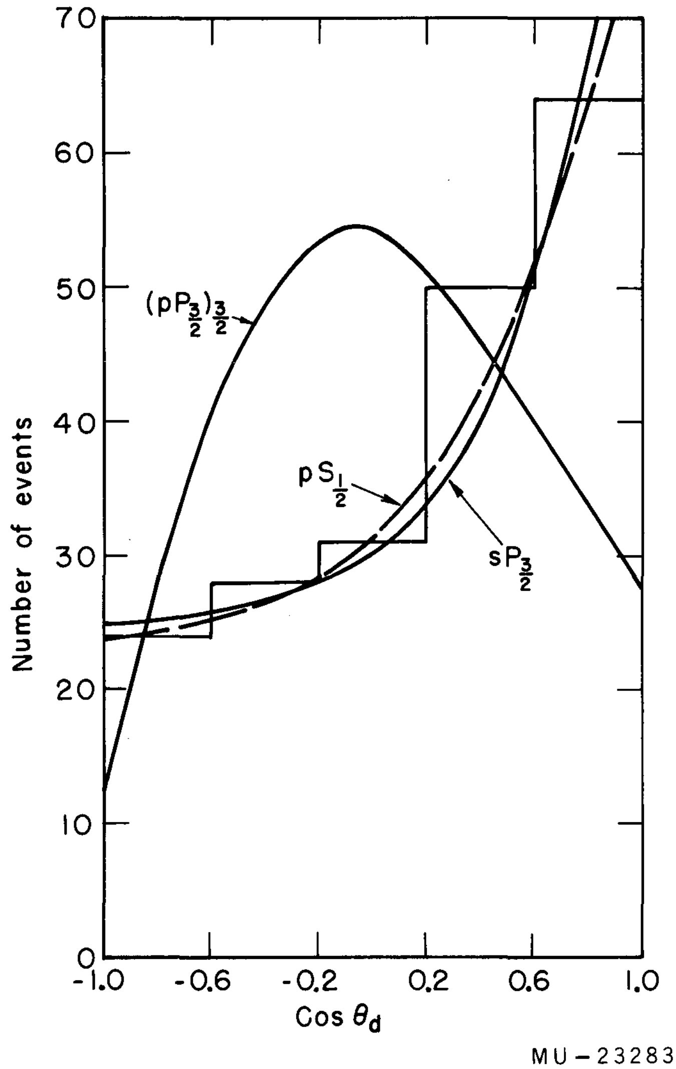

70
60
50
40
30
20
10
0
-1.0
-0.6
-0.2
0.2
0.6
1.0
Number of events
Cos θ_d
$(pP_{\frac{3}{2}})_{\frac{3}{2}}$
$pS_{\frac{1}{2}}$
$sP_{\frac{3}{2}}$
MU-23283

Fig. 3

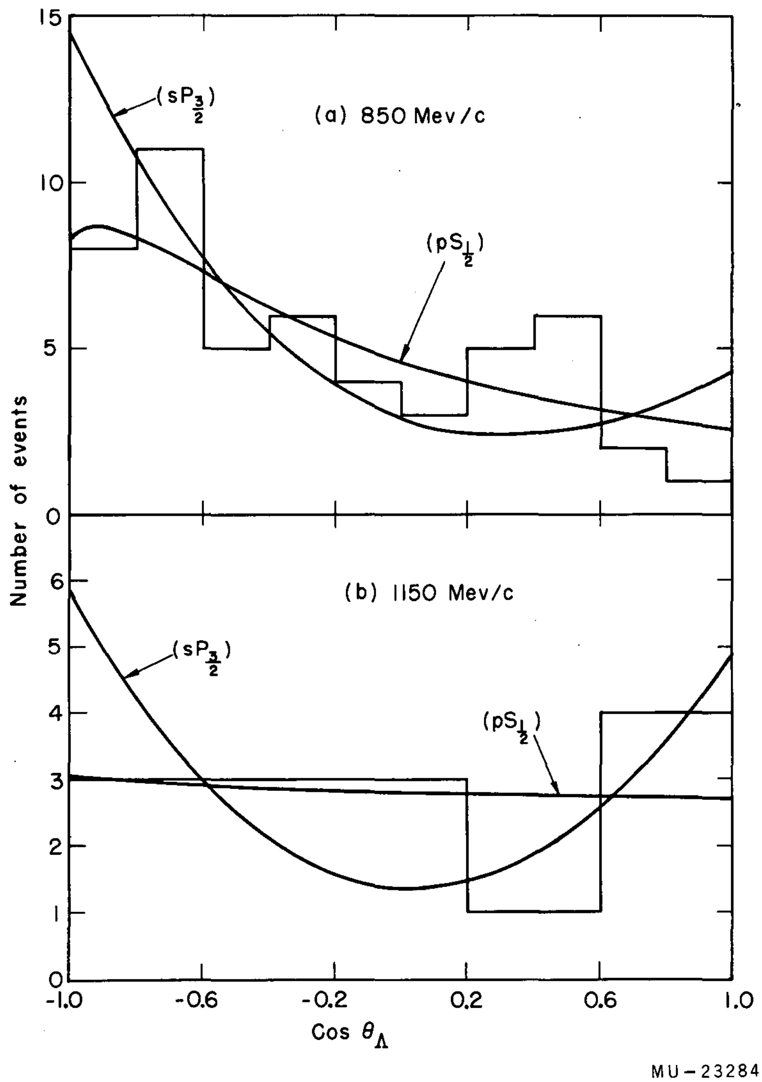

15
10
5
0
6
5
4
3
2
1
0
Number of events
(a) 850 Mev/c
(b) 1150 Mev/c
(sP3/2)
(pS1/2)
-1.0
-0.6
-0.2
0.2
0.6
1.0
Cos θΛ
MU-23284

Fig. 4