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Victor Franco

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Abstract

The possible non-additive character of quark amplitudes and the resulting

additional contributions to total cross sections are investigated. A sum rule is

derived and it is suggested that the non-additive contributions to some cross

sections may be dominant.

High-energy hadron-nucleus cross sections exhibit the effects of multiple

scattering to varying degrees, depending in part upon the reactions measured and

the momentum transfers involved. For example, proton-deuteron elastic scattering

angular distributions exhibit a secondary maximum or shoulder and a backward peak

which appear to be attributable to double scattering.[1-3] Proton-He[4] elastic

scattering exhibits secondary and tertiary maxima which may be attributable to

double and triple scattering, respectively.[4] Double-scattering effects in high-

energy hadron-deuteron <u>total</u> cross sections are not quite so dramatic, but general-

ly do amount to 5-20% of the corresponding hadron-nucleon cross sections.[5-7] The

analyses of multiple scattering at high energies are usually made by means of the

Glauber approximation.[8] In quark models, where hadrons may be thought of as bound

states of three quarks (or antiquarks), or of a quark and an antiquark, one might

expect multiple scattering corrections to hadron-hadron total cross sections to

be significant. We investigate the possible effects of "multiple scattering,"

i.e., non-additivity of quark amplitudes, by applying the Glauber approximation

to the quark model.

Most of the total cross section relations obtained from the quark model

with the assumption of additivity of the quark-quark and quark-antiquark amplitudes[9-15]

appear to be in rather good or very good agreement with the measurements. This agree-

ment need not, however, imply that the quark amplitudes are additive. The treatment

of non-additivity corrections in the quark model may be illustrated for $\pi^+\pi^+$ colli-

sions for which the Glauber approximation leads naturally to a consideration of

single, double, triple, and quadruple scattering. We use the notation pnλ for the

three quarks and we consider the π^+ meson to be a bound state of the antiquark-quark

pair $\bar{n}p$. We assume that the internal velocities of the quarks may be neglected in comparison to the velocity of the incident π^+ meson. We also assume that the π^+ meson has an internal wave function and a corresponding form factor $S(\vec{q})$. The total cross section at a laboratory momentum k, denoted by $\sigma(\pi^+\pi^+,k)$, may be written as

$$\sigma(\pi^+\pi^+,k) = 2\sigma(p\bar{n},\mu k) + 2\sigma(pp,\mu k) + \delta\sigma_2 + \delta\sigma_3 + \delta\sigma_4 \quad , \tag{1}$$

where $\sigma(p\bar{n},\mu k)$ is the $p\bar{n}$ "total cross section" at an incident momentum μk for the p quark in the laboratory system, with a corresponding meaning given to $\sigma(pp,\mu k)$.[16] The factor μ is a constant equal to the ratio of the effective mass of the quasi-free quark p to the mass of the π^+ meson. The quantities $\delta\sigma_2$, $\delta\sigma_3$, and $\delta\sigma_4$ represent respectively the double-, triple-, and quadruple-scattering corrections which we shall investigate.

We let $f(p\bar{n},\vec{q},\mu k)$ denote the $p\bar{n}$ elastic-scattering amplitude for an incident momentum μk and a momentum transfer $\vec{q}$ for the quark p, with a corresponding notation for the pp amplitude. An extension of the Glauber approximation for hadron-deuteron collisions[6] yields[17]

$$\delta\sigma_2 = (2/\mu^2 k^2) \, \text{Re} \int \{4S(\vec{q})f(p\bar{n},\vec{q}) \, f(pp,-\vec{q}) + S^2(\vec{q})[f(pp,\vec{q}) \, f(pp,-\vec{q})$$

$$+ f(p\bar{n},\vec{q})f(p\bar{n},-\vec{q})]\}d^2q \quad , \tag{2}$$

$$\delta\sigma_3 = -(1/\pi\mu^3 k^3)\,\text{Im}\!\int 2S(\vec{q})S(\vec{q}\,')[\,f(p\bar{n},\vec{q}+\vec{q}\,')f(pp,-\vec{q}\,')f(pp,-\vec{q})$$

$$+ f(pp,\vec{q}+\vec{q}\,')f(p\bar{n},-\vec{q}\,')f(p\bar{n},-\vec{q})]\,d^2q\,d^2q' \qquad , \tag{3}$$

and

$$\delta\sigma_4 = -(1/2\pi^2\mu^4 k^4)\,\text{Re}\!\int S(\vec{q})S(\vec{q}\,')f(pp,\vec{q}'' - \tfrac{1}{2}\vec{q} - \tfrac{1}{2}\vec{q}\,')f(pp,\vec{q}'' + \tfrac{1}{2}\vec{q} + \tfrac{1}{2}q')$$

$$\times\, f(p\bar{n},-\vec{q}'' + \tfrac{1}{2}\vec{q} - \tfrac{1}{2}\vec{q}\,')f(p\bar{n},-\vec{q}'' - \tfrac{1}{2}\vec{q} + \tfrac{1}{2}\vec{q}\,')\,d^2q\,d^2q'\,d^2q'' \qquad , \tag{4}$$

for $\pi^+\pi^+$ collisions, where the quark amplitudes are to be evaluated at the momen-
tum. μk (which we have suppressed). We have chosen units in which $\hbar=1$.

Let us consider two extreme cases regarding the binding of the quark-
antiquark pair. If we assume that the $p\bar{n}$ system is very strongly bound, we may
approximate $S(\vec{q})$ by unity in the integrals (2)-(4). Thus $\delta\sigma_2$, for example, would
become

$$\delta\sigma_2^{(s)} = (2/\mu^2 k^2)\,\text{Re}\!\int[\,4f(p\bar{n},\vec{q})f(pp,-\vec{q}) + f(pp,\vec{q})f(pp,-\vec{q}) + f(p\bar{n},\vec{q})f(p\bar{n},-\vec{q})]\,d^2q \quad , \tag{5}$$

where the superscript (s) denotes the strong-binding limit.

The other extreme, in which the $p\bar{n}$ system is very weakly bound, leads to
formulae containing expectation values of expressions involving the quark-antiquark
separation r (or ρ) in the π^+ meson. For example, $\delta\sigma_2$ is given simply by

$$\delta\sigma_2^{(w)} = (4\pi/\mu^2 k^2)\,\text{Re}\Big\{4f(p\bar{n},0)f(pp,0)\langle r^{-2}\rangle_{\pi^+} + [\,f^2(pp,0) + f^2(p\bar{n},0)]$$

$$\times\, \langle\langle(1/2r\rho)\ln[(r+\rho)/|r-\rho|]\rangle_{\pi^+}\rangle_{\pi^+}\Big\} \qquad , \tag{6}$$

where the symbol $\langle\ \rangle_{\pi^+}$ denotes the expectation value in the $p\bar{n}$ bound state, i.e.,
in the π^+ state. The superscript (w) indicates the weak-binding limit.

By including multiple-scattering corrections, cross section sum rules may
be derived under certain simplifying assumptions. It should be clear that for a
given set of assumptions regarding the various quark amplitudes, any sum rule we
obtain by including multiple-scattering effects would have to be a linear combina-
tion of the sum rules obtained from the usual additivity assumption. The reason
for this may be seen by noting that the sum rules are independent of the values of
the various quark amplitudes. Therefore if the multiple-scattering amplitudes
happened to all vanish, we would be in a situation where the additivity assumption
was correct, and consequently the "new" sum rules would not be independent of those
already derived. However, it is important to note that the inverse statement is not
correct.   A sum rule derived from additivity would not in general be any linear
combination of sum rules derived by including multiple-scattering effects. Conse-
quently, a sum rule derived by including multiple-scattering corrections would
perhaps indicate which combinations of the familiar sum rules are to be preferred.

For the remainder of our discussion we consider the strong-binding limit,
treat only double-scattering "corrections" to the additivity assumption, and
assume $f(pp) = f(pn) = f(\bar{p}n) = f(\lambda p) = f(\bar{\lambda}p) \neq f(\bar{p}p)$. We exclude $f(\bar{p}p)$ from this
assumption since an isosinglet annihilation channel may be expected to give signif-
icant contribution and break the Pomeranchuk theorem for amplitudes which can
have an isosinglet component.[12] The sum rule we derive under these assumptions is

$$\sigma(K^-N) - \tfrac{1}{2}\sigma(K^-P) - \tfrac{1}{2}\sigma(K^+P) = \tfrac{1}{4}\sigma(\bar{P}N) - \tfrac{1}{5}\sigma(\bar{P}P) - \tfrac{1}{20}\sigma(PP) \ , \qquad (7)$$

where P = proton and N = neutron. The comparison with the measurements is

shown in Table I. It is seen that five of the six predictions agree well with

experiment. The remaining one lies within two standard deviations of the measure-

ments.[18]

We have calculated separately the single- and double-scattering contribu-

tions to the total cross sections. The single-scattering contributions may con-

sist of two types of terms. One corresponds to $\bar{p}p$ collisions and is proportional

to the antiquark-quark "total cross section" $\sigma(\bar{p}p)$. The other corresponds to pp,

pn, $\bar{p}n$, λp, and $\bar{\lambda}p$ collisions and is proportional to $\sigma(pp)$. A typical calculated

value for the "cross section" $\sigma(\bar{p}p)$ at 12 GeV/c laboratory momentum is (7.6 ± 6.3)mb.

The large uncertainty is due to the large experimental uncertainty in the antiproton-

neutron total cross section measurements. The corresponding value for $\sigma(pp)$ is

calculated to be only (0.2 ± 0.3)mb. These values suggest that within a hadron

quarks rarely engage in single collisions, except possibly for $\bar{p}p$ collisions.

This does not imply, however, that in hadron-hadron collisions/annihilation processes,
                                                                        the

which presumably represent a substantial part of $\bar{p}p$ collisions, necessarily result

mainly from single collisions. That part of $\sigma(\bar{p}p)$ corresponding to absorptive pro-

cesses is equal to the cross section for the absorption of a $\bar{p}p$ pair in hadron-

hadron collisions regardless of what else happens to the remaining quarks and

antiquarks during the collision. For proton-antiproton collisions, for example,

it includes absorptive processes in which three quark-antiquark pairs annihilate.

A relatively large value for $\sigma(\bar{p}p)$ is therefore not incompatible with the proposal of

Kokkedee and Van Hove[15] that antibaryon-baryon annihilation takes place mainly

via the annihilation of three quark-antiquark pairs.

The double-scattering corrections consist of three types of terms. Those which are quadratic in $f(\bar{p}p)$ are found to be negative at the six energies for which data exist. This is to be expected for strongly absorptive processes. For purely absorptive (opaque) interactions it may be explained in part as a "shadow" correction in a manner similar to the interpretation given by Glauber[19] for hadron-deuteron absorption cross sections, and as an additional correction for counting some absorptive processes more than once when using the additivity assumption. The remaining two types of double-scattering corrections, those quadratic in $f(pp)$ and those bilinear in $f(pp)$ and $f(\bar{p}p)$, yield positive contributions.

The two types of single-scattering contributions and their sums, and the sum of the three types of double-scattering contributions are shown in Table II for total cross sections at 12 GeV/c. The uncertainties in the values shown in the second and third columns are 86% and 153%, respectively. We note that the double-scattering contribution alone yields almost the entire PN and K^+P cross sections. This is related to the property that the $\bar{p}p$ (or $\bar{n}n$) combination appears in none of the quark collisions involved in these two cross sections. Empirically, the relative importance of double-scattering appears to decrease with the number of $\bar{p}p$ or $\bar{n}n$ combinations that can be formed from the quark constituents of the two colliding hadrons. The rather large double-scattering contributions we have calculated for some cross sections suggests that triple- or even higher-order multiple-scattering contributions may be significant. We might point out that assumptions regarding the quark amplitudes which are less restrictive than those we have considered may be used to obtain additional sum rules. These sum rules, however, have the disadvantage that they can not be tested so readily as they involve cross sections such as $\sigma(\Lambda p)$, for example, which have not been measured with such great accuracy. Details of such calculations will be reported elsewhere.

Footnotes and References

*This work was done under the auspices of the U. S. Atomic Energy Commission.

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16. Isospin and charge conjugation invariance are assumed throughout.

17. For simplicity we omit those contributions to the hadron-hadron forward elastic scattering amplitudes which arise from multiple interactions in which the identity of three or more quarks change during the collision and the final states of the two bound systems are identical to their initial states. These types of reactions have an analog in proton-deuteron collisions, for example, where an elastic double charge-exchange process may occur. In that case the incident proton charge exhanges with the neutron. The emerging particle, now a neutron, then charge exchanges with the target proton. The final configuration of the target is a neutron-proton system which may be bound, and the final emerging particle is once again a proton. See C. Wilkin, Phys. Rev. Letters $\underline{17}$, 561 (1966) and Ref. 2 for more detailed discussions. A double collision which we omit for $\pi^+\pi^+$ scattering in the quark model is $\bar{n}p \to p\bar{n}$ followed by $p\bar{n} \to \bar{n}p$, with the final state of each of the two $\bar{n}p$ systems being the π^+ state.

18. Our assumptions also imply $\sigma(K^+P) = \sigma(K^+N)$, $\sigma(PP) = \sigma(PN)$, $\sigma(\pi^-P) = \sigma(K^-P)$, and $\sigma(\pi^+P) = \sigma(K^-N)$. The first two relations are in agreement with experiment. In the last two relations a discrepancy of ~ 4 mb exists.

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Table I. Tests of the sum rule $\sigma(K^-N) - \frac{1}{2}\sigma(K^-P) - \frac{1}{2}\sigma(K^+P)$
$= \frac{1}{4}\sigma(\bar{P}N) - \frac{1}{5}\sigma(\bar{P}P) - \frac{1}{20}\sigma(PP)$

Momentum (GeV/c)	$\sigma(K^-N) - \frac{1}{2}\sigma(K^-P) - \frac{1}{2}\sigma(K^+P)$ (mb)	$\frac{1}{4}\sigma(\bar{P}N) - \frac{1}{5}\sigma(\bar{P}P) - \frac{1}{20}\sigma(PP)$ (mb)
6	1.4 ± 0.4	1.0 ± 1.0
8	-0.8 ± 0.4	1.1 ± 1.0
12	0.8 ± 0.4	1.1 ± 0.9
14	0.7 ± 0.4	1.3 ± 0.9
16	1.2 ± 0.6	1.4 ± 0.9
18	1.3 ± 1.2	-1.0 ± 2.4

Table II. Quark single- and double-collision contributions to total cross sections at 12 GeV/c (in mb)

Cross section	$\bar{p}p$ single-collision contributions	$pp, pn, \bar{p}n, \lambda p, \bar{\lambda}p$ single-collision contributions	Total single-collision contribution	Double-collision contribution	Total cross section
$\sigma(PN)$	0	1.9	1.9 ± 2.9	38.5 ± 4.6	40.4
$\sigma(K^+P)$	0	1.2	1.2 ± 1.9	16.1 ± 1.9	17.3
$\sigma(\pi^+P)$	7.6	.1.0	8.6 ± 6.5	15.6 ± 1.6	24.2
$\sigma(\pi^-P)$	15.2	0.8	16.0 ± 12.7	12.9 ± 1.9	28.9 ± 12.8
$\sigma(\bar{P}N)$	30.3	1.0	31.3 ± 25.2	22.5 ± 5.8	53.8
$\sigma(\bar{P}P)$	37.9	0.8	38.7 ± 31.4	13.0 ± 7.7	51.7