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Universality and m_X cut effects in $B \to X_s \ell^+\ell^-$

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The most precise comparison between theory and experiment for the $B \to X_s \ell^+\ell^-$ rate is in the $q^2 < 6\,\mathrm{GeV}^2$ region. The hadronic uncertainties associated with an experimentally required cut on m_X potentially spoil the extraction of short distance flavor-changing neutral current couplings. We compute the m_X cut dependence of $\mathrm{d}\Gamma(B \to X_s \ell^+\ell^-)/\mathrm{d}q^2$ using the $B \to X_s\gamma$ shape function, and show that the effect is universal for all short distance contributions in the limit $m_X^2 \ll m_B^2$. This universality is not spoiled by realistic values of the m_X cut, nor by α_s corrections. Alternatively, normalizing the $B \to X_s \ell^+\ell^-$ rate to $B \to X_u \ell\bar\nu$ with the same cuts removes the main uncertainties. We find that the forward-backward asymmetry vanishes near $q_0^2 = 3\,\mathrm{GeV}^2$.

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I. INTRODUCTION

In the standard model (SM) the flavor-changing neutral current process $B \to X_s \ell^+\ell^-$ does not occur at tree level, and is a sensitive probe of new physics. Predicting its rate involves integrating out the W, Z, and t at a scale of order m_W by matching on to the Hamiltonian [1, 2]

$$H_W = -\frac{G_F}{\sqrt{2}}\, V_{tb}V_{ts}^* \bigg[\sum_{i=1}^{6} C_i\, O_i + \frac{1}{4\pi^2}\sum_{i=7}^{10} C_i\, O_i\bigg], \qquad (1)$$

evolving to $\mu = m_b$, and computing matrix elements of H_W. Here $O_1 - O_6$ are four-quark operators and

$$\begin{aligned} O_7 &= \overline{m}_b\, \bar s \sigma_{\mu\nu} e F^{\mu\nu} P_R b, \qquad O_8 = \overline{m}_b\, \bar s \sigma_{\mu\nu} g G^{\mu\nu} P_R b, \\ O_9 &= e^2 (\bar s \gamma_\mu P_L b)(\bar\ell \gamma^\mu \ell), \quad O_{10} = e^2 (\bar s \gamma_\mu P_L b)(\bar\ell \gamma^\mu\gamma_5 \ell), \end{aligned} \qquad (2)$$

where $P_{L,R} = (1 \mp \gamma_5)/2$. The dilepton invariant mass spectrum, $q^2 = (p_{\ell^+} + p_{\ell^-})^2$, can be calculated in an operator product expansion (OPE), and the leading nonperturbative corrections are suppressed by $\Lambda_{\mathrm{QCD}}^2/m_b^2$ [3, 4]. The matching and anomalous dimension calculations for C_i are known at next-to-next-to-leading log (NNLL) order [5, 6, 7], as are the largest perturbative QCD corrections to the matrix elements of O_i [7].

An important complication in $B \to X_s \ell^+\ell^-$ compared to $B \to X_s\gamma$ is that the long distance contributions, $B \to J/\psi X_s$ and $\psi' X_s$ followed by $J/\psi,\, \psi' \to \ell^+\ell^-$, are an order of magnitude larger than the short distance prediction, a fact which is not well-understood. Therefore, either theory and data are both interpolated, or the short distance calculation is compared with the data for $q^2 < m_{J/\psi}^2$ or $q^2 > m_{\psi'}^2$. The low q^2 region, $q^2 < 6\,\mathrm{GeV}^2$, allows the most precise comparison with the SM, but requires a cut on the invariant mass of the hadronic final state, $m_X < m_X^{\mathrm{cut}}$. In the latest Belle analysis $m_X^{\mathrm{cut}} = 2\,\mathrm{GeV}$ [8], while Babar uses $m_X^{\mathrm{cut}} = 1.8\,\mathrm{GeV}$ [9]. These cuts are to remove backgrounds, and will likely be required for quite some time [10]. The high q^2 region is unaffected by the m_X cut, but the rate is lower, and calculating it involves an expansion in $\Lambda_{\mathrm{QCD}}/(m_b - \sqrt{q^2})$.

In this letter we investigate the effects of the m_X cut on predictions for $B \to X_s \ell^+\ell^-$ decay in the low q^2 region. This was previously studied in the Fermi-motion model in Ref. [11]. For $(m_X^{\mathrm{cut}})^2 = \mathcal{O}(\Lambda_{\mathrm{QCD}} m_b)$, the local OPE breaks down, and is replaced by an OPE involving nonlocal operators, whose matrix elements are b quark distribution functions in the B meson. We define

$$\begin{aligned} \Gamma_{ij}^{\mathrm{cut}} &= \int_{q_1^2}^{q_2^2} \mathrm{d}q^2 \int_0^{m_X^{\mathrm{cut}}} \mathrm{d}m_X\, \mathrm{Re}(c_i c_j^*)\, \frac{\mathrm{d}^2\Gamma_{ij}}{\mathrm{d}q^2 \mathrm{d}m_X} \\ &= \eta_{ij}\big(m_X^{\mathrm{cut}}, q_1^2, q_2^2\big)\, \frac{\Gamma_0}{m_B^5} \int_{q_1^2}^{q_2^2} \mathrm{d}q^2\, \mathrm{Re}(c_i c_j^*)\, \frac{(m_b^2 - q^2)^2}{m_b^3}\, G_{ij}, \end{aligned} \qquad (3)$$

where $ij = \{77,\, 99,\, 00,\, 79\}$ label contributions of time-ordered products $T\{O_j^\dagger, O_i\}$. The η_{ij}'s contain the effects of the m_X cut, and the short distance coefficients $c_{7,9,0}$ track the $C_{7,9,10}$ dependence in Eq. (1). Here $c_7 = C_7^{\mathrm{mix}}(q^2)$, $c_9 = C_9^{\mathrm{mix}}(q^2)$, and $c_0 = C_{10}$ can be obtained from local OPE calculations [12] at each order, as discussed in Ref. [13]. The functions $G_{99,00} = (2q^2 + m_b^2)$, $G_{77} = 4m_B^2(1 + 2m_b^2/q^2)$, and $G_{79} = 12 m_B m_b$ arise from kinematics, where m_b is a short distance mass, such as m_b^{1S} [14], here and below. Finally,

$$\Gamma_0 = \frac{G_F^2 m_B^5}{192\pi^3}\, \frac{\alpha_{\mathrm{em}}^2}{4\pi^2}\, |V_{tb}V_{ts}^*|^2 \,. \qquad (4)$$

We also study $\eta'_{ij}(p_X^{+\mathrm{cut}}, q_1^2, q_2^2)$, which are defined by replacing m_X in Eq. (3) with $p_X^+ = E_X - |\vec p_X|$. The total rate for $B \to X_s \ell^+\ell^-$ with cuts is $\Gamma^{\mathrm{cut}} = \sum_{ij} \Gamma_{ij}^{\mathrm{cut}}$.

At leading order in $\Lambda_{\mathrm{QCD}}/m_b$ and α_s, $\eta_{ij} = 1$ for $m_X^{\mathrm{cut}} = m_B$, and therefore η_{ij} give the fraction of events with $m_X < m_X^{\mathrm{cut}}$. This is altered at subleading order by perturbative corrections, but η_{ij} still determine the rate. In principle, η_{ij} depend in a nontrivial way on ij (and q_1^2 and q_2^2) due to different dependence on kinematic variables, α_s corrections, etc. Working to leading order in $\Lambda_{\mathrm{QCD}}/m_b$, we demonstrate that η_{ij} are independent of the choice of ij, which we call “universality”. We first show this formally at leading order in $p_X^+/m_B \ll 1$ for the p_X^+ cut, η', and then numerically for the experimentally relevant m_X^{cut}, η, including the α_s corrections and all phase space effects. Since the same shape function occurs in $B \to X_s \ell^+\ell^-$, $X_u \ell\bar\nu$, and $X_s\gamma$, the m_X^{cut} or $p_X^{+\mathrm{cut}}$ dependence in one can be determined from the others.

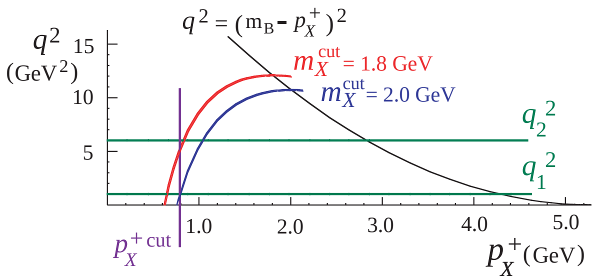

FIG. 1: Phase space cuts. A substantial part of the rate for $q_1^2 < q^2 < q_2^2$ falls in the rectangle bounded by $p_X^+ < p_X^{+\mathrm{cut}}$.

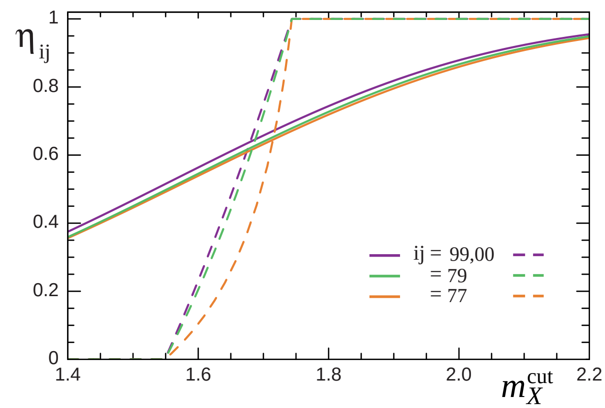

FIG. 2: $\eta_{ij}(m_X^{\mathrm{cut}}, 1\,\mathrm{GeV}^2, 6\,\mathrm{GeV}^2)$ as functions of m_X^{cut}. The dashed curves show the local OPE result, the solid curves include the leading shape function effects. The up-most curves are $\eta_{00} = \eta_{99}$, the middle ones are η_{79}, the lowest ones are η_{77}.

II. m_X CUT EFFECTS AT LEADING ORDER

For simplicity, consider the kinematics in the B meson's rest frame. Since $q = p_B - p_X$,

$$2m_B E_X = m_B^2 + m_X^2 - q^2. \tag{5}$$

If $m_X^2 \ll m_B^2$ and q^2 is not near m_B^2, then $E_X = \mathcal{O}(m_B)$. Since $E_X^2 \gg m_X^2$, p_X is near the light-cone, with $p_X^+ = E_X - |\vec{p}_X| = \mathcal{O}(\Lambda_{\mathrm{QCD}})$ and $p_X^- = E_X + |\vec{p}_X| = \mathcal{O}(m_B)$. Of the variables symmetric in p_{ℓ^+} and p_{ℓ^-} ($p_X^\pm$, E_X, q^2, m_X^2), only two are independent, and we work with q^2 and p_X^+ or m_X. The phase space cuts are shown in Fig. 1.

For the $p_X^+ \ll p_X^-$ region, factorization of the form $\mathrm{d}\Gamma = HJ \otimes \hat{f}^{(0)}$ has been proven for semileptonic and radiative B decays [15], where H contains perturbative physics at $\mu_b \sim m_b$, J at $\mu_i \sim \sqrt{\Lambda_{\mathrm{QCD}} m_b}$, and $\hat{f}^{(0)}(\omega)$ is a universal nonperturbative shape function. This factorization also applies for $B \to X_s \ell^+\ell^-$ with the same universal $\hat{f}^{(0)}$, as long as q^2 is not parametrically small [13].

In the $q^2 < 6\,\mathrm{GeV}^2$ region, $|C_9^{\mathrm{mix}}(\mu_0 = 4.8\,\mathrm{GeV})| = 4.52$ to better than 1%, and can be taken to be constant. We neglect α_s corrections in this section and find

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}p_X^+ \mathrm{d}q^2} &= \hat{f}^{(0)}(p_X^+)\, \frac{\Gamma_0}{m_B^5}\, \frac{[(m_B - p_X^+)^2 - q^2]^2}{(m_B - p_X^+)^3} \\ &\times \Big\{ (|C_9^{\mathrm{mix}}|^2 + C_{10}^2)\big[2q^2 + (m_B - p_X^+)^2\big] \\ &\quad + 4m_B^2\, |C_7^{\mathrm{mix}}|^2 \Big[1 + \frac{2(m_B - p_X^+)^2}{q^2}\Big] \\ &\quad + 12 m_B\, \mathrm{Re}\big[C_7^{\mathrm{mix}} C_9^{\mathrm{mix}*}\big](m_B - p_X^+) \Big\}, \end{aligned} \tag{6}$$

where $\hat{f}^{(0)}(\omega)$ has support in $\omega \in [0, \infty)$. As a function of p_X^+, the kinematic terms in Eq. (6) vary *only* on a scale m_B, while $\hat{f}^{(0)}(p_X^+)$ varies on a scale Λ_{QCD}. Writing $m_B = m_b + \bar{\Lambda}$ and expanding in $(p_X^+ - \bar{\Lambda})/m_B$, decouples the p_X^+ and q^2 dependences in Eq. (6), and gives the local OPE prefactors, $(m_b^2 - q^2)^2\, G_{ij}(q^2)$, in Eq. (3). For $\eta'_{ij}(p_X^{+\mathrm{cut}}, q_1^2, q_2^2)$ the p_X^+ integration is over a rectangle in Fig. 1, whose boundaries do not couple p_X^+ and q^2. Thus, $\eta' = \int \mathrm{d}p_X^+\, \hat{f}^{(0)}(p_X^+)$, independent of ij and $q_{1,2}^2$. While the m_X cut retains more events than the p_X^+ cut, the latter may give theoretically cleaner constraints on short distance physics when statistical errors become small.

The effect of the m_X cut is q^2 dependent, because the upper limit of the p_X^+ integration is q^2 dependent, as shown in Fig. 1. Including the full p_X^+ dependence in Eq. (6), the universality of $\eta_{ij}(m_X^{\mathrm{cut}}, q_1^2, q_2^2)$ is maintained to better than 3% for $1\,\mathrm{GeV}^2 \le q_1^2 \le 2\,\mathrm{GeV}^2$, $5\,\mathrm{GeV}^2 \le q_2^2 \le 7\,\mathrm{GeV}^2$, and $m_X^{\mathrm{cut}} \ge 1.7\,\mathrm{GeV}$, because the region where the p_X^+ and q^2 integration limits are coupled has a small effect on the ij dependence. This is exhibited in Fig. 2, where the solid curves show $\eta_{ij}(m_X^{\mathrm{cut}}, 1\,\mathrm{GeV}^2, 6\,\mathrm{GeV}^2)$ with the shape function set to model-1 of [16] with $m_b^{1S} = 4.68\,\mathrm{GeV}$ and λ_1 from [17]. (Taking $q_1^2 = 1\,\mathrm{GeV}^2$ instead of $4m_\ell^2$ increases the sensitivity to $C_{9,10}$, but one may be concerned by local duality / resonances near $q^2 = 1\,\mathrm{GeV}^2$. To estimate this uncertainty, assume the ϕ is just below the cut and $\mathcal{B}(B \to X_s\phi) \approx 10 \times \mathcal{B}(B \to K^{(*)}\phi)$. Then $B \to X_s\phi \to X_s\ell^+\ell^-$ is $\sim$2% of the $X_s\ell^+\ell^-$ rate.)

The local OPE results for $\eta_{ij}(m_X^{\mathrm{cut}}, q_1^2, q_2^2)$ are obtained by replacing $\hat{f}^{(0)}(p_X^+)$ by $\delta(\bar{\Lambda} - p_X^+)$ in Eq. (6). Performing the p_X^+ integral sets $(m_B - p_X^+) = m_b$ and implies $m_X^2 > \bar{\Lambda}(m_B - q^2/m_b)$. This makes the lower limit on q^2 equal $\max\{q_1^2,\, m_b[m_B - (m_X^{\mathrm{cut}})^2/\bar{\Lambda}]\}$, and so the η_{ij}'s depend on the shape of $\mathrm{d}\Gamma_{ij}$. In Fig. 2 the local OPE results are shown by dashed lines, and clearly $\eta_{77} \neq \eta_{99}$. However, the local OPE is not applicable for $p_X^+ \sim \Lambda_{\mathrm{QCD}}$.

The universality of η_{ij} can be broken by α_s corrections in the hard and jet functions, or by renormalization group evolution, since these effects couple p_X^+ and q^2 and have been neglected so far. We consider these next.

III. CALCULATION AND RESULTS AT $\mathcal{O}(\alpha_s)$

A complication in calculating $B \to X_s\ell^+\ell^-$ compared to $B \to X_u\ell\bar{\nu}$ is that, in the evolution of the effective Hamiltonian down to m_b, $C_9(\mu)$ receives a $\ln(m_W^2/m_b^2)$

enhanced contribution from the mixing of O_2. Thus, formally, $C_9 \sim \mathcal{O}(1/\alpha_s)$, and conventionally one expands the amplitude in α_s, treating $\alpha_s \ln(m_W^2/m_b^2) = \mathcal{O}(1)$ [12]. In the local OPE this is reasonable, since the nonperturbative corrections are small, and at next-to-leading log (NLL) all dominant terms in the rate are included. However, in the shape function region nonperturbative effects are $\mathcal{O}(1)$ and only the rate is calculable. With the traditional counting the C_9^2 contribution to the rate would be needed to $\mathcal{O}(\alpha_s^2)$ before the C_{10}^2 terms could be included.

This would be a bad way to organize the perturbative corrections (numerically $|C_9(m_b)| \approx |C_{10}|$). It can be circumvented by using a "split matching" procedure to decouple the perturbation series above and below the scale m_b [13]. This allows us to consider the short distance coefficients $C_7^{\rm mix}$, $C_9^{\rm mix}$, and C_{10} as $\mathcal{O}(1)$ numbers when organizing the perturbation theory at m_b^2 and $m_b\Lambda_{\rm QCD}$.

The rate and the forward-backward asymmetry are

$$\begin{aligned}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}q^2\mathrm{d}p_X^+} &= \frac{\Gamma_0}{m_B^2}\, H(q^2,p_X^+)\, F^{(0)}(p_X^+,p^-)\,,\\ \frac{\mathrm{d}^2 A_{\rm FB}}{\mathrm{d}q^2\mathrm{d}p_X^+} &= \frac{\Gamma_0}{m_B^2}\, K(q^2,p_X^+)\, F^{(0)}(p_X^+,p^-)\,,\end{aligned} \tag{7}$$

where $p^- = m_b - q^2/(m_B - p_X^+)$. The hard functions H and K were computed in Ref. [13] using SCET [18, 19] and split matching, which factorizes the dependence on scales above and below m_b as $H_1(\mu_0)H_2(\mu_b)$. Here, to the order one is working at, H_1 is μ_0 independent, the μ_b dependence in H_2 and $F^{(0)}$ cancels, and $F^{(0)}$ is μ_i independent. The shape function model is specified at μ_Λ. The convolution of jet and shape functions at NLL including $\mathcal{O}(\alpha_s)$ corrections is

$$\begin{aligned}F^{(0)}(p_X^+,p^-) &= U_H(p^-,\mu_i,\mu_b)\bigg(\hat f^{(0)}(p_X^+,\mu_i) + \frac{\alpha_s(\mu_i)C_F}{4\pi}\bigg\{\Big[2\ln^2\frac{p_X^+p^-}{\mu_i^2} - 3\ln\frac{p_X^+p^-}{\mu_i^2} + 7 - \pi^2\Big]\hat f^{(0)}(p_X^+,\mu_i)\\ &\qquad + \int_0^1 \frac{\mathrm{d}z}{z}\Big[4\ln\frac{zp_X^+p^-}{\mu_i^2} - 3\Big]\Big[\hat f^{(0)}(p_X^+(1-z),\mu_i) - \hat f^{(0)}(p_X^+,\mu_i)\Big]\bigg\}\bigg),\\ \hat f^{(0)}(\omega,\mu_i) &= \frac{e^{V_S(\mu_i,\mu_\Lambda)}}{\Gamma(1+\eta)}\left(\frac{\omega}{\mu_\Lambda}\right)^\eta \int_0^1 \mathrm{d}t\, \hat f^{(0)}\big[\omega(1-t^{1/\eta}),\mu_\Lambda\big],\end{aligned} \tag{8}$$

where U_H was computed in Ref. [18], the one-loop jet function in Ref. [20, 21], and the shape function evolution up to μ_i in Refs. [18, 21] (for earlier calculations, see Refs. [15, 22]). The H and K are

$$\begin{aligned}H(q^2,p_X^+) &= \frac{[(1-\hat p_X^+)^2 - \hat q^2]^2}{(1-\hat p_X^+)^3}\bigg\{\Big[|C_9^{\rm mix}(s,\mu_0)|^2 + C_{10}^2\Big]\Big[2\hat q^2\,\Omega_A^2(s,\mu_b) + (1-\hat p_X^+)^2\,\Omega_B^2(s,\hat p_X^+,\mu_b)\Big]\\ &\qquad + 4|C_7^{\rm mix}(\mu_0)|^2\Big[\Omega_C^2(s,\mu_b) + \frac{2(1-\hat p_X^+)^2}{\hat q^2}\,\Omega_D^2(s,\mu_b)\Big] + 12{\rm Re}\big[C_7^{\rm mix}(\mu_0)C_9^{\rm mix}(s,\mu_0)^*\big](1-\hat p_X^+)\Omega_E(s,\mu_b)\bigg\}\\ K(q^2,p_X^+) &= -\frac{3\hat q^2[(1-\hat p_X^+)^2-\hat q^2]^2}{(1-\hat p_X^+)^3}\,\Omega_A(s,\mu_b)\,{\rm Re}\Big\{C_{10}^*\Big[C_9^{\rm mix}(s,\mu_0)\Omega_A(s,\mu_b) + \frac{2(1-\hat p_X^+)}{\hat q^2}\,C_7^{\rm mix}(\mu_0)\Omega_D(s,\mu_b)\Big]\Big\},\end{aligned} \tag{9}$$

where $s = q^2/m_b^2$, $\hat q^2 = q^2/m_B^2$, $\hat p_X^+ = p_X^+/m_B$, and

$$\begin{aligned}\Omega_A &= 1 + \frac{\alpha_s}{\pi}\,\omega_a^V(s,\mu_b), \qquad \Omega_C = 1 + \frac{\alpha_s}{\pi}\,\omega_a^T(s,\mu_b),\\ \Omega_B &= 1 + \frac{\alpha_s}{\pi}\Big[\omega_a^V(s,\mu_b) + \frac{(1-\hat p_X^+)^2 - \hat q^2}{2(1-\hat p_X^+)^2}\omega_b^V(s) + \omega_c^V(s)\Big],\\ \Omega_D &= 1 + \frac{\alpha_s}{\pi}\big[\omega_a^T(s,\mu_b) - \omega_c^T(s)\big],\\ \Omega_E &= \big(2\Omega_A\Omega_D + \Omega_B\Omega_C\big)/3.\end{aligned} \tag{10}$$

Here $\alpha_s = \alpha_s(\mu_b)$ and $\omega_i^{V,T}$ are defined in Ref. [13].

In Fig. 3 we plot $\eta_{00}(m_X^{\rm cut}, 1\,{\rm GeV}^2, 6\,{\rm GeV}^2)$, including the α_s corrections. For each $\hat f^{(0)}$, the deviations of the η_{ij}'s from being universal is still below 3%. We use five different models for the shape function, constructed to obey the known constraints on its moments [21]. The orange, green and purple (medium, light, dark) curves correspond to $m_b^{1S} = 4.68\,{\rm GeV}$, $4.63\,{\rm GeV}$, and $4.73\,{\rm GeV}$, respectively, using the central values $\mu_0 = \mu_b = 4.8\,{\rm GeV}$ and $\mu_i = 2.5\,{\rm GeV}$. For $m_X^{\rm cut} = 2\,{\rm GeV}$, varying μ_b in the range $3.5\,{\rm GeV} < \mu_b < 7.5\,{\rm GeV}$ changes η_{00} by $\pm 6\%$. We find a $\pm 5\%$ variation for $2\,{\rm GeV} < \mu_i < 3\,{\rm GeV}$. The curves with slightly lower [higher] values of η_{00} at large $m_X^{\rm cut}$ correspond to $\mu_\Lambda = 1.5\,{\rm GeV}$ [$2\,{\rm GeV}$].

The μ_0 dependence of the rate is similar to that in the local OPE, and will be reduced by including the known NNLL corrections [5, 6, 7]. We did not study it here.

Using the c_i's at NLL, for $1\,{\rm GeV}^2 < q^2 < 6\,{\rm GeV}^2$ and $m_X^{\rm cut} = 1.8$ and $2.0\,{\rm GeV}$, we obtain $\Gamma^{\rm cut}\tau_B = (1.20 \pm 0.15)\times 10^{-6}$ and $(1.48 \pm 0.14)\times 10^{-6}$, respectively.

The largest uncertainty in the rate and the largest source of universality breaking in the η_{ij}'s are due to sub-

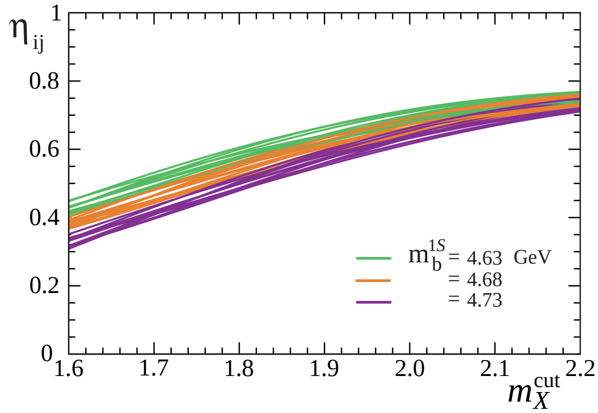

FIG. 3: $\eta_{00}(m_X^{\rm cut}, 1\,{\rm GeV}^2, 6\,{\rm GeV}^2)$ as a function of $m_X^{\rm cut}$. The orange, green and purple (medium, light, dark) curves show $m_b^{1S} = 4.68\,{\rm GeV}$, 4.63 GeV, and 4.73 GeV, respectively.

leading shape functions, which affect the rate by $\sim 5\%$ for $m_X^{\rm cut} = 2\,{\rm GeV}$ and by $\sim 10\%$ for $m_X^{\rm cut} = 1.8\,{\rm GeV}$ [23].

If the $m_X^{\rm cut}$ dependence were not universal, it would modify the zero of the forward-backward asymmetry, $A_{\rm FB}(q_0^2) = 0$. For $m_X^{\rm cut} = 2\,{\rm GeV}$ we find at NLL $\Delta q_0^2 \approx -0.04\,{\rm GeV}^2$, much below the higher order uncertainties [7]. However, we obtain $q_0^2 = 2.8\,{\rm GeV}^2$, lower than earlier results [6]. In the local OPE limit we get $q_0^2 = 2m_b[\overline{m}_b(\mu) C_7^{\rm eff}(\mu)]/{\rm Re}[C_9^{\rm eff}(q_0^2)]$. Here m_b can be taken to be $m_b^{\rm pole}$ or expanded about m_b^{1S}, but to ensure that the μ dependence cancels at the order we are working, we cannot reexpand $\overline{m}_b(\mu)$ in terms of $m_b^{\rm pole}$.

In conclusion, we pointed out that the experimentally used upper cut on m_X makes the observed $B \to X_s \ell^+\ell^-$ rate in the low q^2 region sensitive to the shape function. In this region there is an OPE only for the decay rate and not for the amplitude, necessitating a reorganization of the usual perturbation expansion. Since one can use the shape function measured in other processes, the sensitivity to new physics is not reduced. We found that the η's for the different operators' contributions are universal to a good approximation. The theoretical uncertainties are reduced by raising the $m_X^{\rm cut}$. Another possibility is to keep $m_X^{\rm cut} < m_D$ and measure with the same cuts

$$R = \Gamma^{\rm cut}(B \to X_s \ell^+\ell^-)/\Gamma^{\rm cut}(B \to X_u \ell \bar\nu), \qquad (11)$$

since the effect of $m_X^{\rm cut}$, as well as the m_b dependence, are drastically reduced in this ratio. These results also apply for $B \to X_d \ell^+\ell^-$, which may be studied at a higher luminosity B factory. Subleading $\Lambda_{\rm QCD}/m_b$ as well as NNLL corrections to the rate and the forward-backward asymmetry will be studied in a separate publication [23].

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