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ABSTRACT

We have intercompared geomagnetic field models in B-L space for $0.20 \leq B \leq 0.24$ gauss and $1.2 \leq L \leq 1.8$ R_e (earth radii). Three field models were selected because of their general usage in the analysis of trapped radiation data: Jensen and Whitaker (569 coefficient); Jensen and Cain (48 coefficient); GSFC (9/65)(99 coefficient). These models were compared with the GSFC (12/66) field model (120 coefficient). The geographic coordinates of constant B-L traces were computed using the GSFC (12/66) field in both the southern and northern hemispheres. At each geographical point along the traces thus defined, B and L values were recalculated using different geomagnetic field models. We find that variations in B-L space of the 48- and 569-coefficient models with respect to the 120-coefficient model are great enough to cause significant ambiguities in flux contours of the trapped radiation. We also have examined the effects of temporal variations of the geomagnetic

field on B-L space. The uncertainties in the proton flux contours in B-L space caused by errors in the field models and time variations of the geomagnetic field demonstrate the need for careful reevaluation of existing data that pertain to possible time variations of inner-belt protons. The GSFC (12/66) appears to be sufficiently accurate to undertake such reevaluation.

INTRODUCTION

Since its inception, B-L space has been used extensively in the study of radiation trapped in the earth's magnetic field. B-L space is a two-dimensional, longitude and hemisphere independent, coordinate system, developed by McIlwain, [1961], in which the omnidirectional flux of geomagnetically trapped radiation can be mapped. The B-L coordinate system is applicable to fields that do not possess large azimuthal asymmetries, thereby limiting its use to $L \lesssim 5\ R_e$ in the geomagnetic field. Stone [1963] has shown that the B-L coordinates for mirroring particles accurately reflect the invariant shells (I, B_m) describing the adiabatic invariants of charged particle motion [Northrop and Teller, 1960]. Theoretically, particle flux measurements may be reduced and compared accurately in B-L space. However, a geomagnetic field model must be used in the calculation of B and L, and errors dependent on the model can be significant. If the geomagnetic field model used in calculating B and L does not accurately describe the earth's magnetic field, then a real mirror point trajectory will not be represented by a point in B-L space. Flux mapping in such a poorly defined B-L coordinate system would depend on the longitude and the hemisphere where the data were obtained, and the utility of such a B-L system would break down in regions of steep flux gradients. In order to study variations of measured particle fluxes in B-L space, it is necessary to know the variations which could be introduced by the generating geomagnetic field model.

A number of geomagnetic field models are available for the calculation of B and L values. We have selected the following four field models for the present study:

1. The Jensen and Whitaker 569-coefficient spherical-harmonic expansion model (JW) [Jensen and Whitaker, 1960, Dudziak, et al., 1963];

2. The Jensen and Cain 48-coefficient model (JC) [Jensen and Cain, 1962];

3. The GSFC (9/65) 99-coefficient model [Hendricks and Cain, 1966];

4. The GSFC (12/66) 120-coefficient model [Cain et al., 1967a].

We chose the first three field models because of their use in the analyses of trapped-radiation data. The JW model, for instance, was used in the reduction of the Explorer 4 flux data [McIlwain, 1961]. The JC model is considered to be the standard field model for the interpretation of trapped-particle data [Walt, 1966]. Both the JW and and JC models are static, constructed to represent the geomagnetic field in 1955 and 1960 respectively. The GSFC (9/65) model contains first-time derivatives in the first 48 coefficients and is more accurate than the JW and JC models [Hendricks and Cain, 1966]. The GSFC (12/66) field is the latest field model, and was constructed from OGO-2 satellite magnetic-field measurements as well as the data used in the construction of GSFC (9/65). The GSFC (12/66) contains both first- and second-time derivatives in all its coefficients. Actually, there are two sets of coefficients labeled GSFC (12/66) -- Sets I and II. Both sets were constructed in the same manner using semi-independent samples of the same data. We have chosen arbitrarily to use Set I for our study.

We have limited our study to the region in B-L space bounded by $0.20 \leq B \leq 0.24$ gauss and $1.2 \leq L \leq 1.8\ R_e$ (earth radii). The lower portion of the inner radiation belt is contained in this interval of B-L values. The gradient of the trapped-particle flux is large along B in this region [Valerio, 1964], and small errors in the computation of B can result in significant errors in the calculated particle flux. In this B-L region, ionization and nuclear collision in the atmosphere are the dominant particle loss mechanisms. The solar cycle changes in the atmosphere will affect changes in particle loss rates, and, hence, particle fluxes. Any valid observation of such changes in the particle flux and loss rates with respect to the solar cycle must clearly take account of the accuracy of the flux representations.

Cain, et al. [1965, 1966, 1967a, b] have made direct comparisons of the JC, GSFC (9/65) and GSFC (12/66) field models with the earth's magnetic field over all longitudes and latitudes. It is concluded by Cain, et al., [1967) that the best current model of the main geomagnetic field is the GSFC (12/66). Excluding OGO-2 satellite data, this model fits the magnetic survey measurements to an accuracy of $\sigma = \pm 122\gamma$, ($1\gamma = 10^{-5}$ gauss) where σ is the standard deviation of the residuals of a random 10% of all survey observations (since 1900). This result is to be compared with the accuracies of the GSFC (9/65) and JC models, $\sigma = \pm 220\gamma$ and $\pm 440\gamma$ [Cain, 1966], respectively. However, when a separate distribution of the residuals is calculated for the OGO-2 data, Cain et al. [1967a] find that the GSFC (12/66) field reproduces the OGO-2 data to $\sigma = \pm 11.7\gamma$. Cognizant of the fact that there are possibilities for systematic errors in the satellite data of the order 10-20γ, Cain et al. [1967a] finally conclude that the surface field is probably no further in error than a few tens of gammas.

Of particular relevance to the analysis of particle data is the character of the geomagnetic field in the South Atlantic anomaly. The anomaly is not only the site of large particle flux gradients, but also is the region where the relative variations of the selected geomagnetic field models are greatest. For this reason, we examined this specific region for intercomparing the four field models within the range of B-L values we are considering. We obtained from the U. S. Coast and Geodetic Survey magnetic-field measurements taken in the region of the South Atlantic anomaly (-60° to 0° longitude, -45° to -15° latitude) for the period 1900-1965. OGO-2 satellite measurements were included in the set. Computed values of the geomagnetic field, B, were compared with the survey data points for each field model. The mean difference and standard deviation from the mean were computed for the following periods:

a) 1900 through 1963,
b) 1955 through 1963, and
c) 1965.

The results appear in Table I.

Because we are concerned here with the relative accuracies of the models, we did not weight the U. S. Coast and Geodetic Survey data with respect to reliability of the types of geomagnetic field measurements, nor did we invoke a rejection level to eliminate anomalously high deviations in the data. For 1955 through 1963, the GSFC (9/65) and GSFC (12/66) models describe the anomaly region equally well. For 1900 through 1963 GSFC (12/66) is the best model, and the importance of the second time derivatives in the coefficients is clearly evident. We conclude, therefore, that the GSFC (12/66) field model exhibits the highest accuracy of the several models studied within the anomaly region. Not unexpectedly, the results tabulated in Table I confirm the general conclusions of Cain, et al. [1967a].

The above comparisons of the errors in the magnitude of the geomagnetic field are not, however, a sufficient measure of the model-dependent errors in B-L space. Since a point in B-L space represents the trajectory of the mirror points of a particle trapped in the geomagnetic field, it is necessary to compare the geomagnetic field models over complete B-L traces to reveal the differences that may exist between field models. On the basis of the above discussion, we have selected the GSFC (12/66) field model as the reference field with which the other models were compared. We have made this comparison by (a) calculating the geographic coordinates at various longitudes for a given point in B-L space in the northern and southern hemispheres, as defined by the GSFC (12/66) model, and (b) calculating B and L values for these particular geographic coordinates as computed by the other spherical-harmonic field expansions.

By performing step (b) with the GSFC (12/66), the differences between the given B-L trace and that derived in (b) can be attributable to computational errors only. We find that the maximum computational error in this procedure is ± 10γ in B and ± 0.0005 R_e in L. These errors correspond to a 1-2 kilometer maximum error in locating geographic coordinates given prefixed values of B and L.

METHOD

Using the GSFC (12/66) field model, we calculated sets of geographic coordinates of B-L traces for B values of 0.20 to 0.24 gauss in steps of 0.01 gauss and L values of 1.2 to 1.8 R_e in steps of 0.2 R_e.

We generated these sets for the years 1955 to 1975 in 5-year steps for both the northern and southern hemispheres. The geographic contours were calculated for a given B and L value at 10° intervals in longitude, and in the South Atlantic anomaly region at 5 and 2.5° intervals. By holding B, L, and longitude constant and searching for the altitude and latitude, we located geographic coordinates of a B-L trace with a variation of the computer program SHELL [Roederer and Herod, 1966]. For the computation of B and L we used the computer program INVAR, expanding the subroutine NEWMAG to handle the JW coefficients [McIlwain, 1966]. Given the geographic coordinates computed for the sets of B-L traces defined by the GSFC (12/66) model, we calculated new B and L values using other field models. We have compared the JW and JC fields with the GSFC (12/66) model, both for the years they represent (1955 for JW, 1960 for JC) and for ten years later; and GSFC (9/65) with GSFC (12/66) for 1955, 1965, and 1975. We used GSFC (12/66) for 1965 as the reference field for an examination of the time dependence of the geomagnetic field.

RESULTS

Using the method described above, we made 560 comparisons between the various field representations for selected B-L points in the interval $0.20 \leq B \leq 0.24$ gauss and $1.2 \leq L \leq 1.8\ R_e$, respectively. Figure 1 illustrates one of these comparisons. Plotted are the B and L values computed using the JC model for the geographic coordinates of the B(0.24)-L(1.4) mirror-point trajectory defined by the GSFC (12/66) model for 1960. Figure 1 is typical of all of the comparisons we made because there is no function that will transfer a point in B-L space generated by one geomagnetic field model into a point in B-L space generated by another field model, unless the longitude and hemisphere are known. In other words, flux contours in B-L space cannot be corrected for model-dependent errors without knowing where the data were collected. The flux contours in Fig. 1 are 40 to 110-MeV proton flux contours from INJUN 3 [Valerio, 1964] and are given to indicate the possible range of flux values that can be assigned to the same B-L point.

Let us consider a specific example. Assuming that the INJUN 3 flux contours accurately reflect the shape of the proton flux in the inner radiation belt, then the flux at $B = 0.24$ gauss, $L = 1.4\ R_e$ is 30 protons/cm^2 sec (f_o). If proton flux data were collected in the southern hemisphere at 325° longitude and B-L values were calculated using the JC model, the flux value of f_o would be assigned to the point $B = 0.235$ gauss $L = 1.41\ R_e$. Owing to this (downward) shift in the flux contours, the "expected" flux value at $B = 0.24$ gauss, $L = 1.4\ R_e$ would be approximately 5 protons/cm^{2}sec. If in another flight experiment, flux data were collected at 0° longitude in the southern hemisphere, then f_o would be at $B = 0.243$ gauss, $L = 1.386\ R_e$, and the flux contour would be shifted (upward) so that the flux expected at $B = 0.24$ gauss, $L = 1.4\ R_e$ would be 75 protons/cm^{2}sec. In the two above cases, the apparent 1:15 ratio in flux values at $B = 0.24$ gauss, $L = 1.4\ R_e$ is due only to relative inaccuracies in the JC field model. We find that the B-L contours, such as Fig. 1, change slowly with position of the generating B-L point and can be considered to be constant within $\Delta B = \pm 0.01$ gauss and $\Delta L = \pm 0.05\ R_e$ of the generating point.

Table II gives the maximum deviation in B and L for three examples of B-L points for various field comparisons. The comparison of GSFC (9/65) with GSFC (12/66) confirms the conclusion drawn from the comparisons of these fields with direct geomagnetic-field measurements in the South Atlantic anomaly region -- namely, that the two models agree quite well for 1965 but diverge from each other 10 years before or after. The comparisons between JC and GSFC (12/66) show large variations at all B-L points studied in both 1960, the year the JC field model was generated to represent, and 10 years later. The variations between JW and GSFC (12/66) are even larger.

Table II also shows the range of proton-flux values as deduced from the INJUN 3 proton-flux contours, which could be assigned to the same B-L point due to errors in the geomagnetic-field model. We note that the flux variations caused by errors in the JW and JC fields are comparable to the magnitude of flux changes expected over a solar cycle [Blanchard and Hess, 1964].

The comparison of GSFC (12/66) for 1965 with GSFC (12/66) for 1955 and for 1975 shows another factor which must be taken into consideration in studies of the temporal changes of the trapped radiation. Figure 2 is a comparison of GSFC (12/66) for 1965 with GSFC (12/66) for 1975 at $B = 0.24$ gauss, $L = 1.4\ R_e$. The magnitude of the variation in B-L space of the same geographic coordinates over a 10-year period shows that the geomagnetic field is dynamic. Trapped radiation flux contours constructed in B-L space can contain significant errors if the field model used does not represent the geomagnetic field at the time the data were collected. An important feature of the time variations of the geomagnetic field with respect to the trapped radiation is the change in altitude of the same B-L trace in the South Atlantic anomaly region. Figure 3 shows the B-L trace of $L = 1.4\ R_e$, $B = 0.22$ and 0.24 gauss for 1965 and for 1975. Over a 10-year period the minimum mirror-point altitude at $L = 1.4\ R_e$, $B \geq 0.20$ gauss decreases about 70 kilometers. Particles mirroring at the same B and L values will experience a denser atmosphere in 1975 than in 1965. The flux of trapped radiation at $L = 1.4\ R_e$ and $B = 0.23$ gauss will decrease by about a factor of four, and at $B = 0.24$ gauss will virtually disappear in 10 years, independent of solar activity. The problems introduced by the time dependence of minimum mirror-point altitudes cannot be circumvented by the use of a static field model because of the nature of temporal variations (Fig. 2). The geomagnetic field must be treated as a dynamic field for any long-range studies and predictions of trapped radiation.

The plots of the geomagnetic field comparisons listed in Table II are available in University of California Space Sciences Laboratory Report Series 8 Issue 69, "B-L Space and Geomagnetic Field Models". These plots can be used in converting flux data in B-L space defined by one geomagnetic-field model into B-L space defined by another, provided geographic positions of the flux measurements are known.

CONCLUSION

Model-dependent errors can cause large apparent differences in trapped particle fluxes in B-L space. There is no simple way to transform

B-L points of one field model to B-L points of another field model without using the geographic coordinates used in generating the B-L points. Temporal variations of the geomagnetic field alter the geographic positions of mirror-point trajectories. Since the dominant particle-loss mechanisms for the region of B-L space studied are ionization and nuclear collision in the atmosphere, any computation of loss rates, lifetimes, and temporal changes of particles in the inner belt strongly depends on the use of an accurate time-dependent geomagnetic-field model. The range of model-dependent errors in B-L space demonstrates the need for careful reevaluation of existing data that pertain to possible time variations of inner-belt particle fluxes.

It is our opinion that the GSFC (12/66) field model may possess sufficient accuracy to warrant its use for generating B-L coordinates in such reevaluation. However, the analysis of particle flux data in regions of large flux gradients requires accuracies in the computed values of B that are of the order ± 40 to 50γ. Whether or not this accuracy is actually attained by the GSFC (12/66) model requires experimental confirmation. The intercomparison of the GSFC (12/66) field with the results of the Cosmos 26 and 49 satellites in the South Atlantic anomaly [Cain, Langel, and Hendricks, 1967b] gives partial confirmation for the accuracy of "a few tens of gammas" in the GSFC (12/66) field.

We have shown that the GSFC (9/65) and (12/66) field models describe the geomagnetic field, epoch 1965, in the South Atlantic anomaly equally well. We note, however, that the 99-coefficient GSFC (9/65) field expansion pre-dates the OGO-2 data, yet, fits the OGO-2 observations in the South Atlantic anomaly to an accuracy $\sigma = \pm 14.5\gamma$ (Table I).

Thus, we believe there is evidence that the GSFC (12/66) field model describes the geomagnetic field for epoch 1965 to the desired accuracy of about ± 50γ. Improvements in the secular variations of the coefficients will, in all probability, require updating as later field measurements become available. It is therefore essential that trapped particle data be readily accessible in their original geographic coordinates to allow for any further reevaluation of the data should subsequent improvements in the representation of the geomagnetic field require it.

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Table I. Mean deviation and standard deviation between unweighted magnetic survey data and computed values in the region of the South Atlantic anomaly.

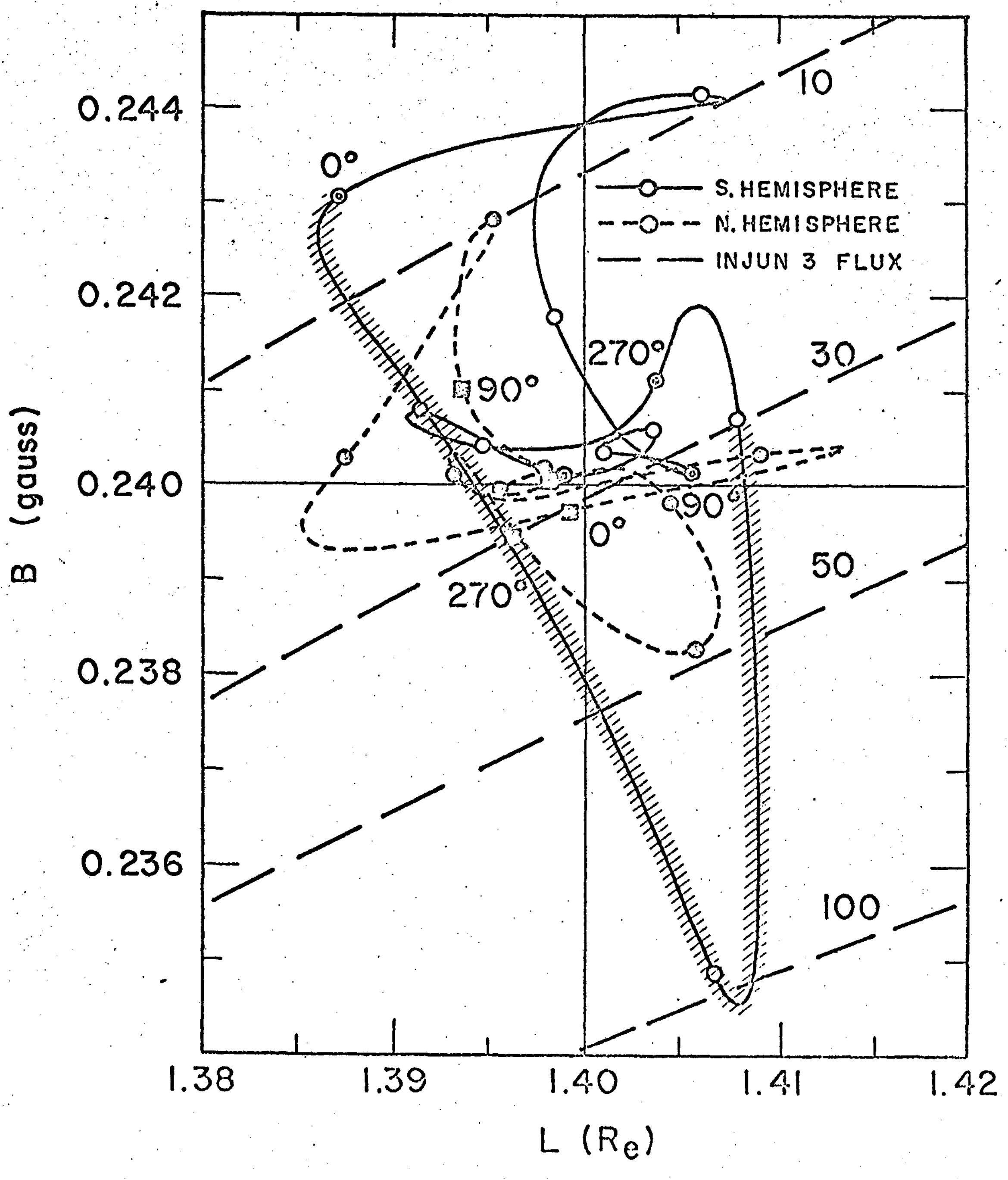
Field model	Number of data points	Mean deviation (γ)	Standard deviation (γ)
Surface and air 1900-1963 geomagnetic-field measurements			
JW	1369	-155.6	1056.3
JC		-560.2	892.0
GSFC (9/65)		297.0	477.6
GSFC (12/66)		-1.2	180.0
Surface and air 1955-1963 geomagnetic-field measurements			
JW	825	461.2	524.4
JC		-20.4	301.5
GSFC (9/65)		15.2	177.2
GSFC (12/66)		9.7	171.9
OGO-2 satellite measurements			
GSFC (9/65)	1330	7.1	14.5
GSFC (12/66)		1.8	7.2

Table II Geomagnetic comparisons of various field model configurations for three B-L points.

B_1 (gauss)	L_1 (R_e)	Year for GSFC(12/66) used in generating B_1-L_1 trajectory	Field for evaluating B-L at geographic coordinates of B_1-L_1 trajectory	Range of differences $\pm\Delta L$, between L and L_1		Range of differences $\pm\Delta B$, between B and B_1		Range of INJUN 3 proton fluxes (J = particles/cm^2 sec allowed by errors in B-L space	
				$+\Delta L$ ($10^{-4}R_e$)	$-\Delta L$ ($10^{-4}R_e$)	$+\Delta B$ (γ)	$-\Delta B$ (γ)	J_{max}	J_{min}
0.21	1.2	1955	GSFC (9/65) for 1955	16	16	61	36	160	130
		1965	GSFC (9/65) for 1965	16	12	37	39	160	140
		1975	GSFC (9/65) for 1975	39	62	149	84	170	130
		1960	JC	91	94	122	208	190	90
		1970	JC	148	190	545	222	310	110
		1955	JW	69	127	685	378	290	70
		1965	JW	77	217	739	440	350	70
		1965	GSFC (12/66) for 1955	75	158	504	107	300	120
		1965	GSFC (12/66) for 1975	199	62	105	600	170	40
0.24	1.4	1955	GSFC (9/65) for 1955	30	33	105	89	35	25
		1965	GSFC (9/65) for 1965	26	16	45	42	30	30
		1975	GSFC (9/65) for 1975	106	83	163	160	40	20
		1960	JC	135	148	416	544	75	5
		1970	JC	323	358	830	211	180	20
		1955	JW	213	312	1028	596	170	5
		1965	JW	230	385	1366	662	325	0
		1965	GSFC (12/66) for 1955	227	323	628	181	140	20
		1965	GSFC (12/66) for 1975	379	226	181	613	45	0
0.24	1.8	1955	GSFC (9/65) for 1955	39	47	119	63	125	110
		1965	GSFC (9/65) for 1965	32	31	51	79	120	110
		1975	GSFC (9/65) for 1975	165	117	288	285	140	95
		1960	JC	121	192	603	584	160	80
		1970	JC	444	468	1189	108	200	105
		1955	JW	363	440	706	605	170	80
		1965	JW	274	439	1110	678	200	75
		1965	GSFC (12/66) for 1955	350	433	693	174	160	105
		1965	GSFC (12/66) for 1975	481	377	180	816	130	75

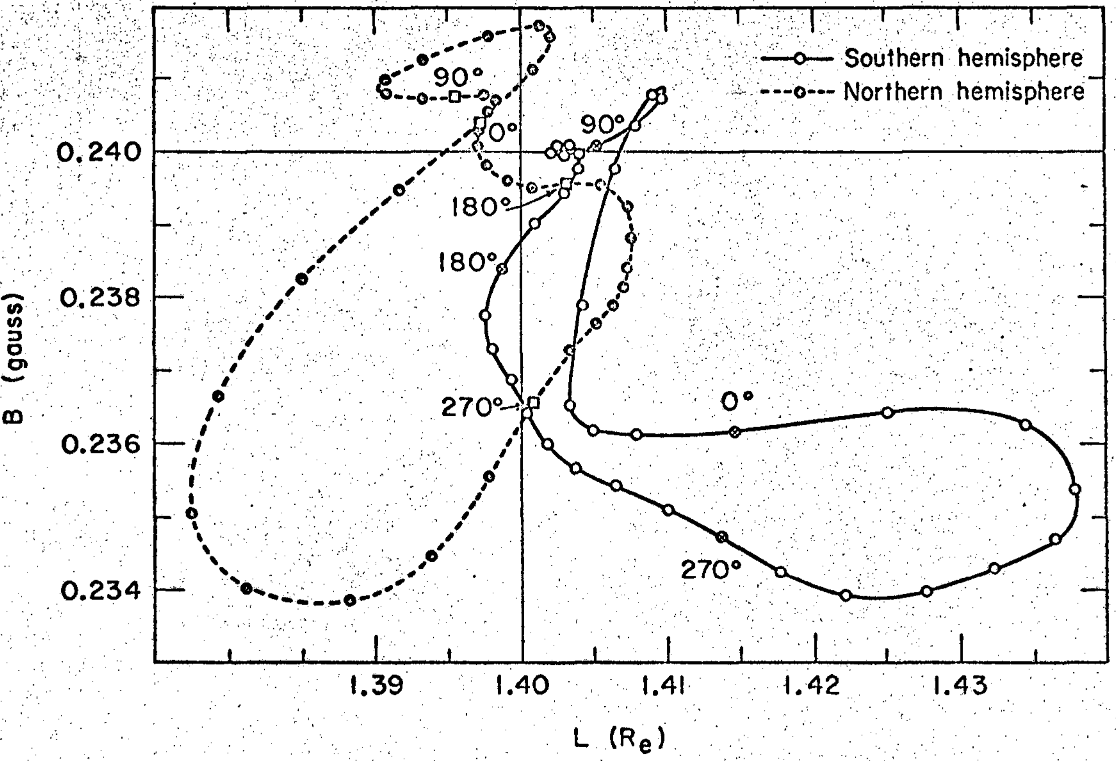
FIGURE CAPTIONS

1. Longitudinal contours of B and L values in the northern and southern hemispheres computed with the JC (48 coefficient) field model given the geographic coordinates of the mirror point trajectory, B = 0.24 gauss and L = 1.4 R_e, defined by the GSFC (12/66) field, epoch 1960. Longitude is indicated at 30° intervals, and the South Atlantic anomaly is identified by the shaded area. Injun 3 proton flux contours, $40 < E < 110$ MeV, are indicated.
2. Temporal change of B and L coordinates over a 10 year period, computed using the GSFC (12/66) field, epoch 1975 given the geographic coordinates of the mirror point trajectory B = 0.24 gauss and L = 1.4 R_e defined by the GSFC (12/66) field, epoch 1965. Longitude is indicated at 10° intervals.
3. Temporal changes in typical B-L traces in the region of the South Atlantic anomaly, computed using the GSFC (12/66) field for epochs 1965 and 1975.



XBL 670-5311-A

Fig. 1



XBL670-5314

Fig. 2

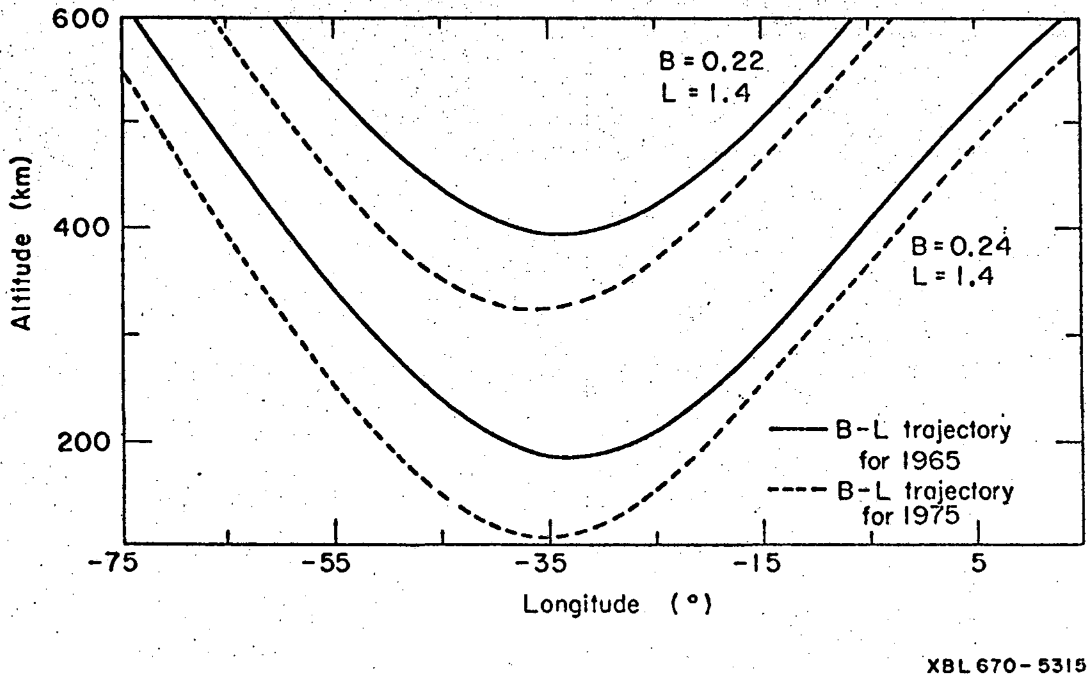

600
400
200
Altitude (km)
-75
-55
-35
-15
5
Longitude (°)
B = 0.22
L = 1.4
B = 0.24
L = 1.4
B-L trajectory for 1965
B-L trajectory for 1975
XBL 670-5315

Fig. 3