Title
Single-view-point omnidirectional catadioptric cone mirror imager

Journal
IEEE Transactions on Pattern Analysis and Machine Intelligence, 28(5)

ISSN
0162-8828

Authors
Lin, S S
Bajcsy, R

Publication Date
2006-05-01

Peer reviewed

Single-View-Point Omnidirectional Catadioptric Cone Mirror Imager

Shih-Schön Lin, *Member, IEEE*, and Ruzena Bajcsy, *Fellow, IEEE*

***Abstract*--We present here a comprehensive imaging theory about the cone mirror in its single-view-point (SVP) configuration and show that an SVP cone mirror catadioptric system is not only practical but also has unique advantages for certain applications. We show its merits and weaknesses, and how to build a workable system.**

***Index Terms*-- Catadioptric camera, imaging geometry, image quality analysis, omnidirectional imaging, optical analysis, panoramic imaging.**

I. INTRODUCTION

MOST ordinary cameras used in machine vision either possess a narrow field of view (FOV) or have a wide FOV but suffer from complex distortion. It can be difficult to unwarp a wide FOV image to perspective projection views accurately. Based purely on the ideal projection imaging model, it has been shown that surfaces of revolution of conic section curves are the only mirror shapes that can be paired with a single converging projection camera to create SVP catadioptric omnidirectional view systems whose omni-view image can be unwarped to perspective projection views without systematic distortions [1]. The pin-hole model based geometry has also been analyzed by others, e.g. [2-6]. The key to being able to unwarp to perspective projection views from a single omni-view image is to satisfy the single-view-point (SVP) condition [1]. The cone shape, although a surface of revolution of a conic section, was not deemed practical before. We discovered and constructed the first practical SVP cone mirror omni-view system in [4;6]; this work is an expansion on the discovery.

The cone mirror has not previously been used to construct an SVP omnidirectional imaging sensor that can reproduce perspective projection views from a single omni-view image.

However, cone mirrors have been used to aid navigation, map building, collision avoidance, and pipe inspections in non-SVP configurations [7-11]. The cone mirror images were used ‘as is’, and no attempt was made to unwarp them to undistorted images. Using multiple normal cameras positioned properly in relation to a plane mirror pyramid, a high resolution SVP wide FOV system can be built [12;13]. The trade offs, though, are the high price and complexities involved with multiple cameras. Bulky size, weight, calibration, synchronization, and gain differences are problems associated with multi-camera systems that are not associated with single camera systems.

SVP is worthwhile to have if the benefits outweigh the drawbacks for a particular application. Only with SVP can a catadioptric omni-cam use a single range-independent look up table or formula for correct unwarping. The SVP cone system is cheap and simple to build, operate, and maintain while retaining a decent vertical resolution and good flexibility in SVP. The SVP cone system is therefore always worth evaluating before considering more complex and expensive omni-view sensors. The main purpose of our work here is to prove that an SVP cone system is both theoretically and physically viable and to present a detailed analysis for cone SVP systems that provides systematic physics-based guidelines for deciding whether the SVP cone is suitable for a particular application. For applications in which SVP is not critical, Swaminathan, *et al.* have shown ways to recover believable perspective views from non-SVP systems [14]. Rees [15], Bogner [8], Hicks *et al.* [16;17], and Chahl *et al.* [10] have shown several types of non-SVP omnidirectional mirror shapes with interesting properties.

The advantages of the single camera SVP catadioptric family of omnidirectional imaging systems come with a price. The most significant trade-off is a much lower image spatial resolution compared to normal cameras, multi-camera omni-view systems [12], or rotating normal camera scanning systems [18] due to the fact that single camera SVP catadioptric systems have an enlarged FOV without a corresponding increase in the number of physical sensing units (e.g. pixels). Nagahara *et al.* [19] thus proposed stitching many omni-view images to form a single picture with better resolution. However scanning and stitching cannot be done in real time, though the extra views may be used for omni-stereo [20-24]. Southwell *et al.* [25], Basu and Baldwin [26] used concentric mirrors to get two views in one picture that sacrifices resolution further in exchange for fast omni-stereo. Multiple omni-views may also be captured simultaneously for omni-stereo with the help of beam splitters [27]. Furthermore, when designing a real optical system that conforms to the SVP condition, it turns out that certain optical aberrations tend to be more visible. However, the analysis of this problem cannot be performed under the pin-hole camera model from which the SVP theory was originally derived. Baker and Nayar [1] analyzed some "defocus blur" problems for hyperbolic and parabolic mirrors using a paraxial (Gaussian) optics model plus a fixed position finite aperture. Yamazawa *et al.* [2] and Yagi *et al.* [7] briefly mentioned some more optical problems for convex mirrors including spherical aberration and astigmatism. Ishiguro [28] gave a qualitative summary of aberrations of various single camera SVP catadioptric systems but not for cone mirrors in the SVP configuration.

We have analyzed the aberrations of SVP cone mirror systems using accurate numerical optical ray tracing. Based on our analysis we show that our optical setup can significantly reduce such aberrations. The cone is among the simplest mirror shapes to produce, and it has much higher meridional (tangential) angular resolution compared with other conic section mirrors for scenes around the horizon [11;28]. It adds the least optical distortion to the resulting meridional images because it is the only omni-view mirror with a non-curved mirror surface in the meridional cross sections.

II. SINGLE-VIEW-POINT CONE MIRROR IMAGING THEORY

The concept of "Single-view-point" (SVP) is well defined in the projective pin-hole camera imaging model, where each lens camera is modeled as a point in space (the "projection center" for the lens camera) and an image plane. By definition, all normal lens cameras in the perspective pin-hole model meet the SVP condition. However the SVP concept becomes increasingly less well defined in the context of more physically accurate optical imaging models. In other words, a real lens camera by itself is not SVP in the strictest mathematical sense. They are numerically good approximations of an ideal pin-hole SVP camera only within their published working distances under intended usage. We have to redefine "SVP" in Gaussian optics and study "defocus" caused by "skew rays" using geometric optics [29;30]. Here we provide the major results and conclusions only. For details see [4;6].

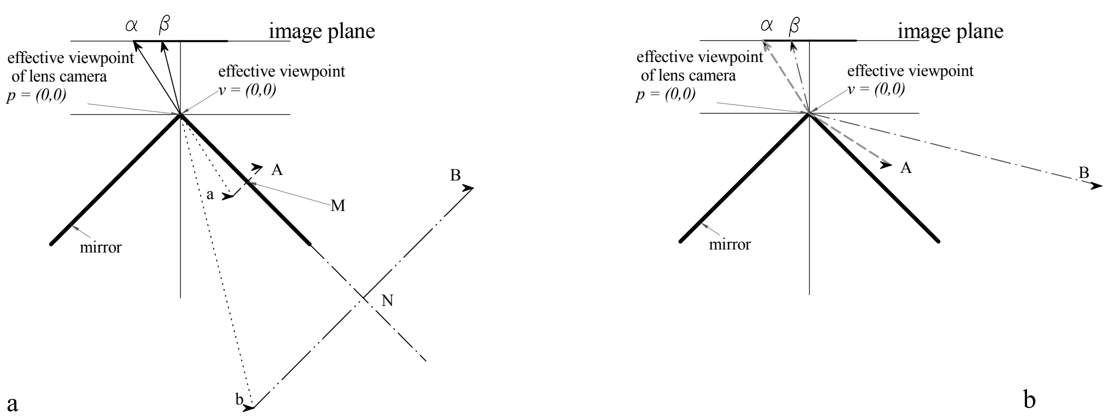

Fig. 1 SVP Cone Mirror imaging model in the pin-hole camera model

A. SVP Catadioptric Cone Omni-cam under the Perspective Pin-hole Model

Fig. 1 illustrates the imaging model of an SVP cone mirror omnidirectional vision sensor system. The imaging process can be described in a few different ways, all of them equivalent but each sheds light on different interesting physical properties. The first description (Fig. 1 a) is based on the concept of a “virtual image.” A “virtual image” of a world point is a point that, when viewed from the position of an observer, seems to be the source point from which all the light of the world point comes. The cross section of cone mirror in any meridional plane as depicted in Fig. 1 a is exactly the same as that of two plane mirrors. Plane mirrors have been proven to be the only mirror shape that produces a perfect virtual image [29;31]. As shown by Baker and Nayar [1], the SVP condition of a cone mirror corresponds to the condition when the viewpoint of a perspective camera coincides with the tip of the cone. The system in Fig. 1 is arranged to have the SVP of the lens camera placed at the SVP of the cone, which is located right at the tip of the cone.

Given the geometric configuration of the mirror (i.e. its shape and position) and the camera (i.e. the viewpoint at the tip of the cone and the position and orientation of the image plane) in an SVP omni-cam system, for any given world point we can find its corresponding image point by finding the virtual object point behind the mirror surface. Once the virtual object position is determined, it can be viewed just like a real object such that its image is found by drawing a line from the virtual object point toward the viewpoint until it intercepts the image plane. The point of interception is the image point we are looking for. For example, in Fig. 1 a, for world point ‘A’, we find its virtual image at the point ‘a’ inside the mirror. Then the camera with its viewpoint at the tip of the cone sees the virtual object point ‘a’ by projecting it at the image point ‘α’. The same principle applies to any other world point, e.g. ‘B’, with virtual object ‘b’, and image point ‘β’. The theory of the working SVP cone catadioptric sensor is as follows: the configuration in Fig. 1 has previously been proven to be SVP [1]. We have just established the theory of how and why arbitrary world points can be imaged under the SVP condition of the cone mirror. Note that we arrive at this conclusion while still assuming pin-hole camera model, i.e. the aperture is considered an ideal dot with no physical size. Thus we have derived the theory of a practical working cone mirror catadioptric omnidirectional sensor system under the pin-hole camera model.

The second way to describe the imaging of the SVP cone mirror system is sometimes called “ray-tracing” (note: “ray-tracing” has a different meaning in the geometric optics model). If we have an algorithm such that given any world point one can trace the light ray via a unique path to a unique image point on the image plane, we have a projection. If every such unique ray path for

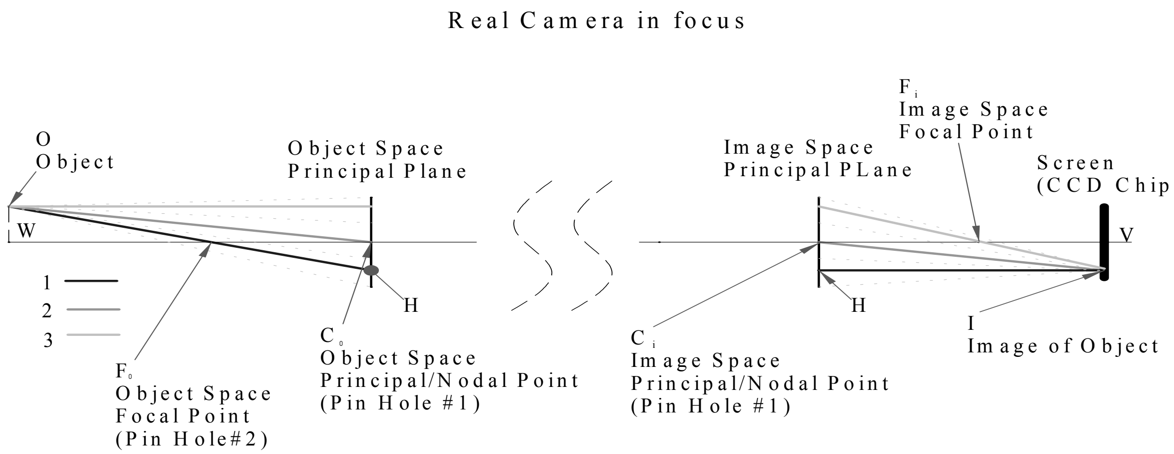

Fig. 2 Multiple “SVPs”: Projection centers and principal points of a focusing camera system in Gaussian optics model.

every given world point passes through the SVP of both the mirror and the camera, we have met the SVP condition. All these ray paths must not violate the law of reflection; however the law of refraction is a non-issue because the lens component is represented by an ideal pin-hole. This is the way the original SVP theory was derived [1]. The cone can also be proven to be SVP by “ray tracing” as shown in Fig. 1 b. The key point is that the tip of the cone serves simultaneously as both the point of reflection on the mirror and as the SVP simultaneously for all scene points. For details see [4;6].

B. SVP Catadioptric Cone Omni-Cam under the Gaussian Optics Model

Gaussian optics, also called first order optics, can be summarized in a concise formula, the Gaussian formula: (s_o: object distance; s_i: image plane distance; f: effective focal length) [29;30]

$$(1/s_o)+(1/s_i)=(1/f) \quad . \quad (1)$$

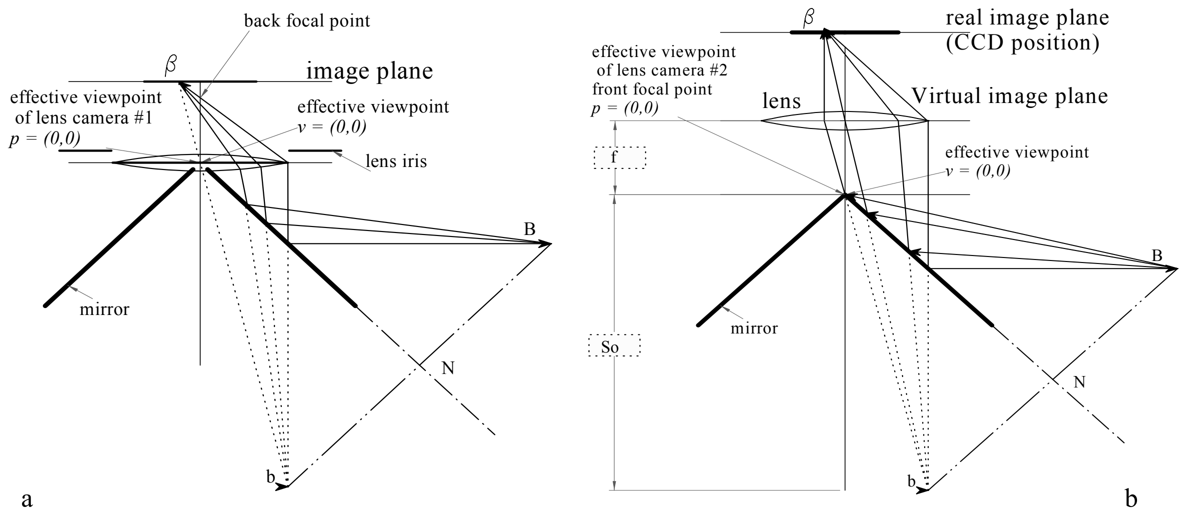

Fig. 3 Meridional ray tracing illustration of two cone SVP configurations. (a) SVP at the cone tip. (b) SVP at front focal point.

The most prominent change in the lens model is that now we can find more than one "effective viewpoint" or "projection center" for a lens or lens system when we try to fit the perspective projection concepts into the Gaussian optics framework. This is why in Fig. 3 we see two different configurations that are both SVP (see [4;6]). In Fig. 2 we show a more generalized lens/lens set having a world point O in focus with an image formed at the point I. The cardinal points of this optical system are F_o (object space focal point), C_o (object space principal and nodal point), C_i (image space principal and nodal point), and F_i (image space focal point). The Gaussian optics model is more realistic than the pin-hole model in that all the rays originating from a world point are considered. The Gaussian optics model is still a simplification from the real world in that it assumes the optical system can perfectly focus all light rays from the same world point and are collected by the optical system to the same image point. Several special properties for rays passing through cardinal points of the system come from such ideal assumption. First, all rays passing through the object space focal point F_o will appear to continue unaltered to the object space principal plane at point H, and then from the same height measured from the optical axis, it will appear to reemerge at the conjugate image space principal plane and continue parallel to the optical axis until it reaches the image plane at the image point I. For all world points at the same object plane (i.e. the plane that is perpendicular to the optical axis and intersects the optical axis at the point W), their images, although actually formed on the screen at V, can be orthographically projected to the object space principal plane along the optical axis. Also from every image point one can draw a straight line from the shifted image point passing the object space focal point F_o and reach the corresponding world point. This is exactly the definition for perspective projection with F_o as the projection center. Although strictly true for only world points in one plane, we can as a practical matter relax the Gaussian optics model a little and treat F_o as the projection center for all world points "inside the depth of field." The concept of depth of field and depth of focus arises because all real imaging devices have finite resolution. The smallest CCD sensing unit is a pixel, so a blurring pattern smaller than a pixel cannot be detected. Similarly, traditional films cannot detect blurring smaller than their light sensing particles/compounds.

In fact, there exists a distance, called the hyperfocal distance, such that all world points farther away from the camera than it can be considered in focus. This assures that we have a practical single image plane (instead of different focus distances for each object distance as suggested by Eq (1)) and a practical single projection center for a lens camera under a slightly relaxed version of the Gaussian optics imaging model.

We can find another practical projection center using the same framework. In Fig. 2 we can use another Gaussian optics rule for the cardinal point C_o, the object side principal point. Namely, any ray that appears to pass through C_o in the object space will appear to emerge from the image side principal point C_i and follows the same propagation direction until it intersects the image plane at I. If we put the two principal planes together, as shown in most illustrations for a single thin lens, we have a perspective projection under the same reasoning as that for F_o.

In addition, another cardinal point pair, called nodal points, can also be regarded as the effective SVP. In simpler optical systems the nodal points coincide with the principal planes. However this is not always the case. The definition of nodal points is that light passing through the object side nodal point will always emerge from the image side conjugate nodal point(s) with the same light path angle relative to the optical axis. The two conjugate nodal points serve the same functions as the two focal points in hyperbolic and ellipsoidal mirrors in preserving SVP. The main difference is that the nodal point properties hold only in the paraxial region, a condition considered met when the optical system is operating inside the depth of field/focus.

We have thus completed the SVP theory for a lens camera under an imaging model of slightly relaxed Gaussian optics. Using our theory, optical engineers will know where exactly to place the cardinal points in order to preserve SVP condition.

III. EXPERIMENTS AND ABERRATION CONSIDERATIONS

The meridional ray patterns as shown in our theory (Fig. 1 to Fig. 3) are perfect with no blurs. To analyze the imaging artifacts or “aberrations” we need to trace “skew rays” (i.e. rays outside the meridional plane), which can only be done numerically [29;30]. We show here only the improvement made possible from our optical analysis (see [4;6] for details).

The most significant aberration for a cone mirror comes from astigmatism [29;30] which can be minimized by using shorter focal length lens, smaller aperture, and positioning the entrance pupil away from the cone. Thus the configuration in Fig. 3 b is better than the configuration in Fig. 3 a. Experiment results shown below are all set up as Fig. 3 b. One must seek a balance between optical parameters because it is difficult to correct for barrel distortion at extremely short focal length and that very small aperture would require very long exposure time or yield a very dark image. Note that it is neither possible nor necessary to eliminate all aberrations from the cone mirror. The lens always introduces its own aberrations; the CCD chip or film also cannot resolve details smaller than its pixel/particle size. From our optical analysis we have found a good balanced set of optical settings for our prototype system. Compare Fig. 4 and Fig. 5. Note that aberrations cannot be removed simply by focusing, e.g. astigmatism means that meridian and saggital rays are focused to different locations so there is no single location to put image plane for all rays to focus perfectly. Both Fig. 4 and Fig. 5 are focused to the best we can.

The pictures shown here are taken by a Casio QV2000UX which allows full manual control of aperture and exposure and up to 3X optical zoom. The cone mirror prototype is machined from a piece of aluminum, polished, and then chrome plated for the mirror shiny finish. The cone mirror has a bottom radius of 2.5 inch (6.35 cm) and tip angle 107 degree. Our machine shop happened to have a spare piece of aluminum cylinder with radius of 2.5 inch when we placed the order. One can make bigger or smaller mirror according to the need of specific applications. The bigger the cone, the brighter the image and the less the vignetting.

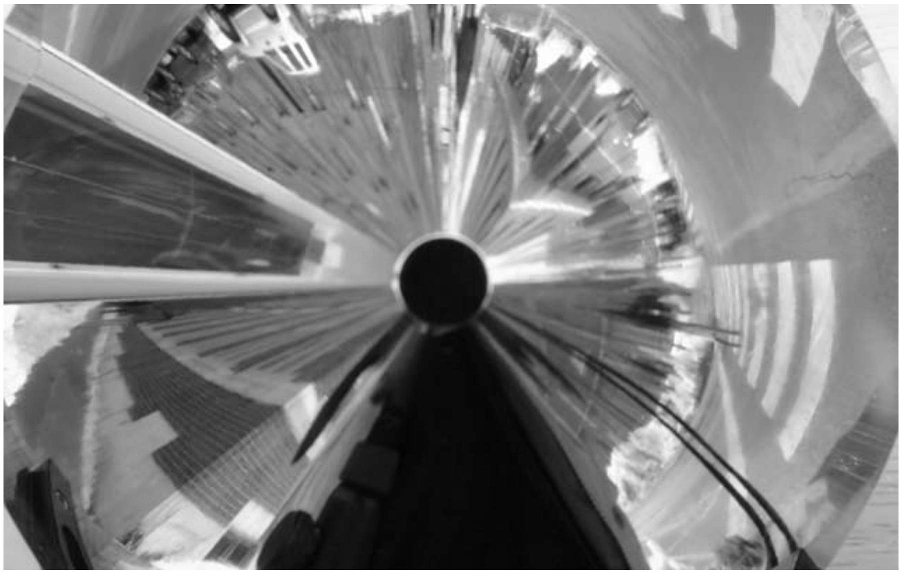

Fig. 4 Matched System. Best focus picture, Casio QV2000UX, f=6.5mm, F/10, 1/30 sec, fair day

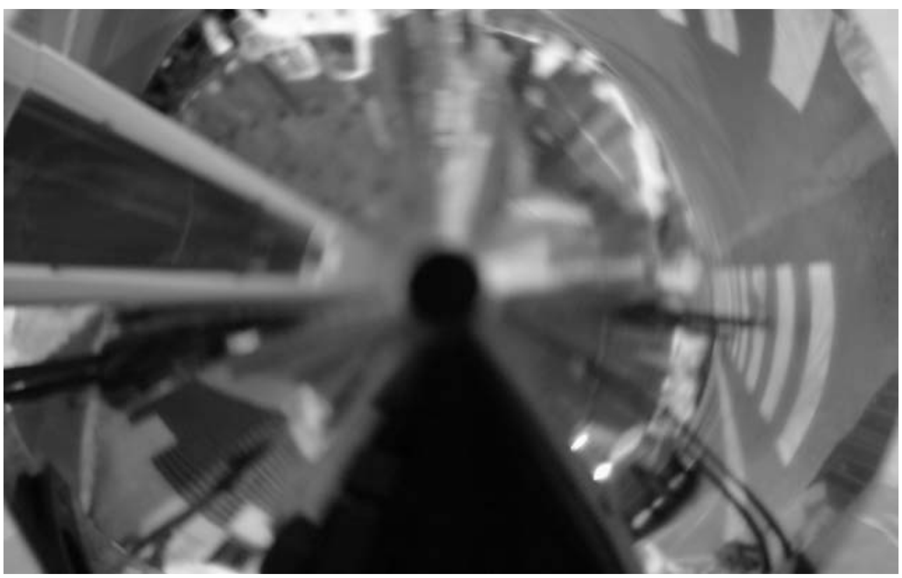

Fig. 5 Ill-Matched system. Best focus picture, Casio QV2000UX, f=6.5mm, F/2, 1/800 sec, fair day

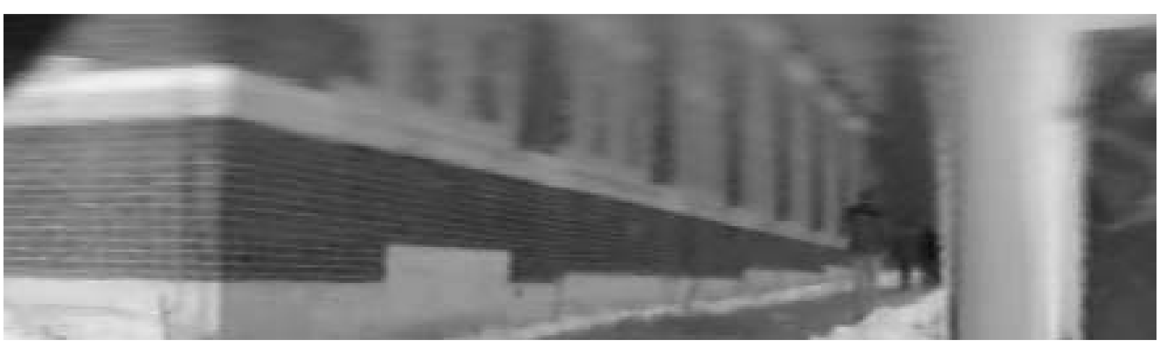

Fig. 6 Unwarped image from bottom left of Fig. 4.

Fig. 7 Unwarped from top left of Fig. 4.

Fig. 6 and Fig. 7 are the unwarped perspective view image from corners of Fig. 4. Except for the very top portion the image quality is close to the normal perspective camera with the same lens and CCD. Near the very top, the same length of picture data is interpolated from smaller and smaller number of actual image data pixels and at the tip of the cone there would only be one real pixel of data. Thus we see the inevitable decrease of horizontal image quality toward the top end of the unwarped picture. Depending on the application one may choose not to unwarp the very top portion at all.

Fig. 8 Best-focus picture, Casio QV2000UX, f=6.5mm, F/8, 1/2 sec, indoor office ceiling fluorescent light.

Fig. 9 Unwarped perspective view from the left of Fig. 8.

Fig. 10 Unwarped perspective view from the top of Fig. 8.

Fig. 8 is a picture taken indoor with overhead fluorescent lighting and close by objects (only a few inches away from the imager). Fig. 9 and Fig. 10 are unwarped perspective views of Fig. 8.

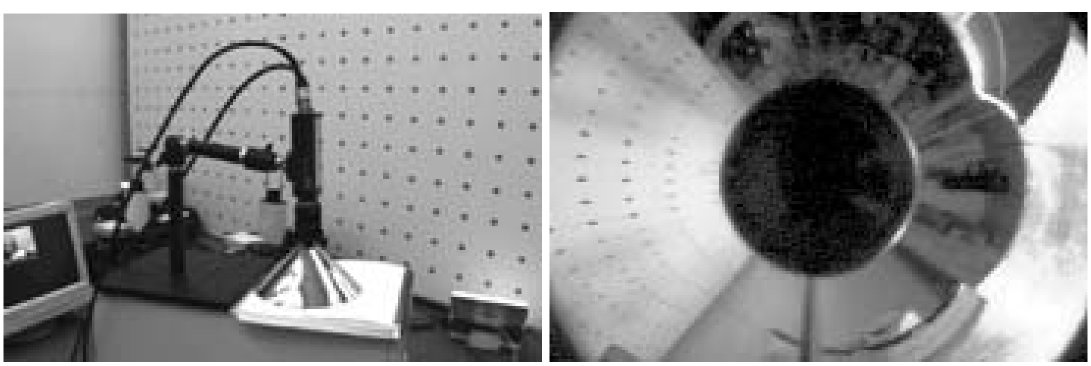

Fig. 11 Left: Overview of the calibration board setup. Right: Omniview of the calibration board.

Fig. 11 and Fig. 12 show the experiment with a calibration board (4 inches dot grid) to demonstrate correct perspective unwarping capability of our SVP cone mirror system.

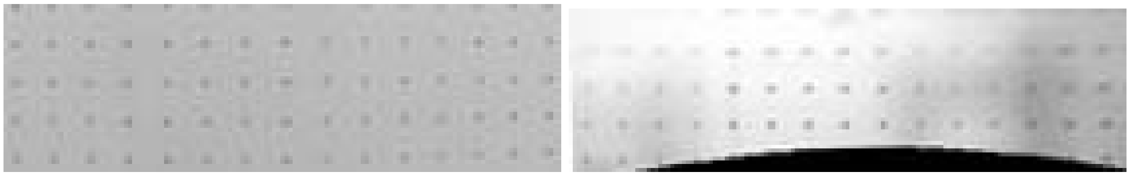

Fig. 12 Left: Calibration board image taken by a real perspective camera. Right: Calibration board image unwarped from SVP cone omni-view picture of Fig. 11.

The system components used in Fig. 11 and Fig. 12 are SONY XC-77 CCD, COSMICAR 6mm CCTV Lens, silver gift wrap paper made cone mirror with bottom radius of 2.5 inch (6.35cm) and tip angle 90 degrees.

IV. Conclusion

The SVP omnidirectional cone mirror system has higher astigmatism and narrower FOV compared to other existing SVP catadioptric systems with similar setups. As a result of narrower FOV, the SVP omnidirecitonal cone mirror has the advantage of more pixels per unit view angle around the horizon than existing SVP catadioptric systems. We do not consider any SVP system being outright superior than any other SVP systems. Each system complements the ability of the other. The SVP cone mirror system introduced here is *not* for everyone. Only consider cone mirror system if its advantages fit your need and its drawbacks are not important for your particular applications. Note that the comparison is valid only between SVP catadioptric systems. SVP condition imposes adverse restrictions to image quality so non-SVP systems can easily achieve better image quality by relaxing or disregarding SVP constraints.

We have established the theory for a practical SVP cone mirror based catadioptric omnidirectional sensor. We have shown why we can see images in an SVP cone omni-cam. We have further shown the potential advantages and disadvantages an SVP cone mirror based system has compared to other existing SVP systems. Real SVP cone mirror images confirm our theory and derivations. Our theory helps people decide whether an SVP cone is suitable or not to their needs. The SVP cone mirror based omni-cam provides the highest meridional image details of any SVP omni-cam that uses only a single fixed planar imager. In contrast, rotating camera systems cannot capture omni-view in real time, while multi-camera systems require more resources to operate. For applications that do not require full hemispherical views such as UAGV, higher resolution around the horizon may be more attractive. The low costs due to simpler mirror shape of the cone may be attractive to low end consumer markets.

REFERENCES

[1] S. Baker and S. K. Nayar, "A Theory of Single-Viewpoint Catadioptric Image Formation," *IJCV*, vol. 35, no. 2, pp. 175-196, Nov. 1999.
[2] K. Yamazawa, Y. Yagi, and M. Yachida, "Omnidirectional Imaging with Hyperboloidal Projection," in *IROS 1993*, vol. 2, pp. 1029-1034.
[3] T. Svoboda, T. Pajdla, and V. Hlavác, "Epipolar Geometry For Panoramic Cameras," in *Proc. ECCV 1998*, pp. 218-232.
[4] S.-S. Lin and R. Bajcsy, "True Single View Point Cone Mirror Omni-Direcitional Catadioptric System," in *Proc. ICCV*, 2001, vol. II, pp. 102-107.
[5] C. Geyer and K. Daniilidis, "Paracatadioptric Camera Calibration," *IEEE T-PAMI*, vol. 24, no. 5, pp. 687-695, May 2002.
[6] S.-S. Lin, "Omni-Directional 3D Stereo Computer Vision Sensor Using Reflective Cone Mirror." PhD Thesis, Computer and Information Science Department, University of Pennsylvania, 2003.
[7] Y. Yagi, S. Kawato, and S. Tsuji, "Real-Time Omnidirectional Image Sensor (COPIS) for Vision-Guided Navigation," *IEEE T-RA*, vol. 10, no. 1, pp. 11-22, Feb. 1994.
[8] S. Bogner, "Introduction To Panoramic Imaging," in *Proc. SMC Conf. 1995*, pp. 3100-3106.
[9] D. Southwell, B. Vandegriend, and A. Basu, "A Conical Mirror Pipeline Inspection System," in *Proc. 1996 ICRA*, vol. 4, pp. 3253-3258.
[10] J. S. Chahl and M. V. Srinvasan, "Reflective Surfaces For Panoramic Imaging," *Applied Optics*, vol. 36, no. 31, pp. 8275-8285, Nov. 1997.
[11] Y. Yagi, "Omnidirectional Sensing And Its Applications," *IEICE Transactions on Information and Systems*, vol. E82D, no. 3, pp. 568-579, Mar. 1999.
[12] H. Hua and N. Ahuja, "A High-Resolution Panoramic Camera," in *Proc. CVPR* 2001, vol. 1, pp. I-960-I-967.
[13] V. S. Nalwa, "Panoramic Viewing System With Offset Virtual Optical Centers," NJ, USA Patent 6219090, Apr. 17, 2001.
[14] R. Swaminathan, M. D. Grossberg, and S. K. Nayar, "A Perspective On Distortions," in *Proc. CVPR 2003*, vol. II, pp. 594-601.
[15] D. W. Rees, "Panoramic Television Viewing System," USA Patent 3505465, Apr. 7, 1970.
[16] R. A. Hicks and R. K. Perline, "Geometric Distributions for Catadioptric Sensor Design," in *Proc. CVPR*, 2001, pp. 584-589.
[17] R. A. Hicks and R. Bajcsy, "Reflective Surfaces As Computational Sensors," *Image and Vision Computing*, vol. 19, no. 11, pp. 773-777, Sept. 2001.
[18] A. Krishna and N. Ahuja, "Panoramic Image Acquisition," in *Proc. CVPR 1996*, pp. 379-384.
[19] H. Nagahara, Y. Yagi, and M. Yachida, "Resolution Improving Method for a 3D Environment Modeling Using Omnidirectinal Image Sensor," in *Proc. ICRA 2002*, vol. 1, pp. 900-907.
[20] H. Ishiguro, M. Yamamoto, and S. Tsuji, "Omnidirectional Stereo," *IEEE T-PAMI*, vol. 14, no. 2, pp. 257-262, Feb. 1992.
[21] D. W. Murray, "Recovering Range Using Virtual Multicamera Stereo," *Computer Vision and Image Understanding*, vol. 61, no. 2, pp. 285-291, 1995.
[22] S. B. Kang and R. Szeliski, "3-D Scene Data Recovery Using Omnidirectional Multibaseline Stereo," *IJCV*, vol. 25, no. 2, Nov. 1997.
[23] R. Benosman and J. Devars, "Panoramic Stereovision Sensor," in *Proc. ICPR 1998*, vol. 1, pp. 767-769.
[24] S. Peleg, M. Ben-Ezra, and Y. Pritch, "Omnistereo: Panoramic Stereo Imaging," *IEEE T-PAMI*, vol. 23, no. 3, pp. 279-290, Mar. 2001.
[25] D. Southwell, A. Basu, M. Fiala, and J. Reyda, "Panoramic Stereo," in *Proc. ICPR 1996*, vol. 1, pp. 378-382.
[26] A. Basu and J. Baldwin, "A Real-Time Panoramic Stereo Imaging System And Its Applications," in *Panoramic Vision: sensors, theory, and applications,* Benosman, R. and Kang, S. B. (eds.) New York: Springer-Verlag, 2001, pp. 123-141.
[27] S.-S. Lin and R. Bajcsy, "High Resolution Catadioptric Omni-Directional Stereo Sensor For Robot Vision," in *Proc. ICRA 2003*, pp. 1694-1699.
[28] H. Ishiguro, "Development of Low-Cost Compact Omnidirectional Vision Sensors," in *Panoramic Vision: sensors, theory, and applications,* Benosman, R. and Kang, S. B. (eds.) New York: Springer-Verlag, 2001, pp. 21-38.
[29] M. Born and E. Wolf, *Principles of Optics:Electromagnetic Theory of Propagation, Interference and Diffraction of Light*, 6 ed. Oxford: Permagon Press, 1984, pp. 1-808.
[30] W. J. Smith, *Modern Optical Engineering: The Design of Optical Systems*, 3 ed. New York: McGraw-Hill, 2000, pp. 1-617.
[31] C. Carathéodory, *Sitzungsberichte der Beyer. Akad. Wiss. Math-naturw. Abt.*, no. 56, pp. 1, 1926.