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Rainfall profile retrieval through spaceborne rain radars utilising a sea surface NRCS model

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Indexing terms: EM models, Sea surface, Rainfall rate vertical profile, Normalised radar cross-section

Abstract: Vertical rainfall profile retrieval over the sea surface, based on reflectivity data collected by spaceborne rain radars, can be improved through existing algorithms that exploit estimates of the power backscattered from that surface. However, prediction errors of the sea surface normalised radar cross-section (NRCS) may significantly affect the performance of such algorithms. This is the first point highlighted in the paper, referring to the K_u band. Consequently, the authors propose the utilisation of an electromagnetic model able to predict with acceptable accuracy the sea surface NRCS under the joint effect of wind- and rainfall-induced corrugation. The core of the paper is the description of an improved algorithm for rainfall rate vertical profile retrieval over the sea surface in the case of a single frequency, nadir-looking radars and the discussion of some simulation results.

1 Introduction

Observation of rainfall over the sea/ocean surface is highly relevant for understanding the earth's climate and its changes. Therefore, spaceborne radars and suitable data processing techniques for providing rainfall rate estimates over those surfaces on a regular basis, as can be made available through an orbiting satellite platform, are gaining increasing interest. Algorithms proposed for rainfall rate vertical profile (RRVP) retrieval exploit backscatter or attenuation estimates derived from radar measurements [1]. Attenuation-based algorithms cope with at least four different basic errors:

- (a) radar calibration errors
- (b) errors related to the uncertainty in the standard attenuation-reflectivity ($k-Z$), reflectivity-rainfall rate ($Z-R$) relationships
- (c) measurement errors of the total PIA (path-

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integrated attenuation) related to the attenuation at the current range cell

(d) errors in the estimate of the surface reflectivity (when exploited).

Included in c is the heavy (and mostly unpredictable) additional attenuation due to the possible presence of the melting layer of precipitation [2, 3].

Several single-frequency algorithms were proposed to reduce the effects of some of the aforementioned errors [4]; surface referenced techniques exploit estimates of the power backscattered by the sea surface as additional reference information to solve the intrinsic ambiguity of the RRVP problem [1]. However, they need sea NRCS estimates that, in practice, are quite rough [3]. It is thus reasonably expected that a well grounded prediction of the backscattering behaviour of the sea surface, accounting for the joint perturbations of wind and rainfall, can be usefully exploited to improve performance of existing RRVP retrieval algorithms of this kind.

In this paper we focus on the K_u band (commonly utilised by spaceborne weather radars) and refer to the EM model described in the companion paper [5], where the relevance of rainfall corrugation at that band is highlighted. We consider the kZS algorithm [6], one of the most effective RRVP retrieval methods. Based on the same numerical simulation setup used in [6], we show that expected variations of NRCS (also those that the EM model ascribes to rainfall) may cause significant errors. To overcome this problem we discuss a possible upgrade of the kZS algorithm that exploits the EM model prediction to improve the accuracy of RRVP retrieval.

2 Principle of the kZS algorithm

We recall here for convenience the basic principle of the kZS algorithm, proposed by Marzoug and Amayenc to estimate RRVPs with a nadir-looking radar operating at a wavelength attenuated by rainfall [6]. The algorithm exploits the ratio between the mean power $P(r)$ backscattered from a generic rain-filled radar resolution cell ranging r and the mean power $P_s(r_s)$ from the resolution cell including the sea surface ranging r_s . The limiting assumption of the nadir incidence angle is needed to ensure that the surface return dominates the total return from the latter cell, thus avoiding any significant contamination of the former cell by the surface return, even through antenna

sidelobes. Analytically $\overline{P(r)}$ is proportional to the rain reflectivity factor $Z(r)$ and to the attenuation factor along the path

$$\overline{P(r)} = \frac{C}{r^2} Z(r) \exp \left[-0.2 \ln 10 \int_0^r k(s) ds \right] \quad (1)$$

and $\overline{P_s(r_s)}$ is proportional to the sea surface NRCS (referred to as σ_s)

$$\overline{P_s(r_s)} = \frac{C_s}{r_s^2} \sigma_s \exp \left[-0.2 \ln 10 \int_0^{r_s} k(s) ds \right] \quad (2)$$

The overbar indicates mean value, r and r_s are in km, $k(s)$ is the specific attenuation coefficient (dB/km) at range s for propagation through rainfall, while C and C_s are the radar constants in the volume and surface backscatter cases, respectively [3, 6]. Assuming that attenuation is due exclusively to rainfall, Z and k are tied by a standard empirical relationship

$$Z(r) = \alpha(r) \cdot k^\beta(r) \quad (3)$$

where α and β depend on frequency and drop size distribution (DSD) [2]. β is supposed to be constant and known along the path, while α is considered variable with range. Introducing the Z - k relationship in eqn. 1, the ratio $\overline{P(r)}/\overline{P_s(r_s)}$ gives

$$k(r) \exp \left(\frac{0.46}{\beta} \int_r^{r_s} k(s) ds \right) = w_0(r) \quad (4)$$

where

$$w_0(r) \triangleq \left[\frac{\overline{P(r)} \cdot r^2 \sigma_s C_s}{\overline{P_s(r_s)} \cdot r_s^2 \alpha(r) \cdot C} \right]^{1/\beta} \quad (5)$$

With respect to

$$g(r) \triangleq \exp \left(\frac{0.46}{\beta} \int_r^{r_s} k(s) ds \right)$$

eqn. 4 is a first-order differential equation of the kind

$$\frac{dg(r)}{dr} = -\frac{0.46}{\beta} w_0(r)$$

Its solution

$$g(r) = 1 + \frac{0.46}{\beta} \int_r^{r_s} w_0(s) ds$$

implicitly provides $k(r)$

$$k(r) = \frac{w_0(r)}{\left[1 + \frac{0.46}{\beta} \int_r^{r_s} w_0(s) ds \right]} \quad (6)$$

The RRVP $R(r)$ is then obtained through a standard frequency-dependent relationship of the kind $R(r) = Ak^B(r)$.

Eqn. 6 shows that kZS, exploiting the power ratio instead of the absolute power, takes the sea surface as the starting point for the integration of radar measurements. Besides avoiding errors due to absolute system calibration errors, rainfall rate at a given altitude is thus obtained by integrating power estimates related to radar cells below that altitude, and is independent of precipitation above. Therefore, bias errors due to ice particles melting in the melting layer can never occur.

Finally, $w_0(r)$ increases with increasing attenuation between the surface and the cell ranging r . This causes a lower sensitivity to σ_s . Instead, when attenuation is low, sensitivity to the accuracy of σ_s becomes relevant [6].

3 Sea NRCS prediction accuracy as the limiting factor for the kZS algorithm

A basic problem is that the actual NRCS σ_s of the sea surface is *a priori* unknown, while it obviously should be well approximated before being used by the kZS algorithm. The 'guess' value of σ_s will be hereafter referred to as σ_0 . In general, a systematic error σ_B (which may also take negative values) is introduced, depending on the way σ_0 is determined

$$\sigma_s = \sigma_0 + \sigma_B \quad (7)$$

To verify the effectiveness of the kZS algorithm, Marzoug and Amayenc simulated spaceborne radar measurements based on the acquisition of 60 independent echo samples [4, 6]. Assuming a 'true' RRVP, they reconstructed 100 RRVPs through the kZS algorithm, accounting for all errors mentioned in Section 1. In the Appendix (Section 9.1) we report the error model they utilised. Doing that, they assumed a climatological fixed value $\sigma_0 = 12$ dB at 13.75 GHz, which can be approximately considered as the mean value of the sea NRCS with respect to wind speed variations [7].

Indeed, resorting to such a fixed 'guess' is the only solution when no other data are available but spaceborne rain radar measurements. However, remarkable variations of σ_s occur with wind velocity. At the K_u band, the EM model described in the companion paper [5] shows that the intensity of rainfall on the sea surface also plays an important role. Fig. 6 of that paper reports the sea surface NRCS against rainfall rate R for some wind velocities at nadir incidence. If σ_0 does not account for such variations, σ_B may directly and heavily affect the RRVP retrieval. The same Figure shows that $\sigma_B = 3$ dB can be reasonably considered as a potential (not maximum) value of the absolute bias error, suitable to evaluate performance degradation of the kZS algorithm.

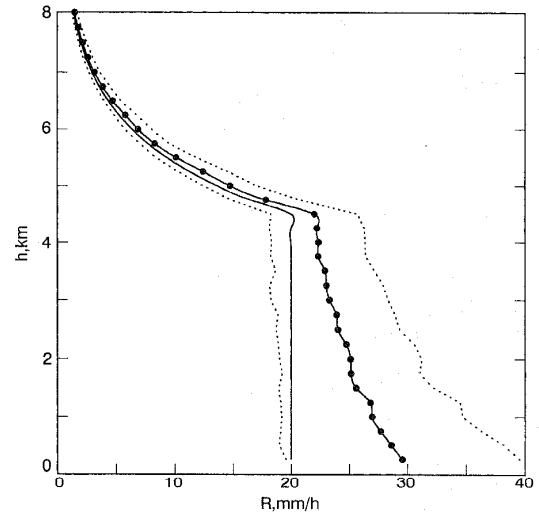


Fig. 1 Simulated reconstruction of a 'true' rainfall profile by means of the kZS algorithm assuming a 'true' NRCS $\sigma_s = 9$ dB and a guess value $\sigma_0 = 12$ dB. Mean value and standard deviation of the 100 reconstructed profiles are plotted for each range cell. — 'true' profile; -•- estimated mean value; estimated standard deviation. $f = 13.75$ GHz

For instance, Fig. 1 shows the error made utilising the kZS algorithm with a particular 'true' profile, for

$\sigma_s = 9\text{dB}$ and $\sigma_0 = 12\text{dB}$, and a rainfall rate of 20mm/h at the sea surface level. Simulations were carried out following the method mentioned above. The 'true' rainfall profile is constant up to 4.5km altitude, then decreases with a corresponding reflectivity decrease rate of 5dB/km . Mean value and standard deviation of the 100 independent RRVP reconstructions are plotted for 32 range cells (range resolution: 250m), up to 8km altitude. The relative uncertainty (error) on R , defined as

$$\varepsilon_R = \frac{\sigma_R}{\bar{R}} \quad (8)$$

where \bar{R} and σ_R are mean value and standard deviation of the estimated rainfall rate, respectively, falls around 34% in the lower region. A still increased error is expected when rainfall effects are not accounted for by σ_0 ; according to Fig. 6 of the companion paper [5], for a maximum expected $R = 100\text{mm/h}$, the maximum NRCS bias due to rainfall is about 3dB , almost independent of wind velocity.

4 Estimating rainfall rate vertical profiles exploiting EM modelling: 'two cells' method

At nadir incidence and for a given surface wind, the EM model described in [5] provides the sea NRCS as a function of rainfall. This is exploited by the simple method introduced here to jointly predict both NRCS and rainfall rate over the sea surface. We call it the 'two cells' method, since it accounts only for echoes from a couple of adjacent radar range cells, namely those closest to the sea surface. For simplicity, suppose that range sidelobes are sufficiently low, so that interference among contiguous range cells can be neglected. Suppose also that the contribution of the sea surface to the first cell return at range r_s is prevailing over that due to rainfall, and that the rainfall rate is the same in the two range cells.

Denoting with Δr the range resolution, with \bar{P}_s and \bar{P} the mean powers related to the first cell (including the sea surface) and second cell (centred at range r), respectively, and with A the total PIA at the second cell, we get, under the aforementioned hypotheses

$$\bar{P} = \frac{C \cdot Z(R)}{r^2} A \quad (9)$$

and

$$\bar{P}_s = \frac{C_s \cdot \sigma_s(R)}{r_s^2} \cdot \exp(-0.46\Delta r k(R)) A \quad (10)$$

where $Z(R)$ and $\sigma_s(R)$ express, respectively, the dependence of the cell reflectivity and of the sea NRCS on the rainfall rate R . Being $r/r_s \approx 1$

$$\frac{\bar{P}_s}{\bar{P}} = \frac{C_s \cdot \sigma_s(R) \cdot \exp(-0.46\Delta r k(R))}{C \cdot Z(R)} \quad (11)$$

Exploiting eqn. 3, we obtain

$$\begin{aligned} & \left(\frac{\alpha(r_s) \cdot \bar{P}_s \cdot C}{\bar{P} \cdot C_s} \right)^{1/\beta} \\ &= \frac{\sigma_s^{1/\beta}(R) \cdot \exp\left(-0.46\frac{\Delta r}{\beta} k(R)\right)}{k(R)} = f(R) \end{aligned} \quad (12)$$

To estimate the rainfall rate in the proximity of the sea surface, $f(R)$ in eqn. 12 can be inverted with respect to R , once estimates of \bar{P}_s , \bar{P} and $\alpha(r_s)$, as well as of the radar constants C and C_s are available. However, the

actual relationship $\sigma_s(R)$ is not known. The EM model is thus exploited to provide the 'guess' law $\sigma_0(R)$ which is introduced in eqn. 12 in place of $\sigma_0(R)$. Notice that one single solution is found by inverting $f(R)$, since it is a monotonically decreasing function. In fact, $k(R)$ (given by the second eqn. 19) increases and $\sigma_0(R)$ decreases with increasing R .

The simple principle described can be exploited by algorithms like kZS, that rely on the sea surface NRCS, at frequency bands, such as the K_u band, where rainfall dependence cannot be neglected. Indeed, the estimate may suffer from the aforementioned approximations. Interference problems between surface and volumetric echoes deriving from echo pulse spreading induced by beamwidth, rough surface and limited system bandwidth could be overcome by applying the 'two cells' method to a couple of non adjacent range cells. Obviously, this requires the stronger assumption of constant rainfall in a higher column over the sea surface. However, the alternative would be to tolerate *a priori* a residual NRCS bias in the standard kZS algorithm.

Summarising, the following steps should be followed for RRVP retrieval through the new proposed method:

- utilise a measured, estimated or predicted value of wind velocity over the sea surface
- for that wind velocity, select the theoretical relationship between rainfall rate R and surface NRCS
- utilise the 'two cells' method to provide the rainfall rate estimate over the sea surface, jointly with the related NRCS estimate,
- utilise the NRCS estimate in the standard kZS algorithm for RRVP retrieval.

5 Numerical simulations: some considerations and results

We present here the results of some simulations carried out to evaluate the performance of the RRVP retrieval based on the described method and referring to 13.75GHz (the frequency of the TRMM radar [3]). As a matter of fact several uncertainties affect the introduced $Z-k$, $Z-R$ and $\sigma_s(R)$ relationships, and simulations need to consider these. Following the example of [6], to compare the simulated RRVP reconstructions with some 'truth' reference, we used as deterministic references the relationships among R , Z and k in eqn. 19 of the Appendix (Section 9.1, eqn. 19), with the values suggested in [4], namely $E = 0.66 \cdot 10^6$, $b = 1.5$, $F = 0.309$ and $d = 1.156$. Integration of 60 independent echo samples for each radar range cell was supposed. Furthermore, we assumed that the 'true' law $\sigma_s(R)$ coincided with $\sigma_0(R)$ as provided by the EM model. Then, random error parameters (see the Appendix (Section 9.3)) were used to simulate uncertainties related to those relationships and to radar power estimates. The result is a modified version of eqn. 12, namely eqn. 27.

A separate discussion is opportune about uncertainties affecting the sea NRCS estimate. These were modelled, after [6], through the random variable σ_m defined as follows:

$$\sigma_m = \sigma_1 \cdot \sigma_0 \quad (13)$$

where σ_1 is the uncertainty in the 'guess' value of eqn. 7. As in [6], σ_1 was taken as a Gamma-distributed r.v. with unitary mean value and 0.5 standard deviation.

tion; thus the NRCS relative uncertainty ε_σ , defined analogously to ε_R of eqn. 8, is 50%.

Notice now that, still in [6], the authors observed that 'possible systematic changes in σ_s due to the effects of raindrops impinging on the ocean surface or to the effects of surface winds' had been neglected. Therefore, through σ_1 they modelled all possible uncertainties around the fixed guess σ_0 , including those due to wind and rainfall. Our formulation of the sea NRCS, which does account for both corrugations due to wind and rainfall, may indeed be affected by EM model approximations related to height and slope distributions, and to the wind and rainfall corrugation spectra, as discussed in the companion paper [5]. However, the new model-based approach leads to the following observations, with direct consequences for the interpretation of the simulation results:

(a) The aforementioned hypothesis $\sigma_s(R) = \sigma_0(R)$ (implying the assumption $\sigma_B = 0$ in the simulations) is consistent with: (i) the evidence that wind and rainfall effects are primary causes of NRCS bias errors at the K_u band; and (ii) the expectation that the residual bias error is lower, on average, than that made adopting an *a priori* fixed guess. Consequently, the average bias error is neglected as a second-order effect, and modelling the other NRCS discrepancies and approximations is left to σ_1 .

(b) Our proposed method requires a wind velocity estimate; a relative uncertainty ε_w on such an estimate, defined analogously to ε_R and ε_σ , generates an uncertainty on the 'guess' law $\sigma_0(R)$, which in all likelihood dominates that due to EM model approximations. For this reason, in our simulations we kept $\varepsilon_\sigma = 50\%$ as in [6]. In any case, the example of Fig. 2 shows that the proposed method is a valid alternative. The performances of (a) the 'standard' kZS algorithm and (b) our model-based method are compared through the following parameter:

$$\eta = \frac{\varepsilon_R^{(a)} - \varepsilon_R^{(b)}}{\varepsilon_R^{(b)}} \quad (14)$$

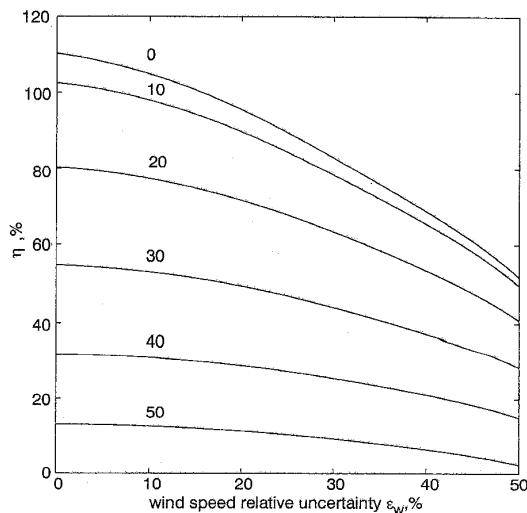


Fig. 2 Performance of the proposed RRVP retrieval method compared to that of the kZS algorithm. η is plotted against the wind speed relative uncertainty, wind velocity is 13m/s at height 19.5m. Curves refer to different values of ε_σ (in%). In the kZS case, a fixed $\varepsilon_\sigma = 50\%$ was assumed

where $\varepsilon_R^{(a)}$ and $\varepsilon_R^{(b)}$ are the relative errors on the final estimate of the rainfall rate in the first cell above the sea surface in the two cases. A constant 'true' RRVP was assumed with $R = 10\text{mm/h}$ and a wind velocity $v_w = 13\text{m/s}$ at 19.5m altitude. In case (i), an unbiased NRCS was assumed, with $\varepsilon_\sigma^{(a)} = 50\%$. Thus, $\varepsilon_R^{(a)} = 41\%$ is the reference performance. In case (ii), the wind estimate was assumed to be unbiased, and both NRCS model uncertainties and wind speed uncertainties were accounted for. Fig. 2 plots η against ε_w for some values of $\varepsilon_\sigma^{(b)}$ supposing the wind velocity to be uniformly distributed around 13m/s. The basic result is that even ascribing 50% NRCS relative uncertainty to the EM model approximations, the model-based method performs better despite wind speed uncertainties of up to 50%. Notice also that for lower $\varepsilon_\sigma^{(b)}$ in model approximations and for $\varepsilon_w = 50\%$, relative performance further improves.

As a marginal observation related to the above issues, notice that in any case further modelling and analysis effort is needed. Surface wave damping effects due to heavy rainfall or to updrafts and downdrafts and nonuniform antenna beam filling [8], when occurring, are primary causes of bias too. However, the 'two cells' method is not related in any way to the complexity of models, but complements them.

The simulation framework described in the Appendix was first used to verify the accuracy of the rainfall rate estimates R_m at sea level obtained through the 'two cells' method. By repeatedly solving eqn. 27 with respect to R_m for several values of the 'true' R , mean values and standard deviations of R_m were estimated. Results are shown in Fig. 3. The line $R_m = R$ shows that a slight bias is present due to the nonlinearity of the second eqn. 19. A wind velocity $v_w = 4.32\text{m/s}$ at 19.5m altitude was assumed. The absolute error increases with R , while ε_R decreases to 16% after a 26% peak reached around $R = 40\text{mm/h}$. It was also verified that a NRCS bias, besides obviously introducing additional bias in the rainfall estimate, does not modify substantially such a behaviour of ε_R , in the same way it does not modify the rainfall estimate accuracy in the RRVP retrieval simulations presented below.

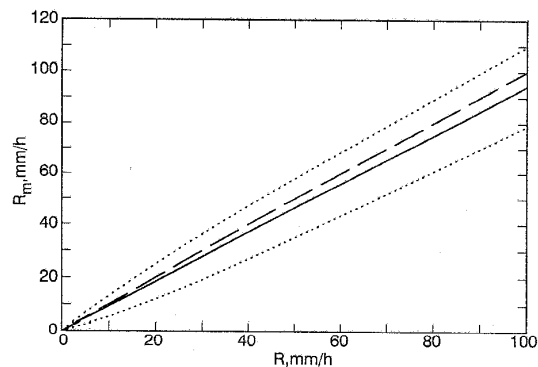


Fig. 3 Mean value and standard deviation curves of the 'estimated' rainfall rate R_m ('two cells' method) against 'true' rainfall rate R at sea level. Wind velocity is 4.32m/s at height 19.5m; $f = 13.75\text{GHz}$. --- exact value, - - - estimated mean value, estimated standard deviation

To simulate RRVP retrieval through our model-based method, we first considered 'true' rainfall profiles like that of Fig. 1. Fig. 4 reports the mean value and standard deviation of 100 independent RRVP recon-

structions for $R = 10\text{mm/h}$ at sea level and $v_w = 4.32\text{m/s}$. Fig. 5 shows analogous results for $R = 50\text{mm/h}$. In both cases, the relative error is comparable at all altitudes. Fig. 6, instead, was obtained for $v_w = 20\text{m/s}$ and $R = 10\text{mm/h}$. Comparing it with Fig. 4, notice that the remarkable difference in wind velocity does not influence the accuracy of profile retrieval (assuming the same ϵ_w).

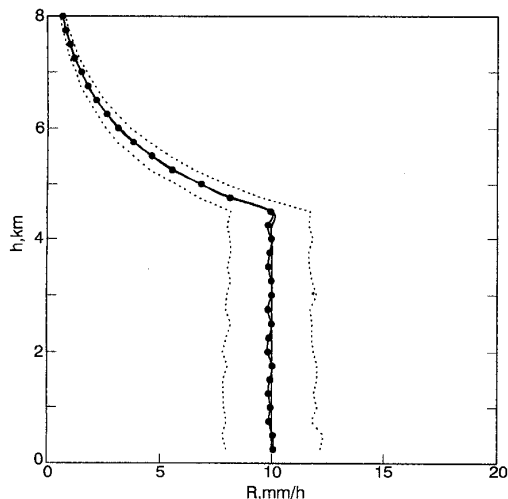


Fig. 4 Rainfall profile obtained through the proposed scheme at K_u band
Rainfall rate at sea level: 10mm/h ; wind velocity: 4.32m/s at height 19.5m ; $f = 13.75\text{GHz}$
Mean value and standard deviation estimated over 100 runs are plotted
— 'true' velocity
-•- estimated mean value
..... estimated standard deviation

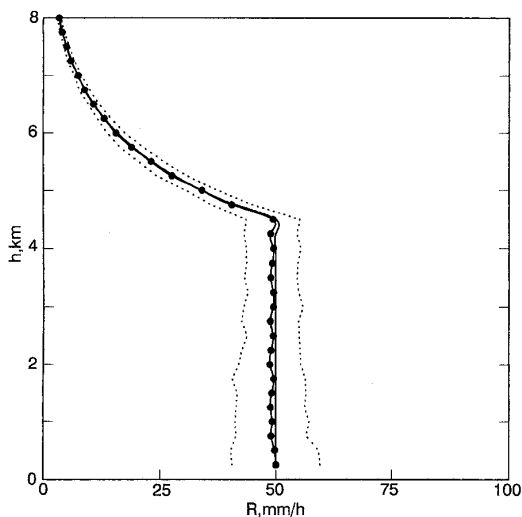


Fig. 5 Rainfall profile obtained through the proposed scheme at K_u band
Rainfall rate at sea level: 50mm/h ; $f = 13.75\text{GHz}$; $v_w = 4.3\text{m/s}$
— 'true' velocity
-•- estimated mean value
..... estimated standard deviation

Through RRVP retrieval, evaporation processes or increase of rainfall rate in proximity to the sea surface can be detected. In these cases, reflectivity gradients occur immediately over the sea surface, that may affect the rainfall profile retrieval. Therefore, in other simulations we employed a different type of profile, with a gradient below 4.5km altitude corresponding to a reflectivity loss rate of 1dBZ/km , and a rainfall rate of 50mm/h at that altitude. Fig. 7 shows the results for

for $v_w = 4.3\text{m/s}$ and a positive reflectivity gradient. Reconstruction performance is still good in terms of mean values, and ϵ_R is comparable to that of previous profiles. Similar results were obtained for a negative gradient.

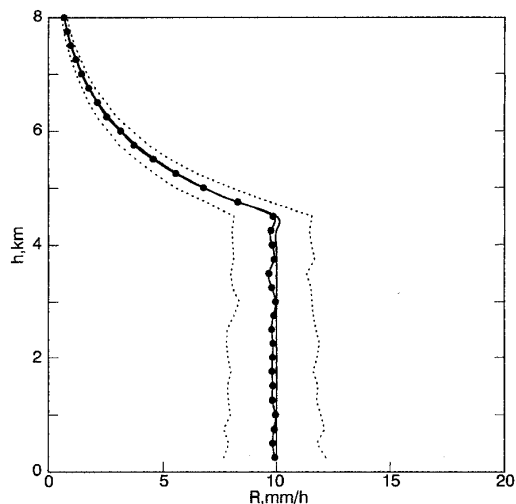


Fig. 6 Rainfall profile obtained through the proposed scheme at K_u band
Rainfall rate at sea level: 10mm/h ; $f = 13.75\text{GHz}$; $v_w = 20\text{m/s}$
— 'true' velocity
-•- estimated mean value
..... estimated standard deviation

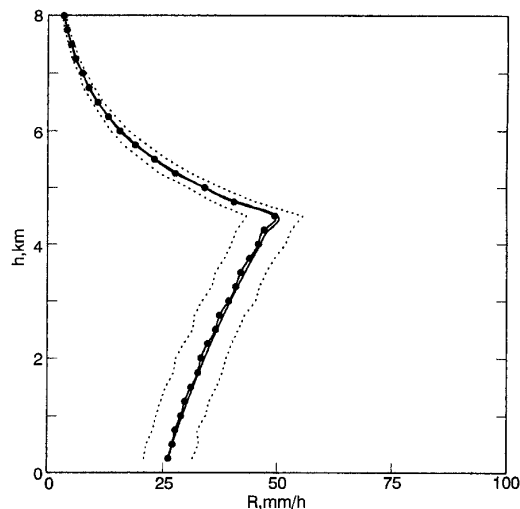


Fig. 7 Rainfall profile obtained through the proposed scheme
Rainfall rate at the referenced altitude of 4.5km : 10mm/h , wind velocity: 4.32m/s at height 19.5m ; $f = 13.75\text{GHz}$
Mean value and standard deviation estimated over 100 independent runs are plotted
— 'true' velocity
-•- estimated mean value
..... estimated standard deviation

6 Conclusions

The problem of an accurate prediction of the power backscattered at nadir incidence by the sea surface is highly relevant to enhance the potentiality of the existing surface referenced RRVP retrieval algorithms, like kZS, for spaceborne weather radars. We highlighted that a direct use of kZS with a 'blind' guess of the sea NRCS (which remains the only solution when no information is available), may cause relevant errors. A chance to increase the reliability of RRVP retrieval comes from predictions based on EM models, like that

proposed in the companion paper [5]. We pointed out that this is particularly important at the K_u band, since rainfall intensity remarkably modifies the NRCS that would be predicted accounting for wind only. A real physical limitation of the adopted EM model is that it does not account for an increase of the NRCS at nadir incidence due to damping of sea waves [9], as resulting from heavy rainfall and associated vertical winds. This problem is partially attenuated by a reduced sensitivity of the kZS algorithm to the NRCS estimate errors in the case of heavy rainfall.

We proposed a method to integrate the NRCS prediction with the kZS retrieval, jointly providing the estimated rainfall rate at sea level; improved performance was demonstrated through simulations, carried out at 13.75GHz. Of course, the foreseeable improvement is real, provided that some additional information allows us to overcome the bare hypothesis of a generic 'standard' average value of sea NRCS. Our proposed approach requires that wind velocity in the area of interest is available, given by either measurements or models, or joint exploitation of both of them. Additional measurements should refer to the same or a contiguous area, provided by an independent sensor, such as a scatterometer.

An interesting perspective for future developments could be to utilise two frequency bands, e.g. C and K_u bands, for measurements over the same area. Actually, in [10] we showed that sea NRCS is not influenced significantly by rainfall at the C band. Exploiting the NRCS estimate at the lower frequency, the wind velocity could be inferred and then integrated with the higher-frequency measurements and the NRCS prediction to estimate the rainfall rate.

7 Acknowledgments

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9 Appendix

9.1 Error model

The error model recalled here is that utilised in [6]. eqns. 5 and 11 exploit a power ratio; assuming the same radar calibration error for all the cells, it can be neglected [6].

The Marshall-Palmer model [2] is considered as 'truth' DSD reference

$$N(D) = N_0 \exp(-\Lambda D) \quad N_0 = 0.8 \times 10^7 \text{ m}^{-4} \quad (15)$$

Random variations of N_0 along the propagation path are accounted for by the r.v. N_{0m}

$$N_{0m} = \nu N_0 \quad (16)$$

where ν is a unitary mean value r.v. following a gamma distribution, with 0.5 standard deviation. The 'measured' powers P_m and P_{Sm} , from cells where rainfall echo and surface echo is prevailing, respectively, are related to the expected powers as follows:

$$P_m = \delta_r \bar{P} \quad \text{and} \quad P_{Sm} = \delta_S \bar{P}_S \quad (17)$$

where δ_r and δ_S are r.v. accounting for estimation errors, with the following pdf [3, 6]:

$$p(x) = N_i^{N_i} x^{N_i-1} \exp(-N_i x) / (N_i - 1)! \quad (18)$$

N_i being the number of independent integrated samples.

The variability of N_0 is considered as the only source of uncertainty in the Z - R , k - R and Z - k relationships. It follows [6]

$$\begin{aligned} Z &= EN_0^{1-b} R^b \\ k &= FN_0^{1-d} R^d \\ Z &= EF^{-\beta} N_0^{1-\beta} k^\beta \end{aligned} \quad (19)$$

where E , F , b , d depend on frequency and $\beta = b/d$. Comparing the third equation of eqn. 19 with eqn. 3 follows $\alpha = EF^{1-\beta} N_0^{1-\beta}$. The uncertainty on α is accounted for by the r.v. α_m

$$\alpha_m = \alpha_1 \alpha \quad (20)$$

where, from eqn. 16, $\alpha_1 = \nu^{1-\beta}$. Finally, as discussed in Section 5, the uncertainty in the NRCS 'guess' $\sigma_0 = \sigma_s - \sigma_B$ is modelled through σ_1

$$\sigma_m = \sigma_1 (\sigma_s - \sigma_B) \quad (21)$$

9.2 Use of the error model parameters in the kZS algorithm simulations

Exploiting the error model, the 'measured' function corresponding to $w_0(r)$ is

$$\begin{aligned} w_{0m}(r) &= \left(\frac{\delta_r(r) \cdot \bar{P}(r) \cdot r^2 \sigma_1 \sigma_0 C_s}{\delta_S \bar{P}_S(r_s) \cdot r_s^2 \alpha_1 \alpha \cdot C} \right)^{1/\beta} \\ &= w_0(r) \cdot T(r) \end{aligned} \quad (22)$$

where $T(r)$ is a random process

$$T(r) = \left(\frac{\sigma_0 \delta_r(r) \sigma_1}{\sigma_S \delta_S \alpha_1} \right)^{1/\beta} \quad (23)$$

and $\delta_r(r)$ accounts for range variations of the mean power estimate errors, supposed independent from cell to cell. Substituting in eqn. 6, and exploiting eqn. 4, the 'estimated' attenuation factor $k_m(r)$ becomes

$$k_m(r) = \frac{k(r)T(r) \exp\left(\frac{0.46}{\beta} \int_r^{r_s} k(s)ds\right)}{1 + \frac{0.46}{\beta} \int_r^{r_s} k(s)T(s) \exp\left(\frac{0.46}{\beta} \int_s^{r_s} k(t)dt\right) ds} \quad (24)$$

Introducing the k - R relationship, we can relate the 'estimated' RRVP $R_m(r)$ to the 'true' reference $R(r)$

$$R_m(r) = R(r) \left[\frac{\nu^{d-1} T(r) \exp\left(\frac{0.46}{\beta} FN_0^{1-d} \int_r^{r_s} R^d(s)ds\right)}{1 + \frac{0.46}{\beta} FN_0^{1-d} \int_r^{r_s} R^d(s)T(s) \times \exp\left(\frac{0.46}{\beta} FN_0^{1-d} \int_s^{r_s} R^d(t)dt\right) ds} \right]^{\frac{1}{d}} \quad (25)$$

9.3 Use of the error model parameters in the 'two cells' method simulations

Rewriting eqn. 12 by inserting all needed error parameters, one gets the following expression, that implicitly defines the 'measured' rainfall rate R_m :

$$\frac{\delta_s \cdot \overline{P}_s \cdot C}{\delta_r \cdot \overline{P} \cdot C_s} = \frac{\sigma_1 \sigma_0(R_m) \cdot e^{-0.46 \Delta r FN_{0m}^{1-d} R_m^d}}{EN_{0m}^{1-b} R_m^b} \quad (26)$$

where \overline{P} and \overline{P}_s are defined in eqns. 9 and 10, respectively, and where $\sigma_B = 0$ for the reasons discussed in Section 5. Introducing eqn. 11, posing $\sigma_s(R) = \sigma_0(R)$ and exploiting again eqn. 19, one gets

$$\begin{aligned} & \frac{\sigma_1 \sigma_0(R_m) \cdot e^{-0.46 \Delta r FN_0^{1-d} \nu^{1-d} R_m^d}}{\nu^{1-b} R_m^b} \\ &= \frac{\delta_s \sigma_0(R) \cdot e^{-0.46 \Delta r FN_0^{1-d} R^d}}{\delta_r R^b} \end{aligned} \quad (27)$$

utilised in the simulations to provide R_m and, in turn, $\sigma_0(R_m)$.