Title
Managerial Learning from Financial Market Prices

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Publication Date
2021

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA,
IRVINE

ESSAYS ON MANAGERIAL LEARNING FROM FINANCIAL MARKET PRICES

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Management

by

Xiaoqi Xu

Dissertation Committee:
Associate Professor Chong Huang, Chair
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2021

DEDICATION

To my parents and grandparents,

who have been there for me from day one.

Thank you for all your love, support, and encouragement.

To my friends,

who make good times better and hard times easier.

Thank you for having my back and adding truckloads of joy in my life.

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ACKNOWLEDGMENTS

I would like to express the deepest appreciation to my committee chair, Professor Chong Huang. His great enthusiasm for Economics and Finance and his rigorous work ethic have been encouraging me to pursue higher goals in this profession. His caring for students has touched me so much. I truly appreciate that he provided endless support whenever I encountered difficulties in research. The countless discussion with him on research in general and on my dissertation was inspirational and fruitful. I am extremely grateful to him for inspiring me to think deeply and training me to present and write more efficiently. But most importantly, he taught me to keep trying alternative ways to overcome shortcomings, breakthrough challenges and always be open and curious to new ideas.

I would like to thank other members on my committee. Professors Jinfei Sheng and Zheng Sun, for their generous support and constructive feedback throughout the development of this dissertation. A special thanks gives to Professor Zheng Sun, who provided chances and guidance for being an instructor to work with students from eclectic cultural and educational backgrounds and offered opportunities to attend conferences. I must also acknowledge Professors David Hirshleifer and David C. Yang for providing valuable finance knowledge and raising constructive questions. I am grateful for their insightful suggestions and comments along the way. In addition, I would like to thank Professor Charles Ka Yui Leung at City University of Hong Kong, and Professors Ren-Raw Chen and Yuewu Xu at Fordham University, who introduced me to research in Economics and Finance.

I would like to thank my academic fellows in the Paul Merage School of Business, particularly Luming Chen, Ziyi Cao, Jie Gao, Shuang Han, Qin Li, Yifan Li, Yue Li, Shengjun Mao, Lin Sun, Yushui Shi, Chenzhe Tian, Joshua Thornton, Qiguang Wang, Shijia Wu, Wanyi Wang, Nan Xu, Yuxuan Zhu, and Youqing Zhou, for a cherished time together. I also want to extend my sincere gratitude to elderly brothers and sisters at Church in Irvine for their care and shepherding.

In addition, I am grateful for all comments and suggestions from Yashar Barardehi, Jiawei Chen, Yao Chen, Philip H. Dybvig, Itay Goldstein, Botond Köszegi, Radhika Lunawat, Claudia Steinwender, Chaojun Wang, Yizhou Xiao, Dong Yan, Liyan Yang, Ádám Zawadowski, Haoxiang Zhu, and conference and seminar participants at 2020 Financial Intermediation Research Society, 2020 Financial Management Association annual meeting, 2020 Australasian Finance & Banking Conference, 2021 Western Conference on Mathematical Finance, 2021 China Meeting of the Econometric Society, 2021 China International Conference in Finance, and Central European University, China-Asean Institute of Financial Cooperation, University of California, Irvine, University of Southern California.

I thank the generous funding from the Paul Merage School of Business at the University of California, Irvine, including Teaching Assistantships, Graduate Research Scholarships, and Graduate Fellowships.

In conclusion, I would like to thank my family and friends for all their love, support, and encouragement. They have changed me and made me a better person.

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ABSTRACT OF THE DISSERTATION

ESSAYS ON MANAGERIAL LEARNING FROM FINANCIAL MARKET PRICES

By

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Doctor of Philosophy in Management

University of California, Irvine, 2021

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Learning from market prices by decision-makers in the real side of the economy affects informed investor trading behavior and financial market efficiency, which feedback to managers' decisions and real efficiency. Such a feedback loop provides some interesting results on what could be equilibrium outcomes in the financial markets. In this doctoral dissertation, I study the role managerial learning from financial market prices plays in various financial and economic settings.

Chapter 1 studies the dark pool effects on price discovery and real efficiency when firm managers who need to make production investment decisions learn information from financial market prices. A strategic informed investor trades on private information and chooses a trading venue between an exchange market and a dark pool. An uninformed noise investor randomly selects a trading venue and randomly trade. Managerial learning leads to wiser real decisions and results in higher firm value, which aggravates the buying profits and alleviates the short-selling profits of the informed investor. The magnitude of managerial learning effects and the execution risk of dark pool trading vary in the noise trading in the exchange market. As a result, the dark pool effects on informed investor trading venue choice, exchange market efficiency, and real efficiency all depend on the noise trading in the exchange market.

Chapter 2 investigates the dark pool effects on investor trading venue choice in a model featuring managerial learning from exchange market prices. The model is essentially the one studied in Chapter 1 with transaction cost in the exchange market and delay cost of the uninformed liquidity investor that make the uninformed liquidity investor trading venue choice endogenous. While the transaction cost and the delay cost affect the informed trading and liquidity trading in the exchange market, the dark pool does not divert investors away from the exchange market and thus does not affect the exchange market efficiency. However, the dark pool may initiate investors' coordination motives to trade in the dark pool whenever trading in the exchange market can not bring higher profits than choosing not to trade.

Chapter 3 analyzes the interaction between secondary financial market efficiency and product market competition in an entry game. A potential entrant learns from an insider's trading in the stock market of a monopoly incumbent, such that the insider and the entrant have conflicting interests. Once the entrant enters, it competes with the incumbent in a Cournot duopoly setting and reduces the incumbent firm value. As a result, entrant learning causes "buy-side" limits to arbitrage. Depending on different entry barriers, transaction costs in the financial market may increase or decrease entry probability. The impact of transaction costs on the entry probability is also affected by economic and informational conditions that the insider faces. A policy of reducing entry barriers has non-monotonic effects on entry probability.

Chapter 1

Dark Pool Effects on Price Discovery and Real Efficiency

Abstract This paper studies dark pool effects on exchange market efficiency and real economic efficiency in a model featuring managerial learning from the exchange market. When the exchange market has low noise trading, an informed investor surely trades in the dark pool when firm fundamentals are bad and randomizes between the exchange market and the dark pool when firm fundamentals are good. Such trading asymmetry is associated with asymmetric firm investments and leads to asymmetric limits to arbitrage. At some noise trading levels in the exchange market, the dark pool increases both exchange market efficiency and real economic efficiency; at some others, the dark pool surprisingly increases real economic efficiency even if it harms exchange market efficiency. Hence, using exchange market efficiency to assess dark pools may overestimate their adverse effects on real economic efficiency. We also find that the effects of managerial learning is non-monotonic in the exchange market noise trading level.

JEL Classification: D83, G11, G14, G18

Key words: Dark pool, managerial learning, exchange market efficiency, real efficiency

1.1 Introduction

Regulatory reform has led to the rapid growth of dark pools.[1] Unlike exchange markets, dark pools do not disclose their bid and ask quotes, and they delay publicly displaying trading information. Such low transparency facilitates institutional investors' block trading by preventing large orders' adverse price effects. Recently, the rise of electronic transactions, together with the almost zero transaction fee, also makes dark pools attractive to retail investors.

In spite of the fast growth of dark pools, their low transparency raises serious concerns in policy circles. In particular, regulators are worried that dark pools may hinder price discovery in exchange markets, a critical role played by exchange markets in modern economies. The SEC (2018), for example, states that price discovery is harmed for high levels of trading on alternative trading systems. Many recent studies, such as Ye (2011), Jiang, McInish, and Upson (2014), and Zhu (2014), come on the heels of the regulatory debate over dark pools, but they draw different conclusions on the dark pool effects on price discovery in exchange markets.

Such a concern, however, is even more disquieting when real decisions are made based on exchange market prices. Many empirical studies, such as Luo (2005), Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), Foucault and Frésard (2012), and Jayaraman and Wu (2019) have provided evidence that firm managers are gleaning information from exchange market prices and making real decisions based on such information.[2] Then, a naïve implication will be that if dark pools hurt exchange market efficiency, they will weaken

[1] As of May 2018, dark pools accounted for 12.8% of average daily trading volume, and as of February 2020, there were more than 50 dark pools registered with the SEC. According to Rosenblatt Securities, an institutional brokerage firm specializing in market structure, dark pool trading roughly rose from 6.5% in 2008 to around 14% in 2012 of U.S. equity volume, and it rose from 3% in 2010 to around 9% in 2017 of European equity volume. We shall provide a brief introduction to dark pools in Section 1.2.

[2] See Bond, Edmans, and Goldstein (2012) for an excellent survey of this literature.

the role of the exchange market in guiding firm investments and thus reduce real economic efficiency.

In this paper, we investigate dark pool effects on exchange market efficiency and real economic efficiency when managers are learning from the exchange market. Several interesting questions arise. How does an informed investor's trading venue choice depend on her private information? Will dark pools necessarily reduce exchange market efficiency? When dark pools reduce exchange market efficiency, do they necessarily reduce real economic efficiency?

We answer these questions in a model in which a firm manager decides to expand, remain, or reduce the firm investment and an informed investor chooses to trade in an exchange market or a dark pool. The firm value is determined by its investment and firm fundamentals. The firm fundamentals are either high or low and are privately known by the informed investor. There is a noise investor who randomly makes trading venue choices and then randomly chooses positions after arriving at a trading platform. Both investors trade simultaneously. If they submit orders to the exchange market, their orders will be surely executed because the exchange market has a competitive market maker who provides liquidity. Alternatively, investors may submit their orders to the dark pool where order execution is not guaranteed and where potential execution price is the concurrent asset price in the exchange market. The manager and the market marker then observe the total trading volume in the exchange market; however, neither observes trading in the dark pool. Based on their information, the market maker sets a price to make herself break even, and the manager makes investment decisions to maximize the firm value.

We find that the dark pool effects on the informed investor trading venue choice, exchange market efficiency, and real economic efficiency all depend on the probability that the noise investor trades in the exchange market. Specifically, when the noise investor is more likely to trade in the exchange market (i.e., the exchange market has high noise trading), the informed investor chooses the exchange market for sure,[3] and the dark pool promotes both exchange market efficiency and real economic efficiency. By contrast, when the exchange market has low noise trading, the informed investor surely sells in the dark pool at low firm fundamentals, while she randomizes between the two trading venues at high firm fundamentals. That is, the informed investor trading venue choice is asymmetric, which is associated with the asymmetric firm investments and leads to asymmetric limits to arbitrage in the exchange market.

Comparing with a benchmark where there is no dark pool, and thereby both investors can trade in the exchange market only, we find that the dark pool reduces exchange market efficiency when the probability of noise investor trading in the exchange market is close to zero or one half but promotes exchange market efficiency otherwise. Furthermore, whenever the dark pool promotes exchange market efficiency, it increases real economic efficiency. Surprisingly, we also identify circumstances when the dark pool harms exchange market efficiency but increases real economic efficiency.

The asymmetric trading venue choices of the informed investor arises from several features of our model. First, managerial learning significantly affects investor trading venue choice since it determines firm investment and thus firm value. As in Edmans, Goldstein, and Jiang (2015), we assume that expanding investment and reducing investment are respectively the correct investment decisions when the firm fundamentals are high and low. Therefore, when firm fundamentals are high, a long position of the informed investor in the exchange market guides the manager to expand investment, leading to a higher firm value and a higher buying profit. When firm fundamentals are low, a short position of the informed investor in the exchange market leads the manager to reduce investment, resulting in a higher firm value and a lower short-selling profit. Hence, managerial learning increases the incentives of the positively informed investor (informed investor observing high firm funda-

[3]In our model, there is no transaction cost in either market. Hence, independent of the market she chooses to trade, the informed investor buys when the firm fundamentals are high and sells otherwise.

mentals) to buy in the exchange market but reduces the incentives of the negatively informed investor (informed investor observing low firm fundamentals) to sell in the exchange market.

However, managerial learning alone is insufficient to explain the asymmetry of the informed investor's equilibrium trading venue choice. Indeed, the key insight in our paper is an opportunity cost of trading in the exchange market, which is the trading profit in the dark pool. Importantly, such an opportunity cost is endogenous: in the dark pool, the trading profit depends on the execution price, which is the concurrent asset price in the exchange market and is determined by the market maker's belief about the informed investor trading venue choice.

When the noise investor is more likely to trade in the exchange market, the informed investor can hardly find a counterparty in the dark pool. The low execution rate in the dark pool results in a low opportunity cost, which is dominated by the trading profit in the exchange market. Hence, the informed investor trades in the exchange market, regardless of the firm fundamentals.

By contrast, when the noise investor is more likely to trade in the dark pool, the informed investor can easily find a counterparty in the dark pool. So, her opportunity cost of trading in the exchange market is high. Importantly, the opportunity cost is asymmetric in this case. Because of managerial learning, the negatively informed investor has strictly weaker incentives to choose the exchange market than the positively informed investor. As a result, the firm manager's posterior belief shifts toward bad fundamentals after learning from the exchange market, making her reduce firm investment. In turn, the negatively informed investor's profit by selling in the exchange market is dominated by that in the dark pool (equivalent, the opportunity cost of trading in the exchange market dominates the trading profit in the exchange market). Ultimately, in equilibrium, the negatively informed investor surely chooses the dark pool. The positively informed investor, on the other hand, is indifferent between the exchange market and the dark pool in equilibrium, so she randomizes between these two trading venues. We further show that the probability of the positively informed investor buying in the exchange market increases in the noise trading in the exchange market.

The investor trading venue choice determines the exchange market efficiency. We measure the exchange market efficiency by *mutual information* introduced in information theory (Shannon, 1948), because the endogenous firm value makes the commonly used variance ratio (Kyle, 1985) implausible in our framework. When the exchange market has high noise trading, the informed investor surely trades in the exchange market. Then, as noise trading increases further, the exchange market efficiency decreases. Therefore, the dark pool increases exchange market efficiency because it helps divert noise trading from the exchange market.

When the exchange market has low noise trading, the dark pool effect on exchange market efficiency is non-monotone as the noise trading increases. In particular, when the noise trading in the exchange market is extremely low, the informed investor prefers the dark pool, so little information is incorporated into the exchange market. As a result, the dark pool reduces exchange market efficiency. As the noise trading increases, the positively informed investor is more likely to trade in the exchange market, increasing exchange market efficiency. (Recall that the negatively informed investor surely chooses the dark pool, regardless of the noise trading in the exchange market.) However, once the noise trading increases beyond a threshold, the positively informed investor surely trades in the exchange market. Then, any further increase in the noise trading will reduce exchange market efficiency.

The dark pool effect on exchange market efficiency then implies its effect on real economic efficiency through managerial learning. We measure real economic efficiency by ex-ante expected firm value. We show that whenever the emergence of a dark pool increases exchange market efficiency, real economic efficiency increases. This is intuitive: a more efficient exchange market can guide the manager to make wiser investment decisions. A more surprising result is that the dark pool promotes real economic efficiency in some circumstances, even if it hurts exchange market efficiency. For example, when the probability of noise investor trading in the exchange market is sufficiently close to one-half, the dark pool reduces exchange market efficiency but increases real economic efficiency. Specifically, with a dark pool, the manager is more likely to reduce corporate investment, which increases the firm value when the firm fundamentals are low. When the firm fundamentals are high, the positively informed investor surely trades in the exchange market, guiding the manager to expand investment; hence, the firm value will also increase.

Our theoretical analyses have important policy implications. First, whether dark pools increase or decrease exchange market efficiency depends on the noise trading in the exchange market. Therefore, to evaluate the dark pool effects on price discovery in the exchange market, we need to calibrate how dark pools affect the noise trading in the exchange market. More importantly, when evaluating the dark pool effects on real economic efficiency, policymakers who aim to improve real economic efficiency should not simply use the exchange market efficiency as a proxy because dark pools may sometimes hurt exchange market efficiency while promoting real economic efficiency.

In addition to the dark pool effects, we also examine managerial learning effects on the investor trading venue choice and generate empirical implications for the managerial learning hypothesis. In a benchmark where the manager does not learn from the exchange market, the informed investor's trading venue choice is symmetric; that is, she chooses the exchange market with a probability that is independent of the firm fundamentals. This probability increases in the likelihood of the noise investor trading in the exchange market. By contrast, when the manager is learning from the exchange market, there are more severe sell-side but less severe buy-side limits to arbitrage in the exchange market, and the buy-side limits to arbitrage are attenuated as the probability of the noise investor trading in the exchange market increases. Finally, managerial learning promotes exchange market efficiency when the noise investor is very unlikely to trade in the exchange market but then hurts it when the probability of the noise investor trading in the exchange market increases beyond a threshold. When the noise investor is very likely to trade in the dark pool, managerial learning does not affect exchange market efficiency.

Our paper contributes to the fast-growing literature on dark pools. Several papers focus on dark pool effects on price discovery in the exchange market.[4] Jiang, McInish, and Upson (2014) find that as uninformed investors can segment their order flow to off-exchange venues, a larger proportion of trades on the exchanges are informed, improving the price discovery in the exchange market. Nimalendran and Ray (2011) reveal that crossing-network trades are informed. Ye (2011) focuses on the strategies of informed investors in the presence of a dark pool and finds that the dark pool deteriorates price discovery. Zhu (2014) suggests that dark pools improve price discovery because they divert noise trading from the exchange market. Our paper contributes to this literature by developing a tractable model featuring managerial learning to analyze the dark pool effect not only on exchange market efficiency but also on real economic efficiency. We show that managerial learning leads to new predictions of dark pool effects on price discovery in the exchange market. We also identify conditions when the emergence of a dark pool can increase the exchange market efficiency and the real economic efficiency. Our analysis then has important policy implications.

Second, our paper belongs to the literature on interactions between financial market and corporate decisions. Some studies find that a firm's investment is sensitive to its own stock price (Baker, Stein, and Wurgler, 2003, Goldstein and Guembel, 2008, Hirshleifer, Subrahmanyam, and Titman, 2006, Khanna and Mathews, 2012), while others document that a firm's investment may be even sensitive to its peers' stock prices (Foucault and Frésard, 2012, 2014, Ozoguz, Rebello, and Wardlaw, 2018). The closest paper to ours in this

[4] Another strand of this literature shows that different trading venues cater to different categories of investors (Buti, Rindi, and Werner, 2017, Degryse, Achter, and Wuyts, 2009, Hendershott and Mendelson, 2000).

literature is Edmans, Goldstein, and Jiang (2015) who find that managerial learning causes an informed investor's asymmetric trading and "sell-side" limits to arbitrage. However, there is no dark pool in their model, and thus the informed investor does not have trading venue choices. In addition, asymmetric trading appears in their model only when the transaction cost is in a certain range. By contrast, we focus on an informed investor's trading venue choice between the exchange market and the dark pool, and we assume that there is no transaction cost in either market. We show that managerial learning and the availability of a dark pool generate endogenous and asymmetric opportunity costs of trading in the exchange market, which in turn lead to the informed investor's asymmetric trading and the firm's asymmetric investment.

Third, the paper contributes to the large literature on the limits to arbitrage. In models without agency problem, arbitragers trade more aggressively when prices move further out of fundamental values (Campbell and Kyle, 1993, DeLong et al., 1990, Grossman and Miller, 1988). In Shleifer and Vishny (1997), the noise trader risk makes investors hardly finance their bets against mispricing. Other studies focus on implementation cost, which includes transaction costs, short-sale constraints, and the costs of discovering or exploiting mispricing (D'Avolio, 2002, Gromb and Vayanos, 2002, Jones and Lamont, 2002, Lamont and Thaler, 2003, Nagel, 2005). We argue that the observed limits to arbitrage in the exchange market may be due to the availability of alternative trading systems. Furthermore, when the manager is learning from the exchange market, the limits to arbitrage is asymmetric and is more severe on the sell-side.

Finally, our paper complements the existing literature on price efficiency. Kyle (1985) defines a measurement of volatility to calibrate how much the insider's information is incorporated into the price. Subrahmanyam (1991) and Spiegel and Subrahmanyam (1992) use the posterior precision of terminal value conditional on the price (or trading volume) to measure price efficiency. All these measurements are developed with the assumption that firm fundamental value is exogenous. Therefore, these measures may be implausible in our model because managerial learning makes the firm value endogenous. We, therefore, propose mutual information, which captures the average information revealed by the stock price (or trading volume), as a measure of exchange market efficiency. We show that mutual information is a robust measure even if cash flows are endogenous.

1.2 An Overview of Dark Pools

Before introducing the model, we first provide an overview of dark pools in this section. We should discuss the special features of dark pools that distinguish themselves from the exchange market. These features play critical roles in our theoretical analysis. First of all, unlike the exchange markets, dark pools do not guarantee order execution. This is because dark pools do not have market makers who provide liquidity. Hence, an order will not be executed until it is successfully matched with a counterparty. This feature has also been highlighted in Zhu (2014). As a result, a dark pool is more attractive to an investor when other investors are more likely to trade on it.

Second, despite the non-execution risk, dark pools are popular among institutional investors because of their relative opaqueness. On the one hand, dark pools do not display their bids and asks, which helps prevent large orders' adverse effects on the stock price. On the other hand, dark pools delay disclosing their trading information. They were not required to disclose trading information until November 2014. Since then, the Financial Industry Regulatory Authority (FINRA) has required alternative trading systems, including all "dark pools," to report their weekly aggregate volume on a security-by-security basis. FINRA will then publish the information regarding Tier 1 NMS stocks (i.e., stocks in the S&P 500 Index, the Russell 1000 Index, and certain ETPs) on a two-week to four-week delayed basis. Information on all other NMS stocks and OTC equity securities will be released two weeks following the publication of information for the Tier 1 NMS stocks. While dark pools' opaqueness may be attractive to certain investors, it hinders information from flowing from dark pools to firms, potentially reducing the efficiency of corporate decisions.[5]

Third, trading conditions in dark pools may depend on exchange markets. Indeed, when a match is formed at a dark pool, the execution price is primarily determined by the concurrent price of the same security at the exchange market. For example, agency broker or exchange-owned dark pools, such as ITG Posit, Liquidnet, and Instinet, derive prices using quotes (e.g., at NBBO midpoint or VWAP) in the exchange market. Therefore, factors that will affect the security price at the exchange market will also affect the execution prices at dark pools, which in turn determine investors' venue choices.

1.3 A Feedback Model with Investor Venue Choice

We now introduce our model. Our model has four agents: a firm manager, an informed investor, a noise investor, and a market maker. The informed investor strategically chooses to trade firm stocks in the exchange market or a dark pool, while the noise investor randomly chooses the trading venue.[6] The market maker is working for the exchange market and clears the exchange market using her own inventories. Therefore, investors in the exchange market will have their orders executed for sure. By contrast, investors who choose the dark pool may not trade successfully. The firm manager and the market maker can observe the total trading volume in the exchange market but can neither identify the trader nor observe the

[5] Nowadays, real-time off-exchange data may be accessible through some data vendors, whose subscribers are mainly high-frequency trading firms. However, subscribing to the whole database is rather costly to a corporation unless the corporation has many security investments. Hence, while firm managers are learning from the exchange market (as documented in empirical studies), they are unlikely to learn from the dark pool.

[6] To focus on the informed investor's strategical venue choice, we model the noise investor as a passive investor. She may randomly make venue choices due to behavioral reasons, such as animal spirits or cognitive errors in processing information. For instance, a noise investor may be a retail investor who simply follows her friends' suggestions to choose a trading venue.

tradings in the dark pool.

1.3.1 Corporate Decisions

The firm value, denoted by $v(\theta, d)$, depends on both the firm investment d and the firm fundamentals $\theta \in \{H, L\}$. In particular, θ is drawn by nature with equal probabilities. The manager can choose to expand the investment ($d = 1$), to keep the current investment ($d = 0$), or to decrease the investment ($d = -1$).[7] If the manager keeps the current investment, the firm value is $v(H, 0) = R_H$ at state H and $v(L, 0) = R_L$ at state L. We assume that $R_H > R_L$. As a convention, we assume that the manager does not change the investment level if doing so cannot bring a strictly higher firm value.

At state H, the "correct" corporate investment is expansion, which creates an additional value g and leads to the firm value $v(H, 1) = R_H + g$; conversely, decreasing the investment is a "wrong" decision at state H, which reduces the firm value (by g) to $v(H, -1) = R_H - g$. By contrast, at state L, decreasing the investment ($d = -1$) is correct and creates the additional value g, while increasing the investment ($d = 1$) is incorrect and reduces the firm value by g. The firm value $v(\theta, d)$ is then summarized in Table 1.1.

		Investment d		
		1	0	-1
State θ	H	$R_H + g$	R_H	$R_H - g$
	L	$R_L - g$	R_L	$R_L + g$

Table 1.1: Firm Value

We assume that state H dominates state L in terms of the fundamentals' effect on the

[7] To focus on the effect of managerial learning effects on the informed investor's venue choice, we follow the literature on informational feedback to abstract away any agency problem. Therefore, the firm manager aims to maximize the expected firm value. As a result, we will use the two terms, the firm and the firm manager, interchangeably.

firm value; that is, even the wrong investment decision at state H brings a higher firm value than the right investment decision at state L; formally, we assume that $v(H,-1) > v(L,-1)$, which is equivalent to $R_H - g > R_L + g$.[8] For simplicity, we further restrict the model parameters to $g = k\frac{(R_H - R_L)}{2}$, where k is strictly less than but arbitrarily close to 1.

1.3.2 Trading Venues

There are two parallel trading venues: an exchange market and a dark pool. We assume that there is no transaction cost in each trading venue. The informed investor perfectly observes the firm fundamentals and then chooses to trade in either the exchange market or the dark pool. Importantly, the trading venue choice of the informed investor is unobservable to other players.

We call the informed investor a "positively" or a "negatively" informed investor if the firm fundamentals are respectively high or low. To focus on her trading venue choice, we assume that after choosing a trading venue, the informed investor buys one share of the firm's stock ($X_I = 1$) if she is positively informed and shorts one share ($X_I = -1$) if she is negatively informed. We denote by β_H and β_L the probabilities that the informed investor chooses the exchange market when she is positively and negatively informed, respectively.

The noise investor chooses trading venues randomly. Specifically, the noise trader may have a positive demand ($\ell = 1$), a negative demand ($\ell = -1$), or no demand ($\ell = 0$) for the firm stock with equal probability. When having a demand, the noise investor chooses the exchange market with probability $\alpha \in [0,1]$; she chooses the dark pool with the complement probability $1-\alpha$. Importantly, α is exogenous. Like the informed investor, once the noise investor chooses a trading venue, she submits a market order $X_L = \ell$.

The informed investor and the noise investor trade simultaneously. In the exchange

[8] We analyze the equilibrium when $R_H - g < R_L + g$ in an Online Appendix.

market, the market maker observes the total trading volume $X = X_I + X_L$, but not their individual orders. Obviously, $X \in \{-2, -1, 0, 1, 2\}$. Since the market maker is competitive, she sets a price based on the total trading volume in the exchange market to keep herself break-even. That is, her pricing strategy is $P(X) = \mathbb{E}(v|X)$. She then clears any excess demand or supply using her own inventory. In the dark pool, there is no market maker, and thus orders are executed only when both the informed investor and the noise investor trade in the dark pool and demand opposite positions. Therefore, the execution price in the dark pool is the concurrent asset price in the exchange market.

1.3.3 Timing

Figure 1.1 presents the timeline of our model. At $t = 0$, nature chooses the firm fundamentals θ. The informed investor observes θ perfectly and chooses a trading venue. The noise investor with a demand $\ell \neq 0$ chooses a trading venue randomly. At $t = 1$, trading occurs in the exchange market or the dark pool. At $t = 2$, the manager observes the total trading volume in the exchange market and then decides to expand the investment, remain the current investment, or reduce the investment. At $t = 3$, all uncertainties are resolved, and all payoffs are realized.

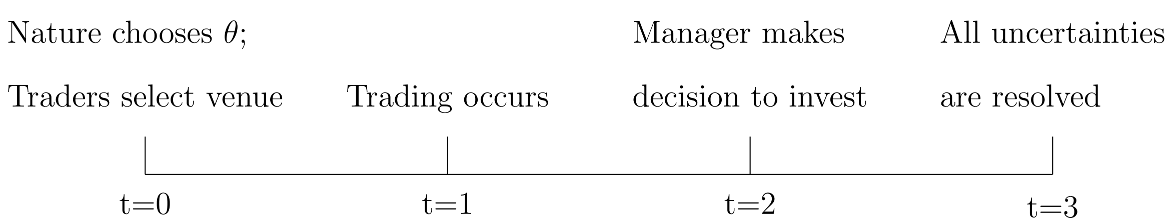

Figure 1.1: Timeline

1.3.4 A Perfect Bayesian Equilibrium

The informed investor's strategy is a mapping from the firm fundamentals θ to the probability of trading in the exchange market. The manager's strategy is a mapping from the total trading volume to her investment decision. Moreover, the market maker's pricing strategy is a mapping from the total trading volume to a price. We are interested in perfect Bayesian equilibrium.

Definition 1.1. The informed investor's strategy of venue choice $\beta^* : \{H, L\} \to [0, 1]$, the manager's investment strategy $d^* : X \to \{-1, 0, 1\}$, and the market maker's pricing strategy $P^*(X)$ constitute a perfect Bayesian equilibrium if:

1. For the informed investor, β^*_θ maximizes her expected final payoff for each $\theta \in \{H, L\}$, given the market maker's pricing strategy and the manager's investment strategy.

2. For the manager, $d^*(X)$ maximizes the expected firm value V given the information in the exchange market and other agents' strategies.

3. For the market maker, the price $P^*(X) = \mathbb{E}(v|X)$ allows her to beak even in expectation for each $X \in \{-2, -1, 0, 1, 2\}$, given all other agents' strategies.

4. The manager and the market maker update their beliefs by Bayes' rule after observing the total trading volume in the exchange market.

1.4 Benchmark Model

To analyze the dark pool effects on the informed investor's venue choice, exchange market efficiency, and real economic efficiency, we establish a benchmark model where a dark pool is not available. Hence, it is straightforward that in this benchmark, the informed investor will trade in the exchange market regardless of her private information about firm fundamentals. The noise investor will also trade in the exchange market if she has a non-zero demand ($\ell \neq 0$); that is, $\alpha = 1$ in this benchmark.

As a result, when the total trading volume in the exchange market is $X = 2$ or $X = 1$, both the market maker and the firm manager believe that the firm is surely at the state $\theta = H$, since if $\theta = L$, the informed investor sells at the exchange market, leading to a total trading volume at most 0. Similarly, when $X = -2$ or $X = -1$, they believe that the firm is at the state L. In a third case that $X = 0$, because of the symmetry of the informed investor's venue choice, the market maker and the firm manager do not update their beliefs about firm fundamentals, and so their posterior belief that $\theta = H$ remains 1/2. Proposition 1.1 then summarizes the agents' equilibrium behavior in the benchmark.

Proposition 1.1. In the benchmark model where a dark pool is not available to investors, the informed investor surely chooses the exchange market ($\beta^H = \beta^L = \beta^E = 1$) regardless of her private information about firm fundamentals. The firm's investment and stock price are

X	-2	-1	0	1	2
$d(X)$	-1	-1	0	1	1
P	$R_L + x$	$R_L + x$	$\frac{1}{2}(R_H + R_L)$	$R_H + x$	$R_H + x$

Table 1.2: Firm Investment and Stock Price in the Benchmark Model

This benchmark model is essentially the model studied by Edmans, Goldstein, and Jiang (2015) with zero transaction cost and the informed investor surely appearing. The informed investor surely trades in the exchange market to make positive profits. Importantly, because of the noise investor's random demand, the informed investor's private information about the firm's fundamentals is not perfectly revealed by her trading when the total trading volume is $X = 0$. Hence, the firm's equilibrium investment is also symmetric in the total trading volume: it reduces investment when $X < 0$, keeps the current investment when $X = 0$, and increases investment when $X > 0$.

1.5 Asymmetric Trading Venue Choices

We now analyze our core model where a dark pool is available to the investors. We start with the market maker and the firm manager's posterior beliefs about firm fundamentals, which will determine the firm's stock price and investment. We then characterize the informed investor's equilibrium venue choice, given the market maker's and the manager's best responses.

The main result in this section is that when the exchange market is lack of noise trading, that is, when $\alpha \in (0, 1/2)$, the informed investor is more likely to choose the exchange market at state H than at state L. Associated with the informed investor's asymmetric venue choice, the firm investment is also asymmetric: It reduces investment when the total trading volume is $X = 0$. These results are contrasting to those in the benchmark model.

1.5.1 Belief Updating, Asset Pricing, and Firm Investment

Since the manager and the market maker have the same prior information in our model, conditional on a total trading volume X, they have the same posterior beliefs in equilibrium. Figure 1.2 helps calculate the posterior belief of the manager and the market maker, given any possible total trading volume.

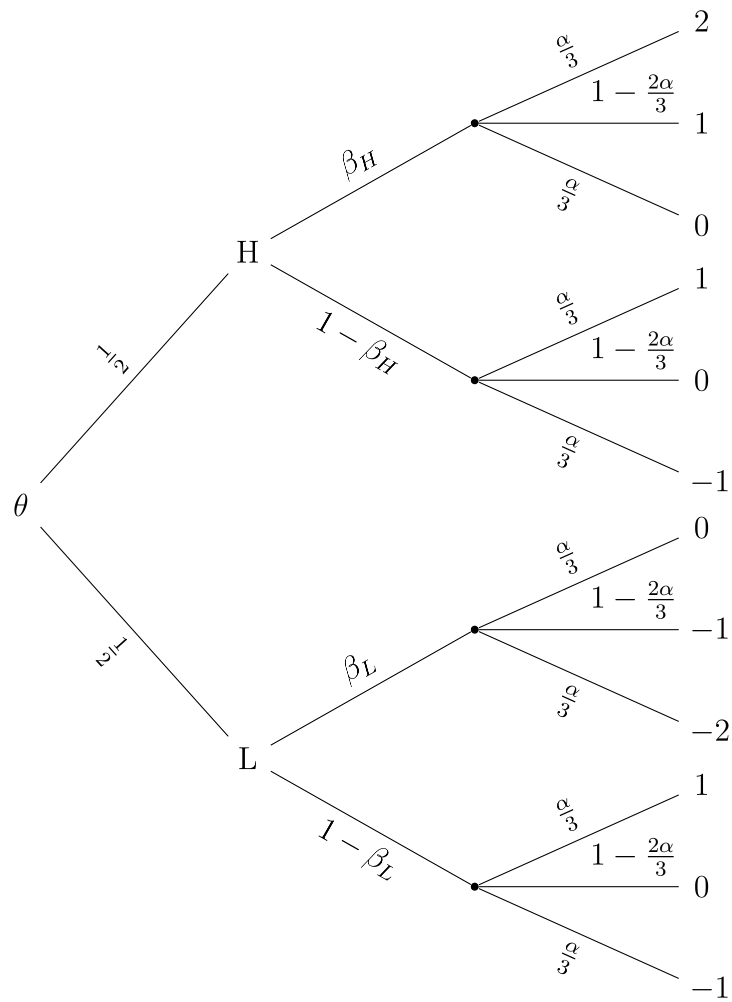

Figure 1.2: Manager and Market Maker's Belief Updating

For example, the total trading volume in the exchange market is zero when either both investors trade in the dark pool (with the probability $\frac{1}{2}[(1-\beta_H)+(1-\beta_L)](1-\frac{2}{3}\alpha)$), or both investors are trading in the exchange market but with opposite positions (with probability $\frac{1}{2}(\beta_H+\beta_L)\frac{\alpha}{3}$). Then, by Bayes' rule, when the total trading volume in the exchange market is zero, the firm manager and the market maker have a posterior belief about $\theta = H$:

$$
\begin{aligned}
\Pr(\theta = H|X = 0) \triangleq q(X = 0) =& \frac{\frac{1}{2}(1-\beta_H)(1-\frac{2}{3}\alpha)+\frac{1}{2}\beta_H\frac{\alpha}{3}}{\frac{1}{2}[(1-\beta_H)+(1-\beta_L)](1-\frac{2}{3}\alpha)+\frac{1}{2}(\beta_H+\beta_L)\frac{\alpha}{3}} \\
=& \frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L)+(3-2\alpha)(2-\beta_H-\beta_L)}.
\end{aligned}
$$

Similarly, we calculate the manager's and the market maker's posterior belief following each possible total trading volume $X \in \{-2, -1, 0, 1, 2\}$. Table 1.3 summarizes the equilibrium posterior belief $q(X)$.

X	-2	-1	0	1	2
q	0	$\frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L}$	$\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L)+(3-2\alpha)(2-\beta_H-\beta_L)}$	$\frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H}$	1

Table 1.3: Manger's and Market Maker's Posterior Beliefs

In equilibrium, the market maker sets prices that make her break-even in expectation. Thus, the pricing function in the exchange market is

$$P(X) = \mathbb{E}(v|X) = q(X)v\left(H, d(X)\right) + \left(1 - q(X)\right)v\left(L, d(X)\right). \tag{1.1}$$

Note that in equation (1.1), the market maker accounts for the manager's investment strategy when setting the price.

On the other hand, the execution price in the dark pool is assumed to be the concurrent stock price at the exchange market. While there are five possible prices in the exchange market (because there are five possible total trading volumes), only the one following a zero total trading volume matters for the execution price at the dark pool. Intuitively, when the informed investor chooses the dark pool, she can have her order executed only when the noise investor is demanding an opposite position in the dark pool. This occurs only when both the informed investor and the noise investor are trading in the dark pool, leading to a zero total trading volume in the exchange market. Lemma 1.1 formally derives the execution price in the dark pool in equilibrium.

Lemma 1.1. The execution price in the dark pool equals the stock price in the exchange market when the total trading volume is zero. Formally, given the informed investor's venue choice strategy (β^H, β^L), the execution price at the dark pool is

$$P^D = P(X = 0), \tag{1.2}$$

which is calculated by equation (1.1).

Lemma 1.1 shows that the informed investor's venue choice strategy affects the execution price at the dark pool, which in turn determines the informed investor's trading profits in the dark pool. Since the trading profits in the dark pool are essentially the opportunity costs of choosing the exchange market, Lemma 1.1 implies that the opportunity cost of trading in the exchange market is endogenous, and if $\beta^H \neq \beta^L$, such an opportunity cost is asymmetric. Hence, the execution price in the dark pool in our model differs from that in Zhu (2014) where the price in the dark pool is zero because it is assumed to be the midpoint of the bid and ask prices in the exchange market.

We now analyze the manager's investment decision. Denote by q_1 and q_{-1} two thresholds in the manager's posterior belief space such that

$$q_1 R_H + (1 - q_1) R_L = q_1(R_H + g) + (1 - q_1)(R_L - g) \tag{1.3}$$

$$q_{-1} R_H + (1 - q_{-1}) R_L = q_{-1}(R_H - g) + (1 - q_{-1})(R_L + g). \tag{1.4}$$

Equation (1.3) indicates that when the manager's posterior belief is exactly q_1, the expected firm value from keeping the current investment equals that from expanding the investment. Similarly, equation (1.4) implies that with a posterior belief q_{-1}, the manager is indifferent between keeping the investment level and reducing the investment. Simple algebra shows that

$$q_1 = q_{-1} = \frac{1}{2}. \tag{1.5}$$

Recall that the manager will keep the investment if changing the investment cannot lead to a strictly higher firm value. Then Lemma 1.2 formally characterizes the manager's investment decision based on her posterior beliefs.

Lemma 1.2. When the manager is learning from the exchange market, his equilibrium investment decision is determined by his posterior belief. In particular,

$$d(X) = \begin{cases} 1, & \text{if } \; q(X) \in \left(\frac{1}{2}, 1\right] \\ 0, & \text{if } \; q(X) = \frac{1}{2} \\ -1, & \text{if } \; q(X) \in \left[0, \frac{1}{2}\right). \end{cases} \tag{1.6}$$

It then follows from Lemma 1.2 and Table 1.3 that the manager's investment decision depends on the informed investor's strategy. Since the informed investor's trading profit at the dark pool is determined by the firm value and the stock price at a zero total trading volume in the exchange market, we specifically show how the informed investor's strategy determines the manager's investment when the total trading volume is $X = 0$ in Corollary 1.1.

Corollary 1.1. In equilibrium, the firm investment at a zero total trading volume in the exchange market, $d(X = 0)$, depends on the informed investor's strategy, (β_H, β_L). Specifically,

$$d(X = 0) = \begin{cases} -1, & \text{if } \; \beta_H > \beta_L \\ 0, & \text{if } \; \beta_H = \beta_L \\ 1, & \text{if } \; \beta_H < \beta_L. \end{cases} \tag{1.7}$$

Corollary 1.1 follows from the assumption that the noise investor's venue choice is independent of her demand; that is, when $\ell = 1$ or $\ell = -1$, the noise investor trades in the exchange market with probability α. Then, if the manager believes that the probability of the informed investor trading in the exchange market is independent of the firm fundamentals, that is, $\beta_H = \beta_L$, he will find that a zero total trading volume occurs equally likely at state

H and at state L. Hence, when the total trading volume is $X = 0$, the manager has a posterior belief about $\theta = H$ equaling to $1/2$ and thus keeps the current investment.

Conversely, Table 1.3 also shows that if $\beta_H \neq \beta_L$, $q(X = 0) \neq 1/2$. In particular, if $\beta_H > \beta_L$, $q(X = 0) < 1/2$, which leads the manager to choose $d(X = 0) = -1$. This is intuitive. With $\beta_H > \beta_L$, the positively informed investor is more likely to trade in the exchange market than the negatively informed investor. So, a zero total trading volume is more likely to occur at state L since the event that both investors trade in the dark pool is more likely to occur at state L. As a result, the manager reduces investment (i.e., $d(X = 0) = -1$). Similarly, if $\beta_H < \beta_L$, $q(X = 0) > 1/2$ and $d(X = 0) = 1$.

1.5.2 Equilibria with Strong Real Effects

We define that the exchange market has "real effects" if the information in the exchange market makes the firm manager expand or reduce investment in some cases. As shown in Table 1.3, when the total trading volume in the exchange market is $X = 2$, the manager's posterior belief about $\theta = H$ is $q(X = 2) = 1$, because the total trading volume $X = 2$ occurs if and only if the informed investor is buying in the exchange market. Therefore, observing the total trading volume $X = 2$, the manager will expand the investment. Similarly, the manager will reduce the investment when the total trading volume is $X = -2$.

The real effects when $X = 2$ and $X = -2$ arise purely from the manager's rationality and are not affected by the informed investor trading venue choice. Therefore, we further define an equilibrium with *strong real effects* and focus on it in the rest of the paper.[9]

Definition 1.2. An equilibrium with strong real effects is a perfect Bayesian equilibrium in which $d(X) \neq 0$ for some $X \in \{-1, 0, 1\}$.

[9] We find that there is an equilibrium in which the exchange market does not affect the manager's investment decision when $X \in \{-1, 0, 1\}$. However, there is no strong real effect in such an equilibrium.

We now characterize equilibria with strong real effects. Consider the positively informed investor. If she decides to trade in the exchange market, her order will lead to a total trading volume $X \in \{0, 1, 2\}$ with probability $(\alpha/3, 1-(2/3)\alpha, \alpha/3)$. The pricing equation (1.1) then implies that the informed investor's expected profit by buying in the exchange market is

$$\left(1 - \frac{2}{3}\alpha\right)[v(H, d(1)) - P(1)] + \frac{\alpha}{3}[v(H, d(0)) - P(0)]. \tag{1.8}$$

If the informed investor chooses the dark pool, on the other hand, she can buy one share only when the noise investor sells in the dark pool, which occurs with probability $(1-\alpha)/3$. Conditional on such a match in the dark pool, the asset price is $P^D = P(0)$, as shown in Lemma 1.1. Therefore, by choosing the dark pool, the informed investor's expected trading profit is

$$\frac{1-\alpha}{3}[v(H, d(0)) - P(0)]. \tag{1.9}$$

As we argue, the informed investor's expected trading profit in the dark pool is essentially her opportunity cost of trading in the exchange market. In addition, it follows from Lemma 1.1 that the conditional trading profit in the exchange market when the total trading volume is $X = 0$ will be the same as the conditional trading profit in the dark pool. (They are both $v(H, d(0)) - P(0)$ as derived in equations (1.9) and (1.8).) Therefore, to simplify the discussion, we define

$$\frac{1-2\alpha}{3}[v(H, d(0)) - P(0)], \tag{1.10}$$

the difference between equation (1.9) and the second term in equation (1.8), as the positively informed investor's *net opportunity cost*. Similarly, we can calculate the negatively informed investor's net opportunity cost of choosing the exchange market as

$$\frac{1-2\alpha}{3}[P(0) - v(L, d(0))]. \tag{1.11}$$

Intuitively, the informed investor's net opportunity cost is decreasing in α, the probability of the noise investor with $\ell \neq 0$ trading in the exchange market. This is so both because the informed investor is easier to hide her private information in the exchange market when α is larger and because it is harder for her to find a counterparty in the dark pool. In particular, when $\alpha \geq \frac{1}{2}$, the net opportunity cost is non-positive, implying that the informed investor should choose the exchange market for sure, independent of her private information about the firm fundamentals.

Proposition 1.2. When the dark pool is available to the investors, if the noise investor with non-zero demand is more likely to trade in the exchange market, the model has a unique equilibrium in which the informed investor surely chooses the exchange market. Formally, when $\alpha \geq 1/2$, $\beta_H = \beta_L = 1$. In such a unique equilibrium, the manager's investment decision is symmetric:

$$d(X) = \begin{cases} -1, & \text{if } \; X = -2 \text{ or } \; X = -1; \\ 0, & \text{if } \; X = 0; \\ 1, & \text{if } \; X = 1 \text{ or } \; X = 2. \end{cases} \tag{1.12}$$

The informed investor's symmetric venue choice in the unique equilibrium when $\alpha \geq 1/2$ arises from the non-positive net opportunity cost of choosing the exchange market. However, when $\alpha < 1/2$, the net opportunity cost is positive. This will dramatically change the informed investor's equilibrium venue choice, as presented in Lemma 1.3.

Lemma 1.3. When the noise investor is less likely to trade in the exchange market, the informed investor's equilibrium venue choice must be asymmetric. Formally, when $\alpha \in (0, 1/2)$, in any equilibrium with strong real effects, $\beta_H \neq \beta_L$.

Lemma 1.3 and Corollary 1.1 then imply that in an equilibrium with strong real effects, the manager will either reduce or expand investment when the total trading volume in the exchange market is zero. Furthermore, Lemma 1.4 argues that the manager's investment decision must be increasing in the total trading volume. This is intuitive. Once the informed investor chooses the exchange market, she buys at state H and sells at state L. Hence, as the total trading volume increases, it is more likely that the firm fundamentals are high.

Lemma 1.4. Both the manager's posterior belief $q(X)$ about $\theta = H$ and the manager's investment decision $d(X)$ are increasing in the total trading volume X.

Lemma 1.3 and Lemma 1.4 largely simplify the equilibrium characterization. In an equilibrium with strong real effects, the firm investment at the total trading volume $X \in (-1, 0, 1)$ can only be $(-1, -1, 0)$, $(-1, -1, 1)$, $(-1, 1, 1)$, and $(0, 1, 1)$. A simple calculation of the posterior beliefs in Table 1.3 shows that it is impossible for the manager's posterior belief to be exactly $1/2$ when the total trading volume is either $X = -1$ or $X = 1$. So, the equilibrium firm investment at the total trading volume $(-1, 0, 1)$ can only be $(-1, -1, 1)$ or $(-1, 1, 1)$.

Proposition 1.3 characterizes equilibria when the noise investor is unlikely to trade in the exchange market ($\alpha < 1/2$).

Proposition 1.3. When $\alpha < 1/2$, there may be multiple equilibria with strong real effects.

1. For any $\alpha \in (0, 1/2)$, there is a unique equilibrium with strong real effects $d(X) = (-1, -1, 1)$ for $X = (-1, 0, 1)$. In such an equilibrium, the informed investor's strategy is

$$\beta_H = \begin{cases} \frac{4\alpha(R_H - R_L) + 16g\alpha(1-\alpha)}{3(1-\alpha)[(1-\alpha)(R_H - R_L) + 2g(3\alpha - 1)]}, & \forall \alpha \in \left(0, \frac{2g - (R_H - R_L)}{2g - 3(R_H - R_L)}\right) \\ 1, & \forall \alpha \in \left[\frac{2g - (R_H - R_L)}{2g - 3(R_H - R_L)}, \frac{1}{2}\right) \end{cases} \tag{1.13}$$

$$\beta_L = 0. \tag{1.14}$$

2. There is an $\hat{\alpha} \in (0, 1/2)$, such that for any $\alpha \in [\hat{\alpha}, 1/2)$, there is an equilibrium with $\beta_H < \beta_L$ and $d(X) = (-1, 1, 1)$ for $X = (-1, 0, 1)$. However, for any $\alpha \in (0, \hat{\alpha})$, there is no equilibrium with the strong real effects $d(X) = (-1, 1, 1)$ for $X = (-1, 0, 1)$.

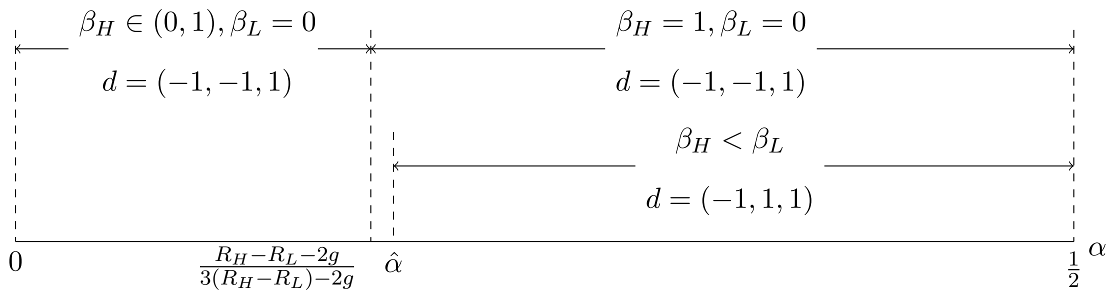

Figure 1.3: Equilibrium Characterization

Figure 1.3 illustrates the equilibria characterized in Proposition 1.3. In particular, for each $\alpha < 1/2$, there is an equilibrium with strong real effects $(-1, -1, 1)$ at the total trading volume $(-1, 0, 1)$. It also follows from equations (1.13) and (1.14) that in such an equilibrium, the informed investor's venue choice is continuous in $\alpha \in (0, 1/2)$. On the other hand, there is a nontrivial set of α's ($\alpha \in (0, \hat{\alpha})$) for which an equilibrium with the real effects $(-1, 1, 1)$ at the total trading volume $(-1, 0, 1)$ does not exist. Therefore, we shall focus on the equilibrium with the real effects $(-1, -1, 1)$ in the rest of the paper.

The most striking property of this equilibrium is the informed investor's asymmetric venue choice and the manager's asymmetric investment. The asymmetry of the informed investor's trading venue choice first arises from the informational feedback effect of the exchange market on firm investment. It directly follows from Table 1.3 that when $\beta_H = \beta_L$, $q(-1) < 1/2 < q(1)$. Therefore, the manager will reduce and increase investment when the total trading volume in the exchange market is $X = -1$ and $X = 1$, respectively. On the other hand, the net opportunity cost is independent of the value of β_H or β_L, since $\beta_H = \beta_L$ leads to the posterior belief $q(0) = 1/2$. Therefore, if trading in the exchange market, the informed investor will guide the manager to make the right investment decision, which will lead to high firm values. This will increase the positively informed investor's expected profit from buying in the exchange market but decrease the negatively informed investor's expected profit from selling in the exchange market.

Therefore, if the manager and the market maker believe that the informed investor employs a symmetric strategy, the negatively informed investor will have weaker incentives to choose the exchange market than the positively informed investor. Hence, $\beta_H = \beta_L$ cannot be part of an equilibrium, unless $\beta_H = \beta_L = 0$ or $\beta_H = \beta_L = 1$. Furthermore, $\beta_H = \beta_L = 0$ implies no strong real effects, and $\beta_H = \beta_L = 1$ makes $X = -1$ and $X = 1$ perfectly revealing and thus leads the informed investor to choose the dark pool (since the opportunity cost of choosing the exchange market is positive). As a result, when $\alpha < 1/2$, the informed investor's equilibrium venue choice must be asymmetric.

While such an informational feedback effect is similar to that in Edmans, Goldstein, and Jiang (2015), the asymmetry of the informed investor's venue choice is also associated with the asymmetry of the net opportunity cost and the asymmetry of the firm investment. Specifically, since the negatively informed investor has strictly weaker incentives to trade in the exchange market than the positively informed investor does, both the market maker and the firm manager believe that the firm fundamentals are more likely to be low at the zero total trading volume. In addition, the firm manager will reduce the investment after observing the zero total trading volume in the exchange market, following from Corollary 1.1. As a result, the negatively informed investor's net opportunity cost increases, further reducing the negatively informed investor's incentives to choose the exchange market. Ultimately, when $\alpha < 1/2$, the negatively informed investor will surely choose the exchange market. Therefore, the mechanism of the asymmetry of the informed investor's equilibrium trading venue choice differs from that in Edmans et al. (2015).

We also notice an interesting discontinuity of the firm investment at $\alpha = 0$. Intuitively, when $\alpha = 0$, the noise investor will surely trade in the dark pool. This maximizes the informed investor's chance to find a counterparty in the dark pool. More importantly, since the market maker knows that there is no noise trading in the exchange market, any non-zero total trading volume is perfectly revealing. Therefore, when $\alpha = 0$, both the positively and negatively informed investor will choose the dark pool, leading the manager to keep the current investment when the total trading volume is zero. However, in the equilibrium characterized in Proposition 1.3, as α converges to zero, the firm investment when the total trading volume is $X = 0$ is always $d(X = 0) = -1$, demonstrating a discontinuity at $\alpha = 0$.

1.5.3 Limits to Arbitrage in Exchange Market

The informed investor's asymmetric venue choice in the equilibrium with strong real effects $d(X) = (-1, -1, 1)$ for $X = (-1, 0, 1)$ causes asymmetric limits to arbitrage in the exchange market.[10] This is formally shown in Corollary 1.2.

Corollary 1.2. The dark pool causes the limits to arbitrage in the exchange market when $\alpha \in (0, 1/2)$. In particular,

1. When $\alpha \in \left(0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}\right)$, the dark pool causes the limits to arbitrage at both state H and state L, and the limits to arbitrage at state L is more severe;
2. When $\alpha \in \left[\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}, 1/2\right)$, the dark pool causes the limits to arbitrage at state L only.

Interestingly, Corollary 1.2 implies that the limits to arbitrage at state H is alleviated as the noise investor is more likely to trade in the exchange market (that is, as α increases from 0 to 1/2). Intuitively, when the noise investor is more likely to trade in the exchange market, it is easier for the informed investor to hide her private information when trading

[10]We emphasize that in our model, the informed investor always trades, either in the exchange market or in the dark pool. However, if we focus only on the trading in the exchange market, as empirical studies usually do, when the informed investor trades in the dark pool, there are limits to arbitrage in the exchange market.

in the exchange market, and it is harder for the informed investor to find a counterparty in the dark pool. Therefore, the positively informed investor has a lower net opportunity cost, increasing her incentives to buy in the exchange market.

On the other hand, the probability of the informed investor trading in the exchange market does not affect the limits to arbitrage at state L. This is due to the fact that the firm investment $d(0) = -1$ and the market maker's posterior belief $q(0) < 1/2$ joint imply that the negatively informed investor has a large net opportunity cost. Since such a net opportunity cost dominates, the negatively informed investor will surely choose the dark pool, independent of the probability that the noise investor trades in the exchange market.

Corollary 1.3. When $\alpha \in (0, 1/2)$, β_H is strictly increasing in α when $\alpha < \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$ and $\beta_H = 1$ for all $\alpha \in \left[\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}, 1/2\right)$. By contrast, at state L, $\beta_L = 0$ for all $\alpha \in (0, 1/2)$.

1.6 Dark Pool Effects on Exchange Market Efficiency

We are now in a position to analyze the dark pool effects on exchange market efficiency and real economic efficiency. These analyses are not only theoretically interesting but also have important policy implications. We shall focus on the effects of the dark pool on exchange market efficiency in this section and leave the analysis of the dark pool effects on real economic efficiency to Section 1.7.

In this section, we also make a technical contribution to the studies of market efficiency. The exchange market trading in our model is essentially a simplified static version of (Kyle, 1985). So, the *variance ratio* developed in Kyle (1985) seems a natural measure of market efficiency. However, we find that the variance ratio is implausible in our model because the firm value is endogenous. We, therefore, develop a firm-value-free measure of the exchange market efficiency, and then apply it to analyze the dark pool effects on exchange market efficiency.

1.6.1 Variance Ratio and Mutual Information

Exchange market efficiency is generally defined as how much information is incorporated into the asset price. Kyle (1985) develops a measure for exchange market efficiency, the variance ratio.

$$\mathbb{E}[\Lambda|\tilde{p}] = \mathbb{E}\left[\frac{\mathbb{V}ar(\widetilde{v}|\widetilde{p})}{\mathbb{V}ar(\widetilde{v})}\right]. \tag{1.15}$$

It measures the variance of the firm value conditional upon the price, scaled by the unconditional variance of the firm value. Therefore, the smaller the variance ratio, the more efficient the exchange market.

When the firm value is exogenous, the variance ratio functions very well in measuring exchange market efficiency. However, our model features the informational feedback effects of the exchange market, so the asset price or the total trading volume in the exchange market will not only reflect but also affect the firm value. Hence, in our model, the variance ratio may be implausible in measuring exchange market efficiency.

We take as an example the exchange market efficiency conditional on the total trading volume $X = 0$ in the exchange market. In the equilibrium characterized in Part 1 of Proposition 1.3, the informed investor's venue choice is asymmetric when $\alpha < 1/2$ and is symmetric when $\alpha \geq 1/2$. As we argued, if the informed investor's venue choice is symmetric, $X = 0$ does not provide any new information. On the other hand, when α is approaching 0, $\beta_H > \beta_L = 0$. Then, the total trading volume $X = 0$ provides the market maker and the firm manager with new information. Hence, conditional on $X = 0$, the exchange market is more efficient when α is close to zero than when $\alpha \geq 1/2$. However, we show that the variance ratio $\mathbb{E}[\Lambda|X = 0]$ is greater in the former case, suggesting that the variance ratio does not measure exchange market efficiency well. We discuss more details about the variance ratio in measuring exchange market efficiency in the Online Appendix.

Therefore, we propose a new measure for exchange market efficiency: We apply the entropy-based *mutual information* from information theory (Shannon, 1948) to measure exchange market efficiency. Mutual information of two random variables quantifies the amount of information obtained by observing one random variable about the other. The amount of uncertainty about one single random variable (Y) is measured by *entropy*. The rest of the uncertainty of the random variable (Y) after observing another random variable (Z) is measured by *conditional entropy*.

Specifically, consider a pair of random variables Y and Z with a joint discrete outcome space $\mathcal{Y} \times \mathcal{Z}$. The entropy $H(Y)$ and the *conditional entropy* $H(Y|Z)$ of the random variable Y conditional on Z are defined as

$$H(Y) \equiv -\sum_{y \in \mathcal{Y}} p(y) log_2 p(y) \tag{1.16}$$

$$H(Y|Z) \equiv -\sum_{z \in \mathcal{Z}} \sum_{y \in \mathcal{Y}} p(y, z) log_2 p(y|z) \tag{1.17}$$

The mutual information is then the amount of information about Y obtained by observing Z and is defined as

$$I(Y|Z) \equiv \mathbb{E}_z I(Z = z) = H(Y) - H(Y|Z). \tag{1.18}$$

Intuitively, mutual information measures the difference between the amount of uncertainty based on the unconditional distribution of Y and that based on the conditional (on Z) distribution of Y. Therefore, if and only if we can obtain more information about Y by observing Z, mutual information becomes larger.

In our model, the mutual information of firm fundamentals (θ) through observing the total trading volume X can well measure how the informed investor's information is incorporated into the exchange market. Importantly, it does not depend on the equilibrium firm value and thus can resolve the issue caused by the exchange market's information feedback effects, which affect the performance of variance ratio. Specifically, the reduction in the uncertainty about firm fundamental θ by observing total trading volume X is given by

$$I(\Theta|X) = \mathbb{E}_x I(X = x) = H(\Theta) - H(\Theta|X), \tag{1.19}$$

in which the expected value is taken over the set of all possible trading volume x.

Since the prior belief about $\theta = H$ is 1/2, the entropy is

$$H(\Theta) = -\left[\frac{1}{2}log_2\frac{1}{2} + \frac{1}{2}log_2\frac{1}{2}\right] = 1.$$

This is consistent with the general result in information theory that entropy reaches its maximum value if the posterior belief is uniform. Then, the mutual information defined in equation (1.19) then depends on the conditional entropy $H(\Theta|X)$ only. The distribution of firm fundamentals conditional on the total trading volume, which is calculated in Table 1.3, is determined in equilibrium and is affected by the economic environments, such as the likelihood of the noise investor trading in the exchange market.

We now compare mutual information with variance ratio in measuring exchange market efficiency by revisiting the example of exchange market efficiency at the total trading volume $X = 0$. A simple calculation shows that when $\alpha \geq 1/2$, $H(\Theta|X = 0) = H(\Theta) = 1$, which is the maximum value of the conditional entropy, implying that the exchange market is least efficient. On the other hand, when $\alpha \in (0, 1/2)$, the conditional entropy $H(\Theta|X) < 1$, showing that the market is much more efficient than in the case when $\alpha \geq 1/2$. This is consistent with the intuitive argument that when $X = 0$, the exchange market is more efficient when $\alpha < 1/2$, contrasting with the implausible conclusion when we measure the exchange market efficiency by the variance ratio.

1.6.2 Dark Pool Effects

In Section 1.4, we establish a benchmark-setting without the dark pool where the informed investor trades in the exchange market for sure regardless of her private information. That is, $\beta_H = \beta_L = \beta^E = 1$. Therefore, the informed investor's private information is fully incorporated into the asset price in the exchange market when the total trading volume is $X \in \{-2, -1, 1, 2\}$. A zero total trading volume, on the other hand, does not reveal any information to the manager: because the informed investor's venue choice is symmetric, the total trading volume $X = 0$ occurs equally likely at state H and state L. Therefore, given that $\alpha = 1$ without the dark pool, the conditional entropy is

$$H^E(\Theta|X) = \frac{1}{3}\left[-log_2\left(\frac{1}{2}\right)\right] = \frac{1}{3}, \tag{1.20}$$

implying that the mutual information is

$$I^E(\Theta|X) = H(\Theta) - H^E(\Theta|X) = \frac{2}{3}. \tag{1.21}$$

With the dark pool, the informed investor also trades in the exchange market for sure when $\alpha \in [1/2, 1)$. Therefore, for any $\alpha \in [1/2, 1)$, the mutual information (or the exchange market efficiency) is

$$I^D(\Theta|X) = H(\Theta) - H^D(\Theta|X) = 1 - \frac{\alpha}{3}, \forall \alpha \in \left[\frac{1}{2}, 1\right). \tag{1.22}$$

Hence, it is straightforward that as the noise investor trades in the exchange market more frequently (i.e., α increases from 1/2 to 1), it is harder to disentangle the orders submitted by the informed investor, implying that the exchange market efficiency decreases. The fact that I^D is strictly decreasing in α in $[1/2, 1)$ but $I^D > I^E$ implies that if the noise investor is likely to trade in the exchange market, the dark pool increases the exchange market efficiency by reducing the noise trading in the exchange market.

When $\alpha \in (0, 1/2)$, However, exchange market efficiency is non-monotonic in α. Proposition 1.3 shows that the negatively informed investor surely chooses the dark pool, while the probability of positively informed investor trading in the exchange market increases from 0 to 1 as α increases from 0 to $\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$ and stays at 1 as α varies between $\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$ and 1/2. By substituting β_H and β_L into Table 1.3, we get the equilibrium posterior beliefs when $\alpha \in (0, 1/2)$ in Table 1.4.

X	-2	-1	0	1	2
q	0	$\frac{1-\beta_H}{2-\beta_H}$	$\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha\beta_H+(3-2\alpha)(2-\beta_H)}$	$\frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H)+(3-2\alpha)\beta_H}$	1

Table 1.4: Equilibrium Posterior Beliefs When $\alpha \in (0, 1/2)$

It follows from Table 1.4 that the conditional entropy $H(\Theta|X = -1)$ is decreasing in α (strictly decreasing when $\alpha \in \left(0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}\right)$). That is, as the noise investor is more likely to trade in the exchange market, the total trading volume $X = -1$ becomes more informative. This is because in equilibrium, as α increases, the positively informed investor is more likely to buy in the exchange market, so $X = -1$ is less likely to occur at state H. Figure 1.4 depicts $H(\Theta|X = -1)$ as a function of α.

Figure 1.4 also depicts $H(\Theta|X = 0)$ and $H(\Theta|X = 1)$ as functions of α. From the figure, we find that for $\alpha \in (0, 1/2)$, while the informativeness of the total trading volume $X = 0$ is non-monotonic in α, the informativeness of the total trading volume $X = 1$ is decreasing in α. The non-monotonicity of the zero trading volume's informativeness is due to the interaction between a higher probability of the positively informed investor trading in the exchange market (which increases exchange market efficiency) and the more noise trading in the exchange market (which reduces exchange market efficiency). In particular, the former dominates when α is very small, but the latter dominates when α is large (when α is greater than a threshold, $\beta_H = 1$, so the former effect disappears). When we consider the total trading volume $X = 1$, the noise trading effect always dominates, so $H(\Theta|X = 1)$ is strictly increasing in α, implying that the informativeness of the total trading volume $X = 1$ decreases in α.

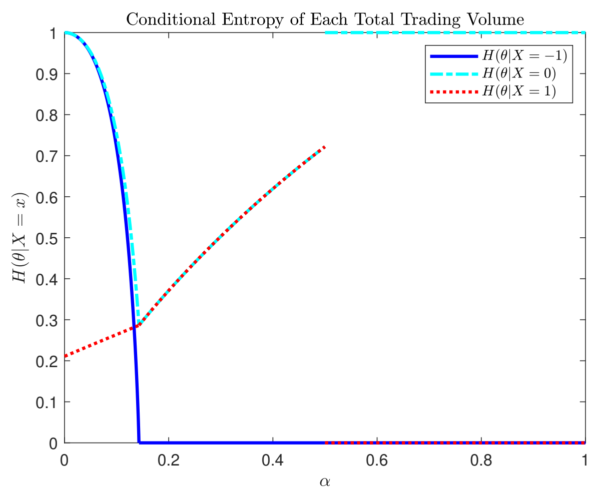

Figure 1.4: Conditional Entropy

We next calculate the expected conditional entropy by taking into account the ex-ante probability of each possible total trading volume X in equilibrium. Such an expected conditional entropy can then measure how efficient the exchange market is on average. Then, we obtain the mutual information when the dark pool is available to investors, which is illustrated by the blue solid curve in Figure 1.5.

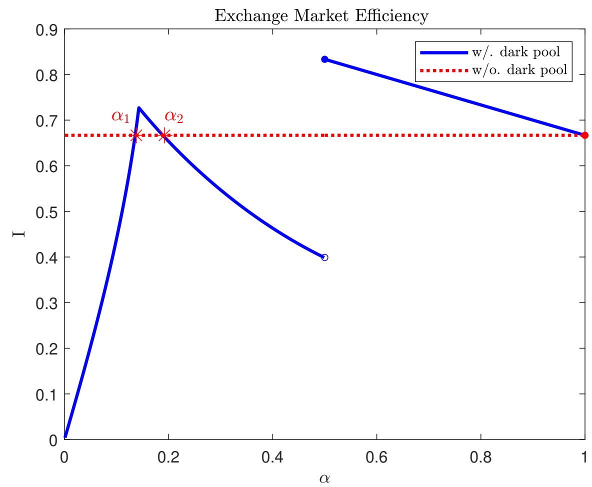

Figure 1.5: Dark Pool Effects on Exchange Market Efficiency.

By comparing the mutual information with and without the dark pool, we study the dark pool effects on exchange market efficiency. Recall that without the dark pool, both the informed investor and the noise investor trade in the exchange market only. So, it follows from equation (1.21) that the exchange market efficiency is quantified to be 2/3. Then, if the mutual information with the dark pool is higher than 2/3, we say that the dark pool promotes the exchange market efficiency. As illustrated in Figure 1.5, there are intervals of α's such that the dark pool does promote the exchange market efficiency.

Proposition 1.4. With the dark pool, there exist α_1 and α_2 with $0 < \alpha_1 < \alpha_2 < 1/2$, such that when the noise investor trades in the exchange market with a probability $\alpha \in (\alpha_1, \alpha_2) \cup [1/2, 1)$, the dark pool promotes the exchange market efficiency.

Proposition 1.4 shows that the dark pool hurts the exchange market efficiency when α is close to but strictly less than 1/2 or when it is close to 0. In the former case, much more noise is added into the exchange market while the probability of the informed investor trading in the exchange market is constant, making the total trading volumes $X = 0$ and $X = 1$ less informative. In the latter case, the probability of the positively informed investor trading in the exchange market is also close to zero. Then, the total trading volumes $X = 0$

and $X = -1$ are almost uninformative, leading to lower exchange market efficiency.

On the other hand, at $\alpha = \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$, β_H reaches one. The total trading volume $X = -1$ becomes perfectly revealing. In addition, at this point, the informativeness of the total trading volume $X = 0$ reaches the highest level. Therefore, the average exchange market efficiency with the dark pool is higher than that without the dark pool. As a result, there are $0 < \alpha_1 < \alpha_2 < 0$ such that the dark pool hurts the exchange market efficiency when $\alpha \in (0, \alpha_1)$ or $\alpha \in (\alpha_2, 1/2)$, but it promotes the exchange market efficiency when $\alpha \in (\alpha_1, \alpha_2)$.

1.7 Dark Pool Effects on Real Economic Efficiency

Proposition 1.4 provides answers to the policy debate about the dark pool effects on exchange market efficiency. As we discuss, however, it is more important to consider the dark pool effects on real economic efficiency, which is measured by the ex-ante expected firm value in our model. Since a more efficient exchange market will guide firm investment better, it is not too surprising that the dark pool promotes real economic efficiency when the dark pool improves the exchange market efficiency.

What is surprising in our model is that in some instances, even if the dark pool hurts exchange market efficiency, it still promotes real economic efficiency. This is illustrated in Figure 1.6 and shown in Proposition 1.5.

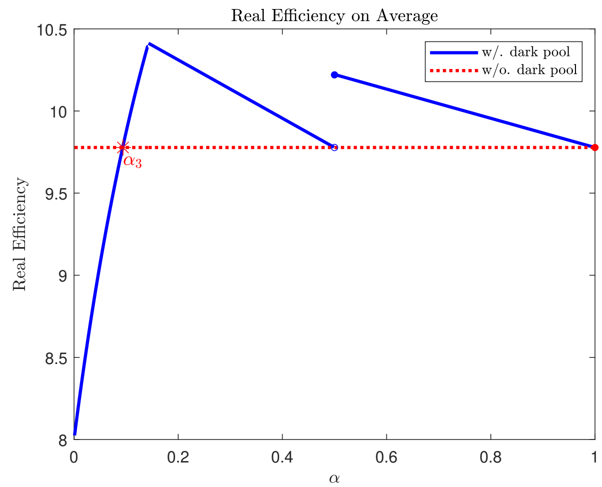

Figure 1.6: Dark Pool Effects on Real Economic Efficiency

Proposition 1.5. There is an $\alpha_3 \in (0, \alpha_1)$ such that the dark pool improves the real economic efficiency for any $\alpha \in (\alpha_3, 1)$.

Figure 1.5 and Figure 1.6 show that for any $\alpha \in (\alpha_3, \alpha_1) \cup (\alpha_2, 1/2)$, the dark pool hurts exchange market efficiency but promotes real economic efficiency. This arises from the firm's asymmetric investment. In particular, with $\alpha \in (0, 1/2)$, the manager reduces the investment when the total trading volume is $X = 0$. That is, the equilibrium firm investment for $X = (-2, -1, 0, 1, 2)$ is $d(X) = (-1, -1, -1, 1, 1)$. Given that $\beta_H \in (0, 1)$ and $\beta_L = 0$ in equilibrium, the probability of each total trading volume in the exchange market is calculated in Table 1.5.

X	-2	-1	0	1	2
d	-1	-1	-1	1	1
H	0	$(1-\beta_H)\frac{\alpha}{3}$	$(1-\beta_H)(1-\frac{2\alpha}{3}) + \beta_H\frac{\alpha}{3}$	$(1-\beta_H)\frac{\alpha}{3} + \beta_H(1-\frac{2\alpha}{3})$	$\beta_H\frac{\alpha}{3}$
L	0	$\frac{\alpha}{3}$	$1-\frac{2\alpha}{3}$	$\frac{\alpha}{3}$	0

Table 1.5: Probability of Total Trading Volume Conditional on Firm Fundamentals for $\alpha \in (0, 1/2)$.

At state H, if the informed investor buys in the dark pool (with a probability $1-\beta_H$), the expected firm value is the weighted average among those when the total trading volume is $X = -1$, $X = 0$, and $X = 1$. On the other hand, if the informed investor buys in the exchange market (with a probability β_H), the expected firm value is the weighted average among those when the total trading volume is $X = 0$, $X = 1$, and $X = 2$. Since reducing investment is the wrong action at state H, the firm value is higher when the informed investor buys in the exchange market. Then, it follows from Corollary 1.3 that as α increases from 0 to $\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$, the probability that the positively informed investor buys in the exchange market increases, leading to a higher expected firm value at state H.

When $\alpha \in \left[\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}, \frac{1}{2}\right)$, the positively informed investor always buys in the exchange market. Since the manager will make the wrong investment decision at state H when $X = 0$, an increase in α decreases the expected firm value because it will increase the probability of a zero total trading volume. (We note that this effect also exists when α increases from 0 to $\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$. It is, however, dominated by the effect of the correct investment when the informed investor buys in the exchange market and $X = 1$, due to the assumption that $R_H - g > R_L + g$.) Importantly, when α is arbitrarily close to $1/2$, compared with the case without the dark pool, the adverse effect of the wrong investment decision when $X = 0$ is just offset by the higher probability of the correct investment decision. Hence, as α converges to $1/2$ from the left, the real economic efficiency with the dark pool converges to that without the dark pool, implying that when $\alpha \in (\alpha_2, 1/2)$, the dark pool promotes the real economic efficiency at state H.

When $\alpha \geq 1/2$, since the informed investor surely trades in the exchange market, $d(X = 0) = 0$, as α increases in this region, the noise investor is more likely to sell in the exchange market, making $X = 0$ occur more frequently. Therefore, the expected firm value is strictly decreasing in α.

The expected firm value at state L is similar to that at state H when $\alpha \in [1/2, 1)$ but differs significantly when $\alpha \in (0, 1/2)$. In particular, as α increases from 0 to $1/2$, the negatively informed investor surely sells in the dark pool, and the total trading volume in the exchange market is governed by the noise investor only. Then, $X = 1$, the total trading volume that leads to the wrong investment decision at state L, occurs more frequently. Given the assumption that $R_H - g > R_L + g$, an increase in α decreases the expected firm value.

We summarize the expected firm value conditional on the firm fundamentals in Lemma 1.5 and depict them in Figures 1.7 and 1.8, respectively.

Lemma 1.5. Without the dark pool, the firm value is $R_H + \frac{2}{3}g$ and $R_L + \frac{2}{3}g$ at state H and state L, respectively; hence, the ex-ante firm value is $(R_H + R_L)/2 + \frac{2}{3}g$. By contrast, with the dark pool, the firm value at each state of the firm fundamentals depends on the probability of noise investor trading in the exchange market α. In particular,

1. The firm value at state H is

$$\begin{cases} R_H + (2\beta_H - \frac{4\alpha\beta_H}{3} + \frac{2\alpha}{3} - 1)g, & \forall\alpha \in \left(0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}\right) \\ R_H + (1 - \frac{2\alpha}{3})g, & \forall\alpha \in \left[\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}, \frac{1}{2}\right) \\ R_H + (1 - \frac{\alpha}{3})g, & \forall\alpha \in \left[\frac{1}{2}, 1\right) \end{cases} \tag{1.23}$$

2. The firm value at state L is

$$\begin{cases} R_L + (1 - \frac{2\alpha}{3})g, & \forall\alpha \in \left(0, \frac{1}{2}\right) \\ R_L + (1 - \frac{\alpha}{3})g, & \forall\alpha \in \left[\frac{1}{2}, 1\right) \end{cases} \tag{1.24}$$

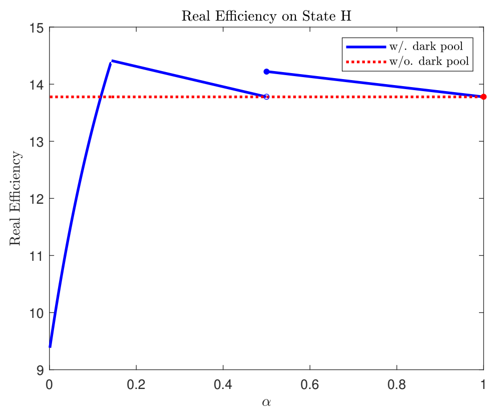

Figure 1.7: Real Economic Efficiency at State H

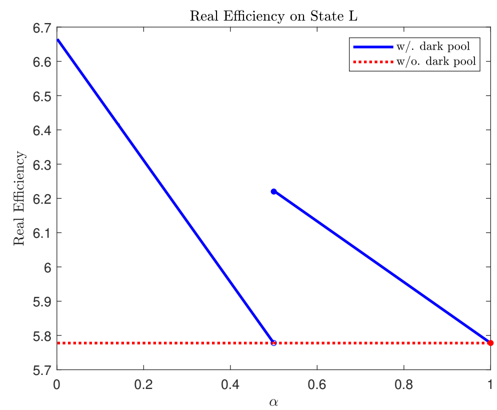

Figure 1.8: Real Economic Efficiency at State L

From Figure 1.7 and Figure 1.8, it is obvious that when α is very close to $1/2$, the dark pool promotes real economic efficiency, even if it hurts the exchange market efficiency by Proposition 1.4; this result is also true when $\alpha \in (\alpha_3, \alpha_1)$. Such a result implies that lower exchange market efficiency does not necessarily imply lower real economic efficiency. Therefore, when regulators design policies for the dark pool, using exchange market efficiency may overestimate the adverse effects of the dark pool on real economic efficiency.

1.8 Managerial Learning Effects

In addition to the dark pool effects on investor venue choice, exchange market efficiency, and real economic efficiency, our model also helps analyze managerial learning effects. Since the literature on managerial learning focuses solely on investors' trading behavior in the exchange market, allowing investors to make trading venue choices may provide new predictions and empirical implications. We, therefore, study managerial learning effects in this section, focusing on the equilibrium with real effects $d(X) = (-1, -1, -1, 1, 1)$ when the total trading volume in the exchange market is $X = (-2, -1, 0, 1, 2)$.

To study the managerial learning effects, we establish a benchmark model in which the manager does not learn from the exchange market for exogenous reasons, such as a high cost of extracting information from the exchange market or the manager's overconfidence. The main result in this section is that for any given probability of the noise investor trading in the exchange market, there is a unique symmetric equilibrium in which the informed investor chooses the exchange market with the same probability regardless of the firm fundamentals. Importantly, this symmetric equilibrium is the only equilibrium that exists at any probability of noise investor trading in the exchange market.

We first consider the manager's investment decision and the market maker's pricing decision. Since the manager does not learn from the exchange market, he will make the investment decision based on his prior belief only. Given that the prior belief about state H is $1/2$, the expected firm value from either expanding the investment or reducing the investment is $(R_H + R_L)/2$, which is the same as that from keeping the current investment. Hence, the manager keeps the current investment in equilibrium. Unlike the manager, the market maker learns from the total trading volume in the exchange market when setting prices and clearing the market. Given her posterior belief about state H for each possible total trading volume X in Table 1.3, her equilibrium pricing function is characterized in equation (1.1).

Then, we derive the informed investor's expected trading profits in the exchange market and in the dark pool. Because the asset price in the exchange market and thus the net opportunity cost of trading in the exchange market are both endogenously determined by the market maker's belief about the informed investor trading venue choice, this benchmark model also features the self-fulfilling prophecy. That is, the informed investor trading venue choice may be caused by the market maker's belief about her choice, which determines the asset price in the exchange market. Because of the self-fulfilling prophecy, multiple equilibria may emerge in our model, which is shown in Proposition 1.6 below.

Proposition 1.6. In the benchmark model without managerial learning, there may be multiple equilibria. In particular:

1. For any $\alpha \in (0,1)$, there is a unique symmetric equilibrium in which the informed investor trading venue choice is independent of the firm fundamentals. Specifically,

$$\beta_H^N = \beta_L^N = \beta^N = \begin{cases} \frac{4\alpha}{3-4\alpha+4\alpha^2}, & \forall \alpha \in \left(0, \frac{1}{2}\right) \\ 1, & \forall \alpha \in \left[\frac{1}{2}, 1\right). \end{cases} \tag{1.25}$$

2. For any $\alpha \in \left(0.3299, \frac{1}{2}\right)$, the model has multiple asymmetric equilibria.

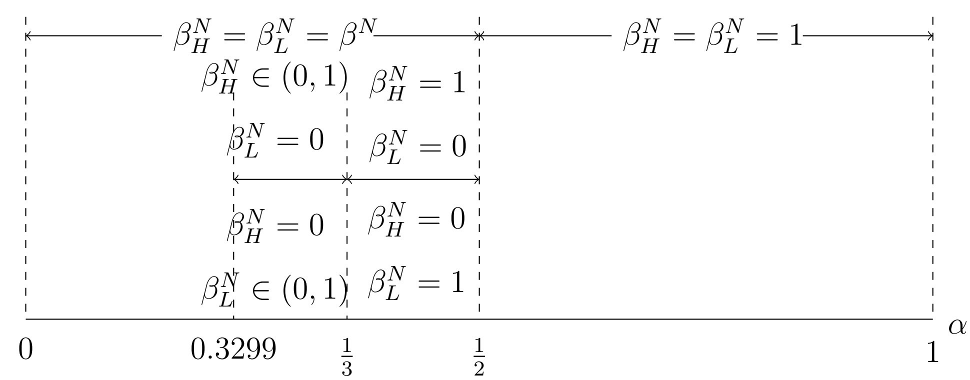

Figure 1.9: Equilibrium Venue Choice without Managerial Learning

Figure 1.9 illustrates the multiple equilibrium strategies of the informed investor in the benchmark model without managerial learning. We notice that except for a small set of parameters, the model has a unique equilibrium in which both the positively and the negatively informed investor choose the exchange market with the same probability β^N. This follows from the symmetric structure of the benchmark model: If the market maker believes that $\beta_H^N = \beta_L^N = \beta^N$, in either the exchange market or the dark pool, both the positively informed investor and the negatively informed investor have the same expected payoffs. Since the only equilibrium that survives for all $\alpha \in (0,1)$ is the one characterized in Part 1 of Proposition 1.6, we focus on it in the rest of this section.

Figure 1.10 depicts the informed investor's equilibrium venue choice strategy, β^N, as a function of the probability that the noise investor trades in the exchange market. For comparison, we also depict the informed investor's equilibrium venue choice strategy, (β_H, β_L), when the manager is learning.

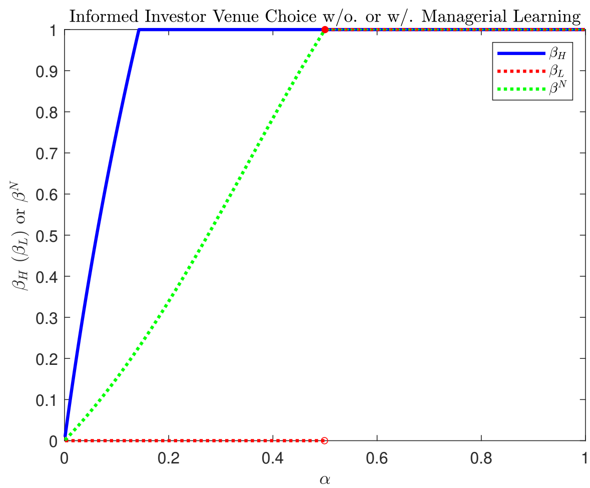

Figure 1.10: Symmetric Equilibrium in Benchmark Model

It is clear in Figure 1.10 that when $\alpha \geq 1/2$, whether the manager is learning from the exchange market or not, the informed investor will trade in the exchange market for sure. This is again due to the negative net opportunity cost. However, $\alpha < 1/2$ is a significantly different case: when the manager is learning, the positively informed investor is more likely to buy in the exchange market, while the negatively informed investor is less likely to sell in the exchange market. This demonstrates the effects of managerial learning on the informed investor's trading venue choice.

Comparing the equilibrium when the manager is learning with that when the manager does not learn, we show the managerial learning effect on exchange market efficiency. Denote by $I^N(\Theta|X)$ and $I(\Theta|X)$ the mutual information of the firm fundamentals conditional upon the total trading volume without and with managerial learning, respectively. Then, the managerial learning effects on exchange market efficiency is measured by $I(\Theta|X) - I^N(\Theta|X)$.

Proposition 1.7. There is an $\alpha_4 \in (0, 1/2)$, such that

$$I(\Theta|X) - I^N(\Theta|X) = \begin{cases} > 0, & \text{if } \alpha \in (0, \alpha_4) \\ = 0, & \text{if } \alpha = \alpha_4 \\ < 0, & \text{if } \alpha \in (\alpha_4, 1/2) \\ = 0, & \text{if } \alpha \in [1/2, 1)\,. \end{cases} \tag{1.26}$$

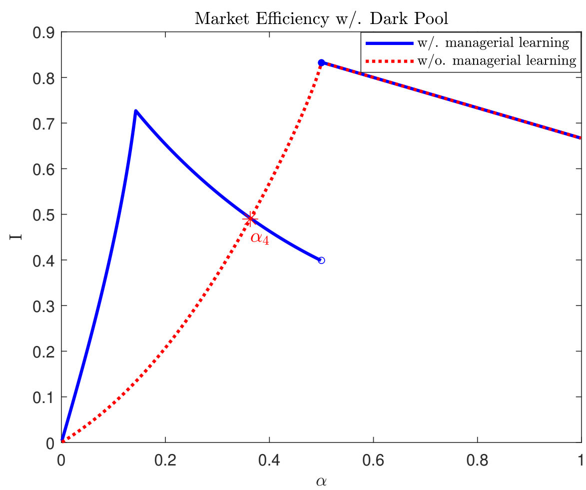

Figure 1.11: Managerial Learning Effects on Exchange Market Efficiency

Figure 1.11 depicts $I^N(\Theta|X)$ and $I(\Theta|X)$ by a red dotted curve and a blue solid curve, respectively. First of all, when the noise investor is likely to trade in the exchange market ($\alpha \geq 1/2$), managerial learning does not affect exchange market efficiency. This is intuitive. Since the managerial learning does not affect the informed investor trading venue choice in this case, it does not affect the exchange market efficiency either.

However, when the noise investor is less likely to trade in the exchange market, that is, $\alpha \in (0, 1/2)$, managerial learning affects exchange market efficiency dramatically. Specifically, when $\alpha \in (0, \alpha_4)$, $I(\Theta|X) > I^N(\Theta|X)$, implying that the managerial learning promotes the exchange market efficiency. This is because the informed trading is higher with managerial learning than that without managerial learning. When $\alpha \in (\alpha_4, 1/2)$, however, managerial learning reduces the exchange market efficiency because the informed trading is lower with managerial learning.

1.9 Conclusion

The fast expansion of alternative trading systems, especially the dark pools, has generated debates about their effects on price discovery in exchange markets in policy circles and among academic scholars. Such a concern is more worrisome when real decision makers are making decisions based on information from the exchange market. In this paper, we develop a model to analyze dark pool effects on investor trading behavior, exchange market efficiency, and real economic efficiency. We show that when the exchange market has low noise trading, the informed investor surely chooses the dark pool when the firm fundamentals are low and randomizes between the exchange market and the dark pool when the firm fundamentals are high. Such an asymmetric trading venue choice, in turn, leads to asymmetric firm investments. Specifically, when the total trading volume is zero, the manager reduces the investment, differing from the case without a dark pool.

The dark pool effects on the exchange market efficiency and real economic efficiency depend crucially on the noise trading in the exchange market. In particular, for a large set of probabilities of the noise investor trading in the exchange market, the dark pool increases both the exchange market efficiency and the real economic efficiency. Surprisingly, in some circumstances, the dark pool hurts the exchange market efficiency but promotes real economic efficiency. These results imply that when evaluating dark pools, policymakers need to calibrate the noise trading in the exchange market and use the correct evaluation criterion. In particular, if policymakers want to maximize the real economic efficiency, using the exchange market efficiency as a measure may overestimate the dark pool's adverse effects on real economic efficiency.

The contributions of this paper are three-fold. First, from an applied perspective, we develop a tractable model to analyze the interaction between managerial learning and investor trading venue choice. The theoretical analysis generates several new empirical and policy implications. In addition, our model captures the main features of dark pools, and thus can be used in future studies of alternative trading systems. Second, from a theoretical perspective, we show that some economic factors may promote real economic efficiency while hurt market efficiency. Therefore, it is not always correct to use market efficiency as a proxy of real economic efficiency. Third, from a conceptual perspective, we demonstrate that when firm cash flow is endogenous, the variance ratio developed by Kyle (1985) is no longer plausible to measure market efficiency (i.e., as more information is incorporated into the asset price, the variance ratio may be larger). We, therefore, propose to use mutual information to measure market efficiency, which not only works well in our model but also helps understand the informativeness of each possible asset price (or total trading volume).

Chapter 2

Dark Pool Effects on Investor Trading Venue Choice

Abstract This paper studies the dark pool effects on investor trading venue choice in a model featuring managerial learning from the exchange market. The dark pool does not divert investors away from the exchange market but initiates investors' coordination incentive when trading in the exchange market does not bring them positive profits. This is because the informed investor goes to the dark pool for making positive profits, and the liquidity investor goes to the dark pool for cost-saving. When the transaction cost in the exchange market is high enough, the informed investor does not trade in the exchange market, regardless of the liquidity investor's trading behavior. When the delay cost of the liquidity investor is higher than the transaction cost, the liquidity investor may trade in the exchange market, regardless of the informed investor's trading behavior. In addition, managerial learning may encourage more informed trading in the exchange market when the transaction cost in the exchange market is low enough.

JEL Classification: D83, G11, G14

Key words: Dark pool, managerial learning, trading venue choice, coordination

2.1 Introduction

Dark pools are private trading systems that provide a platform for anonymous trading of securities. Contrary to the exchange markets, dark pools offer execution prices no worse than the National Best Bid Offer (NBBO). More importantly, they do not display bid and ask quotes, and they delay displaying trading information. Such low transparency facilitates institutional investors' block trading with minimal adverse price impact. Recently, the expansion of electronic trading and the almost zero transaction fee also make dark pools attractive to retail investors. These inherent advantages of dark pools, together with the regulatory reforms, make dark pools occupy a considerable fraction of trading volume in recent years (Figure 2.1). In mid-2011, dark pools accounted for around 30% of average daily trading volume, but this number increased by more than 5% three years later.

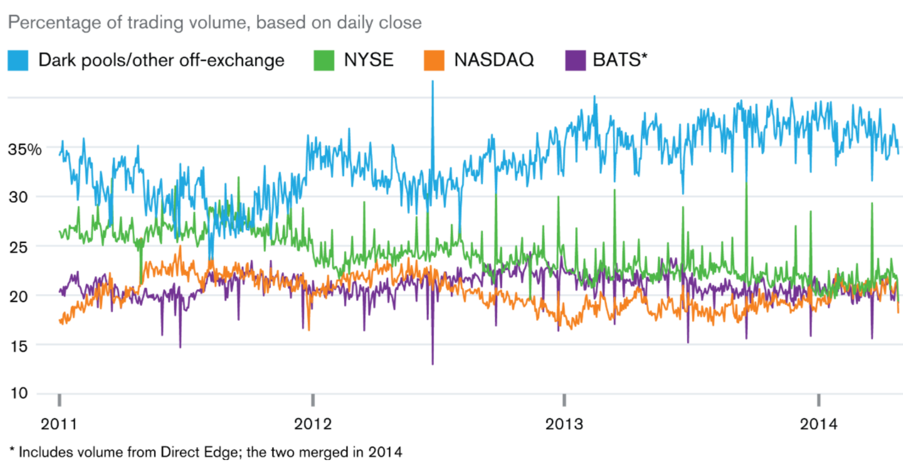

Figure 2.1: Fast Growth of Dark Pools.

The rising market share of dark pool trading catches the eyes of regulators, scholars, and industry professionals and results in serious concerns in policy circles such as loss of price discovery, fragmentation of liquidity, and distribution of welfare between institutional and retail investors, etc. Exchange markets officials are concerned that dark pool trading divert huge volume away from exchanges, and thus they urge the Securities and Exchange

Commission SEC to set rules or amend regulations to hinder dark pool trading.

These concerns, however, are even more worrisome when real decisions are made based on exchange market prices. Many empirical studies, such as Luo (2005), Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), Foucault and Frésard (2012), and Jayaraman and Wu (2019) have provided evidence that firm managers are gleaning information from exchange market prices and making real decisions based on such information.[1] Then, a naïve implication will be that the informed investors will choose dark pools to hide their information and reduce the trading volume in the exchange market. In this paper, we investigate the dark pool effects on investor trading venue choice. Several interesting questions arise. What is the impact of dark pools on investor trading venue choice? Do dark pools divert investors away from exchange markets? If not, which factors cause exchange markets to lose their order flows?

We answer these questions in a model in which a firm manager decides to expand, remain, or reduce the firm investment and investors who select a trading venue between an exchange market and a dark pool, or choose not to trade. The firm value is determined by its investment and firm fundamentals. The firm fundamentals are either high or low and are privately known by an informed investor. There is a liquidity investor who strategically makes trading venue choices or chooses not to trade after receiving a liquidity shock. Both the informed investor and the liquidity investor trade simultaneously. If they submit orders to the exchange market, their orders will be surely executed, because the exchange market has a competitive market maker who provides liquidity. Alternatively, investors may submit their orders to the dark pool where order execution is not guaranteed and potential execution price is the concurrent asset price in the exchange market. The manager and the market marker then observe the total trading volume in the exchange market; however, neither of them observes trading in the dark pool. Based on their information, the market maker sets a

[1]See Bond, Edmans, and Goldstein (2012) for an excellent survey of this literature.

price to make herself break-even, and the manager makes investment decisions to maximize the firm value.

We find that when the dark pool is not available to investors, the investor trading behavior is determined by the transaction cost in the exchange market and the delay cost of the liquidity investor. When the transaction cost in the exchange market is very high, the informed investor chooses not to trade, regardless of the liquidity investor's trading strategy. When the liquidity investor's delay cost is lower than the transaction cost, the liquidity investor does not trade; otherwise, she may choose to trade in the exchange market. Given that the liquidity investor always trades in the exchange market, as the transaction cost reduces, the informed investor's strategy changes from not to trade to trade on one side of information and finally changes to trades on both sides of information when the transaction cost is very low. In addition, managerial learning encourages more informed trading in the exchange market when the transaction cost is low enough.

When the dark pool is available to the investors, all pure strategy equilibria when the dark pool is shut down still hold. However, for those equilibria in which the liquidity investor does not necessarily trade in the exchange market when the dark pool is shut down, the dark pool leads to multiple equilibria in which investors may coordinate to trade in the dark pool. Intuitively, the informed investor goes to the dark pool to make more positive profits, and the liquidity investor goes to the dark pool to save cost.

Therefore, the dark pool does not divert investors from the exchange market. Instead, transaction cost in the exchange market and the delay cost of the liquidity investor significantly affect the trading volume in the exchange market. Given the delay cost of the liquidity investor, exchange markets may increase the trading volume by setting lower transaction fees.

Our paper contributes to the fast-growing literature on dark pools. Several papers show that different trading venues cater to different categories of investors. Hendershott and

Mendelson (2000) find that exchange markets attract more traders who put a high value on the assurance of immediate execution, while crossing networks attract more traders who pursue certainty of execution in return for lower costs. Degryse, Achter, and Wuyts (2009) find that investors with a high willingness to trade are more likely to trade at a dealer market. Buti, Rindi, and Werner (2017) extend Degryse, Achter, and Wuyts (2009) by investigating the competition between a dark pool and a limit order book. Jiang, McInish, and Upson (2014) find that as uninformed investors can segment their order flow to off-exchange venues, a larger proportion of trades on the exchanges are informed. Nimalendran and Ray (2011) reveal that crossing-network trades are informed. Zhu (2014) suggests that dark pools divert noise trading from the exchange market. Our paper contributes to this literature by developing a tractable model featuring managerial learning to analyze the dark pool effects on investor trading venue choice. We show that dark pools do not significantly divert investors from the exchange markets.

Second, our paper belongs to the literature on interactions between financial market and corporate decisions. Some studies find that a firm's investment is sensitive to its own stock price (Baker, Stein, and Wurgler, 2003, Goldstein and Guembel, 2008, Hirshleifer, Subrahmanyam, and Titman, 2006, Khanna and Mathews, 2012), while others document that a firm's investment may be even sensitive to its peers' stock prices (Foucault and Frésard, 2012, 2014, Ozoguz, Rebello, and Wardlaw, 2018). The closest paper to ours in this literature is Edmans, Goldstein, and Jiang (2015) who find that managerial learning causes an informed investor's asymmetric trading and the "sell-side" limits to arbitrage. However, in their model, there is no dark pool, and thus the investors do not have trading venue choices. By contrast, we focus on investors' trading venue choice between the exchange market and the dark pool. We show that managerial learning may encourage more informed trading in the exchange market when the transaction cost is low enough.

Last but not least, the paper complements the large literature on the limits to arbitrage.

In models without agency problem, arbitragers trade more aggressively when prices move further out of fundamental values (Campbell and Kyle, 1993, DeLong et al., 1990, Grossman and Miller, 1988). In Shleifer and Vishny (1997), the noise trader risk makes investors hardly finance their bets against mispricing. Other studies focus on implementation cost, which includes transaction costs, short-sale constraints, and the costs of discovering or exploiting a mispricing (D'Avolio, 2002, Gromb and Vayanos, 2002, Jones and Lamont, 2002, Lamont and Thaler, 2003, Nagel, 2005). We argue that the transaction cost in the exchange market and the delay cost of the liquidity investor cause the limits to arbitrage in the exchange market.

2.2 Model

Our model has four market participants: an informed investor, a liquidity investor, a market maker, and a firm manager. Both the informed investor and the liquidity investor strategically choose to trade firm stocks in the exchange market or the dark pool. The market maker works for the exchange market and clears the market by inventories. Therefore, investors in the exchange market will have their orders implemented for sure. In contrast, investors who choose the dark pool may not trade successfully. The firm manager and the market maker can observe the total trading volume in the exchange market but cannot identify the trader nor observe the tradings in the dark pool.

2.2.1 Corporate Decisions

The firm value, denoted by $v(\theta, d)$, depends on both the manager's investment d and the firm fundamentals $\theta \in \{H, L\}$. In particular, the state θ is drawn by nature with an equal probability. The manager can choose to expand the investment ($d = 1$), to keep the current investment ($d = 0$), or to decrease the investment ($d = -1$). If the manager keeps the current investment, the firm value is $v(H, 0) = R_H$ at state H and $v(L, 0) = R_L$ at state L. We assume that $R_H > R_L$. As a convention, the manager changes the investment level only if doing so can bring a strictly higher firm value. g is the bonus and penalty of making the "correct" and "wrong" action, respectively. Thus, the "correct" action at state H – expanding the investment – creates an additional value g and leads to the firm value $v(H, 1) = R_H + g$. In contrast, the "wrong" action at state H – decreasing the investment – reduces the firm value (by g) to $v(H, -1) = R_H - g$. Similarly, at state L, decreasing the investment is the correct action and creates an additional value g, while increasing the investment is incorrect and reduces the firm value (by g). Table 2.1 summarizes the firm value $v(\theta; d)$.

		Investment d		
		1	0	-1
State θ	H	$R_H + g$	R_H	$R_H - g$
	L	$R_L - g$	R_L	$R_L + g$

Table 2.1: Investment Decision and Firm Value

We assume that state H dominates state L in term of the fundamentals effect on the firm value; that is, even the wrong investment decision at state H brings a higher firm value than the right investment decision at state L; formally, we assume that $v(H, -1) > v(L, -1)$, which is equivalent to $R_H - g > R_L + g$. By this assumption, the firm value is increasing in its fundamentals, so to the informed investor, her private information about the state is more important than the manager's investment decision. For simplicity, we further restrict the model parameters to $g = k\frac{(R_H - R_L)}{2}$, where k is strictly less than but arbitrarily close to 1.

2.2.2 Trading Venues

There are two parallel trading venues: an exchange market and a dark pool. We assume that there is a transaction cost[2] k in the exchange market. The informed investor perfectly observes the firm fundamentals and then selects a trading venue, either the exchange market or the dark pool, or she does not trade. We call the informed investor as a "positively" and a "negatively" informed investor if the firm fundamentals are respectively high and low. The liquidity investor may have a positive liquidity demand ($l = 1$), a negative liquidity demand ($l = -1$), or have no liquidity demand ($l = 0$). We call the liquidity investor as a "positively" and a "negatively" liquidity investor if $l = 1$ and $l = -1$ respectively. After receiving a liquidity shock $l \neq 0$, the liquidity investor may select a trading venue between the exchange market and the dark pool, or she does not trade. We assume that the liquidity investor suffers a delay cost δ if she does not trade or does not trade successfully when $l \neq 0$.

For simplicity, we assume that after choosing a trading venue, both the informed investor and the liquidity investor trade unit share. We denote by β_{HE} and β_{LE} (β_{HD} and β_{LD}) the probabilities that the informed investor chooses the exchange market (the dark pool) when she is respectively positively and negatively informed, and denote by α_{BE} and α_{SE} (α_{BD} and α_{SD}) the probabilities that the liquidity investor chooses the exchange market (the dark pool) when she has a positive and a negative liquidity demand, respectively. The informed investor and the noise investor trade simultaneously. In the exchange market, the market maker observes the total trading volume $X = X_I + X_L$, but not their individual trades. Obviously, $X \in \{-2, -1, 0, 1, 2\}$. The market maker is competitive and sets a price based on the total trading volume in the exchange market to keep herself break-even. That is, her pricing strategy is $P(X) = \mathbb{E}(v|X)$. She then clears any excess demand or supply using her

[2]The transaction cost is exogenous such as commission fees, borrowing costs for short-selling, and the opportunity costs of capital commitment for purchases. While the transaction costs may differ between buying and selling, we assume the same transaction costs in both directions to prevent generating any asymmetry mechanically.

own inventory. In the dark pool, there is no market maker, and thus orders are executed only when both investors trade in the dark pool and demand opposite positions. The execution price in the dark pool therefore is the concurrent asset price in the exchange market.

2.2.3 Timing

Figure 2.2 presents the timeline of our model. At $t = 0$, nature chooses the firm fundamentals θ. The informed investor observes θ perfectly and chooses a trading venue or does not trade. The liquidity investor with a demand $l \neq 0$ chooses a trading venue or does not trade. At $t = 1$, trading may occur in the exchange market or the dark pool. The manager observes the total trading volume in the exchange market and then makes an investment decision. At $t = 2$, all uncertainties are resolved and payoffs are realized.

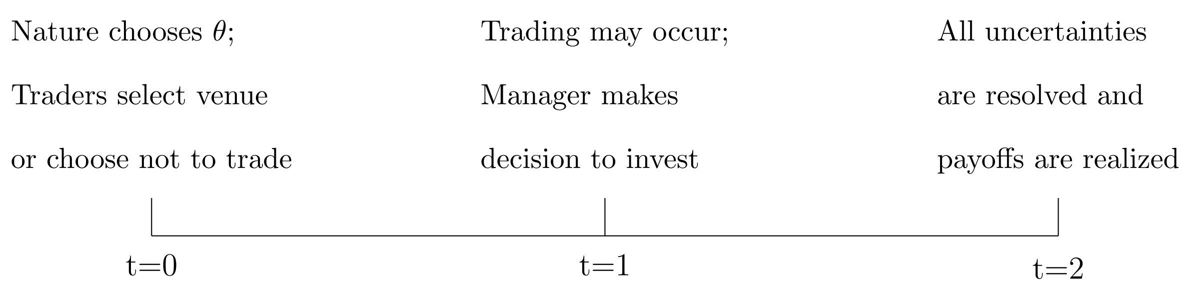

Figure 2.2: Model Timeline

2.2.4 A Perfect Bayesian Equilibrium

The informed investor's strategy is a mapping from the firm fundamentals θ to the probabilities of trading in the exchange market and trading in the dark pool. Similarly, the liquidity investor's strategy is a mapping from the liquidity demand l to the probabilities of trading in the exchange market and trading in the dark pool. The manager's strategy is a mapping from the total trading volume to her investment decision. The market maker's pricing strategy is a mapping from the total trading volume to a price. We are interested in perfect Bayesian equilibrium.

Definition 2.1. The informed investor's strategy of venue choice $\beta^* : \{H, L\} \to [0, 1]$, the liquidity investor's strategy of venue choice $\alpha^* : \{-1, 1\} \to [0, 1]$, the manager's investment strategy $d^* : X \to \{-1, 0, 1\}$, and the market maker's pricing strategy $P^*(X)$ constitute a perfect Bayesian equilibrium if:

1. For the informed investor, β^*_θ maximizes her expected final payoff for each $\theta \in \{H, L\}$, given the liquidity investor's venue choice strategy, the market maker's pricing strategy, and the manager's investment strategy.

2. For the liquidity investor, α^*_l maximizes her expected final payoff, given the informed investor's venue choice strategy, the market maker's pricing strategy and the manager's investment strategy.

3. For the manager, $d^*(X)$ maximizes the expected firm value V given the information in the exchange market and other agents' strategies.

4. For the market maker, the price $P^*(X) = \mathbb{E}(v|X)$ allows her to beak-even in expectation for each $X \in \{-2, -1, 0, 1, 2\}$, given all other agents' strategies.

5. The manager and the market maker update their beliefs by Bayes' rule after observing the total trading volume in the exchange market.

2.3 Benchmark Model

To analyze the dark pool effects on the investor trading venue choice, we establish a benchmark model where a dark pool is not available. Hence, the informed investor will trade in the exchange market if she can earn positive profits and not trade otherwise. The liquidity investor will trade in the exchange market if her profit in the exchange market is higher than the delay cost.

We first analyze the market maker and the firm manager's posterior beliefs for any given trading venue choice strategies of investors, which will determine the firm's stock price and investment. We then characterize investors' equilibrium venue choice strategies, given the market maker's and the manager's best responses.

The main result in this section is that when the dark pool is shut down, depending on the transaction cost in the exchange market and the liquidity investor's delay cost, there are multiple pure strategy equilibria. In particular, when the transaction cost in the exchange market is very high, the informed investor chooses not to trade, regardless of the liquidity investor's trading strategy. For the liquidity investor, when the delay cost is lower than the transaction cost, she may choose not to trade; otherwise, she may choose to trade in the exchange market. When the liquidity investor always trades in the exchange market, as the transaction cost reduces, the informed investor's strategy changes from not to trade to trade on one side of information, and finally changes to trade on both sides of information when the transaction cost is very low. In addition, when the transaction cost is low enough, managerial learning encourages more informed trading in the exchange market.

2.3.1 Belief Updating, Asset Pricing, and Firm Investment

We first analyze the manager's and the market maker's posterior beliefs about state H for each possible total trading volume X, given the investors' strategies of trading venue choice. Since the manager and the market maker have the same prior information in our model, conditional on a total trading volume X, they should have the same posterior beliefs in equilibrium. Figure 2.3 helps calculate the posterior belief of the manager and the market maker given each possible total trading volume.

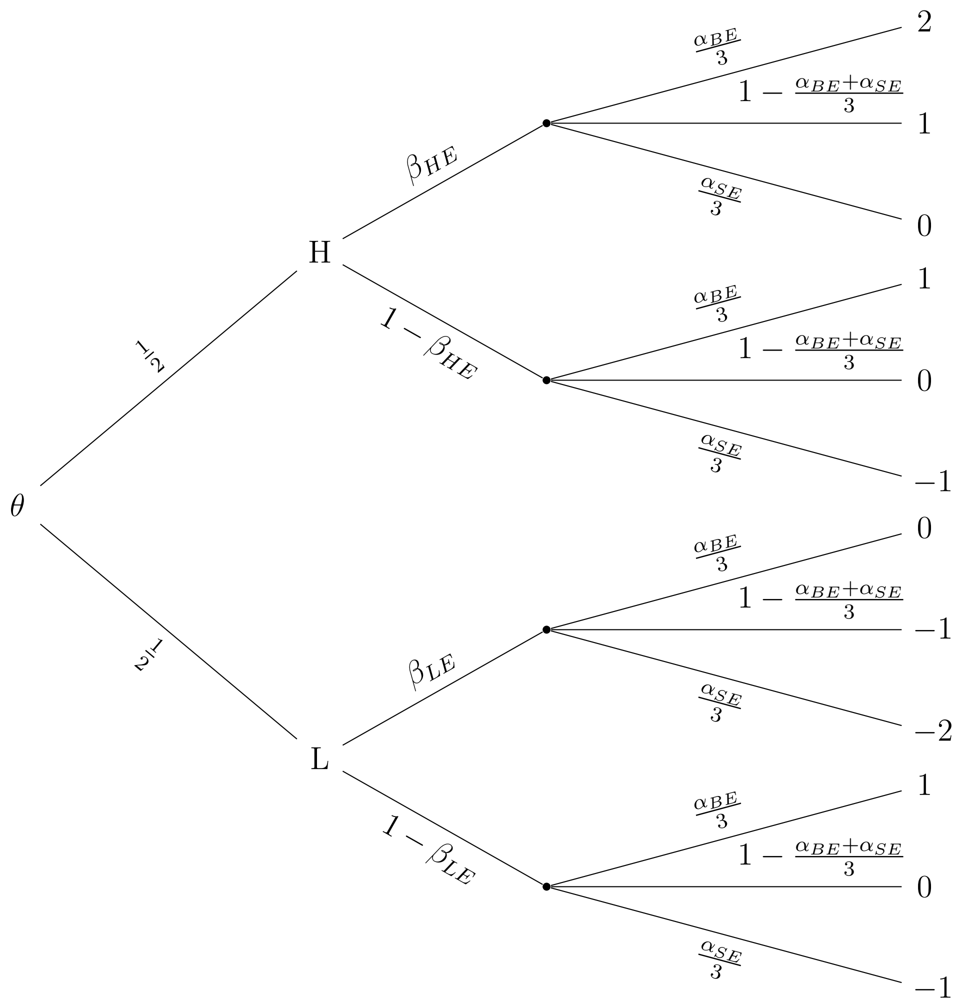

Figure 2.3: Manager and Market Maker's Belief Updating

For example, the total trading volume in the exchange market is zero either when neither of the investors trades in the exchange market (with the probability $\frac{1}{2}[(1-\beta_{HE})+(1-\beta_{LE})](1-\frac{\alpha_{BE}+\alpha_{SE}}{3})$), or when both investors are trading in the exchange market but with opposite positions (with probability $\frac{1}{2}\frac{(\beta_{HE}\alpha_{SE}+\beta_{LE}\alpha_{BE})}{3}$). Then, by Bayes' rule, when the total trading volume in the exchange market is zero, the firm manager and the market maker have a posterior belief about $\theta = H$:

$$\Pr(\theta = H|X=0) \triangleq q(X=0) = \frac{\frac{1}{2}(1-\beta_{HE})(1-\frac{\alpha_{SE}+\alpha_{BE}}{3}) + \frac{1}{2}\frac{\beta_{HE}\alpha_{SE}}{3}}{\frac{1}{2}(2-\beta_{HE}-\beta_{LE})(1-\frac{\alpha_{SE}+\alpha_{BE}}{3}) + \frac{1}{2}\frac{(\beta_{HE}\alpha_{SE}+\beta_{LE}\alpha_{BE})}{3}}$$
$$= \frac{\beta_{HE}\alpha_{SE} + (3-\alpha_{BE}-\alpha_{SE})(1-\beta_{HE})}{\beta_{HE}\alpha_{SE} + \beta_{LE}\alpha_{BE} + (2-\beta_{HE}-\beta_{LE})(3-\alpha_{BE}-\alpha_{SE})}.$$

Similarly, we calculate the manager's and the market maker's posterior belief following each possible total trading volume $X \in \{-2,-1,0,1,2\}$. Equation (2.1) summarizes the equilibrium posterior belief $q(X)$.

$$q(X) = \begin{cases} 0, & X=-2 \\ \frac{\alpha_{SE}(1-\beta_{HE})}{\alpha_{SE}(2-\beta_{HE}-\beta_{LE})+(3-\alpha_{BE}-\alpha_{SE})\beta_{LE}}, & X=-1 \\ \frac{\beta_{HE}\alpha_{SE}+(3-\alpha_{BE}-\alpha_{SE})(1-\beta_{HE})}{\beta_{HE}\alpha_{SE}+\beta_{LE}\alpha_{BE}+(2-\beta_{HE}-\beta_{LE})(3-\alpha_{BE}-\alpha_{SE})}, & X=0 \\ \frac{\alpha_{BE}(1-\beta_{HE})+(3-\alpha_{BE}-\alpha_{SE})\beta_{HE}}{\alpha_{BE}(2-\beta_{HE}-\beta_{LE})+(3-\alpha_{BE}-\alpha_{SE})\beta_{HE}}, & X=1 \\ -1, & X=2. \end{cases} \tag{2.1}$$

In equilibrium, the market maker sets prices that make her break-even in expectation. Thus, the pricing function in the exchange market is

$$P(X) = \mathbb{E}(v|X) = q(X)v\left(H, d(X)\right) + \left(1-q(X)\right)v\left(L, d(X)\right). \tag{2.2}$$

Note that in Equation (2.2), the market maker accounts for the manager's investment strategy when setting the price.

We now analyze the manager's investment decision. Denote by q_1 and q_{-1} two thresholds in the manager's posterior belief space such that

$$q_1 R_H + (1 - q_1) R_L = q_1(R_H + g) + (1 - q_1)(R_L - g) \tag{2.3}$$

$$q_{-1} R_H + (1 - q_{-1}) R_L = q_{-1}(R_H - g) + (1 - q_{-1})(R_L + g). \tag{2.4}$$

Equation (2.3) indicates that when the manager's posterior belief is exactly q_1, the expected firm value from keeping the current investment equals that from expanding the investment. Similarly, Equation (2.4) implies that with a posterior belief q_{-1}, the manager is indifferent between keeping the investment level and reducing the investment. Simple algebra shows that

$$q_1 = q_{-1} = \frac{1}{2}. \tag{2.5}$$

Recall that the manager will keep the investment if changing the investment cannot lead to a strictly higher firm value, then Lemma 2.1 formally characterizes the manager's investment decision based on her posterior beliefs.

Lemma 2.1. When the manager is learning from the exchange market, his equilibrium investment decision is determined by his posterior belief. In particular,

$$d(X) = \begin{cases} 1, & \text{if } q(X) \in \left(\frac{1}{2}, 1\right] \\ 0, & \text{if } q(X) = \frac{1}{2} \\ -1, & \text{if } q(X) \in \left[0, \frac{1}{2}\right). \end{cases} \tag{2.6}$$

2.3.2 Trading Profit in Exchange Market

Consider the informed investor. If the informed investor does not trade, her expected payoff is zero. If the positively informed investor decides to trade in the exchange market, her order will lead to a total trading volume $X \in \{0, 1, 2\}$ with probability $(\alpha_{SE}/3, 1 - \frac{\alpha_{SE}+\alpha_{BE}}{3}, \alpha_{BE}/3)$. Similarly, if the negatively informed investor decides to trade in the exchange market, her order will lead to a total trading volume $X \in \{0, -1, -2\}$ with probability $\{\alpha_{BE}/3, 1 - \frac{\alpha_{SE}+\alpha_{BE}}{3}, \alpha_{SE}/3\}$, respectively. With the pricing Equation (2.2), Lemma 2.2 shows the informed investor's expected profit in the exchange market.

Lemma 2.2. The informed investor's expected payoff in the exchange market is

$$\left(1 - \frac{\alpha_{SE} + \alpha_{BE}}{3}\right) [v(H, d(1)) - P(1)] + \frac{\alpha_{SE}}{3} [v(H, d(0)) - P(0)] - k \tag{2.7}$$

if she is positively informed, and it is

$$\left(1 - \frac{\alpha_{SE} + \alpha_{BE}}{3}\right) [P(-1) - v(L, d(-1))] + \frac{\alpha_{BE}}{3} [P(0) - v(L, d(0))] - k. \tag{2.8}$$

if she is negatively informed.

We now turn to the liquidity investor. If the liquidity investor does not trade when $l \neq 0$, she suffers the delay cost δ. If the positive liquidity investor decides to trade in the exchange market, her order will lead to a total trading volume $X = 0$ in state L with a probability $\frac{\beta_{SE}}{2}$, lead to a total trading volume $X = 1$ in state H with a probability $\frac{1-\beta_{BE}}{2}$ and in state L with a probability $\frac{1-\beta_{SE}}{2}$, and lead to a total trading volume $X = 2$ in state H with a probability $\frac{\beta_{BE}}{2}$. Similarly, if the negative liquidity investor decides to trade in the exchange market, her order will lead to a total trading volume $X = 0$ with probability $\frac{\beta_{BE}}{2}$, lead to a total trading volume $X = -1$ in state H with a probability $\frac{1-\beta_{BE}}{2}$ and in state L with a probability $\frac{1-\beta_{SE}}{2}$, and lead to a total trading volume $X = -2$ in state L with a probability $\frac{\beta_{SE}}{2}$. With the pricing equation (2.2), Lemma 2.3 shows the liquidity investor's expected profit in the exchange market.

Lemma 2.3. The liquidity investor's expected payoff in the exchange market is

$$\frac{1-\beta_{BE}}{2}[v(H,d(1)-P(1)]+\frac{1-\beta_{SE}}{2}[v(L,d(1)-P(1)]+\frac{\beta_{SE}}{2}[v(L,d(0)-P(0)]-k \quad (2.9)$$

if she has a positive liquidity demand, and it is

$$\frac{1-\beta_{BE}}{2}[P(-1)-v(H,d(-1)]+\frac{1-\beta_{SE}}{2}[P(-1)-v(L,d(-1)]+\frac{\beta_{BE}}{2}[P(0)-v(H,d(0)]-k. \quad (2.10)$$

if she has a negative liquidity demand.

2.3.3 Pure Strategy Equilibria

We define that the exchange market has "real effects" if the information in the exchange market makes the firm manager expand or reduce investment. As shown in Equation 2.1, when the total trading volume in the exchange market is $X = 2$, the manager's posterior belief about $\theta = H$ is $q(X = 2) = 1$, because the total trading volume $X = 2$ occurs if and only if the informed investor is buying in the exchange market. Therefore, observing the total trading volume $X = 2$, the manager will expand the investment. Similarly, the manager will reduce the investment, when the total trading volume is $X = -2$.

The real effects when $X = 2$ and $X = -2$ arises purely from the manager's rationality and are not affected by the informed investor trading venue choice. Therefore, we focus on the manager's investment decision when $X \in \{-1, 0, 1\}$.

Lemma 2.4 argues that the manager's investment decision must be increasing in the total trading volume. This is intuitive. Once the informed investor chooses the exchange market, she buys at state H and sells at state L. Hence, as the total trading volume increases, it is more likely that the firm fundamentals are high.

Lemma 2.4. Both the manager's posterior belief $q(X)$ about $\theta = H$ and the manager's investment decision $d(X)$ are increasing in the total trading volume X.

Lemma 2.4 simplify the equilibrium characterization. The firm investment at the total trading volume $X = (-1, 0, 1)$ can only be $(-1, 0, 0)$, $(-1, -1, 0)$, $(-1, 0, 1)$, $(-1, -1, 1)$, $(-1, 1, 1)$, $(0, 0, 1)$, and $(0, 1, 1)$. Simple calculation of the posterior beliefs in Equation (2.1) shows that it is impossible for the manager's investment decision to be $(-1, -1, 0)$ and $(0, 1, 1)$ at the total trading volume $X = (-1, 0, 1)$. So, the equilibrium firm investment at the total trading volume $X = (-1, 0, 1)$ can only be $(-1, 0, 0)$, $(-1, 0, 1)$, $(-1, -1, 1)$, $(-1, 1, 1)$, and $(0, 0, 1)$.

Proposition 2.1 characterizes equilibria with different transaction cost k and delay cost δ.

Proposition 2.1. Depending on transaction cost k and delay cost δ, there may be multiple pure strategy equilibria.

1. When $k > \frac{R_H - R_L}{2}$ and $k > \delta$, the unique pure strategy is that both investors choose not to trade, and the manager's decision is $d = (0, 0, 0)$.

2. When $k > \frac{R_H - R_L}{3}$ and $\delta > k$, the unique pure strategy is that the liquidity investor always trades in the exchange market, the informed investor does not trade, and the manager's decision is $d = (0, 0, 0)$.

3. When $\frac{R_H - R_L}{3} > k > \frac{R_H - R_L}{6}$ and $\delta > \frac{R_H - R_L}{4} + k$, there are two pure strategy equilibria in which the liquidity investor always trades in exchange. If the informed investor only buys but does not sell in exchange, the manager's decision is $d = (-1, 0, 0)$; if the informed investor only sells but does not buy in exchange, the manager's decision is $d = (-1, 0, 0)$.

4. When $\frac{R_H-R_L}{6} > k > 0$ and $\delta > \frac{R_H-R_L}{4} + k$, the unique pure strategy equilibrium is that both investors always trade in exchange, and the manager's decision is $d = (-1, 0, 1)$.

5. When $\frac{2(R_H-R_L)-4g}{9} > k > 0$ and $\frac{R_H-R_L+2g}{2} + k > \delta > \frac{R_H-R_L-2g}{3} + k$, there are two pure strategy equilibria in which the liquidity investor only sells but does not buy in exchange. If the informed investor only buys but does not sell in exchange, the manager's decision is $d = (-1, -1, 1)$; if the informed investor only sells but does not buy in exchange, the manager's decision is $d = (-1, 1, 1)$.

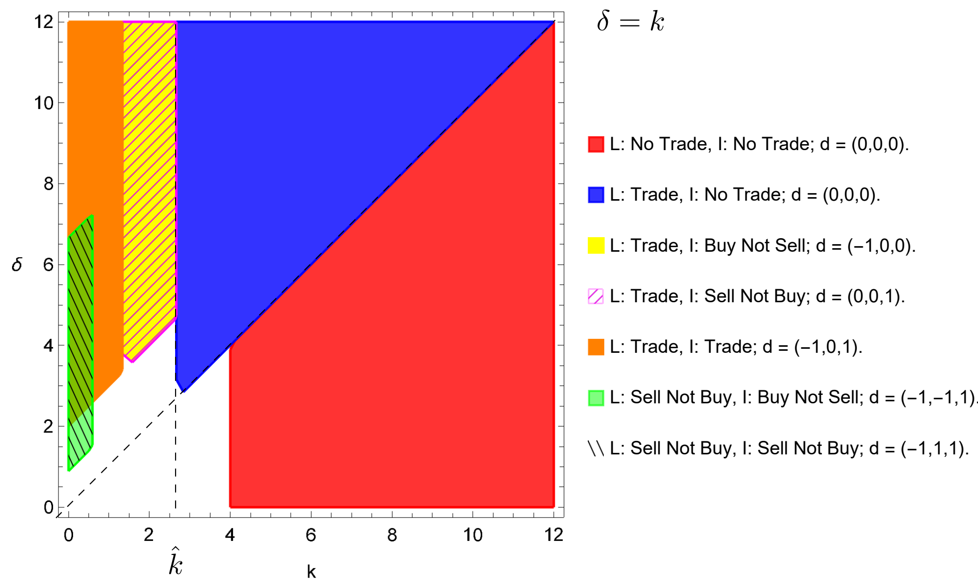

Figure 2.4: Equilibria without Dark Pool

Denote the liquidity investor and the informed investor by "L" and "I" respectively, Figure 2.4 illustrates the equilibria characterized in Proposition 2.1. In particular, for any $k > \hat{k} = \frac{R_H-R_L}{3}$, the informed investor does not trade, regardless of the liquidity investor's trading strategy. It also shows that for any $\delta > k$, there are multiple equilibria in which the liquidity investor always trades in the exchange market, regardless of the informed investor's trading strategy. Besides, if given the transaction cost and delay cost such that the liquidity investor always trades in the exchange market, as transaction cost reduces, the informed investor's trading strategy changes from no trade to asymmetric trading, and finally changes to always trade in exchange market when transaction cost is almost zero.

2.4 Trading Venue Choice

We now analyze our core model where a dark pool is available to the investors. We start with the market maker and the firm manager's posterior beliefs about firm fundamentals, which will determine the firm's stock price and investment. We then characterize investors' equilibrium venue choice, given the market maker's and the manager's best responses.

The main result in this section is that when the dark pool is available to the investors, all pure strategy equilibria when the dark pool is shut down still hold. However, for those equilibria in which the liquidity investor does not necessarily trade in the exchange market when the dark pool is shut down, the dark pool may lead to multiple equilibria in which investors take coordination actions of choosing the dark pool.

2.4.1 Dark Pool and Belief Updating, Pricing, and Investment

We first analyze the manager and the market maker's posterior beliefs about state H for each possible total trading volume X, given the investors' strategies of trading venue choice. Figure 2.5 helps calculate the posterior belief of the manager and the market maker given each possible total trading volume.

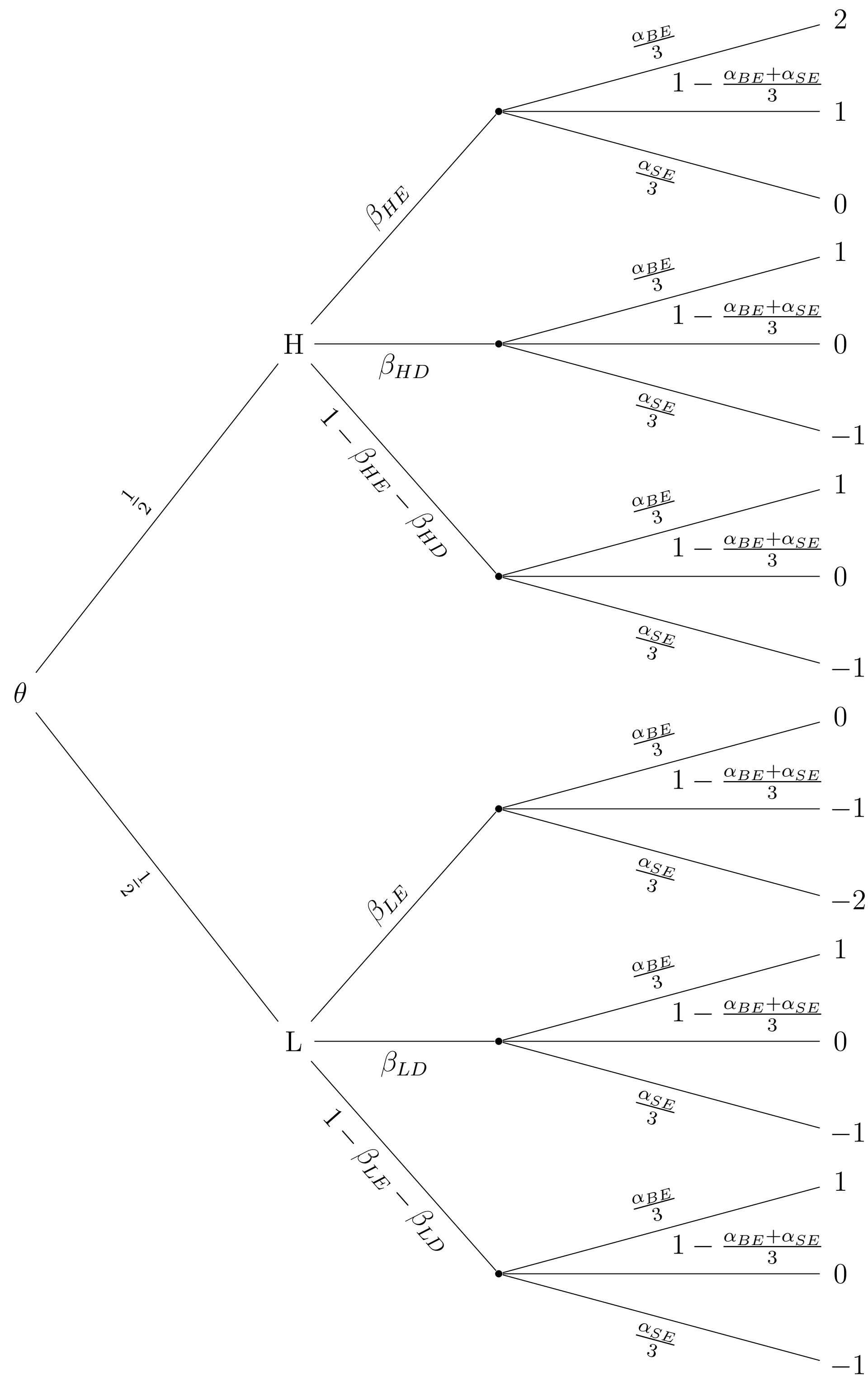

Figure 2.5: Manager and Market Maker's Belief Updating

From Figure 2.5, it is straightforward to see that adding a dark pool does not affect the manager's and the market maker's posterior beliefs about state H, which is stated in Lemma 2.5. Intuitively, the dark pool does not change any information incorporated into the exchange market, given any informed investor's trading strategy.

Lemma 2.5. The dark pool does not affect the manager and the market maker's posterior beliefs about state θ, given each total trading volume in the exchange market X.

Lemma 2.6 argues that given the same total trading volume in the exchange market, the manager's investment decision and the market price after adding a dark pool must be the same as those without the dark pool. This is intuitive. Since the manager's and the market maker's posterior beliefs keep the same after adding a dark pool, the manager will make the same decision, and the market maker will set the same price for each total trading volume in the exchange market.

Lemma 2.6. The dark pool does not affect the manager's investment decision $d(X)$ and market price $P(X)$ given each total trading volume in the exchange market X.

Lemma 2.6 simplify the equilibrium characterization. The firm investment at the total trading volume $X \in (-1, 0, 1)$ can only be $(-1, 0, 0)$, $(-1, -1, 0)$, $(-1, 0, 1)$, $(-1, -1, 1)$, $(-1, 1, 1)$, $(0, 0, 1)$, and $(0, 1, 1)$.

On the other hand, the execution price in the dark pool is assumed to be the concurrent stock price at the exchange market. While there are five possible prices in the exchange market (because there are five possible total trading volumes), only the one following a zero total trading volume matters for the execution price at the dark pool. Intuitively, when an investor chooses the dark pool, she can have her order executed only when the counter-party is demanding an opposite position in the dark pool. This occurs only when both the informed investor and the liquidity investor are trading in the opposite directions in the dark pool, leading to a zero total trading volume in the exchange market. Lemma 2.7 formally derives the execution price in the dark pool in equilibrium.

Lemma 2.7. The execution price in the dark pool equals the stock price in the exchange market when the total trading volume is zero. Formally, given the informed investor's and the liquidity investor's venue choice strategies, the execution price at the dark pool is

$$P^D = P(X = 0), \tag{2.11}$$

which is calculated by equation (2.2).

Lemma 2.7 shows that the investors' venue choice strategies affect the execution price at the dark pool, which in turn determines investors' trading profits in the dark pool. Lemma 2.7 implies that the investors' trading profits in the dark pool are endogenous, and if $\beta_{HE} \neq \beta_{LE}$, or $\alpha_{BE} \neq \alpha_{SE}$, or $\alpha_{BD} \neq \alpha_{SD}$, such trading profits in the dark pool are asymmetric. Hence, the execution price in the dark pool in our model differs from that in Zhu (2014) where the price in the dark pool is zero because it is assumed to be the midpoint of the bid and ask prices in the exchange market.

2.4.2 Payoffs and Venue Choice

If the informed investor does not trade in both markets, she earns zero profit. Similarly, the liquidity investor suffers a delay cost δ if she does not trade when $l \neq 0$. Lemma 2.5 and Lemma 2.6 imply that the dark pool does not change the informed investor's and the liquidity investor's trading profits in the exchange market, given the firm investment d at the total trading volume $X \in (-1, 0, 1)$. Therefore, we focus on the informed investor's and the liquidity investor's trading profits in the dark pool.

If the positively informed investor decides to trade in the dark pool, her order will be executed with probability $\alpha_{SD}/3$. Similarly, if the negatively informed investor decides to trade in the dark pool, her order will be executed with probability $\alpha_{BD}/3$. With the pricing

Equation (2.2), Lemma 2.8 shows the informed investor's expected profit in the dark pool.

Lemma 2.8. The informed investor's expected payoff in the dark pool is

$$\frac{\alpha_{SD}}{3}\left[v(H,d(0))-P(0)\right] \tag{2.12}$$

if she is positively informed, and it is

$$\frac{\alpha_{BD}}{3}\left[P(0)-v(L,d(0))\right]. \tag{2.13}$$

if she is negatively informed.

We now turn to the liquidity investor. If the positive liquidity investor decides to trade in the dark pool, her order can be executed with probability $\beta_{LD}/2$. With the complement probability $1-\beta_{LD}/2$, she can not execute her order successfully and thus has to pay the delay cost δ. Similarly, if the negative liquidity investor decides to trade in the dark pool, her order can be executed with probability $\beta_{HD}/2$. With the complement probability $1-\beta_{HD}/2$, her order can not be executed, and she has to pay the delay cost δ.

With the pricing equation (2.2), Lemma 2.9 shows the liquidity investor's expected profit in the exchange market.

Lemma 2.9. The liquidity investor's expected payoff in the exchange market is

$$\frac{\beta_{SD}}{2}[v(L,d(0)-P(0))]-(1-\frac{\beta_{SD}}{2})\delta \tag{2.14}$$

if she has a positive liquidity demand, and it is

$$\frac{\beta_{BD}}{2}[P(0)-v(H,d(0))]-(1-\frac{\beta_{BD}}{2})\delta. \tag{2.15}$$

if she has a negative liquidity demand.

Since both investors are uncertain about execution risk in the dark pool, which causes them to hesitate in choosing the dark pool, it is straightforward that even if the dark pool is available to the investors, all equilibria when the dark pool is shut down still hold. Proposition 2.2 summarizes such equilibria.

Proposition 2.2. When the dark pool is available to the investors, any pure strategy equilibrium when the dark pool is shut down still holds.

Since compared with no trade, going to the dark pool may bring positive profits to the informed investor and reduce the cost of the liquidity investor, the investors may have some incentives to coordinate to trade in the dark pool. Proposition 2.3 characterizes equilibria when the investors take coordination action.

Proposition 2.3. For those equilibria in which the liquidity investor does not necessarily trade in the exchange market when the dark pool is shut down, the emergence of dark pool may cause multiple equilibria in which investors coordinate to trade in the dark pool.

1. When $k > \frac{R_H - R_L}{2}$ and $k > \delta > \frac{R_H - R_L}{2}$, there are three multiple equilibria. Either the liquidity investor only buys and sells and the informed investor only sells and buys respectively in the dark pool, or both investors always trade in the dark pool. The manager's decision is $d = (0, 0, 0)$.

2. When $k > \frac{R_H - R_L}{2}$ and $\frac{1}{2}(R_L - R_H + 4k) > \delta > k$, the unique pure strategy equilibrium is that both investors always trade in the dark pool. The manager's decision is $d = (0, 0, 0)$.

3. When $\frac{1}{9}(2R_H - 2R_L - 4x) > k > 0$ and $\frac{1}{3}(2R_H - 2R_L + 6k + 8g) > \delta > \frac{1}{3}(R_H - R_L + 3k - 2g)$, the unique pure strategy equilibrium is that the liquidity investor sells in the exchange market and buys in the dark pool, and the informed investor buys in the exchange market and sells in the dark pool. The manager's decision is $d = (-1, -1, 1)$.

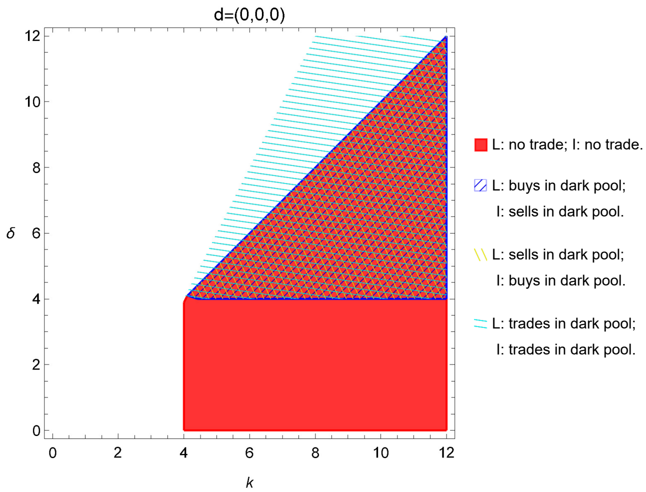

Figure 2.6: Equilibria When $d = (0, 0, 0)$

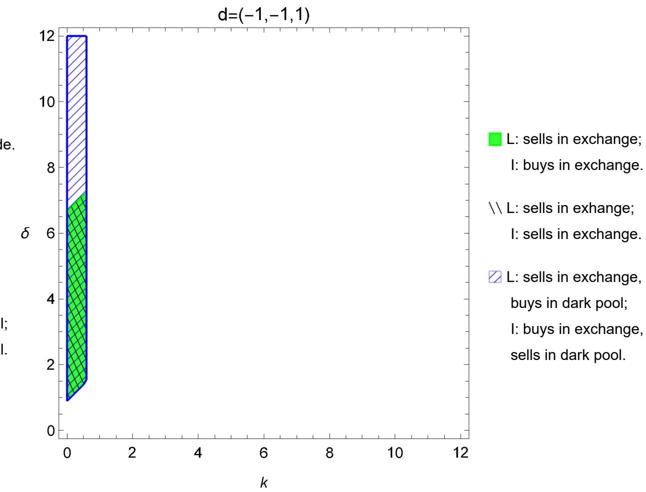

Figure 2.7: Equilibria When $d = (-1, -1, 1)$

Figure 2.6 and Figure 2.7 illustrate the equilibria when investors may take coordination actions given the real effects are respectively $d = (0, 0, 0)$ and $d = (-1, -1, 1)$ at the total trading volume $X = (-1, 0, 1)$. Figure 2.6 shows that with real effects $d = (0, 0, 0)$, when the delay cost is relatively high to the liquidity investor, that is, when $\delta > \frac{R_H - R_L}{2}$, the liquidity investor has coordination incentives of choosing the dark pool to save cost. On the other hand, the informed investor also has coordination incentives of choosing the dark pool to make positive profits. Similarly, Figure 2.7 shows that with real effects $d = (-1, -1, 1)$, the positive liquidity investor and the negatively informed investor may have coordination incentives of trading in the dark pool[3].

2.5 Conclusion

The dark pools have expanded substantially in recent years due to the regulatory reforms and the fast-growing electronic trading methods. Their rising market share caches the eyes of regulators, scholars, and industry professionals and results in serious concerns in policy

[3]The coordination motives of investors are not relevant to negotiations between the informed investor and the liquidity investor. Instead, the coordination actions are strategic complementarity in investors' trading venue choices.

circles. Such concerns are more disquieting when real decision makers are making decisions based on information from the exchange market. In this paper, we develop a model to analyze the dark pool effects on the investor trading behavior. We find that the investors may coordinate with each other and trade in the dark pool when the liquidity investor does necessarily trades in the exchange market. This is because the informed investor can make positive profits in the dark pool instead of no trade and earns nothing. Likewise, the liquidity investor pays less by trading in the dark pool instead of no trade and pays the huge delay costs.

Besides, the paper also shows that when the transaction cost in the exchange market is high enough, the informed investor does not trade in the exchange market, regardless of the liquidity investor's trading behavior. When the delay cost of the liquidity investor is higher than the transaction cost, the liquidity investor may always trade in the exchange market, regardless of the informed investor's trading behavior. In addition, managerial learning may encourage more informed trading in the exchange market when the transaction cost in the exchange market is low enough.

Chapter 3

Informed Trading and Product Market Competition

Abstract This paper studies the interaction between secondary financial market efficiency and product market competition. An insider trades incumbent stocks based on her knowledge about product market demand, which conveys information to a potential entrant. In equilibrium, entrant learning causes a "buy-side" limit to arbitrage. With different entry barriers, entry probability is a function of financial market trading friction and exhibits different patterns. In particular, when the entrant is optimistic ex-ante, its learning will reduce entry probability, which will lead to a higher product price and lower consumer welfare. A policy of reducing entry barriers has non-monotonic effects on entry probability. Furthermore, the product market uncertainty may increase or decrease entry probability, depending upon trading frictions.

JEL Classification: D83, G14, G18, L13, L22, L50

Key words: Oligopoly, managerial learning, limits to arbitrage, product market competition, barriers to entry

3.1 Introduction

How does the financial market affect competition in the product market (or industrial organization)? Previous studies have shown that the primary capital market significantly impacts firms' decisions to enter a product market with incumbents because the primary financial market can supply the necessary entry funds.[1] However, does the secondary financial market also affect competition in the product market? If so, how? Conversely, what are the effects of the potential entry on trading behavior and market efficiency in the incumbent's stock market? How does the secondary financial market play a role when policymakers want to promote product market competition?

This paper addresses these questions. We consider an entrant's learning from the incumbent stock market a key channel through which the incumbent stock market and the product market competition interact. Indeed, since Hayek (1945), researchers have been documenting empirical evidence about how asset prices convey useful information to decision-makers.[2] Since entering a new product market is a core corporate decision, the potential entrant will naturally make entry decisions based on information extracted from the incumbent stock market. Insiders in the incumbent stock market will take into account entrant learning when trading, which determines stock market efficiency that will feedback to the entrant's entry decisions and thus product market competition.

In this paper, we analyze an entry game where a potential entrant learns from an

[1] For instance, in Benoit (1983,1984), predatory pricing may prevent entry if entry decision is financially constrained. Poitevin (1989) shows that incumbent finances with equity, while high-value entrant must issue debt to signal his quality to investors such that it comes into market heavily leveraged. Matveyev and Zhdanov (2019) show that entrant may issue substantially less debt than would be optimal by merely trading off tax benefits against bankruptcy costs to undercut the incumbent in leverage.

[2] For example, Fama and Miller (1972) note "at any point in time market prices of securities provide accurate signals for resource allocation; that is, firms can make production-investment decisions...". Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), and Foucault and Frésard (2012) document that firms learn from their own stock markets. Foucault and Frésard (2014, 2019), and Dessaint, Foucault, Frésard, and Matray (2018) further show that firms are also learning from their peers' stock prices. See Bond, Edmans, and Goldstein (2012) for an excellent survey.

insider's trading in the stock market of a monopoly incumbent. The maximum demand for a homogeneous consumption good is either high or low, which is privately known by the insider. The insider then trades in the incumbent's stock market with a simplified Kyle (1985), which is similar to Edmans, Goldstein, and Jiang (2015) with the difference that the insider and the entrant have conflicting interests. Once the entrant enters (by paying an entry fee), it competes with the incumbent in a Cournot duopoly setting; otherwise, the incumbent remains its monopoly status.

In equilibrium, entrant learning may lead to financial market inefficiency. Since the insider trades shares of the incumbent, she suffers a loss while the entrant makes a profit by entering the product market. To deter the entrant from entering, the insider refrains from buying on positive information to avoid revealing the high state to the entrant. However, in the low state, she sells and reveals to the entrant that entry is non-profitable. This "buy-side" limits to arbitrage may lead to financial market inefficiency and is different from Edmans, Goldstein, and Jiang (2015). In their paper, an insider trades shares of a firm, the firm manager learns from the financial market while making an investment decision. Since the insider and the firm manager have the same interests, she refrains from selling on negative information to avoid disinvestment, but she buys to encourage investment when she receives positive information.

Our key result is that, the transaction cost in the financial market significantly affects the ex-post entry probability, and such an effect is determined by entrant's prior belief. When the entrant is sufficiently optimistic about the market demand ex-ante and enters the market without learning from the incumbent stock market, an increase in the transaction cost weakly increases the entry probability. Conversely, when the entrant is pessimistic about the market demand and does not enter the market without learning, an increase in the transaction cost weakly reduces the entry probability. The intuition of this result lies on the transaction cost effects on the stock market efficiency. In equilibrium, an increase in the transaction cost weakly reduces the informativeness of the stock market. Hence, if the entrant enters the market based on its prior belief, lower stock market efficiency prevents it from learning such that it enters; but if the entrant does not enter the market based on its prior belief, lower stock market efficiency cannot provide the entrant sufficient information for it to enter.

Besides, how the trading cost affects the entrant's entry probability is also affected by economic and informational conditions that the insider faces. The insider is more likely to enter the financial market when she has more information that the entrant does not know. Therefore, when the product market is more uncertain, the entrant has higher incentives to learn from the financial market. In turn, additional information from the financial market may alter the decision more likely, leading to stronger incentives of the insider to enter the financial market. We find that the product market uncertainty may increase or decrease the ex-post entry probability, depending on different trading frictions.

Intuitively, when the product market becomes more uncertain, the insider is more likely to present in the financial market, leading to higher financial market efficiency. As a result, more information in the financial market may guide the entrant to make better entry decisions. We find that when transaction cost is extremely high, the insider does not trade, regardless of the product market uncertainty. Hence, entry probability does not change. By contrast, when transaction cost is extremely low, the insider always trades, regardless of the product market uncertainty. Hence, the entrant alters its decision when learning from the financial market. Besides, when transaction cost is high, the insider only sells on negative information. Therefore, when the product market becomes more uncertain, more negative information will be revealed to the entrant, leading to a lower entry probability. When transaction cost is low, the insider always trades on both directions of information. However, when the product market becomes more uncertain, the insider only trades on the negative information. As a result, entry probability decreases if the entrant is pessimistic ex-ante because less positive information is revealed to the entrant, but it increases if the entrant is optimistic ex-ante because less negative information is revealed to the entrant.

In addition, we find that the relation between entrant's ex-ante entry probability and the entry cost is non-monotonic. It is an inverse "U" shape. With financial market, a public policy that reduces entry barriers may reduce entry probability. If the entrant is pessimistic ex-ante, reducing entry cost increases entry probability. However, if the entrant is optimistic ex-ante, reducing entry cost decreases entry probability. Intuitively, as barriers to entry reduce, the entrant has more incentive to enter the industry, while the insider has less incentive to trade on positive private information to deter the entrant from entering. As a result, entry decision is determined by the net effect.

Our results provide new insights into policy implications. In Stigler (1971) theory of regulatory capture, the stricter regulation of entry raises barriers to entry, keeps out competitors, and raises incumbents' profits. As applies to deregulation of entry, the reduction of barriers to entry may encourage product market competition and improve product market efficiency. However, this capture theory (Peltzman, 1989, Posner, 1974, Stigler, 1971) does not consider the financial market. In the presence of the financial market, if the entrant is optimistic ex-ante, new information may deteriorate entry. Therefore, policies on industry organization should also take into account managerial learning.

Our paper has three main contributions. First, it is among the first papers that connects the financial market with industry organization[3]. An insider trades incumbent stocks based on her private information about product market demand, and an entrant makes its entry decision when learning from the financial market. In equilibrium, entrant learning causes a "buy-side" limits to arbitrage, making the financial market inefficient. This paper finds

[3]For instance, in Yang (2019), firms in a duopoly market faces a trade-off between suffering a proprietary cost and improving learning quality from asset prices when making disclosure decisions. He induces feedback effects by a future market in which speculators trade future contracts based on their private information about product market demand.

that financial market frictions can affect entry decisions: when transaction cost reduces, entry probability decreases if the entrant is optimistic ex-ante, but it increases if the entrant is pessimistic ex-ante. It also reveals that a policy of reducing entry barriers may impede product market competition. In particular, if the entrant is optimistic ex-ante, a reduction of entry cost reduces entry probability. Second, it provides some empirical implications. For example, the correlation between trading friction and entry probability is positive if the entrant is optimistic ex-ante. Third, it provides some policy implications on entry barriers and entry probability. For instance, a policy of reducing entry barriers may decrease product market competition when entry barriers are low enough.

Our paper relates to several important strands of literature. First, it is part of the research agenda that links the financial market to the industry organization. Starting from Titman (1984), firm's capital structure choice affects shareholders' incentive to liquidate when the firm is not bankrupt, so firm's capital structure choice is a determinant of the liquidation policy that it implements. In Brander and Lewis (1986), the financial structure can affect the output market through the limited liability effect of debt financing and the strategic bankruptcy effect. In Chemla and Faure-Grimaud (2001), the strategic use of debt by a durable good monopolist can induce consumers with high valuation to reveal their type. However, none of these researches discuss whether the secondary capital market can affect industry organization.

Second, it relates to the feedback effects from the financial market to the real market. Bond, Edmans, and Goldstein (2012), Ozoguz, Rebello, and Wardlaw (2018), and Foucault and Frésard (2014) document that the investment of a firm is sensitive to its own (or its peers') stock price. In Dessaint, Foucault, Frésard, and Matray (2018), noise in stock prices influences real decisions. It also relates to another feedback literature that features a strategic trader. In Goldstein and Guembel (2008), the feedback effects from the financial market to real investment raises the uninformed speculator's incentive to sell the stock to induce disinvestment, generating a profit on the speculator's short position. Most relevantly, Edmans, Goldstein, and Jiang (2015) study a firm's investment decision in response to trade in the financial market. They are the first finding that feedback effects create the limits to arbitrage on the "sell-side" and affect the investment decision. In their work, a speculator does not sell on bad news since selling activity makes manager dis-invest and thus reduces his profit from selling. Another model where the feedback effects leads to asymmetric trading by strategic investor is Boleslavsky, Kelly, and Taylor (2017). In their work, an authority learns from financial market to guide a bailout, since an intervention erodes the value of private information, informed investors are reluctant to take short positions. Different from their work, we study how the feedback effects affects entrant's entry decision. In our paper, the informed trader refrains from taking a long position to deter the entrant from entering.

Third, it relates to the literature on the limits to arbitrage. Campbell and Kyle (1993) study fundamental risk. The risk that fundamentals may change in the process of pursuing arbitrage. DeLong, Shleifer, Summers, and Waldmann (1990) focus on noise trader risk. They find that mispricing may get worse in the short run, making early liquidation at a loss. As Shleifer and Vishny (1997) note, this can make it hard for investors to finance their bets against mispricing. Other authors (D'Avolio (2002), Gromb and Vayanos (2002), Lamont and Thaler (2003), Jones and Lamont (2002), Nagel (2005), etc.) focus on implementation costs such as the costs of discovering or exploiting mispricing, transactions costs, and short-sale constraints. More recently, Edmans, Goldstein, and Jiang (2015) and Boleslavsky, Kelly, and Taylor (2017) find that the feedback effects generates limits to arbitrage on the short side. However, different from previous works, our paper is the first that finds limits to arbitrage on the long side due to the feedback effect.

Fourth, it relates to the research about how financial variables such as trading volume, the volatility of idiosyncratic returns and the informativeness of stock prices are related to market power. In Stoughton, Wong, and Zechner (2001), consumers infer product quality from stock price, so firms may signal their quality through going public. Gaspar and Massa (2006) argue that competition increases return volatility through raising profit volatility or decreasing uncertainty about the firm's future performance. In Hou and Robinson (2006), either barriers to entry or less innovation enables highly concentrated industries to earn higher risk-adjusted returns. Tookes (2008) predicts that informed traders may have incentives to make information-based trades in the stocks of competitors, especially when private information events occur at individual firms with large market shares. In Irvine and Pontiff (2009), increased competition is a source of increased idiosyncratic volatility. Peress (2010) argues that firms with more market power can pass on shocks to consumers, which encourages stock trading, expedites the capitalization of private information into stock prices, and improves capital allocations.

Finally, the paper also belongs to the large body of research on trading under asymmetric information. To the best of our knowledge, the role of the financial market to the competition of the product market has not been studied in this context. Our paper contributes in particular to policymakers. They indicate that product market deregulation can encourage market competition, but our findings reveal that such reforms should be conducted in combination with reforms of improving the financial market efficiency.

3.2 Model

In this section, we present a model to analyze the learning and entry game. We modify the Kyle (1985) model to consider informed trading in a stock that derived its value from an imperfectly competitive industry. We first analyze firm value and competition in the product market, and then we analyze financial market participants. Finally, we combine these two markets through entrant learning of financial market total trading volume.

3.2.1 Competition in Product Market

In the product market, there is an incumbent. A potential entrant makes its entry decision. The incumbent is a public firm whose shares are traded in the stock market, and the entrant is a private firm. If the entrant does not enter the market, it is a monopoly market with a single producer. Otherwise, it is a standard Cournot duopoly with a single homogeneous good and symmetric constant marginal costs. Given marginal cost $C_i(q_i) = cq_i$ and a linear inverse demand function $p(Q) = S - Q$, where S is affected by the state $\theta \in \{H, L\}$ and $S_H > S_L > c$, Q is total demand. If the entrant does not enter, $Q = q_1$, where q_1 is the optimal quantity of the incumbent. Otherwise, firms simultaneously choose quantities q, where $Q = q_1 + q_2$, and q_2 is the optimal quantity of the entrant. Equilibrium profits are returned to shareholders.

If the entrant does not enter the market, the incumbent maximize its firm value by choosing optimal quantity q_1 in the monopoly market:

$$\max_{\{q_1\}} \quad q_1(S - q_1) - cq_1$$

The first-order condition gives optimal quantity and profit:

$$q_{1\theta}^M = \frac{S_\theta - c}{2}; \qquad V_{1\theta}^M = \frac{(S_\theta - c)^2}{4} \tag{3.1}$$

It is clear from equation 3.1 that high state results in a higher quantity ($q_{1H}^M > q_{1L}^M$) and higher profit ($V_{1H}^M > V_{1L}^M$).

If the entrant enters the market, firms maximize their firm value by choosing optimal quantities q_1 and q_2 in the Cournot duopoly market.

$$\max_{\{q_1,q_2\}} \quad q_i(S - q_1 - q_2) - cq_i$$

The first-order condition gives optimal quantity and profit:

$$q^C_{i\theta} = \frac{S_\theta - c}{3}; \quad V^C_{1\theta} = \frac{(S_\theta - c)^2}{9}; \quad V^C_{2\theta} = \frac{(S_\theta - c)^2}{9} - E \tag{3.2}$$

where E is an exogenous entry cost suffered by the entrant.

Clearly, $V^M_{1H} > V^C_{1L}$ but we can not compare V^M_{1L} and V^C_{1H} without knowing S_H and S_L. If the high state dominates the low state, $V^M_{1H} > V^M_{1L} > V^C_{1H} > V^C_{1L}$. Otherwise, $V^M_{1H} > V^C_{1H} > V^M_{1L} > V^C_{1L}$. We first consider the scenario in which the high state dominates the low state, and then we analyze the second scenario. Indeed, as we will explain later, the two scenarios have the same results.

3.2.2 Trading in Financial Market

In the financial market, there are three types of equity market participants: an insider who may arrive in the financial market with a probability $0 < \beta < 1$; an uninformed noise trader; and a risk-neutral market maker.

The insider is risk-neutral. Whether she presents or not is unknown to anyone else. We assume that if the insider presents, she receives a perfect private signal regarding the state of the product market. She acts as an inter-temporal monopolist in the stock market. If the insider observes a high and low state, we describe the insider "positively" and "negatively" informed insider when she respectively. Similar to Glosten and Milgrom (1985), the insider endogenously trades unit share $s \in \{-1, 0, 1\}$, and she suffers a transaction cost k by selling one share or buying one share. The noise trader is an uninformed liquidity trader whose trades are unrelated to the realization of the true state of the industry. The noise trader trades $z \in \{-1, 0, 1\}$ with equal probability. The insider and noise trader trade simultaneously. The competitive market maker collects total trading volume $X \in \{-2, -1, 0, 1, 2\}$ from the insider and the noise trader, but it cannot identify traders. It absorbs any excess demand or supply out of its inventory and sets a price by the pricing function $P(X) = \mathbb{E}(V|X)$ and earns on average zero profit.

3.2.3 Timing

Figure 3.1 presents the timeline of the events. At $t = 0$, nature chooses the state of the incumbent's industry, a potential entrant appears in the product market, a risk neutral speculator may present in the financial market. If she presents, she receives a perfect private signal regarding incumbent's industry. If it is a high state, the signal reveals a high state; if it is a low state, the signal reveals a low state. At $t = 1$, trading occurs in the stock market. At $t = 2$, the entrant makes entry decision after learning from the stock market. If it enters, it suffers an entry cost E. At $t = 3$, production begins. Profits are realized in the product market and returned to shareholders.

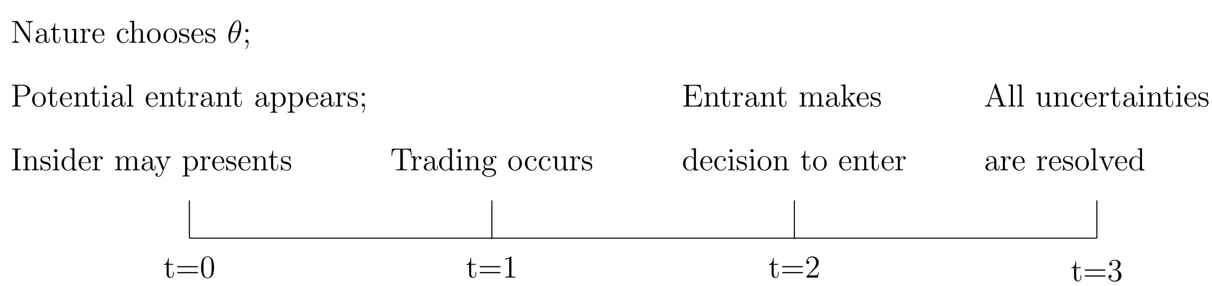

Figure 3.1: Timeline of Events

3.2.4 Information Distribution and Learning

In our model, the insider has exclusive information that the entrant does not have. If the entrant does not learn from the stock market for some exogenous reasons, it makes entry decisions based on its prior α_0 and the entry cost E. At $t = 2$, the entrant makes its entry decision when learning from the stock market.

The setting subsumes the traditional belief that the entrant has internal information and the insider's information is a subset of that of the entrant. The old view corresponds to an empty insider unique information set. The key insight of our setting is that the optimal real decision depends on both internal information to the firm and external information, so the entrant has incentives to learn as long as the insider's unique information set is not empty.

3.3 Equilibrium Characterization

The insider's strategy is a mapping from the firm fundamentals θ to the trading strategy in the stock market. The entrant's strategy is a mapping from the total trading volume to its entry decision. The market maker's pricing strategy is a mapping from the total trading volume to a price. We are interested in perfect Bayesian equilibrium.

Definition 3.1. The insider's trading strategy $s^* \to \{-1, 0, 1\}$, the entrant's entry decision $d^* \to \{\text{Enter, Not Enter}\}$, and the market maker's pricing strategy $P^*(X)$ constitute a perfect Bayesian equilibrium if:

1. For the insider, s^* maximizes her expected gross gain $s(V - P) - |s|k$, given the market maker's price setting rule and the entrant's entry decision;

2. For the entrant, $d^*(X)$ maximizes the expected firm value, given the information in

the stock market and other agents' strategies.

3. For market maker, the price $P^*(X) = \mathbb{E}(v|X)$ that results in zero profit for each $X \in \{-2, -1, 0, 1, 2\}$, given all other agents' strategies.

4. The entrant and the market maker update their beliefs by Bayes' rule after observing the total trading volume in the stock market.

3.3.1 Trading Strategies

As we will show, depending on different values of transaction cost k, four pure-strategy equilibrium outcomes can raise:

1. No Trade Equilibrium NT: the insider does not trade.

2. Trade Equilibrium T: the insider buys when she knows $\theta = H$ and sells when she knows $\theta = L$.

3. Buy-Not Sell Equilibrium BNS: the insider buys when she knows $\theta = H$ and does not trade when she knows $\theta = L$.

4. Sell-Not Buy Equilibrium SNB: the insider does not trade when she knows $\theta = H$ and sells when she knows $\theta = L$.

3.3.2 Entry Decision

Without learning, the entrant makes the entry decision based on its prior α_0 and the entry cost E. In the presence of the financial market, the entrant makes the entry decision based on its prior α_0, the trading volume X in the financial market, and the entry cost E. We first consider how does the entrant update its posterior α when learning from the financial market, then we discuss how does the entrant make its entry decision with and without learning.

Learning With learning, the entrant updates its posterior belief α based on its prior belief α_0 about the state of the industry and the trading volume X. For simplicity, we assume that $\alpha_0 = \frac{1}{2}$, which means the entrant believes that the industry is in the high state and the low state with equal probability. We calculate its posteriors α by Bayes' rule under different trading strategies.

No Trade Equilibrium NT: This equilibrium is straightforward because there is nothing to learn from the stock market based on this trading strategy. As a result, the posterior should be identical to the prior. Since $X = -2$ and $X = 2$ are off equilibrium path with posteriors 0 and 1.

Trade Equilibrium T: For the trade equilibrium, the insider buys when she receives positive information, and sells when she receives negative information. Thus $X \in \{-2, -1, 0, 1, 2\}$ are on the equilibrium path. The posteriors can be calculated by Bayes' rule.

Under the *T*, *X=-1* includes three situations. First, the industry is in the high state H, the insider does not presents in the financial market with probability $1 - \beta$, and the noise trader sells. The probability of this situation is $\frac{1}{2} * (1 - \beta) * \frac{1}{3}$. Second, the industry is in the low state L, the insider does not present in the financial market with probability $1 - \beta$, and the noise trader sells. The probability of this situation is $\frac{1}{2} * (1 - \beta) * \frac{1}{3}$. Third, the industry is in the low state L, the insider presents in the financial market with probability β and sells, and the noise trader does not trade. The probability of this situation is $\frac{1}{2} * \beta * \frac{1}{3}$. Based on these three situations, the probability that *X=-1* and the industry is on the high state H is $P(X = -1|H) * P(H) = \frac{1}{2} * (1 - \beta) * \frac{1}{3}$, the probability that *X=-1* and the industry is on the low state L is $\frac{1}{2} * \left(\beta * \frac{1}{3} + (1 - \beta) * \frac{1}{3}\right)$, thus the probability that *X=-1* is $\frac{1}{2}\left((1-\beta)*\frac{1}{3}+\beta*\frac{1}{3}+(1-\beta)*\frac{1}{3}\right)$. We can solve for the probability that the industry is in the high state H conditional on observing trading volume *X=-1*: $\alpha(-1) = Pr(H|X = -1) = \frac{1-\beta}{2-\beta}$. Following the same logic, we can solve for the probability that the industry is in the high state H conditional on observing trading volume *X=0*: $\alpha(0) = Pr(H|X = 0) = 1/2$, and the probability that the industry is in the high state H conditional on observing trading volume *X=1*: $\alpha(1) = Pr(H|X = 1) = \frac{1}{2-\beta}$. The probability that the industry is in the high state H conditional on observing trading volume *X=2* and *X=-2* are respectively *1* and *0*.

Similarly, we can solve for the posteriors under other trading strategies *Partial Trade Equilibrium SNB,* and *Partial Trade Equilibrium BNS.* Table 3.1 summarizes the posteriors after learning given insider's different trading strategies.

Strategy \ X	-2	-1	0	1	2
NT	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
SNB	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2-\beta}$	1
BNS	0	$\frac{1-\beta}{2-\beta}$	$\frac{1}{2}$	$\frac{1}{2}$	1
T	0	$\frac{1-\beta}{2-\beta}$	$\frac{1}{2}$	$\frac{1}{2-\beta}$	1

Table 3.1: Posterior Belief of the Entrant

Table 3.1 shows that when $X = 1$, the posterior is higher under SNB and T than NT and BNS. This is because the SNB and T exclude the case in which the insider presents and receives negative information revealing the low state. Similarly, when $X = -1$, the posterior is higher under NT and SNB than under BNS and T. This is because the BNS and T exclude the case in which the insider presents and receives positive information revealing the high state.

Entry Decision with Learning Let α_I denote the entrant's posterior of the high state that makes no difference between entering and not entering. Since the profit of not entering is zero, the following condition must hold.

$$\alpha_I V_{2H}^C + (1-\alpha_I)V_{2L}^C - E = 0$$

This condition implies $\alpha_I = \frac{E - V_{1L}^C}{V_{1H}^C - V_{1L}^C}$. The entrant makes entry decision based on the following criteria:

$$\textit{Entry decision } d(X) = \begin{cases} \textit{Enter } E, & \textit{if } \alpha > \alpha_I; \\ \textit{Not Enter } NE, & \textit{if } \alpha \leq \alpha_I. \end{cases} \tag{3.3}$$

Entry Decision without Learning Given that the entrant's prior of the high state H is α_0, which is common knowledge, without learning, the entrant's expected payoff from entering is

$$\alpha_0 V_{2H}^C + (1-\alpha_0)V_{2L}^C$$

Because of the entry cost E, its profit after entry is

$$\alpha_0 V_{2H}^C + (1-\alpha_0)V_{2L}^C - E$$

If $E \geq \alpha_0 V_{2H}^C + (1-\alpha_0)V_{2L}^C$, entering generates a negative profit, and thus the entrant *does not enter* without learning. Otherwise, it *enters*.

3.3.3 Price Setting Strategies

From the previous analysis, we have already known the incumbent's firm value with and without competition. If the entrant does not enter the market, the firm value of the incumbent is $V_{1\theta}^M = \frac{(S_\theta - c)^2}{4}$; if the entrant enters, the firm value of the incumbent is $V_{1\theta}^C = \frac{(S_\theta - c)^2}{9}$, where $\theta \in \{H, L\}$. The market maker sets prices such that $\mathbb{E}(X(P-V)|X) = 0$, so the price function is $P = \mathbb{E}(V|X)$, then we calculate prices P given the insider's different trading strategies.

No Trade Equilibrium NT: For the no trade equilibrium, the trading volume $X \in \{-1, 0, 1\}$ are on the equilibrium path with posteriors $\alpha \in \{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$. If the entrant is pessimistic and does not enter the market without learning, it does not enter when its posterior $\alpha = \alpha_0 = 1/2$. Hence, the entry decisions are $d^* \in \{NE, NE, NE\}$. However, if the entrant is optimistic and enters without learning, it enters the market when its posterior $\alpha = \alpha_0 = 1/2$. Hence, the entry decisions are $d^* \in \{E, E, E\}$. Trading volume $X \in \{-2, 2\}$ are off equilibrium path with posteriors $\alpha \in \{0, 1\}$, and entry decisions are $d^* \in \{NE, E\}$. Based on this information, we solve for the equilibrium price corresponds to trading volumes.

If the entrant enters the market without learning, when trading volume X=-2, the posterior $\alpha = 0$, the entry decision $d = NE$, and then the price $P = 0 * V_{1H}^M + 1 * V_{1L}^M = V_{1L}^M$; when trading volume $X \in \{-1, 0, 1\}$, the posterior $\alpha \in \{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$, the entry decision $d^* \in \{NE, NE, NE\}$, and then the price $P = \frac{1}{2} * V_{1H}^M + \frac{1}{2} * V_{1L}^M$; when trading volume X=2, the posterior $\alpha = 1$, the entry decision $d = E$, and then the price $P = 1 * V_{1H}^C + 0 * V_{1L}^C = V_{1H}^C$.

If the entrant does not enter the market without learning, when trading volume X=-2, the posterior $\alpha = 0$, the entry decision $d = NE$, and then the price $P = 0 * V_{1H}^M + 1 * V_{1L}^M = V_{1L}^M$; when trading volume $X \in \{-1, 0, 1\}$, the posterior $\alpha \in \{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\}$, the entry decision $d^* \in \{E, E, E\}$, and then the price $P = \frac{1}{2} * V_{1H}^C + \frac{1}{2} * V_{1L}^C$; when trading volume X=2, the posterior $\alpha = 1$, the entry decision $d = E$, and then the price $P = 1 * V_{1H}^C + 0 * V_{1L}^C = V_{1H}^C$.

Similarly, we can solve for the prices under other trading strategies *Partial Trade Equilibrium SNB, Partial Trade Equilibrium BNS,* and *Trade Equilibrium T.* Table 3.2 summarizes the prices after learning given the insider's different trading strategies.

w/o Learning	X / S.	-2	-1	0	1	2
Not Enter	NT	V_{1L}^M	$\frac{1}{2}(V_{1L}^M + V_{1H}^M)$	$\frac{1}{2}(V_{1L}^M + V_{1H}^M)$	$\frac{1}{2}(V_{1L}^M + V_{1H}^M)$	V_{1H}^C
	SNB	V_{1L}^M	$\frac{1}{2}(V_{1L}^M + V_{1H}^M)$	$\frac{1}{2}(V_{1L}^M + V_{1H}^M)$	$\frac{1}{2-\beta}V_{1H}^M + \frac{1-\beta}{2-\beta}V_{1L}^M$	V_{1H}^C
	BNS	V_{1L}^M	$\frac{1-\beta}{2-\beta}V_{1H}^M + \frac{1}{2-\beta}V_{1L}^M$	$\frac{1}{2}(V_{1L}^M + V_{1H}^M)$	$\frac{1}{2}(V_{1L}^M + V_{1H}^M)$	V_{1H}^C
	T	V_{1L}^M	$\frac{1-\beta}{2-\beta}V_{1H}^M + \frac{1}{2-\beta}V_{1L}^M$	$\frac{1}{2}(V_{1L}^M + V_{1H}^M)$	$\frac{1}{2-\beta}V_{1H}^M + \frac{1-\beta}{2-\beta}V_{1L}^M$	V_{1H}^C
Enter	NT	V_{1L}^M	$\frac{1}{2}(V_{1L}^C + V_{1H}^C)$	$\frac{1}{2}(V_{1L}^C + V_{1H}^C)$	$\frac{1}{2}(V_{1L}^C + V_{1H}^C)$	V_{1H}^C
	SNB	V_{1L}^M	$\frac{1}{2}(V_{1L}^C + V_{1H}^C)$	$\frac{1}{2}(V_{1L}^C + V_{1H}^C)$	$\frac{1}{2-\beta}V_{1H}^C + \frac{1-\beta}{2-\beta}V_{1L}^C$	V_{1H}^C
	BNS	V_{1L}^M	$\frac{1-\beta}{2-\beta}V_{1H}^C + \frac{1}{2-\beta}V_{1L}^C$	$\frac{1}{2}(V_{1L}^C + V_{1H}^C)$	$\frac{1}{2}(V_{1L}^C + V_{1H}^C)$	V_{1H}^C
	T	V_{1L}^M	$\frac{1-\beta}{2-\beta}V_{1H}^C + \frac{1}{2-\beta}V_{1L}^C$	$\frac{1}{2}(V_{1L}^C + V_{1H}^C)$	$\frac{1}{2-\beta}V_{1H}^C + \frac{1-\beta}{2-\beta}V_{1L}^C$	V_{1H}^C

Table 3.2: Price after Learning.

3.3.4 Benchmark Setting without Feedback Effects

The entrant may or may not enter without learning. We analyze both cases and summarize the equilibrium of the benchmark settings in the following lemma.

Lemma 3.1. If the entrant does not enter without learning, and posteriors $\frac{1}{2} < \frac{1}{2-\beta} \leq \alpha_I$, there is no feedback effects, but there exist transaction costs $k_{NF} < k_{NT}$(defined in the analysis) such that insider trading strategy has the following pure-strategy equilibria:

1. The only pure-strategy equilibrium is NT when $k \geq k_{NT}$.
2. The two pure strategy equilibria are SNB and BNS when $k_{NF} \leq k < k_{NT}$.
3. The only pure strategy equilibrium is T when $k < k_{NF}$.

From the previous analysis, we know that after learning from stock m arket, the entrant may have posteriors $\alpha \in \{0, \frac{1-\beta}{2-\beta}, \frac{1}{2}, \frac{1}{2-\beta}, 1\}$. Intuitively, if it does not enter the market without learning, it also does not enter the market after learning when its posterior $\alpha = \alpha_0 = 1/2$, where α_0 is its prior. When its posterior $\alpha = \frac{1-\beta}{2-\beta} < \alpha_0 = \frac{1}{2}$, it does not enter the market. It also does not enter when its posterior $\alpha = \frac{1}{2-\beta} > \alpha_0 = \frac{1}{2}$, but $\alpha = \frac{1}{2-\beta} \leq \alpha_I$, where α_I is the posterior that the entrant is indifferent between entering and not entering. The only case that the entrant enters the market is when its posterior $\alpha = 1$, which means the H state is fully revealed. Thus when its posteriors $\frac{1}{2} < \frac{1}{2-\beta} \leq \alpha_I$, there is no feedback effects. Information in the financial market can not change the entrant's entry decision *NE*.

Now turn to analyze the conditions for the equilibrium to be sustainable, we focus on the insider's expected payoffs. Under *the pure-strategy equilibrium NT*, the insider's expected gross gain is 0 when she receives positive information. If she deviates to buying, trading volume $X = 2$ with probability $p = \frac{1}{3}$, and she is fully revealed with payoff 0; trading volume $X \in \{0, 1\}$ with probability $p = \frac{2}{3}$, she pays the stock price $\frac{1}{2}(V_{1H}^M + V_{1L}^M)$ and receives V_{1H}^M per share, thus her expected gross gain from deviating to buying is given by: $\frac{1}{3}(V_{1H}^M - V_{1L}^M) \equiv k_{NT}$. Following the same logic, the insider's expected gross gain is 0 when she receives negative information. If she deviates to selling, her expected gross gain from deviating is also k_{NT}. Thus, if and only if $k \geq k_{NT}$, the no trade equilibrium is sustainable.

Similarly, we can find conditions for *the two pure strategy equilibria SNB and BNS*, and *the pure-strategy equilibrium T* to be sustainable. There exist transaction costs $k_{NF} < k_{NT}$, where $k_{NF} \equiv \frac{1}{3} * \frac{1}{2} * (V_{1H}^M - V_{1L}^M) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$, such that The two pure strategy equilibria SNB and BNS exist when $k_{NF} \leq k < k_{NT}$, and the only pure strategy equilibrium T is sustainable when $k < k_{NF}$.

Lemma 1 gives the equilibria when the entrant does not enter the market without learning and when there is no feedback effects. We now turn to consider the equilibria when the entrant enters the market without learning and when there is no feedback effects in the following ***Lemma 2***.

Lemma 3.2. If the entrant is optimistic and enters without learning, and posteriors $0 <$

$\alpha_I < \frac{1-\beta}{2-\beta} < \frac{1}{2}$, there is no feedback effects, but there exist transaction costs $k^C_{SNB} < k^C_{NT}$(defined in the analysis) such that insider trading strategy has the following pure-strategy equilibria:

1. The only pure-strategy equilibrium is NT when $k \geq k^C_{NT}$.

2. The two pure strategy equilibria are SNB and BNS when $k^C_{SNB} \leq k < k^C_{NT}$.

3. The only pure strategy equilibrium is T when $k < k^C_{SNB}$.

The entrant may have posteriors $\alpha \in \{0, \frac{1-\beta}{2-\beta}, \frac{1}{2}, \frac{1}{2-\beta}, 1\}$ after learning. Intuitively, if it enters the market optimistically without learning, it also enters the market after learning when its posterior $\alpha = \alpha_0 = 1/2$, where α_0 is its prior. It enters when its posterior $\alpha = \frac{1-\beta}{2-\beta} < \alpha_0 = \frac{1}{2}$, but $\alpha = \frac{1-\beta}{2-\beta} > \alpha_I$, where α_I is the posterior that the entrant is indifferent between entering and not entering. The only case that the entrant does not enter the market is when its posterior $\alpha = 0$, which means the L state is fully revealed. Thus when its posteriors $0 < \alpha_I < \frac{1-\beta}{2-\beta} < \frac{1}{2}$, there is no feedback effects. Information in the financial market can not change the entrant's entry decision E.

Similar to previous analysis, we can find conditions for *the pure-strategy equilibrium* NT, *the two pure strategy equilibria* SNB *and* BNS, and *the pure-strategy equilibrium* T to be sustainable. There exist transaction costs $k^C_{SNB} < k^C_{NT}$, where $k^C_{SNB} \equiv \frac{1}{3} * \frac{1}{2} * (V^C_{1H} - V^C_{1L}) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V^C_{1H} - V^C_{1L})$, and $k^C_{NT} \equiv \frac{1}{3}(V^C_{1H} - V^C_{1L})$ such that *the only pure strategy equilibrium* NT is sustainable when $k \geq k^C_{NT}$, *the two pure strategy equilibria* SNB *and* BNS exist when $k^C_{SNB} \leq k < k^C_{NT}$, and *the only pure strategy equilibrium* T is sustainable when $k < k^C_{SNB}$.

Lemma 1 and ***Lemma 2*** analyze the equilibria when the entrant is respectively pessimistic and optimistic ex-ante and when there is no feedback effects. Now we analyze the equilibria when there is feedback effects in the following subsection.

3.3.5 Product Market Equilibrium and Financial Market Equilibrium

After learning from the financial market, the entrant alters its entry decision. We summarize the results in the following propositions.

Proposition 3.1. If the entrant does not enter without learning, and posteriors $\frac{1}{2} \leq \alpha_I < \frac{1}{2-\beta}$, there is feedback effect, and there exist transaction costs $k_{SNB} < k_{NF} < k_{NT}$(defined in the analysis) such that insider trading strategy has the following pure-strategy equilibria:

1. The only pure-strategy equilibrium is NT when $k \geq k_{NT}$.
2. The two pure-strategy equilibria are SNB and BNS when $k_{NF} \leq k < k_{NT}$.
3. The only pure-strategy equilibrium is SNB when $k_{SNB} < k < k_{NF}$.
4. The only pure strategy equilibrium is T when $k < k_{SNB}$.

The entrant may have posteriors $\alpha \in \{0, \frac{1-\beta}{2-\beta}, \frac{1}{2}, \frac{1}{2-\beta}, 1\}$ after learning. Intuitively, if it does not enter the market without learning, it also does not enter the market after learning when its posterior $\alpha = \alpha_0 = 1/2$, where α_0 is its prior. When its posterior $\alpha = \frac{1-\beta}{2-\beta} < \alpha_0 = \frac{1}{2}$, it does not enter the market. However, it enters when its posterior $\alpha = \frac{1}{2-\beta} > \alpha_0 = \frac{1}{2}$, and $\alpha = \frac{1}{2-\beta} > \alpha_I$, where α_I is the posterior that the entrant is indifferent between entering and not entering. It also enters the market when its posterior $\alpha = 1$, which means the H state is fully revealed. Thus when its posteriors $\frac{1}{2} \leq \alpha_I < \frac{1}{2-\beta}$, there is feedback effects. Information in the financial market changes the entrant's entry decision from NE to E.

Similar to previous analysis, we can find conditions for *the pure-strategy equilibrium* NT, *the two pure strategy equilibria* SNB *and* BNS and *the pure-strategy equilibrium* T to be sustainable. There exist transaction costs $k_{SNB} < k_{NF} < k_{NT}$, where $k_{SNB} \equiv \frac{1}{3} * \frac{1}{2} * (V_{1H}^M - V_{1L}^M) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C)$, $k_{NF} \equiv \frac{1}{3} * \frac{1}{2} * (V_{1H}^M - V_{1L}^M) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$ and $k_{NT} \equiv \frac{1}{3}(V_{1H}^M - V_{1L}^M)$ such that *the only pure strategy equilibrium* NT is sustainable when $k \geq k_{NT}$, *the two pure strategy equilibria* SNB *and* BNS exist when $k_{NF} \leq k < k_{NT}$, *the only pure strategy equilibrium* SNB is sustainable when $k_{SNB} < k < k_{NF}$, and *the only pure strategy equilibrium* T is sustainable when $k < k_{SNB}$.

Proposition 1 gives the equilibria when the entrant does not enter the market without learning and when there is feedback effects. We now turn to consider the equilibria when the entrant enters the market without learning and when there is feedback effects in the following ***Proposition 2***.

Proposition 3.2. If the entrant is optimistic and enters without learning, and $\frac{1-\beta}{2-\beta} < \alpha_I < \frac{1}{2}$, there is feedback effects, there exist transaction costs $k_{SNB}^C < k_{NF}^C$ and $k_{SNB}^C < k_{NT}^C$(defined in the analysis) such that the insider trading strategy has the following pure-strategy equilibria:

1. The only pure-strategy equilibrium is NT when $k \geq k_{NT}^C$.
2. The pure-strategy equilibrium is SNB when $k_{SNB}^C \leq k < k_{NT}^C$.
3. The only pure strategy equilibrium is T when $k < k_{SNB}^C$.
4. If $k_{NF}^C < k_{NT}^C$, BNS is also a pure-strategy equilibrium; Otherwise, the pure-strategy equilibrium BNS does not exist.

If the entrant enters the market optimistically without learning, it also enters the market after learning when its posterior $\alpha = \alpha_0 = 1/2$, where α_0 is its prior. It enters when its posterior $\alpha = \frac{1}{2-\beta} > \alpha_0 = \frac{1}{2}$, but does not enter when $\alpha = \frac{1-\beta}{2-\beta} < \alpha_I$, where α_I is the posterior that the entrant is indifferent between entering and not entering. It also does not enter the market when its posterior $\alpha = 0$, which means the L state is fully revealed. Thus when its posteriors $\frac{1-\beta}{2-\beta} < \alpha_I < \frac{1}{2}$, there is feedback effects. Information in the financial market changes the entrant's entry decision from E to NE.

Similar to the previous analysis, we can find conditions for *the pure-strategy equilibrium NT*, *the pure strategy equilibrium SNB*, *the pure-strategy equilibrium T* to be sustainable. In particular, as we will show if $1 > \beta > \frac{2\left[V_{1H}^M - V_{1L}^M - (V_{1H}^C - V_{1L}^C)\right]}{2(V_{1H}^M - V_{1L}^M) - (V_{1H}^C - V_{1L}^C)}$, we can also find conditions for the *the pure strategy equilibrium BNS* to be sustainable. There exist transaction costs $k_{SNB}^C < k_{NT}^C$ and $k_{SNB}^C < k_{NF}^C$, where $k_{SNB}^C \equiv \frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C)$, $k_{NF}^C \equiv \frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$ and $k_{NT}^C \equiv \frac{1}{3}(V_{1H}^C - V_{1L}^C)$ such that *the only pure strategy equilibrium NT* is sustainable when $k \geq k_{NT}^C$, *the pure strategy equilibrium SNB* is sustainable when $k_{SNB}^C \leq k < k_{NT}^C$, and *the only pure strategy equilibrium T* is sustainable when $k < k_{SNB}^C$. However, the existence of *the pure strategy equilibrium BNS* requires the condition $k_{NF}^C < k_{NT}^C$ to be satisfied, which gives the range of β: $1 > \beta > \frac{2\left[V_{1H}^M - V_{1L}^M - (V_{1H}^C - V_{1L}^C)\right]}{2(V_{1H}^M - V_{1L}^M) - (V_{1H}^C - V_{1L}^C)}$. If $\frac{2\left[V_{1H}^M - V_{1L}^M - (V_{1H}^C - V_{1L}^C)\right]}{2(V_{1H}^M - V_{1L}^M) - (V_{1H}^C - V_{1L}^C)} > \beta > 0$, *the pure strategy equilibrium BNS* does not exist.

3.3.6 Discussion on the Limits to Arbitrage

Based on the above analysis, we can find two sources of the limits to arbitrage no matter whether there is feedback effects or not.

Transaction Cost As transaction cost k increases, insider's trading strategy moves from *Trade* to *Sell-Not Buy* or *Buy-Not Sell*, and finally move to *No Trade.* The transaction cost impedes insider's trading on her private information. When the transaction cost is sufficiently small, the insider earns profits from trading her private information on both directions, so she always trades; when the transaction cost is sufficiently high, the insider suffers a loss from trading her private information on both directions, so she does not trade; when transaction cost is in between, the insider only trades one side of her private information.

Price Impact The price impact makes the insider trade one type of information but not the other. For instance, in *the pure-strategy equilibrium* SNB, if the market maker believes that the insider does not trade when she receives positive information, after observing a positive market order flow, the market maker knows that the state is good, and then sets a higher price to reflect the true fundamental, and the insider does not buy.

Besides, similar to Edmans et al. (2015), we also find that the feedback effects creates a source of the limits to arbitrage:

Feedback Effect By comparing *Lemma 1* and *Proposition 1*, we can find that if the entrant does not enter the market without learning, in the range of transaction cost $k_{SNB} < k < k_{NF}$, the feedback effects changes the insider's trading strategy from T to SNB, which means that the feedback effects leads to asymmetric trading. Similarly, by comparing *Lemma 2* and *Proposition 2*, if the entrant enters the market optimistically without learning, in the range of transaction cost $k^C_{SNB} < k < k^C_{NF}$, the feedback effects leads to asymmetric trading in which selling is more common than buying.

3.3.7 Equilibrium when Firm Value is Non-Monotonic in States

In the second scenario, $V^M_{1H} > V^M_{1L} > V^C_{1H} > V^C_{1L}$, the firm value may be higher in state L. In this case, entry not only mitigates the firm value in the low state but is sufficiently powerful to overturn the value in the high state. As firm value is higher in the state L, a positively-informed insider may find it optimal to sell to pretend it is in L state, but a negatively-informed insider does not have any incentive to pretend it is in the H state to avoid the entrant enter the market. Hence, there seems to be six possible pure-strategy equilibria. Except for the four equilibria in the first scenario $V^M_{1H} > V^C_{1H} > V^M_{1L} > V^C_{1L}$ that the high state dominates the low state, the insider may sell when $\theta = H$, so the insider may both sell when the state is H and L, or the insider sells when $\theta = H$ and not trade when $\theta = L$. We discuss the equilibrium in these situations.

Proposition 3.3. There is no equilibrium with trading against information. The trading game has no pure-strategy equilibrium where the insider sells when she knows $\theta = H$.

The reason why the positively-informed insider never sells in equilibrium is that she cannot gain from selling when the market maker and the entrant believe that she sells in the stock market. However, she still has incentives to deviate to selling in any of the four equilibria. When she sells, she may mislead the market maker and the entrant to believe that the negatively-informed insider is present, which causes the entrant does not enter the market. This decision increases the firm value of the incumbent such that the insider earns a positive profit.

However, if the insider sells when she knows that $\theta = H$, then $X \in \{-2, -1, 0\}$. In each of these nodes, the price will incorporate the possibility that $\theta = L$. As the firm value is lower under $\theta = L$ than under $\theta = H$, the price is always smaller than the firm value of the incumbent. Therefore, the insider's expected payoff will be negative if she sells when $\theta = H$, which means she suffers a loss. Thus both the equilibrium that the insider sells when $\theta = H$ and $\theta = L$, and the equilibrium that the insider sells when $\theta = H$ and does not trade when $\theta = L$ are not sustainable.

3.4 Real Effects

In this section, we analyze the real effects of the entry game in the presence of the financial market. First, we analyze how financial market efficiency affects real market efficiency by studying the relationship between the transaction costs and the entry probability conditional on different posteriors. Second, we analyze how the ex-post entry probability changes with the market uncertainty given the transaction costs. Last, we investigate the relationship between the entrant's entry decision and the barriers to entry by studying the relationship between ex-ante entry probability and the barriers to entry.

3.4.1 Trading and Product Market Efficiency

From the previous analysis, we know that there may be feedback effects if the entrant learns from the financial market while making its entry decision. As transaction cost reduces, the insider trades more such that the financial market becomes more efficient. If the entrant does not enter without learning, the feedback effects changes its entry decision from *Not Enter NE* to *Enter E*, which creates the competition in the real market and makes the real market more efficient; if the entrant enters without learning, the feedback effects changes its entry decision from *Enter E* to *Not Enter NE*, which impedes the competition in the real market and makes the real market less efficient. We analyze how financial market efficiency affects real market efficiency by studying the relation between the transaction costs and the entry probability conditional upon different posteriors.

Entry Probability Table 3.3 gives the entry probabilities corresponds to the trading strategies under different posteriors.

Posteriors	No Trade	SNB	BNS	Trade
$\frac{1}{2} < \frac{1}{2-\beta} \leq \alpha_I$	0	0	$\frac{1}{6}\beta$	$\frac{1}{6}\beta$
$\frac{1}{2} \leq \alpha_I < \frac{1}{2-\beta}$	0	$\frac{1}{6}(2-\beta)$	$\frac{1}{6}\beta$	$\frac{1}{3}$
$\frac{1-\beta}{2-\beta} < \alpha_I < \frac{1}{2}$	1	$1-\frac{\beta}{6}$	$\frac{1}{6}(2+\beta)$	$\frac{2}{3}$
$0 < \alpha_I < \frac{1-\beta}{2-\beta} < \frac{1}{2}$	1	$1-\frac{\beta}{6}$	1	$1-\frac{\beta}{6}$

Table 3.3: Conditional Entry Probability

If posteriors $\frac{1}{2} < \frac{1}{2-\beta} \leq \alpha_I$, entrant does not enter under strategies NT and SNB. Under strategy BNS, it enters when $X = 2$. This is the case when the state is high, the insider presents in the financial market and buys one share, and noise trader buys one share, so the conditional probability is $\frac{1}{2} * \beta * \frac{1}{3} = \frac{\beta}{6}$. Under strategy T, entrant enters when $X = 2$. This is the case when the state is high, insider presents in the financial market and buys one share, and noise trader buys one share, so the conditional probability is: $\frac{1}{2} * \beta * \frac{1}{3} = \frac{\beta}{6}$.

Proposition 3.4. When posteriors $\frac{1}{2} < \alpha_I < 1$, the entry probability may increase as transaction cost reduces. When posteriors $0 < \alpha_I < \frac{1}{2}$, the entry probability may decrease as transaction cost reduces.

Figure 3.2 and Figure 3.3 plot the relationship between transaction cost and entry probability when the entrant does not enter without learning and posteriors $\frac{1}{2} < \alpha_I < 1$, Figure 3.4 and Figure 3.5 plot the relationship between transaction cost and entry probability when the entrant enters without learning and posteriors $0 < \alpha_I < \frac{1}{2}$. We assume values for different variables $V_{1H}^M = 100$, $V_{1H}^C = 80$, $V_{1L}^M = 70$, $V_{1L}^C = 65$ and $\beta = 0.4$.

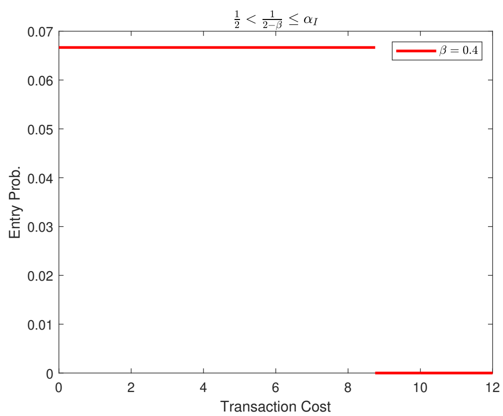

Figure 3.2: Pessimistic: No Feedback Effects

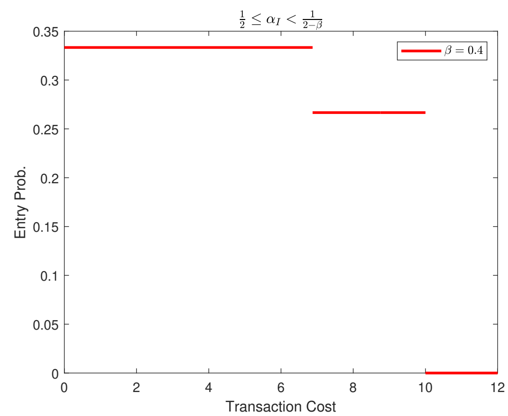

Figure 3.3: Pessimistic: Feedback Effects

Seeing from Figure 3.2 and Figure 3.3, if the entrant is pessimistic ex-ante, when transaction cost is extremely high, the insider does not trade on her private information, so that the entrant learns nothing from the financial market and does not enter. As transaction cost reduces, the insider starts to trade, and then positive information may be revealed to the entrant. Thus, entry probability may increase. As transaction cost reduces a lot, since more positive information is revealed to the entrant, the entry probability may increase even more. In these cases, a reduction in transaction cost improves the financial market efficiency, leading to higher competition and real market efficiency.

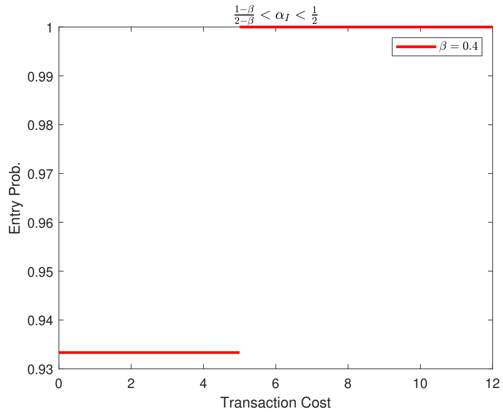

Figure 3.4: Optimistic: No Feedback Effects

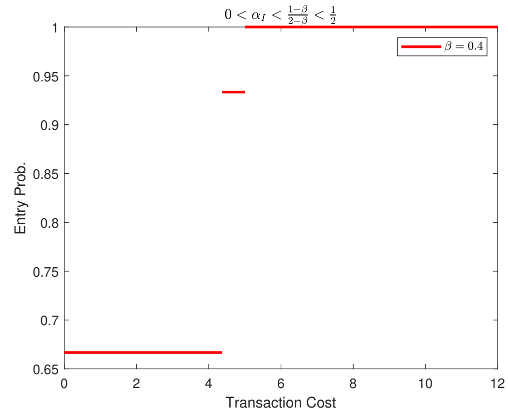

Figure 3.5: Optimistic: Feedback Effects

Seeing from Figure 3.4 and Figure 3.5, if the entrant is optimistic and enters without learning, when transaction cost is extremely high, the insider does not trade on her private information, the entrant learns nothing from financial market and always enters the market. As transaction cost reduces, the insider may start to trade, and then negative information may be revealed to the entrant. Thus, the entry probability may decrease. As transaction cost reduces a lot, since more negative information is revealed to the entrant, the entry probability may decrease even more. In these cases, a reduction in transaction cost improves the financial market efficiency, leading to lower competition and real market efficiency.

3.4.2 Market Uncertainty and Entry Probability

The relation between the ex-post entry probability and trading friction varies with economic and informational conditions that the insider faces. The insider is more likely to enter the financial market when she is expected to have more information that the entrant does not know. Since the insider receives perfect private information that fully reveals the state of the industry, she has higher incentives to present in the financial market when the product market is more uncertain. We investigate how the entry decision changes with the changes of market uncertainty. Since the insider is more likely to present in the financial market with probability β, β may represent the product market uncertainty, the higher the β, the more uncertain the product market is. We analyze how the ex-post entry probability changes with the market uncertainty given the transaction costs.

Proposition 3.5. The relation between market uncertainty and ex-post entry probability depends on the trading frictions. Market uncertainty may create the limits to arbitrage on the "buy side".

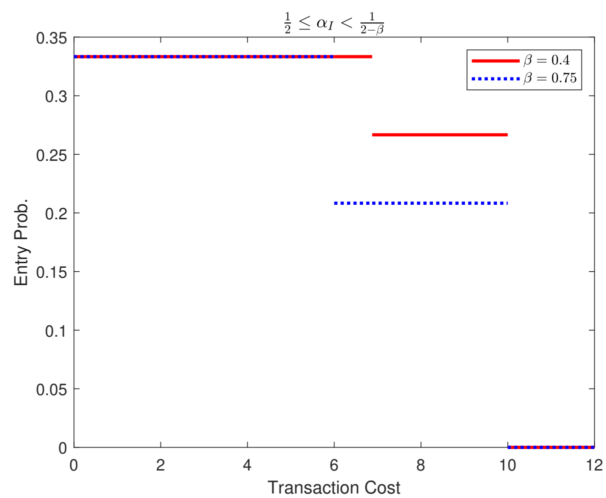

Figure 3.6: Pessimistic Entrant: Entry Probability and Entry Barriers

Figure 3.6 shows the impact of product market uncertainty on the relation between ex-post entry probability and transaction cost when the entrant does not enter the market without learning. When the entrant does not enter the market without learning, if the insider does not trade her private information on both directions, the entrant cannot learn from the financial market, it makes entry decision based on its prior belief and entry cost, and it does not enter the market; if the insider only trades on her negative information, the entrant enters the market when the total trading volume $X = 1$; if the insider trades on both directions of her private information, the entrant enters the market when the total trading volume $X = \{1, 2\}$.

When transaction cost is sufficiently high, the insider does not trade on information regardless of market uncertainty, and the entrant does not enter the market. When transaction cost decreases, the insider starts to trade on negative information, and the entrant enters the market when it observes positive trading volume. If the market becomes more uncertain, then the insider is more likely to present in the financial market and trade her negative private information. Since more negative information is revealed, the entry probability may decrease. When transaction cost reduces a lot, as the market becomes more uncertain, the insider's trading strategy changes from trade on both positive and negative information to only trade on negative information. Since less positive information is revealed, entry probability may decrease. When transaction cost is sufficiently low, the entrant enters the market regardless of the market uncertainty, and entry probability does not change.

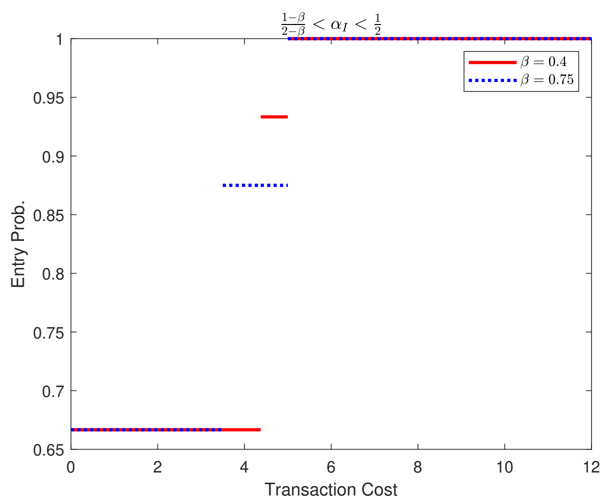

Figure 3.7: Optimistic Entrant: Entry Probability and Entry Barrier

Figure 3.7 shows the impact of product market uncertainty on the relation between ex-post entry probability and transaction cost when the entrant enters the market without learning. When the entrant enters the market without learning, if the insider does not trade on both positive and negative information, the entrant cannot learn from the financial market, it makes entry decision based on its prior belief and entry cost, it enters the market regardless of the market uncertainty; if the insider only trades on her negative information, the entrant does not enter the market when the total trading volume $X = -2$. If the insider trades on both directions of her private information, the entrant does not enter the market when the total trading volume $X = \{-1, -2\}$.

When transaction cost is sufficiently high, the insider does not trade on information regardless of market uncertainty, and the entrant enters the market. When transaction cost decreases, the insider starts to trade on negative information, and the entrant does not enter when the total trading volume $X = -2$. If the market becomes more uncertain, the insider is more likely to present in the financial market and trades her negative private information. Since more negative information is revealed, the entry probability may decrease. When transaction cost decreases a lot, as the market becomes more uncertain, the insider's trading strategy changes from trade on both positive and negative information to only trade on negative information. Since less negative information is revealed, the entry probability may increase. When transaction cost is sufficiently low, the insider trades on both positive and negative information regardless of market uncertainty, and entry probability does not change.

The Limits to Arbitrage Based on the above analysis, we know that given transaction cost, if increase in market uncertainty makes trading strategy changes from T to SNB, it may create a limits to arbitrage on the "buy side."

3.4.3 Entry Cost and Product Market Efficiency

Intuitively, when the barriers to entry reduces, entrant's incentives to enter the market increases. However, since the entrant becomes more likely to enter, the insider's incentive to deter the entrant from entering also increases. The relation between ex-ante entry probability and the barriers to entry seems uncertain. We analyze the relationship between the entrant's entry decision and the barriers to entry by studying the relationship between the ex-ante entry probability and the barriers to entry.

Barriers to Entry Since there are different types of barriers to entry, we do not consider any certain type of barriers to entry, but instead, we regard entry barriers as a whole and investigate the relationship between the mean of barriers to entry and ex-ante entry probability. We assume that the entry cost E follows a log normal distribution with mean μ and variance σ^2, that is $LnE \sim N(\mu, \sigma^2)$. Thus we have $E \sim LN(e^{\mu+\frac{1}{2}\sigma^2}, e^{2\mu+\sigma^2}(e^{\sigma^2}-1))$ and $E \sim N(Ln\mu - \frac{1}{2}\sigma_E^2, \sigma_E^2)$, where $\sigma_E = \sqrt{ln[(\frac{\sigma}{\mu})^2+1]}$. Then we calculate the ex-ante entry probability under different ranges of transaction cost k by:

$$P(\beta, E, \mu, \sigma, k) = \int_0^\infty p(\beta|E, k) f(E, \mu, \sigma) dE$$

where $p(\beta|E, k)$ is the conditional probability, and $f(E, \mu, \sigma)$ is the density function.

Proposition 3.6. The relation between entry barriers and ex-ante entry probability is non-monotonic. As entry barriers reduce, entry probability first increases then decreases.

Figure 3.8 shows the relation between the entry cost and ex-ante entry probability when transaction cost $k_{SNB} < k < k_{NF}$ by taking values for parameters $V_{1H}^M = 100$, $V_{1H}^C = 80$, $V_{1L}^M = 70$, and $V_{1L}^C = 65$.

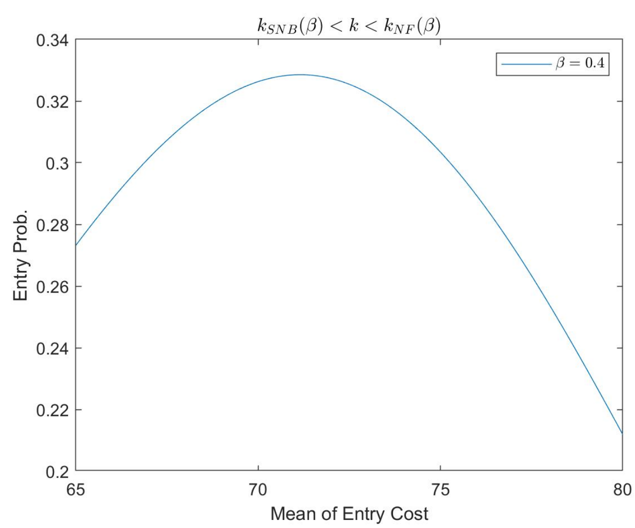

Figure 3.8: Entry Probability and Entry Barrier

From Figure 3.8, we find that the relationship between entry barriers and entry probability is non-monotonic. A reduction in the entry barriers does not necessarily increase the entry probability. When the entry barriers reduces, there are two effects. First, it increases entrant's net-profit, thus increases entrant's incentives to enter, which increases entry probability. Second, since entry may reduce insider's expected gross gain, it further reduces insider's incentives to trade on positive information, which makes the limits to arbitrage on the "buy side" more severe and reduces entry probability. When the entrant does not enter without learning, the first effect dominates the second effect, so that entry probability increases as the entry cost reduces. When the entrant enters without learning, the second effect dominates the first effect, so that entry probability decreases as entry cost reduces.

3.5 Empirical Implications

This paper provides some empirical implications, which are untested. First, due to entrant learning, the limits to arbitrage on the "buy side" may exist in the incumbent stock market. Second, there is a negative correlation between entry probability and trading cost when the entrant is pessimistic ex-ante, but such a correlation is positive when the entrant is optimistic ex-ante. Third, how the trading cost affects entry probability is affected by economic and informational conditions that the insider faces. Fourth, when entry barriers are low enough, a policy of reducing entry barriers decreases entry probability.

3.6 Policy Implications

Our results provide new insights into the policy implications. In Stigler (1971) theory of regulatory capture, the stricter regulation of entry raises barriers to entry, keeps out competitors, and raises incumbents' profits. As applies to deregulation of entry, the reduction of barriers to entry may encourage market competition and improve market efficiency. However, this capture theory (Peltzman, 1989, Posner, 1974, Stigler, 1971) does not take into account the financial market. Based on our analysis, if the entrant is optimistic and enters the market without learning, deregulation reduces entry probability.

In addition to government regulation, financial market policies affect financial market efficiency and thus affect entry probability. When the insider trades only her negative information, less positive information will be revealed to the entrant when the financial market becomes more efficient, leading to lower entry probability and product market competition. Therefore, policy on the financial market should also consider its impact on industry organization.

3.7 Conclusion

This paper studies the interaction between secondary financial market efficiency and product market competition. An insider trades incumbent stocks based on her knowledge about product market demand. A potential entrant learns from the financial market when making the entry decision. Because of entrant learning, the insider may refrain from trading on positive information to avoid revealing good information to the entrant, leading to the "buy-side" limits to arbitrage and financial market inefficiency. With different priors, entry probability as a function of financial market trading friction exhibits different patterns. Reducing transaction costs may make the insider change her trading strategies. When transaction cost is extremely high, the insider does not trade on her private information, the entrant cannot learn from the financial market and makes entry decision based on its prior. When transaction cost is extremely low, the insider buys when she receives positive information and sells when she receives negative information. When transaction cost is in between, the insider only sells on negative information but does not buy on positive information. As transaction cost reduces, entrant learning increases and reduces entry probability when the entrant is respectively optimistic and pessimistic ex-ante. Therefore, increase in financial market efficiency may encourage or impede product market competition, depending on entrant's prior belief. Besides, a policy of reducing entry barriers has non-monotonic effects on entry probability. In particular, when entry barriers are sufficiently low, reducing entry barriers may reduce entry probability. Furthermore, the product market uncertainty may increase or decrease entry probability, depending upon trading frictions.

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Appendix to Chapter 1

Proof of Table 1.3

Proof. Since the probability that the state is high and the total trading volume is $X = -1$ is $\frac{1}{2}\frac{\alpha}{3}(1-\beta_H)$, and the probability that the total trading volume is $X = -1$ is $\frac{1}{2}\frac{\alpha}{3}(1-\beta_H) + \frac{1}{2}\frac{\alpha}{3}(1-\beta_L) + \frac{1}{2}(1-\frac{2\alpha}{3})\beta_L$, the posterior belief that the state is H given the total trading volume is $X = -1$ is $q(-1) = p(H|X = -1) = \frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L}$.

Similarly, given that the probability that the state is high and the total trading volume is 1 is $\frac{1}{2}\frac{\alpha}{3}(1-\beta_H) + \frac{1}{2}(1-\frac{2\alpha}{3})\beta_H$, and the probability that the total trading volume is 1 is $\frac{1}{2}\frac{\alpha}{3}(1-\beta_H) + \frac{1}{2}(1-\frac{2\alpha}{3})\beta_H + \frac{1}{2}\frac{\alpha}{3}(1-\beta_L)$, the posterior belief that the state is H given the total trading volume is 1 is $q(1) = p(H|X = 1) = \frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H}$.

Given that the probability that the state is high and the total trading volume is $X = 0$ is $\frac{1}{2}\frac{\alpha}{3}\beta_H + \frac{1}{2}\frac{\alpha}{3}\beta_L + \frac{1}{2}(1-\beta_L)(1-\frac{2\alpha}{3}) + \frac{1}{2}(1-\frac{2\alpha}{3})(1-\beta_H)$, and the probability that the total trading volume is $X = 0$ is $p(X = 0) = \frac{1}{2}\frac{\alpha}{3}\beta_H + \frac{1}{2}(1-\frac{2\alpha}{3})(1-\beta_H)$, the posterior belief that the state is H given $X = 0$ is $q(0) = p(H|X = 0) = \frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L)+(3-2\alpha)(2-\beta_H-\beta_L)}$.

Since $X = -2$ or $X = 2$ fully reveal the low state or the high state, $q(-2) = p(H|X = -2) = 0$ and $q(2) = Prob(H|X = 2) = 1$. □

Proof of Corollary 1.1

Proof. Taking the first order derivative of $q(X = 0)$ with respect to β_H, the numerator is $3(\alpha - 1)\big(\alpha\beta_L + (1-\beta_L)(3-2\alpha)\big) < 0$, thus $q(X = 0)$ decreases as β_H increases. We have already shown that if $\beta_H = \beta_L$, $q(X = 0) = \frac{1}{2}$ and $d(X = 0) = 0$, thus if $\beta_H > \beta_L$, $q(X = 0) < 1/2$, and $d(X = 0) = -1$. Similarly, $q(X = 0)$ increases as β_H decreases. If $\beta_H < \beta_L$, $q(X = 0) > \frac{1}{2}$ and $d(X = 0) = 1$. □

Proof of Proposition 1.2

Proof. Assume that manager's investment decision is $d = (-1, -1, 1)$ when $X = (-1, 0, 1)$. The informed investor's trading profit from the exchange market is

$$V_H = \frac{\alpha}{3}\frac{\big(\alpha\beta_L + (3-2\alpha)(1-\beta_L)\big)(R_H - R_L - 2g)}{\alpha(\beta_H+\beta_L) + (3-2\alpha)(2-\beta_H-\beta_L)} + \left(1 - \frac{2\alpha}{3}\right)\frac{\alpha(1-\beta_L)(R_H - R_L + 2g)}{\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_H}$$

when she is positively informed, and it is

$$V_L = \frac{\alpha}{3}\frac{\big(\alpha\beta_H + (3-2\alpha)(1-\beta_H)\big)(R_H - R_L - 2g)}{\alpha(\beta_H+\beta_L) + (3-2\alpha)(2-\beta_H-\beta_L)} + \left(1 - \frac{2\alpha}{3}\right)\frac{\alpha(1-\beta_H)(R_H - R_L - 2g)}{\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_L}$$

when she is negatively informed.

Her trading profit from the dark pool is

$$DP_H = \frac{1-\alpha}{3}\big(R_H - g - P(X=0)\big) = \frac{1-\alpha}{3}\frac{\big(\alpha\beta_L + (3-2\alpha)(1-\beta_L)\big)(R_H - R_L - 2g)}{\alpha(\beta_H+\beta_L) + (3-2\alpha)(2-\beta_H-\beta_L)}$$

when she is positively informed, and it is

$$DP_L = \frac{1-\alpha}{3}\big(P(X=0) - R_L - g\big) = \frac{1-\alpha}{3}\frac{\big(\alpha\beta_H + (3-2\alpha)(1-\beta_H)\big)(R_H - R_L - 2g)}{\alpha(\beta_H+\beta_L) + (3-2\alpha)(2-\beta_H-\beta_L)}.$$

when she is negatively informed.

Hence, the informed investor's net opportunity cost is

$$\frac{1-2\alpha}{3}\frac{\big(\alpha\beta_L+(3-2\alpha)(1-\beta_L)\big)(R_H-R_L-2g)}{\alpha(\beta_H+\beta_L)+(3-2\alpha)(2-\beta_H-\beta_L)}$$

if she is positively informed, and it is

$$\frac{1-2\alpha}{3}\frac{\big(\alpha\beta_H+(3-2\alpha)(1-\beta_H)\big)(R_H-R_L-2g)}{\alpha(\beta_H+\beta_L)+(3-2\alpha)(2-\beta_H-\beta_L)}$$

if she is negatively informed.

Since when the noise investor with non-zero demand is more likely to trade in the exchange market, that is, $\alpha \in (\frac{1}{2}, 1]$, the net opportunity cost is non-positive for $\forall \beta_H, \beta_L \in [0, 1]$, it is straightforward that $DP_H - V_H < 0$ and $DP_L - V_L < 0$, which means that the informed investor earns a higher profit in the exchange market. Therefore, she prefers to trade in the exchange market, regardless of firm fundamentals. The equilibrium is $\beta_H = \beta_L = 1$, which contradicts $d = (-1, -1, 1)$.

Similarly, we can proof that given the manager's investment decision is $d = (-1, 0, 1)$, $DP_H - V_H < 0$ and $DP_L - V_L < 0$ for $\forall \alpha \in (\frac{1}{2}, 1]$. Therefore, there is a unique equilibrium in which the informed investor's venue choice strategy is $\beta_H = \beta_L = 1$, and the manager's decision is $d = (-1, 0, 1)$ at $X = (-1, 0, 1)$.

□

Proof of Lemma 1.4

Proof. When the total trading volume is $X = -2$, the low state $\theta = L$ is fully revealed, and thus the posterior belief about the high state is $q(-2) = 0$, which is the lowest posterior belief. When the total trading volume is $X = 2$, the high state $\theta = H$ is fully revealed, and thus the posterior belief about the high state is $q(2) = 1$, which is the highest posterior belief.

$$\begin{aligned}
q(X=1) - q(X=0) &= \frac{\alpha(1-\beta_H) + (3-2\alpha)\beta_H}{\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_H} - \frac{\alpha\beta_H + (3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)} \\
&= \frac{\alpha\beta_L(\alpha + 3\beta_H(1-\alpha)) + 3\beta_H(1-\beta_L)(3-\alpha)(1-\alpha)}{(\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_H)(\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha))}
\end{aligned}$$

Since $\alpha \in (0,1)$, $\{\beta_H, \beta_L\} \in [1,0]$, the numerator is positive and thus $q(X = 1) - q(X = 0) > 0$.

$$\begin{aligned}
q(X=0) - q(X=-1) &= \frac{\alpha\beta_H + (3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)} - \frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_L} \\
&= \frac{3\beta_L(1-\beta_H)(3-\alpha)(1-\alpha) + \alpha^2\beta_H(1-\beta_L) + \alpha\beta_H\beta_L(3-2\alpha)}{(\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_L)(\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha))}
\end{aligned}$$

Since $\alpha \in (0,1)$, $\{\beta_H, \beta_L\} \in [1,0]$, the numerator is positive, and thus $q(X = 0) - q(X = -1) > 0$.

Therefore, the posterior belief is increasing in the total trading volume X, and thus the real effects are monotonic on the total trading volume. □

Proof of Proposition 1.3

Proof. The proof of *Proposition 1.3* relies on the self-fulfilling hypothesis. There are two steps to find out equilibrium with strong real effects. Given the informed investor venue choice strategy β_H and β_L, and the manager's investment decision d at $X = (-1, 0, 1)$, the first step is to check if we can find $\forall\alpha \in (0, 1]$ that support for the manager's investment decision d at $X = (-1, 0, 1)$. If such α exists, we further check if the informed investor's venue choice strategy β_H and β_L could be sustainable, given the informed investor's expected payoffs from the exchange market and the dark pool.

For example, given the equilibrium $\beta_H = 1, \beta_L = 0$, the manager's posterior beliefs are $q = (0, \frac{\alpha}{3-\alpha}, \frac{3-2\alpha}{3-\alpha})$ at $X = (-1, 0, 1)$ respectively. If the manager's investment decision is $d = (-1, -1, 1)$ at $X = (-1, 0, 1)$, q must satisfy the condition $0 \leq q(X = 0) = \frac{\alpha}{3-\alpha} < \frac{1}{2}$, and the condition $\frac{1}{2} < q(X = 1) = \frac{3-2\alpha}{3-\alpha} \leq 1$. Then we can find $\alpha's$ support for these two conditions.

Given the $\alpha's$, we further check if the informed investor's venue choice strategy $\beta_H = 1, \beta_L = 0$ is sustainable, that is, if $V_H \geq DP_H$ and $V_L \leq DP_L$ survive for some $\alpha's$.

When $g \to \frac{R_H - R_L}{2}^{-}$, we have

$$V_H = \frac{\alpha}{3}\frac{3-2\alpha}{3-\alpha}(R_H - R_L - 2g) + (1 - \frac{2\alpha}{3})\frac{\alpha}{3-\alpha}(R_H - R_L + 2g)$$

$$V_L = \frac{\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L - 2g) + (1 - \frac{2\alpha}{3})0 = \frac{\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L - 2g),$$

$$DP_H = \frac{1-\alpha}{3}\frac{3-2\alpha}{3-\alpha}(R_H - R_L - 2g),$$

$$DP_L = \frac{1-\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L - 2g).$$

The condition $V_H \geq DP_H$ and $V_L \leq DP_L$ survive for $\forall\alpha \in \left[\frac{R_H - R_L - 2g}{3(R_H - R_L) - 2g}, \frac{1}{2}\right)$, thus the asymmetric equilibrium in which the informed investor's venue choice strategy is $\beta_H = 1, \beta_L = 0$, and the manager's investment decision is $d = (-1, -1, 1)$ exists.

Following the same logic, we first check the pure strategy $\beta_H = 1$ and $\beta_L = 0$, and the pure strategy $\beta_H = 0$ and $\beta_L = 1$. In the pure strategy $\beta_H = 1$ and $\beta_L = 0$, the posterior belief $q(X = -1) = 0$, thus the corresponding investment decision is $d(X = -1) = -1$. Given Lemma 1.2 and Lemma 1.4, there are two potential manager's investment decision $d \in \{(-1, -1, 0), (-1, -1, 1)\}$ at $X = (-1, 0, 1)$ to consider. However, only $d = (-1, -1, 1)$ survives for $\forall\alpha \in (0, 1)$. Thus we further check if the informed investor's venue choice strategy $\beta_H = 1$ and $\beta_L = 0$ survive for $\forall\alpha \in \left[\frac{R_H - R_L - 2g}{3(R_H - R_L) - 2g}, \frac{1}{2}\right)$, that is, if $V_H \geq DP_H$

and $V_L \leq DP_L$ survive for $\forall \alpha \in \left[\frac{R_H - R_L - 2g}{3(R_H - R_L) - 2g}, \frac{1}{2}\right)$. We have show this proof in the above example, thus there is a pure strategy equilibrium in which the informed investor's venue choice is $\beta_H = 1$ and $\beta_L = 0$, and the manager's investment decision is $d \in \{-1, -1, 1\}$ for $\forall \alpha \in \left[\frac{R_H - R_L - 2g}{3(R_H - R_L) - 2g}, \frac{1}{2}\right)$.

In the pure strategy $\beta_H = 0$ and $\beta_L = 1$, the manager's posterior beliefs are $q = (\frac{\alpha}{3-\alpha}, \frac{3-2\alpha}{3-\alpha}, 1)$ at $X = (-1, 0, 1)$, then given Lemma 1.4 and $d(X = 1) = 1$, there are two potential investment decision $d \in \big\{(0, 1, 1), (-1, 1, 1)\big\}$ to consider. After checking, only $d = (-1, 1, 1)$ survives for $\forall \alpha \in (0, 1)$. Thus we further check if the informed investor's venue choice strategy $\beta_H = 0$ and $\beta_L = 1$ survive for $\forall \alpha \in (0, 1)$, that is, if $V_H \leq DP_H$ and $V_L \geq DP_L$ survive for $\forall \alpha \in (0, 1)$.

When $g \to \frac{R_H - R_L}{2}^-$, we have

$$V_H = \frac{\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L + 2g) + (1 - \frac{2\alpha}{3})0 = \frac{\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L + 2g),$$

$$V_L = \frac{\alpha}{3}\frac{3-2\alpha}{3-\alpha}(R_H - R_L + 2g) + (1 - \frac{2\alpha}{3})\frac{\alpha}{3-\alpha}(R - H - R_L - 2g),$$

$$DP_H = \frac{1-\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L + 2g),$$

$$DP_L = \frac{1-\alpha}{3}\frac{3-2\alpha}{3-\alpha}(R_H - R_L + 2g).$$

We find that for $\forall \alpha \in \left[\frac{R_H - R_L + 2g}{3(R_H - R_L) + 2g}, 1\right)$, $V_H \leq DP_H$ and $V_L \geq DP_L$, thus there is a pure strategy equilibrium in which informed investor's venue choice is $\beta_H = 0$ and $\beta_L = 1$ and the manager's decision is $d = (-1, 1, 1)$ for $\forall \alpha \in \left[\frac{R_H - R_L + 2g}{3(R_H - R_L) + 2g}, 1\right)$.

Now we check the mixed strategy. In the mixed strategy $\beta_H = 1, \beta_L \in (0, 1)$, the manager's posterior beliefs are $q = (0, \frac{\alpha}{\alpha(1+\beta_L) + (1-\beta_L)(3-2\alpha)}, \frac{3-2\alpha}{\alpha(1-\beta_L) + 3 - 2\alpha})$ at $X = (-1, 0, 1)$, then given Lemma 1.2 and Lemma 1.4, $d(X = 0) = -1$ and $d(X = -1) = -1$, there are two potential investment decision $d \in \big\{(-1, -1, 0), (-1, -1, 1)\big\}$ to consider. However, only

$d = (-1,-1,1)$ survives for $\forall\alpha \in (0,1)$. Then, we check if the informed investor venue choice strategy $\beta_H = 1$ and $\beta_L \in (0,1)$ is sustainable for $\forall\alpha \in (0,1)$, that is, if $V_H \geq DP_H$ and $V_L = DP_L$ survive for $\forall\alpha \in (0,1)$.

When $g \to \frac{R_H-R_L}{2}^-$, we have

$$V_H = \frac{\alpha}{3}\frac{\alpha\beta_L + (3-2\alpha)(1-\beta_L)}{\alpha(1+\beta_L)+(1-\beta_L)(3-2\alpha)}(R_H-R_L-2g)+(1-\frac{2\alpha}{3})\frac{\alpha(1-\beta_L)}{\alpha(1-\beta_L)+3-2\alpha}(R_H-R_L+2g)$$

$$\begin{aligned} V_L &= \frac{\alpha}{3}\frac{\alpha}{\alpha(1+\beta_L)+(1-\beta_L)(3-2\alpha)}(R_H-R_L-2g)+(1-\frac{2\alpha}{3})0 \\ &= \frac{\alpha}{3}\frac{\alpha}{\alpha(1+\beta_L)+(1-\beta_L)(3-2\alpha)}(R_H-R_L-2g), \end{aligned}$$

$$DP_H = \frac{1-\alpha}{3}\frac{\alpha\beta_L + (3-2\alpha)(1-\beta_L)}{\alpha(1+\beta_L)+(1-\beta_L)(3-2\alpha)}(R_H-R_L-2g),$$

$$DP_L = \frac{1-\alpha}{3}\frac{\alpha}{\alpha(1+\beta_L)+(1-\beta_L)(3-2\alpha)}(R_H-R_L-2g).$$

We can not find $\forall\alpha \in (0,1)$ to satisfy these conditions, thus there is no asymmetric equilibrium $\beta_H = 1, \beta_L \in (0,1)$ with strong real effects $d = (-1,-1,1)$.

In the mixed strategy $\beta_H \in (0,1), \beta_L = 0$, we know the manager's posterior beliefs are $q = (\frac{1-\beta_H}{2-\beta_H}, \frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha\beta_H+(3-2\alpha)(2-\beta_H)}, \frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H)+(3-2\alpha)\beta_H})$ at $X = (-1,0,1)$, then there are two potential investment decision $d \in \big\{(-1,-1,0),(-1,-1,1)\big\}$ to consider. Similarly, only $d = (-1,-1,1)$ survives for $\forall\alpha \in (0,1)$. After further check if $\beta_H \in (0,1)$ and $\beta_L = 0$ is sustainable for $\forall\alpha \in (0,1)$, that is, if $V_H = DP_H$ and $V_L \leq DP_L$ survive for $\forall\alpha \in (0,1)$.

When $g \to \frac{R_H-R_L}{2}^-$, we have

$$V_H = \frac{\alpha}{3}\frac{3-2\alpha}{\alpha\beta_H+(2-\beta_H)(3-2\alpha)}(R_H-R_L-2g)+(1-\frac{2\alpha}{3})\frac{\alpha}{\alpha(2-\beta_H)+(3-2\alpha)\beta_H}(R_H-R_L+2g)$$

$$V_L = \frac{\alpha}{3}\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha\beta_H+(3-2\alpha)(2-\beta_H)}(R_H-R_L-2g)+(1-\frac{2\alpha}{3})\frac{1-\beta_H}{2-\beta_H}(R_H-R_L-2g),$$

$$DP_H = \frac{1-\alpha}{3}\frac{3-2\alpha}{\alpha\beta_H+(2-\beta_H)(3-2\alpha)}(R_H-R_L-2g),$$

$$DP_L = \frac{1-\alpha}{3}\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha\beta_H+(3-2\alpha)(2-\beta_H)}(R_H-R_L-2g).$$

The conditions $V_H = DP_H$ and $V_L \leq DP_L$ hold for $\forall\alpha \in \left(0, \frac{R_H-R_L-2g}{3(R_H-R_L)-2g}\right)$, thus the informed investor's venue choice strategy $\beta_H \in (0,1)$ and $\beta_L = 0$ holds for $\forall\alpha \in \left(0, \frac{R_H-R_L-2g}{3(R_H-R_L)-2g}\right)$.

In the mixed strategy $\beta_H \in (0,1), \beta_L = 1$, we know the manager's posterior beliefs are $q = (\frac{\alpha(1-\beta_H)}{\alpha(1-\beta_H)+3-2\alpha}, \frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(1+\beta_H)+(3-2\alpha)(1-\beta_H)}, 1)$ at $X = (-1,0,1)$, then there are two potential investment decision $d \in \{(0,1,1),(-1,1,1)\}$ to consider. Only $d = (-1,1,1)$ survives for $\forall\alpha \in (0,1)$. Then, further check if the informed investor's venue choice $\beta_H \in (0,1)$ and $\beta_L = 1$ is sustainable for $\forall\alpha \in (0,1)$, that is, if $V_H = DP_H$ and $V_L \geq DP_L$ survive for $\forall\alpha \in (0,1)$.

When $g \to \frac{R_H-R_L}{2}^-$, we have

$$\begin{aligned} V_H &= \frac{\alpha}{3}\frac{\alpha}{\alpha(\beta_H+1)+(1-\beta_H)(3-2\alpha)}(R_H-R_L+2g)+(1-\frac{2\alpha}{3})0 \\ &= \frac{\alpha}{3}\frac{\alpha}{\alpha(\beta_H+1)+(1-\beta_H)(3-2\alpha)}(R_H-R_L+2g) \end{aligned}$$

$$V_L = \frac{\alpha}{3}\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(1+\beta_H)+(3-2\alpha)(1-\beta_H)}(R_H-R_L+2g)+(1-\frac{2\alpha}{3})\frac{\alpha(1-\beta_H)}{\alpha(1-\beta_H)+3-2\alpha}(R_H-R_L-2g)$$

$$DP_H = \frac{1-\alpha}{3}\frac{\alpha}{\alpha(\beta_H+1)+(1-\beta_H)(3-2\alpha)}(R_H-R_L+2g),$$

$$DP_L = \frac{1-\alpha}{3}\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(1+\beta_H)+(3-2\alpha)(1-\beta_H)}(R_H-R_L+2g).$$

The condition $V_H = DP_H$ and $V_L \geq DP_L$ survives only when $\alpha = \frac{1}{2}$, thus the informed investor's venue choice strategy $\beta_H \in (0,1)$ and $\beta_L = 1$ does not exist. Therefore, there is no mixed strategy $\beta_H \in (0,1)$ and $\beta_L = 1$ for $\forall\alpha \in (0,1)$.

In the mixed strategy $\beta_H = 0, \beta_L \in (0,1)$, we know the manager's posterior beliefs are $q = (\frac{\alpha}{\alpha(2-\beta_L)+(3-2\alpha)\beta_L}, \frac{3-2\alpha}{\alpha\beta_L+(3-2\alpha)(2-\beta_L)}, \frac{1}{2-\beta_L})$ at $X = (-1,0,1)$, then given Lemma 1.4, there are two potential investment decision $d \in \{(0,1,1),(-1,1,1)\}$ to consider. Only $d = (-1,1,1)$ survives for $\forall\alpha \in (0,1)$. Then we further check if the informed investor venue choice $\beta_H = 0$ and $\beta_L \in (0,1)$ is sustainable for $\forall\alpha \in (0,1)$, that is, if $V_H \leq DP_H$ and $V_L = DP_L$ survive for $\forall\alpha \in (0,1)$.

When $g \to \frac{R_H - R_L}{2}^-$, we have

$$V_H = \frac{\alpha}{3}\frac{\alpha\beta_L + (3-2\alpha)(1-\beta_L)}{\alpha\beta_L + (3-2\alpha)(2-\beta_L)}(R_H - R_L + 2g) + (1 - \frac{2\alpha}{3})\frac{\alpha\beta_L}{\alpha(2-\beta_L)}(R_H - R_L + 2g)$$

$$V_L = \frac{\alpha}{3}\frac{3-2\alpha}{\alpha\beta_L + (3-2\alpha)(2-\beta_L)}(R_H - R_L + 2g) + (1 - \frac{2\alpha}{3})\frac{\alpha}{\alpha(2-\beta_L) + (3-2\alpha)\beta_L}(R_H - R_L - 2g)$$

$$DP_H = \frac{1-\alpha}{3}\frac{\alpha\beta_L + (3-2\alpha)(1-\beta_L)}{\alpha\beta_L + (3-2\alpha)(2-\beta_L)}(R_H - R_L + 2g),$$

$$DP_L = \frac{1-\alpha}{3}\frac{3-2\alpha}{\alpha\beta_L + (3-2\alpha)(2-\beta_L)}(R_H - R_L + 2g).$$

The condition $V_H \leq DP_H$ and $V_L = DP_L$ hold for $\forall\alpha \in \left(\frac{8g-2(R_H-R_L)}{6g+R_H-R_L}, \frac{R_H-R_L+2g}{3(R_H-R_L)+2g}\right)$, thus the informed investor's venue choice strategy $\beta_H = 0$ and $\beta_L \in (0,1)$ exists for $\forall\alpha \in \left(\frac{8g-2(R_H-R_L)}{6g+R_H-R_L}, \frac{R_H-R_L+2g}{3(R_H-R_L)+2g}\right)$.

In the mixed strategy $\beta_H \in (0,1) > \beta_L \in (0,1)$, the manager's posterior beliefs are $q = (\frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L}, \frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L)+(3-2\alpha)(2-\beta_H-\beta_L)}, \frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H})$ at $X = (-1,0,1)$. As $\beta_H > \beta_L$, there are two potential investment decision $d \in \{(-1,-1,0),(-1,-1,1)\}$ to consider. After checking the posterior beliefs that satisfy the investment decision, only $d = (-1,-1,1)$ survives for $\forall\alpha \in (0,1)$. Then, we further check if the informed investor's venue choice strategy $\beta_H \in (0,1)$, $\beta_L \in (0,1)$, and $\beta_H > \beta_L$ is sustainable for $\forall\alpha \in (0,1)$, that is, if $V_H = DP_H$ and $V_L = DP_L$ survive for $\forall\alpha \in (0,1)$.

When $g \to \frac{R_H - R_L}{2}^-$, we have

$$V_H = \frac{\alpha}{3}\frac{\alpha\beta_L + (3-2\alpha)(1-\beta_L)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)}(R_H - R_L - 2g)$$
$$+ (1 - \frac{2\alpha}{3})\frac{\alpha(1-\beta_L)}{\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_H}(R_H - R_L + 2g)$$

$$V_L = \frac{\alpha}{3} \frac{\alpha\beta_H + (3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)} (R_H - R_L - 2g)$$
$$+ (1 - \frac{2\alpha}{3}) \frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_L} (R_H - R_L - 2g)$$

$$DP_H = \frac{1-\alpha}{3} \frac{\alpha\beta_L + (3-2\alpha)(1-\beta_L)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)} (R_H - R_L - 2g),$$

$$DP_L = \frac{1-\alpha}{3} \frac{\alpha\beta_H + (3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)} (R_H - R_L - 2g).$$

The condition $V_H = DP_H$ and $V_L = DP_L$ does not hold for $\forall\alpha \in (0,1)$, thus the informed investor's venue choice strategy $\beta_H \in (0,1) > \beta_L \in (0,1)$ does not exist. Therefore, there is no mixed strategy $\beta_H \in (0,1) > \beta_L \in (0,1)$ for $\forall\alpha \in (0,1)$.

In the mixed strategy $\beta_H \in (0,1) < \beta_L \in (0,1)$, we know the manager's posterior beliefs are $q = (\frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L}, \frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L)+(3-2\alpha)(2-\beta_H-\beta_L)}, \frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H})$ at $X = (-1,0,1)$. As $\beta_H < \beta_L$, there are two potential investment decision $d \in \{(0,1,1),(-1,1,1)\}$ to consider. After checking the posterior beliefs that satisfy the investment decision, only $d = (-1,1,1)$ survives for $\forall\alpha \in (0,1)$. Thus we further check if the informed investor's venue choice strategy $\beta_H \in (0,1)$, $\beta_L \in (0,1)$, and $\beta_H < \beta_L$ is sustainable for $\forall\alpha \in (0,1)$, that is, if $V_H = DP_H$ and $V_L = DP_L$ survive for $\forall\alpha \in (0,1)$.

When $g \to \frac{R_H - R_L}{2}^-$, we have

$$V_H = \frac{\alpha}{3} \frac{\alpha\beta_L + (3-2\alpha)(1-\beta_L)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)} (R_H - R_L + 2g)$$
$$+ (1 - \frac{2\alpha}{3}) \frac{\alpha(1-\beta_L)}{\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_H} (R_H - R_L + 2g),$$

$$V_L = \frac{\alpha}{3} \frac{\alpha\beta_H + (3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)} (R_H - R_L + 2g)$$
$$+ (1 - \frac{2\alpha}{3}) \frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L) + (3-2\alpha)\beta_L} (R_H - R_L - 2g),$$

$$DP_H = \frac{1-\alpha}{3} \frac{\alpha\beta_L + (3-2\alpha)(1-\beta_L)(3-2\alpha)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)} (R_H - R_L + 2g),$$

$$DP_L = \frac{1-\alpha}{3} \frac{\alpha\beta_H + (3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L) + (2-\beta_H-\beta_L)(3-2\alpha)}(R_H - R_L + 2g).$$

The condition $V_H = DP_H$ and $V_L = DP_L$ holds for $\forall\alpha \in (\hat{\alpha}, \frac{1}{2})$, in which $\hat{\alpha} = \frac{(R_H-R_L)^2-7g(R_H-R_L)+42g^2}{4g(10g-R_H+R_L)} - \frac{\sqrt{84g^3(R_H-R_L)+(85g^2-4R_HR_L)(R_H-R_L)^2-14g(R_H-R_L)^3+(R_H^2-R_L^2)-156g^4}}{4g(10g-R_H+R_L)}$. Thus the informed investor's venue choice strategy $\beta_H \in (0,1) < \beta_L \in (0,1)$ exists for $\forall\alpha \in (\hat{\alpha}, \frac{1}{2})$. □

Discussion of Kyle's Market Efficiency with Real Effects

After observing $X = (-2, -1, 0, 1, 2)$, the manager makes the decision $d = (-1, -1, -1, 1, 1)$.

We know $\mathbb{V}ar(V|X = x) = \mathbb{E}[(V - \mu_{V|X=x})^2|X = x]$, where $\mu_{V|X=x} = \mathbb{E}(V|X = x)$. Thus the price $P(X = x) = \mu_{V|X=x}$.

The price informativeness is:

$$\mathbb{E}[\alpha(P)] = \mathbb{E}\big[\frac{\mathbb{V}ar[V|P]}{\mathbb{V}ar[V]}\big]$$

First, solve for the conditional variance under different total trading volume. Second, solve for the unconditional variance.

However, price fully reveals the true value when $X = (-2, 2)$, thus

$$\mathbb{V}ar(V|X = -2) = \mathbb{V}ar(V|X = 2) = 0$$

The unconditional variance is $\mathbb{V}ar(V) = \mathbb{E}(V^2) - [\mathbb{E}(V)]^2$, given that

$$\mathbb{E}[V] = (\frac{1}{2} - \frac{\beta_H}{2})\big[\frac{\alpha}{3}V_H(1) + (1 - \frac{2\alpha}{3})V_H(0) + \frac{\alpha}{3}V_H(-1)\big] + \frac{\beta_H}{2}\big[\frac{\alpha}{3}V_H(0) + (1 - \frac{2\alpha}{3})V_H(1) + \frac{\alpha}{3}V_H(2)\big] + (\frac{1}{2} - \frac{\beta_L}{2})\big[\frac{\alpha}{3}V_L(1) + (1 - \frac{2\alpha}{3})V_L(0) + \frac{\alpha}{3}V_L(-1)\big] + \frac{\beta_L}{2}\big[\frac{\alpha}{3}V_L(0) + (1 - \frac{2\alpha}{3})V_L(-1) + \frac{\alpha}{3}V_L(-2)\big]$$

and

$$\mathbb{E}[V^2] = (\frac{1}{2} - \frac{\beta_H}{2})\big[\frac{\alpha}{3}V_H(1)^2 + (1 - \frac{2\alpha}{3})V_H(0)^2 + \frac{\alpha}{3}V_H(-1)^2\big] + \frac{\beta_H}{2}\big[\frac{\alpha}{3}V_H(0)^2 + (1 - \frac{2\alpha}{3})V_H(1)^2 + \frac{\alpha}{3}V_H(2)^2\big] + (\frac{1}{2} - \frac{\beta_L}{2})\big[\frac{\alpha}{3}V_L(1)^2 + (1 - \frac{2\alpha}{3})V_L(0)^2 + \frac{\alpha}{3}V_L(-1)^2\big] + \frac{\beta_L}{2}\big[\frac{\alpha}{3}V_L(0)^2 + (1 - \frac{2\alpha}{3})V_L(-1)^2 + \frac{\alpha}{3}V_L(-2)^2\big].$$

Thus the price informativeness is:

$$\mathbb{E}[\alpha(P)] = \frac{1}{2}\mathbb{E}[\alpha_H(P)] + \frac{1}{2}\mathbb{E}[\alpha_L(P)]$$

in which

$$\begin{aligned}\mathbb{E}[\alpha_H(P)] &= \mathbb{E}_H\big[\frac{\mathbb{V}ar[V|P]}{\mathbb{V}ar[V]}\big] \\ &= \frac{1-\beta_H}{\mathbb{V}ar(V)}\big[\frac{\alpha}{3}\mathbb{V}ar(V|X=1) + (1 - \frac{2\alpha}{3})\mathbb{V}ar(V|X=0) + \frac{\alpha}{3}\mathbb{V}ar(V|X=-1)\big] \\ &+ \frac{\beta_H}{\mathbb{V}ar(V)}\big[\frac{\alpha}{3}\mathbb{V}ar(V|X=0) + (1 - \frac{2\alpha}{3})\mathbb{V}ar(V|X=1)\big]\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}[\alpha_L(P)] &= \mathbb{E}_L\big[\frac{\mathbb{V}ar[V|P]}{\mathbb{V}ar[V]}\big] \\ &= \frac{1-\beta_L}{\mathbb{V}ar(V)}\big[\frac{\alpha}{3}\mathbb{V}ar(V|X=1) + (1 - \frac{2\alpha}{3})\mathbb{V}ar(V|X=0) + \frac{\alpha}{3}\mathbb{V}ar(V|X=-1)\big] \\ &+ \frac{\beta_L}{\mathbb{V}ar(V)}\big[\frac{\alpha}{3}\mathbb{V}ar(V|X=0) + (1 - \frac{2\alpha}{3})\mathbb{V}ar(V|X=-1)\big]\end{aligned}$$

With managerial learning, the firm fundamental are endogenous, but the Kyle (1985) variance ratio does not take it into account, thus the variance ratio is not a plausible measure for the exchange market efficiency. We use the variance ratio $\mathbb{E}[\Lambda|X=0]$ as an example.

First, when $\alpha \to 0^+$, the condition variance when $X=0$ is given by

$$lim_{\alpha\to 0^+}\mathbb{V}ar(V|X=0) = lim_{\alpha\to 0^+}q(0)(1-q(0)(R_H - R_L - 2g)^2$$

Substitute $lim_{\alpha\to 0^+}q(0) = lim_{\alpha\to 0^+}(1-q(0)) = \frac{1}{2}$ into the above equation, $lim_{\alpha\to 0^+}\mathbb{V}ar(V|X=0) = \frac{(R_H-R_L-2g)^2}{4}$.

Since when $\alpha \to 0^+$, $lim_{\alpha\to 0^+}\beta_H \to 0^+$ and $\beta_L = 0$, we have

$$lim_{\alpha\to 0^+}\mathbb{E}(V) = \frac{1}{2}V_H(0) + \frac{1}{2}V_L(0) = \frac{R_H + R_L}{2}$$

and

$$lim_{\alpha\to 0^+}\mathbb{E}(V^2) = \frac{1}{2}V_H(0)^2 + \frac{1}{2}V_L(0)^2 = \frac{R_H^2 + R_L^2 + 2g^2 - 2gR_H + 2gR_L}{2}.$$

Thus, the unconditional variance is given by

$$lim_{\alpha\to 0^+}\mathbb{V}ar(V) = lim_{\alpha\to 0^+}[\mathbb{E}(V^2) - (\mathbb{E}(V))^2] = \frac{(R_H - R_L - 2g)^2}{4}.$$

The variance ratio $lim_{\alpha\to 0^+}\mathbb{E}[\Lambda|X=0]$ is given by

$$lim_{\alpha\to 0^+}\mathbb{E}[\Lambda|X=0] = lim_{\alpha\to 0^+}\frac{\mathbb{V}ar(V|X=0)}{\mathbb{V}ar(V)} = 1.$$

Similarly, when $\alpha \to 1^-$, the condition variance when $X=0$ is given by

$$lim_{\alpha\to 1^-}\mathbb{V}ar(V|X=0) = lim_{\alpha\to 1^-}q(0)(1-q(0)(R_H - R_L - 2g)^2$$

Substitute $lim_{\alpha\to 1^-} q(0) = lim_{\alpha\to 1^-}(1-q(0)) = \frac{1}{2}$ into the above equation, $lim_{\alpha\to 1^-}\mathbb{V}ar(V|X = 0) = \frac{(R_H-R_L-2g)^2}{4}$.

Since when $\alpha \to 1^-$, $\beta_H = 1$ and $\beta_L = 0$, we have

$$lim_{\alpha\to 1^-}\mathbb{E}(V) = \frac{1}{2}[\frac{1}{3}V_H(0)+\frac{1}{3}V_H(1)+\frac{1}{3}V_H(2)]+\frac{1}{2}[\frac{1}{3}V_L(0)+\frac{1}{3}V_L(-1)+\frac{1}{3}V_L(-2)] = \frac{3R_H+3R_L+4g}{6}$$

and

$$lim_{\alpha\to 1^-}\mathbb{E}(V^2) = \frac{3R_H^2+3R_L^2+2R_Hg+6R_Lg+6g^2}{6}.$$

Thus, the unconditional variance is given by

$$lim_{\alpha\to 1^-}\mathbb{V}ar(V) = lim_{\alpha\to 1^-}[\mathbb{E}(V^2)-(\mathbb{E}(V))^2] = \frac{9R_H^2+9R_L^2-12g(R_H-R_L)+20g^2-18R_HR_L}{36}.$$

The variance ratio $lim_{\alpha\to 1^-}\mathbb{E}[\Lambda|X = 0]$ is given by

$$lim_{\alpha\to 1^-}\mathbb{E}[\Lambda|X = 0] = lim_{\alpha\to 1^-}\frac{\mathbb{V}ar(V|X = 0)}{\mathbb{V}ar(V)} = \frac{1}{1+\frac{24g(R_H-R_L)-16g^2}{9(R_H-R_L-2g)^2}}.$$

Since $g \to \frac{R_H-R_L}{2}^-$, $\frac{24g(R_H-R_L)-16g^2}{9(R_H-R_L-2g)^2} > 0$ such that $lim_{\alpha\to 1^-}\mathbb{E}[\Lambda|X = 0] < 1 = lim_{\alpha\to 0^+}\mathbb{E}[\Lambda|X = 0]$. Therefore, it is clear to see due to the real effects, the variance ratio does not work anymore.

Proof of Proposition 1.4

Proof. With real effects, the conditional entropy is given by:

$$
\begin{aligned}
H(\Theta|X) &= \sum_{x\in X} p(x)H(\Theta|X=x) \\
&= -\sum_{x\in X} p(x)\sum_{\theta\in\Theta} q(\theta|x)log_2 q(\theta|x) \\
&= -p(-2)\big[q(H|-2)log_2 q(H|-2) + q(L|-2)log_2 q(L|-2)\big] \\
&- p(-1)\big[q(H|-1)log_2 q(H|-1) + q(L|-1)log_2 q(L|-1)\big] \\
&- p(0)\big[q(H|0)log_2 q(H|0) + q(L|0)log_2 q(L|0)\big] \\
&- p(1)\big[q(H|1)log_2 q(H|1) + q(L|1)log_2 q(L|1)\big] \\
&- p(2)\big[q(H|2)log_2 q(H|2) + q(L|2)log_2 q(L|2)\big]
\end{aligned}
$$

in which the ex ante probabilities are $p(-2) = \frac{\alpha\beta_L}{6}$, $p(2) = \frac{\alpha\beta_H}{6}$, $p(-1) = \frac{1}{2}(1-\beta_H)\frac{\alpha}{3} + \frac{1}{2}\beta_L(1-\frac{2\alpha}{3}) + \frac{1}{2}(1-\beta_L)\frac{\alpha}{3}$, $p(1) = \frac{1}{2}(1-\beta_H)\frac{\alpha}{3} + \frac{1}{2}\beta_H(1-\frac{2\alpha}{3}) + \frac{1}{2}(1-\beta_L)\frac{\alpha}{3}$, and $p(0) = \frac{1}{2}\beta_H\frac{\alpha}{3} + \frac{1}{2}(1-\beta_H)(1-\frac{2\alpha}{3}) + \frac{1}{2}\beta_L\frac{\alpha}{3} + \frac{1}{2}(1-\beta_L)(1-\frac{2\alpha}{3})$. The ex post probabilities are $q(H|-2) = q(-2) = 0$ and $q(L|-2) = 1-q(-2) = 1$, $q(H|-1) = q(-1) = \frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L}$ and $q(L|-1) = 1-q(-1) = \frac{\alpha(1-\beta_L)+(3-2\alpha)\beta_L}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L}$, $q(H|0) = q(0) = \frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L)+(2-\beta_H-\beta_L)(3-2\alpha)}$ and $q(L|0) = 1-q(0) = \frac{\alpha\beta_L+(3-2\alpha)(1-\beta_L)}{\alpha(\beta_H+\beta_L)+(2-\beta_H-\beta_L)(3-2\alpha)}$, $q(H|1) = q(1) = \frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H}$ and $q(L|1) = 1-q(1) = \frac{\alpha(1-\beta_L)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H}$, and $q(H|2) = q(2) = 1$ and $q(L|2) = 1-q(2) = 0$.

Hence, the conditional entropy with managerial learning is given by

$$
\begin{aligned}
H(\Theta|X) = & -\frac{\alpha(2-\beta_H-\beta_L)+\beta_L(3-2\alpha)}{6} \\
& \left[\frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L} log_2 \frac{\alpha(1-\beta_H)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L}\right. \\
& \left. + \frac{\alpha(1-\beta_L)+(3-2\alpha)\beta_L}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L} log_2 \frac{\alpha(1-\beta_L)+(3-2\alpha)\beta_L}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_L}\right] \\
& - \frac{\alpha(\beta_H+\beta_L)+(2-\beta_H-\beta_L)(3-2\alpha)}{6} \\
& \left[\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L)+(2-\beta_H-\beta_L)(3-2\alpha)} log_2 \frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha(\beta_H+\beta_L)+(2-\beta_H-\beta_L)(3-2\alpha)}\right. \\
& \left. + \frac{\alpha\beta_L+(3-2\alpha)(1-\beta_L)}{\alpha(\beta_H+\beta_L)+(2-\beta_H-\beta_L)(3-2\alpha)} log_2 \frac{\alpha\beta_L+(3-2\alpha)(1-\beta_L)}{\alpha(\beta_H+\beta_L)+(2-\beta_H-\beta_L)(3-2\alpha)}\right] \\
& - \frac{\alpha(2-\beta_H-\beta_L)+\beta_H(3-2\alpha)}{6} \\
& \left[\frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H} log_2 \frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H}\right. \\
& \left. + \frac{\alpha(1-\beta_L)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H} log_2 \frac{\alpha(1-\beta_L)}{\alpha(2-\beta_H-\beta_L)+(3-2\alpha)\beta_H}\right]
\end{aligned} \tag{4}
$$

Given Equation 1.13 and Equation 4, it is easy to find out when $\alpha \to 0^+$, $\beta_H \to 0$, and then $H(\Theta|X) = 1$. Hence the exchange market efficiency is 0 when $\alpha \to 0^+$.

When $\alpha \in \left(0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}\right)$, $\beta_L = 0$ by Equation 1.14, $\beta_H = \frac{4\alpha(R_H-R_L)+16g\alpha(1-\alpha)}{3(1-\alpha)[(1-\alpha)(R_H-R_L)+2g(3\alpha-1)]}$

by Equation 1.13, which is strictly increasing in α. Hence,

$$\begin{aligned}H(\Theta|X) = &-\frac{2-\beta_H}{6}\left[\frac{1-\beta_H}{2-\beta_H}log_2\frac{1-\beta_H}{2-\beta_H}+\frac{1}{2-\beta_H}log_2\frac{1}{2-\beta_H}\right]\\ &-\frac{\alpha\beta_H+(2-\beta_H)(3-2\alpha)}{6}\left[\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha\beta_H+(2-\beta_H)(3-2\alpha)}log_2\frac{\alpha\beta_H+(3-2\alpha)(1-\beta_H)}{\alpha\beta_H+(2-\beta_H)(3-2\alpha)}\right.\\ &\left.+\frac{3-2\alpha}{\alpha\beta_H+(2-\beta_H)(3-2\alpha)}log_2\frac{3-2\alpha}{\alpha\beta_H+(2-\beta_H)(3-2\alpha)}\right]\\ &-\frac{\alpha(2-\beta_H)+\beta_H(3-2\alpha)}{6}\left[\frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H)+(3-2\alpha)\beta_H}log_2\frac{\alpha(1-\beta_H)+(3-2\alpha)\beta_H}{\alpha(2-\beta_H)+(3-2\alpha)\beta_H}\right.\\ &\left.+\frac{\alpha}{\alpha(2-\beta_H)+(3-2\alpha)\beta_H}log_2\frac{\alpha}{\alpha(2-\beta_H)+(3-2\alpha)\beta_H}\right],\end{aligned} \tag{5}$$

which is strictly decreasing in α such that the exchange market efficiency is strictly increasing in α when $\alpha \in \left(0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}\right)$. When $\alpha \to \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}^-$, $\beta_H \to 1$, and then $H(\Theta|X) < \frac{1}{3}$ always hold for $\forall g \to \frac{R_H-R_L}{2}^-$, thus the exchange market efficiency is highest when $\alpha \to \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}^-$, and it is higher than $\frac{2}{3}$.

When $\alpha \in \left[\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}, \frac{1}{2}\right)$, $\beta_H = 1$ by Equation 1.13 and $\beta_L = 0$ by Equation 1.14. Hence $H(\Theta|X) = -\frac{3-\alpha}{3}\left[\frac{\alpha}{3-\alpha}log_2\frac{\alpha}{3-\alpha}+\frac{3-2\alpha}{3-\alpha}log_2\frac{3-2\alpha}{3-\alpha}\right]$, which is strictly increasing in α such that the exchange market efficiency is strictly decreasing in α. Hence, for $\forall g \to \frac{R_H-R_L}{2}^-$, $H(\Theta|X) < \frac{1}{3}$ always hold when $\alpha = \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$, thus the exchange market efficiency when $\alpha = \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$ is the highest, and it is higher than $\frac{2}{3}$.

Besides, We can get

$$lim_{\alpha\to\frac{1}{2}^-}H(\Theta|X) = -\frac{1}{6}\left[log_2\frac{1}{5}+4log_2\frac{4}{5}\right] = 0.6016, \tag{6}$$

then the exchange market efficiency when $\alpha \to \frac{1}{2}^-$ is 0.3984, which is lower than $\frac{2}{3}$.

Thus, the exchange market efficiency is single-peaked when $\alpha \in (0, \frac{1}{2})$, and it reaches the peak when $\alpha = \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}$. Therefore, there exist α_1 and α_2 with $0 < \alpha_1 < \alpha_2 < \frac{1}{2}$, such that Proposition 1.4 holds. □

Proof of Proposition 1.5

Proof. When $\alpha \in \left(0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}\right)$, the informed investor's venue choice is $\beta_L = 0$ and $\beta_H = \frac{4\alpha(H-L)+16g\alpha(1-\alpha)}{3(1-\alpha)[(1-\alpha)(H-L)+2g(3\alpha-1)]}$. By Proposition 5, we know α_1 is the cutoff such that $1 - H(\Theta|X)$ in Equation 5 equals to $\frac{2}{3}$. That is,

$$1 - H(\Theta|X) = \frac{2}{3}. \tag{7}$$

From Equation 1.23 and Equation 1.24, we know the real economic efficiency on average is $\frac{R_H+R_L}{2} + \beta_H g(1 - \frac{2\alpha}{3})$ with dark pool. It is $\frac{R_H+R_L}{2} + \frac{2g}{3}$ without dark pool. Thus α_3 is the cutoff such that

$$\frac{R_H + R_L}{2} + \beta_H g(1 - \frac{2\alpha}{3}) = \frac{R_H + R_L}{2} + \frac{2g}{3},$$

or simply,

$$\beta_H(1 - \frac{2\alpha}{3}) = \frac{2}{3}. \tag{8}$$

Figure 9 compares α_1, which makes the exchange market efficiency indifferent without and with dark pool, with α_3, which makes the real economic efficiency indifferent without and with dark pool. It is clearly that $\alpha_3 < \alpha_1$ because $\beta_H(1 - \frac{2\alpha}{3}) > 1 - H(\Theta|X)$ for $\forall\alpha \in \left(0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}\right)$.

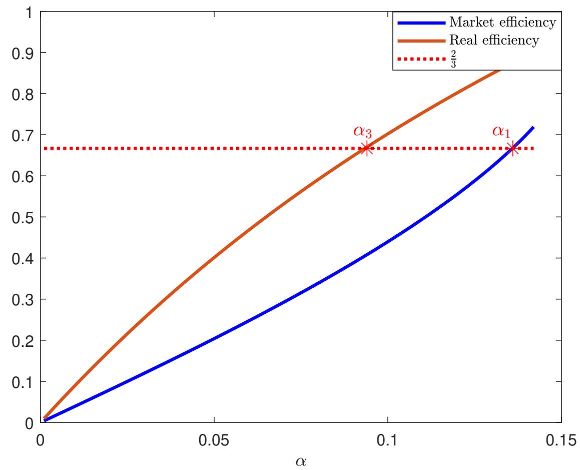

Figure 9: α of market efficiency and real economic efficiency

We assume $g = k * (R_H - R_L)$ where $k \to \frac{1}{2}^-$, then solve for Equation 8 gives

$$\alpha_3 = \frac{7+22k}{48k} + \frac{-196+1072k-1936k^2}{192k(-343+2814k-1524k^2+10088k^3+144\sqrt{5\sqrt{-49k^2+404k^3-768k^4+1520k^5-112k^6}})^{1/3}} - \frac{(-343+2814k-1524k^2+10088k^3+144\sqrt{5\sqrt{-49k^2+404k^3-768k^4+1520k^5-112k^6}})^{1/3}}{48k} \tag{9}$$

Plug Equation 9 into $1 - H(\Theta|X)$, and then we can find that $lim_{k \to \frac{1}{2}^-} 1 - H(\Theta|X) < \frac{2}{3}$ for $\forall \alpha \in \left(0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}\right)$. Therefore, $\alpha_1 > \alpha_3$. $\square$

Proof of Lemma 1.5

Proof. Without dark pool, $\alpha = 1$ and $\beta_H = \beta_L = 1$, state H is fully revealed for $X = (1, 2)$ and state L is fully revealed for $X = (-2, -1)$. Thus the firm value at state H is $R_H + \frac{2}{3}g$ and the firm value at state L is $R_L + \frac{2}{3}g$. Hence, the ex-ante firm value is $\frac{R_H+R_L}{2} + \frac{2}{3}g$.

With managerial learning, the informed investor's venue choice is $\beta_L = 0$ and $\beta_H = \frac{4\alpha(H-L)+16g\alpha(1-\alpha)}{3(1-\alpha)[(1-\alpha)(H-L)+2g(3\alpha-1)]}$ when $\alpha \in (0, \frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)})$. Given Table 1.5, the firm value at state H is $R_H + \left(2\beta_H - \frac{4\alpha\beta_H}{3} - 1 + \frac{2\alpha}{3}\right) g$, and the firm value at state L is $R_L + (1 - \frac{2\alpha}{3})g$.

The informed investor's venue choice is $\beta_H = 1, \beta_L = 0$ when $\alpha \in [\frac{2g-(R_H-R_L)}{2g-3(R_H-R_L)}, \frac{1}{2})$. Hence, the firm value is $R_H + (1-\frac{2\alpha}{3})g$ at state H and $R_L + (1-\frac{2\alpha}{3})g$ at state L.

The informed investor's venue choice is $\beta_H = 1$ and $\beta_L = 1$ when $\alpha \in [\frac{1}{2}, 1)$. Hence, the firm value is $R_H + (1-\frac{\alpha}{3})g$ at state H and $R_L + (1-\frac{\alpha}{3})g$ at state L. □

Proof of Proposition 1.6

Proof. Payoffs in exchange market and dark pool The positively informed investor's expected payoff is

$$V_H^N = \frac{\alpha}{3}\frac{\big(\alpha\beta_L^N + (3-2\alpha)(1-\beta_L^N)\big)(R_H - R_L)}{\alpha(\beta_H^N + \beta_L^N) + (3-2\alpha)(2-\beta_H^N-\beta_L^N)} + (1-\frac{2\alpha}{3})\frac{\alpha(1-\beta_L^N)(R_H-R_L)}{\alpha(2-\beta_H^N-\beta_L^N)+(3-2\alpha)\beta_H^N}$$

For a negatively informed investor, with probability $\frac{\alpha}{3}$, the total trading volume is -2, and the expected payoff is 0. With probability $\frac{\alpha}{3}$, the total trading volume is -1, and the expected payoff is $\frac{\alpha}{3}\frac{(\alpha\beta_H^N+(3-2\alpha)(1-\beta_H^N))(R_H-R_L)}{\alpha(\beta_H^N+\beta_L^N)+(3-2\alpha)(2-\beta_H^N-\beta_L^N)}$. With probability $1-\frac{2\alpha}{3}$, the total trading volume is 0, the expected payoff is $(1-\frac{2\alpha}{3})\frac{\alpha(1-\beta_H^N)(R_H-R_L)}{\alpha(2-\beta_H^N-\beta_L^N)+(3-2\alpha)\beta_L^N}$.

Thus the negatively informed investor's expected payoff is

$$V_L^N = \frac{\alpha}{3}\frac{\big(\alpha\beta_H^N + (3-2\alpha)(1-\beta_H^N)\big)(R_H - R_L)}{\alpha(\beta_H^N + \beta_L^N) + (3-2\alpha)(2-\beta_H^N-\beta_L^N)} + (1-\frac{2\alpha}{3})\frac{\alpha(1-\beta_H^N)(R_H-R_L)}{\alpha(2-\beta_H^N-\beta_L^N)+(3-2\alpha)\beta_L^N}$$

The informed investor's expected payoff from trading in the dark pool is

$$DP_H^N = \frac{1-\alpha}{3}\big(R_H - P(X=0)\big) = \frac{1-\alpha}{3}\frac{\big(\alpha\beta_L^N + (3-2\alpha)(1-\beta_L^N)\big)(R_H-R_L)}{\alpha(\beta_H^N+\beta_L^N)+(3-2\alpha)(2-\beta_H^N-\beta_L^N)}$$

when she is positively informed, and it is

$$DP_L^N = \frac{1-\alpha}{3}\big(P(X=0) - R_L\big) = \frac{1-\alpha}{3}\frac{\big(\alpha\beta_H^N + (3-2\alpha)(1-\beta_H^N)\big)(R_H-R_L)}{\alpha(\beta_H^N+\beta_L^N)+(3-2\alpha)(2-\beta_H^N-\beta_L^N)}.$$

when she is negatively informed.

Pure Strategy Equilibrium If the informed investor's expected payoff from both buying and selling activities in the exchange market is higher than those from the dark pool, she prefers the exchange market regardless of the information type. When she prefers both buying and selling in the exchange market, $\beta_H^N = \beta_L^N = 1$ for $\forall\alpha \in [\frac{1}{2}, 1)$ if and only if $\frac{\alpha}{3}\frac{R_H-R_L}{2} > \frac{1-\alpha}{3}\frac{R_H-R_L}{2}$, given that her expected payoff is $\frac{\alpha}{3}\frac{R_H-R_L}{2}$ in the exchange market and $\frac{1-\alpha}{3}\frac{R_H-R_L}{2}$ in the dark pool.

Similarly, if the informed investor's expected payoff from both buying and selling activities in the dark pool is higher than those from trading in the exchange market, she prefers the dark pool regardless of the information type and $\beta_H^N = \beta_L^N = 0$ for $\forall\alpha \in (0, 1)$. Thus $(1 - \frac{\alpha}{3})\frac{R_H-R_L}{2} < \frac{1-\alpha}{3}\frac{R_H-R_L}{2}$, given that her expected payoff is $(1 - \frac{\alpha}{3})\frac{R_H-R_L}{2}$ in the exchange market and $\frac{1-\alpha}{3}\frac{R_H-R_L}{2}$ in the dark pool. However, we can not find any parameters to satisfy the conditions, thus there is not pure strategy $\beta_H^N = \beta_L^N = 0$.

If the informed investor prefers buying in the exchange market ($\beta_H^N = 1$) and selling in the dark pool ($\beta_L^N = 0$), then $\frac{\alpha}{3}\frac{3-2\alpha}{3-\alpha}(R_H - R_L) + (1 - \frac{2\alpha}{3})\frac{\alpha}{3-\alpha}(R_H - R_L) > \frac{1-\alpha}{3}\frac{3-2\alpha}{3-\alpha}(R_H - R_L)$ and $\frac{\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L) < \frac{1-\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L)$, which means her expected payoff from buying in the exchange market is higher than that from the dark pool and her expected payoff from selling in the exchange market is lower than that from the dark pool. Both conditions hold for $\forall\alpha \in (\frac{1}{3}, \frac{1}{2})$.

If the informed investor prefers buying in the dark pool ($\beta_H^N = 0$) and selling in the exchange market ($\beta_L^N = 1$), $\frac{\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L) < \frac{1-\alpha}{3}\frac{\alpha}{3-\alpha}(R_H - R_L)$ and $\frac{\alpha}{3}\frac{3-2\alpha}{3-\alpha}(R_H - R_L) + (1 - \frac{2\alpha}{3})\frac{\alpha}{3-\alpha}(R_H - R_L) > \frac{1-\alpha}{3}\frac{3-2\alpha}{3-\alpha}(R_H - R_L)$, which means her expected payoff from buying in the exchange market is lower than that from the dark pool and her expected payoff from selling in the exchange market is higher than that from the dark pool. Both conditions hold for $\forall\alpha \in (\frac{1}{3}, \frac{1}{2})$.

Based on the above analysis, there are three pure strategy equilibrium: $\beta_H^N = \beta_L^N = 1$ for $\forall \alpha \in (\frac{1}{2}, 1)$, $\beta_H^N = 1, \beta_L^N = 0$ for $\forall \alpha \in (\frac{1}{3}, \frac{1}{2})$, and $\beta_H^N = 0, \beta_L^N = 1$ for $\forall \alpha \in (\frac{1}{3}, \frac{1}{2})$.

Mixed Strategy Equilibrium If the informed investor is indifferent between trading in the exchange market and trading in the dark pool, then her expected gross gains are the same from trading in both venues and there are mixed strategies. There are three possible cases to consider: $\beta_H^N > \beta_L^N$, $\beta_H^N < \beta_L^N$, and $\beta_H^N = \beta_L^N = \beta^N \in (0, 1)$.

When $\beta_H^N > \beta_L^N$, $V_H^N > V_L^N$. If $V_H^N = DP_H^N$ and $DP_L^N > V_L^N$, $\beta_H^N = \frac{12\alpha - 8\alpha^2}{9 - 24\alpha + 21\alpha^2 - 6\alpha^3} > \beta_L^N = 0$ for $\forall \alpha \in (0.3299, \frac{1}{3})$. If $V_H^N > DP_H^N$ and $V_L^N = DP_L^N$, $\beta_H^N = 1 > \beta_L^N \in (0, 1)$ only exists when $\alpha = \frac{1}{2}$, thus we do not take it into account.

When $\beta_H < \beta_L^N$, $V_H^N < V_L^N$. If $V_L^N = DP_L^N$ and $DP_H^N > V_H^N$, $\beta_L = \frac{12\alpha - 8\alpha^2}{9 - 24\alpha + 21\alpha - 6\alpha^3} > \beta_H^N = 0$ for $\forall \alpha \in (0.3299, \frac{1}{3})$. If $V^N(-) > DP_L^N$ and $DP_H^N = V_H^N$, $\beta_L^N = 1 > \beta_H \in (0, 1)$ hold only when $\alpha = \frac{1}{2}$, thus we do not take it into account.

When $\beta_H^N = \beta_L^N \in (0, 1)$, there are $V_H^N = V_L^N = DP_H^N = DP_L^N$. Thus $\beta_H^N = \beta_L^N = \beta^N = \frac{4\alpha}{3 - 4\alpha + 4\alpha^2}$ for $\forall \alpha \in (0, \frac{1}{2})$.

Monotonicity in β^N When $V_H^N = DP_H^N$ and $V_L^N = DP_L^N$, $\beta_H^N = \beta_L^N = \beta^N = \frac{4\alpha}{3 - 4\alpha + 4\alpha^2}$ for $\forall \alpha \in (0, \frac{1}{2})$. Taking the first order derivative of β^N with respect to α gives: $\frac{d\beta^N}{d\alpha} = \frac{12 - 16\alpha^2}{(3 - 4\alpha + 4\alpha^2)^2}$. The numerator $12 - 16\alpha^2 > 0$ for $\forall \alpha \in (0, \frac{1}{2})$. Thus for $\forall \alpha \in (0, \frac{1}{2})$, $\frac{d\beta^N}{d\alpha} > 0$. We find $lim_{\alpha \to \frac{1}{2}^-} \beta^N = 1$. β^N is continuously increasing in α. □

Proof of Proposition 1.7

Proof. We measure the impact of managerial learning on price informativeness through the following measure:

$$\triangle I = I(\Theta|X) - I^N(\Theta|X) = H^N(\Theta|X) - H(\Theta|X)$$

which is the difference between mutual information with and without managerial learning. If $\triangle I > 0$, managerial learning improves the exchange market efficiency; if $\triangle I < 0$, managerial learning reduces the exchange market efficiency; otherwise, managerial learning does not change the exchange market efficiency. $\square$

Appendix to Chapter 2

Proof of Proposition 2.1

Proof. Given the liquidity investor's trading venue choice α_{BE} and α_{SE}, and the informed investor's trading venue choice β_{HE}, β_{LE}, there are 16 potential possible pure strategy equilibria. After checking the real effects d at the total trading volume $X = (-1, 0, 1)$, we have:

1. When $\alpha_{BE} = 0$, $\alpha_{SE} = 1$, $\beta_{HE} = 1$, and $\beta_{LE} = 0$, $q(H|-1) = 0$, $q(H|0) = \frac{1}{3}$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, -1, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned} VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L - 2g) - k \\ &= \frac{2(R_H - R_L - 2g)}{9} - k \end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned} VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L - 2g) - k \\ &= -k. \end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\
&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H + 2g) - k \\
&= \frac{R_L - R_H}{2} - k - g
\end{aligned}
$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\
&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H + 2g) - k \\
&= \frac{R_L - R_H + 2g}{3} - k
\end{aligned}
$$

Solving $LVH < -\delta$, $LVL > -\delta$, $VH > 0$, and $VL < 0$ gives $0 < k < \frac{2(R_H - R_L - 2g)}{9}$, and $\frac{1}{3}(R_H + 3k - R_L - 2g) < \delta < \frac{1}{2}(R_H + 2k - R_L + 2g)$.

2. When $\alpha_{BE} = 0$, $\alpha_{SE} = 1$, $\beta_{HE} = 1$, and $\beta_{LE} = 1$, $q(H|-1) = 0$, $q(H|0) = 1$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, 1, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$
\begin{aligned}
VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L + 2g) - k \\
&= -k
\end{aligned}
$$

and the negatively informed investor's trading profit is given by

$$
\begin{aligned}
VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L + 2g) - k \\
&= -k.
\end{aligned}
$$

The positive liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\
&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H - 2g) - k \\
&= \frac{R_L - R_H}{2} - k - g
\end{aligned}
$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\
&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H - 2g) - k \\
&= -k
\end{aligned}
$$

Solving $LVH < -\delta$, $LVL > -\delta$, $VH > 0$, and $VL > 0$ leads to contradiction.

3. When $\alpha_{BE} = 1$, $\alpha_{SE} = 0$, $\beta_{HE} = 0$, and $\beta_{LE} = 0$, $q(H|-1)$ does not exist.

4. When $\alpha_{BE} = 1$, $\alpha_{SE} = 0$, $\beta_{HE} = 0$, and $\beta_{LE} = 1$, $q(H|-1) = 0$, $q(H|0) = \frac{2}{3}$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, 1, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$
\begin{aligned}
VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L + 2g) - k \\
&= -k
\end{aligned}
$$

and the negatively informed investor's trading profit is given by

$$
\begin{aligned}
VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L + 2g) - k \\
&= \frac{2(R_H - R_L + 2g)}{9} - k
\end{aligned}
$$

The positive liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\
&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H - 2g) - k \\
&= \frac{R_L - R_H - 2g}{3} - k
\end{aligned}
$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\
&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H - 2g) - k \\
&= \frac{R_L - R_H}{2} - k + x
\end{aligned}
$$

Solving $LVH > -\delta$, $LVL < -\delta$, $VH < 0$, and $VL > 0$ gives $0 < g < \frac{R_H - R_L}{10}$, $0 < k < \frac{2(R_H - R_L + 2g)}{9}$,and $\frac{R_H - R_L + 2g + 3k}{3} < \delta < \frac{1}{2}(R_H - R_L) + k - x$.

5. When $\alpha_{BE} = 1$, $\alpha_{SE} = 0$, $\beta_{HE} = 1$, and $\beta_{LE} = 0$, $q(H|-1) = 0$, $q(H|0) = 0$, and $q(H|1) = \frac{2}{3}$, and the manager's decision is $d = (-1, -1, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$
\begin{aligned}
VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L - 2g) - k \\
&= \frac{2(R_H - R_L + 2g)}{9} - k
\end{aligned}
$$

and the negatively informed investor's trading profit is given by

$$
\begin{aligned}
VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L - 2g) - k \\
&= -k.
\end{aligned}
$$

The positive liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\
&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H + 2g) - k \\
&= \frac{R_L - R_H - 2g}{3} - k
\end{aligned}
$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\
&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H + 2g) - k \\
&= \frac{R_L - R_H}{2} - k + g
\end{aligned}
$$

Solving $LVH > -\delta$, $LVL < -\delta$, $VH > 0$, and $VL < 0$ gives $0 < k < \frac{2(R_H - R_L + 2g)}{9}$, and $\frac{R_H - R_L + 2g}{3} + k < \delta < \frac{R_H - R_L}{2} + k - g$.

6. When $\alpha_{BE} = 1$, $\alpha_{SE} = 0$, $\beta_{HE} = 1$, and $\beta_{LE} = 1$, $q(H|-1) = 0$, $q(H|0) = 0$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, -1, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$
\begin{aligned}
VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L - 2g) - k \\
&= -k
\end{aligned}
$$

and the negatively informed investor's trading profit is given by

$$
\begin{aligned}
VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L - 2g) - k \\
&= -k.
\end{aligned}
$$

The positive liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\
&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H + 2g) - k \\
&= -k
\end{aligned}
$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$
\begin{aligned}
LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\
&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H + 2g) - k \\
&= \frac{R_L - R_H}{2} - k + x.
\end{aligned}
$$

Solving $LVH > -\delta$, $LVL < -\delta$, $VH > 0$, and $VL > 0$ leads to contradiction.

7. When $\alpha_{BE} = 0$, $\alpha_{SE} = 0$, $\beta_{HE} = 0$, and $\beta_{LE} = 0$, $q(H|-1) = \frac{1}{2}$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = \frac{1}{2}$, and the manager's decision is $d = (0, 0, 0)$. The positively informed investor's trading profit in the exchange market is given by

$$
\begin{aligned}
VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L) - k \\
&= \frac{R_H - R_L}{2} - k
\end{aligned}
$$

and the negatively informed investor's trading profit is given by

$$
\begin{aligned}
VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L) - k \\
&= \frac{R_H - R_L}{2} - k.
\end{aligned}
$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned} LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L) \\ &+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H) - k \\ &= -k \end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned} LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L) \\ &+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H) - k \\ &= -k \end{aligned}$$

Solving $LVH < -\delta$, $LVL < -\delta$, $VH < 0$, and $VL < 0$ gives $k > \delta > 0$ and $k > \frac{R_H - R_L}{2}$.

8. When $\alpha_{BE} = 0$, $\alpha_{SE} = 0$, $\beta_{HE} = 0$, and $\beta_{LE} = 1$, $q(H|-1) = 0$, $q(H|0) = 1$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, 1, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned} VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L + 2g) - k \\ &= -k \end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned} VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L + 2g) - k \\ &= -k. \end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\ &+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H - 2g) - k \\ &= \frac{R_L - R_H}{2} - k - g\end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\ &+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H - 2g) - k \\ &= \frac{R_L - R_H}{2} - k + g.\end{aligned}$$

Solving $LVH < -\delta$, $LVL < -\delta$, $VH < 0$, and $VL > 0$ leads to contradiction.

9. When $\alpha_{BE} = 0$, $\alpha_{SE} = 0$, $\beta_{HE} = 1$, and $\beta_{LE} = 0$, $q(H|-1) = 0$, $q(H|0) = 0$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, -1, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned}VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L - 2g) - k \\ &= -k\end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned}VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L - 2g) - k \\ &= -k.\end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g)\\&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H + 2g) - k\\&= \frac{R_L - R_H}{2} - k - g\end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g)\\&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H + 2g) - k\\&= \frac{R_L - R_H}{2} - k + g.\end{aligned}$$

Solving $LVH < -\delta$, $LVL < -\delta$, $VH > 0$, and $VL < 0$ leads to contradiction.

10. When $\alpha_{BE} = 0$, $\alpha_{SE} = 1$, $\beta_{HE} = 0$, and $\beta_{LE} = 0$, $q(H|1) = 1$ does not exist;

11. When $\alpha_{BE} = 0$, $\alpha_{SE} = 0$, $\beta_{HE} = 1$, and $\beta_{LE} = 1$, $q(H|-1) = 0$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, 0, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned}VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L) - k\\&= -k\end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned}VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L) - k\\&= -k.\end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned} LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\ &+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H) - k \\ &= -k \end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned} LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\ &+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H) - k \\ &= -k. \end{aligned}$$

Solving $LVH < -\delta$, $LVL < -\delta$, $VH > 0$, and $VL > 0$ leads to contradiction.

12. When $\alpha_{BE} = 0$, $\alpha_{SE} = 1$, $\beta_{HE} = 0$, and $\beta_{LE} = 1$, $q(H|-1) = \frac{1}{3}$, $q(H|0) = 1$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, 1, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned} VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L + 2g) - k \\ &= -k \end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned} VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L + 2g) - k \\ &= \frac{2(R_H - R_L - 2g)}{9} - k. \end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g)\\&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H - 2g) - k\\&= \frac{R_L - R_H}{2} - k - g\end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g)\\&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H - 2g) - k\\&= \frac{R_L - R_H + 2g}{3} - k.\end{aligned}$$

Solving $LVH < -\delta$, $LVL > -\delta$, $VH < 0$, and $VL > 0$ gives $0 < k < \frac{2(R_H - R_L - 2g)}{9}$, and $\frac{1}{3}(R_H - R_L - 2g + 3k) < \delta < \frac{1}{2}(R_H - R_L) + k + g$.

13. When $\alpha_{BE} = 1$, $\alpha_{SE} = 1$, $\beta_{HE} = 0$, and $\beta_{LE} = 0$, $q(H|-1) = \frac{1}{2}$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = \frac{1}{2}$, and the manager's decision is $d = (0, 0, 0)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned}VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L) - k\\&= \frac{R_H - R_L}{3} - k\end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned}VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L) - k\\&= \frac{R_H - R_L}{3} - k.\end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}
LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L) \\
&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H) - k \\
&= -k
\end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}
LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L) \\
&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H) - k \\
&= -k
\end{aligned}$$

Solving $LVH > -\delta$, $LVL > -\delta$, $VH < 0$, and $VL < 0$ gives $k > \frac{R_H - R_L}{3}$, and $\delta > k$.

14. When $\alpha_{BE} = 1$, $\alpha_{SE} = 1$, $\beta_{HE} = 0$, and $\beta_{LE} = 1$, $q(H|-1) = \frac{1}{2}$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = 1$, and the manager's decision is $d = (0, 0, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned}
VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L) - k \\
&= \frac{R_H - R_L}{6} - k
\end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned}
VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L) - k \\
&= \frac{R_H - R_L}{3} - k.
\end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\ &+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H) - k \\ &= \frac{R_L - R_H}{4} - k\end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L) \\ &+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H) - k \\ &= -k\end{aligned}$$

Solving $LVH > -\delta$, $LVL > -\delta$, $VH < 0$, and $VL > 0$ gives $\frac{R_H - R_L}{6} < k < \frac{R_H - R_L}{3}$, and $\delta > \frac{1}{4}(R_H - R_L + 4k)$.

15. When $\alpha_{BE} = 1$, $\alpha_{SE} = 1$, $\beta_{HE} = 1$, and $\beta_{LE} = 0$, $q(H|-1) = 0$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = \frac{1}{2}$, and the manager's decision is $d = (-1, 0, 0)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned}VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L) - k \\ &= \frac{R_H - R_L}{3} - k\end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned}VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L) - k \\ &= \frac{R_H - R_L}{6} - k.\end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned} LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L) \\ &+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H) - k \\ &= \frac{R_L - R_H}{4} - k \end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned} LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\ &+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H) - k \\ &= \frac{R_L - R_H}{4} - k \end{aligned}$$

Solving $LVH > -\delta$, $LVL > -\delta$, $VH > 0$, and $VL < 0$ gives $\frac{R_H - R_L}{6} < k < \frac{R_H - R_L}{3}$, and $\delta > \frac{1}{4}(R_H - R_L + 4k)$.

16. When $\alpha_{BE} = 1$, $\alpha_{SE} = 1$, $\beta_{HE} = 1$, and $\beta_{LE} = 1$, $q(H|-1) = 0$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, 0, 1)$. The positively informed investor's trading profit in the exchange market is given by

$$\begin{aligned} VH &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(L|1)(R_H - R_L + 2g) + \frac{\alpha_{SE}}{3}P(L|0)(R_H - R_L) - k \\ &= \frac{R_H - R_L}{6} - k \end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned} VL &= (1 - \frac{\alpha_{BE} + \alpha_{SE}}{3})P(H|-1)(R_H - R_L - 2g) + \frac{\alpha_{BE}}{3}P(H|0)(R_H - R_L) - k \\ &= \frac{R_H - R_L}{6} - k. \end{aligned}$$

The positive liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}
LVH &= (\frac{1-\beta_{LE}}{2})P(H|1)(R_L - R_H - 2g) + \frac{1-\beta_{HE}}{2}P(L|1)(R_H - R_L + 2g) \\
&+ \frac{\beta_{LE}}{2}P(H|0)(R_L - R_H) - k \\
&= \frac{R_L - R_H}{4} - k
\end{aligned}$$

and the negative liquidity investor's trading profit in the exchange market is given by

$$\begin{aligned}
LVL &= (\frac{1-\beta_{HE}}{2})P(L|-1)(R_L - R_H + 2g) + \frac{1-\beta_{LE}}{2}P(H|-1)(R_H - R_L - 2g) \\
&+ \frac{\beta_{HE}}{2}P(L|0)(R_L - R_H) - k \\
&= \frac{R_L - R_H}{4} - k.
\end{aligned}$$

Solving $LVH > -\delta$, $LVL > -\delta$, $VH < 0$, and $VL < 0$ gives $0 < k < \frac{R_H - R_L}{6}$, and $\delta > \frac{1}{4}(R_H - R_L + 4k)$.

□

Proof of Proposition 2.3

Proof. Intuitively, the informed investor has incentives to trade in the dark pool to earn positive profits, and the liquidity investor has incentives to trade in the dark pool to save cost. Therefore, possible equilibrium with dark pool trading should only include cases in which both investors trade in the dark pool and demand opposite positions. Besides, the dark pool does not affect the investors' trading profits from the exchange market. Thus, in the presence of the dark pool, the informed investor's trading profits from the exchange market are the same as Equation 2.7 and Equation 2.8, and the liquidity investor's trading profits from the exchange market are the same as Equation 2.9 and Equation 2.10. We summarize these cases as follows:

1. When $\alpha_{BE} = 0$, $\alpha_{SE} = 1$, $\beta_{HE} = 1$, $\beta_{LE} = 0$, $\alpha_{BD} = 1$, $\alpha_{SD} = 0$, $\beta_{HD} = 0$, and $\beta_{LD} = 1$, $q(H|-1) = 0$, $q(H|0) = \frac{1}{3}$, and $q(H|1) = 1$, and the manager's decision is $d = (-1, -1, 1)$. The positively informed investor's trading profit in the dark pool is given by

$$\begin{aligned} DH &= \frac{\alpha_{SD}}{3} P(L|0)(R_H - R_L - 2g) \\ &= 0 \end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned} DL &= \frac{\alpha_{BD}}{3} P(H|0)(R_H - R_L - 2g) \\ &= \frac{R_H - R_L - 2g}{9} \end{aligned}$$

The positive liquidity investor's trading profit in the dark pool is given by

$$\begin{aligned} LDH &= \frac{\beta_{LD}}{2} P(H|0)(R_L - R_H + 2g) + (1 - \frac{\beta_{LD}}{2})(-\delta) \\ &= \frac{R_L - R_H}{6} - \frac{\delta}{2} + \frac{g}{3} \end{aligned}$$

and the negative liquidity investor's trading profit in the dark pool is given by

$$\begin{aligned} LDL &= \frac{\beta_{BD}}{2} P(L|0)(R_L - R_H + 2g) + (1 - \frac{\beta_{BD}}{2})(-\delta) \\ &= -\delta \end{aligned}$$

Solving $LDH > LVH$, $LDH > -\delta$, $LVL > LDL$, $LVL > -\delta$, $VH > DH$, $VH > 0$, $DL > VL$, and $DL > 0$ gives $\frac{1}{9}(2R_H - 2R_L - 4x) > k > 0$ and $\frac{1}{3}(2R_H - 2R_L + 6k + 8g) > \delta > \frac{1}{3}(R_H - R_L + 3k - 2g)$.

2. When $\alpha_{BE} = 0$, $\alpha_{SE} = 0$, $\beta_{HE} = 0$, $\beta_{LE} = 0$, $\alpha_{BD} = 1$, $\alpha_{SD} = 0$, $\beta_{HD} = 0$, and $\beta_{LD} = 1$, $q(H|-1) = \frac{1}{2}$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = \frac{1}{2}$, and the manager's decision is $d = (0, 0, 0)$. The positively informed investor's trading profit in the dark pool is given

by

$$DH = \frac{\alpha_{SD}}{3} P(L|0)(R_H - R_L)$$
$$= 0$$

and the negatively informed investor's trading profit is given by

$$DL = \frac{\alpha_{BD}}{3} P(H|0)(R_H - R_L)$$
$$= \frac{R_H - R_L}{6}$$

The positive liquidity investor's trading profit in the dark pool is given by

$$LDH = \frac{\beta_{LD}}{2} P(H|0)(R_L - R_H) + (1 - \frac{\beta_{LD}}{2})(-\delta)$$
$$= \frac{R_L - R_H}{4} - \frac{\delta}{2}$$

and the negative liquidity investor's trading profit in the dark pool is given by

$$LDL = \frac{\beta_{BD}}{2} P(L|0)(R_L - R_H) + (1 - \frac{\beta_{BD}}{2})(-\delta)$$
$$= -\delta$$

Solving $LDH > LVH$, $LDH > -\delta$, $-\delta > LVL$, $-\delta > LDL$, $VH \leq 0$, $DH \leq 0$, $DL > 0$, and $DL > VL$ gives $k > \frac{R_H - R_L}{2}$ and $k > \delta > \frac{R_H - R_L}{2}$.

3. When $\alpha_{BE} = 0$, $\alpha_{SE} = 0$, $\beta_{HE} = 0$, $\beta_{LE} = 0$, $\alpha_{BD} = 0$, $\alpha_{SD} = 1$, $\beta_{HD} = 1$, and $\beta_{LD} = 0$, $q(H| - 1) = \frac{1}{2}$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = \frac{1}{2}$, and the manager's decision is $d = (0, 0, 0)$. The positively informed investor's trading profit in the dark pool is given by

$$DH = \frac{\alpha_{SD}}{3} P(L|0)(R_H - R_L)$$
$$= \frac{R_H - R_L}{6}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned} DL &= \frac{\alpha_{BD}}{3} P(H|0)(R_H - R_L) \\ &= 0 \end{aligned}$$

The positive liquidity investor's trading profit in the dark pool is given by

$$\begin{aligned} LDH &= \frac{\beta_{LD}}{2} P(H|0)(R_L - R_H) + (1 - \frac{\beta_{LD}}{2})(-\delta) \\ &= -\delta \end{aligned}$$

and the negative liquidity investor's trading profit in the dark pool is given by

$$\begin{aligned} LDL &= \frac{\beta_{BD}}{2} P(L|0)(R_L - R_H) + (1 - \frac{\beta_{BD}}{2})(-\delta) \\ &= \frac{R_L - R_H}{4} - \frac{\delta}{2} \end{aligned}$$

Solving $LDH \leq -\delta$, $LVH \leq -\delta$, $LDL > LVL$, $LDL > -\delta$, $DH > VH$, $DH > 0$, $0 \geq DL$, and $0 \geq VL$ gives $k > \frac{R_H - R_L}{2}$ and $k > \delta > \frac{R_H - R_L}{2}$.

4. When $\alpha_{BE} = 0$, $\alpha_{SE} = 0$, $\beta_{HE} = 0$, $\beta_{LE} = 0$, $\alpha_{BD} = 1$, $\alpha_{SD} = 1$, $\beta_{HD} = 1$, and $\beta_{LD} = 1$, $q(H|-1) = \frac{1}{2}$, $q(H|0) = \frac{1}{2}$, and $q(H|1) = \frac{1}{2}$, and the manager's decision is $d = (0, 0, 0)$. The positively informed investor's trading profit in the dark pool is given by

$$\begin{aligned} DH &= \frac{\alpha_{SD}}{3} P(L|0)(R_H - R_L) \\ &= \frac{R_H - R_L}{6} \end{aligned}$$

and the negatively informed investor's trading profit is given by

$$\begin{aligned} DL &= \frac{\alpha_{BD}}{3} P(H|0)(R_H - R_L) \\ &= \frac{R_H - R_L}{6} \end{aligned}$$

The positive liquidity investor's trading profit in the dark pool is given by

$$\begin{aligned} LDH &= \frac{\beta_{LD}}{2} P(H|0)(R_L - R_H) + (1 - \frac{\beta_{LD}}{2})(-\delta) \\ &= \frac{R_L - R_H}{4} - \frac{\delta}{2} \end{aligned}$$

and the negative liquidity investor's trading profit in the dark pool is given by

$$\begin{aligned} LDL &= \frac{\beta_{BD}}{2} P(L|0)(R_L - R_H) + (1 - \frac{\beta_{BD}}{2})(-\delta) \\ &= \frac{R_L - R_H}{4} - \frac{\delta}{2} \end{aligned}$$

Solving $LDH > -\delta$, $LDH > LVH$, $LDL > LVL$, $LDL > -\delta$, $DH > VH$, $DH > 0$, $DL > 0$, and $DL > VL$ gives $k > \frac{R_H - R_L}{2}$ and $\frac{1}{2}(R_L - R_H + 4k) > \delta > k$.

□

Appendix to Chapter 3

Proof of Table 3.1

Proof. Under the *NT*, *X=-1* includes four situations. First, the industry is in the high state *H*, the insider presents in the financial market with probability β and does not trade, and the noise trader sells. The probability of this situation is $\beta * \frac{1}{3}$. Second, the industry is in the high state *H*, the insider does not present in the financial market with probability $1-\beta$, and the noise trader sells. The probability of this situation is $(1-\beta) * \frac{1}{3}$. Third, the industry is in the low state *L*, the insider presents in the financial market with probability β and does not trade, and the noise trader sells. The probability of this situation is $\beta * \frac{1}{3}$. Last, the industry is in the low state *L*, the insider does not present in the financial market with probability $1-\beta$, and the noise trader sells. The probability of this situation is $(1-\beta) * \frac{1}{3}$. Based on these four situations, the probability that *X=-1* and the industry is on the high state *H* is $P(X=-1|H) * P(H) = \left(\beta * \frac{1}{3} + (1-\beta) * \frac{1}{3}\right) * \frac{1}{2}$, the probability that *X=-1* and the industry is on the low state *L* is $P(X=-1|L) * P(L) = \left(\beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3}\right) * \frac{1}{2}$, thus the probability that *X=-1* is $\frac{1}{2} * \left(\beta * \frac{1}{3} + (1-\beta) * \frac{1}{3} + \beta * \frac{1}{3} + (1-\beta) * \frac{1}{3}\right)$. We can solve for the probability that the industry is in the high state *H* conditional on observing trading volume *X=-1*: $\alpha(-1) = Pr(H|X=-1) = 1/2$. Following the same logic, we can solve for the probability that the industry is in the high state *H* conditional on observing trading volume *X=0*: $\alpha(0) = Pr(H|X=0) = 1/2$, and the probability that the industry is in the high state H conditional on observing trading volume *X=1*: $\alpha(1) = Pr(H|X = 1) = 1/2$. *X=-2* and *X=-1* are off equilibrium with posteriors *0* and *1*.

Under the *SNB*, *X=-1* includes four situations. First, the industry is in the high state H, the insider presents in the financial market with probability β and does not trade, and the noise trader sells. The probability of this situation is $\frac{1}{2} * \beta * \frac{1}{3}$. Second, the industry is in the high state H, the insider does not present in the financial market with probability $1-\beta$, and the noise trader sells. The probability of this situation is $\frac{1}{2}*(1-\beta)*\frac{1}{3}$. Third, the industry is in the low state L, the insider presents in the financial market with probability β and sells, and the noise trader does not trade. The probability of this situation is $\frac{1}{2}*\beta*\frac{1}{3}$. Last, the industry is in the low state L, the insider does not present in the financial market with probability $1-\beta$, and the noise trader sells. The probability of this situation is $\frac{1}{2}*(1-\beta)*\frac{1}{3}$. Based on these four situations, the probability that *X=-1* conditional on the high state H is $\frac{1}{2}*\beta*\frac{1}{3}+\frac{1}{2}*(1-\beta)*\frac{1}{3}$, the probability that *X=-1* conditional on the low state L is $\frac{1}{2} * \beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3}$, thus the probability that *X=-1* is $\frac{1}{2} * \beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3} + \frac{1}{2} * \beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3}$. We can solve for the probability that the industry is in the high state H conditional on observing trading volume *X=-1*: $\alpha(-1) = Pr(H|X = -1) = 1/2$. Following the same logic, we can solve for the probability that the industry is in the high state H conditional on observing trading volume *X=0*: $\alpha(0) = Pr(H|X = 0) = 1/2$, and the probability that the industry is in the high state H conditional on observing trading volume *X=1*: $\alpha(1) = Pr(H|X = 1) = \frac{1}{2-\beta}$. The probability that the industry is in the high state H conditional on observing trading volume *X=-2*: $\alpha(-2) = 0$, and *X=2* are off equilibrium with posterior *1*.

Under the *BNS*, *X=-1* includes three situations. First, the industry is in the high state H, the insider does not presents in the financial market with probability $1-\beta$, and the noise trader sells. The probability of this situation is $\frac{1}{2} * (1-\beta) * \frac{1}{3}$. Second, the industry is in the low state L, the insider does not present in the financial market with probability $1-\beta$, and the noise trader sells. The probability of this situation is $\frac{1}{2} * (1-\beta) * \frac{1}{3}$. Third, the industry is in the low state L, the insider presents in the financial market with probability β and sells, and the noise trader does not trade. The probability of this situation is $\frac{1}{2} * \beta * \frac{1}{3}$. Based on these three situations, the probability that *X=-1* conditional on the high state H is $\frac{1}{2}*(1-\beta)*\frac{1}{3}$, the probability that *X=-1* conditional on the low state L is $\frac{1}{2}*\beta*\frac{1}{3}+\frac{1}{2}*(1-\beta)*\frac{1}{3}$, thus the probability that *X=-1* is $\frac{1}{2} * (1-\beta) * \frac{1}{3} + \frac{1}{2} * \beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3}$. We can solve for the probability that the industry is in the high state H conditional on observing trading volume *X=-1*: $\alpha(-1) = Pr(H|X = -1) = \frac{1-\beta}{2-\beta}$. Following the same logic, we can solve for the probability that the industry is in the high state H conditional on observing trading volume *X=0*: $\alpha(0) = Pr(H|X = 0) = 1/2$, and the probability that the industry is in the high state H conditional on observing trading volume *X=1*: $\alpha(1) = Pr(H|X = 1) = \frac{1}{2}$. The probability that the industry is in the high state H conditional on observing trading volume *X=2*: $\alpha(2) = 1$, and *X=-2* are off equilibrium with posterior *0*.

Under the T, *X=-1* includes three situations. First, the industry is in the high state H, the insider does not presents in the financial market with probability $1-\beta$, and the noise trader sells. The probability of this situation is $\frac{1}{2} * (1-\beta) * \frac{1}{3}$. Second, the industry is in the low state L, the insider does not present in the financial market with probability $1-\beta$, and the noise trader sells. The probability of this situation is $\frac{1}{2} * (1-\beta) * \frac{1}{3}$. Third, the industry is in the low state L, the insider presents in the financial market with probability β and sells, and the noise trader does not trade. The probability of this situation is $\frac{1}{2} * \beta * \frac{1}{3}$. Based on these three situations, the probability that *X=-1* conditional on the high state H is $\frac{1}{2}*(1-\beta)*\frac{1}{3}$, the probability that *X=-1* conditional on the low state L is $\frac{1}{2}*\beta*\frac{1}{3}+\frac{1}{2}*(1-\beta)*\frac{1}{3}$, thus the probability that *X=-1* is $\frac{1}{2} * (1-\beta) * \frac{1}{3} + \frac{1}{2} * \beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3}$. We can solve for the probability that the industry is in the high state H conditional on observing trading volume *X=-1*: $\alpha(-1) = Pr(H|X = -1) = \frac{1-\beta}{2-\beta}$. Following the same logic, we can solve for the probability that the industry is in the high state H conditional on observing trading volume *X=0*: $\alpha(0) = Pr(H|X = 0) = 1/2$, and the probability that the industry is in the high state H conditional on observing trading volume *X=1*: $\alpha(1) = Pr(H|X = 1) = \frac{1}{2-\beta}$.

The probability that the industry is in the high state H conditional on observing trading volume *X=2* and *X=-2* are fully revealed with posteriors *1* and *0*. □

Proof of Lemma 1

Proof. Sell-Not Buy Equilibrium: SNB

Under *the pure strategy equilibrium SNB*, the insider's expected gross gain is 0 when she receives a positive information. If she deviates to buying, with probability $p = \frac{1}{3}$, $X = 2$, and he is fully revealed, and her payoff is 0. With probability $p = \frac{1}{3}$, $X = 1$, her payoff is $\frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$. With probability $p = \frac{1}{3}$, $X = 0$, her payoff is $\frac{1}{2}(V_{1H}^M - V_{1L}^M)$, thus her expected gross gain from deviating to buying is given by: $\frac{1}{3} * \frac{1}{2} * (V_{1H}^M - V_{1L}^M) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M) \equiv k_{NF}$. If the insider receives negative information and sells, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{-1, 0\}$, she receives $\frac{1}{2}(V_{1H}^M + V_{1L}^M)$ and pays V_{1L}^M per share, her payoff is $\frac{1}{2}(V_{1H}^M - V_{1L}^M)$, thus her expected gross gain from selling is $\frac{1}{3}(V_{1H}^M - V_{1L}^M) \equiv k_{NT}$. If she deviates to not trading, her expected gross gain is 0. Thus her expected gross gain from deviating to not trading is $-k_{NT}$.

Thus if if and only if $k_{NF} \leq k < k_{NT}$, the SNB equilibrium is sustainable.

Buy-Not Sell Equilibrium: BNS

Under *the pure strategy equilibrium BNS*, under positively informed insider's equilibrium strategy of buying, with probability $p = \frac{1}{3}$, $X = 2$, she is fully revealed and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{0, 1\}$, she receives V_{1H}^M and pays $\frac{1}{2}(V_{1H}^M + V_{1L}^M)$ per share, her payoff is $\frac{1}{2}(V_{1H}^M - V_{1L}^M)$, her expected gross gain is $\frac{1}{3}(V_{1H}^M - V_{1L}^M) \equiv k_{NT}$ from buying. If she deviates to not trading, her expected gross gain is 0. Thus her expected gross gain from deviating to not trading is $-k_{NT}$. Under negatively informed insider's equilibrium strategy of not trading, her expected payoff is 0. If she deviates to selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed, and her payoff is 0. With probability $p = \frac{1}{3}$, $X = -1$, her payoff is $\frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$. With probability $p = \frac{1}{3}$, $X = 0$, and her payoff is $\frac{1}{2}(V_{1H}^M - V_{1L}^M)$. Her expected gross gain from deviating to selling is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^M - V_{1L}^M) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M) \equiv k_{NF}$.

Thus, if and only if $k_{NF} \leq k < k_{NT}$, the BNS equilibrium is sustainable.

Trade Equilibrium: T

Under *the pure strategy equilibrium T*, under negatively informed insider's equilibrium strategy of selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed and her payoff is 0. With probability $p = \frac{1}{3}$, $X = -1$, she receives $\frac{1-\beta}{2-\beta}V_{1H}^M + \frac{1}{2-\beta}V_{1L}^M$ and pays V_{1L}^M per share. With probability $p = \frac{1}{3}$, $X = 0$, she receives $\frac{1}{2}(V_{1L}^M + V_{1H}^M)$ and pays V_{1L}^M per share. Her expected gross gain is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^M - V_{1L}^M) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M) \equiv k_{NF}$. If she deviates to not trading, her expected gross gain is 0. Thus her expected gross gain from deviating to not trading is $-k_{NF}$. Under positively informed insider's equilibrium strategy of buying, her payoff is k_{NF}. If she deviates to not trading, her payoff is 0. His expected gross gain from deviating to not trading is $-k_{NF}$.

Thus, if and only if $k < k_{NF}$, T equilibrium is sustainable. □

Proof of Lemma 2

Proof. Sell Not Buy Equilibrium: SNB

Under *the pure strategy equilibrium SNB*, the insider's expected gross gain is 0 when she receives a positive information. If she deviates to buying, with probability $p = \frac{1}{3}$, $X = 2$, and she is fully revealed, and her payoff is 0. With probability $p = \frac{1}{3}$, $X = 1$, her payoff is $\frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C)$. With probability $p = \frac{1}{3}$, $X = 0$, her payoff is $\frac{1}{2}(V_{1H}^C - V_{1L}^C)$, thus her expected gross gain from deviating to buying is given by: $\frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C) \equiv k_{SNB}^C$. If the insider receives negative information and sells, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{-1, 0\}$, she receives

$\frac{1}{2}(V_{1H}^C + V_{1L}^C)$ and pays V_{1L}^C per share, her payoff is $\frac{1}{2}(V_{1C}^M - V_{1L}^C)$, thus her expected gross gain from selling is $\frac{1}{3}(V_{1H}^C - V_{1L}^C) \equiv k_{NT}^C$. If she deviates to not trading, her expected gross gain is 0. Thus her expected gross gain from deviating to not trading is $-k_{NT}^C$.

Thus if if and only if $k_{SNB}^C \leq k < k_{NT}^C$, the SNB equilibrium is sustainable.

Buy-Not Sell Equilibrium: BNS

Under *the pure strategy equilibrium BNS*, under positively informed insider's equilibrium strategy of buying, with probability $p = \frac{1}{3}$, $X = 2$, she is fully revealed and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{0, 1\}$, she receives V_{1H}^C and pays $\frac{1}{2}(V_{1H}^C + V_{1L}^C)$ per share, her payoff is $\frac{1}{2}(V_{1H}^C - V_{1L}^C)$, her expected gross gain is $\frac{1}{3}(V_{1H}^C - V_{1L}^C) \equiv k_{NT}^C$ from buying. If she deviates to not trading, her expected gross gain is 0. Thus her expected gross gain from deviating to not trading is $-k_{NT}^C$. Under negatively informed insider's equilibrium strategy of not trading, her expected payoff is 0. If she deviates to selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed, and her payoff is 0. With probability $p = \frac{1}{3}$, $X = -1$, her payoff is $\frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C)$. With probability $p = \frac{1}{3}$, $X = 0$, and her payoff is $\frac{1}{2}(V_{1H}^C - V_{1L}^C)$. Her expected gross gain from deviating to selling is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C) \equiv k_{SNB}^C$.

Thus, if and only if $k_{SNB}^C \leq k < k_{NT}^C$, the BNS equilibrium is sustainable.

Trade Equilibrium: T

Under *the pure strategy equilibrium T*, under negatively informed insider's equilibrium strategy of selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed and her payoff is 0. With probability $p = \frac{1}{3}$, $X = -1$, she receives $\frac{1-\beta}{2-\beta}V_{1H}^C + \frac{1}{2-\lambda}V_{1L}^C$ and pays V_{1L}^C per share. With probability $p = \frac{1}{3}$, $X = 0$, she receives $\frac{1}{2}(V_{1L}^C + V_{1H}^C)$ and pays V_{1L}^C per share. Her expected gross gain is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C) \equiv k_{NF}$. If she deviates to not trading, her expected gross gain is 0. Thus her expected gross gain from deviating to not trading is $-k_{NF}^C$. Under positively informed insider's equilibrium strategy of buying, her payoff is k_{SNB}^C. If she deviates to not trading, her payoff is 0. Her expected gross gain from deviating to not trading is $-k^C_{SNB}$.

Thus, if and only if $k < k^C_{SNB}$, T equilibrium is sustainable.

□

Proof of Proposition 1

Proof. No Trade Equilibrium: NT

Now turn to the insider's payoff. Under the positively-informed insider's equilibrium strategy of not trading, her payoff is 0. If she deviates to buying, with probability $p = \frac{1}{3}$, $X = 2$, and she is fully revealed, and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{0, 1\}$, she pays $\frac{1}{2}(V^M_{1H} + V^M_{1L})$ and receives V^M_{1H} per share, thus her expected gross gain from deviating to buying is given by: $\frac{1}{3}(V^M_{1H} - V^M_{1L}) \equiv k_{NT}$. Similarly, if the negatively informed insider deviates to selling, her expected gross gain from deviating is also k_{NT}.

Thus, if and only if $k \geq k_{NT}$, the no trade equilibrium is sustainable.

Sell Not Buy Equilibrium: SNB

Under negatively informed insider's equilibrium strategy of selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{-1, 0\}$, she receives $\frac{1}{2}(V^M_{1H} + V^M_{1L})$ and pays V^M_{1L} per share, her payoff is $\frac{1}{2}(V^M_{1H} - V^M_{1L})$, her expected gross gain is $\frac{1}{3}(V^M_{1H} - V^M_{1L}) \equiv k_{NT}$. If she deviates to not trading, her payoff is 0. Thus her expected gross gain from deviating to not trading is $-k_{NT}$. Under positively informed insider's equilibrium strategy of not trading, her expected payoff is 0. If she deviates to buying, with probability $p = \frac{1}{3}$, $X = 2$, and she is fully revealed with payoff 0. With probability $p = \frac{1}{3}$, her payoff is $\frac{1-\beta}{2-\beta}(V^C_{1H} - V^C_{1L})$. With probability $p = \frac{1}{3}$, $X = 0$, and her payoff is $\frac{1}{2}(V^M_{1H} - V^M_{1L})$. Her expected gross gain from deviating to buying is $\frac{1}{3} * \frac{1}{2} * (V^M_{1H} - V^M_{1L}) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V^C_{1H} - V^C_{1L}) \equiv k_{SNB}$.

Thus, SNB equilibrium is sustainable if and only if $k_{SNB} < k < k_{NT}$.

Buy Not Sell Equilibrium: BNS

Under positively informed insider's equilibrium strategy of buying, with probability $p = \frac{1}{3}$, $X = 2$, she is fully revealed and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{0, 1\}$, she receives V_{1H}^M and pays $\frac{1}{2}(V_{1H}^M + V_{1L}^M)$ per share, her payoff is $\frac{1}{2}(V_{1H}^M - V_{1L}^M)$, her expected gross gain is $\frac{1}{3}(V_{1H}^M - V_{1L}^M) \equiv k_{NT}$. If she deviates to not trading, her payoff is 0. Thus her expected gross gain from deviating to not trading is $-k_{NT}$. Under negatively informed insider's equilibrium strategy of not trading, her expected payoff is 0. If she deviates to selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed, and her payoff is 0. With probability $p = \frac{1}{3}$, $X = -1$, her payoff is $\frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$. With probability $p = \frac{1}{3}$, $X = 0$, and her payoff is $\frac{1}{2}(V_{1H}^M - V_{1L}^M)$. Her expected gross gain from deviating to selling is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^M - V_{1L}^M) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M) \equiv k_{NF}$.

Thus, BNS equilibrium is sustainable if and only if $k_{NF} \leq k < k_{NT}$.

Trade Equilibrium: T

Under negatively informed insider's equilibrium strategy of selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed and her payoff is 0. With probability $p = \frac{1}{3}$, $X = -1$, she receives $\frac{1-\beta}{2-\beta}V_{1H}^M + \frac{1}{2-\beta}V_{1L}^M$ and pays V_{1L}^M per share. With probability $p = \frac{1}{3}$, $X = 0$, she receives $\frac{1}{2}(V_{1L}^M + V_{1H}^M)$ and pays V_{1L}^M per share. Her expected gross gain is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^M - V_{1L}^M) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M) \equiv k_{NF}$. If she deviates to not trading, her payoff is 0. Thus her expected gross gain from deviating to not trading is $-k_{NF}$.

Similarly, under positively informed insider's equilibrium strategy of buying, her payoff is k_{SNB}. If she deviates to not trading, her payoff is 0. Her expected gross gain from deviating to not trading is $-k_{SNB}$.

Thus, T equilibrium is sustainable if and only if $k < k_{SNB}$.

□

Proof of Proposition 2

Proof. No Trade Equilibrium: NT

Now turn to the insider's payoff. Under the positively-informed insider's equilibrium strategy of not trading, her payoff is 0. If she deviates to buying, with probability $p = \frac{1}{3}$, $X = 2$, and she is fully revealed, and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{0, 1\}$, she pays $\frac{1}{2}(V_{1H}^C + V_{1L}^C)$ and receives V_{1H}^C per share, her payoff is $\frac{1}{2}(V_{1H}^C - V_{1L}^C)$, and her expected gross gain from deviating to buying is given by: $\frac{1}{3}(V_{1H}^C - V_{1L}^C) \equiv k_{NT}^C$. Similarly, if the negatively informed insider deviates to selling, her expected gross gain from deviating is also k_{NT}^C. Thus, if and only if $k \geq k_{NT}^C$, the no trade equilibrium is sustainable in the case.

Sell Not Buy Equilibrium: SNB

Under negatively informed insider's equilibrium strategy of selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{-1, 0\}$, she receives $\frac{1}{2}(V_{1H}^C + V_{1L}^C)$ and pays V_{1L}^C per share, her expected gain is $\frac{1}{3}(V_{1H}^C - V_{1L}^C) \equiv k_{NT}^C$. If she deviates to not trading, her payoff is 0. Thus her expected gross gain from deviating to not trading is $-k_{NT}^C$.

Under positively informed insider's equilibrium strategy of not trading, her expected payoff is 0. If she deviates to buying, with probability $p = \frac{1}{3}$, $X = 2$, and she is fully revealed, her payoff is 0. With probability $p = \frac{1}{3}$, $X = 1$, her payoff is $\frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C)$. With probability $p = \frac{1}{3}$, $X = 0$, and her payoff is $\frac{1}{2}(V_{1H}^C - V_{1L}^C)$. Her expected gross gain from deviating to buying is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C) \equiv k_{SNB}^C$.

Thus, SNB equilibrium is sustainable if and only if $k_{SNB}^C \leq k < k_{NT}^C$.

Buy Not Sell Equilibrium: BNS

Under positively informed insider's equilibrium strategy of buying, with probability $p = \frac{1}{3}$, $X = 2$, she is fully revealed and her payoff is 0. With probability $p = \frac{2}{3}$, $X \in \{0, 1\}$, she receives V_{1H}^C and pays $\frac{1}{2}(V_{1H}^C + V_{1L}^C)$ per share, her expected gain is $\frac{1}{3}(V_{1H}^C - V_{1L}^C) \equiv k_{NT}^C$. If she deviates to not trading, her payoff is 0. Thus her expected gross gain from deviating to not trading is $-k_{NT}^C$. Under negatively informed insider's equilibrium strategy of not trading, her expected payoff is 0. If she deviates to selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed, and her payoff is 0. With probability $p = \frac{1}{3}$, $X = -1$, her payoff is $\frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$. With probability $p = \frac{1}{3}$, $X = 0$, and her payoff is $\frac{1}{2}(V_{1H}^C - V_{1L}^C)$. Her expected gross gain from deviating to selling is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M) \equiv k_{NF}^C$.

As $k_{NF}^C \equiv \frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$ and $k_{NT}^C \equiv \frac{1}{3}(V_{1H}^C - V_{1L}^C)$, to make this equilibrium sustainable, we require $k_{NF}^C < k_{NT}^C$, this gives the condition for β: $\beta > \frac{2\left[V_{1H}^M - V_{1L}^M - (V_{1H}^C - V_{1L}^C)\right]}{2(V_{1H}^M - V_{1L}^M) - (V_{1H}^C - V_{1L}^C)}$.

Therefore, when $\beta > \frac{2\left[V_{1H}^M - V_{1L}^M - (V_{1H}^C - V_{1L}^C)\right]}{2(V_{1H}^M - V_{1L}^M) - (V_{1H}^C - V_{1L}^C)}$, BNS equilibrium is sustainable if and only if $k_{NF}^C \leq k < k_{NT}^C$.

Trade Equilibrium: T

Under negatively informed insider's equilibrium strategy of selling, with probability $p = \frac{1}{3}$, $X = -2$, she is fully revealed and her payoff is 0. With probability $p = \frac{1}{3}$, $X = -1$, her payoff is $\frac{1-\beta}{2-\beta}(V_{1H}^M - V_{1L}^M)$. With probability $p = \frac{1}{3}$, $X = 0$, her payoff is $\frac{1}{2}(V_{1L}^C - V_{1H}^C)$. Her expected gain is $\frac{1}{3} * \frac{1}{2} * (V_{1H}^C - V_{1L}^C) + \frac{1}{3} * \frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C) \equiv k_{SNB}^C$. If she deviates to not trading, her payoff is 0. Thus her expected gross gain from deviating to not trading is $-k_{SNB}^C$ if $\alpha < \frac{1-\beta}{2-\beta}$. Similarly, under positively informed insider's equilibrium strategy of buying, with probability $p = \frac{1}{3}$, $X = 2$, she is fully revealed and her payoff is 0. With probability $p = \frac{1}{3}$, $X = 1$, she receives V_{1H}^C and pays $\frac{1}{2-\beta}V_{1H}^C + \frac{1-\beta}{2-\beta}V_{1L}^C$ per share. With probability $p = \frac{1}{3}$, $X = 0$, she receives V_{1H}^C and pays $\frac{1}{2}(V_{1L}^C + V_{1H}^C)$ per share. Her expected payoff is k_{SNB}^C. If she deviates to not trading, her payoff is 0. Therefore, her expected gross gain from deviating to not trading is $-k^C_{SNB}$.

Thus, T equilibrium is sustainable if and only if $k < k^C_{SNB}$. □

Proof of Entry Probability

Proof. If $\alpha_0 < \frac{1}{2-\beta} < \alpha < 1$, entrant does not enter under strategies NT and SNB. Under strategy BNS, it enters when $X = 2$, this is the case when the state is high, insider presents in financial market and buys one shares, and noise trader buys one share, the conditional probability is: $\frac{1}{2} * \beta * \frac{1}{3} = \frac{\beta}{6}$. Under strategy T, entrant enters when $X = 2$, this is the case when the state is high, insider presents in financial market and buys one shares, and noise trader buys one share, the conditional probability is: $\frac{1}{2} * \beta * \frac{1}{3} = \frac{\beta}{6}$.

If $\alpha_0 < \alpha < \frac{1}{2-\beta} < 1$, entrant does not enter under strategy NT. Under strategy BNS, it enters when $X = 2$, this is the case when the state is high, insider presents in financial market and buys one shares, and noise trader buys one share, the conditional probability is: $\frac{1}{2} * \beta * \frac{1}{3} = \frac{\beta}{6}$. Under strategy SNB, it enters when $X = 1$, this is the case when the state is high, insider presents but does not trade, noise trader buys; or the case when the state is high, insider does not present, noise trader buys; or the state is low, insider does not present, noise trader buys. The conditional entry probability is: $\frac{1}{2}*\beta*\frac{1}{3}+\frac{1}{2}*(1-\beta)*\frac{1}{3}+\frac{1}{2}*(1-\beta)*\frac{1}{3} = \frac{1}{6}(2-\beta)$.

If $0 < \frac{1-\beta}{2-\beta} < \alpha < \alpha_0$, entrant always enters. Under strategy NT, entrant enters regardless of trading volume. Under strategy SNB, entrant enters when $X \in \{-1, 0, 1\}$. If $X = -1$, it is the case when state is high, insider presents but does not trade, noise trader sells; or the case when state is high, insider does not present and noise trader sells; or the case when state is low, insider sells and noise trader does not trade; or the case when state is low, insider does not present and noise trader sells, the conditional probability is: $\frac{1}{2} * \beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3} + \frac{1}{2} * \beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3} = \frac{1}{3}$. If $X = 0$, it is the case when state is high, insider presents but does not trade, noise trader does not trade; or the case when state is high, insider does not present and noise trader does not trade; or the case when state is low, insider sells and noise trader buys; or the case when state is low, insider does not present and noise trader does not trade, the conditional probability is: $\frac{1}{2} * \beta * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3} + \frac{1}{2} * (1-\beta) * \frac{1}{3} + \frac{1}{2} * \beta * \frac{1}{3} = \frac{1}{3}$. If $X = 1$, it is the case when state is high, insider presents but does not trade, noise trader buys; or the case when state is high, insider does not present and noise trader buys; or the case when state is low, insider does not present and noise trader buys, the conditional probability is $\frac{1}{6}(2-\beta)$. Therefore, the conditional probability is: $\frac{1}{3} + \frac{1}{3} + \frac{1}{6}(2-\beta) = 1 - \frac{\beta}{6}$. Under the equilibrium BNS, entrant enters when $X \in \{0, 1, 2\}$. If $X = 0$, it is the case when state is high, insider buys, noise trader sells; or the case when state is high, insider does not present and noise trader does not present; or the case when state is low, insider does not trade and noise trader does not trade; or the case when state is low, insider does not present and noise trader does not trade, the conditional probability is $\frac{1}{3}$. If $X = 1$, it is the case when state is high, insider buys, noise trader does not trade; or the case when state is high, insider does not present and noise trader buys; or the case when state is low, insider does not trade and noise trader buys; or the case when state is low, insider does not present and noise trader buys, the conditional probability is $\frac{1}{3}$. If $X = 2$, it is the case when state is high, both insider and noise trader buys, the conditional probability is $\frac{1}{6}\beta$. Therefore, the conditional probability is: $\frac{1}{3} + \frac{1}{3} + \frac{1}{6}\beta = \frac{1}{6}(4+\beta)$. Under equilibrium T, entrant enters when $X \in \{0, 1, 2\}$. If $X = 0$, it is the case when state is high, insider buys, noise trader sells; or the case when state is high, insider does not present and noise trader does not present; or the case when state is low, insider sells and noise trader buys; or the case when state is low, insider does not present and noise trader does not trade, the conditional probability is $\frac{1}{3}$. If $X = 1$, it is the case when state is high, insider buys, noise trader does not trade; or the case when state is high, insider does not present and noise trader buys; or the case when state is low, insider does not present and noise trader buys, the conditional probability is $\frac{1}{6}(2-\beta)$. If

$X = 2$, it is the case when state is high, both insider and noise trader buys, the conditional probability is $\frac{1}{6}\beta$. Therefore, the conditional probability is: $\frac{1}{3} + \frac{1}{6}(2-\beta) + \frac{1}{6}\beta = \frac{2}{3}$.

If $0 < \alpha < \frac{1-\beta}{2-\beta} < \frac{1}{2}$, entrant always enters under strategies NT and BNS. Under strategy SNB, the conditional probability is $1 - \frac{\beta}{6}$. Under strategy T, it enters when $X \in \{-1, 0, 1, 2\}$, the conditional probability is $1 - \frac{\beta}{6}$.

□

Proof of Proposition 4

For simplicity, denote $E_1 = V_{1L}^C$, $E_2 = V_{1L}^C + \frac{1-\beta}{2-\beta}(V_{1H}^C - V_{1L}^C)$, $E_3 = \frac{1}{2}(V_{1H}^C + V_{1L}^C)$, $E_4 = \frac{1}{2-\beta}(V_{1H}^C - V_{1L}^C) + V_{1L}^C$ and $E_5 = V_{1H}^C$, we calculate the ex-ante entry probability by $P(\beta, E, \mu, \sigma, k) = \int_0^\infty p(\beta|E,k)f(E,\mu,\sigma)dE$. Given certain ranges of transaction cost k, taking the conditional probability and density function into it, we obtain:

Transaction cost	Ex-ante entry probability
$k_{NT} < k$	$\frac{1}{2}[erf(\frac{lnE_3-\mu}{\sigma\sqrt{2}}) - erf(\frac{lnE_5-\mu}{\sigma\sqrt{2}})]$
$k_{NF} < k < k_{NT}$	$\frac{2-\beta}{12}erf(\frac{lnE_2-\mu}{\sigma\sqrt{2}}) + \frac{4+\beta}{12}erf(\frac{lnE_3-\mu}{\sigma\sqrt{2}}) - \frac{1}{2}erf(\frac{lnE_5-\mu}{\sigma\sqrt{2}})$
$k_{SNB} < k < k_{NF}$	$\frac{\beta}{12}erf(\frac{lnE_5-\mu}{\sigma\sqrt{2}}) + \frac{1-\beta}{3}[\frac{1}{2}erf(\frac{lnE_4-\mu}{\sigma\sqrt{2}})] + \frac{4+\beta}{12}erf(\frac{lnE_3-\mu}{\sigma\sqrt{2}}) - \frac{1}{2}erf(\frac{lnE_1-\mu}{\sigma\sqrt{2}})$
$k_{NT}^C < k < k_{SNB}$	$\frac{\beta}{12}erf(\frac{lnE_1-\mu}{\sigma\sqrt{2}}) + \frac{2-\beta}{12}erf(\frac{lnE_2-\mu}{\sigma\sqrt{2}}) + \frac{1}{6}erf(\frac{lnE_3-\mu}{\sigma\sqrt{2}}) - \frac{1}{2}erf(\frac{lnE_5-\mu}{\sigma\sqrt{2}})$
$k_{SNB}^C < k < k_{NT}^C$	$\frac{\beta}{12}erf(\frac{lnE_1-\mu}{\sigma\sqrt{2}}) + \frac{2-\beta}{12}erf(\frac{lnE_2-\mu}{\sigma\sqrt{2}}) - \frac{\beta}{12}erf(\frac{lnE_3-\mu}{\sigma\sqrt{2}}) - (\frac{1}{2} - \frac{\beta}{12})erf(\frac{lnE_5-\mu}{\sigma\sqrt{2}})$
$k < k_{SNB}^C$	$\frac{\beta}{12}erf(\frac{lnE_1-\mu}{\sigma\sqrt{2}}) + \frac{2-\beta}{12}erf(\frac{lnE_2-\mu}{\sigma\sqrt{2}}) + \frac{\beta}{6}erf(\frac{lnE_3-\mu}{\sigma\sqrt{2}}) + \frac{2-\beta}{12}erf(\frac{lnE_4-\mu}{\sigma\sqrt{2}})$ $-(\frac{1}{2} - \frac{\beta}{12})erf(\frac{lnE_5-\mu}{\sigma\sqrt{2}})$

Table 4: Ex-ante entry probability