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No Star has ever Passed through our Planetary System

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Abstract

Passage of a field star or a solar companion (such as the hypothetical Nemesis[1,2]) through our planetary system would be a singular event, with far reaching implications. It is shown that no such close passage has taken place since the formation of the planetary system. The Jovian planets are in nearly circular, coplanar orbits, and would have been perturbed into inclined, eccentric orbits by any close stellar passage. We also find that the orbital inclination *i* and eccentricity *e* of the outer planets are relics of the early solar system, and need to be explained by theories of planetary system formation.

1. Introduction

The inclinations and eccentricities of the planetary orbits vary with time due to gravitational interactions between the planets which cause exchange of energy and angular momentum. Applegate *et al.*[3] have numerically integrated the motions of the outer planets more than 100 million years into the past and future. The inclinations and eccentricities of the planets were found to undergo quasi-periodic variations but remained small. The range of variations of i and e and their time averages $<i>$ and $<e>$ over this interval are given in Table I. No secular trend in i and e was seen (except for a possible decrease in the i of Pluto). The orbits did not tend to circularize or align, and we expect this behavior to extend over the 4.6×10^9 yr age of the solar system. A stellar encounter which increased the instantaneous values of i and e would result in similar increases in $<i>$ and $<e>$. Thus the present values of $<i>$ and $<e>$ must either be relics of the formation of the solar system, or the result of a close stellar encounter. Extension of the numerical simulations over 4.6×10^9 yr, and inclusion of a sudden perturbation to i and e, could be used to confirm these assumptions.

Neptune provides the best constraint on a hypothetical close stellar passage because it is in a large, relatively weakly bound orbit and yet has small $<i>$ and $<e>$. There are two scenarios which could account for this: 1) i and e were initially small and have never been increased by a passing star, 2) i and/or e were originally larger and have been reduced to their present values by a passing star. It will be shown [see equations (6), (8) and (9)] that the second scenario is less likely to have led to the low $<i>$ and $<e>$ of Neptune observed today, and only the first scenario need be considered when establishing limits on close stellar passages.

In the first scenario, $<i>$ and $<e>$ are taken to be small initially, so the changes Δi and Δe caused by the closest stellar passage cannot have been much greater than the present $<i>$ and $<e>$. We will show that for a stellar passage to be consistent with the present $<i>$ and $<e>$, the change ΔV in orbital velocity of the planet caused by the star must have been smaller than about ΔV_{max} =

0.084 km/s for Neptune and 0.3 km/s for Uranus. We then compare ΔV_{max} for Neptune with the ΔV generally produced by the passage of a star of mass M_* and initial velocity V_* to find a constraint on S, the distance of closest approach of the star. We will use the dimensionless quantities $m_* = M_*/M_O$ and $v_* = V_*/46$ km/s. [The RMS velocity of stars in the solar neighborhood, relative to the sun, is 46 km/s (ref. 4 and Appendix I)].

The limits we place on the passage of unbound and weakly bound objects are valid for typical interaction geometries, but a star passing parallel to the sun-planet direction produces a much smaller perturbation, and therefore could have come closer. We will show that the probability F that a randomly-directed stellar passage changes the orbital velocity by less than ΔV_{max} is very small for significant violations of the constraint on S (see equations 6 and 7).

2. The Change in Orbital Velocity

For typical stellar velocities ($V_* >\sim 10$ km/s) the change ΔV in orbital velocity occurs in a time much shorter than the orbital period. If ΔV is small compared to the orbital velocity $V_P = (GM_O/a)^{1/2}$, the change in the orbital angular momentum **L** is approximately $\Delta \mathbf{L} = M_P \mathbf{R} \times \Delta \mathbf{V}$ (ref. 6). Here M_P is the planet's mass and **R** is its position relative to the sun. The change in i is given by the change in the direction of **L**: *i.e.*, $|\Delta i| = \Delta V_L / V_P$ (see Appendix II). Here ΔV is given by three orthogonal components: ΔV_L (in the direction of L), ΔV_t (tangential, in the direction of $\mathbf{R} \times \mathbf{L}$) and ΔV_R (in the direction of **R**). The change in the Runge-Lenz vector **A** is $\Delta \mathbf{A} = M_P (\Delta \mathbf{V} \times \mathbf{L} + \Delta \mathbf{L} \times \mathbf{V}_P)$ (ref. 6). Because $e = A/(GM_O M_P^2)$, it is changed by the amount $\Delta e = \Delta A/(GM_O M_P^2)$ during the star's passage[6]. This gives $|\Delta e| = (\Delta V_R^2 + 4\,\Delta V_t^2)^{1/2}\, V_P^{-1}$ (see Appendix II). As mentioned earlier, the perturbation would have increased $<i>$ and $<e>$ above their present values unless $|\Delta i| \leq <i>$ and $|\Delta e| \leq <e>$. If the passage geometry produces a ΔV in the direction of **L**, then $\Delta V_R = \Delta V_t = 0$ and $\Delta e = 0$. However, in this case $|\Delta i|$ is maximized. Conversely, in passage geometries where $|\Delta i|$ is small, $|\Delta e|$ is large.

Thus, a planet's i and/or e would have been increased above the observed values unless $\Delta V \leq \Delta V_{max}$, where

$$\Delta V_{max} \equiv (\langle i \rangle^2 + \langle e \rangle^2)^{1/2} V_P. \qquad (1)$$

Using the values of $\langle i \rangle$ and $\langle e \rangle$ from Table I, we find $\Delta V_{max} = 0.084$ km/s for Neptune and $\Delta V_{U,max} = 0.3$ km/s for Uranus.

3. The Passage of a Star at a Distance Greater than 30 AU

We first treat the case of a star passing outside the orbit of Neptune. The change ΔV in orbital velocity caused by a passing star is the difference between the velocity changes of the planet ΔV_P and of the sun ΔV_O. This difference arises because the star passes at different distances and directions from the sun and the planet (see fig. 1). The change in orbital velocity is $\Delta V = \Delta V_P - \Delta V_O = 2\, G\, M_*\, V_*^{-1}\, (\mathbf{P} / P^2 - \mathbf{S} / S^2)$ in the impulse approximation. Here **P** is the position of the star relative to the planet when they are closest (see fig. 1). When $S >> a$, ΔV is typically about equal to ΔV_{typ}, where

$$\Delta V_{typ} \equiv 2\, G\, M_*\, a\, S^{-2}\, V_*^{-1} \qquad (2)$$

(Appendix III). [For example, if the three bodies are aligned at the time of closest approach, then $P = S \pm a$ and $\Delta V = 2\, G\, M_*\, V_*^{-1}\, (P^{-1} - S^{-1}) \approx \Delta V_{typ}$.] Equation (2) is accurate within a factor of two for $S \geq a$, as can be shown (see Appendix III) by comparison with numerical simulations by Hills (fig. 8 of ref. 7). Thus, we expect that the stellar passage would increase $\langle i \rangle$ and $\langle e \rangle$ above the observed values unless $\Delta V_{typ} \leq \Delta V_{max}$. Using the orbital parameters of Neptune, we find that the star must have passed at a distance

$$S \geq 117\ m_*^{1/2}\ v_*^{-1/2}\ \mathrm{AU}. \qquad (3)$$

The dependence of the smallest permissible value of S on the stellar mass is plotted in Figure 3 for $V_* = 46$ km/s, the RMS velocity of stars with respect to the sun.

4. The Passage of a Low-Mass Star at a Distance Smaller than 30 AU

According to equation (3) the passage of a star of mass $m_* < 0.07\ v_*$ within Neptune's orbit is not excluded. In this case, approximating the perturbation by the differential impulse is incorrect since the distance of closest approach of the star to the sun and to Neptune can be significantly different, and equations (2) and (3) no longer apply.

The perturbation caused by a star passing very close to the sun is easily evaluated. In this case the impulse on the sun is much larger than that on Neptune and dominates the change in orbital velocity. A star passing close enough to the sun will be deflected from its initial trajectory, so the impulse approximation requires modification for stars passing inside the orbit of Neptune. However, the sun's velocity is still changed impulsively—*i.e.*, in a time short compared to the planet's orbital period. Using the hyperbolic stellar trajectory, the change in the sun's velocity can be calculated (Appendix IV) with the result,

$$\Delta V = 2\ G\ M_*\ V_*^{-1}\ [S + a_c]^{-1}, \qquad (4)$$

where $a_c \equiv G\ (M_O + M_*)\ V_*^{-2}$ is the accretion radius[7]. When $S >> a_c$ this expression reduces to $\Delta V = 2\ G\ M_*\ S^{-1}\ V_*^{-1}$, the impulse approximation. When $S << a_c$ and $M_* << M_O$, $\Delta V \approx 2\ m_*\ V_*$: the star delivers a momentum $2\ M_*\ V_*$ to the sun, twice its own initial momentum. This is because the star swings around the sun and its final momentum is the exactly opposite its initial momentum. This is the largest possible change in the star's momentum, and therefore in the sun's as well.

Equation (4) is accurate within a factor of two when $S < a$, by comparison (see Appendix IV) with Hills' numerical simulation (fig. 8 of ref. 7). The perturbation by the star must have satisfied $\Delta V \leq \Delta V_{max}$ to be consistent with the low <*i*> and <*e*> of Neptune (equation 1). Using equation (4), we can place a lower limit on S for a small object of mass $0.0009\ v_*^{-1} \leq m_* <\sim 0.07\ v_*$, given by

$$S \geq [4.55\ (M_* / 0.01\ M_O)\ v_*^{-1} - a_c]\ \text{AU}. \qquad (5)$$

This is plotted in Figure 3 over the appropriate mass range; the velocity V_* is taken as 46 km/s. When $m_* >> 0.0018\ v_*^{-1}$, the star is not deflected appreciably, and equation (5) reduces to $S \geq 4.55\ (M_* / 0.01\ M_O)\ v_*^{-1}$ AU. The orbit of Neptune places no limit on the passage of an object of mass $m_* \leq 0.0009\ v_*^{-1}$, since such an object carries insufficient momentum to change the sun's velocity by more than ΔV_{max}, no matter how close it passes.

5. The Perihelion Passage of a Weakly Bound Object

Equations (3) and (5) are valid only for unbound perturbers, since they were derived assuming hyperbolic stellar trajectories. Hills[8] carried out numerical simulations giving the change in a planet's *e* produced during the perihelion passage of a weakly bound 0.05 M_O or 0.005 M_O object such as Nemesis (the hypothetical brown dwarf solar companion)[1,2]. He concluded that no Oort cloud object as massive as 0.05 M_O has passed through the planetary system since the dissipation of the solar nebula[8]. (He reached the same conclusion for the passage of a 0.05 M_O field star.) Since the change in *e* at each perihelion distance studied by Hills is proportional to the mass of the intruder we can extend his results by scaling Δe to find the largest mass that could have passed without increasing the e of Neptune above the observed $<e>$ (see Appendix V). The results are shown in Figure 3. We find that $M_* < 0.01\ M_\odot$ for weakly bound objects passing through the planetary system.

6. Passage in Special Geometries

The limits on S in equation (3) or (5) apply to stars passing in most directions. However, a star passing closer but parallel to **R** can not be ruled out, since in this case $\Delta V = 0$. The sun and the planet experience identical impulses because the star's path relative to the sun is the same as that relative to the planet except for a difference R / V_* in the time of passage. In a randomly-directed passage it is unlikely that the angle θ between $\mathbf{V}_*$ and **R** would be small enough that a star could pass much closer than the limit from (3) or (5) without noticeably perturbing Neptune. In a passage with small θ, $\Delta V \approx \Delta V_{typ}\,\theta$ (see fig. 2 and Appendix VII). For the perturbation to satisfy $\Delta V \le \Delta V_{max}$, we find $\theta \le \theta_c$ where $\theta_c \equiv \Delta V_{max} / \Delta V_{typ}$. Thus, the solid angle within which the star perturbs the planet by less than ΔV_{max} consists of two cones of opening angle $2\ \theta_c$. The probability that a stellar passage in a random direction would give Neptune a $\Delta V \le \Delta V_{max}$, is the fraction of solid angle subtended by these cones:

$$F_N = 2\ (\pi\ \theta_c^2) / (4\ \pi) = 0.5\ (S / 117\ \mathrm{AU})^4\ v_*^2\ m_*^{-2}. \qquad (6)$$

From equation (1), any change in the orbital velocity of Uranus has been less than $\Delta V_{U,max} = 0.3$ km/s. A star passing close to the sun would change the orbital velocity of Uranus by more than $\Delta V_{U,max}$ unless it passed nearly parallel to the Uranus-sun direction. For the star to leave both Neptune and Uranus in low i and e orbits, it would have to pass within θ_c of Neptune's **R** and within $\theta_{U,c}$ of Uranus's **R**, where $\theta_{U,c} \equiv \Delta V_{U,max} / \Delta V_{typ}$. This is only possible if Uranus, Neptune and the sun are aligned at the time of stellar passage. The fraction of Uranus's orbit for which the alignment would have been sufficiently accurate is $F_U = 4\,\theta_{U,c} / (2\,\pi) = 0.64\ (S / 47.7\ AU)^2\ v_*\ m_*^{-1}$. When $F_U \leq 1$, the probability that the star would have disturbed neither Uranus or Neptune is the product of F_N and F_U:

$$F = F_N\,F_U = 0.0088\ (S / 47.7\ AU)^6\ v_*^3\ m_*^{-3}. \qquad (7)$$

For a very close passage or a very massive star, Saturn would also have to be aligned with Neptune and the sun, giving $F = 2.3\times10^{-4}\ (S / 26\ AU)^8\ v_*^4\ m_*^{-4}$. We see that only a very small fraction F of randomly-directed close stellar passages would occur in the special geometry that leaves the Jovian planets unperturbed.

We will show that, considering the density and velocity distribution of stars in the solar neighborhood, equations (6) and (7) permit us to practically rule out the passage of any star near the sun since the formation of the solar system. The probability that a star with speed near V_* would pass at a distance near S during a time t, is $dp = 2\pi\, n\, t\, V_*\, S\, dS\, dV_*$ (ref. 7), where $n\, dV_*$ is the number density of such stars. We determined dp for the 20 stellar mass classes given in Heisler, Tremaine and Alcock[4]. For our calculations, the velocity distribution $n\, dV_*$ of each stellar class in the sun's frame was approximated by an isotropic Maxwellian. The RMS velocity of the stars *in the sun's frame* was found by adding (in quadrature) the stars' RMS velocity and the sun's velocity (17 km/s), both measured in the local standard of rest. Integrating dp over V_* and S gives the long dash curve in Figure 4. The probability that a star has passed near S without perturbing the Jovian planets is the product $F \times dp$. This was integrated for the 20 stellar species to give the solid curves in Figure 4. It is clear that significant violations of the limits given in Figure 3 are extremely unlikely.

The above analysis was carried out for the first scenario described at the beginning of this note. In the second scenario, Neptune originally had arbitrarily high values of *i* and/or *e*, which were subsequently reduced by a passing star. In this case, the star could have passed closer and changed i and e by more than $\langle i \rangle$ and $\langle e \rangle$, but it is unlikely that large changes of i and e would reduce $\langle i \rangle$ and $\langle e \rangle$ to their present low values. This can only occur for special geometries of the sun, star and Neptune which give ΔV_L, ΔV_R and ΔV_t of the magnitudes and signs appropriate to cancel the previous $\langle i \rangle$ and $\langle e \rangle$ (Appendix VI). The probability of a sufficiently precise geometry is

$$f \leq 2.3\times10^{-7}\ (V_P\ /\ \Delta V_{typ})^3 = 0.062\ (S\ /\ 117\ \mathrm{AU})^6\ v_*^3\ m_*^{-3}. \qquad (8)$$

for passages outside Neptune's orbit, and

$$f \leq 2.3\times10^{-7}\ (V_P\ /\ \Delta V)^3 = 0.00064\ v_*^3\ (S + a_c)^3\ (M_*\ /\ 0.01\ M_\odot)^{-3}. \qquad (9)$$

for closer passages (Appendix VI). Both results are smaller than F_N given in equation (6). Hence, under scenario 2, it is even less likely that a star has passed near the sun.

7. Perturbations to the Orbits of the Other Planets

The limits derived above imply that the i and e of the planets other than Neptune have not changed significantly since the formation of the solar system. It is easily shown that the ΔV of the planets interior to Neptune would be on the same order or smaller than Neptune's. If the star passed at $S > 30$ AU, then this result follows from the linear dependence of ΔV_{typ} on a (equation 2). If the star passed at $S < 30$ AU, then the planets with $a \geq S$ experience the same ΔV, since in this case ΔV is independent of a (equation 4). We have shown earlier that for Neptune, $\Delta V \leq \Delta V_{max} = 0.084$ km/s, so the same limit applies to the ΔV's experienced by the planets interior to Neptune. The largest changes in i and e that could be produced by these ΔV's are:

$\Delta i_{max} = \Delta V_{max}/V_P = 1.55\times10^{-2}\ (a\ /30\ \mathrm{AU})^{1/2}$ and $\Delta e_{max} = 2\ \Delta V_{max}/V_P = 3.1\times10^{-2}\ (a\ /30\ \mathrm{AU})^{1/2}$. The values for the different planets are listed in Table 1.

The ΔV of Pluto can be larger than that of Neptune (equation 2). But, from the constraint on the ΔV of Neptune, the maximum changes in Pluto's i and e are $\Delta i_{max} = 1.55\times10^{-2}\ (40\ \mathrm{AU}\ /\ 30\ \mathrm{AU})^{3/2} = 0.02$ and $\Delta e_{max} = 3.1\times10^{-2}\ (40\ \mathrm{AU}\ /\ 30\ \mathrm{AU})^{3/2} = 0.05$. The present values of Pluto's $\langle i \rangle$ and $\langle e \rangle$ are much larger, and therefore are almost certainly relics of the early solar system and not the result of a stellar passage.

From Table 1 it is clear that the inclinations and eccentricities of Saturn, Uranus and Pluto and the eccentricity of Jupiter almost certainly have not changed significantly since the formation of the solar system. The probability that a star or a disc dark matter object has ever come close enough to change the i or e of any of these planets by more than Δi_{max} or Δe_{max} is no greater than about 1%. The probability that the i of Jupiter and the i and e of Neptune have been changed by more than half of their present values is ~ 3%.

8. Conclusions

The orbit of Neptune has very small inclination angle i and eccentricity e, both of which would be larger if a star had ever passed near the solar system. Using analytic approximations of the changes in i and e produced by a passing star, we have found that no star with mass $\geq 0.1\ M_{\odot}$ has passed through our planetary system, and no object with mass $\geq 0.003\ M_{\odot}$ (*i.e.*, 3 Jupiter masses) has passed within the Earth's orbit (see fig. 3). No weakly bound object of mass $\geq 0.01\ M_{\odot}$ (such as Nemesis) has passed through the planetary system. It is very unlikely that any star or dark matter object has passed close enough to significantly change the i and e of any of the outer planets, so theories of planetary system formation need to explain them.

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TABLE I

Orbital Elements of the Outer Planets

	Values from 200 Myr numerical integration[a]						Maximum perturbations due to passing stars[b]	
	Inclination i (10^{-2} rad)			Eccentricity e (10^{-2})				
Planet	i_{min}	i_{max}	$<i>$	e_{min}	e_{max}	$<e>$	Δi_{max} (10^{-2} rad)	Δe_{max} (10^{-2})
Jupiter	0.3	1.0	0.6	2.5	6.2	4.6	0.6	1.3
Saturn	1.2	1.9	1.6	0.8	8.9	5.4	0.9	1.8
Uranus	1.4	2.2	1.8	0.1	7.6	4.4	1.2	2.5
Neptune	0.8	1.5	1.2	0.01	2.3	1.0	---	---
Pluto	26	30	28	21	28	24	2.3	4.7

[a]From Table III of reference 3.

[b]Largest values consistent with Neptune's $\Delta V \leq \Delta V_{max}$ (see text)

Fig. 1 Diagram of a stellar passage. The orbital velocity of the planet is changed because the passing star accelerates the planet and the sun differently. Vectors **S** and **P** are directed to the star's positions at its closest approach to the sun and to the planet, respectively. The velocity impulses on the sun and on the planet are $\Delta \mathbf{V}_{O} = 2\, G\, M_{*}\, \mathbf{S} / (S^2\, V_{*})$ and $\Delta \mathbf{V}_{P} = 2\, G\, M_{*}\, \mathbf{P} / (P^2\, V_{*})$ in the impulse approximation. For symbols, see text.

Fig. 2 A stellar passage nearly parallel to **R**, the sun-planet separation (*i.e.*, $a\,\theta << S$). Vectors **S** and **P** give the positions of the star's closest approach as measured from the sun and from the planet, respectively. Two views are represented: the upper part of the figure shows the view in the direction opposite $\mathbf{V}_{*}$ (the star's velocity), the lower part of the figure gives the view along the vector $-\mathbf{S}\times\mathbf{V}_{*}$. As can be seen from the lower part of the figure, the circle has radius $a \sin\theta \approx a\,\theta$ so $|\mathbf{P} - \mathbf{S}| \approx a\,\theta$. The angle between **S** and **R** in the plane of the circle is ψ. In this stellar passage, the change in the planet's orbital velocity is $\Delta V \approx (2\, G\, M_{*}\, S^{-2}\, V_{*}^{-1})\, \theta$ (see Appendix VII).

Fig. 3 Minimum distances of passing stars and of weakly bound objects which are consistent with the observed low *i* and *e* of Neptune when the passage direction is not parallel to **R**. The velocity of the passing star is taken as 46 km/s (the present RMS stellar velocity with respect to the sun). The square and circle data points indicate limits for weakly-bound objects derived by scaling the mass from numerical simulations of Hills[8].

Fig. 4 The probability that an intruder has passed within a distance D of the sun. The long dash line indicates the probability that a main sequence star or white dwarf has come closer to the sun than the distance D during the 4.6×10^9 yr age of the solar system, based on the number density and RMS stellar velocities from ref. 4. The solid lines show the probabilities of stars in various mass ranges having come closer than D and, at the same time, not perturbing the Jovian planets sufficiently to increase their *i* and *e* to greater than the observed values. The heavy line is the probability for all main sequence stars and white dwarfs. The short dash curve gives the probability that any dark matter object has come closer than D, assuming that all disc dark matter is 0.07 solar mass brown dwarfs.

Fig. 5 A stellar passage. Vectors **S** and **P** are directed to the star's positions at its closest approach to the sun and to the planet, respectively. The difference in the star's position at its closest approaches to the sun and to the planet is the projection of the planet-sun separation **R** along the star's velocity $\mathbf{V}_{*}$: $(\mathbf{R} \cdot \mathbf{V}_{*})\, \mathbf{V}_{*}/V_{*}^2$. See Appendix III.

Fig. 6 The passage of a light star so close to the sun that the star is appreciably deflected. The position of the star relative to the sun is $\mathbf{R}_*$, which equals $\mathbf{S}$ when they are closest. The angle between $\mathbf{R}_*$ and $\mathbf{S}$ is labeled φ, whose value is φ_0 when R_* is very large. The angle between the incoming and outgoing asymptotes is $2\varphi_0$. The star's direction is deflected through an angle $\chi = 2\varphi_0 - \pi$.

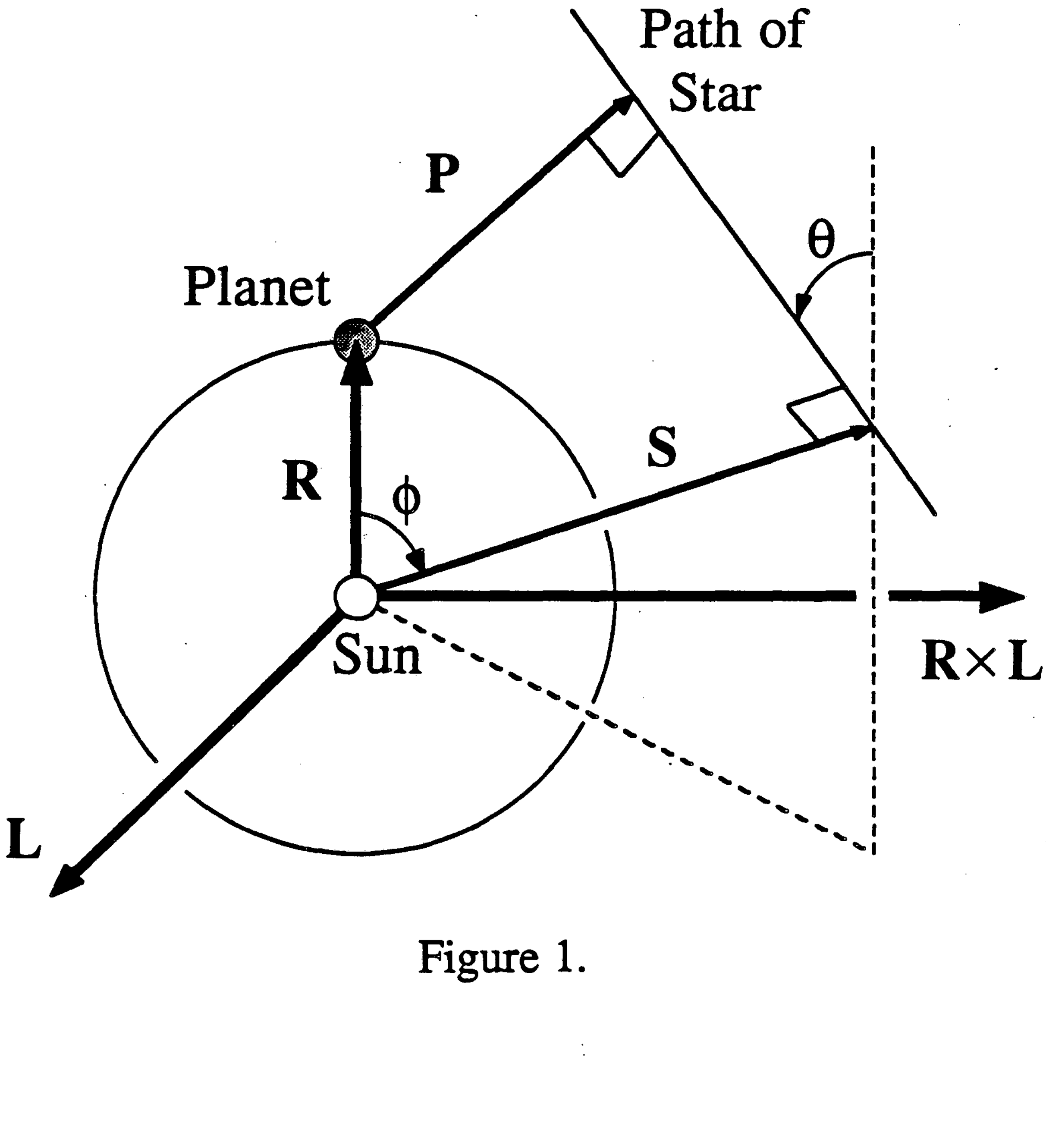

Figure 1.

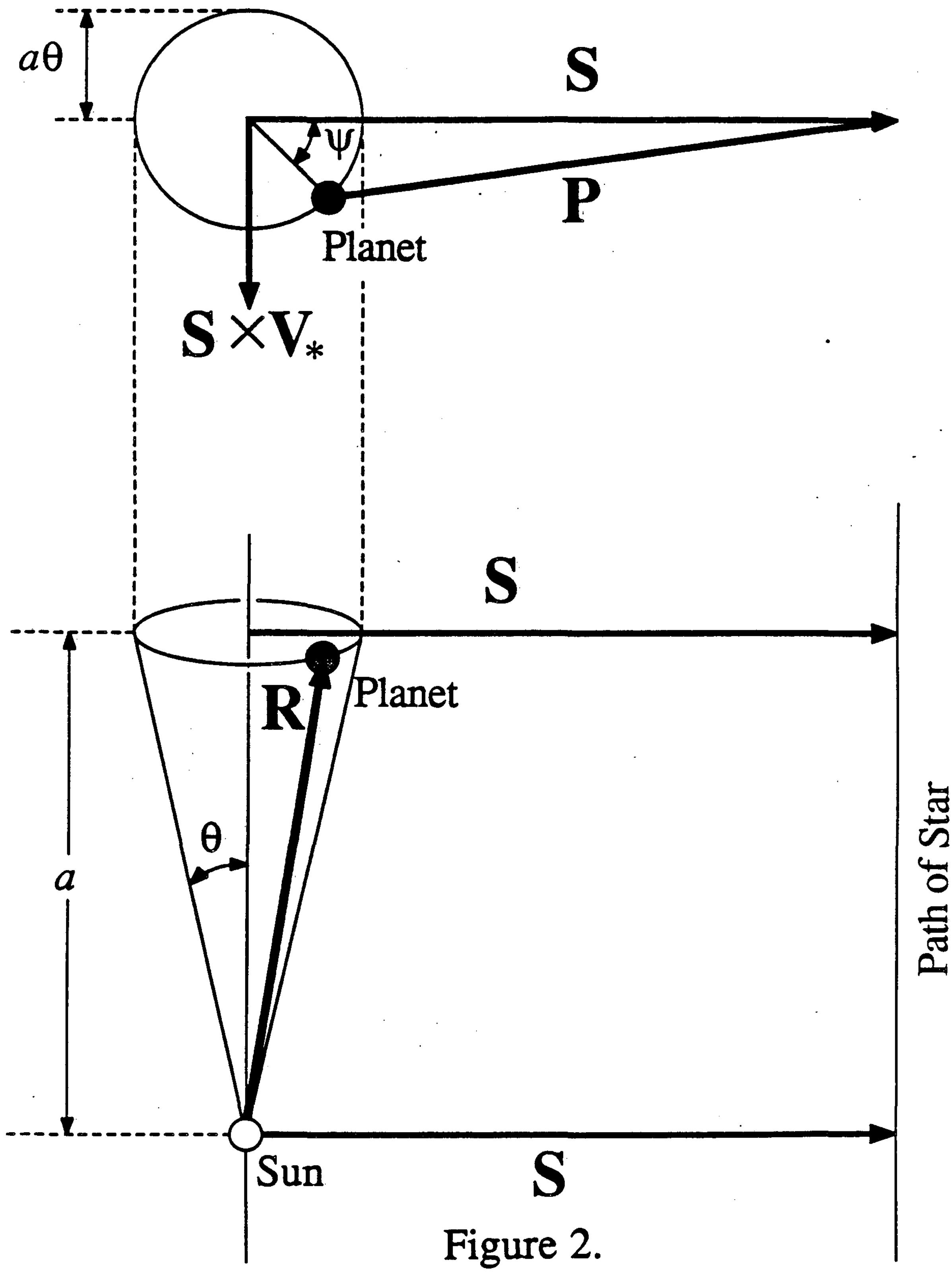

Figure 2.

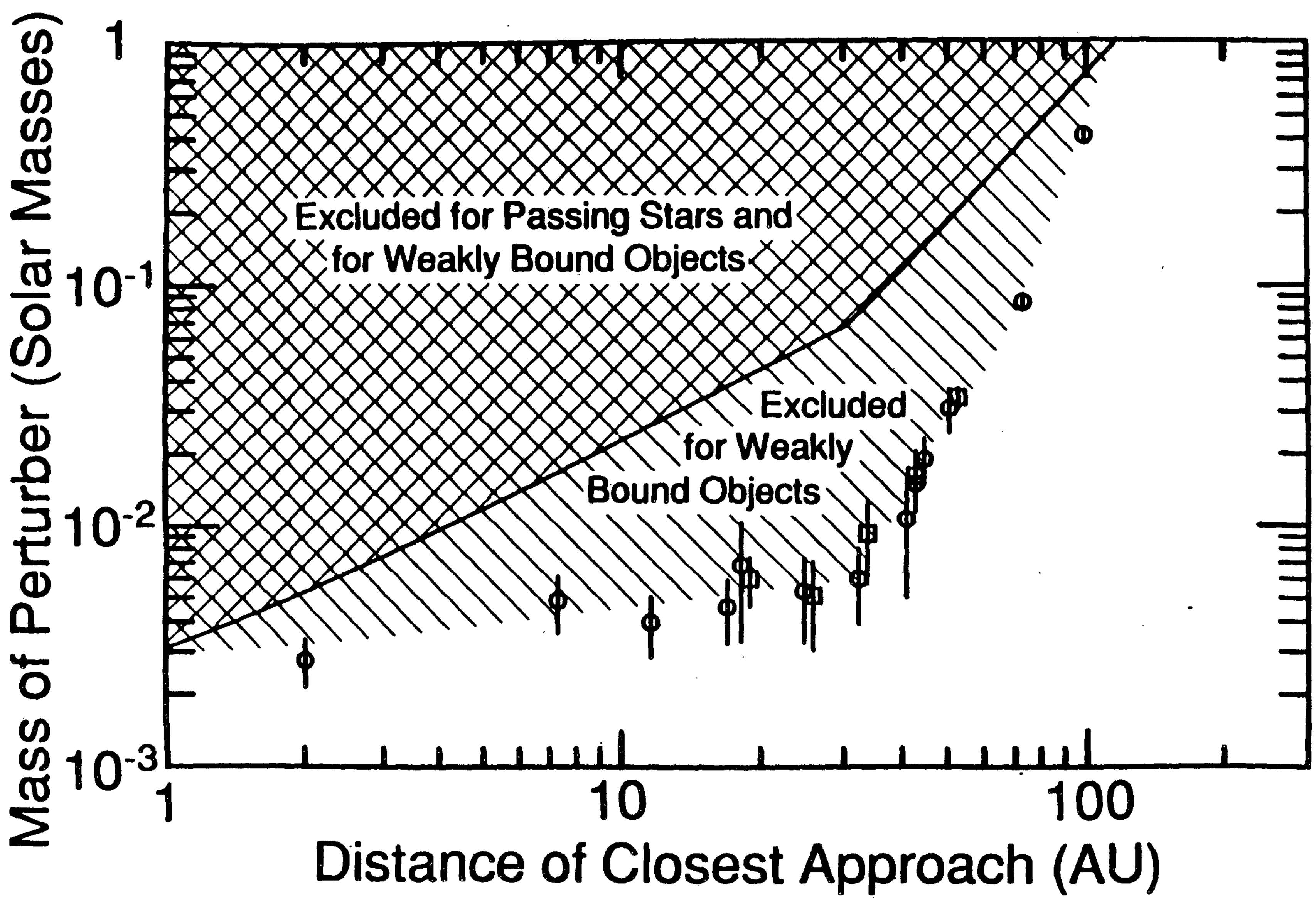
Mass of Perturber (Solar Masses)
1
10-1
10-2
10-3
1
10
100
Distance of Closest Approach (AU)
Excluded for Passing Stars and
for Weakly Bound Objects
Excluded
for Weakly
Bound Objects

Figure 3.

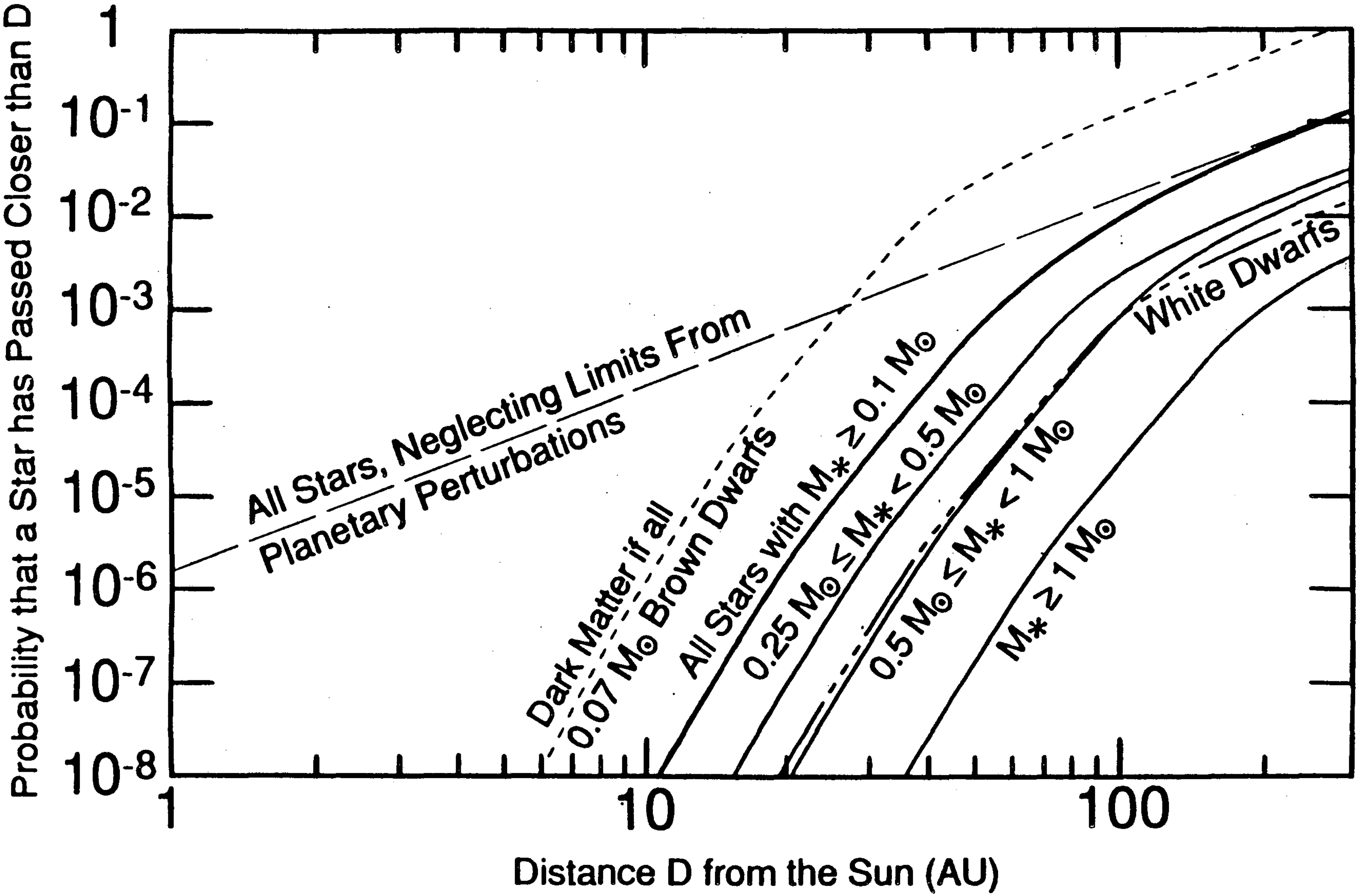
Probability that a Star has Passed Closer than D
1
10-1
10-2
10-3
10-4
10-5
10-6
10-7
10-8
1
10
100
Distance D from the Sun (AU)
All Stars, Neglecting Limits From
Planetary Perturbations
Dark Matter if all
0.07 M⊙ Brown Dwarfs
All Stars with M* ≥ 0.1 M⊙
0.25 M⊙ ≤ M* < 0.5 M⊙
0.5 M⊙ ≤ M* < 1 M⊙
M* ≥ 1 M⊙
White Dwarfs

Figure 4.

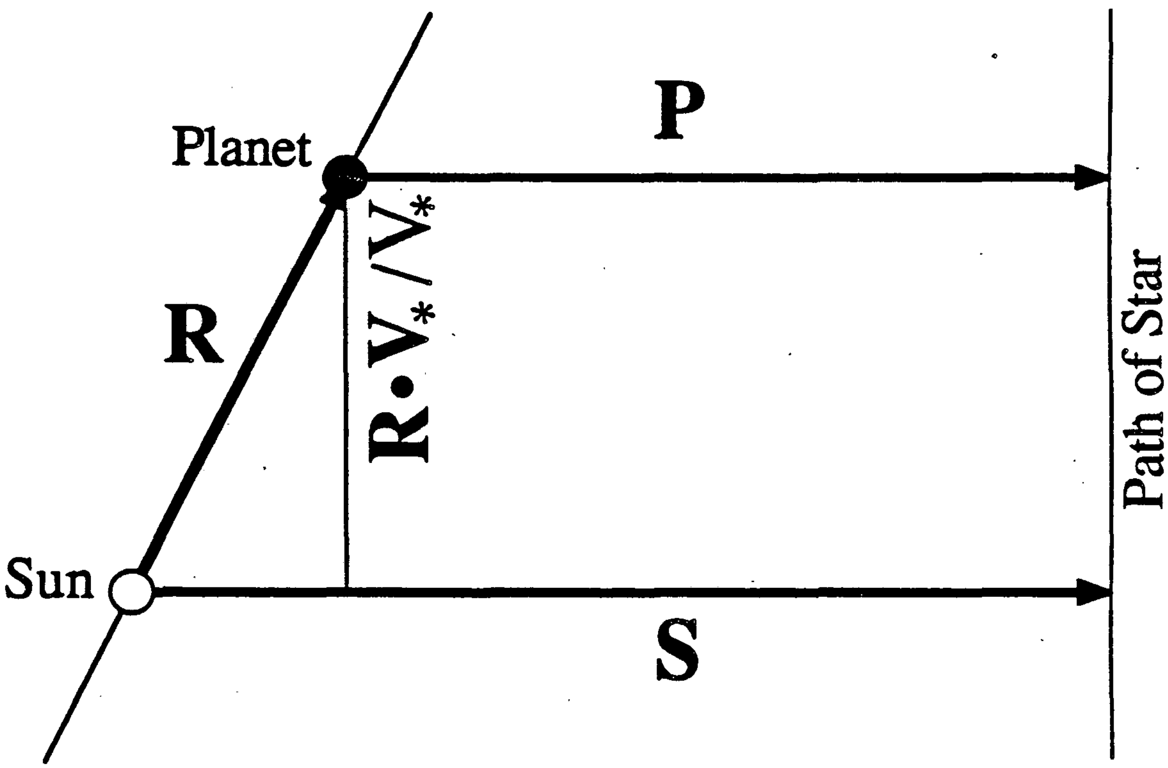

Figure 5.

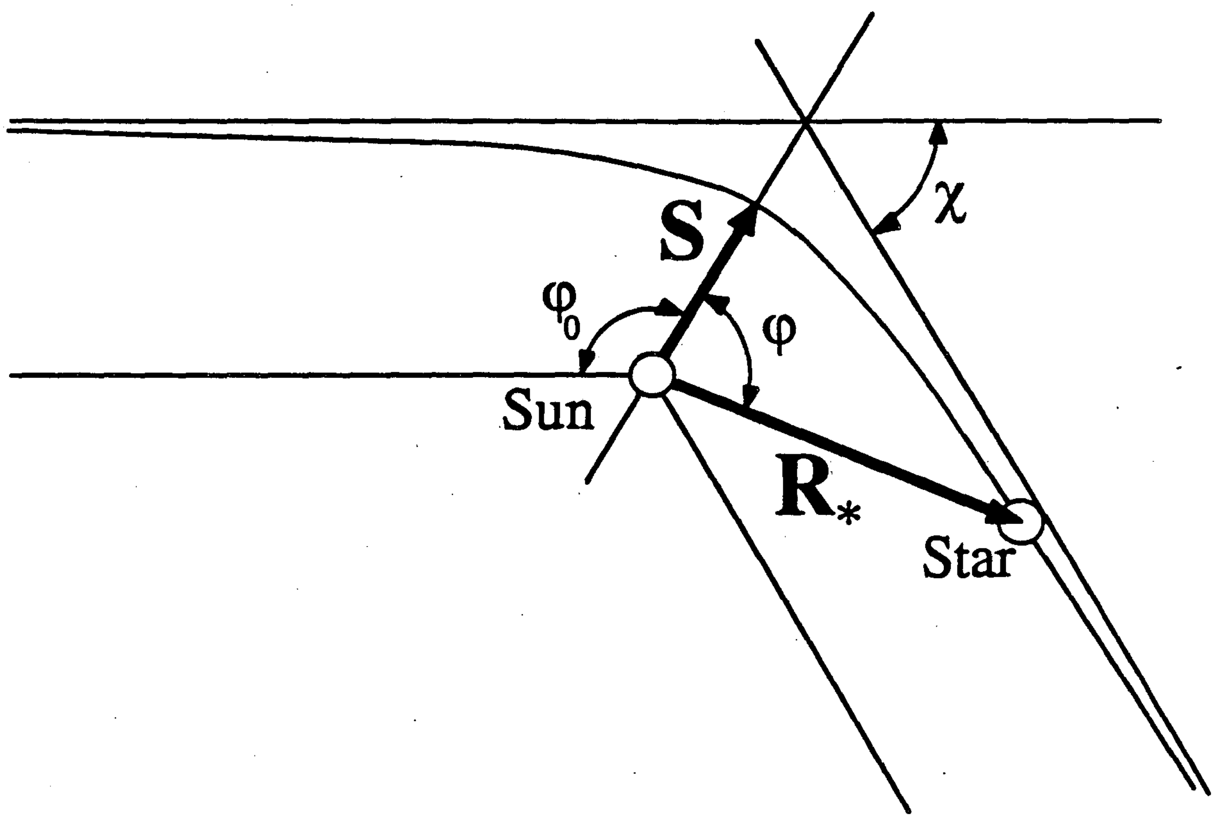

Figure 6.

Appendix I

Mass and Velocity Distribution of Stars in the Solar Neighborhood

The distribution of stellar velocities in the solar neighborhood is given by Heisler, Tremaine and Alcock[5]. For the stars in each of 20 different stellar classes i, they give the the number density n_i and one-dimensional RMS velocity in the local standard of rest (LSR) $\sigma_{i,LSR}$. They approximate the velocity distribution for each class by an isotropic Maxwellian in the LSR. We model the velocity distribution *in the sun's frame* as being isotropic Maxwellian with one-dimensional RMS velocity $\sigma_i = [\sigma_{i,LSR}^2 + (17 \text{ km/s})^2 / 3]^{1/2}$ where 17 km/s is the sun's velocity in the LSR:

$$n(V_*)\, dV_* = \sqrt{2/\pi}\, n_i\, \sigma_i^{-3}\, V_*^2\, e^{-V_*^2/2\sigma_i^2}\, dV_*$$

where $n(V_*)\, dV_*$ is the number density of stars in the i^{th} stellar class with velocity between V_* and $V_* + dV_*$. The true distribution is not isotropic, since the sun's velocity in the LSR introduces a direction preference, but the sun's LSR velocity is small, so this does not produce significant errors in our model. We used this model to calculate the probabilities in Figure 4.

The RMS velocity of all stars in the solar neighborhood was calculated as follows:

$$\langle V_*^2 \rangle = \frac{\sum_{i=1}^{20} I_i}{\sum_{i=1}^{20} J_i} = 46 \text{ km/s.}$$

Here, $I_i = \int_0^\infty V_*^2 n(V_*)\, dV_*$ is the average for the i^{th} class of V_*^2 weighted by $n(V_*)$. We find $I_i = 3\,\sigma_i^2\, n_i$ because the three-dimensional RMS velocity of the i^{th} stellar class is $\sqrt{3}$ times the one-dimensional RMS velocity σ_i. The sum over I_i gives the weighted average of V_*^2 for the stars in all 20 stellar classes. The average is normalized by the sum over $J_i = \int_0^\infty n(V_*)\, dV_* = n_i$.

Appendix II

The Change in i and e Due to a Passing Star

A change ΔV in orbital velocity results in changes in i and e. Formulae for Δi and Δe are found for perturbations where ΔV is small compared to the orbital velocity V_P. This is an elaboration of the analysis sketched out in the text leading to equation (1). We will also show that an upper limit ΔV_{max} can be set on the strength of the perturbation of the closest passing star.

Our coordinate system is centered on the sun at the time of the perturbation. The planet's orbital angular momentum is **L** and its position with respect to the sun is **R**. The axes of the coordinate system are in the directions of **L**, **R** × **L** and **R**, and are called the L, t and R axes, respectively. The orbit is nearly circular, so **R** ≈ (0, 0, a). The polar coordinates of ΔV are the magnitude ΔV, polar angle $\theta_{\Delta V}$ (measured from **R**) and azimuthal angle $\phi_{\Delta V}$ (measured from **L** in the L,t plane).

The inclination i is the angle between the orbital and invariant planes. Since the invariant plane is by definition orthogonal to the total angular momentum **J** of the solar system, i is also the angle between **L** and **J**. A passing star would cause a much larger change in the direction of **L** than in **J**. (The direction of **J** is determined largely by Jupiter's orbital angular momentum, and the tightly bound Jupiter is less sensitive to outside perturbations than Neptune and Uranus, the planets we use to constrain any close stellar passage.) Hence, Δi is approximately given by the change in the direction of **L**. Now the vector change in **L** is $\Delta \mathbf{L} = M_P (\mathbf{R} \times \Delta \mathbf{V}) = a M_P (-\Delta V_t, \Delta V_L, 0)$. The initial magnitude of **L** is $L \approx a M_P V_P$, so the change in the direction of **L** is $\Delta i = \arctan \Delta L_t/L \approx \Delta L_t/L = \Delta V_L / V_P$. This is the result cited in the text. The mean square change in any component of ΔV is $\langle \Delta V_L^2 \rangle = \langle \Delta V_R^2 \rangle = \langle \Delta V_t^2 \rangle = (\Delta V)^2/3$, since the sum of these must equal ΔV^2. Thus, the RMS change in inclination for perturbations of strength ΔV is $\langle \Delta i^2 \rangle^{1/2} = \langle \Delta V_L^2 \rangle^{1/2} / V_P = (1/\sqrt{3})\, \Delta V / V_P$.

The planet's e is proportional to the magnitude of its Runge-Lenz vector, $\mathbf{A} = M_P (\mathbf{V}_P \times \mathbf{L}) - GM_O M_P (\mathbf{R}/R)$. The change in e can therefore be determined by finding the change in $\mathbf{A}$. Because the perturbation is impulsive $\Delta\mathbf{R}=0$. We find $\Delta\mathbf{A} = M_P (\Delta\mathbf{V} \times \mathbf{L}) + M_P (\mathbf{V}_P \times \Delta\mathbf{L}) = M_P L (0, \Delta V_R, -\Delta V_t) + aM_P^2 V_P (0, 0, -\Delta V_t) = aM_P^2 V_P (0, \Delta V_R, -2\,\Delta V_t)$. Since $e = A/(GM_O M_P^2)$ and $V_P = (GM_O/a)^{1/2}$, the change in eccentricity is $|\Delta e| = (\Delta V_R^2 + 4\,\Delta V_t^2)^{1/2}\, V_P^{-1}$, as cited in the text. Therefore, for perturbations of strength ΔV, the RMS change in eccentricity is $\langle\Delta e^2\rangle^{1/2} = (\langle\Delta V_R^2\rangle + 4\,\langle\Delta V_t^2\rangle)^{1/2} / V_P = [(1/3)\Delta V^2 + (4/3)\Delta V^2]^{1/2} / V_P = \sqrt{5/3}\,\Delta V / V_P = \sqrt{5}\,\langle\Delta i^2\rangle^{1/2}$.

We can find an upper limit $\Delta V \leq \Delta V_{max}$ on the change in a planet's orbital velocity. This limit follows from the inequalities $|\Delta i| \lesssim \langle i\rangle$ and $|\Delta e| \lesssim \langle e\rangle$, which must be satisfied for the close passage of the star to be consistent with the observed orbits of the Jovian planets. The limit can be found by substituting the above expressions for $|\Delta i|$ and $|\Delta e|$ into these inequalities: $\Delta V_L V_P^{-1} \leq \langle i\rangle$ and $(\Delta V_R^2 + 4\,\Delta V_t^2)^{1/2}\, V_P^{-1} \leq \langle e\rangle$. Squaring these equations and adding them yields $(\Delta V_L^2 + \Delta V_R^2 + 4\,\Delta V_t^2)\, V_P^{-2} \leq (\langle i\rangle^2 + \langle e\rangle^2)$. Since $\Delta V^2 = \Delta V_L^2 + \Delta V_R^2 + \Delta V_t^2 \leq \Delta V_L^2 + \Delta V_R^2 + 4\,\Delta V_t^2$, we have $\Delta V^2 \leq \Delta V_{max}^2 \equiv (\langle i\rangle^2 + \langle e\rangle^2)\, V_P^2$. Thus, we find equation (1).

Appendix III

ΔV Produced by a Star Passing at a Distance Greater than 30 AU

In this appendix we find the change in orbital velocity produced by the close passage of a field star when S is much greater than *a*.

Using the impulse approximation, we find: $\Delta \mathbf{V} = \Delta \mathbf{V}_P - \Delta \mathbf{V}_O = 2GM_* V_*^{-1} (\mathbf{P}/P^2 - \mathbf{S}/S^2)$. It is important to note that the velocity impulse on the planet is in the direction of **P**, not of **S**, because the directions differ to first order in $\alpha \equiv a/S$. We now determine **P** and P^2 for a given **S** in order to find the corresponding $\Delta \mathbf{V}$. The position of the star relative to the planet when the star is closest to the *sun* is $\mathbf{S} - \mathbf{R}$. As can be seen in Figure 5, the position of the star when it is closest to the *planet* differs from this by the component of **R** along $\mathbf{V}_*$, $\mathbf{R} \cdot \mathbf{V}_*/V_*$.

Thus, the position of the star relative to the planet when they are closest is $\mathbf{P} = \mathbf{S} - \mathbf{R} + ((\mathbf{R} \cdot \mathbf{V}_*)/V_*)\mathbf{V}_*/V_*$. Using $\mathbf{V}_* \perp \mathbf{S}$, we find $P^2 = S^2 + R^2 - 2\,\mathbf{S} \cdot \mathbf{R} - (\mathbf{R} \cdot \mathbf{V}_*)^2/V_*^2$. It is convenient to use polar angles θ_S and θ_V (called simply θ in the text) and azimuthal angles ϕ_S and ϕ_V of **S** and $\mathbf{V}_*$ as measured in the coordinate system introduced in Appendix II.

Substituting $\mathbf{R} = (0, 0, a)$ from Appendix II, we find $\mathbf{P} = \mathbf{S} + (0, 0, -a) + a \cos\theta_V \mathbf{V}_*/V_*$. To first order in α, $P^2 \approx S^2 (1 - 2\alpha \cos\theta_S)$. So, to first order in α,

$$\Delta \mathbf{V} \approx 2GM_* V_*^{-1} S^{-2} \{(1 + 2\alpha \cos\theta_S)[\mathbf{S} + (0, 0, -a) + a \cos\theta_V \mathbf{V}_*/V_*] - \mathbf{S}\}$$

$$\approx 2GM_* V_*^{-1} S^{-2} \{2\alpha \cos\theta_S \mathbf{S} + (0, 0, -a) + a \cos\theta_V \mathbf{V}_*/V_*\}$$

We now define $\Delta V_{typ} \equiv 2\, G\, M_*\, a\, V_*^{-1}\, S^{-2}$ and expand **S** in rectangular coordinates by converting from its polar coordinates:

$$\Delta \mathbf{V} \approx \Delta V_{typ} \{(2\sin\theta_S \cos\theta_S \cos\phi_S, 2\sin\theta_S \cos\theta_S \sin\phi_S, 2\cos^2\theta_S - 1) + \cos\theta_V \mathbf{V}_*/V_*\}$$

$$\approx \Delta V_{typ} \{(\sin 2\theta_S \cos\phi_S, \sin 2\theta_S \sin\phi_S, \cos 2\theta_S) + \cos\theta_V \mathbf{V}_*/V_*\}$$

The perturbation ΔV is significantly smaller than ΔV_{typ} only when θ_V is small. In that case the $\cos \theta_V$ term nearly cancels the first term. This can also be seen by reference to Appendix VII. Otherwise, $\Delta V \approx \Delta V_{typ}$.

This result can be compared to the results of Hills (fig. 8 of ref. 7), though not directly. Hills[7] gives the average change in eccentricity $|\Delta e|_{av}$ produced by such encounters, but not the average change in orbital velocity. So to compare our result with the simulations by Hills[7], we use the RMS value of Δe found in Appendix II: $<\Delta e^2>^{1/2} \approx \sqrt{5/3}\, \Delta V_{typ} / V_P$. This agrees with $|\Delta e|_{av}$ from Hills[7] within a factor of 2 as long as $S \geq a$.

Appendix IV

ΔV Produced by a Star Passing at a Distance Less than 30 AU

When S is much smaller than a, $\Delta V \approx \Delta V_O$, and the impulse approximation does not necessarily apply. The deflection of the star must be considered in determining the impulse it delivers to the sun. The impulse on the sun can easily be found by analysing the star-sun orbit in the star-sun center of mass frame. We begin by finding the equations for the orbit in the CM frame. Then we determine the change in the sun's momentum.

Define the reduced mass $\mu \equiv (M_O M_*) / (M_O + M_*)$. In the CM frame, the angular momentum $\mathbf{L}_*$, energy E_*, eccentricity e_* and radial separation $\mathbf{R}_*$ of the star-sun orbit are given by[7]:

$$\mathbf{L}_* = \mu \, \mathbf{R}_* \times d\mathbf{R}_*/dt, \qquad \text{(A1)}$$

$$E_* = \frac{1}{2}\mu\left(\frac{dR_*}{dt}\right)^2 + \frac{L_*^2}{2\mu R_*^2} - \frac{GM_O M_*}{R_*}, \qquad \text{(A2)}$$

$$e_*^2 = 1 + \frac{2\,E\,L_*^2}{\mu(GM_O M_*)^2}, \qquad \text{(A3)}$$

$$\frac{1}{R_*} = \frac{GM_O M_* \mu}{L_*^2}(1 + e_* \cos\varphi). \qquad \text{(A4)}$$

Here φ is the angle between $\mathbf{R}_*$ and S (see Figure 6). If the star has initial velocity (at great distance) V_* and impact parameter B with respect to the sun, then L_*, E_* and e_* are given by

$$L_* = \mu \, B \, V_* \qquad \text{(A5)}$$

$$E_* = \mu \, V_*^2 / 2 \qquad \text{(A6)}$$

$$e_*^2 = 1 + [\mu^2 B^2 V_*^4 / (G\, M_O\, M_*)^2] = 1 + B^2 a_c^{-2} \qquad \text{(A7)}$$

where $a_c \equiv G\,(M_O + M_*)\,V_*^{-2} = GM_OM_*\,\mu^{-1}V_*^{-2}$. When the star reaches its closest approach to the sun ($R_* = S$), its radial velocity dR_*/dt is zero. At this time, equation (A2) reduces to $\mu V_*^2/2 = \mu B^2V_*^2/2S^2 - GM_OM_*/S$. Multiplying both sides by $2\mu^{-1}V_*^{-2}$, we find $S^2 + 2a_cS - B^2 = 0$. With this result and equation (A7), we find the more useful equation: $e_*^2 = 1 + 2Sa_c^{-1} + S^2a_c^{-2} = (1 + Sa_c^{-1})^2$. That is,

$$e_* = 1 + Sa_c^{-1} \qquad \text{(A8)}$$

Now we can determine the angle through which the sun is deflected in the CM frame, which allows us to find the change in its momentum. The asymptotic angles φ_0 of the star-sun separation $\mathbf{R}_*$ can be found using equation (A4) in the limit of large R_*: $(1 + e\cos\varphi_0) = 0$ or $\cos\varphi_0 = -1/e_*$. Since φ_0 is the angle between one asymptote and S, the angle between the two asymptotes is $2\varphi_0$. The angle between the initial and final momenta of the sun is $\chi = 2\varphi_0 - \pi$, where the π is subtracted because the initial momentum is directed *towards* the center of mass along one asymptote but the final momentum is directed *away from* the CM along the other (see Figure 6). Note that $\sin(\chi/2) = \cos\varphi_0 = -1/e_*$.

The final momenta have the same magnitude as the initial [let R_* go to infinity in equation (A2)]: $M_OV_{Oi} = M_OV_{Of} = \mu V_*$. That is, in the CM frame, the momenta of the sun and the star are redirected, but not changed in magnitude. It follows that the impulse on the sun is

$$M_O\Delta V_O = 2\,M_OV_{Oi}\,|\sin(\chi/2)| = 2\mu V_*/e_* = 2\mu V_*\,(1 + Sa_c^{-1})^{-1} \qquad \text{(A9)}$$

where we have used equation (A8) for e_*. Using $a_c = GM_O M_* \mu^{-1} V_*^{-2}$, we finally arrive at the desired result, $\Delta V_O = 2\, G\, M_*\, V_*^{-1}\, [S + a_c]^{-1}$. Due to the symmetry of the hyperbolic orbit, the impulse is in the direction of the star at its closest approach ($\Delta V_O \parallel S$).

This result can be compared to the results of Hills (fig. 8 of ref. 7). As in Appendix III, we compare the RMS value $<\Delta e^2>^{1/2} \approx \sqrt{5/3}\, \Delta V_O / V_P$ of Δe with the average value $|\Delta e|_{av}$ found by Hills[7]. The values agree within a factor of 2 as long as $S \leq a$.

Appendix V

Excluded Passages of Weakly Bound Objects

The results of the numerical simulations of Hills[9] for 0.05 $M_\odot$ and 0.005 $M_\odot$ weakly bound objects can be used to derive limits on the close passage of weakly bound objects of various masses. Hills found the change in *e* produced by the passage of a weakly bound object at several different small perihelion distances. Since the change in *e* is proportional to the mass of the passing object we can, for each perihelion distance studied by Hills[9], determine the largest mass consistent with the low presently observed $<i>$ and $<e>$ of the Jovian planets. To set limits on the perihelion passages of weakly bound objects that correspond to those set for passing stars, we estimate the value of ΔV from the Δe found by Hills[9] and require $\Delta V \leq \Delta V_{max}$.

For example, if a weakly bound object of mass 0.05 $M_\odot$ passing at S is found to produce an average change $|\Delta e|_{av}$, then a star of mass M_* passing at S would produce a change $\Delta V \approx \sqrt{3/5}\, V_P\, |\Delta e|_{av}\, (M_*/0.05\ M_\odot)$ in orbital velocity. The largest mass that could pass without changing the orbital velocity by more than ΔV_{max} is $M_* = \sqrt{5/3}\,(0.05\ M_\odot)\,\Delta V_{max}\, V_P^{-1}\, |\Delta e|_{av}^{-1}$. Using the ΔV_{max} and V_P of Neptune, we find $M_* = (1\times10^{-3}\ M_\odot / |\Delta e|_{av})$. The open circle data points in Figure 3 were determined using this formula for M_*. The square data points were determined by applying the analogous formula $M_* = (1\times10^{-4}\ M_\odot / |\Delta e|_{av})$ to Hills's[9] simulations of passages of 0.005 $M_\odot$ objects.

Appendix VI

Probability of Reduction of i and/or e from Initially Large Values by a Close Stellar Passage. (the Second Scenario)

If the planet's i and/or e were initially large, a close passage could only reduce them to values as low as observed today if the passage occured with a special geometry. We will show that the likelihood of a sufficiently precise special geometry of the passage is extremely small. We will first consider the case in which either i or e was initially large but the other was no larger than the present average value. Then, the passing star would have to reduce the large orbital element without increasing the other orbital element above its present average value. We calculate the probability of this happening. We will show that the probability that a passing star would simultaneously reduce i and e from large initial values is always smaller than the probability that one orbital element was reduced from an initially large value while the other, initially small, remained so.

We will show that of the possible initial conditions allowed in scenario 2, the one most likely to lead to an orbit with the low $\langle i \rangle$ and $\langle e \rangle$ observed today is the case of a large initial e but a small inital i not much greater than $\langle i \rangle$, the present average value. Of course, as stated in the text after equation (9), this is still less likely to have occured than Scenario 1, where i and e were initially small and no stellar perturbation has taken place of sufficient strength to increase i and e above their present average values.

Reduction of i from a large initial value to a near-zero value would require that the planet be passing through the invariant plane when the perturbation occurs, and that the perturbation be of the correct strength to nearly cancel V_L, the component of orbital velocity V_P orthogonal to the invariant plane. Only then would the planet begin to orbit in the invariant plane, because in subsequent orbits the planet returns to the position it was at when the perturbation occured. When the planet is passing through the invariant plane, $V_L = \pm V_P \sin i_0 \approx \pm V_P i_0$. The cancellation of V_L must occur to an accuracy of $V_P <i>$ if the final inclination is to be no greater than $<i>$. That is, the change in V_L is $-V_L \pm V_P<i>$. The probability of this is roughly $<i>/i_0$.

This probability is further reduced by the requirement that the planet be at a distance $\leq <i>a$ from the invariant plane when the perturbation occurs. Since the maximum distance from the plane of an orbit with inclination i is $a \sin i$, the final inclination would be larger than $<i>$ if the planet was any farther from the plane. In the initial inclined orbit, the amount of time that the planet spends within this distance from the plane is $4(<i>a \,/\, V_L)$ per orbital period. The probability that the planet is this close to the plane when the star passes is therefore $(2/\pi)$ $(GM_O/a)^{1/2}$ $(<i>/i_0)$. Thus, the probability that the planet is passing through the plane *and* that the perturbation cancels V_L accurately enough to leave the planet with $|i\,| \leq <i>$ is approximately $(2/\pi)$ $(GM_O/a)^{1/2}$ $(<i>/i_0)^2$. Of course, we cannot know the value of i_0, but for a perturbation ΔV, Δi is typically about $\sqrt{3}\ \Delta V/V_P$, (see appendix II) so if such a cancellation occured, then i_0 would have been $\sim \sqrt{3}\ \Delta V/V_P$. Thus, the probability that a perturbation ΔV would be consistent with the observed low i is $C_i \approx (2/3\pi)\ (GM_O/a)^{1/2}\ (<i>V_P/\Delta V)^2$.

We next consider the case in which the initial eccentricity e_0 was large, but the initial orbital inclination was no larger than at present. Then, the perturbation had to reduce the magnitude of the Runge-Lenz vector A from its initial value $A_0 \equiv GM_O M_P^2\, e_0$ to its present value $A_1 \equiv GM_O M_P^2 <e>$ or less. For the cancellation to occur, the change ΔA in A has to be nearly opposite A_0: that is, it must have both the proper magnitude and direction. Since A points towards the planet's perihelion, A_0 and A_1 are both in the orbital plane. Thus, the probability of ΔA being in the proper direction is about $(1/\pi)(A_1/A_0)$. This is the fraction of the circle of possible final values of A (with radius $\Delta A \approx A_0$) that coincides with the target disk of width $2A_1$. If the perturbation is in the proper direction, the probability of ΔA having the proper magnitude is about (A_1/A_0). Thus, the probability of scattering e down to below $<e>$ is $C_e \approx (1/\pi)(A_1/A_0)^2 = (1/\pi)\ (<e>/e_0)^2$. If the perturbation is of strength ΔV, then $e_0 \sim \sqrt{5/3}\ \Delta V/V_P$, so $C_e \approx (3/5\pi)$ $(<e>V_P/\Delta V)^2$.

If only one of the orbital elements was initially much larger than the observed value, then the other must have been nearly unaffected by the interaction. Otherwise it would have been increased by the passage, and would have a present value much larger than that observed. Either the initial value of i or e could have been large. Of these two alternatives, the one most likely to lead to the circular, coplanar orbits observed today is a large initial e and an initial value of i on the same order as the present $\langle i \rangle$. In order that the encounter not change i substantially, $|\Delta i| \leq \langle i \rangle$. The probability K_i of this occuring is the fraction of solid angle for which a perturbation of strength ΔV satisfies $\Delta V_L \leq \langle i \rangle V_P$. Integrating the solid angle differential $d\Omega = (2\pi/\Delta V)\, d(\Delta V_L)$ from $- \langle i \rangle V_P$ to $+ \langle i \rangle V_P$ gives the fraction as $K_i = \langle i \rangle (V_P / \Delta V)$.

The probability that the passing star would reduce e but leave i small is approximately $K_i C_e$, The alternative, with initial $i \gg \langle i \rangle$ and $e_0 \leq \langle e \rangle$, is less likely to have led to the present orbits, since the probability K_e that the passing star would not alter the already low e is much smaller than K_i. (For ΔV to leave i virtually unchanged, only the component ΔV_L had to be small, but for it to leave e unchanged, both ΔV_t and ΔV_R had to be small.) The probability that a passing star would simultaneously reduce i and e from large initial values is approximately $C_i C_e$.which is always smaller than $K_i C_e$,, and need not be considered further.

Thus, under scenario 2, the probability f that the close passage of a star would be consistent with the observed low $\langle i \rangle$ and $\langle e \rangle$ is no greater than $K_i C_e$:

$$f \leq K_i C_e = (3/5\pi) \; \langle i \rangle \; \langle e \rangle^2 \, (V_P/\Delta V)^3$$

For Neptune, $f \leq 2.3\times10^{-7} \, (V_P / \Delta V)^3$, as cited in equations (8) and (9).

Appendix VII

ΔV Produced by a Star Passing Nearly Parallel to the Direction of the Planet from the Sun

A star passing nearly parallel to the sun-planet separation **R** would leave the planet's orbital velocity virtually unchanged. Such a passage would have negligible effect on i and e, and cannot therefore be excluded by the arguments in this paper. However, the passage could only have escaped detection if the angle θ_V (called θ in the text) between $\mathbf{V}_*$ and **R** was very small, a very unlikely situation. We will show that when $a\,\theta_V$ is small compared to S, $\Delta V \approx \Delta V_{typ}\,\theta_V$.

By reference to Figure 2, we see that $|\mathbf{P} - \mathbf{S}| = a \sin\theta_V \approx a\,\theta_V$. Let ψ be the angle between **S** and the projection of **R** onto the plane orthogonal to $\mathbf{V}_*$. Then, the position of the star relative to the planet when they are closest is $\mathbf{P} \approx \mathbf{S} - a\,\theta_V\,(\sin\psi\,\mathbf{S}\times\mathbf{V}_*/SV_* + \cos\psi\,\mathbf{S}/S)$. The magnitude of **P** is $P \approx S - a\,\theta_V \cos\psi$. Thus, to first order in $a\,\theta_V/S$, the change in orbital velocity is

$$\Delta\mathbf{V} = 2\,G\,M_*\,V_*^{-1}\,(\mathbf{P}/P^2 - \mathbf{S}/S^2)$$

$$\approx \frac{2GM_*}{V_*}\left[\frac{\mathbf{S} - a\theta_V(\sin\psi\,\mathbf{S}\times\mathbf{V}_*/SV_* + \cos\psi\,\mathbf{S}/S)}{(S - a\theta_V\cos\psi)^2} - \frac{\mathbf{S}}{S^2}\right]$$

$$\approx \frac{2GM_*}{V_*}\left\{\frac{[(S - a\theta_V\cos\psi)\mathbf{S}/S - a\theta_V\sin\psi\,\mathbf{S}\times\mathbf{V}_*/SV_*][S + 2a\theta_V\cos\psi]}{S^3} - \frac{\mathbf{S}}{S^2}\right\}$$

$$\approx \frac{2GM_*}{S^2V_*}\left\{(2a\theta_V\cos\psi - a\theta_V\cos\psi)\mathbf{S}/S - a\theta_V\sin\psi\,\mathbf{S}\times\mathbf{V}_*/SV_*\right\}$$

$$\approx \frac{2GM_*a\theta_V}{S^2V_*}\left\{\cos\psi\,\mathbf{S}/S - \sin\psi\,\mathbf{S}\times\mathbf{V}_*/SV_*\right\}$$

The magnitude of this result is $\Delta V \approx 2GM_*a\theta_V/S^2V_* = \Delta V_{typ}\,\theta$, as cited in the text and in the caption of Figure 2.

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