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Developing Teacher Candidates' Self-Awareness and Vision for Equitable Mathematics Teaching through a Researcher-Teacher Educator Partnership

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UNIVERSITY OF CALIFORNIA,
IRVINE

Developing Teacher Candidates' Self-Awareness and Vision for Equitable Mathematics
Teaching through a Researcher-Teacher Educator Partnership

DISSERTATION

submitted in partial satisfaction of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in Education

by

Jiwon Lee

Dissertation Committee:
Professor Rossella Santagata, Chair
Associate Professor Hosun Kang
Associate Professor June Ahn
Associate Professor Lindsey Richland
Professor of Teaching Alessandra Pantano

2021

DEDICATION

To

my parents and family
here in California and in Korea

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VITA
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Lee, J. & Santagata, R. *Mathematical Knowledge for Teaching and Mathematical Quality of Instruction of Novice Elementary School Teachers*, Poster presentation, Association of Mathematics Teacher Educators, Houston, Texas, February 2018

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ABSTRACT OF THE DISSERTATION

Developing Teacher Candidates' Self-Awareness and Vision for Equitable Mathematics

Teaching through a Researcher-Teacher Educator Partnership

by

Jiwon Lee

Doctor of Philosophy in Education

University of California, Irvine, 2021

Professor Rossella Santagata, Chair

Decades of scholarly work continue to document the marginalization of children from nondominant backgrounds in mathematics classrooms. Learning to equitably teach children from linguistically, ethnically, racially, and culturally diverse backgrounds remains a priority. This dissertation approaches this broader problem in society by understanding how to prepare elementary teacher candidates to center equity in their mathematics teaching. The first study explores how an innovative pedagogical activity elicits and has candidates confront their expectations, assumptions, and biases about children and their mathematical thinking. The study conceptualizes a framework that captures the extent of candidates' self-awareness of their assumptions and biases. The second study conceptualizes teacher candidates' conceptions of equity in mathematics teaching and explores their prior mathematics experiences. The third study tells the story of how a researcher-practitioner collaboration developed and shifted the mathematics course to integrate an equity lens over time. Research data include candidates' written reflections, interviews, and other artifacts they produced in the methods course; field

notes of class observations; teacher educators' interviews and syllabi; and other documentations of idea exchanges between the teacher educator and researcher. Qualitative analytic methods were employed.

The study findings reflect the importance of self-confrontation and critical self-reflection in a mathematics methods course to support candidates to center equity in their practice. The three studies contribute to the field of teacher preparation in the following ways: (1) a conceptual framework for candidates' self-awareness of their biases and assumptions and a conceptual framework for centering equity in mathematics teaching; (2) a typology for teacher candidates' conceptions of equity in mathematics teaching and a more nuanced understanding of candidates' prior mathematical experiences; (3) an illustrative example of how a collaborative relationship between a researcher and practitioner developed over shared sensemaking experiences.

INTRODUCTION

Given the ever-increasing population of students from linguistically, ethnically, racially, and culturally diverse backgrounds, there continues to be an urgent need to better support teacher candidates to center equity in their mathematics teaching to ensure all students receive high quality education (Nieto, 2000; Sleeter, 2001; Hollins & Guzman, 2005; Villegas & Lucas, 2002). In 2013, California was reported to have a higher proportion of students of color (75%) than White students (25%) (U.S. Department of Education). Scholars have raised concerns of the cultural mismatch between teachers and students (Gomez, 1993; Villegas, 1988), as well as implicit and explicit biases or expectations teachers have of students of color (Copur-Gencturk et al., 2020; Nosek & Smyth, 2011; Peterson et al., 2016) and how these contribute to students' learning opportunities and outcomes.

Studies have reported the role of teacher expectations in shaping differential learning outcomes among students from diverse racial/ethnic backgrounds (Ferguson, 1998; Weinstein et al., 2004). Scholars have reported how teachers' implicit prejudices predicted student achievement and seemed to have an ethnic bias (Peterson et al., 2016). In a study of K-8 teachers, scholars have found teachers exhibited gender bias as well as racial bias when considering student mathematical ability in ways that aligned with and were contrary to findings from prior studies (Copur-Gencturk et al., 2019). They reported a bias among teachers of color favoring White students, White teachers favoring male students. These biases emerged when the student solutions they reviewed were unclear and/or incorrect. Scholars have clarified the distinction between explicit and implicit bias in that people with explicit prejudices are consciously aware and have control over their biases and attitudes (Gawronski & Bodenhausen, 2006), while implicit prejudices “emerge via automatic processing and are typically

unconscious” (Peterson et al., 2016, p. 124). However, research suggests that these automatic implicit biases can be controlled through intentional efforts, such as “motivation to maintain a positive self-image or have positive relationships with others,” “strategic efforts to reduce stereotypes or promote counter stereotypes,” “focus of attention,” and “contextual cues” (Blair, 2002, p. 255). These findings suggest that controlling implicit biases and expectations can be learned.

Teacher preparation programs have been one avenue for better preparing teachers for classrooms with diverse student populations. Multicultural education courses have been included in programs as a way to address the issue of better preparing teachers to teach for diverse student populations. However, these isolated efforts are often surface-level and not integrated across the learning experiences within the teacher preparation program (Irvine, 2003; Ladson-Billings, 1999; Villegas & Lucas, 2002). Anti-bias education has emerged in an effort to develop teacher candidates’ critical cultural consciousness (Lin, Lake, & Rice, 2008), recognize and reflect on their own identities (Milner, 2003), and engage in field-based experiences where they interact with children from backgrounds different from their own (Brown, 2005). The importance of engaging in habitual reflection and critical reflection in teaching has been emphasized by scholars for decades (Brown, Vesely, & Dallmann, 2016; Dewey, 1933; Liu, 2015; Lucas, 2012; Mezirow, 1990; Ward & McCotter, 2004). The way in which candidates learn to engage in reflective practice varies across programs and across courses. Valli (1997) reviewed the literature and various teacher preparation programs and conceptualized five different types of reflection: technical reflection, reflection-in and on-action, deliberative reflection, personalistic reflection, and critical reflection. Valli asserted that these different types of reflection should be used in combination as each type supplements the limitation of another. Valli also provided a list of

processes that would support teacher candidates with learning to become reflective. These include action research, journaling, case studies, supervision, and classroom activities and discussions. Given Valli's comprehensive review of the typologies of reflection and the activities that support candidates in the different types of reflection, one that is not included is a self-confrontation approach to reflection, which is explored in the first study (Chapter 1).

The context of the present three-study dissertation project is in a mathematics methods course within a teacher preparation program at a research university in California. The teacher educator of the mathematics methods course graciously welcomed me into her class. She expressed excitement about my ideas of how to integrate an equity lens into her coursework and wanted to learn together to improve her practice. I explored the work of critical reflection of teacher candidates' own expectations, biases, and assumptions of students in the disciplinary context of mathematics for a couple of reasons. First, my prior experiences and expertise lied in mathematics teaching and interpreting students' mathematical thinking. My research experiences prior to this dissertation study were in the context of mathematics teaching and learning. I felt most knowledgeable and interested in mathematics teaching and learning. Second, mathematics continues to be a sorting mechanism in society for "smartness," college entrance, careers, and socioeconomic status to name a few. There are serious implications for students if they lack access to high quality mathematics education. Teachers play a significant role in shaping students' educational experiences. Therefore, teacher candidates need to be prepared to teach high quality equitable mathematics lessons to empower students with knowledge and choice. In order to be prepared, teacher candidates need to confront themselves, who they are, what experiences shaped their lenses, how they perceive children and their mathematical thinking, and reflect on how their whole selves shape their teaching and implicate student learning.

Given my motivations, my first study aimed to understand: (1) how teacher candidates engage in a self-confrontation activity that prompted them to critically self-reflect on their biases and assumptions, and (2) how their noticing of their biases and assumptions related to the ways they center equity in their teaching. In the second study (Chapter 2), I explored elementary teacher candidates' conceptions of equity and their past mathematical experiences with the aim of understanding how the two may inform one another. The first two studies required great coordination with the teacher educator to implement activities and collect data. Our collaboration went smoothly and motivated my third study (Chapter 3), which examined how we worked together and how our collaboration changed her course design during the two years we worked together. Study 1 is situated in the first year of our collaboration. Study 2 is situated in the second year of our collaboration.

The broader purpose of all three studies is to better understand how to prepare teacher candidates to center equity in their mathematics teaching. A focus on equity consists of attending to the inequities that exist in and across learning environments and broader society. In American society and in the education system, a central root of inequities is race and racism (Ladson-Billings & Tate, 1995). The intertwined nature of race with ethnicity, gender, language, sexuality, class, and other issues are important to attend to when aiming to work towards disrupting inequities. Scholars have argued for the importance of centering a course on race, as well as integrating conversations of race across all courses, such as the methods courses and content courses (Ladson-Billings, 1999, 2000; Watson, 2012). Multicultural education courses often play the role of including the topic of race; however, it is necessary to support teacher candidates to understand the role and conceptions of race in teaching and learning (Bales & Saffold, 2011). Given all this, I had intended to design this dissertation study to elicit candidates'

assumptions and biases related to race among other identity markers in the context of the mathematics methods course; however, race had not been explicitly discussed during the years I joined to collaborate. Simultaneously, I assumed that if the course community was ready to engage in discourse about race, then it would surface in the data I collected. However, only a few candidates across both years of study discussed issues related to race and racism. In addition, my understanding of the complexities of race relations was developing at the time, and the conversations I could initiate and engage in, as well as integrate into the research design decisions were limited. The decisions I made with the teacher educator were in response to what surfaced from teacher candidates' responses in assignments and activities. The teacher candidates in the study most commonly attended to their students' language and socioeconomic status. Therefore, I could not explore candidates' conceptions of race and racism to the extent I had hoped. I share this note to acknowledge the limited focus on race and racism and recognize the importance of creating opportunities and experiences to have conversations about race in teacher education (Bales & Saffold, 2011; Howard & del Rosario, 2000). Without attending to race, I recognize it is challenging to address inequities in schools and in society. However, this study provides an entry point for candidates to confront and engage in discourse about inequities due to their own biases and assumptions. Then, centering oneself as one of the forces that perpetuate inequities may serve as a foundation to begin conversations of race and racism in the context of mathematics methods.

The dissertation study findings push the field to consider the following. The first study provides teacher educators with a tool that creates opportunities for candidates to recognize their implicit biases and assumptions and to reflect on the implications of their preconceived notions on student learning. The second study provides a conceptual framework for teacher candidates'

equity conceptions within the context of mathematics education, which teacher educators may find useful as they consider ways to push candidates' thinking to be expansive and inclusive. The second study also illuminates the importance of understanding who the teacher candidates are and extending the notion of knowing our learners to teacher candidates and their histories with mathematics. The third study conceptualizes different forms of collaboration and considers a new collaborative arrangement within a School of Education that facilitates productive collaboration between graduate students and practitioners towards building knowledge and improving practice.

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CHAPTER 1

Confronting Oneself to Notice for Equity in Mathematics: An Exploratory Study of Teacher Candidates

Abstract

This study aims to understand: (1) how elementary teacher candidates notice their own biases and assumptions about children and mathematics learning through a video-based activity, and (2) how their noticing of their own biases and assumptions relate to how they center equity in their practice. The study examined 21 elementary teacher candidates in the context of a mathematics methods course in a research public university. Two case studies are examined. The analyses revealed variation in how candidates noticed their own biases and assumptions and the two case studies portray the alignment between noticing of biases and centering equity in practice. The findings of this study reveal the importance of confronting one's underlying assumptions and biases to be able to critically self-reflect in order to learn to center equity in teaching mathematics.

Keywords: mathematics teaching and learning, self-confrontation, equity, critical reflection, teacher candidates

Confronting Oneself to Notice for Equity in Mathematics: An Exploratory Study of Teacher Candidates

This study aims to understand the extent to which a task designed for teacher candidates to confront their assumptions, biases, and expectations about children and their mathematics thinking prompted them to consider how these implicate students' identities in mathematics classrooms. This study emerged from a collaborative effort between the author and a teacher educator to address a problem of practice in the context of an elementary mathematics methods course. The teacher educator designed the methods course to have a central focus on developing teacher candidates' competencies to learn from their teaching and to understand children's mathematical thinking (Santagata & Guarino, 2010; Santagata, Yeh, & Mercado, 2018). This framework of understanding children's thinking is grounded in research and has shown to positively impact teachers' knowledge, beliefs, and practice, as well as students' learning (Carpenter et al., 1989; Fennema et al., 1996). Despite this central focus, the teacher educator noticed candidates exhibiting a pattern of teacher-centered interactions with children during course activities. To better understand the disconnect between the student-centered discourse during class and enacted practices with children, the teacher educator agreed to utilize an innovative task that prompts candidates to engage in critical self-reflection. Our collaborative work was paralleled by the broader efforts in the teacher preparation program around issues of equity. The program was in its third year of improvement efforts to build coherence in their commitments to equity. During this time, the teacher educator and I were developing our understanding of equity in the context of mathematics teaching and learning and ways to support candidates' learning to center equity in their mathematics teaching. Thus, the newly designed

task became a tool that supported the development of our understanding of ways to make equity an integral part of the mathematics methods course.

Inequities Perpetuated in Mathematics Classrooms

Mathematics classrooms are critical spaces in which children develop their identities as mathematics learners; the development of this identity is a critical component of learning mathematics (Anderson, 2007; Nasir, 2002). However, not all students are provided equitable learning opportunities in mathematics classrooms (Louie, 2017; Stinson, 2008). In the U.S., the culture of mathematics education reflects an underlying belief that mathematics is a fixed set of facts a learner must acquire, and the teacher's role is to transmit this knowledge to the learner (Schoenfeld, 1988; Stigler & Hiebert, 2009). This underlying belief creates missed opportunities for students to make sense of the mathematics, and any other way of participating in mathematics that is not normative may be discounted and label students as being mathematically incompetent (Boaler & Staples, 2008; Louie, 2017).

Many scholars have illuminated how schools and teachers perpetuating narrow and rigid conceptions of mathematics learning bring about injustices in mathematics classrooms for learners from nondominant, diverse backgrounds (Boaler & Staples, 2008; de Freitas, 2008; Louie, 2017). Hence, these students are being denied the opportunity to develop an identity as a mathematics learner. Louie (2017) conceptualized this as the notion of culture of exclusion, which "limits all students' access to rich and meaningful mathematics learning experiences and further limits many students' opportunities to develop identities as mathematically capable learners and thinkers" (p. 489). This culture of exclusion promotes the legitimacy of ability groupings, which correlate with the hierarchies within socially constructed categories based on

race, gender, socio-economic status, and linguistic and cultural background (Gutiérrez, 2002; Martin, 2012).

As someone who holds power and influence in the classroom, the teacher plays a fundamental role in creating equitable learning opportunities for all students by attending to and responding appropriately to the inequities in the classroom. From a sociocultural lens, teachers shape the cultural norms of the mathematics classroom, and students typically adopt and appropriate the mathematical practices dominant in the classroom as they learn to participate in a community of learners. Thus, if a teacher's practice conveys mathematics learning as absorbing facts and applying formulas and that teachers are the primary knowers while students are receivers of knowledge, then students may develop identities as competent mathematicians only if they can successfully memorize and apply facts or incompetent if they cannot.

Preparing Teacher Candidates for Equitable Practices in Mathematics

Studies have documented that teachers are underprepared to teach mathematics in ways that provide access to diverse learners in the classroom (Sleeter, 2001; Wiggins & Follo, 1999). Scholars suggest teachers can better prepare to teach students from diverse backgrounds by “developing knowledge, dispositions, and practices that support building on children’s mathematical thinking, as well as their cultural, linguistic, and community-based knowledge” (Turner et al., 2012, p. 68). One group of scholars documented the changes made in teacher knowledge, belief, and practice (and student learning) when teachers learned to understand children’s mathematical thinking (Carpenter et al., 1989; Fennema et al., 1996). Other scholars (González et al., 2001; Ladson-Billings, 1994) argued for teachers to develop their competencies in understanding and building on children’s *funds of knowledge*, which refers to the “historically

and culturally based knowledge, skills, and practices found in students' homes and communities" (Turner & Drake, 2016, p. 32).

The varying impact of teacher preparation programs on shifting candidates' understandings of teaching for equity and social justice has been documented. Fragmented learning experiences within the teacher preparation program have been documented (Feiman-Nemser, 1990). However, there has been growing evidence of programs efforts to integrate multicultural education as a programmatic effort instead of taking the approach of adding a single course into an existing program structure (Enterline, Cochran-Smith, Ludlow, & Mitescu, 2008; McDonald, 2007). Scholars have reported the positive shifts in beliefs and dispositions related to social justice (Enterline et al., 2008; Frederick et al., 2010; Mills, 2013). Frederick et al. (2010) reported shifts from the experiences within one course of analyzing and reflecting on various learning experiences (e.g., course readings, class discussions, course simulations, and school observations) supported candidates to shift their lens to recognize the broader context education is situated in. Mills (2013) reported that the positive changes occurred as candidates' transition to their first year of teaching, which highlights the importance of contextualized learning experiences of attending to social justice and equity. In one study that documented changes in candidates' understandings of teaching for social justice during a one-year teacher preparation program, Lee (2011) reported that while some candidates showed evidence of changes in their understandings, for others there was limited evidence of change because their understandings of social justice conflicted with their practice.

Specific to the context of mathematics teaching, Garii and Appova (2013) reported elementary teacher candidates' challenges in integrating mathematics and social justice practices due to their limited knowledge about both, even after three semesters of coursework that had a

focus on developing social justice ideas and pedagogical practices. Although candidates recognized concerns of diversity and cultural sensitivity in a classroom community, they struggled to draw broader connections to systems and structures, as “social justice requires a wider frame of reference to address issues of marginalization, power, and access” (p. 206). For teachers to develop the knowledge, dispositions, and practices needed to effectively teach diverse learners, they need to recognize the power hierarchies and dynamics that exist within the education system and how those shape their students’ experiences (Santoro, 2009). Therefore, teachers must critically self-reflect and actively attend to the ways in which they are reproducing inequities in their own classrooms, and more so within the context of mathematics teaching and learning.

The importance of candidates’ self-examination as cultural beings in relation to their students have been highlighted in a review of studies by Anderson and Stillman (2013) (e.g., Brock et al., 2007; Downey & Cobbs, 2007; Goodwin, 1997; Valli, 1995). In particular, Downey and Cobbs (2007) explored the affordances of an interview protocol for candidates to use in their field experience. The semi-structured interview protocol was designed to interview children (from cultural backgrounds different than that of candidates) about their beliefs and experiences (Downey, 2002). While candidates were able to gain new insight about themselves as mathematics teachers, the study did not explore candidates’ reflection on their assumptions and biases that permeate their practice. In order to support candidates to connect the role they themselves play as cultural beings in teaching children from diverse backgrounds, critical self-reflection is necessary. Goodwin (1997) highlights the importance of teacher preparation programs’ role in supporting candidates to “examine their own assumptions, expectations, and perceptions of children of color” as well as self-reflecting on their own understandings about

diversity, power, racial identity should candidates encounter “misconceptions, hidden assumptions, and prejudices about the competencies and capabilities of visible racial/ethnic children” (p. 143).

Thus, the present study explores the implementation of a video-based activity that was designed to support teacher candidates to recognize their own assumptions, biases, and expectations and how these implicate students’ identities and learning opportunities in mathematics classrooms. This activity, originally created by a science teacher educator (the first author of Kang & Lee, 2019), is the first of its kind to be implemented in the context of elementary mathematics education.

Self-Confrontation Noticing Activity

The Self-Confrontation Noticing Activity was adapted from a prior project that designed an innovative activity for secondary science candidates to critically reflect on themselves by focusing on their professional discourse about students’ science thinking/learning (Kang & Lee, 2019). In that study, candidates self-recorded their discussions with colleagues about their students’ work. Candidates watched their own video with prompts that required them to focus on the extent to which they recognized their language use when describing student thinking, their students’ identities, and their own assumptions/biases. Candidates then wrote reflections in response to the prompts. This activity was among many experiences the instructor designed to develop candidates’ capacity to enact equitable practices. Findings revealed that candidates recognized the ways in which they perpetuate inequities from their assumptions and biases, and some candidates furthered their recognition by problematizing and proposing actions to disrupt inequities. The adaptation to the mathematics methods course centered on connecting the task to the course content and its structure. I adapted two key features of this study: the use of videos to

engage candidates in critical reflection, and the videos capturing one's own discourse with colleagues about students' work. The present study aims to understand candidates' learning to enact equitable practices when they critically reflect on how their discourse illuminates their assumptions and biases of students and mathematics, position students as (in)competent, and implicate students' identities and opportunities to learn. The following questions guided this study:

Research Questions

1. How do teacher candidates notice their own biases and assumptions about children and mathematics learning?
2. In what ways does teacher candidates' noticing of their own biases and assumptions relate to the ways they center equity in their teaching?

Literature Review

This study draws on three bodies of work: (1) noticing for equity; (2) critical reflective practice; and (3) learning to learn from teaching.

Defining Equity

First, I describe how I conceptualize equity, as it guided my study design and analyses. As Gutiérrez (2002) claims, equity in mathematics education is when one cannot “predict mathematics achievement and participation based solely on student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language” (p. 153). This indicates that there is fairness and true equal opportunity to achieve; however, the goal is not for all students to achieve towards the same place. Gutiérrez (2012) further elaborates on equity by describing four dimensions: access, achievement, identity, and power. The former two

dimensions have been the dominant conception of equity in the 1980s and 1990s. The latter two dimensions have become more prominent in recent years. Gutiérrez argues that all four dimensions are critical for equity, as it is necessary to learn the dominant mathematics to learn how to “play the game” to then be able to have a critical lens when analyzing the world so that students can change the world.

Access refers to “tangible resources” that are available to students to be able to participate in mathematics. Gutiérrez provides examples of such resources: “high-quality mathematics teachers, adequate technology and supplies in the classroom, a rigorous curriculum, a classroom environment that invites participation, reasonable class sizes and support for learning outside of class hours” (p. 19).

Achievement refers to student outcomes, which includes not only test scores or grades, but also students’ participation, their coursework trajectory or patterns, and their participation in the trajectory of the discipline. Gutiérrez argues that there are “serious economic and social consequences” for not achieving (i.e., not enough math credits to graduate, not strong enough of a standardized test score to be accepted to a college, and not graduating with a STEM degree to have access to higher salary or greater social prestige) (p. 19).

Identity refers to the opportunities students have to see their belonging in the discipline as well as its relevance and meaningfulness to their own lives. This means students should have “opportunities to draw upon their cultural and linguistics resources (e.g. other languages and dialects, algorithms from other countries, different frames of reference) when doing mathematics” (p. 20).

Power refers to the opportunities to recognize and push on the mathematics reflective of the status quo in society and the ways social transformation is taken up. Some examples

Gutiérrez provides include: whose voice is heard in the classroom, opportunities to critique society using mathematics, and acknowledging the alternative ways of knowing and participating (p. 20).

Noticing for Equity

Scholars have conceptualized the construct of noticing within the context of developing teachers' skills in attending to and making sense of student thinking. Jacobs, Lamb, & Philipp, (2010) conceptualize the noticing work as expertise that require "three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 172). The authors suggest that these are the skills that occur in the background as teachers make in-the-moment decisions during instruction.

The first skill ("attending to children's strategies") refers to attending to "noteworthy aspects of complex situations" (p. 172). The authors suggest that teachers with deeper understanding of children's mathematical thinking have skills to "discern patterns and chunk information in complex situations" that allow them to better remember the details of children's mathematical strategies (p. 172). The second skill ("interpreting children's understandings") refers to "how teachers interpret children's understandings as reflected in their strategies" (p. 172) and the extent to which it aligns with existing research on children's mathematical development as well as the details in the child's strategies. The third skill ("deciding how to respond on the basis of children's understandings") refers to "the reasoning that teachers use when deciding how to respond" (p. 173).

Scholars have extended the work of noticing student thinking to noticing for equity by capturing whether teachers attend to status, participation, access, and opportunity and how these

intertwine with the mathematics and instructional practice (Hand, 2012; Jilk, 2016; Louie, 2017; McDuffie et al, 2014; van Es, Hand, & Mercado, 2017; Wager, 2014).

Wager (2014) examined how teachers' positionality towards equitable mathematics pedagogy was connected to what they noticed about children's participation in mathematics classrooms. Wager extended the noticing framework (Jacobs et al., 2010) and integrated equity to define a framework on attending, interpreting, and responding to participation. Attending to participation is defined as "what teachers notice about participation, and as a first step, it may or may not lead to assigning meaning to what is observed (Mason, 2002) or to action" (p. 316). Interpreting participation is defined as "how teachers explained and assigned meaning to what they noticed (Mason, 2002)" (p. 316). Responding to participation is defined as the intended response or change in practice in response to inequities observed in children's participation. Wager reported that teachers who were "positioned as thinking deeply about equity and mathematics" tended to notice inequitable participation patterns the most (p. 342) and noticing participation patterns supported teachers with providing equitable practices in mathematics classrooms.

Jilk (2016) emphasized the importance of noticing students' assets to disrupt a focus on students' deficits in the classroom. However, the cultural norm of focusing on deficits in an effort to "close the gap with what they need to understand" is a dominant and commonly held notion by many teachers in America and is continued to be perpetuated to students, who may become future teachers (p. 189). Given that teaching is a cultural activity and teachers are cultural beings, there is a further need for teachers to attend to the cultural norms they take for granted and reflect on how they shape children's mathematical experiences.

This study made central a focus on noticing the taken-for-granted assumptions and biases teacher candidates might be holding about children and mathematics by engaging them in the aforementioned Self-Confrontation Noticing Activity. Candidates watched self-recorded videos of their discussion with their colleagues about the student's mathematical work and reflected on their discourse, assumptions, and biases. This task engaged candidates in confronting their talk about the student's mathematical thinking and whether it was influenced by some assumption or bias.

Critical Self-Reflective Practice

This study combines the theoretical perspectives on *critical reflection* and *self-confrontation* to inform the design and analysis of the video-based pedagogical activity in this study. *Self-confrontation* is “a method consisting of confronting a person with his or her own image, behavior, or experience by means of an artifact” and entails either revealing one's affective profile, construct one's identity, and/or transform behavior patterns; or analyzing activities (Rix & Lièvre, 2010, p. 2). In the teacher education context, the candidates confront their own practice when student teaching so that they can reflect on their teaching with a mentor teacher or supervisor to improve their practice. Instead of focusing on their practice, this study engages candidates to confront and *critically reflect* on their discourse with colleagues.

Liu (2015) defines *critical reflection* as:

a process of constantly analyzing, questioning, and critiquing established assumptions of oneself, schools, and the society about teaching and learning, and the social and political implications of schooling, and implementing changes to previous actions that have been supported by those established assumptions for the purpose of supporting student learning and a better schooling and more just society for all children (pp. 144-145).

This study examines how critical reflection might be a vehicle for developing noticing for equity. Many scholars have documented that reflective practices support the development and transformation of teachers and their practices (Acquah & Commins, 2015; Liu, 2015). Furthermore, many studies utilized videos of teachers' own practice to reflect on (among others, Harford & MacRuairc, 2008; van Es, Hand, & Mercado, 2017). However, very few studies created experiences for teachers to do the work of noticing on their own reflections. This study engaged teacher candidates in an additional layer of reflection, which aimed to engage candidates in the work of critical reflection.

Teacher candidates must *critically reflect* to focus on themselves *within* a broader system (Brookfield 2015; Liu, 2015) and not reflect generally, as routine reflective practices tend to be centered on oneself (Finlay, 2002). A step towards enacting equitable practices entails focusing on how broader sociopolitical issues shape teacher candidates' experiences and worldviews and implicate their practice.

When candidates from dominant backgrounds (e.g., white, female, monolingual, etc.) engage in critical reflection, prior studies have found candidates to have increased empathy (Houser, 2008; Nieto, 2006) and consider their contributions to students' struggles (Saito & Khong, 2017). However, critical reflection absent of disciplinary context can make it difficult to transfer multicultural awareness to equitable teaching in a specific discipline. Kang & Zinger (2019) emphasize a need to consider critical reflection in the context of the discipline by calling for the curriculum and pedagogy of science teacher education (especially in methods courses) to be designed in ways to develop candidates' "multicultural and flexible frames of reference along with critical consciousness about structural inequity" (p. 27). Critical reflection in a disciplinary context is also necessary, since primary teachers teach multiple disciplines, and each discipline

has its own set of assumptions and normalized practices. In this study, I argue that teachers must confront their underlying assumptions and biases and critically self-reflect in order learn to center equity in their teaching.

Methods

Research Goals

The goals of this study are: (1) to understand how teacher candidates engage in an activity that is designed to have them confront and make sense of their own assumptions and biases; and (2) to examine how their noticing of their biases and assumptions relates to how they center equity in their teaching. In what follows, I will describe the context, research design, and the Self-Confrontation Noticing Activity.

Research Context

The context of this study is in a mathematics methods course within a 14-month, post-baccalaureate teacher preparation program at a large public university on the west coast of the United States. Study participants are 21 teacher candidates enrolled in the first (Fall term) of the two-quarter sequence of the mathematics methods course. Twenty identified as female and one as male. Seven identified as White, five as Asian, seven as Latinx, and two as Unidentified. During the fall term, each teacher candidate is assigned to a mentor teacher's classroom at a local elementary school for their first of two student teaching placements in this program.

Research Design

The collaboration began a year prior to this study, during which I attended the fall course as an observer with the purpose of understanding the context and the course's learning progression. I recognized my positionality as a researcher who was entering a community of learners with existing practices and norms. I wanted to ensure that conducting my study in this

context did not create disruptions to candidates’ learning, as the course had been designed with activities that align with the course’s learning goals. By collaborating with the teacher educator, we negotiated how to best integrate the activity designed to elicit and understand candidates’ confrontation of their biases and assumptions. The teacher educator and I co-designed the sequence of the course activities to best integrate the Self-Confrontation Noticing Activity without losing the design intent of the activities (see Figure 1.1).

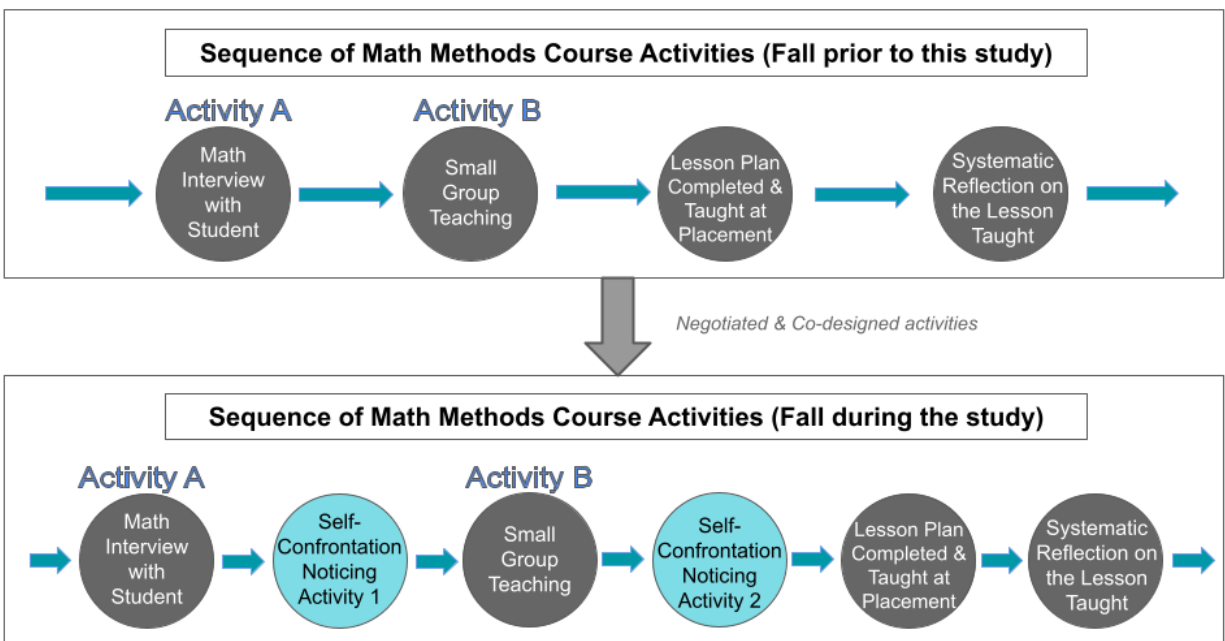


Figure 1.1. Changes in sequence of mathematics methods course activities

Self-Confrontation Noticing Activity Implementation

As described earlier, the Self-Confrontation Noticing Activity was adapted from a project that had originally created this activity for secondary science candidates to engage them in critical self-reflection on their professional discourse about student thinking and learning (Kang & Lee, 2019). In this study, the teacher educator and I made a few modifications that were appropriate for the participants of this study. Specifically, we modified the framing of this

activity to use the language that provided candidates with access, as well as the timing and the frequency of the activity implementation.

Since there were two terms of the mathematics methods course with different instructors in each term, my study was limited to one quarter with the teacher educator. Given that the sequence of course activities had been designed without the Self-Confrontation Noticing Activity in mind (Figure 1.1), we had to consider which of the existing activities created opportunities for candidates to reflect on their interactions with students. In the math methods course, there were two key activities that allowed candidates to interact with students over mathematics. The first is the Student Problem Solving Interview Activity, during which each candidate is paired with a student who is given two open-ended word problems. Each candidate unpacks the problem. Then the candidate observes the student solving the problem and asks questions to elicit their thinking. The second is the Small Group Teaching during which three candidates work together to develop a mini lesson plan to teach to three students they were previously paired with for the math interview. Both activities occurred over two classroom visits in a first grade classroom, whose teacher is a long time collaborator with the teacher educator. The classroom visits were a week apart. The Self-Confrontation Noticing Activity took place during the following class session after each of the two key activities (see Figure 1.2).

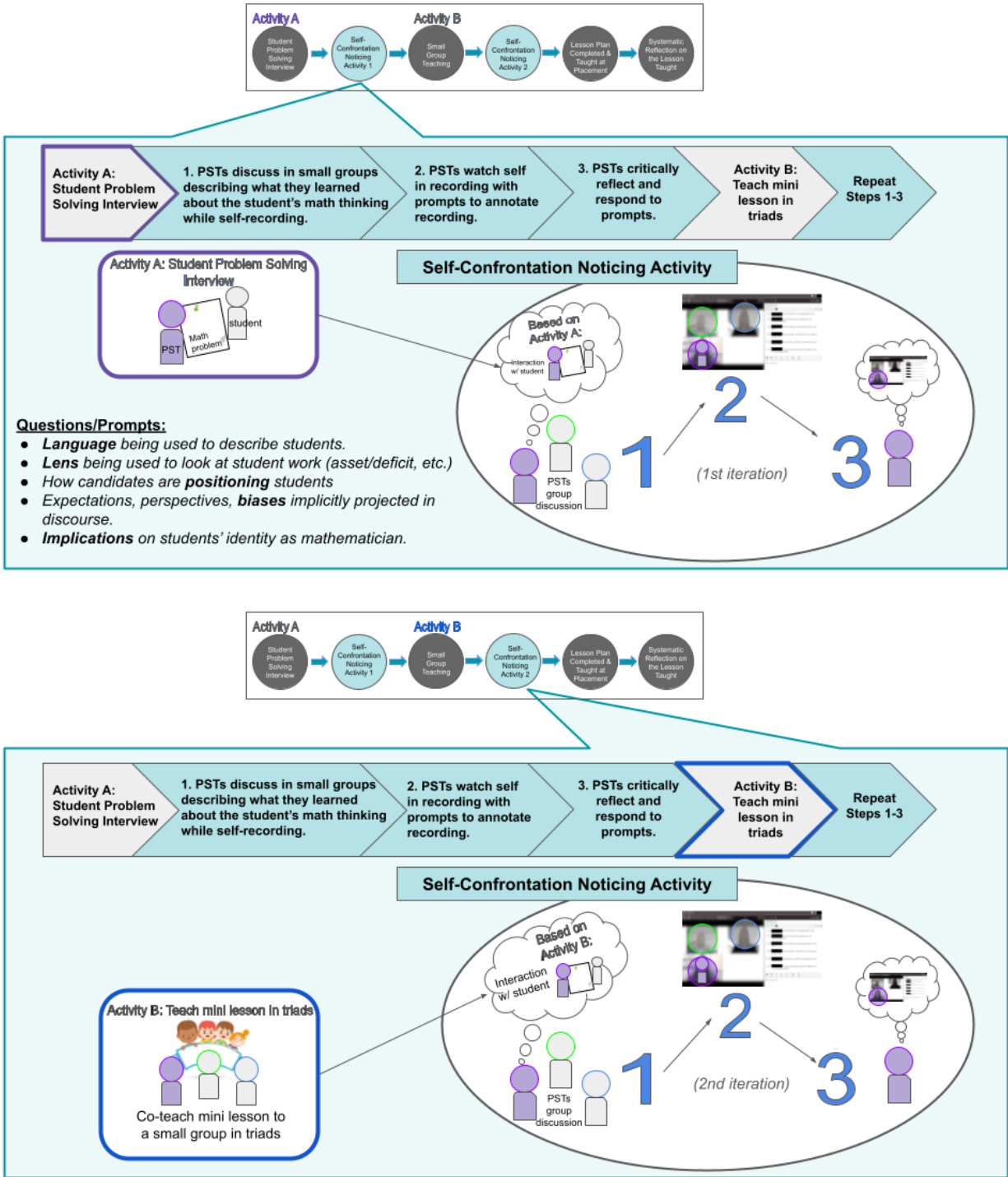


Figure 1.2. Visualization of the course sequence with the Self Confrontation Noticing Activity

Data Sources

The primary sources of data for this study were the 42 written reflections (two from each candidate) from the Self-Confrontation Noticing Activity and the 21 written reflections (one from each candidate) from the final lesson plan. The reflection responses from the Self-Confrontation Noticing Activity were completed as part of candidates' assignment after completing the video task with their colleagues. The reflections from the final lesson plan were reflections candidates completed after teaching their lesson at their student teaching placement, under the supervision of their mentor teachers. The lesson plan consists of multiple parts. The first part is the planning for the lesson and requires candidates to outline the key content standards, learning objectives, language objectives, assessments, and lesson resources or materials. The second part requires candidates to outline the instructional sequence from opening to closure. The third part asks candidates to specify the ways in which they are incorporating academic language. Finally, the fourth part requires candidates to annotate on their lesson plan after teaching it to indicate what worked, what didn't, missed opportunities, where evidence of student learning was collected, etc. Candidates are asked to indicate the learning goal and the evidence that the student met the learning objective, as well as evidence of students struggling to meet or making progress toward the goal, and the next steps in future instruction. I examined only the fourth part to capture the reflection aspect of practice as it also provided rich information on candidates' reflections, their decision making in-the-moment as well as in the future. The interviews of five teacher candidates were used to supplement information about teacher candidates' experiences with mathematics and their implications on how candidates engaged in the Self-Confrontation Noticing Activity. The five interviewed candidates were

selected based on the different ways they engaged in the Self-Confrontation Noticing Activity, to capture variation.

Data Analysis

Data Analysis Plan for the Self-Confrontation Noticing Activity

A total of 42 written reflection responses were analyzed to examine the extent this activity supported teacher candidates to attend to equity in mathematics. Using a random sample of responses, the coding scheme was developed, tested, and revised with a research assistant. After finalizing the coding scheme, the written reflection responses were double coded with a research assistant. Reliability was difficult to reach, specifically the threshold of 80% (agreement=65%). Thus, I decided to double code with the research assistant and discuss disagreements until a consensus was reached. The unit of analysis was one idea unit (Jacobs et al., 1997), which typically ranged from a clause to a sentence. The number of idea units identified in each reflection response ranged from 4 to 22. The coding scheme (Table 1.1) was developed iteratively and drew from an existing noticing framework specific to children's mathematical thinking (Jacobs et al., 2010). Jacobs et al. (2010) conceptualized the noticing work as expertise that require "three interrelated skills: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understandings" (p. 172). The authors suggested that these are the skills that occur in the background as teachers make in-the-moment decisions during instruction.

In this study, I conceptualize *attending* as the act of attending to one's own biases and assumptions about children and their mathematical thinking. (I hypothesize that when candidates have deeper understanding of children's mathematical thinking and the ways biases and assumptions shape how they interact with children, they will be better able to notice their own

biases and assumptions in the way they describe children’s mathematical thinking.) I conceptualize interpreting as how teacher candidates interpret and make sense of the nature of their assumptions and biases that they attended to as well as the implications of such assumptions and biases on children’s learning opportunities. I conceptualize *responding* as how candidates respond to what was noticed or what they have done to respond to their own assumptions/biases. While the authors focus on the intended and not the execution of a response, in this study, I include both enacted responses as well as intended responses. The primary reason for this is to acknowledge that some candidates have begun the program with some depth of understanding of issues of inequities and they provided evidence of having responded in-the-moment to their own biases and assumptions during the activity. Thus, the coding rubric consisted of Attending, Interpreting, and Responding codes. An additional way candidates reflected on their biases and assumptions was that they did not notice their own assumptions or biases, or they conveyed that they held objective views, thus were unbiased and assumption free. This fourth code was named “No Noticing.” As I developed the coding rubric, I found an ordinal relationship among the codes in Table 1.1. Each noticing competency built on each other. Therefore, each code was assigned a numerical value from 0 to 3. Since candidates completed the activity twice, two responses were collected from each candidate. I added their two scores and created summed scores, which ranged from 1 to 6.

Table 1.1. Coding rubric for how candidates noticed their biases and assumptions about students and their mathematical thinking

Code	Score	Description
No Noticing	0	Does not notice own assumptions or biases about children or their mathematical thinking, or conveys being objective, thus not holding biases or assumptions.

Attending	1	Describes what one notices, such as discrepancies between judgments made about students or their mathematical thinking and what was observed.
Interpreting	2	Recognizes own assumptions/biases and provides an interpretation of what was recognized, as well as the implications of assumptions/biases on students' mathematics identity and equitable learning opportunities. The assumption or bias that was recognized needs to be accompanied with an explanation of why one behaved/spoked/expressed in a particular way.
Responding	3	Responds to what one attends to and interprets. Describes actual action taken or proposes specific next steps in relation to what was noticed & interpreted.

Based on the summed scores, four distinct groups emerged, which will be discussed in the findings (see Table 1.2). After forming the four groups, I confirmed that the responses in each group did have a qualitative distinction and created descriptive names for each group. Table 1.2 illustrates the differences among the four groups. The percentages of the occurrence of each code were calculated to examine the variation across reflection responses (Figure 1.3). Interviews were reviewed for data triangulation.

Table 1.2. Variation in extent of awareness of own assumptions and biases

Extent of Awareness	Summed Scores	Description
Minimal or no awareness of biases and assumptions	1-2	Describing events that had occurred without attending to teacher candidates' own biases and assumptions; conveying objectivity when reflecting on biases.
Passive awareness of biases and assumptions	3	Attending to teacher candidates' own biases and assumptions. Noticing or recognizing that they hold particular ideas of mathematics or of their students without explaining how these notions implicate equitable learning opportunities.

Interpretive awareness of biases and assumptions	4	Interpreting what teacher candidates attend to, which means they are analyzing, making sense of the “why” behind their biases and assumptions <i>and/or</i> recognizing the implications of their language/actions on students’ math identities <i>and/or</i> equitable learning opportunities.
Responsive awareness of biases and assumptions	5-6	Responding to what teacher candidates attend to and interpret . Actual action taken in addressing inequitable learning opportunities or specific next steps proposed in relation to what was noticed and interpreted. Vague next steps, such as “I want to grow in this,” without linking these to implications on students’ equitable learning opportunities did not meet the criteria of specificity.

Data Analysis Plan for the Post-lesson Reflections

A total of 42 written reflections from candidates’ final lesson plans were initially analyzed. The unit of analysis was one idea unit (Jacobs et al., 1997), which typically ranged from a clause to a sentence. With the post-lesson reflection, each idea unit typically corresponded to each question prompt in the reflection portion of the lesson plan, except for two questions that asked candidates to list examples of teacher action along with evidence of students making progress (or struggling to make progress) towards the learning objective. For these two questions, each teacher action and its corresponding evidence of student learning was coded as an idea unit, as candidates listed as many as they could think of, each being distinct ideas. Thus, across candidates, there was a range of nine to 13 idea units.

I drew on Gutiérrez’s (2012) equity framework, which consists of four dimensions: (1) the extent to which candidates are supporting students to develop positive *identity*, to become and see themselves becoming better individuals, (2) *access* to tangible resources and learning opportunities, (3) *achievement* in the learning objectives when participating in quality mathematics classes, and (4) *power* to challenge issues of social transformation. The framework

guided my analytical lens to document the themes and patterns that emerged from the post-lesson reflections.

Each idea unit was categorized under one of the four equity dimensions. Then, among the idea units in each dimension, I identified themes, which revealed two distinct ways candidates reflected. They either perpetuated traditional, narrow views that maintain social hierarchies and limited how smartness and competency is defined, or they acknowledged the multiple ways of knowing and doing mathematics and expanded how smartness and competency is defined (which aligns with the current state standards of mathematical practices). Thus, I coded each idea unit as either a productive or unproductive view. See Table 1.3 for the coding rubric.

Then, I examined whether there were patterns in each candidate's post-lesson reflection. I noticed that some candidates were consistent in their thinking—they held only productive or only unproductive views in one or more dimensions. A large number of candidates varied in their thinking by holding both productive and unproductive views in a dimension as they reflected on their lesson. An overall code was assigned to each candidate's post-lesson reflection. If candidates only had "productive" or "unproductive" codes for a specific dimension, then they were assigned an overall code of "productive" or "unproductive", respectively, for that dimension. If candidates had both "productive" and "unproductive" themes for a specific dimension, then they were assigned "mixed" for that dimension. Figure 1.4 displays a data matrix to examine patterns in how candidates center equity based on their extent of awareness of their assumptions and biases. Each row represents each candidate.

Table 1.3. Coding rubric of equity dimensions for post-lesson reflections

Equity Dimension	Codes*	Description	Themes
Access	Productive	The ways students are given access to participate and learn are expansive and inclusive; supports students' development of positive identities as doers and knowers of mathematics.	<ul style="list-style-type: none"> ● Provides multiple ways to provide access to learn and participate ● Student collaboration as a form of providing access to learning ● Recognizes own role in preventing access
	Unproductive	The ways students are given access to participate and learn are limiting opportunities to learn and develop positive identities as mathematicians. (e.g. simplifying problems, providing direct instruction, etc.).	<ul style="list-style-type: none"> ● Simplifying problems (lowering cognitive demand of task) ● Resorting to direct telling of what to do, creating structures and sequence of steps to follow
Achievement	Productive	Perceptions of achievement (e.g. what counts as learning in math), measures of achievement, and the supports provided (or planning to provide) for students to achieve that foster inclusive learning opportunities and disrupt dominant hierarchies in math classrooms.	<ul style="list-style-type: none"> ● Achievement as beyond correctness, focus on strategies/multiple ways of doing math ● Multiple ways to measure learning ● Can support learning in ways that empowers students
	Unproductive	Perceptions, measures, and supports provided for students that maintain a culture of exclusion (Louie, 2017), which assumes hierarchy among students and limits learning opportunities for all students.	<ul style="list-style-type: none"> ● Achievement as correctness ● Limited ways of measuring learning (e.g. no work shown means no understanding), through a deficit lens ● Needs to model and support for students to learn
Identity	Productive	Candidates' views and actions reflect efforts to	<ul style="list-style-type: none"> ● Sees students as math contributors/knowers

		create opportunities for students to see themselves as knowers, doers and contributors of mathematics, see meaningful relevance, and understand the broader world.	<ul style="list-style-type: none"> ● Wants students to have cultural and personal connections to problems (“mirror” seeing selves in curriculum) ● Attends to socioemotional learning/comfort/safe environment to share in class ● Recognizes the need to have diverse thinking/approaches from students
	Unproductive	Static views of students and actions reflect an effort to categorize hierarchically and assign labels to students.	<ul style="list-style-type: none"> ● Tendency to hierarchically categorize kids ● Labels assigned to kids (e.g. ELLs, low/mid/high levels, struggling students, behaviors, participation, age) ● Not attending to identity of ss (at all)
Power	Productive	Candidates’ views and actions convey efforts to empower students by recognizing (and/or addressing) unequal distribution of power among students and between the student(s) and teacher.	<ul style="list-style-type: none"> ● Recognizes unequal distribution of power among kids ● Wants equal distribution of power among by hearing different student voices ● Wants students to see each other as resource ● Wants to position kids as contributors of knowledge ● Sees own role in perpetuating hierarchies (e.g. selectively calling only on some kids) ● Recognizes teacher’s own power and tries to share power with students (positions kids as contributors of knowledge) ● Starting to recognize own power (but still learning how to distribute power)
	Unproductive	Views and actions don’t show evidence of attending to power dynamics in class, perpetuates	<ul style="list-style-type: none"> ● Doesn’t attend to power dynamics in the classroom at all

		<p>hierarchies among students and between teacher and student(s).</p>	<ul style="list-style-type: none"> ● Sees and perpetuates hierarchy among kids by positioning smarter/advanced kids as knowers ● Doesn't recognize own power and maintains it. ● Primary holder of knowledge, checks for confusion/ struggles, makes the connections for students (math ideas) ● Guides students towards the 'right path' of solving math problems
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**Note.* If a post-reflection response was assigned both productive and unproductive codes within a dimension, then an overall “mixed” code was assigned.

Unpacking the Coding Framework for Centering Equity in Practice

To better understand the coding framework to capture the ways candidates center equity in their practice, I will unpack excerpts from two teacher candidates' post-lesson reflections. The first example conveys a mixed view across several dimensions of equity. The second example conveys a consistent, productive view across multiple dimensions of equity.

In the first example, a teacher candidate described an action she took during the lesson (*"Teacher only called on students who rose their hand and chose to participate in the lesson"*) and reflected on her action (*"As an educator, I know that those students were focused/on task, I should have chosen sticks to get perspectives from others, even those who were unsure of finding the right answer."*). This idea unit demonstrates an unproductive view on access as the candidate suggests using "equity sticks" for equality or equal participation and on identity by assuming that students who raise their hands are the ones focused and on task. Simultaneously one productive view on power is illustrated through the candidate's recognition of selectively calling on students to only let a few voices be heard. In the same post-lesson reflection, the following excerpt exemplifies a productive view on power: *"If I could, I would want to begin by having different participants in my lesson, even those who rarely speak up."* Then, an unproductive view on students' identities is demonstrated: *"It will give me a clearer understanding of where they are and how I, as the educator, could guide them down the right path. I would also want something to help my "quicker" students not get as bored in the lesson."* Labeling students as "quicker" conveys an unproductive view on students' identities and simultaneously needing to guide her students "down the right path," as if there is one right way to do the mathematics is an unproductive view of power and access. Then, an unproductive view of access is conveyed through a broad assumption that reading learning objectives will give access for all students to

understand the lesson: *“At the beginning of my lesson I have noted that in all my subjects, not just math, I called on selected students to read the objectives, and re-watching my videos that tells me, that only those students, specifically, comprehend our lesson. I would want to have all of the students read the objectives so they can [have] some engagement and comprehension of what the lesson will be about.”* The basis for the broad assumption was determined by successful students.

In the second example, a consistent productive view of power, access, and identity is conveyed in the following excerpt:

“Something that I would work to do next time is include student methods for answers that might not be right. I think a lot of my thought process is shared among many new teachers where I think that if I share a wrong strategy then it might confuse students. But what I noticed is that students are fairly quick to disagree with answers that they deem “not correct.” In this class, it’s shown silently and students never say “you’re wrong.” I think when you bring to light students with different thinking, it gives their classmates opportunities to change their thinking by creating relatable and student friendly methods of learning. It also needs to be an idea that is reinforced in the class that missteps add to our growth mindset and help us learn.”

Positioning students as competent and knowers of mathematics who can contribute to the knowledge building happening during the lesson demonstrates a productive view on identity. Implicitly recognizing the power held by the teacher and planning to create opportunities for students to be the knowledge contributors of mathematics conveys a productive view of power. Also recognizing that students learn by engaging in each other’s thinking, as opposed to having the teacher explain a single, correct strategy is another example of a productive view of power.

Findings

Research Question 1 Results: Variation in how candidates notice their own biases and assumptions about children and mathematics learning

Teacher candidates exhibited a variation in the way they engaged with the Self-Confrontation Noticing Activity (Figure 1.3). Among 21 candidates, 14% (n=3) were engaged in the Self-Confrontation Noticing Activity with *minimal to no awareness* of their assumptions and biases; 24% (n=5) were engaged in the activity with *passive awareness*; 33% (n=7) were engaged in the activity with *interpretive awareness*; and 29% (n=6) were engaged in the activity with *responsive awareness*.

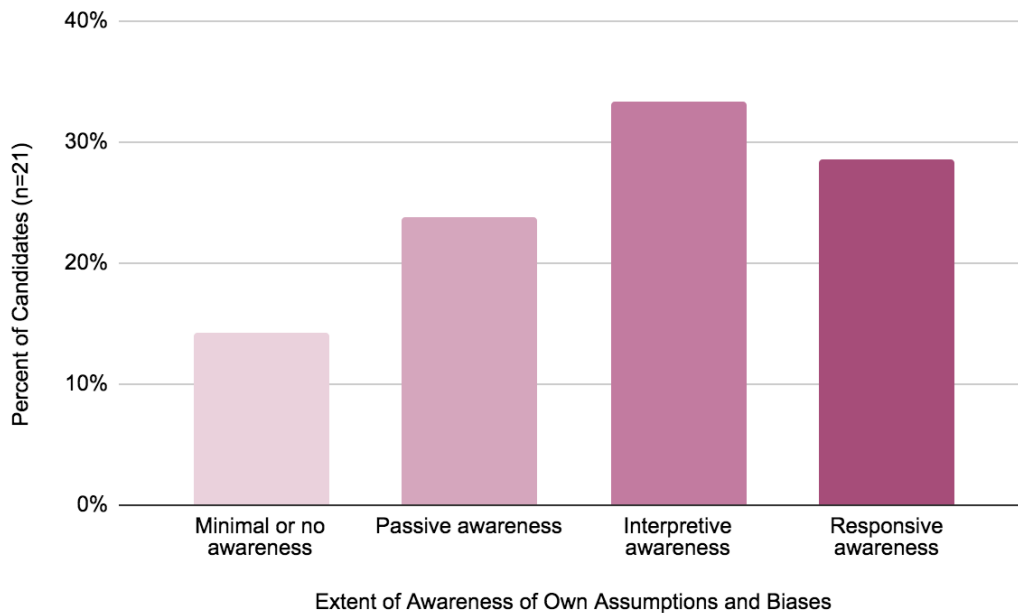


Figure 1.3. Teacher Candidates' Extent of Self-Awareness (Total of 42 responses)

Variation in Candidates' Engagement in Self-Confrontation Noticing Activity

Minimal or no awareness. One way candidates engaged in the self-confrontation noticing task was by not attending to their own assumptions or biases. Candidates were given a set of questions that prompted them to reflect on their language use, their lens when looking at

student work, how they position their student(s), their expectations/perspectives/biases projected in their discourse, and the implications on students' identity as a mathematician. Even though candidates were prompted to reflect on their own assumptions and biases, they responded to the prompts by not focusing on themselves but rather on students or justifying an expectation/bias they identified. For example, a candidate wrote, "*I expected [student] to get distracted which is why expectations and modeling were important to the lesson.*" This quote exemplifies the candidate's assumption that the student will be distracted. Thus, the candidate claims the need to model how to do the problems to help the student focus, which takes away the student's agency and imposes the teacher's thinking on the student.

Passive awareness. The second way candidates engaged in this activity was by showing passive awareness of their own biases and assumptions of children and their learning. This means they were attending to their own biases and assumptions and recognizing that holding such ideas is not good for their students. For example, a candidate wrote, "*Although in my head I was not purposefully positioning her as mathematically smart, I believe I made some observations that characterized her as being competent. For example, I said that [student] was good at using manipulative to demonstrate what she did. I said she was good at talking out the first problem. I did not consider [student's] funds of knowledge at all. I think I could have done better at seeing her as mathematically competent.*" This quote demonstrates that the candidate is aware that they are holding both an asset and deficit lens by juxtaposing contrasting statements. However, she does not make sense of why she was not purposefully positioning the student and not considering the student's fund of knowledge. The candidate also does not consider the implications of biases on equitable learning opportunities.

Interpretive awareness. The third way candidates engaged in this activity was by interpreting and making sense of the why behind their biases and assumptions and/or how these implicate students' math identities and learning opportunities. For example, a candidate wrote,

“When I’m describing her performance and behavior, sometimes I have a puzzled look or tone of voice...With this language and puzzled expression I am, in a sense, labeling her as someone different that I have yet to figure out. I’m labeling her as someone who has thinking that isn’t normal and who doesn’t belong... While I do validate her thinking and acknowledge that she has mathematical sense and capabilities, my labeling of her as an “other” or a different/special case could be dangerous. Identifying her needs and her different thinking is important to understanding who she is, but categorizing her as a separate entity could affect how I and others see her.”

Quote shows how the candidate's bias/assumption is coming out of their expressions and tone as they talk and reflects on the fact that they are holding a bias internally by seeing the student as an “other.” The candidate recognizes that othering is dangerous and implicates the child's identity.

Responsive awareness. The fourth way candidates engaged in this activity was by showing responsive awareness. These individuals not only recognized and deeply made sense of their biases and assumptions, they also strategically thought about what they could do next to address their problematic actions and/or thoughts. For example, a candidate wrote,

“I think there was a broad assumption that he was an ELL because he interjected 40panish numbers into the count. Looking back, I don’t think this necessarily means that the student is ELL if he adds 40panish numbers. He understood everything I said to him and he translated it back well... I think the one thing I would change about the way I am teaching is how much prompting I am doing with students. In some parts of the video I

voice that I got nervous that the method the student was using would confuse him so that's why I asked him to give more space in between his manipulatives but that is also an assumption that he might make a mistake in his counting. So I guess it shows that I am not fully letting students create their own thinking. I think this can lead to a more teacher centered classroom if I'm not careful, I should be more conscious to allow students to explore their learning and have more questions to guide their thinking rather than actually telling them what to do."

This quote shows how this candidate recognized an assumption they were making about the child's language ability and noticed how the candidate was doing much of the mathematical thinking by prompting the student frequently to guide him. The candidate recognized that their moves were rooted in being nervous that the kid might make a mistake and how that led the candidate to control the situation. The candidate plans to respond by prompting less and creating space for the kids to explore and create. The candidate plans to develop questions to provide scaffolds instead of telling students what to do.

Research Question 2 Results: Variation in the relationship between teacher candidates' noticing of their own biases and assumptions and how they center equity in their teaching

Teacher candidates exhibited different patterns of centering equity in their teaching based on their extent of awareness of their own biases and assumptions. Candidates who demonstrated *responsive awareness* were more likely to consider all four dimensions of equity in their practice. For teacher candidates who demonstrated *interpretive awareness* in their reflections of their own assumptions and biases, most considered achievement and access productively and most considered identity and power in unproductive ways. There was less of a clear pattern

among teacher candidates who demonstrated *passive awareness* or *no to minimal awareness* of their own biases and assumptions. See Figure 1.2 for more detail.

A couple of clear patterns emerged from this analysis, as displayed in Figure 1.2. When candidates were able to make sense of their assumptions and biases by considering why they hold those views, the implications of their views on children's learning opportunities, and how to respond to their problematic views, their practices were more likely to center equity. Candidates who attended to and made sense of their assumptions and biases but did not yet consider how to respond, they were more likely to demonstrate productive views of achievement and access. Candidates who either attended to their assumptions and biases only or did not at all were showing variation in how they center equity in their practice. Among these candidates, most held productive views of achievement and mixed views of power and access with a few holding productive views of identity. To provide a more nuanced explanation of the patterns found in the analysis, I present two cases of teacher candidates and unpack the ways each candidate engaged in the pedagogical activities and the influence of their prior lived experiences and their concurrent student teaching experiences on how they confront and reflect on their own biases and assumptions.

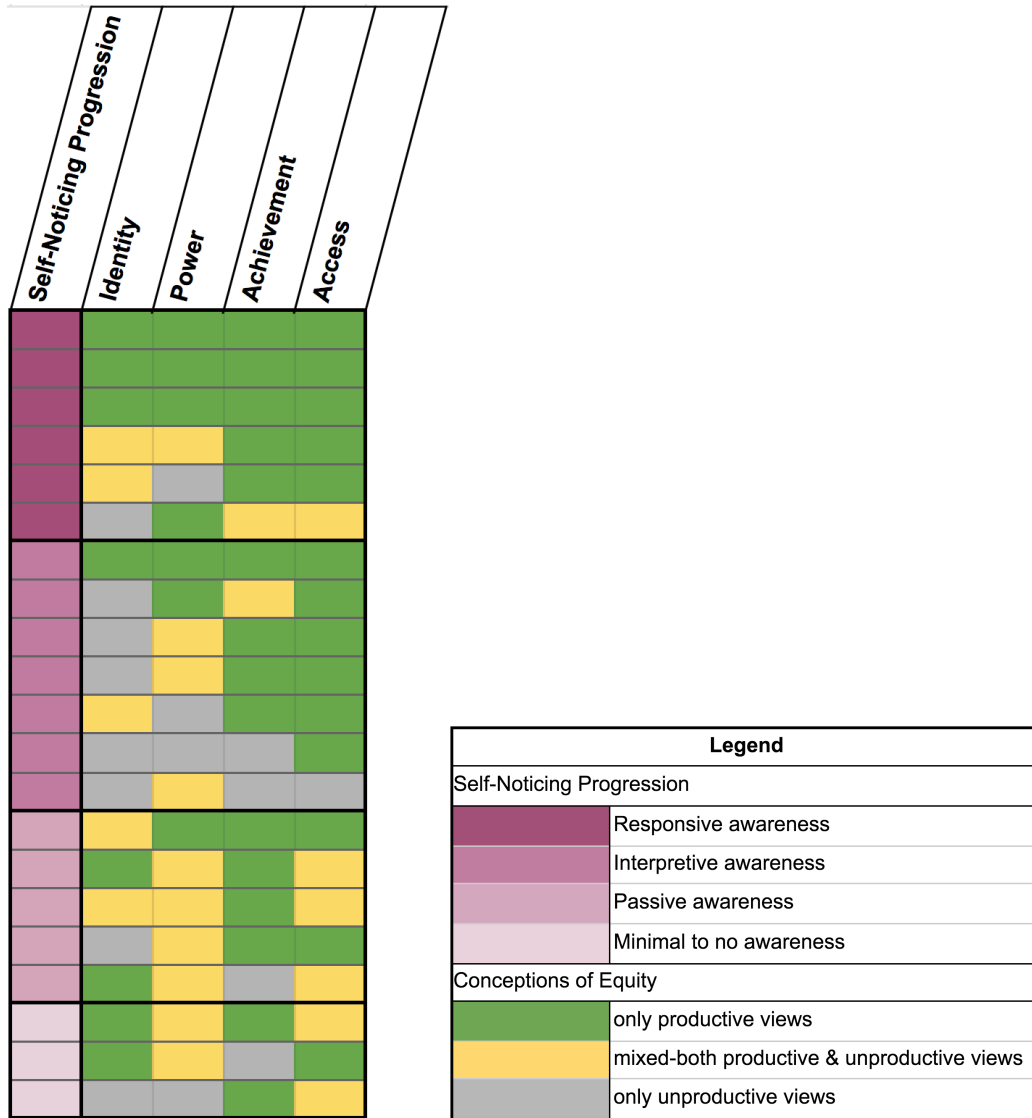


Figure 1.4. Patterns of candidates' extent of awareness of assumptions/biases and how equity is centered in practice

Case of Blake

Blake's Background

Blake is a 22-year-old bilingual candidate who identifies as Hispanic/Latina. She entered the teacher preparation program soon after she graduated with bachelor's degrees in Sociology and Education. She shared that once she knew she wanted to go into teaching, she switched from

a Biology to an Education major. Her placement was in a bilingual elementary school, which is located near a city she grew up in.

Blake's Experience with the Self-Confrontation Noticing Activity

Blake's experience with the Self-Confrontation Noticing Activity illustrated a case of a candidate demonstrating minimal self-awareness due to limited learning opportunities to confront her biases from being intentionally objective. Although she demonstrated minimal awareness of her assumptions and biases, in her interview, Blake admitted that she held deficit thoughts. However, she explained she was being cautious with her language by "intentionally being more matter of fact on purpose... because I didn't want to use another word, so like when you're describing a child you want to use certain words to not offend anyone or like disrespect the child." By doing so, she recognized how discourses that characterize students as incompetent were not appropriate and she believed that she was not participating in such discourses. Thus, in her self-confrontation noticing reflection, she described her tone as "matter-of-fact" and did not describe any preconceived notions she held about students. This may likely be because she did not perceive herself as using deficit-oriented language. Her framing of her reflection response was that she was using a "teacher lens" in addition to being "matter-of-fact." However, her perception of being objective was misleading her to believe she did not convey biases or assumptions. In her reflection responses, she did not respond to the prompt about the implications of her bias and assumptions on students' identities in the way it was designed. Instead, she described the implications of students' lack of preparation to solve problems on their confidence and identity. Her reflection responses were missing the 'self' in self-reflection and primarily consisted of describing observed or anticipated student struggle and what she should do mathematically during instruction to help students understand.

When examining her videorecording of her describing her student's mathematical thinking, there were multiple instances where she missed the opportunity to build on what the student already knew and understood. In one example, Blake described how she showed her student a penny and asked if he knew what it was. Blake narrated how the student first called it a quarter and then a nickel after being asked to try again. Blake shared with her colleagues her interpretation that this student "doesn't have that much experience with [coins]." She then asked if he had experience with money, to which he said he had a dollar, which helped him purchase a video game. Blake explained to her colleagues her claim that "he doesn't know a lot about money or how money works" because video games cannot be purchased with a dollar "unless if it is from a thrift store." Blake demonstrated her sensemaking of the situation through a deficit lens by jumping to the conclusion that the student did not have access to the problem because of his lack of knowledge of money and lack of experience with purchases, despite his responses showing partial understanding and some experience with money. However, Blake described her depiction of the student as being objective and focusing on only his mathematical understanding.

When triangulating her reflection response, the actual video recording of her discourse, and her interview transcript, it was clear that she did not notice the subtle ways her deficit-oriented lens was evident in her language. Her reflections focused on her teaching and what more she needs to work on to help students understand instead of focusing on her biases and assumptions and how they implicate students' identities.

How Blake's Noticing is Related to her Centering Equity in Teaching

Her limited noticing of her own biases and assumptions aligned with how she centered equity in her practice. Blake did not engage in the pedagogical activities in ways that show the influence and power teachers hold in shaping student learning opportunities and classroom

culture. During the interview, she provided more insight on others (mentor teachers, students, her colleagues) than herself by pointing out her colleagues' assumptions and biases. Reflective practice may be something she is learning to engage in, as she mentioned in the interview that "I don't do a lot of reflection when I'm teaching. I just teach my lesson." Blake's tendency to focus on action without reflection along with her minimal awareness of her own biases/assumptions created an obstacle for her by limiting her opportunities to learn and improve. Moreover, Blake's student teaching experiences also shaped her vision of high quality instruction.

At the time of the interview, Blake transitioned from her first student teaching placement (kindergarten) to her second student teaching placement (4th grade). She reflected on the stark contrast between the two mentor teachers. Blake described her first mentor teacher's practices as being student-centered and adopted her mentor teacher's language, which suggested that Blake found her instructional practices agreeable, or at the very least not disagreeable. She described a practice of ability grouping students into "lower group" and "higher group" to teach in small groups. Blake shared her concern about leading a small warm-up activity, which was an open-ended task that engaged kids in looking for patterns which of four images did not belong, because of her concerns that the students may not understand how to engage in this activity. To her surprise, students were able to understand and engaged in a mathematics discussion, and "even the lower group was able to understand it, so I was like truly amazed how like the lower group was able to understand my concepts that I was trying to teach." She contrasted the performance between the two groups of students, "...it was still a difference the higher group saw more, and the lower group were like, 'oh well that one doesn't belong,' and so they saw like one of two options and the higher group saw like all 4."

Then, Blake narrated her second mentor teacher's practices, describing it as a stark contrast due to "intense direct instruction." In this class, Blake described the teacher-centeredness: "what the kids do is wait until the teacher gives them the answer or they'll wait for the other two peers who always answer it correctly be like "this is the answer, this is the answer, this is the answer." Blake's describing the two experiences with a stark contrast in tone and language suggested that Blake found her first mentor teacher's practices as favorable and agreeable. Blake characterized the first mentor teacher's practice as being student-centered because nearly all students participated, and the mentor teacher was open to multiple ways of engaging in mathematics. Blake's lesson plan reflection conveyed productive views on identity and achievement, which aligned with her first mentor teacher's practices. When considering equitable instruction, Blake focused on participation patterns and access to support participation, which Blake described as the key characteristics of her mentor teacher's student-centered practice.

Blake's two student teaching placements were her only practice-based experiences that informed her vision of high-quality, equitable mathematics teaching. She compared the two student teaching experiences and described agreeable practices demonstrated by one of her mentor teachers. These agreeable practices appeared in her lesson plan reflection. When I examined her lesson plan reflections through the equity framework (Gutiérrez, 2012), she conveyed both productive and unproductive views on the power and access dimensions. For example, she showed evidence of positioning students as contributors of knowledge during the lesson by providing opportunities for students to share their thinking with each other to build understanding together; however, she also conveyed that she is ultimately the knower in the class and needs to provide explanations on the board for students. She conveyed a productive view on

access by providing multiple tools and strategies for students to participate, and she saw the strength in students communicating their thinking with each other as it provides another form of access to the content. However, she conveyed an unproductive view when she described simplifying the work or problems (by lowering the cognitive demand) to provide access to the learning opportunities. She stated that simplifying how students complete the worksheet by allowing students to choose and copy the explanations provided by the teacher with simpler words was appropriate, especially since the students were kindergarteners.

Case of Kelly

Kelly's Background

A direct contrast to Blake, Kelly is a candidate who enjoys engaging in reflective practices and has a heightened sense of awareness of her assumptions and biases. Kelly is a 25-year-old White Female teacher candidate who completed both her Bachelor's and Master's in a STEM discipline prior to the teacher education program. She described her prior graduate school experience as equipping her with experience and skills to connect theory to practice. Her placement is in a magnet elementary school. She described that she did not interact with students identified as "English language learners" until she moved out-of-state to this teacher preparation program.

Kelly's Experience with the Self-Confrontation Noticing Activity

In her critical self-reflection she demonstrated responsive awareness of her assumptions and biases. In her reflection response, she noticed her use of the label "English Language Learner" when describing her student and her facial expressions "that may have conveyed annoyance or frustration about the [Student Problem Solving] interview." She attended to the subtle changes in her tone from matter of fact when describing the student to negative when

describing “the more challenging aspects [she] faced working with an English language learner.” She noticed her deficit-oriented discourse, which consisted of what the student was unable to do or struggled with. She reflected on how she positioned her student as “mathematically incompetent” and noticed how when student does not respond quickly enough, she jumps in, “cutting off his thought process.”

When reviewing her videorecording, the facial expressions she made, the focus on the student’s deficits she described were all observable. As an observer, it is challenging to interpret her mannerisms and the intent and thinking behind those; thus, her reflection responses provided deep insights into her underlying thinking. Her self-recorded video served as a useful tool for her to reflect at a more meta-level. Kelly described her familiarity in the importance of reflecting on biases and assumptions and their impact on students, but to actually see it, she stated “it really surprised me. I was like, wow, I’m using a lot of negative language. Is that coming across in how I’m teaching this student?”

Her interview conveyed a narrative coherent with her videorecording and reflection response. She described the same assumptions she mentioned in her reflection response—that she “made a lot of assumptions that his math skills were related to his inability to translate his thinking. I was like oh, he doesn’t understand it ‘cause he’s not talking to me, which is not the case at all.” She reflected on how the Self-Confrontation Noticing Activity “was helpful for me to catch myself. I feel like that’s a really bad assumption to have. But now I see that I have that.”

How Kelly’s Noticing is Related to her Centering Equity in Teaching

Kelly’s engagement in the Self-Confrontation Noticing Activity did not strongly relate to how she centered equity in teaching. Kelly’s responsive awareness surfaced in how she

considered the power dynamics in the classroom, but she conveyed mixed views on achievement and access dimensions and unproductive views on identity.

In regard to the identity dimension, Kelly attended to the labels associated with the students. In her lesson reflection, she did not provide evidence of considering the multiple identities of her students nor the ways to support positive math identity development, which was unexpected as she explicitly discussed this in both her interview and in her reflection response for the Self-Confrontation Noticing Activity. Her concerns for students labeled as ELL and/or with IEPs emerge in her reflection and considers ways to utilize tools to provide access to the content.

When considering access and achievement, she conveyed mixed views. She considered practices that promote multiple access points to the content by considering intentional use of tools, having an expansive conception of what counts as math learning, valuing multiple solution strategies, and providing opportunities for students to engage in each other's thinking. At the same time, she considered lowering cognitive demand by simplifying problems or creating more structure and scaffolds as ways to provide access and promote achievement. Her approach to supporting students with an IEP or an ELL label was one that positioned students as needing guidance from the teacher, which aligned with the way she perceived her "ELL student" with a deficit lens when discussing with her colleagues.

Kelly attended very closely to the power dynamics within the classroom between her and her students and among students. She created multiple opportunities to shift the power to the students so that they are positioned as knowers and contributors to the learning. She recognized that there was an unequal distribution of power among kids and made intentional moves to position kids competently.

Her lesson plan reflections appeared to be focused on the details of mathematics and did not reflect the same thinking reflected in her other data sources. It may be that the template of and the questions in the lesson plan reflection did not provide the same prompts to critically reflect as did the other activities. This template is generalized across all content areas and used in all courses in the teacher preparation program. The teacher educator shared that the lesson planner template ends up becoming an assignment that candidates complete for the sake of completing the assignment and teachers typically do not use a template as such when lesson planning. If Kelly shared the same sentiment, this could be a potential reason for lacking the same kind of reflection she provided in other data sources. Additionally, Kelly's experience with "English language learners" is new, as she had never interacted with the population prior to this program. She was learning to interact with students with specific learning needs in productive ways that position them as competent and promote equitable learning opportunities.

Summary of Findings

This study answered the following two research questions: (1) How do teacher candidates notice their own biases and assumptions about children and mathematics learning? (2) In what ways does teacher candidates' noticing of their own biases and assumptions relate to the ways they center equity in their teaching?

To summarize, this study characterizes the different ways teacher candidates engaged in an activity that required them to confront their own biases and assumptions about children and mathematics learning. Candidates demonstrated four ways they engaged in the Self-Confrontation Noticing Activity: little to no awareness, passive awareness, interpretive awareness, and responsive awareness. Most candidates demonstrated their deep engagement and awareness of their assumptions and biases.

The study also examined the extent of alignment between noticing one's own biases and assumptions and how equity is centered in one's practice. There was some variation in the alignment, which depended on how deeply candidates were self-aware of their own biases and assumptions. Candidates with deeper self-awareness were more likely to center equity in their practice by considering all four equity dimensions—identity, power, access, and achievement—in productive ways that promote equitable learning opportunities and support positive mathematics identity development. Candidates with little to no self-awareness or with passive awareness demonstrated more variation in how they centered equity in their practice. These candidates had more productive views on what counts as achievement/learning but conveyed more mixed views on power and access than in other dimensions. Candidates' notion of achievement considers various forms of evidence of learning and recognizes that mathematics is beyond knowing the correct solution and the most efficient strategy. Their conception of achievement is consistently pushed to expand in the mathematics methods course, as it is one of the instructor's themes for the course.

Interpretation of the Case of Blake

The case of Blake illustrated how having an intent to not engage in problematic discourse misled the candidate to believe that her discourse was free of biases and assumptions. In her actual reflection response for the Self-Confrontation Noticing Activity, her language conveyed her assumption that the student had limited understanding and knowledge based on the fact that the student did not know what a penny was: "I thought he wasn't going to get that one penny meant one cent, so I was worried if he would've known one to one correspondence." She over-generalized what the student knew and understood and decided to move away from the student's thinking and try multiple strategies to support their understanding. She conveyed a teacher-

centered approach in her practice by “[changing her] tactics right away if the child wasn’t getting the first strategy” and believed the child was “headstrong” and “stubborn” because he “wanted to do things (math strategy) the first way.” Rather than investigating why the student wanted to work through his strategy and building on what the student understood, Blake tried to introduce other strategies and believed that the appropriate approach for a “stubborn” student was to “have multiple strategies ready because there’s no one fits all strategy for everyone.”

However, she did not recognize that she was assuming and over-generalizing what the student understood and could do. With other candidates who were franker in their characterization of students, unveiling asset-oriented and deficit-oriented discourses, they were more successful in recognizing their assumptions and biases and reflected more deeply about their own role in students’ learning experiences.

Interpretation of the Case of Kelly

The case of Kelly illustrates the power of having deep self-awareness of one’s own biases and assumptions. Among those interviewed, Kelly was the only candidate who reflected metacognitively about her own reflections to better understand where her biases and assumptions were coming from and ways to challenge them. She had an epiphany during the interview that this assignment was promoting the work of “reflecting on the reflection,” as “you shouldn’t only reflect on your lesson but reflect on how you’re thinking about it.” Kelly’s developing understanding of the learning trajectories of emerging bi- or multi-lingual learners was due to her limited experience with such students prior to the program. Her developing knowledge led her to place a focus on how to provide support for them—but in ways that further limit equitable learning opportunities. However, the Self-Confrontation Noticing Activity created an opportunity for her to notice and confront her assumptions and biases about emerging bilinguals.

Despite her self-reflective disposition, biases and assumptions surfaced when Kelly had no prior experiences to draw on. Due to her self-reflectiveness, she was able to quickly notice and work towards expanding her conception of asset-oriented teaching approaches that position all students as competent. The case of Kelly supports findings by Garmon (1998) in that, candidates who demonstrated self-awareness/self-reflectiveness and openness were more likely to develop favorable attitudes towards diversity and openness to new or conflicting ideas.

Discussion

The findings from this study suggest how a tool that elicits assumptions and biases and engages candidates to confront their assumptions and biases and creates learning opportunities to reflect on the lens through which they see children and their mathematical thinking. One hypothesis is that candidates who are more blunt or open to sharing their thinking may have more success in recognizing the fact that they hold particular assumptions and biases. However, simply recognizing is not enough for candidates to reflect and learn. They need to be posed with questions that push their thinking and draw connections to their role in children's learning experiences and opportunities. Blake's approach to centering equity in her teaching could be strengthened if she had opportunities to engage in critical reflections with mentor teachers or teacher educators. Her student teaching was an important experience she drew from as she made sense of her learnings in the program.

The innovative video-based Self-Confrontation Noticing Activity served as a helpful tool in prompting Blake and Kelly to revisit their thinking when they described their students' mathematical thinking to their colleagues. They were able to remember their thoughts during the activity. However, it was the depth of awareness and familiarity in reflective practices that allowed Kelly to engage in the pedagogical activities as intended. Blake shared her assumptions

without realizing they were assumptions that could implicate her student's learning opportunity and identity development.

The findings from the case studies point to the importance of engaging in critical reflection that is centered on oneself instead of having self-reflection be a byproduct of a pedagogical activity. This is not to create a negative experience for candidates to feel guilt for being who they are but rather to recognize that becoming a culturally competent teacher means becoming someone who recognizes there are tensions and struggles to encounter and process (Buehler et al., 2009). It is important for candidates to recognize that they are not neutral, objective beings but are cultural beings with diverse, rich histories and experiences.

It is critical for teacher educators to also learn who their teacher candidates are to better understand ways to support candidates to learn more about themselves as cultural beings and the role they play in shaping children's learning opportunities. This can be done through creating assignments and activities that engage both teacher educators and candidates to learn about themselves and each other, in a way that models the kinds of classrooms we would want to see in elementary schools. In addition, creating such experiences in the context of the discipline is crucial for candidates to consider their role as *mathematics* teachers in shaping students' learning opportunities. There are specific ways access, achievement, identity, and power play out within the context of mathematics classrooms. The equity framework developed in this study is specific to productive and unproductive perspectives in mathematics teaching and learning.

Teacher educators need to recognize that some candidates may continue to maintain unproductive practices for various reasons. Candidates may feel uncomfortable with the mathematics content as elementary candidates have been documented to enter programs with highest levels of mathematics anxiety (compared to students pursuing other majors) (Hembree,

1990). Struggling with the content makes it more challenging for candidates to meaningfully integrate equity and social justice lens in mathematics (Garii & Appova, 2013). Another reason may be that their field placement experiences align with their unproductive practices, which are then validated and reinforced. Field experiences have been documented as one of the weakest areas of teacher preparation programs due to the wide variation of the quality of student teaching placements (Wideen, Mayer-Smith, & Moon, 1998). Even among the candidates in this study, conversations and questions raised during class revealed the wide variation to be true. Teacher preparation programs can investigate ways to examine the alignment between the program and field experiences, as coherence across all experiences create powerful learning opportunities for candidates (Koerner, Rust, & Baumgartner, 2002).

Conclusion and Limitations

In conclusion, there were variations in how candidates engaged in the pedagogical activities in this study. Using the described pedagogical activities could serve a useful purpose for teacher educators as they prepare candidates to develop a strong vision for equitable, high-quality instruction and enact the equitable practices in mathematics teaching. Making visible candidates' thinking allows teacher educators to see the kinds of assumptions and biases they may hold to then be able to consider ways to support candidates to recognize and challenge problematic views and practices. The findings of this study support Kagan's (1992) claim that "the practice of classroom teaching remains forever rooted in personality and experience and that learning to teach requires a journey into the deepest recesses of one's self-awareness, where failures, fears, and hopes are hidden" (p. 196).

This study also builds on studies that have found the approach of adding culturally relevant content into an existing teacher preparation structure or curriculum is not enough to

support candidates' learning to integrate multicultural education (Barry & Lechner, 1995; Goodwin, 1997; Tatto, 1996). Furthermore, learning to center equity in teaching likely looks different in each discipline as every discipline has its own set of disciplinary practices.

There were some limitations to the study. The lack of coherence in how candidates engaged in the multiple activities may be due to their differing and/or conflicting practice-based experiences as well as the different foci and messaging sent by the different teacher educators within the program. Some of these contextual challenges may be difficult to address by a single teacher educator. These challenges require program-level improvement efforts to build coherence for candidates' learning. Another limitation is that this study did not consider ways to support candidates who have stronger filters and present themselves in the way they want to be perceived, especially when they are recorded. This would not create opportunities for candidates to notice the subtle ways biases and assumptions shape their thinking and decisions. A possible way to address this is to interact with candidates throughout the course by responding to their thinking in their coursework and pose specific prompts that push them to self-interrogate and self-reflect. The teacher educator and I discussed this exact issue of gaining access to their thinking when working with students. Our solution was to create an ongoing fieldwork noticing journal, which was implemented in the next year, to gain access to their thinking and respond in ways that push candidates to question the lens they use to observe and interpret students' mathematical thinking and engagement in class.

Future research could explore the role of the Self-Confrontation Noticing Activity in eliciting biases and assumptions and whether they are linked to particular racial or ethnic identities of students and ways to support candidates to critically self-reflect and draw connections to their efforts to center equity in their mathematics teaching.

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CHAPTER 2

Understanding elementary teacher candidates' visions of equitable mathematics teaching and learning

Abstract

Challenges in learning to integrate equity and social justice in teaching have been widely documented. This study aimed to explore teacher candidates' conceptions and visions of equitable mathematics teaching and their prior mathematics learning experiences. A total of 37 elementary teacher candidates in a mathematics methods course participated in the study. Drawing on the equity framework by Gutiérrez (2012), candidates' survey responses were analyzed. In addition, candidates' mathematics autobiographies were analyzed. Findings revealed three types of equity conception: (1) focus on access only, (2) integrating an understanding of diverse student identities to position them as competent mathematics thinkers, (3) recognizing inequities related to power relations in classrooms. How the different types of equity conceptions and prior learning experiences inform one another is explored.

Keywords: equity, teacher candidates, elementary mathematics methods, equity in mathematics teaching

Understanding elementary teacher candidates' visions of equitable mathematics teaching and learning

Teacher candidates typically enter a teacher preparation program with particular ideas of what constitutes good, effective teaching, which is based on their own learning experiences and shaped by various actors (Lortie, 1975). These prior experiences provide candidates with a frame of reference through which they interpret their teaching experiences and make decisions on how to respond to particular situations (Kennedy, 1999). Studies have frequently characterized the teaching force as predominantly White, female, monolingual, middle-class (Sleeter, 2001, 2012), particularly for multiple-subject, or elementary, teacher candidates. Since the student body is increasingly racially/ethnically diverse, especially in California (U.S. Department of Education, 2013; California Department of Education, 2021), scholars have called for recruiting more candidates of color with the presumption that they bring expansive frames of reference for teaching due to their culturally, linguistically, and racially diverse backgrounds. The increase in the representation of teachers of color from diverse backgrounds may benefit students' learning experiences. Not only would teachers from diverse backgrounds be able to serve as relatable role models but they may be better positioned to recognize the assets and funds of knowledge (Moll et al., 1992) children of color bring to classrooms (Nevarez, Jougantos, & Wood, 2019; Sleeter, 2008; Villegas, 2008). However, simply increasing the number of candidates who identify as persons of color will not lead to equitable learning outcomes for students (Dee, 2005). Preparing candidates to teach equitably requires a range of competencies.

Challenges in learning to integrate equity and social justice in teaching have been widely documented. One effort by teacher preparation programs to address this challenge of increasing cultural sensitivity and multicultural awareness in teacher candidates was the integration of

multicultural education coursework. However, the content of the course and how it is taught varies widely (Garmon, 2004; Gorski, 2009) and the impact of such courses on shifting candidates' dispositions has been mixed (Sleeter, 2001). Other documented challenges that candidates experience with integrating equity and social justice into teaching include candidates' limited content knowledge (Garii & Appova, 2013), candidates' strong dispositions and varied experiences prior to entering the program or course (Clarke & Drudy, 2006; Garmon, 2004; Pohan, 1996), and lack of shared vision or coherence within a program (McDonald, 2007).

Decades of research have documented the importance of teacher knowledge, such as pedagogical content knowledge and knowledge of student thinking, for high instructional quality (Copur-Gencturk, 2015; Fennema et al., 1996; Hill et al., 2008; Lee & Santagata, 2020; Santagata & Lee, 2021). In addition, scholars have also documented the importance of creating culturally relevant and responsive pedagogy (Gay, 2000; Ladson-Billings, 2001; Villegas & Lucas, 2002). However, the work of developing the specialized knowledge for mathematics teaching and the knowledge of students' identities is often separated (Aguirre et al., 2013). The separation is evident by the typical structure of teacher preparation programs in which equity and diversity or multicultural education courses are standalone (Cochran-Smith, Davis, & Fries, 2004; Zeichner, 2012). The importance of situating equity in the context of disciplinary teaching and learning has been highlighted by multiple scholars (Ballantyne & Mills, 2008; Garii & Appova, 2013; Kang & Zinger, 2019). Garii and Appova (2013) emphasized the importance of both a standalone course on social justice and integrated throughout the teacher preparation curriculum, as candidates in the study found it challenging to incorporate social justice into mathematics lessons because they were still struggling with the mathematics content and "trying to confront and ... rectify their own mathematical conceptions" (p. 206). The present study

considers candidates' perceptions of their own mathematical competencies as an important consideration when designing activities to support their learning to center equity in practice.

In addition, the work of learning to center equity in practice requires deep self-reflection of one's own biases, assumptions, and expectations (see Study 1 of this dissertation). Many studies have documented candidates entering teacher preparation programs with preconceived notions of children and mathematics teaching and learning that are deficit-oriented (Baldwin et al., 2007; Sleeter, 2000; 2017). Sleeter (2008) describes how candidates interpret "teaching, students and communities" through the lens of their "prior life experience, beliefs, and assumptions" (p. 1950). Depending on candidates' prior experiences, the lens through which they make sense of teaching and learning throughout the teacher preparation may vary from a deficit-oriented lens to an asset-oriented, equitable lens. Since teacher candidates are learners in the context of a teacher preparation program, it is important for teacher educators to understand who their learners are and how candidates conceptualize equity to then be able to build on and support their learning to center equity in teaching (Clarke & Drudy, 2006). The present study aims to understand conceptions of equity to contribute to broader efforts within the program to support candidates' learning to center equity in their practice.

Specifically, I aim to explore and understand how candidates conceptualize equity *in the context of mathematics teaching and learning* to theorize a possible learning progression for teacher candidates to center equity in their practice. I also aim to understand candidates' prior mathematics experiences. The following research questions guided the study design and analysis:

Research Questions

1. How do elementary school teacher candidates conceptualize equity in mathematics teaching and learning?

2. How did candidates experience mathematics as students prior to the teacher preparation program?
3. What patterns emerged between candidates' prior experiences with mathematics learning and their conceptions and vision for equitable mathematics teaching?

Study Context

This study was situated in a broader context of a teacher preparation program that has been engaged in program-wide improvement efforts to center equity in teaching for over five years. One of the efforts included conveying coherent messages to emphasize the program's commitment to equity to prospective applicants as well as the candidates enrolled in the program. Another effort included organizing courses to be thematically connected under specific strands, one of which was the equity strand. Regardless of how candidates understood equity at any point in time during the program and in this study, having a focus on equity resonated with all candidates in this study, as they knew equity was a central program commitment and focus.

An important contextual information to note is that this study was conducted during the year of the unprecedented pandemic in 2020. One of the major impacts of the pandemic on candidates' learning experiences was that they had limited access to students at their teaching field placements. Initially, many schools were virtual in the beginning of the fall. When some districts decided to transition into a hybrid model of learning, depending on the district policies, some candidates joined their student teaching classroom virtually with the mentor teacher carrying their laptop around and placing it near some groups of students. Other candidates were able to physically enter classrooms. Many classrooms were reconfigured to follow safety protocols and much of the collaborative work that naturally happens when children sit closely next to each other could not happen.

Theoretical Framework

Framing Equity

There exists the constant challenge of supporting the growth of all learners in increasingly diverse classrooms. Practices and norms have been created and reified to support the dominant populations, which have historically been White, monolingual, in the United States. However, catering to a dominant population actively excludes and disregards children who identify in nondominant ways as it excludes, thus invalidating ‘other’ ways of knowing and doing mathematics (Louie, 2017). Given the urgent call to shift teaching and learning to be inclusive for all students to foster rich, equitable learning opportunities, scholars have conceptualized equity in various ways. This study draws on the framework of equity in the context of teacher education and learning for mathematics education.

In particular, this study draws on Rochelle Gutiérrez’s (2012) definition and framework for equity. Equity is about being fair and just, not same and equal. Equity is not achieved by equal approaches or reaching equal outcomes or achievement but is achieved through equitable approaches resulting in equitable outcomes. Gutiérrez (2007) describes equity as consisting of three aspects: (1) race, class, ethnicity, gender, beliefs, and proficiency in the dominant language do not predict *students’ mathematics achievement and participation*; (2) race, class, ethnicity, gender, beliefs, and proficiency in the dominant language do not predict *students’ ability to engage in mathematical practices*, such as analyzing, reasoning about, and critiquing knowledge and events occurring in the world; and (3) there no longer exists inequities between people, mathematics, and the world (pp. 41-45).

To further elaborate, Gutiérrez’s definition of equity highlights that there may be variation *within* groups, but the observed student patterns should not be associated with power or

status in society *between* different groups. This does not mean seeking sameness across all students or “eras[ing] cultural markers in the process of erasing power relations” (p. 42). The first aspect, which emphasizes that students’ characteristics do not predict mathematics achievement and participation, refers to the need to gain cultural capital to fully participate economically in society. However, participating in an unjust society is not fair. Thus, the second aspect, which emphasizes that students’ characteristics do not predict their ability to engage in mathematical practices, refers to preparing students to develop a critical lens when approaching knowledge and analyzing world data with an orientation towards justice. Some ways the second aspect can be observed in practice is when teachers create opportunities for students to recognize the current, dominant mathematics they engage in has Western origins and see if students engage in discourse that explores issues of power within mathematics, differential purposes in learning mathematics (i.e., going to college is not the only reason for learning mathematics), and connecting mathematics as a tool to explore their personal worlds. The third aspect emphasizes a long-term goal of reforming mathematics education.

Analytical Framework for Equity

The analytical work is guided by Gutiérrez’s (2012) four dimensions of equity, which are access, achievement, identity, and power. *Access*, which has been a dominant conception of equity in the past several decades, refers to “tangible resources that students have available to them to participate in mathematics” (p. 19). Some examples Gutiérrez provided include having access to “high-quality mathematics teachers, adequate technology and supplies in the classroom, a rigorous curriculum, a classroom environment that invites participation, reasonable class sizes and support for learning outside of class hours” (p. 19). *Achievement* refers to tangible student outcomes in mathematics. Some examples provided include, but are not limited to,

“participation in a given class, course-taking patterns, standardized test scores and participation in the math ‘pipeline’ (e.g. majoring in mathematics in college, having a math-based career)” (p. 19). This second dimension became the next prominent conception of equity during the late 80s and 90s, with much of the discourse shifting towards gaps and focusing on narrowing achievement gaps to achieve equity. However, the discourse among equity scholars has shifted away from achievement gaps, as it has been problematic and framed students as deficient. Achievement has been redefined to be more expansive and inclusive while maintaining that all students must achieve.

The third dimension is *identity*, and it has played a significant role in the way scholars have conceptualized equity. Considering identity as another dimension means “understanding mathematics as a cultural practice in ways that might further develop the appreciation of one’s ‘roots’” (p. 19). Gutiérrez provides a metaphor of a window and mirror to elaborate on the idea of identity as finding a balance between oneself and others: “students need to have opportunities to see themselves in the curriculum (mirror), as well as have a view onto a broader world (window)” (pp. 19-20). This means students see mathematics as practical for the ‘real world’ and personally meaningful. The fourth dimension is *power*, which points to “issues of social transformation at many levels” (p. 20). Some examples include who has power in the classroom to voice their thinking, whether students can use mathematics as an analytical tool to critique society, and acknowledging there are more than one way of knowing.

The four dimensions are mapped along two axes. The dominant axis includes access and extends to achievement, as student achievement is dependent on their access to learning opportunities. The critical axis maps identity and extends to power, which indicates that developing a lens for identity precedes seeing power.

Developing a Vision for Equitable Mathematics Teaching

This study conceptualizes instructional vision as an ideal image of classroom practice teachers strive to reach (Hammerness, 2001, 2006). When teachers' aspirations for instructional practices are believed to be attainable, they are more likely to stay committed, while unattainable visions are more likely to be discouraging (Hammerness, 2001). Instructional vision differs from beliefs in that it is more concrete and specific than it is philosophical and abstract (Hammerness, 2006; Jansen et al., 2020). Also, this study's conceptualization of instructional vision differs from professional vision as noticing, which entails using three interrelated skills, attending, interpreting, and responding (Sherin & van Es, 2009). Since instructional vision typically consists of practices that reflect what the teacher values and intends to teach (Hammerness, 2001), this study follows work by Jansen and colleagues (2020) and Munter (2014) in capturing teacher candidates' vision through written discourse when they are asked to provide their ideal high quality instructional practices with a focus on equitable practice for this study.

Prior studies have shown the role of sophisticated instructional vision in practice. Scholars documented the relationship between practicing teachers' inquiry-oriented instructional vision and their instructional practices specific to choosing cognitively demanding mathematical tasks (Munter, 2015; Wilhelm, 2014), as well as the relationship between their instructional vision and improvement in practice over time (Munter & Correnti, 2017). Furthermore, the more precisely teachers' visions are articulated, the more they are likely to reach their vision and find a community in which they can enact the vision.

Scholars have also documented the importance of knowledge and community of which teachers are members in developing a sophisticated instructional vision. Munter and colleagues (2021) reported the relationship between mathematical knowledge for teaching (the specialized

knowledge teachers develop and use in practice) and teachers' instructional vision. Specifically, teachers with greater specialized knowledge for teaching mathematics and/or instructional practices that emphasize cognitively demanding tasks and discussion of student ideas were more likely to develop a more sophisticated instructional vision. Furthermore, their findings suggested that if teachers had colleagues whose instructional visions were more sophisticated on average, the more likely teachers individually were to develop a more sophisticated vision, especially if their interactions with colleagues were more frequent. The authors also found the reverse to be true. When instructional vision of colleagues was less sophisticated on average than the individual teacher's, then that teacher's instructional vision was more likely to decrease (and even more so if the community of teachers interacted frequently). This finding highlights the importance of being engaged in a community of practice (Lave & Wenger, 1991), in which there is shared discourse and development of a shared vision of high-quality mathematics teaching and learning through shared experiences.

Teacher preparation programs play an important role in developing the instructional vision of early-career teachers. Jansen and colleagues (2020) documented the instructional vision of teachers two or three years after graduating from a teacher preparation program. They found that beginning teachers' instructional vision of teaching mathematics was aligned with the intentions of faculty members of mathematics education in the teacher preparation program they graduated from. The literature thus points to the importance of designing experiences that support teacher candidates to develop an attainable shared vision of high-quality mathematics teaching and learning that centers equity.

Prior studies primarily examined the vision of high-quality mathematics teaching for practicing teachers and very few studies examined that of teacher candidates, and even more so

their vision of equitable mathematics teaching. According to Hammerness (2001), if visions are far reaching, teachers may be discouraged. This thinking may also apply to teacher candidates. If their visions of equitable mathematics teaching feel impossible, then they may feel discouraged to pursue. However, understanding the reasoning of why certain aspects of their visions seem impossible is an important question to explore. Scholars have documented the deficit-based understandings that teacher candidates hold of underserved communities of color (McKenzie & Phillips, 2016; Williams et al., 2016) from being exposed to deficit-based narratives of these communities. Scholars have also documented the mathematics anxiety that elementary teacher candidates developed prior to entering the teacher preparation program (Hembree, 1990).

One Challenge of Developing a Strong Vision of Equitable Mathematics Teaching: Prior Anxiety-inducing Negative Experiences with Mathematics

The literature documents the shared negative experiences many primary teacher candidates had with mathematics prior to entering the teacher preparation program. Scholars have documented that elementary school marks the beginning of a negative perception of mathematics. Students begin to develop mathematical anxiety--“a feeling of tension, apprehension, or fear that interferes with math performance” (Ashcraft, 2002, p. 181)--in elementary school from the high stakes learning experiences and a narrow conception of what it means to do mathematics (Harper & Daane, 1998; Jackson & Leffingwell, 1999; Swars, 2006). In fact, Hembree (1990) reported primary teacher candidates as having the highest levels of mathematics anxiety compared to students pursuing other majors. Given the slow changes to mathematics teaching, many candidates are likely to have entered the teacher preparation program with limited experiences in meaningful engagement of mathematics (Ball & Bass, 2000). As students, candidates were more likely to have received the traditional instruction that

places a strong emphasis on correctness, speed, and memorization of algorithms as opposed to meaning making and valuing the diverse thinking of learners. Thus, if candidates draw on prior experiences to construct their vision of mathematics teaching, their instruction will lack depth and meaning. In addition to the high-stakes, high pressure learning environment (evidenced by this study's candidates' frequent reference to timed tests and good grades), candidates had to have completed increasingly more abstract and difficult mathematics to graduate from high school to then enter and graduate from college. All these experiences contribute to the development of mathematics anxiety and a negative attitude towards mathematics, due to both the increasing complexity in a subject matter they do not like and having experiences being positioned as incompetent.

Implications of Unresolved Challenges

It is important for teacher preparation to support candidates to reconcile their negative experiences with mathematics by creating multiple opportunities to engage in transformative learning experiences combined with self-reflective tasks in order to shift their conception of learning mathematics, and thus teaching mathematics.

There are serious implications to having mathematics anxiety as future elementary school teachers: 1) their negative sentiment towards mathematics may pass down to the elementary grade students they teach (Middleton & Spanias, 1991; Richland et al., 2020), which is a concern candidates in this study also expressed; 2) the very narrow conception of mathematics teaching and learning that produced their anxiety may be perpetuated as they teach, unless candidates have experienced alternative, inclusive teaching methods that shifted or expanded their vision of mathematics teaching and learning; 3) quality and time spent on high quality mathematics instruction will be compromised and trickle into other related content area, such as science

(Bursal & Paznokas, 2006). Scholars who have examined the implications of mathematics anxiety recommend being aware of the different levels of math anxiety when designing teacher preparation programs to support candidates with developing strong knowledge for teaching mathematics and reflecting on their personal histories with mathematics and their implications on instruction (Olson & Stoehr, 2019).

Both the literature and the data in this study reveal many candidates had limited meaningful learning experiences in mathematics, thus limiting their vision of mathematics teaching and learning that centers equity. Holding a strong vision of teaching and learning plays an important role in practice as teachers are more likely to shift their practice towards the sophisticated vision they articulated (Cobb et al., 2018), hence suggesting the importance of developing a sophisticated vision of *equitable* teaching as a starting point towards learning to enact equitable practices. In addition, Burton (2012) reported that when candidates associate mathematics with school, they hold negative perceptions, while candidates who associate mathematics with the ‘real world’ hold positive perceptions of mathematics. Therefore, it is important for candidates to develop a vision of mathematics teaching and learning that is meaningful and relevant beyond the classroom.

Methods

Context and Participants

The study was conducted in a mathematics methods course for elementary teacher candidates at a teacher preparation program in a large research university in California. Thirty-seven teacher candidates participated in this study. Demographic information of the teacher candidates is in Table 2.1. About a third of the candidates identify as Asian/Asian American, another third as Hispanic/Latinx, 21% as White, and 10% as Two or More Races.

Table 2.1. Demographic characteristics of participants

Characteristics	n	%
Gender		
Female	34	89.5%
Male	3	7.9%
Race		
White, not Hispanic	8	21.1%
Asian/Asian American	13	34.2%
Hispanic/Latinx	12	31.6%
Two or more races ^a	4	10.5%
Hispanic/Latinx, White	2	5.3%
Asian/Asian American, White	1	2.6%
Education - Major		
BA/BS in Education Sciences or related (e.g. Psychology, Child Development)	19	50.0%
BA/BS in Other Humanities or Social Sciences	12	31.6%
BA/BS in STEM	1	2.6%
BA/BS, unspecified	5	13.2%
Education - Highest Degree Completed		
BA/BS	36	94.7%
MA/MS/MEd	1	2.6%

Note. N=37. Participants were on average 23.9 years old (SD = 2.6).

^a Total of 4 participants self-identified as Two or more races, with 3 specifying their races.

Data Sources

The primary data sources for this study were the candidates' survey responses and mathematics autobiographies. Course artifacts and observation field notes were used to

supplement the analysis. Data sources were collected throughout the methods course. I describe below each data source in detail.

Mathematics Autobiography

Prior to the first class session, candidates were asked to complete their mathematics autobiography and post it on Canvas, the course learning platform. This assignment was co-designed in the first year of our partnership to better understand candidates' experiences with mathematics as they remember it. The assignment prompts candidates to respond to two questions: (1) What was your experience with learning mathematics growing up, specifically at school, with your family, and/or in your community?; and (2) How do you think your prior experiences learning math (the ones you described in question 1) may shape the way you teach mathematics? The two questions were intentionally posed to gain insight into prominent memories that shaped candidates' narrative about how they experienced mathematics and their perceptions of mathematics teaching and learning. The second question also asks candidates to draw connections between their experiences and intended teaching practices. The second question provided supplementary information if the first question response was not clear and was not systematically analyzed.

Survey Measure

A short survey was designed and administered towards the end of the course. The survey collected information on candidates' demographic information, such as their gender identity, race/ethnicity, age, education (major and highest degree completed), and prior teaching experience. The survey posed two questions related to equity: (1) What does equity mean? Please provide examples, and (2) If you were asked to observe a teacher's math classroom what would you look for to decide whether the mathematics instruction is equitable? Please provide a

few examples. The second question was adapted from a question that Munter (2014) designed to capture practitioners' vision of high-quality mathematics instruction. The two questions are posed together to capture candidates' conception of equity because the first question simply elicits a definition of equity, and the second question situates their conception of equity in mathematics instruction. I will refer to candidate's responses to both questions as their conception *and* vision of equity in mathematics teaching.

Data Analysis of Research Question 1: How do elementary school teacher candidates conceptualize equity in mathematics teaching and learning?

Candidates' survey responses were analyzed to examine their conception of equity and vision of equitable practices in mathematics classrooms. Candidates responded to two questions in the survey: (1) *What does equity mean? Please provide examples,* and (2) *If you were asked to observe a teacher's math classroom what would you look for to decide whether the mathematics instruction is equitable? Please provide a few examples.* Candidates' responses to the two questions were combined and analyzed together to capture their conception of equity holistically, as the first question simply elicits a definition and the second question elicits how equity is envisioned to be enacted in mathematics classrooms. Responses ranged from two sentences (i.e., a sentence for each question) to a total of 19 sentences. The unit of analysis was one idea unit (Jacobs et al., 1997), which typically ranged from a clause to a few sentences.

First Coding Phase

I first approached the data using the open-coding method and created codes on MAXQDA, a qualitative software program. Then, I sorted these codes into Gutiérrez's (2012) framework of equity, which consists of four dimensions (Access, Achievement, Power, and Identity). Many of the codes fell under Access, Power, and/or Identity dimensions (Table 2.2).

There were not many codes related to Achievement; thus, the dimension, Achievement, was not included in the analysis.

Table 2.2. Coding rubric for equity dimensions

Dimension of Equity	Description	Example Codes
Access	The ways students are given access to participate and learn are expansive and inclusive; supports students' development of positive identities as doers and knowers of mathematics.	<ul style="list-style-type: none"> ● Creating access to participate during class ● Differentiating small groups by needs, different methods/strategies to teach ● Different student strategies/methods accepted ● Removing barriers ● Multiple strategies or representations are used by teacher ● Focus on resources (financial aid, food), tools (accommodation/UDL, manipulatives)
Identity	PSTs' views and actions reflect efforts to create opportunities for students to see themselves as knowers, doers and contributors of mathematics, see meaningful relevance, and understand the broader world.	<ul style="list-style-type: none"> ● Focus on students' strengths and assets ● Builds students math competency by being inclusive of different ideas ● Getting to know students better in a way that supports learning ● Cultural reference/relevance
Power	PSTs' views and actions convey efforts to empower students by recognizing (and/or addressing) unequal distribution of power among students and between the student(s) and teacher.	<ul style="list-style-type: none"> ● All students have a voice, can be heard equally ● Students build knowledge through collaboration & interactions (not only from teachers) ● Creating opportunities for students to contribute to learning/lesson

In candidates' responses, there were two primary ways candidates described issues of access. The first way candidates described access issues was in terms of material resources, such as lack of resources (financial, food) outside the classroom and/or the use of tools (manipulatives) inside the classroom. The second way was in terms of pedagogical practices and

multiple ways to create opportunities to participate and learn. Identity included the ways candidates discussed the importance of focusing on students' strengths, assets, and the unique experiences they bring, in ways that support the development of positive math identities. Power included the ways candidates discussed ways to balance power dynamics between students and position students as contributors of knowledge building. In addition, there were two other recurring codes that did not fall into any of the four dimensions. One of the codes was named "evidence of deficit lens," as candidates described practices that reinforced hierarchy within classrooms by ability grouping, or their rationale for particular 'equitable practices' perceived students in deficient ways. The second code was a recurring phrase that appeared across many candidates' responses, which was, "Equity is being fair, not equal."

Second Coding Phase

Given that there were a few codes that seemed relevant but did not necessarily fall under one of Gutiérrez's equity dimensions, I revisited the data using a ground up approach to allow for additional codes to emerge. Then, I systematically coded the responses with the new codes (along with the straggler codes from the first coding phase) to notice additional patterns. I recorded the occurrence of each pattern in candidates' responses.

Several additional themes emerged. One theme was that candidates were making explicit statements about how equity is not equality but fairness or differentially supporting students based on their needs (color coded as brown). The second theme was candidates' deficit lens surfacing in their responses (color coded as red). The most common statement that made visible their deficit lens was defining equity as needing to help struggling kids catch up to "further ahead" students, which positioned the "further ahead" kids as more competent and directed the struggling kids to be more like the other kids.

To paint a holistic picture of which dimensions candidates focused on and the intensity of each dimension relative to their entire response, I created a visual representation of candidates' conceptions using the Document Portrait feature on MAXQDA. The Document Portrait provided a visual impression of the written responses and the intensity with which candidates discussed particular themes or codes (See Figure 2.1). Each portrait represented each candidate's survey responses, which varied in length. If one candidate provided a brief response that consisted of two sentences, their portrait represented the frequency and intensity of the codes in the two sentences. I assigned a color for each dimension: Access as blue (dark blue for the first way and light blue for the second way), Identity as pink, and Power as purple. The code that captured evidence of deficit lens was assigned the color red, and the recurring phrase of equity as fairness not equality was assigned the color brown.

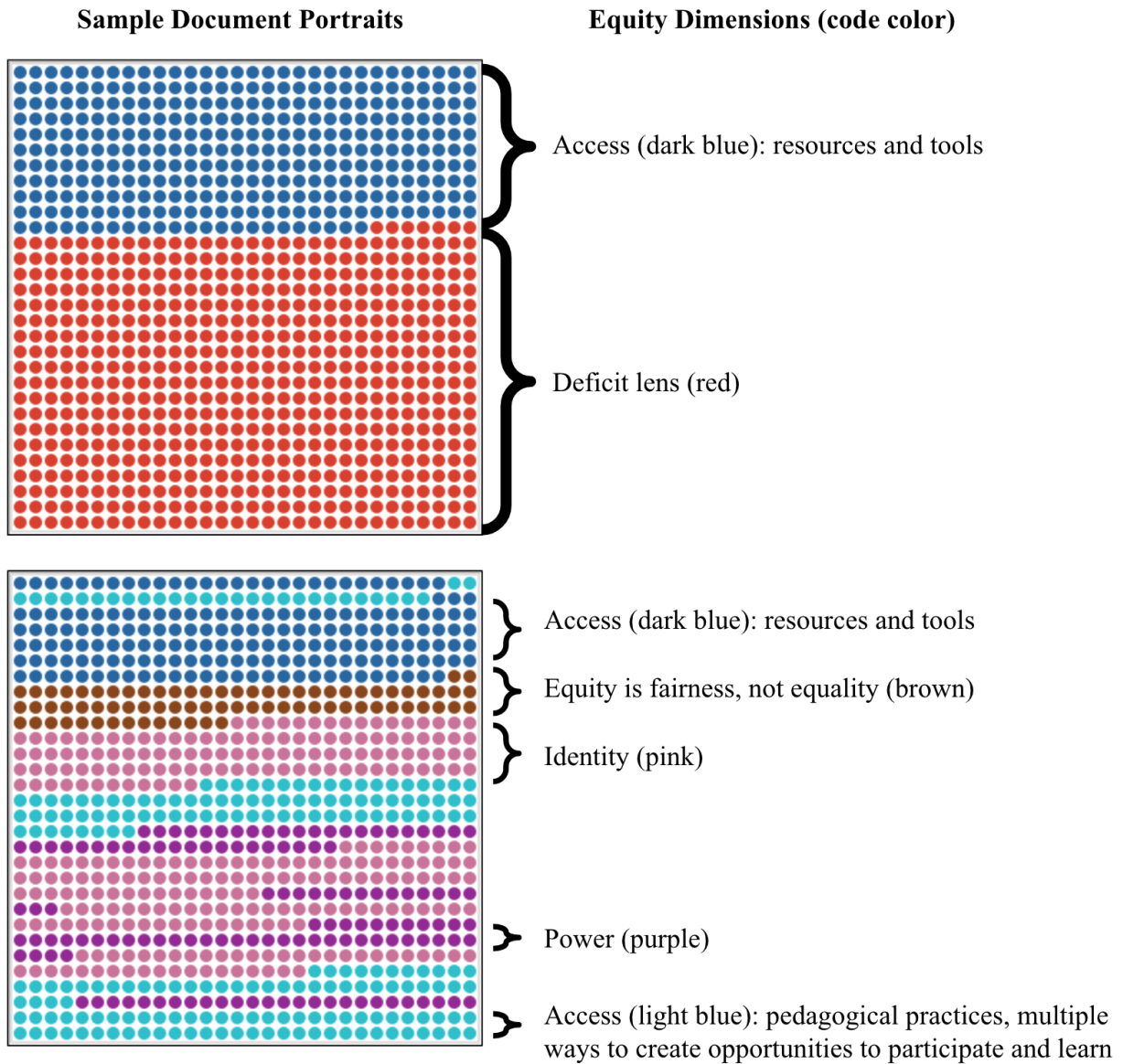


Figure 2.1. Sample document portraits with code labels and their colors

Data Analysis of Research Question 2: How did candidates experience mathematics as students prior to the teacher preparation program?

To understand teacher candidates’ prior mathematics learning experiences, I analyzed 37 candidates’ mathematics autobiography assignments, which were written responses to two questions that engaged them to reflect on their prior mathematics learning experiences and how

their experiences might have shaped their mathematics teaching. I employed a thematic analysis approach (Braun & Clarke, 2006) for their mathematics autobiography. I first approached the autobiography data inductively to let candidates' stories and ideas surface. Candidates typically described prominent memories that shaped their view of mathematics teaching and learning, such as positive or negative experiences, specific interactions with teachers or family, and/or how they recall being positioned by teachers. Then, I created analytic memos for each autobiography, focusing on whether candidates described a positive or negative experience with mathematics, the specific experiences they described that made their experiences positive or negative, and how they narrate these experiences. I examined the analytic memos for themes.

Data Analysis of Research Question 3: What patterns emerged between candidates' prior experiences with mathematics learning and their conceptions and vision for equitable mathematics teaching?

To examine emerging patterns between candidates' prior experiences and equity conceptions and vision, the autobiography (raw data) along with the annotated memos of the autobiography were juxtaposed with their coded and raw written responses of their equity conception and vision. After organizing the data in such way, I employed the thematic analysis approach by reviewing the data within candidate, then across candidates. I wrote annotated memos for each candidate to detect salient patterns. Then, after the patterns were identified, I reviewed the data for commonalities in equity conceptions or in prior experiences.

Findings

Research Question 1 Results: Variation in Candidates' Conceptions of Equity in Mathematics Teaching and Learning

There were 15 candidates whose conceptions of equity focused only on issues of access (“Type 1”). There were nine candidates whose conception of equity considered both access and student identity as competent mathematics thinkers (“Type 2”). There were 13 candidates whose conceptions of equity considered access, student identity, and power relations in mathematics classrooms (“Type 3”). From hereon I will refer to these three patterns as Type 1, Type 2, and Type 3 equity conceptions, respectively. Examples of each type are displayed in Figure 2.2 and the visualization for all candidates are in the Appendix. Representative example excerpts and proportion of candidates in each type is presented in Table 2.3.

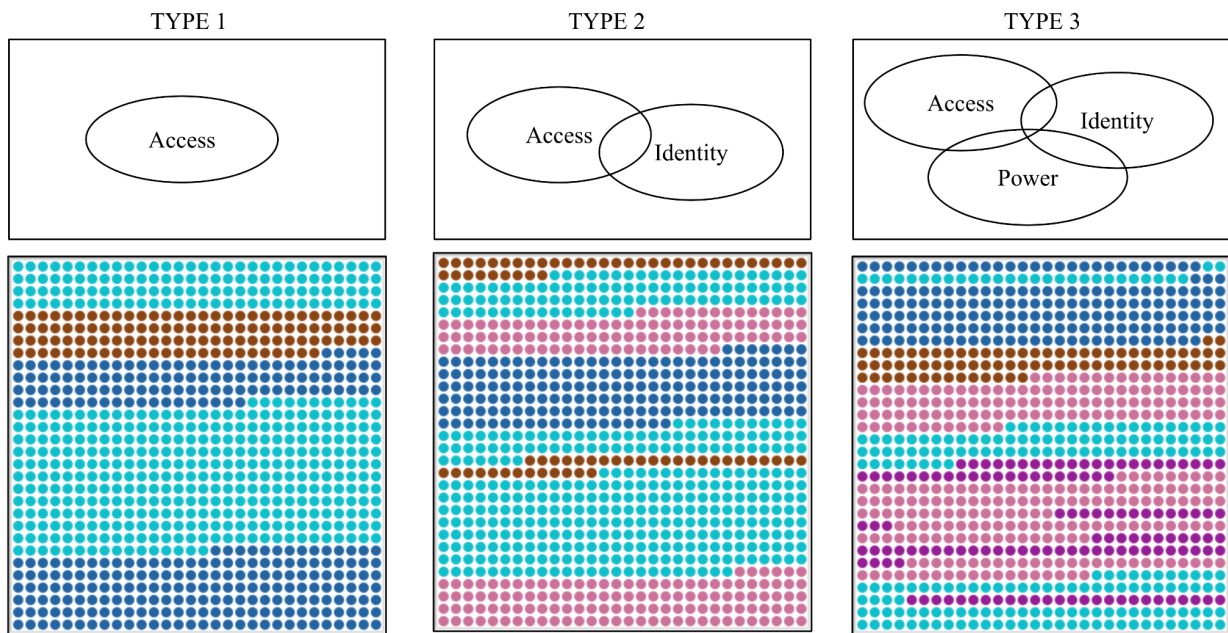


Figure 2.2. Three types of equity conceptions

Type 1: Exclusive Focus on Access

Fifteen of the 37 participants conceived equity in terms of access only. As Gutiérrez (2012) stated, the dimension of *Access* has been a dominant conception of equity and refers to “tangible resources that students have available to them to participate in mathematics” (p. 19).

There were two ways candidates considered access under this type. In one way, four candidates demonstrated an isolated focus on tools, resources, and/or accommodations. For example, one candidate described equity as giving extra time to a student with a learning disability to complete a task. Another candidate described equity as “providing extra resources for a student that needs additional help so that they have the same opportunities as someone who does not need help and is “further ahead” than the other student.” These responses did not include much specificity of what kind of tools and resources and for what purpose.

In the second way, nine candidates discussed the importance of not only students having access to resources or tools to learn but also having their socioemotional needs addressed to have access to the learning and be successful in classrooms. The distinction from the first group was that the second group stated the importance of allowing children to share their diverse mathematical thinking during lessons without further elaboration on why it was important.

Within both groups, some candidates (two in each group) demonstrated a deficit-oriented lens permeating in their responses. For example, one candidate in the first group stated, “I would look if the teacher was including all students and had small groups based on mathematical ability.” While the candidate recognized the importance of “including all students,” she also valued having ability-based groups to support learning, which contradicted the work of being inclusive and positioned certain groups of children as incompetent. Both groups also provided brief descriptions without further elaboration on the “how” and “why” of the resources they consider.

Type 2: Integrating an Understanding of Diverse Student Identities to Position Them as Competent Mathematics Thinkers

Nine candidates conceived equity as integrating concerns around students' identities in addition to considering access by attending to both tangible resources and their socioemotional needs to access learning opportunities. Candidates discussed that it was important to recognize and value each child's unique experiences and backgrounds. They also considered practices that promoted the development of positive math identities, which meant children perceived themselves as capable and competent when doing mathematics. For example, one candidate described the importance of "recognizing student differences, instead of ignoring them, so you can support them and work with them. But it also looks like holding the same confidence in each students' worth and ability and having a strengths-based pedagogy that builds on what students DO know and understand." This candidate considered the differences in a way that positioned students competently, as opposed to considering quick solutions ("give extra time") for students with the typical labels assigned to them ("ELL," "special needs"). Candidates with Type 2 equity conception differed from those with Type 1 in that they considered ways to position competently given their differential learning needs. This was evident through the rationale or reasoning they provided with the accommodations they had in mind: "a student may need more time on a test or assignment because of an intellectual disability, so the teacher gives the student more time. This is not unfair, rather this is allowing every student to have the same opportunity to complete the assignment in a way that makes sense to them." This elaboration was distinct from someone only stating that giving time accommodations to a child with learning disabilities as an example of equitable practice, as it was a quick solution that did not consider thoughtful ways of positioning the student as competent within instruction. Instead, she described the importance of being inclusive and valuing students' thinking and allowing them to choose their approach to solve the math problems.

One pattern emerged among the responses by the nine candidates. Two candidates maintained ideas that conveyed a deficit-based thinking about children's learning, such as labeling certain groups of students as falling behind and needing to be caught up to the larger group, ("Equity means that we are giving extra support to those who need it, so they can get to the same level as the other students in the group"). For these candidates, despite the fact that they recognized children come with varied lived histories and experiences and diverse thinking, there was a norm (established by the majority in the classroom, not learning standards) that struggling children needed to reach.

Type 3: Recognizing Power Relations in Mathematics Classrooms

Thirteen participants conceived equity as including issues of access, attention to student identity, but also consideration of issues of power in the classroom as contributing or creating inequities. For this group, candidates' recognition of power centered on who took up space in the classroom, whose ideas were heard and valued, and whether everyone had the opportunity to contribute to the learning. For example, one candidate described, "I would also look for equity of voice in strategy sharing, so being clear that EVERY student voice is valuable. Students are so perceptive, and can feel the difference in the teacher's view of each student's contributions to the class. So I would look for intentional valuing of each student voice, even if it looks different for different students like sharing verbally vs. showing their handwritten work." She recognized the power she held as a teacher in explicitly and subtly positioning some kids as more competent than others and expressed her desire to be more intentional with distributing power, or who could contribute to the knowledge building during the lesson, among students. Similarly, another candidate described how students have different comfort levels to share their thinking during whole class discussion; thus, teachers could consider other ways for students to share their ideas

through small group or partner talk, as this would “deepen their math skills and be more engaged in the math lesson.” She explained that “classroom discussion should involve everyone, not just the ones who always get the answers right or [have] the confidence to share in class.” Candidates in this group also described the need to address inequities due to access and to understand who their students are.

Table 2.3. Three types of candidates' understanding of equity and their vision of equitable mathematics teaching and learning

Patterns	n (%)	Representative Example Excerpts
Type 1: Exclusive focus on access	15 (41%)	<p>"Equity means providing a fair education to all students. For example, a student is given extra time to complete a task due to having a learning disability. I would look if the teacher was including all students and had small groups based on mathematical ability."</p> <p>"Equity means giving every student the tools they need to succeed. This looks different for every student and can be in the form of resources they receive, time such as individual meetings or differentiated instruction. It also depends on how you define success and for me it includes academic success (meeting/exceeding grade level expectations) as well as social and emotional. To see visually if all students are engaged and understanding the material. Also if the material is presented in the variety of different ways (visual, auditory..) and if there is scaffolding in place to help all students reach proficient understanding." Screen reader support enabled.</p>
Type 2: Integrating an understanding of diverse student identities to position them as competent mathematics thinkers	9 (24%)	<p>"Equity is truly being fair which means giving everyone the resources and supports they need to all start at the same place. For example in school a student may need more time on a test or assignment because of an intellectual disability, so the teacher gives the student more time. This is not unfair, rather this is allowing every student to have the same opportunity to complete the assignment in a way that makes sense to them. ... I would look to see if their instruction was fair and providing every student the same opportunity to learn that makes sense to them. For example, is the teacher allowing students to solve problems in a way that makes sense to them (equitable) or are they only letting students use one strategy (not equitable)? Is the teacher allowing students to work out the problems at their own pace (equitable) or making all the students move along for the sake of time and not offering support to students who may still not understand (not equitable)?"</p>

<p>Pattern 3: Recognizing inequities related to power relations in classrooms</p>	<p>13 (35%)</p>	<p>"Equity means fair opportunity for all. In education, we must ensure equitable opportunities for students to ensure that every student has the support they need to be successful. Furthermore, being equitable in the classroom means recognizing each student has their own unique strengths and struggles. We must be attentive to their strengths and struggles when designing curriculum and when establishing a classroom environment. To decide whether mathematics instruction is equitable in the classroom, I would look for the materials available to the students and pay attention to how the instructor incorporates the materials in class. For instance, if there are manipulatives available, are there enough manipulatives for everybody? Does the teacher set expectations of how to use these materials? In addition, I would focus on student engagement in the classroom. I believe it would be most equitable for all students to be allowed to participate in discourse. For instance, are all students able to share their thoughts and ideas? Are their thoughts being recognized by the teacher? // Learning math becomes meaningful to students if they have the opportunities to engage in their learning, whether it is through the use of manipulatives (or other materials) or through discourse. To ensure equity in the classroom, ensure that all students have access to these materials and that all students are able to contribute to the lesson."</p>
<p>Total</p>	<p>37</p>	

Research Question 2 Results: Varying Narratives of Prior Mathematics Experience

The analysis of candidates' autobiographies revealed varying prior experiences with mathematics. In general, candidates described their prior experiences positively and/or negatively. They either focused on particular memories that shaped their view of mathematics teaching and learning or provided an overall view by characterizing experiences across K-12 and college. I will describe below prominent themes that emerged in the analyses.

Five candidates only described their negative experiences and their struggles with mathematics. They expressed that they “didn’t enjoy math” or “hated math” and recalled feeling “anxiety and frustration.” Some continued to perceive mathematics negatively and reflected on their desire to provide an environment where students could see mathematics positively, feel confident, enjoy learning it, and see their potential to do well in class and in a future career. The common experiences among candidates who continued to carry a negative perception were: (1) being positioned as incompetent through multiple experiences of failure, (2) struggling to understand the mathematics, (3) having been exposed to similar teaching styles that focused on procedures, correctness, and grades. For these candidates, they expressed having empathy towards students who struggle with mathematics and expressed a dedication to increase access to learning mathematics by building relevance, helping students develop “growth mindset” about themselves, and “creat[ing] an environment that has more discourse.”

For all other 32 candidates, they described having both positive and negative experiences. There were four ways candidates attended to their prior experiences: (1) a focus on outcomes; (2) a focus on tools and instructional practices that shaped their experiences to be positive or negative; (3) a focus on how they were positioned as learners in mathematics classrooms and the expectations people around them held about their mathematics abilities; and (4) a critical focus

on their prior experiences in which they recognized particular problematic practices that attributed to unproductive experiences.

The first theme that emerged was that candidates attended to outcome-oriented markers of achievement when describing their experiences. For example, one candidate described the challenges she experienced at each phase of her mathematics experience by recalling her failed exams (e.g., “failed almost all my tests and visibly struggled to keep up with everyone else”; “took regular Trigonometry and almost failed every test again”) and coursework grades. Even in years she described positively, she focused on outcomes that made her the “top student” in her class. Other candidates who focused on outcomes described the grades they received as justification for why they did not do well in mathematics without further explaining other factors that could have shaped their experience.

The second theme that emerged was that candidates attended to tools and/or specific instructional practices that contributed to either their negative or positive experiences in mathematics classrooms. For example, one candidate described how he loved mathematics because he enjoyed “finding strategies to solve all kinds of math problems.” He shared one of his favorite mathematics memories, which was “figuring out a new strategy to a math problem that [his] teacher hasn’t seen before.” His high school mathematics experience was primarily being shown steps to find solutions and instructed to replicate. He described struggling and no longer enjoying mathematics in high school, as he recalled primarily finding formulas and plugging in numbers without understanding why. Recounting how his favorite teachers were ones that allowed him to be creative with his strategies, he stated his belief about mathematics and that “there is not one single way to solve a math problem. It’s my job to help my students learn and develop multiple strategies so they can apply them when they see fit.” The practices that

positioned him as competent made his learning experience positive, thus his plan to provide access by creating similar experiences for his students. This candidate conceptualized ways to increase access to learn mathematics by broadening participation and engagement in ways that validate students' ways of thinking and build relevance. Other candidates who attended to tools and/or specific practices all commonly experienced feeling competent and successful in mathematics, and the rationale they all provided for the positive experience ranged from particular effective teacher interactions and out-of-classroom support. Their visions of equitable mathematics teaching included not only tools but also more general pedagogical strategies that create access to learning.

The third theme that emerged was that candidates described how they were positioned in mathematics by teachers, family, and/or class community. For example, some candidates described being positioned as the best in mathematics by teachers (e.g., "I was constantly praised by my teachers for "always knowing my math""; "Teachers and family members kind of implanted a fixed mindset on me because I grew up hearing that I am simply good at math since it comes easy to me and that I should find a career that involves math. Therefore, I was confident about my math skills") and by peers (e.g., "because I was always one of the top students in math, many students would portray me as the stereotypical smart Asian kid"). The reasoning candidates provided for being positioned as mathematically smart and competent was because they "picked up concepts easily" or "quick to learn addition, subtraction, multiplication and division." One candidate described feeling both "proud, but also burdened" from pressure to meet others' expectations of her. Their positioning as the best in class was short lived because candidates were placed in advanced or accelerated tracks and encountered difficulty and were quick to judge their own capabilities. One candidate said she fell behind her peers after being

placed in an advanced track, and “labeled [her]self as someone who was naturally bad at math.” Another candidate described her challenges when she encountered situations that required her to utilize conceptual understanding: “even though I knew how to solve equations, I never understood the math behind those equations and how I could apply them to real-world problems.” Thus, her “confidence at doing math declined drastically and [her] confidence ended up completely gone with 12th grade AP calculus.”

Most candidates whose experiences resonated with this third theme were quick to shift from having positive perceptions of mathematics and their own abilities to a negative one in which they doubted their ability to be successful with mathematics. The adults labeled these candidates as smart and competent students based on behavioral characteristics that were not necessarily grounded in mathematical thinking. When candidates recalled being positioned as smart, they referred to how easy mathematics was, how quickly they solved problems with accuracy, and how they understood everything. While there is value in having strategies that demonstrate candidates’ procedural fluency, learning mathematics goes beyond memorization and speed. Solely labeling and positioning children as competent for more superficial reasons created a fragile sense of competency that easily shifted candidates to perceive themselves as lacking the skills to be successful in mathematics. They also described experiencing external pressure to maintain their status of being ‘good at mathematics.’ One candidate referred to the pressure she felt from her teacher and family who constantly described her as someone who is good at mathematics, thus molding her sense of identity to be one affiliated with mathematics. In contrast, another candidate’s father would “encourage [her] to keep trying and ask for help or do it [her] own way as long as it made sense to me.” However, her classroom teacher held a narrow conception of mathematics teaching as she “taught one way,” which was not helpful for the

candidate. When the candidate approached the teacher for more support, she was told to simply “study more,” after which the candidate received failing grades. The candidate reflected, “I was really discouraged from math and I honestly hate it. It gives me anxious feeling anytime I look at a problem and I can't automatically think of a strategy to solve it.”

The fourth theme that emerged was that some candidates provided a critical reflection of their mathematics experiences by providing a narrative beyond grades, outcomes, and how they felt as students. Instead, they commented on problematic instructional practices and norms in the classroom. Multiple candidates commented on normalized practices that they find problematic for learning and developing positive dispositions towards mathematics. For example, candidates described their secondary learning experience as becoming “less about learning and more about simply getting the grade.” One candidate described how her teachers “rushed through all topics” and gave “packets of work... to learn majority of the content on [their] own.” Another candidate shared her experience with her AP Calculus teacher “reading off of a PowerPoint, not answering our questions, and getting mad when we tried to talk to each other to figure out what we were learning.” In another example, one candidate reflected on how not being “good at math” was normalized in both her family and at school, which contributed to her decreasing curiosity for mathematics. She was often told that her “brother had all the math smarts” leaving none for her. However, she demonstrated her recognition of the subtle ways students’ struggles become reinforced by adults as an innate trait by sharing her goal as a teacher to normalize struggle and perseverance to help build her students’ confidence in their abilities and never feel that they are “naturally bad at math.” Two candidates reflected on how their successes were within a limited framing of teaching and learning mathematics. The first candidate described how she was “lucky that the step-by-step procedural method of teaching math without any real conceptual

understanding happened to work for my brain.” She then narrated how as an adult, she realized she does not conceptually understand the automatized procedures she easily performed, such as multiplying fractions for recipes. Similarly, the second candidate described how she enjoyed the memorization of times tables and simple computation but realized she “didn’t understand why “tricks we were being taught worked.” Although she “liked getting things “right,” and was “good at “guessing” what we were supposed to do to solve problems in math” she recognized that she “lacked some basic number sense, and it led to a lot of misconceptions and misunderstandings about math.” Both candidates recognized that while they felt successful and liked math over the years, they had developed a thin understanding of mathematics as adults. Their realization prompted them to engage in learning experiences to expand their own understanding of mathematics in hopes of creating enjoyable experiences that connect and build on children’s prior understandings.

One interesting commonality among the four candidates who commented on the existing structures that normalize problematic practices in mathematics was that they were older than their peers. Their average age was 28.3, ranging from 25 to 34, while their younger counterpart’s average age was 23.3, ranging from 21 to 27. The four candidates had spent at least more than a year prior to the teacher preparation program completing a masters, working in an industry, or working full-time with children.

RQ3: Salient patterns between candidates’ prior experiences with mathematics and their conception and vision of equitable mathematics teaching

When juxtaposing the candidates’ mathematics autobiographies with their conceptions and vision of equitable mathematics teaching, several patterns emerged. (I will refer to “conception and vision of equitable mathematics teaching” as “equity conceptions” for short).

The first pattern that emerged was among the candidates who described primarily negative mathematics experiences. Their conception and vision of equitable mathematics teaching generally focused on tools and resources to give access to students (Type 1). They did not specify the pedagogical purpose for the tools and resources within the context of mathematics teaching and learning. Instead, they described creating a safe classroom environment that allows students to feel competent and capable of doing well in mathematics, without specifying how they may go about creating such an environment.

Another pattern that emerged was for the group of candidates whose common experience was being placed on a pedestal by teachers and/or family members for being mathematically talented and experiencing external pressures to maintain their status. For these candidates, their conception and vision of equitable mathematics teaching (Type 2) demonstrated that they attended to the labels given to children (e.g., visual learners, English language learners, special education learners, low-income students) and ways to support diverse learners. For the candidates who were not positioned as smart for superficial reasons by adults, their vision considered the role of teachers in getting to know students deeply to then understand their learning needs. While they mentioned the labels given to students, they also recognized the importance of being inclusive of the different strategies/ideas and positioning all students competent in mathematics classrooms. These candidates' experiences illuminate the importance of having an expansive conception of competency and 'smartness' in mathematics classrooms-- one that focuses on the mathematical practices instead of how fast or correct a student is, as well as perceiving different thinking children bring as contributing to the learning instead of imposing one way of solving problems.

A prominent pattern that emerged was that candidates with a more expansive equity conception reflected on their prior mathematical experience with a critical lens. Candidates whose autobiographies conveyed a more critical reflection were those with Type 3 equity conception, which was the most expansive conception among the three types. These candidates did not position inequitable, problematic practices as helpful for learning. Many of the candidates recognized and specified the helpful practices they have observed or experienced. Although many candidates described enjoying timed tests, which was a task that contributed to developing anxiety and fear of mathematics for the candidates who only shared negative experiences, they did not list timed tests as a task they wanted to use. In addition, the same candidates described that they were positioned competently because they were typically the first to finish math tasks, the only one with all correct answers, or the one designated to help others. The common takeaway these candidates expressed was their desire to generate the same feelings of competency and enthusiasm without using the same activities and markers of success they benefited from. Implied in their reasoning was their recognition and understanding that what worked for them and positioned them as competent does not always work for other students. This was also reflected in their conception and vision of equitable mathematics teaching, as candidates were cognizant of the power dynamics among students and considered different ways to position students as mathematically competent. For example, candidates attended to equity of voice (i.e., whose voice is heard and not heard, who tends to dominate or shy away), as well as strategic sharing of student thinking and strategies in a way that demonstrated “intentional valuing of each student voice,” which would look different for different students. They also provided a multicultural lens on children’s identities by specifying in their conception and vision of equitable teaching the importance of drawing on community knowledge, recognizing

marginalization in classrooms, and learning the cultural background of students to understand their strengths and needs.

In addition to recognizing unproductive structures and practices, candidates with Type 3 equity conception also demonstrated their critical lens by reframing their negative experiences by highlighting experiences that empowered them to overcome negative memories. Candidates' autobiographies illustrated that the negative experiences still had a lasting impact, as some candidates described past experiences as "traumatic," with a strong emotional response--"hated [math]" or "dreaded practicing [math]"--and felt a sense of shame--"too embarrassed to ask for help." However, the impact was not to the extent of feeling constant anxiety and fear of the content. Their cumulative experiences prior to entering the teacher education program had equipped them with a better understanding of mathematics. Some experiences included taking mathematics education courses that developed their knowledge of *how to teach* mathematics, tutoring close family friends (i.e., knowledge developed from teaching others), receiving support from parents, family members, and/or close family friends who strengthened their knowledge and confidence. Except for one candidate, all candidates expressed enthusiasm to be teachers who they needed when they struggled with mathematics by providing specific practices they hoped to enact. The one candidate expressed concerns not about the mathematics content but about how to teach the content, alluding to her recognition that she needed to develop her mathematical knowledge *for teaching*, which is a specialized knowledge teachers develop that is distinct from knowing how to do the mathematics.

Discussion

A possible learning progression?

This study illustrates candidates' conceptions and vision of equitable mathematics teaching as ranging in focus and depth. When examining the three types of equity conceptions altogether, each type seemed to build on one another. When comparing Type 2 with Type 1 equity conceptions, Type 2 seemed to be a more expansive conception of equity than Type 1, as it considers both Access and Identity dimensions of equity. Similarly Type 3 seemed to be even more of an expansive conception of equity than Type 1 or Type 2, as it considers Access, Identity, and Power dimensions of equity. Moreover, there were fewer candidates who showed evidence of deficit-oriented thinking the more expansive the conception of equity was. The number of candidates dropped from 8 (Type 1) to 3 (Type 2) to 1 (Type 3). For Type 3 equity conception, only one candidate conveyed deficit-oriented thinking while other candidates showed no evidence of thinking in deficit ways about students and their abilities. One conjecture is that there may be a progression in expanding one's conception of equity, from access only to access and identity to access, identity, and power. However, this study was not designed to capture changes within each candidate. Thus, there is not enough evidence to suggest that candidates progress across these three types of equity conceptions.

Key Takeaways

This study portrays the different ways candidates reflected on their prior experiences with mathematics. The findings suggest that candidates with an expansive conception and vision of equitable mathematics teaching reflected on their prior experiences with a more critical lens. More specifically, candidates whose conception and vision of equitable practice included access, identity, and power (Type 3), reflected on problematic structures and practices of mathematics teaching and learning in their autobiographies. The commentary on problematic existing structures typically came from older candidates, who had multiple experiences prior to entering

the teacher preparation program, which aligned with findings from a study that documented how older teacher candidates primarily drew from principles and learnings developed from teacher preparation coursework, while their younger colleagues drew from recent school experiences (Powel & Riner, 1992, as cited in Wideen, Mayer-Smith, & Moon, 1998).

Importance of Diverse Experiences for Strong Vision of Equitable Mathematics Teaching

In addition, having limited experiences in mathematics classrooms that embrace inclusive and expansive ways of participating created a ceiling for candidates' conceptions of equity. What I mean by ceiling is that if candidates had limited opportunities to engage in multiple learning experiences that could shift their conceptions of what equitable mathematics teaching and learning could look like, then their vision of equitable practices--or what they strive to achieve--will be constructed within their limited frame of reference. Their vision would likely be characteristic of a superficial effort to address equity. As reported by MIST researchers, teachers with more sophisticated vision of teaching and learning showed growth than their counterparts (Cobb et al., 2018). Most candidates who define equity only within the access dimension did not convey an expansive conception of competency in mathematics and of equitable mathematics teaching. Almost half of the candidates with Type 1 equity conception provided contradictory definitions and examples of equity or equitable practices in mathematics teaching.

However, establishing a strong conception and vision of equitable mathematics teaching and learning is not always associated with enacting equitable practices. Sandoval and colleagues (2020) documented the limited evidence of alignment between candidates' conceptualizations of equity and their practices. They argued the importance of building coherence in teacher preparation program across the various learning experiences for candidates, such as other

coursework, student teaching fieldwork, and programmatic activities within the preparation program, to better support candidates to connect theory to practice.

Getting to Know the Candidates

In addition, there is a need to consider and attend closely to candidates' varying prior mathematics experiences. For some candidates, their negative experiences with mathematics were traumatic and anxiety-inducing. These experiences should not be taken lightly, as they directly impacted candidates' sense of competency with not only mathematics but also with teaching mathematics. The implications of novice teachers carrying negative sentiments towards mathematics may impact their practice and interactions with students, which may contribute to how students experience and perceive mathematics as well as their own competency (Middleton & Spanias, 1991; Richland et al., 2020). For some candidates with Type 3 equity conceptions, post-secondary learning experiences that focused on developing their knowledge for teaching supported them to reconcile with the negative sentiments and shift their perception of mathematics teaching and learning. These learning experiences were prior to teacher preparation and/or during teacher preparation program. Learning opportunities beyond K-12 schooling played an important role in shifting these candidates' perceptions. This finding provides optimism that prior anxiety-inducing negative experiences can be reconciled during teacher preparation if learning experiences are intentionally designed to position candidates as competent and capable. This study pushes for efforts to deeply understand candidates' experiences and initial conceptions of equity in an effort to better support candidates through their journey into the teaching profession.

Conclusion

The observed patterns suggest that candidates may appropriate language they learned to define equity; however, for some candidates, particularly those with Type 1 equity conceptions, more learning opportunities were needed to clarify the distinction between fairness and equality. Unless candidates are asked to describe their vision of equitable mathematics teaching, a teacher educator may incorrectly presume candidates have a strong understanding of equitable practices if they make their judgment based on candidates' definitions of equity. The findings of this study reinforce the importance of eliciting candidates' conceptions of equity in strategic ways. The findings also highlight the importance of engaging candidates in diverse learning experiences that strengthen and expand their visions of equitable mathematics teaching as well as the importance of developing strong content knowledge, as it facilitates the development of a more sophisticated vision for high quality mathematics instruction that centers equity.

The contribution of this study is to understand teacher candidates' conceptualization of equity in mathematics teaching through their articulation of an equitable instructional vision and their prior mathematics experiences. The findings of this study would provide teacher educators with insight on ways to elicit candidates' visions and experiences to better understand who they are as learners and future teachers. The findings will also support teacher educators and other faculty members engaged in program-wide improvement efforts to center equity while considering ways to better support teacher candidates.

Limitations and Future Research

Although this study utilized multiple sources of evidence to triangulate and support findings, this study was conducted and analyzed solely by the author, and there may be limitations to having a single researcher, as I acknowledge that I am an individual with particular lenses, experiences and history. Another limitation is that this study captured teacher candidates'

thinking at specific moments. I recognize that the conceptions of equity were captured at one point in time and may not fully be inclusive of all candidates' ideas and thinking. Future research could explore more nuanced ways of capturing candidates' narratives and conceptions of equity. One other limitation is that this study is based on teacher candidates who were enrolled in a specific course. Findings in this study may not be generalizable to all teacher candidates, as they are situated in different teacher preparation contexts.

In addition, although racial/ethnic demographic information was collected, analyses revealed that there were no prominent themes in prior experiences and vision of equitable mathematics teaching among candidates from different racial/ethnic backgrounds. In other words, there was limited evidence of an intersection between candidates' prior mathematics experiences and their vision for equitable mathematics instruction based on their racial background. It is unclear the extent to which candidates perceived their race or racialized experiences in mathematics classrooms. It is also important to note that almost all (but two) candidates commented explicitly about their race and the privileges they had/experienced. This may be due to several reasons. I designed this study with an open definition of equity and elicited candidates thinking about equity without explicitly specifying issues of race. Additionally, the mathematics methods course did not have explicit discussions of race. Hence, the questions posed to candidates may not have prompted candidates to consider their own or their students' races.

Another limitation is the racial categories used for this study can be limiting when grouping individuals with varied experiences and lived histories. For example, for the Asian/Asian American group, there are multiple ethnicities and cultures, spoken languages, significant differences in immigration history that shape candidates' education experience. To

extend the current study, expanding or reconceptualizing the racial categories can be further explored. Use of interviews may provide greater and deeper insight into candidates' thinking around equity and their vision of equitable practices. Furthermore, whether these different types of equity conceptions translate to a potential progression can be explored.

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CHAPTER 3

Developing Collaboration between a Teacher Educator and Doctoral Student Researcher to Center Equity in a Mathematics Methods Course

This study explores whether and how a teacher educator and graduate student researcher collaborated to integrate an equity lens into a mathematics methods course. The following research questions are addressed: (1) How can a mathematics methods course be redesigned through a researcher-practitioner collaboration to integrate an equity lens? (2) How did the research-practice collaboration evolve over time to result into a productive partnership centered around the course design? The findings illustrated revisions made in the course design due to the partnership efforts. The findings revealed three forms of collaboration that the practitioner and researcher progressed through, from peripheral collaboration to complementary collaboration, to integrative collaboration. Constant shared sensemaking experiences and the unique role of a graduate student researcher contributed to the partnership's progression through the different forms of collaboration.

Developing Collaboration between a Teacher Educator and Doctoral Student Researcher to Center Equity in a Mathematics Methods Course

Introduction

The practice-theory connection is no better served than when it is lived. We can learn *from* as well as *about* practice. Our challenge is to create a community that educates all of us, those in the university and those in the schools, a community that expands our relationships with one another and, in so doing, our knowledge and our effectiveness.

(Lieberman, 1992, p. 11)

The work of bridging theory and practice is at the heart of improvement in the field of education, both for researchers and practitioners. Historically, building theory was the work of researchers and applying it to practice was delegated to practitioners. This arrangement divides and isolates communities that have shared goals of improving experiences for learners (Gore & Gitlin, 2004). Furthermore, it is illogical to have practitioners do the work of translating and bridging theory to practice when they were not part of the process of developing theory. Scholars have challenged this arrangement and pushed for changes in the way we conceptualize the relationship between researchers and practitioners (Bickel & Hatrup, 1995; Cochran-Smith & Lytle, 1999; Lieberman, 1992).

One way of bridging theory and practice has been through research practice partnerships (RPPs), which has emerged as a new, equitable model of collaborative work between researchers and practitioners. RPPs are mutually beneficial, “long-term collaboration aimed at educational improvement or equitable transformation” (Farrell, Penuel, Coburn, Daniel, & Steup, 2021, p. 5). RPPs have been touted as ideal arrangements for collaboration towards educational improvements, as they are “intentionally organized to bring together a diversity of expertise,

which allow for shifts in power relations and position practitioners as experts and contributors of knowledge when engaging in joint work with researchers on shared problems of practices (Farrell et al., 2021, p. iv). In addition, engaging in research activities is central to RPPs, which includes making sense of data or findings together (Coburn et al., 2008), co-designing learning activities that draw on the research findings (Johnson et al., 2016). Not only do research activities advance the knowledge base of the field, but they also help build relevance for practitioners.

To understand how researchers and practitioners collaborate to bridge theory and practice, the conceptual work of boundaries and boundary crossing has been an important topic of study within education (Akkerman & Bakker, 2011; Penuel et al., 2015; Suchman, 1994) and emerging within the RPP community (Penuel et al., 2015; Wegemer & Renick, 2021). The present study draws on the research-practice partnership to frame the collaborative work between a researcher (graduate student) and practitioner (teacher educator) and on the conceptualizations of boundary crossing to understand how the participants engaged in collaboration to improve the design of a mathematics methods course. In addition, there is a special focus placed on the roles of the participants to highlight the affordances of a collaboration between a doctoral student researcher and a teacher educator. Graduate students have a unique positionality in the context of a School of Education, which typically houses both researchers and practitioners. Within RPPs, they are positioned in complex ways, both as novices and experts and as boundary spanners. Wegemer & Renick (2021) highlight the “social affordances that enable [graduate students] to effectively enact boundary roles” (p. 4). As a doctoral student researcher, I designed the present study to understand how my interactions with a teacher educator developed over the course of

our collaboration and share learnings from this work to build on the literature on the affordances of positioning graduate students in such roles.

The present study is situated in a mathematics methods course of a teacher preparation program. Within a broader context of program-wide improvement efforts to center equity in practice, I initiated the collaboration with the teacher educator to understand how to integrate an equity lens into a mathematics methods course. The research questions are organized to first examine course design changes during the partnership (integration) and then how the form of collaboration changed over time (process). This study addresses the following questions:

1. How can a mathematics methods course be redesigned through a researcher-practitioner collaboration to integrate an equity lens?
2. How did the research-practice collaboration evolve over time to result into a productive partnership centered around the course design?

Theoretical Framework

This study primarily draws on two complementary bodies of literature to guide the analysis: (1) boundary crossing and (2) collaboration as co-learning. Given that this study focused on the development of a partnership, the interactions between research and practice within a partnership required boundary crossing (Penuel et al., 2015). Though not exactly an RPP, our partnership work was collaborative and generated learning opportunities for ourselves. The body of literature on collaboration by Wagner (1997) frames the analysis of this study.

Boundary Crossing

I draw on conceptualizations of boundaries and boundary crossing to understand the roles in collaborative work between researchers and practitioners. Boundary is defined as “a sociocultural difference leading to discontinuity in action or interaction” and can

“simultaneously suggest sameness and continuity in the sense that within discontinuity two or more sites are relevant to one another in a particular way” (Akkerman & Bakker, 2011, p. 133).

Teacher preparation programs are sites with boundaries especially when teacher candidates begin student teaching. Teacher candidates experience the sameness between both sites given the focus on pedagogy and their learning as student teachers (Alsup, 2006). They also experience the discontinuity from competing perspectives and shifts in their roles between the two sites.

Similarly, the context of a teacher preparation program and the research community within the School of Education are two sites with shared yet competing priorities and values. Boundary crossing refers to situations of “enter[ing] onto territory which we are unfamiliar and, to some significant extent therefore unqualified” (Suchman, 1994, p. 25). The discontinuity or tensions indicate the existence of and help identify the boundaries (Wegemer & Renick, 2021). The collaborative work between researchers and practitioners is typically located in the practitioners’ context and engaging in joint work implies that the collaborators are boundary crossing as they “encounter multiple boundaries that separate the world of research from the world of practice” (Penuel et al., 2015, p. 183). As Penuel and colleagues (2015) summarize, “boundary crossing involves the intentional efforts of partners on both sides to make space for and enter into joint work with partners whose work involves responsibilities, expertise, pressures, and strategies differing from one’s own” (p. 190).

Forms of Collaboration between Researchers and Practitioners

This study draws on Wagner’s (1997) conceptualization of collaboration (which Wagner refers to as “cooperation”) to frame the interactions between the researcher and the practitioner. Wagner describes three ways researchers and practitioners work together. The first refers to a data-extraction agreement, which is the most traditional form of collaboration. In this form,

researchers make the necessary logistical and legal arrangements with practitioners whose context is being studied. In this arrangement, researchers are considered outsiders, as they are socially located outside of the school, and their primary role is to engage in the inquiry process. Practitioners are inside the school context with their primary role being the enactment. There is little expectation for researchers to gain a deeper understanding of the world of practitioners. This form of collaboration resembles an arrangement in which researchers generate knowledge and communicate findings to other researchers. Practitioners play no role in contributing to the knowledge generation. The boundaries are clearly defined, but there is no effort to cross boundaries. The generated research output from this arrangement reaches practitioners through the decisions made by institutions at the district or state-level. Naturally, there is little to no agency of practitioners and there is a risk of the research findings lacking relevance, thus likely being unusable.

The second form of working together is called clinical partnerships, which entails the perspective of exploring how researchers and practitioners can collaborate for improvement in schools and learning experiences. Since this arrangement entails cooperation and negotiation, researchers and practitioners both develop a shared understanding of their respective world through research. In this form of collaboration, researchers and practitioners engage in joint work that cross boundaries. As a result, the value and relevance of the research output is higher as the work is interactive and findings can be directly communicated to practitioners. Despite being more interactive, this form of collaboration indicates that the work of inquiry is done by the researcher and the practitioner's work is the focus of study. Practitioners may partake in the inquiry but to assist. There still exists an imbalance of power between the researcher and

practitioner, given that the researcher's inquiry is not necessarily co-constructed with the practitioner.

The third form is called the co-learning agreement, which is defined as:

“In a co-learning agreement, researchers and practitioners are both participants in processes of education and systems of schooling. Both are engaged in action and reflection. By working together, each might learn something about the world of the other. Of equal importance, however, each may learn something more about his or her own world and its connections to institutions and schooling.” (Wagner, 1997, p. 16)

In this third arrangement, the work is far more interactive and the “division of labor between researchers and practitioners becomes much more ambiguous, as both researchers and practitioners are regarded as agents of inquiry and as objects of inquiry” (p. 16). This form of collaboration resonates with the literature on boundary crossing. Practitioners play a more integral role in the inquiry process. Both engage in action and reflection and changes in practice occur for both practitioners and researchers in their respective contexts. There is more of a balance in power between the researcher and practitioner and the research output is relevant and usable. In this study, the co-learning agreement reflects the collaborative relationship that the teacher educator and I reached.

Role of Graduate Students

Only a few studies highlight the role of graduate students in the context of an RPP. Wegemer and Renick (2021) conceptualized a framework of their boundary spanning roles as graduate students as they engaged in three long-term research practice partnerships. Their framework considers boundary spanning roles along five spectrums: institutional focus, task orientation, expertise, disposition, and agency (p. 6). As graduate students, they assert the need

to position graduate students through an asset-based lens and leverage the unique skill sets they bring to facilitate partnership work.

As graduate students, we learn about the importance of collaboration with practitioners or building research practice partnerships, as well as the histories of harm of research on marginalized communities to ensure that we work towards developing an equitable relationship as we engage in research inquiry. However, there is limited information on what this collaboration looks like in practice, and moreover, how the collaboration is initiated and developed. The published papers graduate students read are typically narratives of successful partnerships that are large in scale and not as many studies on failures that highlight lessons learned. To address this gap in the literature, this study aims to depict the beginnings of a collaboration, which began more like a clinical partnership with unclearly defined roles for myself and transitioned to a co-learning agreement in which our roles were clear and dynamic. This study also aims to conceptualize intentional moves that support developing a strong collaboration.

Methods

Study Context

The duration of this study was two years in a mathematics methods course at a teacher preparation program in a large public research university. The teacher preparation program is a 14-month long master's and credentialing program for elementary grade teacher candidates. Candidates begin the program in the summer term and finish in the following summer term. In the fall term of each academic year, the mathematics methods course is taught by the teacher educator. The duration of the course is 10 weeks, one three-hour session per week. The interactions with the teacher educator had started prior to the first year of the present study. An

important contextual note is that the second year of this study was conducted during the year of the unprecedented pandemic. There were several adjustments that were made. The intended learning experiences were adapted to ensure all candidates had equitable access to participate. For example, candidates had varying contextual complications, such as having limited access to students at their student teaching field placement site. Arrangements were made to ensure candidates could experience the course content with a practice-based context to draw connections to by pairing candidates with colleagues who have access to children.

Participants

There are two participants in this study: the teacher educator of the mathematics methods course within a teacher preparation program and me, the graduate student researcher, who was enrolled in the doctoral program at the School of Education. Both of us were situated in a large, public research university in California.

The teacher educator is a White woman and teaches as an adjunct faculty member at the university. She is one of three mathematics methods course instructors. She also teaches other courses in the program, depending on the needs of the program. Compared to other teacher educators, she has taught a diverse range of courses throughout the year. During the time frame of this study, the teacher educator had seen the candidates across every academic term because she taught at least one course every term. She is also a full-time mathematics coordinator at a County of Education office.

I am a Korean American woman and a graduate student researcher at the university. As a graduate student, I worked on multiple research projects with different professors. Given my research interests in teacher learning to center equity in mathematics teaching and my prior experiences with mathematics teaching and learning, the faculty members I have worked with

typically shared perspectives on learning theories and conceptions of equity. Thus, I was able to develop a coherent understanding of equity and learning theories as well as research skills.

Prior to our collaborative work, I had opportunities to interact with the teacher educator briefly as our interests overlapped. For example, I was introduced to the teacher educator during my first year of the doctoral program by my advisor, and we applied as a team to attend a workshop to learn about mathematics teachers' specialized content knowledge. I also had the opportunity to interact with her for a course I had taken in my second year. However, I had not collaborated with the teacher educator on a research project prior to this study.

Data Sources

The primary data sources were field notes generated from course observations, course slides and syllabi from each year, interview transcripts between the participants, written response to an interview protocol, meeting notes, my calendar for records of meetings, and other documented forms of communication, such as emails and comments on Google Docs and Slides.

A total of 20 field notes of class meeting observations were created, ten for the Fall 2019 and ten for Fall 2020. In each field note, I took notes of the sequencing of experiences and captured some interesting comments made by candidates or by the teacher educator that were related to their conceptions of mathematics teaching and learning and equity. I also summarized conversations I had with the teacher educator during class. There were frequent interactions between us during breaks or when candidates were working in small groups. She shared what she had noticed and revisions she planned to make during class or for the next session. The teacher educator also reflected on the unanticipated responses or reactions from candidates. I took note of her comments in the field notes, as I imagined they might be useful for us when we debrief and reflect.

Interview protocols were developed for the two semi-structured interviews. The first interview was conducted for an hour about two months after the Fall 2019 course had ended. The second interview was conducted for a total of two hours about six months after the Fall 2020 course had ended. There were two parts to the second interview. I developed an extensive interview protocol to capture our reflections of both first and second year of collaboration. The protocol had been shared with the teacher educator a few days before the interview, to provide the teacher educator an opportunity to reflect on the past two fall terms. The protocol had three parts: (1) questions about the first year of collaboration (Fall 2019), key learning goals, tensions from collaboration, key moments when trust was built or might have been broken, takeaways from working together; (2) same set of questions as first year but for the second year of collaboration (Fall 2020); (3) questions that prompted the teacher educator to reflect on both years. The teacher educator typed out some responses, ranging from one to two paragraphs for the first part, directly onto the protocol because she had immediate responses to share and to save some time for the interview. I also provided my perspective and responses onto the protocol in a different color for the questions she responded to. This pre-activity of writing out some responses to interview questions helped us to get through more questions on the protocol. Then, the actual interview was conducted for a total of two hours. Both interviews were transcribed for analysis.

Analytic Strategy

The analytic approach for this study was a longitudinal case study design (Yin, 2003). For both research questions, I employed content analysis (Stemler, 2001) to the data sources to reduce the data into fewer, meaningful content categories to then examine for patterns in the data. I describe below the specific procedures for each research question.

Data Analysis of Research Question 1: How can a mathematics methods course be redesigned through a researcher-practitioner collaboration to integrate an equity lens?

To examine changes in the course design between the first and second years of collaboration, I reviewed the course syllabus, specifically the course schedule of activities and assignments. I also examined my field notes on the days the key assignments occurred to review notes on enactment or how the teacher educator and I engaged with candidates (if at all) during the activities. I reviewed my meeting notes for the design decision conversations we engaged in. After constructing a table that visually shows differences between each year, I reviewed the interview transcript for conversations related to the changes in the course structure and comments on the work of integrating an equity lens.

Data Analysis of Research Question 2: How did the research-practice collaboration evolve over time to result into a productive partnership centered around the course design?

I first created a spreadsheet and created a column for each class session. I created a row for each type of interaction or conversation between the teacher educator and me to record when reviewing data sources. The field notes were reviewed for notes on teacher educator and my interactions during each session, specifically the content and nature of the conversations, and any researcher reflections that I wrote down during or at the end of each class session. These excerpts were recorded onto the spreadsheet. Then, I reviewed my emails, calendar, and course slide decks comments section to record on the spreadsheet the nature of the conversation, who initiated, and when it occurred. For every new day of interaction, I created a new column, keeping the date order. Then, after recording all documented interactions, I coded for the type of interaction between the teacher educator and myself, which included: course session, scheduled meetings to plan outside of course, and interview. See Figure 3.4 for more information.

The nature of the emails and asynchronous conversations through the comment feature on Google Slides were characterized and coded as one of the following: research-related, planning/logistics, co-designing, and assignment-related. Who initiated the communication was also recorded. Then, a visual was created for each year to compare how our interactions specific to the planning of course content changed between the first and second year of collaboration. Course slides and syllabi were reviewed for supplementary information on the course content, its sequencing, and assignments.

Data were analyzed using a thematic analysis approach (Braun & Clarke, 2006) for the interview transcripts. The transcripts were annotated with memos. Then, the notes were reviewed for patterns or emerging themes.

Findings

Research Question 1 Findings: Mathematics Methods Course Re-Design

Our collaborative work led to redesigning the mathematics methods course at two levels. The first was at the level of the course design. The second was at the level of an experience within a session. To illustrate the changes in the course design between the first and second year of working together, I highlight key learning experiences and assignments each year. I first briefly describe the course learning experiences and assignments in the year prior to the study to set the context. Then, descriptions of the first and second year of collaboration follow. See Table 3.1 to see the changes across the two years. To illustrate the changes at the level of an experience within a session, I provide two vignettes of a moment in the first and second year of our collaborative work. The course content is parallel between the two years and facilitates comparison.

Changes at the Course-level

Prior to the study: Pre-dissertation (Year 0)

Course learning experiences. There were several key experiences in the mathematics methods course. The teacher educator arranged with two first grade teachers at different schools to bring teacher candidates into their classrooms. Candidates each paired with a student to complete the Student Problem Solving Interview activity, which engaged candidates in unpacking the story problem and asking questions that elicited student thinking. In the second teacher's classroom, candidates formed small groups with each other and taught a mini lesson to a small group of children, with the same lens of creating opportunities to elicit and learn more about children's mathematical thinking. The main assignment in this course was developing a lesson plan that they revised over a few weeks. Candidates then taught this lesson at their student teaching placement site and self-recorded their teaching. Candidates were asked to select a clip to engage in a reflection activity in small groups on the last day of class. Their supervisors were invited to join the conversations.

Assignments. For assignments, candidates engaged in weekly readings and assignments from the Elementary and Middle School Mathematics Curriculum: Teaching Developmentally (Van de Walle et al., 2018). The assignments aimed to develop candidates' knowledge of the mathematics content and effective ways to teach through developmentally appropriate, problem-based activities.

Collaboration Year 1

Course learning experiences. The key learning experiences that continued from the previous year were the one-to-one Student Problem Solving Interview, Small Group Mini Lesson, developing and teaching a full mathematics lesson, and engaging in a reflection activity in small groups with supervisors using a clip from their teaching. The interview and mini lesson

occurred in one first grade teacher's classroom (instead of two). For my dissertation study, we agreed to integrate a new learning experience called the Self-Confrontation Noticing Activity, which engaged candidates in confronting their own biases and assumptions about children and their mathematics thinking as a way to center equity in their conception of mathematics teaching and learning (see Study 1 for more detail). This activity was linked to their one-on-one Student Problem Solving Interview. While in the first grade teacher's classroom, candidates recorded their Student Problem Solving Interview using GoReact video platform and were tasked to watch their interview and be prepared to share what they learned about their student's thinking.

Then, in the next class session, candidates formed groups of three and took turns sharing and spent up to five minutes each. When each candidate shared, they opened the GoReact video platform and self-recorded their conversation using their computer camera. After self-recording and engaging in the share out in their group, candidates were given prompts to read and watch their own video to annotate, which is a feature in GoReact. The prompt asked candidates to identify moments in their discourse when specific **language** was used to describe student's performance and behavior, particular **lens** (asset, deficit, funds of knowledge, etc.) used to look at student work, how they **positioned** students with respect to mathematics learning, and the **expectations, biases, or assumptions** were implicitly projected. After annotating on their video, candidates were asked to write a 300-500 word response to the same prompt questions. This activity was completed twice. Candidates went back to the same first grade classroom to teach a mini-lesson in the same group of three they formed for the Self-Confrontation Noticing Activity. Since each candidate worked with one student, the three candidates in each group taught a mini lesson to the three students they interviewed. This lesson was also recorded using GoReact and candidates were asked to complete the same set of procedures: (1) watch the mini lesson to

prepare for a share out in the same group of three, (2) self-record share out of what they learned about their students' thinking during the mini lesson, (3) annotate their self-recording, using the same prompt (language, lens, positioning, biases/assumptions); (4) write a 300-500 word response to the same prompt. Their responses became a primary data source for the first dissertation study. On the final day of class, candidates brought back clips they wanted to use to engage in the reflection activity in small groups with their supervisors.

Assignments. The same weekly readings and assignments from the Van de Walle et al. (2018) textbook were assigned. In addition, a new assignment was introduced and given to candidates in their third class session. This assignment was the Mathematics Autobiography, which was drawn from work by the TEACH Math Group (Drake et al., 2015). The original assignment had a set of questions, but to avoid adding too much time being spent on completing assignments for the sake of my dissertation, the teacher educator and I shortened the autobiography to two key questions of interest: (1) What was your experience with learning mathematics growing up, specifically at school, with your family, and/or in your community?; and (2) How do you think your prior experiences learning math might shape the way you teach mathematics? The responses were collected as part of a homework assignment due before the fourth class session. Another ongoing classwork was the Interactive Journal; candidates were given resources (e.g., content standards, mathematical practices standards, etc.) to paste into specific pages. Candidates were given prompts to write down reflections into the journal as well. The intent of this journal was to create a tool for candidates to take into their classroom practice.

Collaboration Year 2

Course learning experiences. All activities continued in the second year. Some adjustments were made due to the pandemic, but the key course activities (Student Problem

Solving Interview, Small Group Mini Lesson, Self-Confrontation Noticing Activity, and the Interactive Journal) continued to be assigned. Given the integration of a new assignment and the methods course becoming an online course, the candidates did not go into a first grade classroom to complete the interview and mini lesson activity. Instead, candidates were asked to identify a student or two they could interview at their student teaching placement. For the mini lesson, candidates formed small groups with one another on Zoom and taught each other. One new activity was added. This was the “Getting to know you” Student Interview. Learnings from prior year prompted the teacher educator and I to add in another interview that candidates conducted before the Student Problem Solving Interview to provide candidates with the opportunity to get to know the student at a personal level.

Assignments. A change in the second year was that the mathematics autobiography assignment was assigned and due before the first class session. We realized we did not make good use of last year’s autobiographies and decided that learning in advance who the candidates are in terms of their mathematics experiences will be critical to creating meaningful learning experiences for candidates, especially in a virtual learning environment. In response to our learnings from the previous year, we co-designed and integrated a new assignment, which will be referred to as the Field Notes Journal.

The Field Notes Journal assignment was originally inspired by a module developed by the TEACH Math group (Drake et al., 2015) for teacher educators to use in their course. This is different from the Interactive Journal. The Case Study Module has four activities and was designed to support teacher candidates to: (1) “learn to observe and examine learning in more detail”; (2) “expand thinking about children as mathematical learners including which skills, knowledge, practices and experiences they see as relevant to children’s learning”; and (3) “think

about how the knowledge, skills and competencies that children demonstrate in different contexts (school, after school, home, community) might support their school mathematics learning” (Foote et al., 2015, para. 2). The teacher educator and I added the first activity, “Getting to Know You” Interview, as part of candidates’ Week 5 Field Notes Journal assignment. The third activity, “Problem Solving Interviews” was very similar to an activity the teacher educator had already been assigning to candidates the past two years (referred to as the Student Problem Solving Interview). The Student Problem Solving Interview assignment was integrated into week 7 of the Field Notes Journal.

The Field Notes Journal assignment was developed to address a need for supporting candidates to draw connections from course content to practice, gain insight on how candidates were making sense of students’ mathematical thinking, as well as how their noticing developed over time. Candidates were asked to follow two students at their student teaching placement site and record their observations and interpretations of each student’s mathematical thinking. Candidates completed the two interviews with their students. We included weekly reflection questions that prompted candidates to reflect on what they newly learned about the student during the week, what revisions they want to make in their thinking or understanding of their student, and evidence to support their noticing. Candidates were given an online document that they could make a copy of and begin using. The document had two columns for them to complete. The first column asked candidates to jot down what they noticed their student(s) did mathematically (“What did you notice the student do mathematically?”). The second column required candidates to interpret/make sense of what they noticed in the first column (“How are you making sense of what you noticed”) in the student’s work. The Interactive Journal continued to be an ongoing assignment that candidates would build over the course.

Table 3.1 Changes in Mathematics Methods Course in First and Second Year of Collaboration

	Fall 2018 Before collaboration	Fall 2019 First year of collaboration	Fall 2020 Second year of collaboration
Key Themes	Positioning students as mathematically competent.	Reflecting on ourselves to recognize our biased perceptions and how that shapes students' learning opportunities.	How can we get to know our students deeply beyond their mathematical thinking?
Course Activities	Key Learning Activities: <ul style="list-style-type: none"> • 1:1 Student Problem Solving Interview • Small group mini lesson (groups of 3). 	<ul style="list-style-type: none"> • Added Self-confrontation Noticing Activity (Dissertation Study 1) 	<ul style="list-style-type: none"> • Added a 1:1 "getting to know you" student interview
Assignments	<ul style="list-style-type: none"> • Math content assignments 	<ul style="list-style-type: none"> • Added Math Autobiography 	<ul style="list-style-type: none"> • Added Ongoing Field Notes Journal with intentionally crafted reflection questions at the end of each week

Changes at a Micro Moment Level

Vignette from the first year of collaboration

In our first year of collaboration, candidates walked in on the first day of class and were asked to bring a notebook to create the Interactive Journal. After being instructed to number the pages, candidates were shown the following slide (Figure 3.1) to discuss norms from a book called *Intentional Talk* (Kazemi & Hintz, 2014) for our community in mathematics methods class, highlighting the norms that linked to the Standards of Mathematical Practices. The teacher educator then transitioned into a story problem to engage candidates in solving a problem to intentionally model the mathematical practices that position learners competently. After the learning experience, candidates were asked "What does it mean to "do math?" and the teacher educator typed out bullet points as candidates shared their responses aloud. Then, candidates

were asked to read a page in the book, which lists each norm and a supporting rationale, and with their partner discuss what was interesting and what they were wondering about. Candidates were posed with the following questions: “How do these norms create a culture of learning?” The field notes indicated that the key message of the conversations was the importance of seeing children as sense makers, as that helps them see themselves build a “math identity,” and that there is “no such thing as math people.” Then, candidates were asked to reflect on “Which norms were present in our experience with the [story problem] task? In what ways?” and write their thoughts down into the Interactive Journal. Then the teacher educator moved onto the next learning experience.

Although I made no contribution to the planning of this portion of the session, I took notes of what was occurring play by play during the session. I also walked around when candidates were working on the problem to hear the different ways they were solving the problem and asked questions to convey my interest in their strategies. The teacher educator shared her in-the-moment reflection on what had happened and what she thought she could have done better. I was not sure how an equity lens could be integrated as I was learning how the teacher educator was thinking about equity. In the beginning of our first year of collaboration, I noticed that this course regarded equity as inclusion of diverse children’s thinking to position children competently in mathematics. Drawing from Gutiérrez’s (2012) framework of equity, the teacher educator was considering the equity dimension of identity, which refers to the opportunities that students are given to feel belonging in the discipline and find relevance and meaningfulness to their own lives.

Norms

- **Make sense of mathematics**
- **Keep trying even when problems are challenging**
- **Remember that it's okay to make mistakes and revise our thinking**
- Share our mathematical ideas (whether we are using words, numbers, pictures, gestures, or tools)
- **Listen to understand some else's idea; give each other time to think**
- Ask questions that help us better understand the mathematics
- **Agree and disagree with mathematical ideas, not each other**
- **Remember that everyone has good mathematical ideas**
- Contribute to a learning environment in which its safe not to know
- Attend to the language of the standards

Page 6 of interactive journal

Figure 3.1. Norms for the class from year 1 of collaboration.

Vignette from the second year of collaboration

In our second year of collaboration, we re-designed how we would introduce norms. We agreed it was unclear whether the norms resonated with candidates and that there were some missed opportunities to connect back to the norms during course activities. Given what we had observed at the last class session of our first year of collaboration, we felt that some of these norms could be better connected to their own student teaching practice. During the first session, candidates engaged in the story problem first. After, candidates were posed with the question “What does it mean to “do math?”” As candidates typed out their responses in the Zoom chat (course was virtual in 2020), I typed in themes from their chat responses into the slides. Then, candidates were asked “what supported your participation in these experiences?” We pasted in or typed out what they shared in the chat directly onto the slide (Figure 3.2). The intent of collecting their experiences was to then group their experiences strategically into the norm

categories (the same ones shared in the previous year). We thought this approach would better support candidates with seeing how they experienced particular norms that align with efforts to position learners as competent.

What supported your participation in these experiences?

- Individual time to think before sharing
- Using Desmos
- Didn't feel alone, others were working on it and may not get the right answer
- Open ended
- Sharing ideas in small groups
- Being able to learn from the strategies of others in the small groups
- Individual work time first, had to try it
- Not feeling pressure of achieving the answer right away
- We never talked about the answer; we talked about the thinking
- We revised our thinking
- Asked people to "revoice" what others had said; use the words of others
- T questions... what did you notice?
- Introduction of the problem - What are you thinking about?
- Problem was unpacked line by line
- Pictures of lizards and beetles -- could have made sure everyone knows how many legs each creature has
- Ideas shared and being built upon each other
- Do you collect anything? - relevance to students
- Started with visual, built up strategies
- Intentional selection of strategies
- No strategy was positioned as "right" or "best"
- We compared strategies; similarities and differences

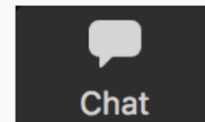


Figure 3.2. Candidates' share out in the Zoom chat during Session 1 from Year 2 of collaboration

For the second session we built out a slide (Figure 3.3) to show how their experiences were connected to norms we hoped to establish in mathematics classrooms. During the second session, the teacher educator then posed the question, "From reading your autobiographies, I'm wondering if some of these norms could have made your experiences with math different? Or impacted your identity with math?" Using the comment feature on slides, I had suggested engaging candidates in reflecting on their prior experiences with mathematics. Since I had read through their autobiographies (and shared themes to the teacher educator), I recognized an opportunity for candidates to reflect on unnoticed ways their prior experiences shaped their

practice. Many of the experiences highlighted how certain mathematical practices made them feel incompetent, thus creating a negative mathematics learning experience. In an effort to be intentional and build coherence across assignments, activities, and learning goals, we decided to include reflection questions as the one before throughout the sessions. In other places on the slides, I had left suggestions with possible questions to pose that would prompt reflection on things we as adults may do unintentionally that positions children incompetently.

What supported your participation in these experiences? → **NORMS**

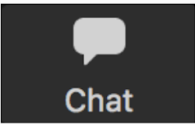
<p>Listen to understand some else’s idea; give each other time to think</p> <ul style="list-style-type: none"> ● Sharing ideas in small groups ● Being able to learn from the strategies of others in the small groups ● Ideas shared and being built upon each other <p>Make sense of mathematics</p> <ul style="list-style-type: none"> ● Individual time to think before sharing ● Individual work time first, had to try it ● Introduction of the problem - What are you thinking about?, Problem unpacked line by line, do you collect anything (relevant to students) <p>Remember that it’s okay to make mistakes and revise our thinking</p> <ul style="list-style-type: none"> ● We revised our thinking ● Not feeling pressure of achieving the answer right away ● We never talked about the answer; we talked about the thinking <p>Keep trying even when problems are challenging</p> <ul style="list-style-type: none"> ● Didn’t feel alone, others were working on it and may not get the right answer <p>Remember that everyone has good ideas!</p>	<p>Teacher Moves</p> <ul style="list-style-type: none"> ● Asked people to “revoice” what others had said; use the words of others ● T questions... what did you notice? ● Pictures of lizards and beetles -- could have made sure everyone knows how many legs each creature has ● Intentional selection of strategies ● Started with visual and built up strategies ● No strategy was positioned as “right” or “best” ● We compared strategies; similarities and differences <div style="text-align: right; margin-top: 20px;">  </div>
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Figure 3.3. Collectively built norms slide during Session 2 from Year 2 of collaboration

By the second year, it became clearer to me that the teacher educator’s lens for equity focused on identity and access. In an interview, she described her thinking around equity,

“I am heavily influenced by the work of the CGI [Cognitively Guided Instruction] community, so I believe that kids bring intuitive knowledge, and I believe that it's our job ... to elicit that and build upon that. So I think I see kids in a particular way. And then also want to, I guess, convey that or push on that in the methods class and then thinking, I mean, I've said this probably a million times in front of you, but everything that we do and say, like every single move we make I think impacts kids' identity in one way or another. So how we're talking about them, or just even how we think about them. So looking at things like the--Jilk reading that I use, all the time, or just those bigger lenses and having all kinds of access, I just think it's all connected together...”

Although the interview was conducted after the second year of collaboration, her thinking was reflected in the experiences she had designed.

My role collaborating with her was eliciting her design intentions and suggesting moments to make those explicit connections in the course activities during class. One example during Session 2 that demonstrated my role was suggesting how the teacher educator could frame a slide with a set of student work examples, “frame this slide with: what do the kids **know**? Focusing candidates' attention on kids' assets.” In the following slide, I commented on an opportunity to bring in equity language, ““scaffold” is interpreted differently (by candidates), opportunity to bring up power--important dimension of equity.” I had anticipated the different ways candidates might think of scaffolding. Based on my observations from the previous year, I noticed candidates' examples of scaffolding was sequencing close-ended questions to lead children to the right answer, as opposed to providing tools or enacting moves that push their thinking in rigorous ways towards a learning goal. As stated by a professional mathematics teaching community, “Scaffolding and supporting students make tasks accessible without

lowering the cognitive demand, the type of questioning used, or ways of promoting mathematical discourse” (ASSM, NCSM, & NCTM, 2021, p. 3). The opportunity to engage candidates in considering power relations and whose idea contributes to the learning exemplified my role in the collaborative work.

From the vignettes, it was evident that by the second year, I was drawing on my knowledge of the overall course design and learning goals. Living the learning experiences in the first year helped me recognize moments to integrate questions or talking points that prompted candidates to reflect through an equity lens. By the second year, I was able to cross boundaries and utilize my research skills to co-design with the teacher educator both at a micro level and at a macro level.

Research Question 2 Findings: Three Forms of Collaboration

There were three ways the teacher educator and I collaborated to integrate an equity lens into the mathematics methods course. The first form of collaboration can be characterized as *peripheral collaboration*, which is when the research partner is in the periphery and interactions between the practice and research partners do not significantly alter the course learning experiences. The second is referred to as *complementary collaboration*, which is when the researcher’s role shifts to contribute to the learning community but does not play a significant role in the designing of learning experiences. The researcher’s contributions complement the existing course design and structure. The third form is called *integrative collaboration* because the researcher transitions to become integral and contributes substantially to the course design. This relationship highlights the co-laboring aspect of collaboration, as the teacher educator and researcher become co-dependent and share the cognitive labor to achieve their shared goals.

Figure 3.4 displays two years of collaboration with the first year as the top row and the second year in the bottom row. The first half of the first year of collaboration (from sessions 1 to 4) provides a vivid representation of peripheral collaboration with sparse communication patterns and infrequent co-sensemaking experiences. Complementary collaboration is best represented in the latter half of the first year of collaboration (from sessions 5 to 10) with more frequent communication patterns and structured meetings to co-plan. Integrative collaboration is notably visible in the second year of collaboration with not only increased communication patterns but also an increase in communication for co-designing each session. I expand on each form of collaboration below.

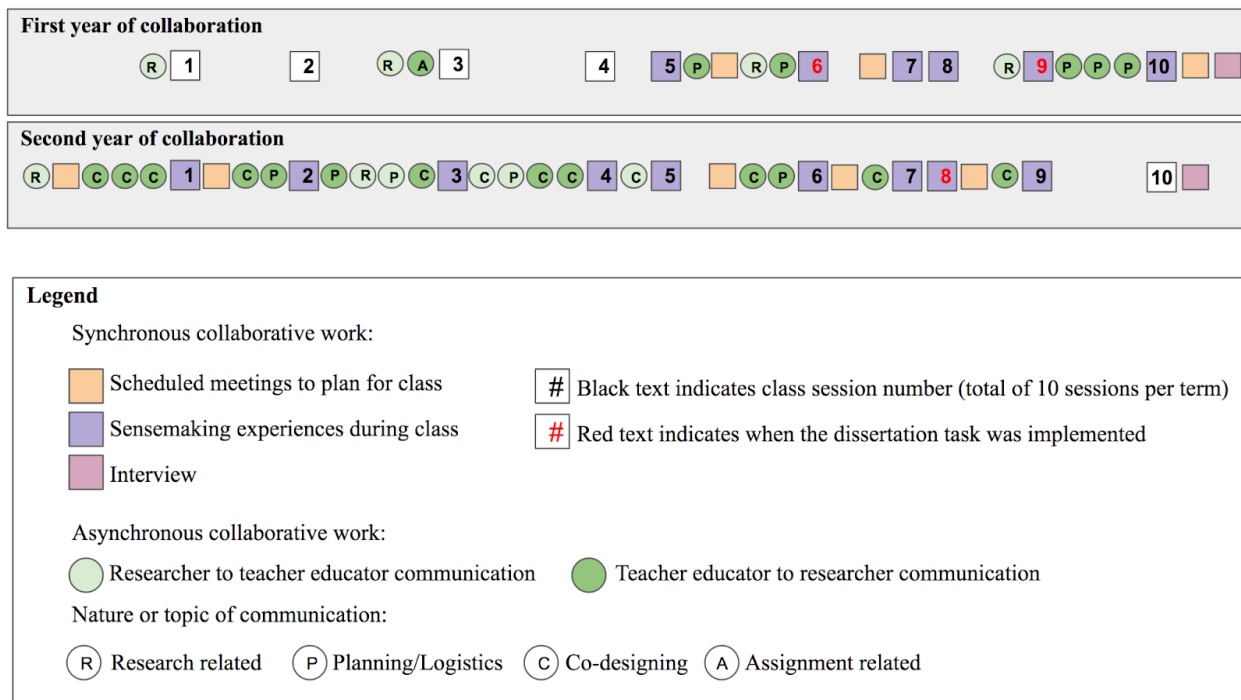


Figure 3.4. Visual representation of collaboration between teacher educator and researcher

Peripheral Collaboration

Peripheral collaboration began during the first year of our partnership. In this form of collaboration, my role was *non-consequential* for the course design, and the interactions between

us specific to the course design were *minimal and focused on logistical aspects*. Since my goal was to pilot an activity for my dissertation, conversations regarding the integration of this activity were focused on the “when” and “how” to pilot the activity and collect data.

Interactions minimally consequential to course design

One of the first documented interactions between the teacher educator and I was a long email I had sent before the course started. I listed out some goals with activities to integrate in her class for my dissertation. I wrote out my rationale for each goal and how I intended to achieve them through the activity and/or assignment based on my understanding of the mathematics methods course at the time. The tone of the email was to seek permission and elicit feedback in the case some of the goals were not appropriate for the course. I intentionally introduced my dissertation goals through an email to ensure a clear communication of the activities (and their purpose) I had in mind. Verbally introducing all parts is harder to make sense of and respond to on the spot. Detailing the purpose provides an opportunity for the teacher educator to decide if she sees commonalities with her course goals. My permission-seeking tone was to provide the space for the teacher educator to reject or share concerns of what may not work, given the current class context. Other documented interactions were email exchanges and comments left on the course slide deck. These interactions were related to logistical aspects of the course design. The nature of our communication was for the teacher educator to prime me with her plan for the next session so that I could plan accordingly if I needed to add something in relation to my dissertation. Although her messages typically ended on an open-ended note or a question asking for my thoughts, I did not contribute much thought as I did not know enough to respond in a substantive way. I was also concerned about sharing input that goes against her intended learning goals, so I chose to observe further to build a better understanding of the

course. The substance of the communication was not centered in candidates' learning experiences, thus the minimal changes in the course design.

The interactions that occurred during class sessions were typically during breaks. The field notes documented several instances of the teacher educator sharing her thoughts with me in response to what she saw unfold in class. In the second class, the teacher educator shared her comments on how “the interactive journal takes up too much time in the beginning of class” and “how to consider this and not spend time doing cutting/pasting type tasks.” While she reflected on particular aspects of the session that seemed to take up more time than she had anticipated, she also shared that she “wouldn't have cut out anything that was done earlier in class today, they all served a purpose.” During the short break, she made sense of what unfolded by sharing with me her thoughts and reactions. Then, she realized something and walked over to her computer to revise the wording on one of her session slides. As she read aloud the original question, she said, “this is a bad question” and immediately revised the wording to prompt candidates to reflect on an activity they engaged in before the short break and write into their interactive journal. During this moment, I was on my computer, taking notes of her thoughts and her actions. I told her I would record her thinking in case she might want to retrieve it. I made an intentional move of suggesting a role I could take because I wanted to shift my positioning from an outsider to someone who might be useful during class for the teacher educator. These field notes were saved in a shared folder for her to access. This form of interaction, in which I was positioned as a mirror for the teacher educator, continued until the fifth class session, which documented the first sensemaking interaction between the both of us.

Modeling sensemaking experiences

Making sense of class events was an important practice the teacher educator constantly engaged in. Up until the fifth class session, much of the sensemaking was unidirectional in that the teacher educator thought aloud as I recorded her thinking. The first four session field notes included the line “[teacher educator] is thinking aloud” followed by what she was thinking. In retrospect, her thinking aloud were invitations to engage in shared sensemaking. However, I vividly recalled feeling uncertain how to respond to her thoughts and instead recorded her thinking, as I thought it might be a useful tool for her. At the same time, I was still developing my own understanding of the learning experiences candidates engaged in within a mathematics methods course.

During the first four sessions, the moments of sensemaking that the teacher educator engaged in evolved into shared experiences that helped me see how I could participate. The teacher educator perceived me as a thought partner who could make sense of the class events together. When asked how she had envisioned my role in her class, she stated, *“I’m not sure I had a particular vision of your role or how we’d work together. You had observed my class the year before and I loved having you there to talk with, share some of what I was doing and why, my noticings and wonderings, and just have someone there to experience the learning with me.”* However, I did not contribute to the sensemaking in ways consequential to candidates’ learning experiences. Rather, the process was unidirectional, and the teacher educator’s thinking aloud supplemented with my recording of her thoughts supported her reflection process. However, these moments became shared experiences in which I learned to participate in the sensemaking process.

Researcher role as an observer and a mirror

My primary role was mirroring and recording the teacher educator's thinking and sequence of learning experiences. She would share what she noticed, make sense of her noticing, and then I would respond minimally or ask questions to better understand her comments, as I wanted to correctly capture her thinking in my field notes. My moves reflect how I typically engage in spaces as a graduate student and a researcher. As a researcher who was entering a space with pre-existing practices and routines, I positioned myself as a distant observer during class. As a graduate student in an unfamiliar context, I needed to learn about the course context and how candidates engaged in the learning experiences before suggesting the integration of new activities.

As an observer in the periphery, the amount of agency I had to contribute to course planning was minimal, as I was still learning about the course. It was unclear how I might meaningfully integrate the dissertation task. I identified the point in which the task could be integrated after examining the syllabus and offered my ideas to the teacher educator. I relied on the teacher educator to make suggestions or grant me permission with what could be integrated. The caution I took when approaching the teacher educator with ideas reflected a lack of clarity in my role and my desire to signal respect for her space. Learning about the intrusiveness of researchers in practitioner's spaces, I wanted to also signal that I did not intend to overstep. However, the teacher educator mentioned in her interview that perceived me as a thought partner who could sense-make with her, though it is unclear if she had developed that perception over time or had that perception from the beginning of our collaboration. The incongruity in how we perceived each other did not lead to shared sense-making experiences, and the nature of collaboration was constrained.

Complementary collaboration

From the fifth session of our first year of collaborative work, the nature of our collaboration changed to be *complementary* because I was able to utilize my research background and expertise in ways that made sense for the course. In this form of collaboration, the interactions between us in relation to course content development became more frequent. The conversations we engaged in involved more *sensemaking* of class events together and *co-reflection* on missed learning opportunities for the candidates. My role in the course shifted to be more involved as a member of the course community, both as a learner and a knowledgeable other. Having been immersed in the course learning experiences and engaged in debriefs with the teacher educator, I had developed a better understanding of the overarching goals for the course and how that played out in each session. From a researcher lens, I was better able to see how my dissertation activity could be integrated while still maintaining the key course learning goals.

Shared sensemaking experiences

The teacher educator and I engaged in our first shared sensemaking moment during the fifth session. On this day, candidates engaged in the Student Problem Solving Interview with first graders at a local elementary school. Prior to the interview activity, candidates did a pre-activity with their students (prompted by the first grade teacher). Each candidate and student pair were given a small collection of items to count. The teacher educator and I walked around the room to listen in on the interactions between the candidates and students. We both shared our noticing of candidates imposing their own ideas of how to count efficiently, taking the children out of their original thinking. We noticed some candidates creating addition problems or suggesting how to count or group the quantities when this activity was framed as an opportunity for candidates to learn about children's mathematical thinking. Then, when the interviews took place, the teacher educator and I noticed two types of interactions. The first was candidates being

intent about questions they posed to elicit the student's thinking. The second was candidates making suggestions to students, giving them hints to lead them to understand and solve the problem correctly. After the interview activity, the candidates gathered to debrief with the teacher educator on their experiences and what they had learned. While there were asset-oriented thoughts, there were prominent shared thoughts around what the children lacked to successfully solve the story problem. Some ideas included the issue of children's "language level" and how "holding information is hard for them (the students)." During this debrief, the teacher educator highlighted the importance of listening to children to understand their thinking is different from asking questions with the intent of correcting their thinking. She drew candidates' attention to the implications of teachers telling children how they should solve problems, and some candidates recognized that doing so implicitly tells children "that their idea doesn't matter." Having debriefed our noticing during the candidate-student interactions prompted the teacher educator to bring up these issues during the whole class debrief.

Changes in nature of interactions

There were increased interactions between us as we debriefed during and after class and made sense of what we had noticed during each session. We communicated more frequently outside of class and by sending each other emails or leaving comments on slides that initiated a conversation (as opposed to informing what is to come). When previously I initiated emails to check if certain aspects of my dissertation activity could be integrated, the communication was now initiated by both of us for the purpose of making intentional decisions about the course learning experiences, while taking into account my dissertation study. For example, before the sixth session, the teacher educator commented on a shared slide deck for the next session (during which the dissertation activity is first implemented) with some initial thoughts and asked for

input: “*Here are my beginning thoughts of next steps from student interviews. Does this align with your thoughts? Feel free to make suggestions here, or add slide with your ideas. Do you have specific questions? I’m happy to design or happy to have you design. Also happy to talk through all of this sometime tomorrow or Monday night.*” We then scheduled an hour-long meeting the following evening to think through the learning experiences of the next session. I had written out an outline that explicated the goals of the dissertation activity and the data that would be produced. We went through each goal and discussed the rationale of each to ensure we had a shared understanding of the intention of the activities. Then, we discussed affordances of some activities and the teacher educator suggested some changes given her knowledge of the class dynamics and her learning goals. Prior to the session, I had sent an email to the teacher educator for approval of the talking points to introduce my dissertation.

While my interactions with the teacher educator have increased and we engaged in co-planning, much of these interactions were in relation to how I could support the teacher educator with addressing her learning goals and how my dissertation activity could be integrated in a way that makes sense given the course design. In other words, the contributions I was able to make did not shift or significantly influence the course design. Rather, the contributions were in addition to the existing course structure. Another way our interaction changed was that I was able to provide more substantive comments or responses to the teacher educator’s reflections. Given my intent to understand how candidates learn to integrate an equity lens in mathematics teaching, I weighed in my thoughts and shared my noticing of how candidates engaged in the learning experiences through an equity lens.

Researcher role as a contributor to course community

My role shifted to be more of a contributing member in the course community and less of an observer. One way I contributed more to the course learning experiences was by checking in with candidates and pushing their thinking during some of the activities. For example, in the sixth session, the teacher educator had asked me to help with a couple of things. She noticed how candidates were confusing learning goals (i.e., understandings to develop) with activity goals (i.e., sequence of activities to complete in a lesson) in their lesson planner and asked to help her push on those ideas. In addition, I personally noticed that I was able to take a more proactive role during the sessions because my understanding of the learning experiences within the course context had grown. The combination of being immersed in the learning experiences as well as having shared experiences with the teacher educator to make sense together developed a strong enough understanding for me to recognize opportunities to integrate an equity lens. For example, during the fifth session when the teacher educator and I debriefed what we noticed, she had described particular moves candidates made when interacting with the first graders, while I commented on the implications of children's identity development in mathematics and the power teachers hold. Then, as described above, the teacher educator drew candidates' attention to positioning children competently in mathematics and the implications of strictly direct instruction.

Collaborating throughout the course had developed a better sense for the both of us how research could become an integral part of improving the course design. After the course had ended, I conducted interviews with five candidates for one of my dissertation studies. I shared back findings related to their learning with the teacher educator. Some of the key findings the teacher educator found useful was candidates' feedback on the learning experiences. *“There were certain instances where they didn't make sense of what I had asked them to do, or like my*

goal of something, they didn't perceive the goal, or they were like I don't really know why we did that, like they didn't see the coherence or the connections. And that somehow came out in your first year interviews.” The teacher educator recognized the value of some of the research activities I conducted, *“I also had this huge opportunity to learn from what you were learning to improve my own practice right ... I teach this class [and I] have never thought about that. So I think I had this [reaction] like, ‘oh my gosh I can't even wait for the second year.’”* The teacher educator initially saw me as a thought partner to reflect about class events. Then, she began to see the ways my research expertise could be used to improve her own practice.

An advantage of my positionality as a graduate student was that during the interviews candidates felt comfortable enough to share their honest input on what did not make sense in class. As a graduate student, I was able to receive theoretical support from professors with developing an interview protocol that would elicit thoughts that align with my study and be useful for the teacher educator. At the same time, I experienced a shift in the power I perceived myself to have. My dissertation study task was being implemented and I was most knowledgeable about it. Naturally, I saw this opportunity to step in and contribute more to our interactions. This would create a moment of shift in the power relations between us and provide an opportunity for both the teacher educator and I to see if our collaboration stayed stable (in terms of trust and the synergy) when I became a knowledgeable other. It was also a safe moment to test this out because it would be a temporary shift, since the moment is only during the implementation of the dissertation activity, which was for about 30-45 minutes.

Integrative Collaboration

In our second year of collaborating, the nature of our collaboration changed to be more codependent and integrative. The shared sensemaking experiences became central to our

collaboration. Our roles in the course and in how we work together became clear. The lens through which we co-plan and co-design was specific: the teacher educator's lens focused on developing candidates' knowledge of both content and pedagogy, while my lens focused on seeing opportunities to emphasize or integrate an equity perspective. Because we saw a need for each other's expertise, we leaned on each other's input for the purpose of creating a coherent learning experience to integrate an equity lens in mathematics teaching. Since our roles were clearly defined, we were efficient in our collaborative work and routines were established. In a way, the nature of our collaboration highlighted the co-laboring and the shared cognitive labor for building experiences for the course.

Constant sensemaking interactions as consequential to course design

Our second year of collaborative work significantly changed since the first year. We established a routine for sensemaking and reflecting on each session, co-planning each session, leveraging our strengths to work together. The level of detail in the co-planning was higher. For example, for each session, as we revised the slides from the previous year, we specified the learning goals within the session and created talking points to be intentional in building coherent experiences for candidates. This was illustrated in the vignette.

After the previous course ended, we had an opportunity to share our work with the new task we piloted by writing a manuscript for a journal. I worked on the data analysis and went through several iterations of making sense of the data together, re-analyzing, and sensemaking until there were salient findings. This one-month long process provided an opportunity for us to think through the course experience and gave us ideas of how we could improve the course. Then, several months passed and we reconvened to think through the mathematics methods course for the new cohort of teacher candidates. When we reflected together, there were several

takeaways from the previous course we intended to address the second year. First takeaway was that there were missed opportunities to make explicit the intent of the course activities and how the different activities are connected to broader course objectives. We worked on this by creating talking points at specific moments to draw connections. I took on the role of creating a draft of talking points, as it was easier for me to recognize moments that lacked clarity when recalling my experience as a newcomer who had experienced all the course activities. Second takeaway was that the dissertation activity I piloted (Self-Confrontation Noticing Activity) provided insights on the kinds of assumptions and biases candidates held about children and their mathematical thinking. Therefore, we wanted to implement the activity again but build additional learning experiences that could complement the dissertation activity. We co-created an ongoing assignment called the Field Notes Journal to engage candidates in weekly noticing and interpreting of students' mathematical thinking. The journal prompted candidates to reflect on their knowledge of their students each week and revisions they want to make about their students. Some of the reflection questions also prompted candidates to draw connections to some of the key learning experiences (e.g., Student Problem Solving Interview and Self-Confrontation Reflection Activity) and the role they play in positioning children as competent in mathematics.

The teacher educator appreciated how we collaborated, which involved frequent communication, sensemaking, and working towards the same goal: *“just this thoughtfulness and going back and forth, and I think, also this honesty of, like I don't know what you mean here, that's not really making sense, or like this whole idea of like co-constructing and that we're in both of these situations like we're working toward a common goal, and we certainly don't have the same, you know the same experiences, by any means, but like we're moving in the same direction and we have enough of a shared vision of like what's that thing at the end or what's*

that thing we're reaching for.” Our collaborative work was effective and efficient because whoever was the knowledgeable other in what we were trying to develop or had the time would take the lead. The teacher educator reflected on one of our processes, “I love that you always like started developing something. And then share the ideas and then either I gave feedback, or we talked through them or whatever, but I really appreciated that. Like you made it really easy for me, like I wasn't having to come up with like, ‘what would this look like.’”

Researcher role as integral to course design

My role became more integral to the course, as I shifted from being an observer to having a direct role in planning the learning experiences. I was positioned as a knowledgeable other regarding course content (beyond my dissertation task) by both the teacher educator and the candidates, on the basis of increased interactions and questions from candidates compared to prior years. Candidates would ask me questions about the course content, logistics, and assignments during class and outside of class, presuming I was the Teaching Assistant (TA). In addition, as with other forms of collaboration, I positioned myself as a learner, thus allowing candidates to perceive me as someone they could talk to with minimal social and academic risks, since I was not there to evaluate.

As a researcher, I had a stronger sense of agency to directly contribute to the course learning experiences. I gained deeper insight from the collaboration and was enabled to integrate the new activities into the course design in a seamless way. Since I co-designed some of the course content with the teacher educator, the dissertation tasks no longer became added-on work but one of the core learning experiences and assignments for candidates.

Discussion

The collaborative work between the teacher educator and I evolved over time. We began our collaborative work with an interest in each other's work and context. I had been interested in understanding the teacher educator's course context, while the teacher educator was interested in the activity, I wanted to pilot in the first year. As we engaged in shared experiences of sensemaking, which evolved, we developed a shared vision and understanding of the experiences that would foster candidates' learning to center equity in mathematics teaching. We also developed clearer roles of how each of us could leverage our expertise to contribute to our collaborative efforts. This finding is supported by Farrell et al. (2019), who asserted the importance of clear roles for collaborative efforts to make progress. The process and outcome of our collaboration relates closely to Wagner's (1997) conception of "co-learning," in which researchers and practitioners not only gain insight about each other's worlds but also gain deeper insight in their respective worlds.

Shared Sensemaking as Opportunities to Learn and Build Stronger Collaboration

The role of shared sensemaking was critical in developing a shared vision and understanding of teacher learning and how to collaborate effectively. Our engagement in frequent sensemaking of class events transitioned from a unidirectional experience in which the teacher educator reflected while I served as a mirror or a thinking board to a bidirectional experience. As I gained more understanding, I was able to contribute to the sensemaking experiences, bringing in my research and equity lens. These shared experiences created opportunities to learn together and build a shared vision, which allowed us to naturally progress from peripheral to an integrative collaboration.

When engaged in integrative collaboration, the teacher educator and I shared the cognitive load of thinking about the course design. Given that our roles became clearer, I was

able to think about the learning experiences across the course through an equity lens, while the teacher educator was able to think about the learning experiences from a mathematics content lens. Working together afforded us with opportunities to see the connection between our two lenses and places to bridge equity with the mathematics content. Again, what was underlying our collaboration was a strong interest and intention to integrate an equity lens. Because the teacher educator saw the affordances of the activity that was piloted in the first year, she wanted to better integrate the activity the following year by building coherence in language and experiences across the different learning experiences. Although I could not continue collaborating with the teacher educator for a third year of collaboration as I transitioned to graduate, some of the structures we built in our second year continued to stay.

Doctoral Student Researcher as a Conduit

This study highlights the affordance of graduate student researchers who can cross boundaries between the teacher preparation context and the research community within the School of Education. I was uniquely positioned to bridge the conceptual aspects of the work between teacher educators who are adjunct faculty and tenure track faculty members. Though not explicitly highlighted in this study, the nature of my experience as a doctoral student is one that is immersed in the doctoral program. I have taken courses, engaged in research projects, and interacted with several faculty members who have influenced my thinking and supported me to learn and develop as a research scholar. My doctoral student experiences were occurring simultaneously. An example is taking a portion of my analysis using the data from this study and receiving feedback from a weekly research group meeting. The feedback then informed my next iteration of analysis, how I might take my expanded understanding into the study context, or what I might share with the teacher educator that would be useful for her practice. In the context

of this particular teacher preparation program, many of the courses are taught by adjunct faculty, who bring diverse expertise but may not naturally fall into collaborative work. With the teacher educator in my study, she mentioned in her interview that collaboration is not an integral part of her full-time job. Unless there is scheduled collaborative work at the teacher preparation program, she simply drives to campus to teach and then leaves.

In addition to being a bridge between the research side and the practitioner side within the School of Education, I was also a bridge between the teacher candidates and the teacher educator. Given the structure and design of teacher preparation programs, the teacher educator holds more power than the candidates. Although the teacher educator makes the effort to distribute power, the hierarchy is perceived because in the end candidates are evaluated with a grade in the class. This was evident in the kind of questions candidates frequently asked about grades and logistical aspects of the course. At the same time, candidates were learning to shift from their identities as students to teachers. My identity of being a student was shared with the candidates, and I intentionally positioned myself as a learner in the study context so that I could be perceived as an approachable figure. Since I played no role in candidates' evaluation (e.g., grading assignments), there was less social and academic risk in interacting with me and sharing their thinking.

Furthermore, my positionality as a graduate student allowed me flexibility in how I positioned myself, which was signaled through my intentional moves. I positioned myself as a novice in the beginning of our partnership, giving more power to the teacher educator, since the methods course is her context that she is most knowledgeable about. My positioning signaled my intent to learn from her and her class. I made intentional moves throughout to shift my positionality from an outsider to a useful contributor and from being a novice to a knowledgeable

other. I identified opportunities to make intentional moves using my developing understanding of the course context and how my dissertation-related goals could be integrated into the course.

Conclusion and Future Directions

This study offers one example of how a collaborative relationship between a researcher and a practitioner begins and develops within the School of Education at a research university. The findings demonstrate how the different forms of collaboration shapes the learning experiences within a mathematics methods course. The structural changes to the course design were sustainable changes that supported the teacher educator to continue using the assignments and learning experiences that we co-designed.

Through this study, I emphasize the importance of recognizing the existing resources within the School of Education, specifically in the teacher preparation program and the doctoral program and creating spaces and opportunities for collaboration to foster co-learning for both practitioners and researchers. This study builds on prior work that asserts an asset-based view of graduate students within the context of research practice partnerships (Wegemer & Renick, 2021). Doctoral student researchers as conduits between the two contexts or programs can be one way to bridge relationships and build shared understandings. However, such arrangements may depend on each institution's context. In my case, this study was not funded. I spent many hours in addition to my graduate student researcher position for another project to participate in the way I did. It was only possible to complete this work due to the supportive and understanding community and mentors. This is likely not a sustainable structure for graduate students to pursue. There is a need for financial support for an intensive partnership project as this study.

In addition, this study contributes to the literature by conceptualizing the work of identifying the opportunities to make intentional moves, as well as enacting them. In the case of

my collaboration with the teacher educator, my intentional moves created opportunities for both of us to envision my role as a researcher in the course, shift power dynamics in the partnership, test for stability in our collaboration, and engage in shared sensemaking in routine ways. My intentional moves in positioning myself in different ways that seemed socially acceptable conveyed my respect to the community I entered and allowed for me to become a contributor.

This study opens new possibilities to further explore and understand how collaboration among members of a School of Education can build coherence within courses and across multiple learning experiences. Future studies should explore ways to systematically study the roles of graduate students within researcher-practitioner collaborations. I recognize that this study did not explore the role of the teacher educator's and my identities beyond our institutional roles. Although my study explores the dynamics of power based on knowledge and expertise, there is likely dynamics of power based on our race, age, and other aspects of our identities at play (Denner et al., 2019). Another future direction for related studies would be to center a focus on the dynamics of power, race, and culture in collaborative efforts.

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CONCLUSION

The overarching goal of this dissertation was to understand how teacher candidates learn to center equity in their practice. The literature conceptualizes many ways to support teacher candidates to develop a sophisticated conception of equity and learn to enact equitable practices in mathematics classrooms. I focused on the role of reflection, specifically critical self-reflection, in developing nuanced self-awareness to recognize the subtle ways we as educators and adults shape the worlds of children in mathematics classrooms both in good and bad ways.

The first study focused on the development of teacher candidates' self-awareness and how their extent of self-awareness was related to their practice. A novel pedagogical activity (Kang & Lee, 2019) was adapted to be implemented into an elementary mathematics methods course. The activity was video-based and elicited candidates' assumptions and biases about children and their mathematical thinking. Candidates engaged in this activity in four ways: (1) Very few candidates did not notice their assumptions or biases (minimal to no awareness); (2) Some candidates simply recognized their own biases and assumptions and did not consider implications of holding such ideas of children and their mathematical thinking (passive awareness); (3) About a third of candidates not only recognized their assumptions and biases but also analyzed why they held such thoughts and discussed the implications of their biases and assumptions on students' opportunities to learn and develop a positive identity in mathematics (interpretive awareness); (4) About a third of the candidates not only recognized and interpreted their assumptions and biases but also responded by discussing actual actions they took to address inequitable learning opportunities or specified next steps to take (responsive awareness). Candidates' engagement revealed the extent of their self-awareness of their own biases and assumptions. There was a variation in the alignment between their self-awareness and how

equity is centered in their practice. Candidates who had deeper self-awareness were more likely to center equity in their practice by considering the four dimensions of equity (Gutiérrez, 2012) in their practice: access, achievement, identity, and power. Candidates with minimal awareness were more likely to consider the equity dimensions of access and achievement in their practice. The findings suggested that candidates needed more support to consider the importance of the role of identity and power in mathematics classrooms. One future direction for this study is to engage in explicit conversations using the equity framework after candidates complete the Self-Confrontation Noticing Activity. Designing specific learning experiences to complement the Self-Confrontation Noticing Activity to put an explicit focus on the role of identity and power in mathematics classrooms may better support candidates to deepen their self-awareness. Furthermore, the equity coding framework developed to capture productive and unproductive perspectives in mathematics teaching and learning can be translated into a tool for teacher educators to use as they support candidates to develop more nuanced understandings of centering equity in practice.

The second study focused on the development of teacher candidates' conception of equity and vision of equitable mathematics teaching and understanding their past experiences with mathematics in relation to their equity conceptions and visions. There were three types of equity conceptions that emerged from the analysis, which drew on the equity framework by Gutiérrez (2012). In the first type, candidates considered only the access dimension of equity. In the second type, candidates considered access and identity dimensions of equity. In the third, candidates considered equity, identity, and power dimensions of equity. The findings also revealed candidates' varying experiences as students in mathematics classrooms. Some candidates had unresolved negative mathematics experiences and their equity conceptions and visions focused

on the dimensions of access (Type 1). Candidates with this type of equity conception tended to have fewer opportunities to engage in rich mathematics learning in the past. Other candidates had been positioned as competent in mathematics for superficial reasons, which shaped the way they perceived their competency when they experienced challenges that they could not resolve in the past. A prominent pattern that emerged was that a group of candidates who had the most expansive conception of equity reflected on their prior mathematics experiences by focusing on the structural issues in education that perpetuate problematic, inequitable mathematics practices. The findings of this study assert the importance of getting to know candidates as individuals with particular mathematics histories and creating experiences that support them with reconciling negative experiences and developing confidence in the mathematics content. Then, candidates may develop more nuanced and complex ways of thinking about centering equity in their mathematics teaching. One future direction of this study is to consider designing a longitudinal comparative case study to better understand how candidates with varying conceptions of equity in mathematics teaching make sense of the course content in the mathematics methods course. Another direction could be creating a usable tool for teacher educators that support them with understanding how their candidates understand and envision equity in mathematics teaching. Knowing where candidates are would provide teacher educators with concrete ideas of which dimensions of equity to push on candidates' thinking.

The third study examined how the mathematics methods course changed over the two years of partnership and how the form of our collaboration changed simultaneously. In the first year of our collaboration work, we engaged in peripheral collaboration and then transitioned to complementary collaboration. In our second year, we engaged in integrative collaboration. The key distinctions among the three forms of collaboration were the extent to which and how my

ideas and contributions were taken up by the teacher educator and my role as a research collaborator. In peripheral collaboration, my role was to observe and learn and minimal ideas and contributions were made, as I was still trying to understand the context. In complementary collaboration, my role was to support the existing structure of the course, and my ideas (which was the Self-Confrontation Noticing Activity) were inserted into what the teacher educator had already planned. In integrative collaboration, my role was more integral to course decisions and my ideas and contributions were consequential to the course design. Given the integrative nature of our collaboration, the ownership of ideas and products became unclear, as the course learning experiences became co-designed. My role became more clearly defined as I understood better how I could contribute and what expertise I needed to draw on to work effectively with the teacher educator. The findings revealed the importance of shared sensemaking experiences with the teacher educator. Being in the same context and living the same experiences allowed us to draw on our respective lenses and make sense of our observations of teacher candidates' learning. The shared sensemaking developed a shared vision of our goals and our roles, thus making our collaboration stronger. The findings of this study also revealed an important network that could be leveraged within Schools of Education that houses both teacher preparation programs and doctoral programs. Graduate students have unique, flexible positionalities that allow them to be both experts and learners, as well as supporters of and co-designers with practitioners. They also bridge the two programs and worlds--the worlds of research and practice. Building on prior work by other graduate students (Wegemer & Renick, 2021), I argue for opportunities and settings for collaboration among stakeholders within the School of Education.

Both Studies 1 and 2 propose a possible learning trajectory. Study 1 proposes a possible trajectory of growing self-awareness of one's own biases and assumptions. Study 2 proposes a possible trajectory of expanding and deepening the conception of equity. Since the study was not designed to capture longitudinal data of candidates beyond their time in the course, I cannot claim that there is a clear learning trajectory of becoming deeply self-aware and of expanding one's conception of equity. Future research can be intentionally designed to follow teacher candidates during the course of the 14-month teacher preparation program and possibly into their beginning years of teaching. The concepts central to my study, such as equity, are not simple ideas that can be understood deeply and internalized in a short period of time. The 10-week long course may not have been sufficient time unless every activity, every experience including student teaching, every word spoken by the teacher educator was tightly designed to convey coherent messaging about equity. There were many factors that were outside of our control to align with the core purpose of this study. This points to another limitation. Ultimately the efforts were between the teacher educator and me. Despite the significant progress we made that had a lasting impact on her course, these efforts to build teacher candidates' understanding of equity in practice should be a collective, coherent effort across the teacher preparation program (McDonald, 2007).

The third study conceptualizes forms of collaboration between a researcher and a practitioner. This study provides one example of how a collaborative partnership was initiated and how it developed over two years. Given that the limited information on how to begin partnership work with practitioners was an obstacle, 'learning by doing' was my lens. I hope this study can illuminate one process for fellow colleagues who are wondering how they can develop a partnership relationship with practitioners. While much of the efforts of our collaboration

yielded sustainable results (e.g., new assignments continued to stay in the course), there were challenges I had to overcome. Because this study was not funded, I had spent many hours in addition to my typical duties as a graduate student researcher. What made this possible was the supportive environment I was in that allowed me to pursue my genuine interest in the research topic and develop a dynamic relationship with the teacher educator. The excitement and joy made the work worthwhile. However, this structure may not be feasible and sustainable and requires financial support. A limitation to consider for this study is the temporariness of graduate students. A typical time frame for graduate students to complete their degree is 5 to 6 years. The first 2 years consist of intensive coursework and learning of research methods. For the next few years, we work towards transitioning to candidacy and then work on completing their dissertation. There is a need to be strategic about timing should graduate students consider partnership work for their dissertation. However, some relationships cannot be forced to develop quickly.

To conclude, my dissertation study contributes to existing work on critical self-reflection, supporting teacher candidates' learning to center equity in mathematics teaching, and developing collaborative relationships between practitioners and researchers. This study marks a conclusion as well as a beginning for future investigations and collaborations. The work of systematically improving the learning experiences of historically marginalized students from diverse racial, ethnic, linguistic, and cultural backgrounds never stops. Until we can say that race, class, ethnicity, gender, beliefs, and proficiency in the dominant language do not predict children's mathematics achievement and participation as well as their ability to engage in mathematical practices (Gutiérrez, 2007), we will need to continue working towards ensuring all children have equitable learning opportunities.

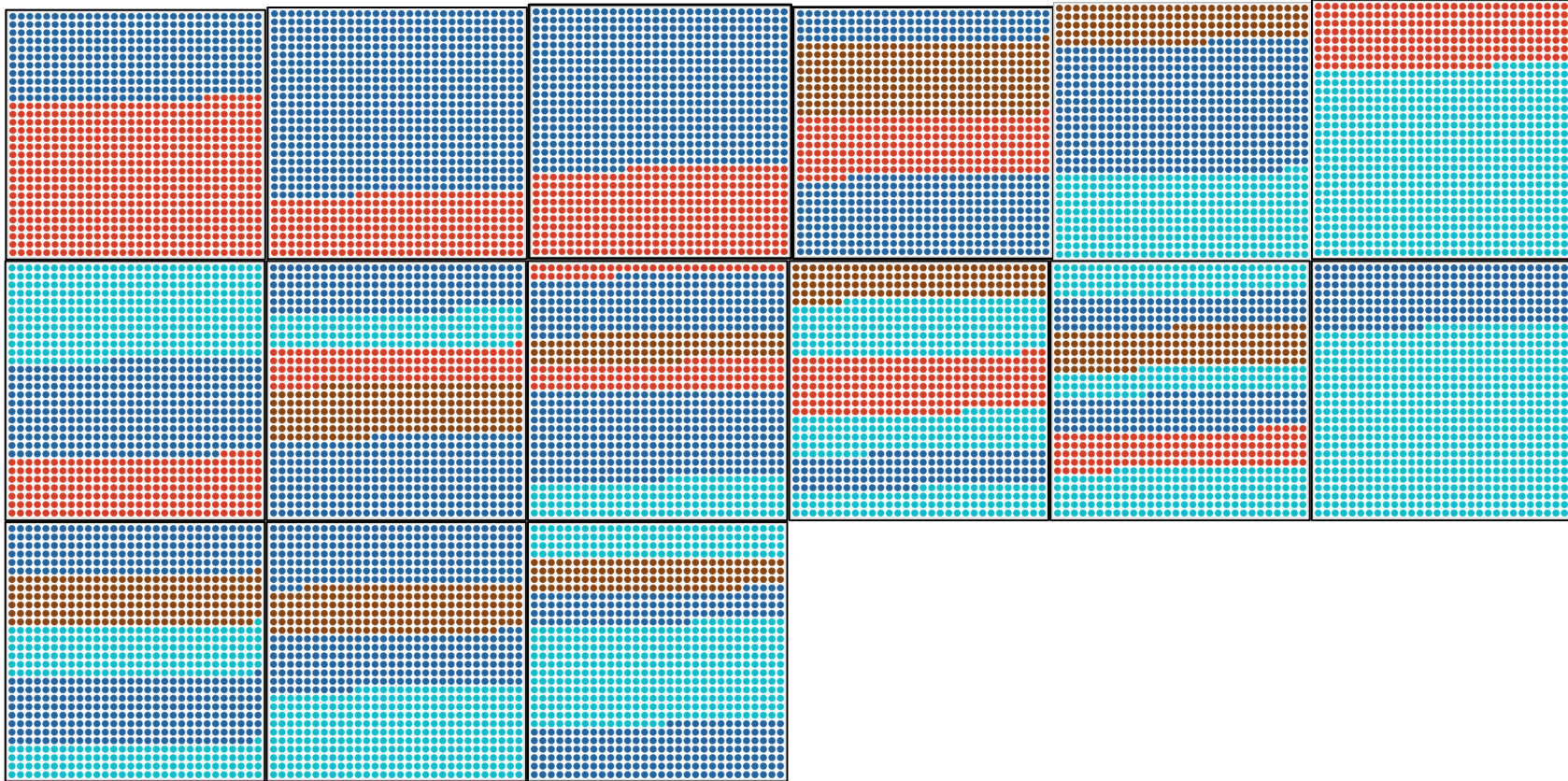
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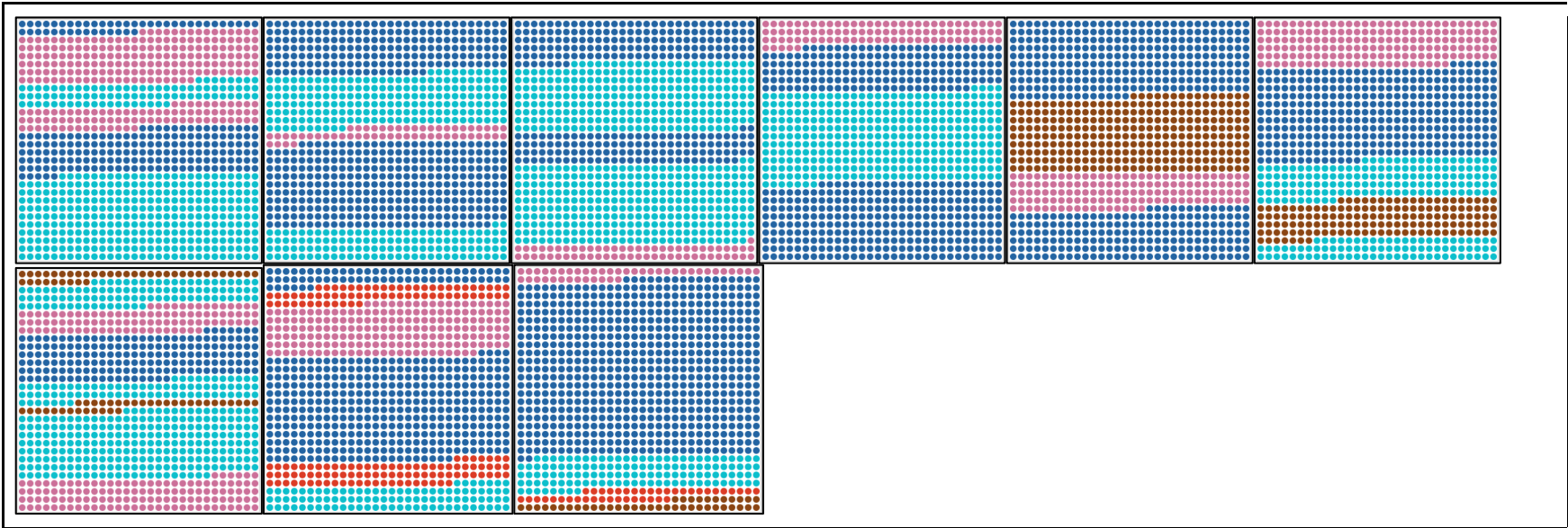
Appendix

Appendix A. Color map representation of candidates' understanding of equity and vision of equitable practices

Pattern 1: Exclusive focus on access



Pattern 2: Integrating an understanding of diverse student identities



Pattern 3: Recognizing power relations in mathematics classrooms

