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RESPONSE OF TRAPPED PARTICLES TO A COLLAPSING DIPOLE MOMENT

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ABSTRACT

Particle motion in the secularly varying geomagnetic field is investigated in terms of a dipolar magnetic field with decreasing magnetic moment, M. For $\dot{M}$ equal to the rate of decay of the earth's dipole component, we find there is drift in B-L space, resulting in an inward drift of particles accompanied with increased energy and unidirectional intensity. Secular variation of the geomagnetic field appears to be a dominant mechanism for radial drift in the inner radiation belt.

RESPONSE OF TRAPPED PARTICLES TO A COLLAPSING DIPOLE MOMENT

In a recent survey of models of the geomagnetic field, Cain [1971] pointed out that the decay rate of the effective dipole moment of the field has increased dramatically since 1950. Although the origins of the accelerated secular variation of the dipole component of the earth's field are unknown, the phenomenon is an active demonstration of the mechanism by which gross changes in the field can develop, possibly leading to the eventual reversal of the field. Clearly, such dynamic secular changes in the geomagnetic field will introduce equally dynamic changes in the structure of the Van Allen radiation zones.

In Figure 1, we have plotted the rates of change of the dipole moment as estimated from magnetic observatory and satellite data, as given by Cain [1971]. In terms of the spherical harmonic coefficients, g and h, of the earth's field,

$$\dot{M} \cong \dot{H}_0 = [g_1^{\ 0} \dot{g}_1^{\ 0} + g_1^{\ 1} \dot{g}_1^{\ 1} + h_1^{\ 1} \dot{h}_1^{\ 1}] \ [(g_1^{\ 0})^2 + (g_1^{\ 1})^2 + (h_1^{\ 1})^2]^{-1/2},$$

where the dipole moment M is taken to be the equatorial value of the field H_0 for the eccentric dipole. The secular variation of M based on magnetic observatory data [Hurwitz and Fabiano, 1969] and the GSFC and POGO models thus appears to have nearly double since about 1950. The GSFC (12/66) model, which incorporates data from 1900 through 1965.7, gives an average $\dot{M} = -16 \ \gamma/\text{yr}$ $= -16 \times 10^{-5}$ gauss/yr for this period. The POGO models, based on satellite data collected between 1965 and 1968, clearly show an increased rate of dipole collapse, with $\dot{M} = -27 \ \gamma/\text{yr}$.

We wish to show that such changes in the dipole moment result in significant systematic drifts of the particles in both B-L and geographic space, and that these "secular" drifts are particularly important to the inner radiation belt. To estimate the magnitude of the secular change in the B-L coordinate and energy of a particle, we shall consider motion in a dipolar magnetic field under the assumption that the adiabatic invariants of particle motion μ, J, and Φ, are conserved.

From the conservation of the third (flux) invariant, Φ, for a dipole field,

$$\Phi = \oint \underset{\sim}{A} \cdot d\underset{\sim}{l} = \oint \frac{M}{L^2} \, L \, d\phi,$$

we find that

$$\Phi = \frac{2\pi M}{L},$$

where L is the equatorial distance to the field line in units of earth radii, i.e., the McIlwain L coordinate. Thus,

$$\frac{L}{M} = \text{constant}, \tag{1}$$

or, in terms of rates of fractional change,

$$\frac{\dot{L}}{L} = \frac{\dot{M}}{M}. \tag{2}$$

The change in M also changes the magnetic field value, B, along the particle's trajectory. To demonstrate this, consider the dependence of B along a field line versus magnetic latitude, λ,

$$B(\lambda) = \frac{M}{L^3} \, \frac{[1 + 3\sin^2\lambda]^{1/2}}{\cos^6\lambda} = \frac{M}{L^3} \, G(\lambda). \tag{3}$$

By (1), $M^2 B(\lambda) = (\frac{M}{L})^3 G(\lambda) = \text{constant} \times G(\lambda)$.

Thus, for any given λ,

$$M^2 B = \text{constant}, \tag{4}$$

and $$\frac{\dot{B}}{B} = -2 \frac{\dot{M}}{M} . \tag{5}$$

A collapsing dipole thus alters the B and L coordinates of trapped particles, the changes in B and L being given in terms of the change in M by (2) and (5).

We now consider the consequences of the conservation of the first and second adiabatic invariants,

$$\mu = \frac{p^2 \sin^2 \alpha}{B} = \frac{p^2}{B_m} \tag{6}$$

and $$J = 4p \int_0^{s_m} [1 - \frac{B}{B_m}]^{1/2} ds = 4pI. \tag{7}$$

In (6) and (7), p is the particle's momentum, α its pitch angle, and B_m is the value of the field, B, at the particle's mirror point, $\alpha = \pi/2$. Eliminating p from (6) and (7), we obtain

$$\text{constant} = B_m^{1/2} I = \int_0^{s_m} [B_m - B]^{1/2} ds = \int_0^{\lambda_m} [B(\lambda_m) - B(\lambda)]^{1/2} \frac{ds}{d\lambda} d\lambda . \tag{8}$$

Using (3), we have

$$B_m^{1/2} I = \frac{M^{1/2}}{L^{3/2}} \int_0^{\lambda_m} [G(\lambda_m) - G(\lambda)]^{1/2} \frac{ds}{d\lambda} d\lambda = \frac{M^{1/2}}{L^{3/2}} F(\lambda_m) . \tag{9}$$

Denoting quantities at later times by primes, we may write, from (8) and (9),

$$B_m^{1/2} I = B_m'^{1/2} I' = \frac{M'^{1/2}}{L'^{3/2}} \int_0^{\lambda_m'} [G(\lambda_m') - G(\lambda)]^{1/2} \frac{ds'}{d\lambda} d\lambda. \tag{10}$$

Because the differential path length along a field line, $ds/d\lambda$, is

proportional to L, $ds'/d\lambda = (L'/L)(ds/d\lambda)$. With this expression, (10) becomes

$$B_m'^{1/2} I' = (\frac{M'}{L'})^{1/2} \cdot \frac{1}{L} F(\lambda_m').$$

From (1), $M'/L' = M/L$, and

$$B_m'^{1/2} I' = \frac{M^{1/2}}{L^{3/2}} F(\lambda_m').$$ (11)

Comparing (9) and (11), we conclude that

$$\lambda_m' = \lambda_m = \text{constant}.$$ (12)

Furthermore, since $\sin^2\alpha = B/B_m = \text{const} \times G(\lambda)$, we note that the pitch angle α is a function of λ only, independent of changes in M. Hence, the mirror-point latitude, λ_m, and pitch angles $\alpha(\lambda)$ of a particle are invariant during secular changes in the dipole moment. Now, consider μ, Equation 6,

$$\text{constant} = \frac{p^2}{B_m} = \frac{L^3 p^2}{MG(\lambda_m)} = (\frac{L}{M})^3 \frac{(pM)^2}{G(\lambda_m)}.$$

Because L/M and $G(\lambda_m)$ are invariants (Equations 1 and 12) we find that

$$pM = \text{constant}$$ (13)

independent of L and λ_m. This may also be expressed as

$$\frac{\dot{p}}{p} = -\frac{\dot{M}}{M} \quad \text{or} \quad \frac{\dot{E}}{E} = -\frac{2\dot{M}}{M},$$ (14)

where E is the nonrelativistic kinetic energy.

The Liouville Theorem also has something to say about secular change in M:

$$\frac{dN}{p^2 \, dA \, dE \, d\Omega \, dt} = \frac{j}{E} = \text{constant},$$

where j is the differential directional particle intensity. Applying

Equation 14 to Liouville's Theorem, we find that along the particle's
trajectory,

$$\frac{\dot{j}}{j} = -\frac{2\dot{M}}{M} \, . \tag{15}$$

The result of a decaying dipole moment is to increase at the same rela-
tive rates both the particle energy and directional intensity, j, at the
increased energy.

Thus, we find the effect of the collapsing dipole is to produce
an inward radial drift of the trapped particle with an attendant
increase in energy and directional flux. The rate of this inward drift
can be compared with that given by the diffusion coefficient deduced by
Farley, Tomassian, and Walt [1970], who employed the Fokker-Planck
diffusion equation to interpret the structure of inner belt protons in
the equatorial plane. The diffusion coefficient that Farley et al.
find to fit best the unidirectional flux data [Thede, 1969] for protons,
$15 \le E \le 170$ MeV; $1.15 \le L \le 1.7$, is

$$D = 5 \times 10^{-9} L^{10} (R_e)^2/\text{day}.$$

The quantity D is related to D_1, the mean radial displacement of a
particle per unit time by the relation $D_1 = 8 D/L$ [Fälthammar, 1966].
Hence, $D = 4 \times 10^{-8} L^9 R_e/\text{day} = 1.46 \times 10^{-5} L^9 R_e/\text{yr}.$

In Figure 2 we plot $\dot{L}$ and D_1 as a function of equatorial distance L.
Given $\dot{M}$(POGO) $= -27$ γ/yr, we find that inward radial drift $\dot{L}$ owing to
dipole collapse, is equal to the coefficient of radial diffusion, D_1,
at L = 1.67, exceeding D_1 for radial distances less than this value.
At L = 1.25, for example, the secular drift is an order of magnitude

greater than the drift obtained by diffusion theory.

The secular drift in B-L space is therefore an effective mechanism for lowering the mirror points of particles throughout the inner radiation belt. Furthermore, concurrent with this drift in B-L space is the actual lowering in altitude of the entire B-L coordinate system owing to secular change in the geomagnetic field. As shown by Lindstrom and Heckman [1968], the mirror-point trajectories in the vicinity of B = 0.24, L = 1.4, given by the GSFC (12/66) field model, decrease at a rate of about 7 km/yr. To this is added the effect of secular drift in B-L space. At B = 0.24, L = 1.4 we find that the secular drift rates are

$$\dot{L} = L \frac{\dot{M}}{M} = -1.2 \times 10^{-3} \, R_e/yr$$

and

$$\dot{B} = -2B \frac{\dot{M}}{M} = 41 \, \gamma/yr.$$

These changes in B and L result in a decrease in mirror-point altitude at the rate of $\dot{h}_{min}$ = 5 km/yr, independent of the secular motion of a B-L trajectory.

The rate at which the mirror points physically move is therefore 12 km/yr. This is an exceedingly high mirror point drift for inner-belt particles. During a 10-year period, particles at B = 0.22 and L = 1.4, for instance, will have their minimum mirror point altitudes lowered by 120 km, effecting a tenfold increase in the atmospheric density encountered by the particles. Finally, the fractional rates of the increase in B, E, and j are equal, and are, for $\dot{M}$ = -27 γ/yr, given by

$$- 2\frac{\dot{M}}{M} = +1.7 \times 10^{-3}/\text{yr}.$$

Quantities that remain invariant during changes in M, derivable from the discussion above, are L/M, BM^2, pM, λ_m, $\alpha(\lambda)$ and j/p^2. In terms of the gyroradius ρ, the conserved quantity pM can be expressed as ρ/M. This leads us to the interesting observation that, because (a) λ_m and $\alpha(\lambda)$ are constant, and (b) the L-shell geometry of the dipole field and particle trajectory both scale as M^{-1}, the configuration of field lines and trajectories in units L/M and ρ/M, respectively, is static for dipole fields. In this coordinate system, the physical distances satisfy rM = constant, and the (variable) earth's radius is $R_e' = R_e M/M'$.

In summary, then, we have shown that the diminution of the earth's dipole moment leads to significant changes in the B-L and spatial coordinates for geomagnetically trapped particles. The rates of these changes dominate the computed diffusion rate for inner-belt protons at low altitudes. An obvious conclusion from this report is that it is unlikely that a theory of the inner radiation belt can be developed under the assumption of a static geomagnetic field.
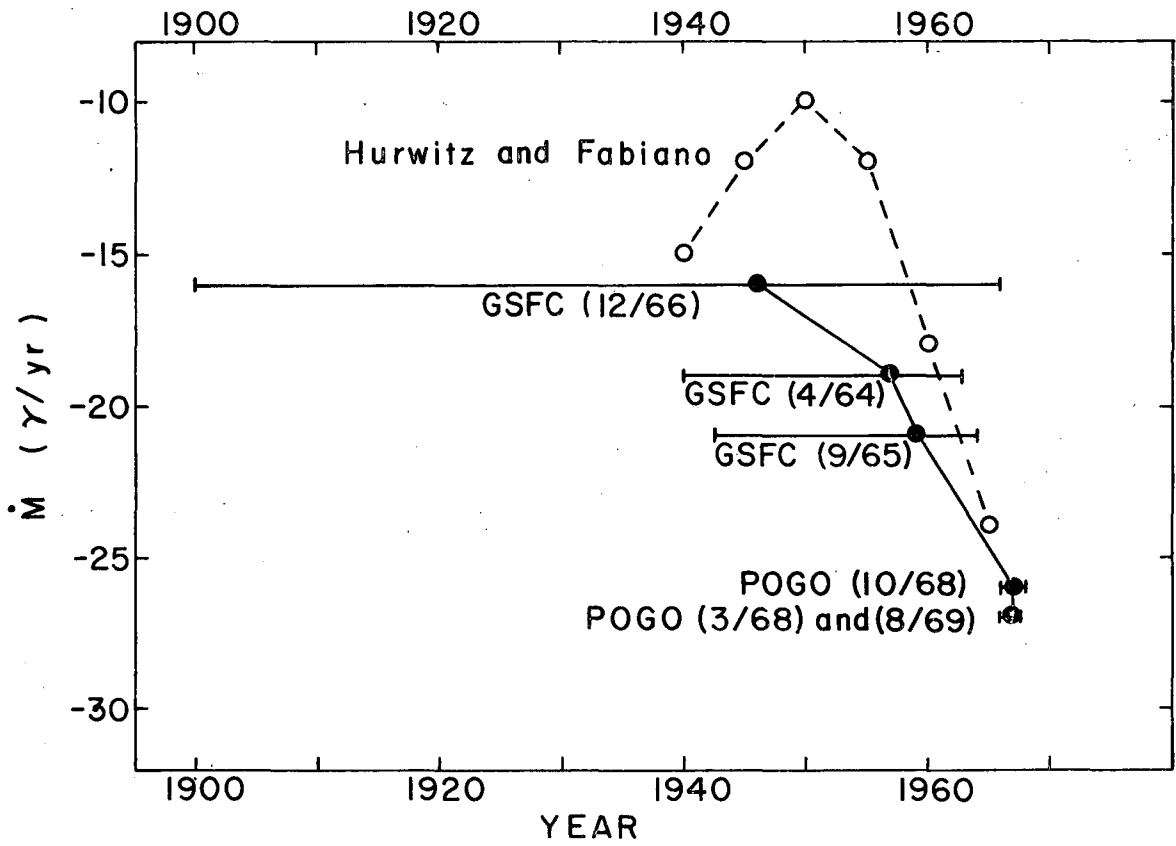
ACKNOWLEDGMENTS

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   1969 (unpublished).

FIGURE CAPTIONS

Fig. 1. Rates of change of the earth's magnetic dipole moment from

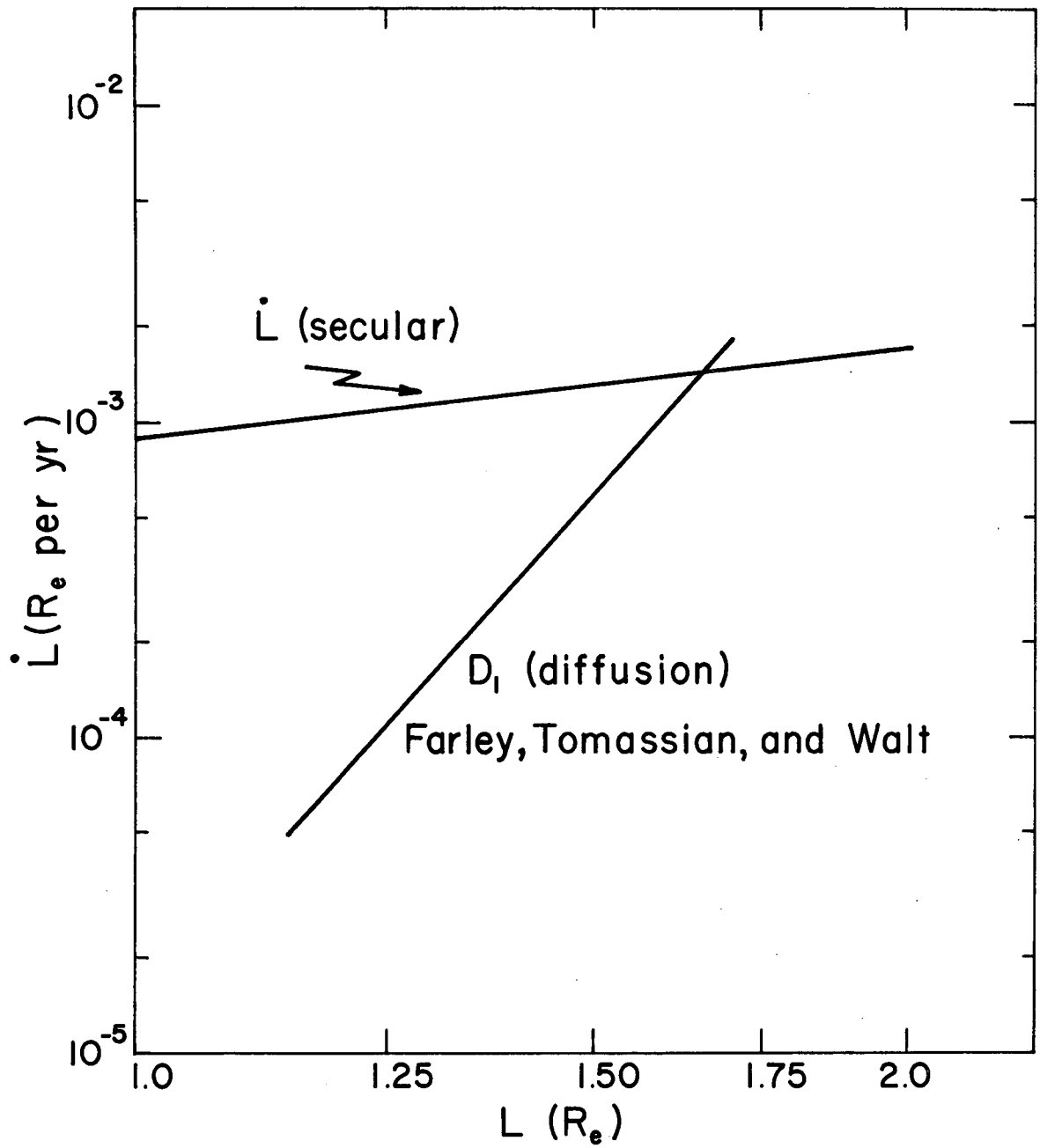magnetic observatory and satellite data.

Fig. 2. Secular drift rate in L coordinate, given by Equation 2.

The diffusion rate $D_1 = 8\ D/L$, where the diffusion coefficient

$D = 5 \times 10^{-9}\ L^{10}\ R_e^2/d^{-1}$ is from **Farley, Tomassian, and Walt**

[1970] for the region bounded by $1.15 \leqq L \leqq 1.7$.

Fig. 1

XBL 717-1195

$\dot{L}$ (secular)

D_1 (diffusion)

Farley, Tomassian, and Walt

Fig. 2

XBL 717-1194