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High Frequency Green's Function for Tapered Planar Arrays

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I. INTRODUCTION

The array Green's function (AGF) is the basic building block for the full-wave analysis of planar phased array antennas. Its representation in terms of element-by-element summation over the individual dipole radiations can be replaced by a more efficient global representation constructed via Poisson summation. The resulting Poisson-transformed integrals can be interpreted as the radiation from continuous equivalent Floquet wave (FW)-matched source distributions extending over the array aperture [1], [2]. Applying high-frequency asymptotics to each FW-matched array aperture casts the AGF in the format of a generalized Geometrical Theory of Diffraction (GTD) which includes conical wavefront edge diffracted rays as well as spherical wavefront vertex diffracted rays. In this paper, the results in [1], valid for equiamplitude excitation, are extended to accommodate tapered illumination, which also includes dipole amplitudes tending to zero at the edges. This extension, which has been performed in [3] with a pure numerical technique based on the discrete Fourier transform (DFT), is herein carried out by a direct Poisson-transformed asymptotic evaluation of the strip-array GF, with the inclusion of asymptotically subdominant "slope" diffracted fields, in addition to the dominant diffracted fields for appreciable edge illumination. Comparisons between the hybrid DFT-(Floquet ray) algorithm of [3] and the present formulation has been shown in [4] for a tapered strip-array of dipoles. In this paper the examples are more realistic antenna modelings of those presented in [4].

II. FORMULATION

We consider a strip periodic array of linearly phased dipoles located in the x,z-plane (Fig.1a). The array is infinite in the z direction and finite in the x direction, with interelement spatial period along the x and z directions given by d_x and d_z, respectively, and interelement phase gradient γ_x and γ_z, respectively. All dipoles are oriented along the unit vector $\hat{\mathbf{J}}_0$ (a bold character denotes a vector quantity, and a caret $\hat{}$ denotes a unit vector). Superimposed upon that background is a x-dependent amplitude tapering function $f(x)$, sampled at the dipole locations, $J(nd_x, md_z) = f(nd_x) \exp(j(\gamma_x nd_x + \gamma_z md_z))$, with $J(x', z')$ denoting the dipole current amplitude, and $(x', z') = (nd_x, md_z)$ denoting the location of (m, n)th dipole (the time dependence $\exp(j\omega t)$ is suppressed). Without compromising practical utility, we assume $f(x)$ real and positive in the domain $x \in [0, L]$, where $L = (N_x - 1)d_x$ is the dimension of the strip array with N_x dipoles (Fig.1a). The electromagnetic vector field at any observation point $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ can be derived from the $\hat{\mathbf{J}}_0$-directed vector potential $A(\mathbf{r})$ by summing over the individual n, m dipole radiations $g(\mathbf{r}; nd_x, md_z)f(nd_x)e^{j(\gamma_x nd_x + \gamma_z md_z)}$ where, $g(\mathbf{r}; nd_x, md_z) = \exp(-jkR)/(4\pi R)$, with $R = |\mathbf{r} - nd_x\hat{\mathbf{x}} - md_z\hat{\mathbf{z}}|$, is the free-space scalar Green's function. We employ the k_x, k_z spectral Fourier representation of the free space Green's function $g(\mathbf{r}; x', z')$ as shown in [1]. Then, the m-series is summed into closed form via the infinite Poisson sum formula which reduces the k_z integration to a q-series $A(\mathbf{r}) = \sum_{q=-\infty}^{\infty} A_q$, with

$$A_q(\mathbf{r}) = \frac{e^{-jk_{zq}z}}{4\pi j d_z} \int_{-\infty}^{\infty} I(k_x) \frac{e^{-j(k_x x + k_{yq}y)}}{k_{yq}} dk_x, \quad I(k_x) = \sum_{n=0}^{N_x-1} e^{j(k_x-\gamma_x)nd_x} f(nd_x), \quad (1)$$
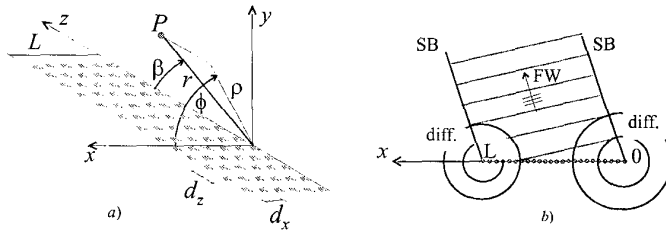
Fig. 1. (a) Geometry of the strip array; (b) Transverse $x-y$ view. Shadow boundaries (SBs) truncates the domain of existence of FWs

where $k_{zq} = \gamma_z + 2\pi q/d_z$ is the Floquet wave (FW) wavenumber in the z-direction , and the branch of $k_{yq} = \sqrt{k^2 - k_x^2 - k_{zq}^2}$ is chosen such that $\Im m\, k_{yq} < 0$ on the top Riemann sheet of the k_x-plane. The n-sum $I(k_x)$ in (1) is manipulated via the truncated Poisson sum formula into a p-sum of Fourier transformed f-functions, translated by the FW wavenumbers in the x direction, $k_{xp} = \gamma_x + 2\pi p/d_x$,

$$I(k_x) = \frac{f(0)}{2} + \frac{f(L)}{2} + \frac{1}{d_x} \sum_{p=-\infty}^{\infty} \widetilde{f}(k_x - k_{xp}), \quad \widetilde{f}(k_x') = \int_0^L e^{jxk_x'} f(x)dx. \tag{2}$$

III. HIGH-FREQUENCY SOLUTION FOR SLOWLY VARYING $f(x)$

Henceforth, we assume (legitimately for actual tapering functions for large arrays) that $f(x)$ varies slowly with respect to the wavelength λ. Thus, adiabatic methods can be applied, based on perturbation about $f(x) = const.$, which is discussed first.

Equiamplitude excitation. Now, the n-series $I(k_x)$ in (1),(2) is evaluated in closed form as $I(k_x) = B(k_x)(1 - e^{-j(k_x-\gamma_x)L})$ which has no singularities, although $B(k_x) = [1 - exp(jd_x(k_x - \gamma_x)]^{-1}$ has poles at $k_x = k_{xp}$. The semi-infinte array treated in [1] has $I(k_x) = B(k_x)$, which is also obtained from (2) when $N_x \to \infty$. The strip array Green's function can be synthesized from the semi-infinite AGF by omitting the dipole contributions from N_x to ∞; i.e., by subtracting the AGF of a semi-infinite array with spectral shift $exp(-j(k_x - \gamma_x)L)$ which corresponds to a space translation. For the semi-infinite array, a uniform asymptotic evaluation of (1) is carried out [1] via deformation of the original integration contour into steepest descent paths (SDP) through the saddle points of the phase in the integrand, with extraction of the residues at intercepted poles [1].

Weakly tapered excitation. When $f(x)$ is weakly tapered and positive real in the domain $x \in (0, L)$, the spectrum of $\widetilde{f}(k_x')$ in (2) is localized around $k_x' = 0$, thereby enhancing contributions to $I(k_x)$ from $k_x = k_{xp}$, $p = 0, \pm 1, ...$ Consequently, the integral in (1) for A_q is dominated asymptotically by $a)$ saddle points (SPs) at $k_x = k_{x,s}$, that satisfy $d/dk_x(k_x x + k_{yq}y)|_{k_{x,s}} = 0$; $b)$ spectral points $k_x = k_{xp}$, that possess the same phenomenology and localization property as the poles for the semi-infinite array [1], and are therefore called "quasi poles". Uniform evaluation is necessary when a saddle point $k_{x,s}$ approaches one of the k_{xp} quasi poles ($p = 0, \pm 1, \pm 2, ...$). The asymptotic evaluation of A_q is addressed by initially assuming that every k_{xp} is "far enough" from $k_{x,s}$ to validate nonuniform asymptotics. Alternatively, we assume $k_{x,s}$ close to k_{xp}, and obtain a *locally uniform* asymptotic solution, which has been demonstrated to patch onto the non uniform solution far from the shadow boundary.

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Floquet Wave Contributions. Inserting (2) into (1), the contributions due to the critical points at $k_x = k_{xp}$ are found by expanding the exponent of the integrand in Taylor series in a neighborhood of $k_x = k_{xp}$ (see [4]). Retaining only the dominant asymptotic term of the remaining integral, yields

$$A'_q(\mathbf{r}) \sim \sum_p A^{FW}_{pq} f(x_{pq}) U(x_{pq}) U(L - x_{pq}) + \dots, \quad A^{FW}_{pq} = \frac{e^{-j(k_{xp}x + k_{ypq}y + k_{zq}z)}}{2jd_x d_z k_{ypq}}, \quad (3)$$

where $k_{ypq} = \sqrt{k^2 - k_{xp}^2 - k_{zq}^2}$ (branches chosen according to (1)), and U is the Heaviside unit step function ($U(x) = 1$ or 0 if $x > 0$ or $x < 0$, respectively). Criteria for the asymptotic validity of the expansion obtained in (3) will be given elsewhere. In (3), A^{FW}_{pq} is the pqth FW of the equiamplitude excitation [1] which is multiplied in (3) by the tapering function $f(x_{pq})$ evaluated at the footprint x_{pq} of the pqth FW. We note that stationary phase evaluation, as in [5], of the radiation integral associated with each p, qth equivalent FW-matched aperture distribution would provide the same result, and in this case, x_{pq} would have been the stationary phase point of the p, qth spatial radiation integral. Limiting the sum $\sum_p$ to the propagating contributions, x_{pq} is real (because k_{ypq} is real) and the existence condition $U(x_{pq})U(L - x_{pq})$ is automatically imposed since $f(x_{pq}) = 0$ for $x_{pq} < 0$ and $x_{pq} > L$. In the ϕ-angular domain (ϕ is the observation angle, see Fig.1a) $U(x_{pq}) = U(\phi_{pq} - \phi)$, where for propagating FWs, $\phi^{SB}_{pq} = \phi_{pq} = \cos^{-1}(k_{xp}/k_{\rho q})$ is the pqth shadow boundary (SB) plane angle that truncates the ϕ-domain of existence of the FW$_{pq}$ (see Fig. 1b). The discontinuity of the truncated FW is repaired by the diffracted field that arises from the saddle point evaluation of (1).

FW-modulated diffracted field. It will be convenient to find an asymptotic expansion of $I(k_x)$ that highlights the behavior of $f(x)$ at the truncations $x = 0$ and $x = L$. For simplicity we will consider only the end point at $x = 0$. To this end, the FT expression in (2) is inserted into the p-series in $I(k_x)$; then, the integration is performed by parts. Using the identities $B(k_x) = [1 - \exp(jd_x(k_x - \gamma_x))]^{-1} = \frac{1}{2} + \frac{1}{d_x}\sum_{p=-\infty}^{\infty}[-j(k_x - k_{xp})]^{-1}$, and $jB'(k_x) = \frac{1}{d_x}\sum_{p=-\infty}^{\infty}[-j(k_x - k_{xp})]^{-2}$, we have

$$I(k_x) \sim f(0)B(k_x) + jf'(0)B'(k_x) + O((k_x - k_{xp})^{-3}). \quad (4)$$

Similar considerations apply to the truncation at $x = L$ after inclusion of the phase term $\exp(-j(k_x - \gamma_x)L)$. Diffracted fields arising from the truncation at $x = 0$ are obtained from the SDP uniform evaluation at the saddle point $k_x = k_{xs} = xk_{\rho q}/\rho$ of the integral in (1) together with (4). Thus, the total high-frequency solution is

$$A^{d,0}_q(\mathbf{r}) = \frac{e^{-j(k_{\rho q}\rho + k_{zq}z)}}{2d_z\sqrt{2\pi j\rho k_{\rho,q}}} \left[f(0)B(k_{x,s})F(\delta^2_{pq}) + jf'(0)B'(k_{x,s})F_s(\delta^2_{pq})\right], \quad (5)$$

were $F(x)$ is the standard UTD transition function, and $F_s(x) = 2jx[1 - F(x)]$ is the slope UTD transition function with argument $\delta_{pq} = (2k_{\rho q}\rho)^{-1/2}\sin((\phi - \phi_{pq})/2)$. It can be shown that when the nondimensional parameter $\delta^2_{pq} \gg 1$, $F \to 1$ and $F_s \to 1$, and the locally uniform diffracted field A^d_q tends to the non uniform result which can be obtained by a straightforward saddle point evaluation of (1) with (4). $A^{d,0}_q$ is a conical wave decaying along ρ, discontinuous at the SBs planes. The dominant asymptotic term (the first in (5)) is the same as that for the uniform case [1], except for multiplication by the tapering function evaluated at the edge. The second contribution is of higher asymptotic order since $B(k_x) \equiv \frac{1}{2} + O(k_x^{-1})$ and $B'(k_x) \equiv O(k_x^{-2})$.
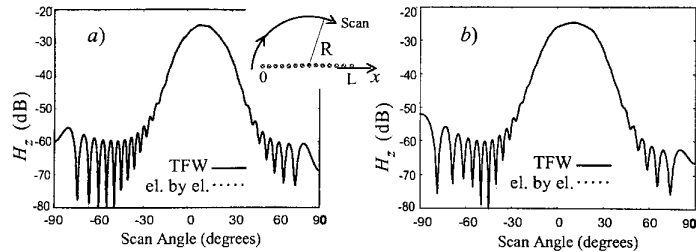
Fig. 2. H_z component of the magnetic field radiated by a slot array antenna, at a distance $R = 20\lambda$ from the center of the array ($L = 17\lambda$). (a) Gaussian excitation; (b) Sine excitation

IV. APPLICATION TO ACTUAL ANTENNAS

In order to show the applicability of the above method to actual cases, let us consider a strip-array of z-directed resonant slots on an infinite ground plane. The test array (24 elements along x and 600 along z, $d_x = d_z = 0.7\lambda$, $\gamma_x = 1.1\lambda^{-1}$, i.e., beam tilted of 10° in the E-plane) is chosen to simulate typical dimensions of a X-band synthetic aperture radar (SAR) antenna (12x0.6 m at 10GHz). For such dimensions, contributions from x-directed edges and corners are negligible. The results are calculated via the truncated Floquet wave (TFW) asymptotics in (3) and (5), with inclusion of the diffracted field arising from the truncation at $x = L$, and compared with a reference solution obtained by an element-by-element summation over the radiation due to each slot. In order to model the element pattern more realistically, in contrast to [4], the AGF here is multiplied by the spectral element factor, as is typically done in the far field regime by the pattern multiplication law. To this end, defining as $\tilde{J}(k_x, k_z)$ the element current transformed in the wavenumber domain, the pqth FW contribution in (3) is multiplied by $\tilde{J}(k_{xp}, k_{zq})$ while the $q$the diffracted field in (5) is multiplied by $\tilde{J}(k_{x,s}, k_{zq})$. Two cases of $f(x)$ tapering functions are considered: a Gaussian profile $f(x) = \exp(-0.9[(x - L/2)/L]^2)$ with 10% edge illumination (Fig. 2a), and a sine profile $f(x) = \sin(\pi x/L)$ (Fig.2b). For this latter case, $f(0) = f(L) = 0$; thus the diffracted field in (5) is given only by the term with $f'(0) \neq 0$, therefore representing a good test case for the additional "slope diffracted field". The electromagnetic quantities are evaluated from the vector potential $A(\mathbf{r})$ as in [1]; only the H_z component is shown in Fig.2 along a scan at $R = 17\lambda$ from the center of the array ($x = L/2$). Element-by-element and asymptotic solutions are not distinguishable on the scale of the plot.

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