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The least action principle is used to determine the semiclassical path in the penetration of a multidimensional fission barrier using the gap parameter as a dynamical variable. An increase in the gap parameter, the larger the deeper the penetration of the barrier is predicted.

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The standard pairing formalism generates a Hamiltonian, diagonal in quasi-particle space, whose expectation value can be expressed as follows:

$$\langle H \rangle = \langle H(n_k,\Delta) \rangle = 2\sum v_k^2 (\varepsilon_k-\lambda) + 2\sum n_k (\varepsilon-\lambda)(u_k^2-v_k^2) - G\left[\sum u_k v_k (1-2n_k)\right]^2 \quad (1)$$

where the n_k are the quasi-particles occupation numbers, Δ is the gap parameter which expresses the diffuseness of the Fermi surface due to the residual interaction, $\varepsilon_k = \varepsilon_k(\alpha)$ are the single particle levels, λ is the chemical potential, $v_k^2 = 1 - u_k^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_k - \lambda}{E_k}\right)$ and $E_k = [(\varepsilon_k - \lambda)^2 + \Delta^2]^{\frac{1}{2}}$. The gap parameter is usually determined by the condition:

$$\frac{\partial \langle H \rangle}{\partial \Delta} = 0 \quad , \quad (2)$$

which is called the gap equation. Such a condition, which requires $\langle H \rangle$ to be stationary with respect to Δ, is relevant to physical situations where the static, or equilibrium properties of a system are to be determined.

This is not the case when the quantum-mechanical penetration of a fission barrier is considered. As was pointed out by Pauli and Ledergerber [1], the penetration of a multidimensional barrier implies a dynamical motion whose semiclassical trajectory must be determined not by considering the potential energy only, but by applying a dynamical principle such as the least action principle.

It follows that, the application of the gap equation (2) to the problem of barrier pentrability is not appropriate and a new gap equation, defining the gap parameter in the framework of the more general dynamical problem, should be obtained.

The action integral for the problem of barrier penetration is:

$$S = \int_a^b \sqrt{2B(V-E)}\ d\alpha \qquad (3)$$

where α is the deformation coordinate, a and b are classical turning points, B is the inertia associated with the coordinate α, V is the potential energy and E is the total energy. Such an expression depends upon the gap parameter Δ through both the inertia B and the potential energy V.

The potential energy V can be identified with the expectation value of the pairing Hamiltonian expressed as a function both of the deformation α and of the gap parameter Δ:

$$V \equiv \langle H \rangle = 2 \sum v_k^2 (\varepsilon_k' - \lambda) - G \left[\sum u_k v_k \right]^2 \quad . \qquad (4)$$

The dependence of $\langle H \rangle$ upon Δ and α is now to be determined. In order to approximate the dependence of $\langle H \rangle$ upon Δ it is useful to calculate the second derivative of $\langle H \rangle$ with respect to Δ when $\Delta = \Delta_o$, Δ_o being the stationary value of Δ:

$$\left.\frac{\partial \langle H \rangle}{\partial \Delta^2}\right|_{\Delta=\Delta_o} = \Delta_o^2 \sum E_k^{-3} - \frac{1}{2}\Delta_o^4 G \left[\sum E_k^{-3}\right]^2 \quad . \tag{5}$$

If the single particle Hamiltonian is approximated by the uniform model, one obtains:

$$\left.\frac{\partial^2 \langle H \rangle}{\partial \Delta^2}\right|_{\Delta=\Delta_o} = 2g\left(1 - \frac{gG}{2}\right) \simeq 2g \tag{6}$$

where g is the total density of the doubly degenerate signle particle levels inclusive of neutrons and protons.

The potential energy $V(\alpha)$ can be expressed in quadratic approximation as:

$$V(\alpha) = V_o(\alpha) + g(\Delta - \Delta_o)^2 \tag{7}$$

where $V_o(\alpha)$ is the "shape" of the barrier which corresponds to a value of Δ equal to its stationary value Δ_o. Similarly the inertia, which can be obtained by the cranking model, is given, for the uniform model, by the expression:

$$B = \hbar^2 \left\langle \left(\frac{\partial \varepsilon_k}{\partial \alpha}\right)^2 \right\rangle \frac{g}{3\Delta^2} \sim \frac{K}{\Delta^2} \quad . \tag{8}$$

In order to correct the limiting form of this equation at large Δ, the following expression can be postulated:

$$B = \frac{K}{\Delta^2} + \beta \tag{9}$$

where β is the irrotational limit of the inertia.

By substituting (9) and (7) in (3) one obtains:

$$S = \int_a^b d\alpha \sqrt{2\left(\frac{K}{\Delta^2} + \beta\right)\left\{V_o(\alpha) - E + g(\Delta - \Delta_o)^2\right\}} \quad . \tag{10}$$

The least action principle requires that the integral S be an extremum, namely $\delta S=0$. In other words a function $\Delta(\alpha)$ must be found which minimizes the integral (10).

Since eq. (10) does not contain the derivative of Δ with respect to α, the variational condition reduces to the algebraic equation:

$$\frac{d}{d\Delta}\left[\left(\frac{K}{\Delta^2} + \beta\right)\left\{V_o(\alpha) - E + g(\Delta - \Delta_o)^2\right\}\right] = 0 \quad . \tag{11}$$

This equation is the gap equation relevant to the dynamical problem. By setting $\beta = 0$, a very simple expression is obtained:

$$\frac{\Delta}{\Delta_o} = 1 + \frac{V_o(\alpha) - E}{g\,\Delta_o^2} \quad . \tag{12}$$

Before penetrating the barrier, at the classical turning point given by the equation $V_o(\alpha) - E = 0$, the gap parameter is $\Delta = \Delta_o$: in other words the solution for Δ is the same as that given by eq. (2). As the system dives into the barrier, the least action principle tends to decrease the inertia by increasing Δ. The gap parameter is prevented from increasing indefinitely by the restoring force originated by the potential energy (7).

Since in order to obtain eq. (12) β has been set equal to zero, the gap parameter given by (12) is somewhat overestimated. Still, one can see that the effect is indeed very large. By using the following round numbers: $g = 7\ \text{MeV}^{-1}$, $\Delta_o = 1$ MeV, and $V_o(\varepsilon) - E = 7$ MeV, one obtains $\Delta = 2\Delta_o$. The effect of such a pairing increase can be incorporated into an effective potential V^*:

$$V^*(\alpha) - E = \frac{V_o(\alpha) - E}{1 + \dfrac{V_o(\alpha) - E}{g\,\Delta_o^2}} \quad . \tag{13}$$

Again, by using the above mentioned parameters, it appears that, the deeper the system dives into the barrier, the more the effective barrier is reduced: in particular by using the numerical values of the parameters mentioned above, the height of the effective barrier is reduced by a factor of two with respect to the true barrier. Again, although eq. (13) is overestimating the effect, it is clear that such a dramatic reduction of the barrier must have a substantial effect on the spontaneous fission half-lives.

In fig. 1, a plot of $\frac{\Delta}{\Delta_o}$ is shown as a function of α. This calculation applied to the uniform model, has been performed by substituting eq. (9) and eq. (4) into eq. (3), and the following parameters have been used: $B = 1666 \frac{\Delta_o^2}{\Delta^2} + 225\, \hbar^2\ \mathrm{MeV}^{-1}$, $\Delta_o = 0.775$ MeV, $V = 6 + \frac{1}{2}\, 269\ (\alpha - \alpha_o)^2$ MeV and $E = 0$. These quantities are expected to be realistic for an actinide nucleus. In fig. 2, the effective potential energy is shown as a function of deformation. The overall features are still the same as those estimated by eq. (12) and eq. (13).

A limitation of the above treatment is related to the fact that Δ should be considered as a true dynamical variable instead of a simple parameter. In other words, one should account for the kinetic energy associated with Δ as well as for the potential energy. Consequently, the inertia becomes a tensor of the form:

$$B = \begin{vmatrix} B_{\alpha\alpha} & B_{\alpha\Delta} \\ B_{\Delta\alpha} & B_{\Delta\Delta} \end{vmatrix} \tag{14}$$

and the action becomes:

$$S = \int \sqrt{2 \left[B_{\alpha\alpha} + 2B_{\alpha\Delta} \frac{d\Delta}{d\alpha} + B_{\Delta\Delta} \left(\frac{d\Delta}{d\alpha}\right)^2\right] [V_o(\alpha) - E + g(\Delta - \Delta_o)^2]}\ d\alpha \quad . \tag{15}$$

For the uniform model:

$$B_{\alpha\Delta} \simeq 0 \qquad\qquad B_{\Delta\Delta} = \hbar^2 \frac{g}{6\,\Delta^2} \quad .$$

In this case the variational equation $\delta S = 0$ becomes a rather complicated differential equation. Fortunately, it turns out that $B_{\Delta\Delta} \left(\frac{d\Delta}{d\alpha}\right)^2 << B_{\alpha\alpha}$, as can be checked by using eq. (12) to calculate $\frac{d\Delta}{d\alpha}$. Furthermore, a substitution of eq. (12) into eq. (15) produces a negligible difference in the action as compared with that calculated by means of eq. (10). Therefore, it is concluded that eq. (12) and eq. (13) as well as the numerical results presented in figs. 1 and 2 should be reasonably accurate.

The problem of the barrier pentrability needs to be further pursued by means of more realistic models. Some indication of the effect discussed above may exist in low energy induced fission. Fission fragment angular distributions, measured very close to the barrier of the compound nuclei ^{210}Po and ^{211}Po have led to the tentative conclusion of anomalously large values of the gap parameter at the saddle point [2]. Such conclusions, however, seem to be incompatible with higher energy fission excitation function [2]. This effect could find a possible justification along the following lines. The penetration into higher levels in induced fission is expected to depend upon the rate of increase of the level density. This rate is substantial at energies close to the top of the barrier thus leading to a larger value of the gap parameter. At higher excitation energies the rate of increase of the level density diminishes thus reducing the penetration and accounting for a normal excitation function.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

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2. L. G. Moretto, R. C. Gatti, S. G. Thompson, J. R. Huizenga and J. O. Rasmussen, Phys. Rev. 178 (1969) 1845.

Figure Captions

Fig. 1. Enhancement of the gap parameter in the penetration of the fission barrier.

Fig. 2. Effective fission barrier $V^*(\alpha)$ (thick line) as compared with the true fission barrier $V(\alpha)$ (thin line).

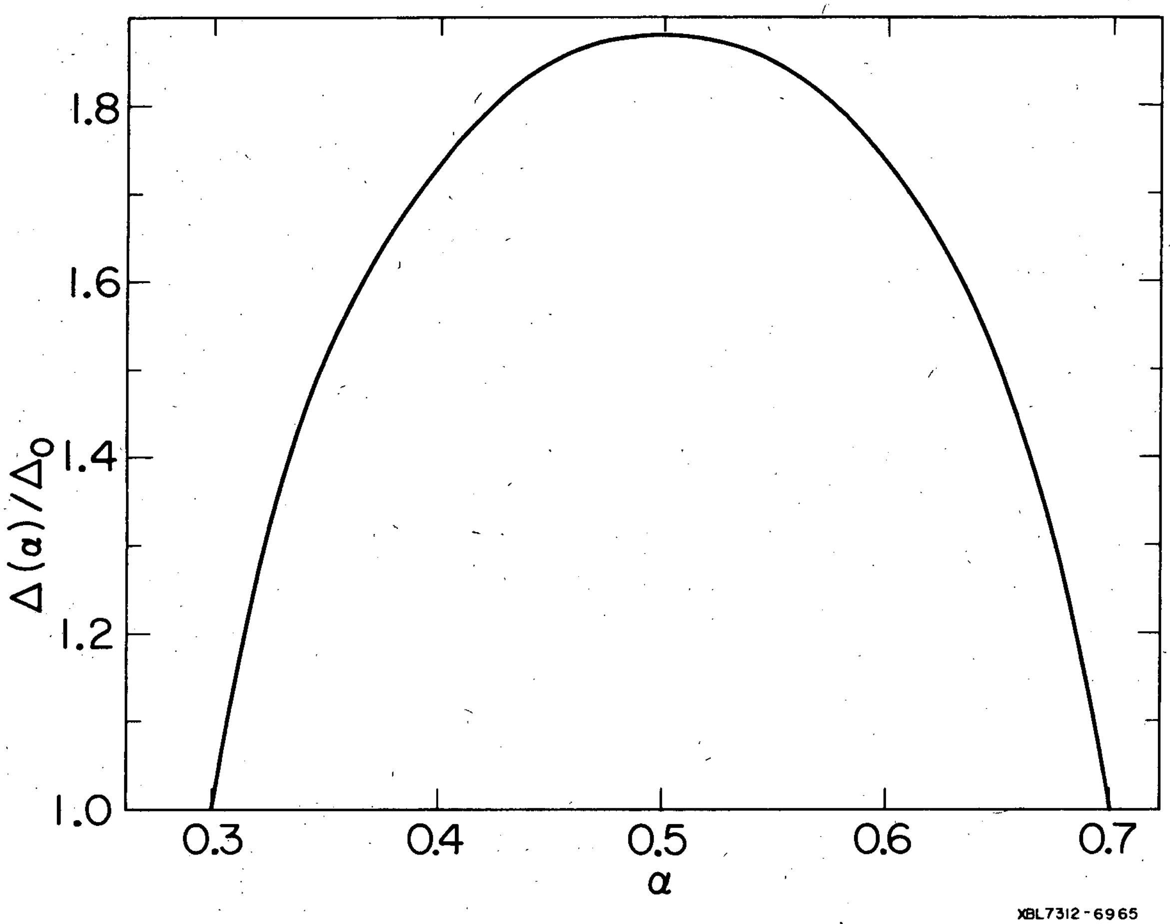

Fig. 1

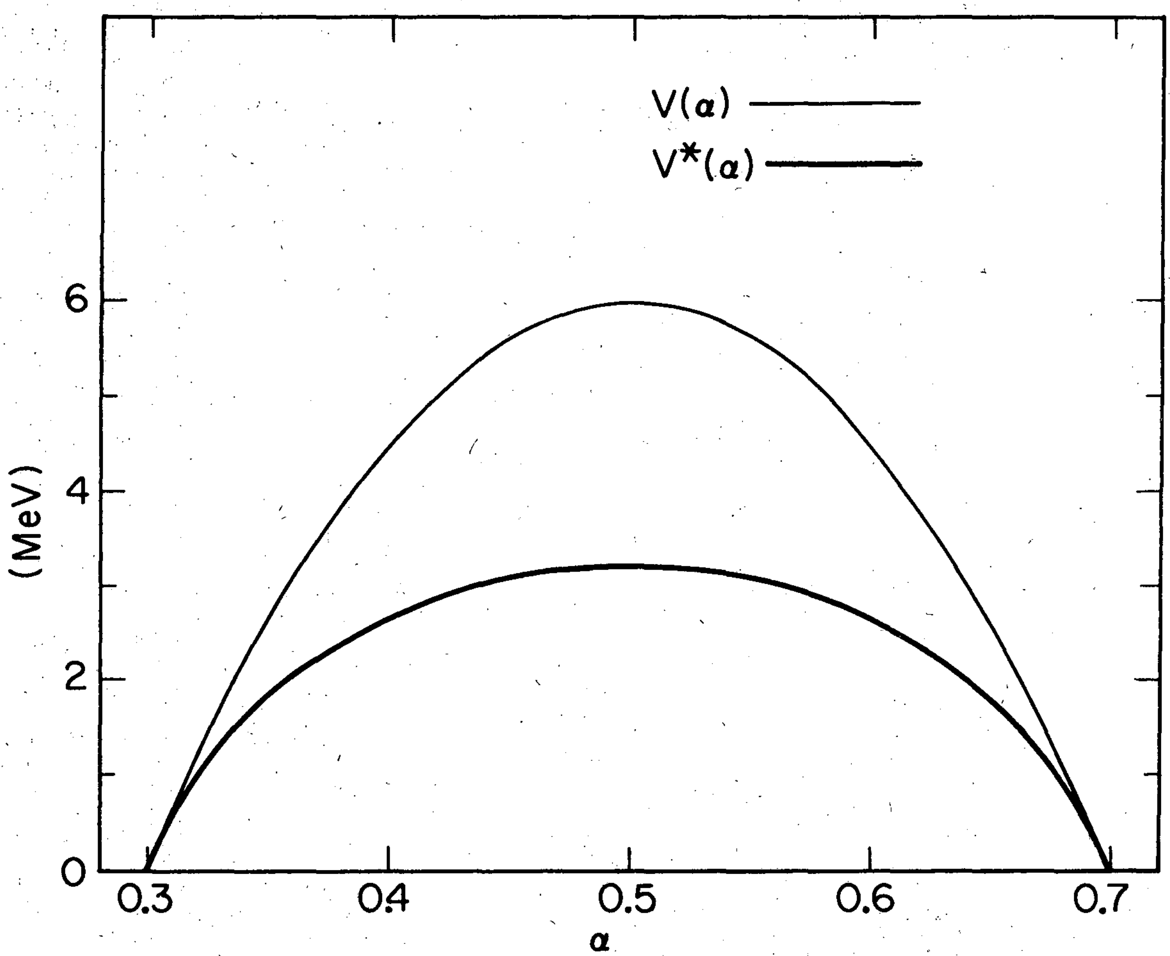

Fig. 2

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