Illusory inferences: disjunctions, indefinites, and the erotetic theory of reasoning

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Abstract

Work in the mental model tradition has shown that human reasoners are subject to fallacious inferences from very simple premises that have been described as tantamount to cognitive illusions (Walsh & Johnson-Laird, 2004; Khemlani & Johnson-Laird, 2009). We present four experiments that show that these phenomena are much more general and systematic than has previously been thought. Among other results, we find that premises using 'some' mirror premises using 'or' in generating fallacious inferences, showing that there are interesting facts about reasoning with quantifiers beyond syllogisms that have been the main focus in the literature. Neither mental model theory nor other familiar theories of reasoning account for the results we present. However, the novel illusory inferences we present are predicted by the erotetic theory of reasoning (Koralus and Mascarenhas, 2013). The key idea is that, by default, we reason by interpreting successive premises as questions and maximally strong answers to those questions, which generates the observed fallacies.

Keywords: illusory inferences; disjunction; quantifiers; erotetic theory; reasoning; mental models; fallacies

Introduction

The capacity for reasoning is central to modern human endeavors. The most widely discussed approaches to understanding this capacity are mental logic (Rips, 1994), Bayesianism (Oaksford & Chater, 2007), and mental model theory (Johnson-Laird, 1983). Each of these approaches has important advantages. Mental logic approaches incorporate the important insight that a fully satisfactory theory of reasoning should be formally precise in the sense that it is possible to calculate predictions of the theory for an unbounded number of possible reasoning problems from a clearly specified set of axioms. Bayesian approaches are a heterogeneous family, and some theorists place themselves broadly within the mental logic or the mental models approach. Nevertheless, they uniformly have the advantage of seeking to account for our reasoning performance in ways that would flow from a simple core idea of what our reasoning capacity is aiming at, e.g. rationally updating a probability distribution in light of evidence. Mental model theory has the advantage of making central the nature of the representations we build as we interpret premises and how those representations can be less detailed than the premise statements in terms of their information content. We have argued elsewhere for a new theory of reasoning that we believe unites those advantages, the erotetic theory of reasoning (henceforth ETR) (Koralus & Mascarenhas, 2013). The core idea behind ETR is the notion that we reason by raising questions and seeking to answer them as directly as possible. This intuitive notion has been given a mathematically rigorous description (Koralus & Mascarenhas, 2013), which we showed captures core existent data on propositional reasoning. Here, we present four new experiments on reasoning documenting novel systematic fallacies, and argue that those results support the erotetic theory over existing approaches.

Illusory inferences from disjunction

Inferences from disjunctive statements involving 'or' may well be the simplest non-trivial cases of reasoning. For example, if we accept that either there is an apple on the table or else an orange, and we further accept that there isn't an orange on the table, we may straightforwardly conclude that there is an apple. Now, we are subject to systematic failures of reasoning with premises that are similarly simple. Walsh and Johnson-Laird (2004) presented participants in an experiment with the following problem:

(1) P_1: Either Jane is kneeling by the fire and she is looking at the TV or otherwise Mark is standing at the window and he is peering into the garden.
P_2: Jane is kneeling by the fire.
Concl.: Jane is looking at the TV.

Only 10% of participants gave correct answers to problems of this form, prompting the authors to call them "illusory inferences." Most participants say that it follows from the two premises that Jane is looking at the TV. However, on reflection, we can see that this is a fallacy. The truth of the premises is compatible with a situation in which Jane is kneeling by the fire but *not* looking at the TV, while Mark is standing by the window and is peering into the garden.[1]

To date, the only systematic accounts of illusory inferences of this kind are offered by the mental model theory and by the erotetic theory of reasoning. The mental-model based explanation Walsh and Johnson-Laird propose for this illusory inference is based on the idea that we build mental models

[1] Notice that the conclusion in (1) is illicit even if we interpret 'or' exclusively (i.e. "*A* or *B* but not both").

of only some of the alternative possibilities compatible with the first premise, notably of the alternatives in which Jane is kneeling and looking, the alternative in which Mark is standing and peering, and the alternative which combines both of the former. Walsh and Johnson-Laird suggest that when we encounter the categorical premise we "match" that premise to the alternatives that are partly co-referential with the second premise in the disjunction. This match is then treated as definitely establishing that one of these alternatives holds. In this case, all envisaged alternatives that "match" Jane kneeling by the fire also include that Jane is looking at the TV, yielding the illusory inference.

ETR proposes a rather different explanation of illusory inferences. The core idea is that a disjunctive premise statement raises the question of which of the disjuncts is the case. Effectively, P_1 is interpreted as akin to the question, "am I in a kneeling and looking situation or in a standing and peering situation?" P_2 is then interpreted as akin to the answer, "you are in a kneeling situation!" Interpreted *strongly*, this answer would lead us to conclude that we are in a kneeling and looking situation. This question/answer-based explanation of the illusory inference pattern does not require co-reference between expressions in the premise statements. Moreover, ETR predicts an effect of premise order that the mental models account does not. If reasoners are being led to errors by their interpreting the disjunctive premise as a question and the simpler premise as an answer, then this tendency should be mitigated by a reversal in the order of the premises. It ought to be somewhat more difficult to apply a question asking/answering strategy to the problem if the (putative) answer occurs first and the question second. By contrast, the mental model matching strategy ought not to be sensitive to order in this sense. In Experiment 1, to be presented shortly, we show that indeed this effect of premise order exists.

From disjunctions to indefinites

One natural way to look at sentences with indefinite expressions as in (2a), schematized in (2b), is as generalized (and theoretically infinitary) disjunctions as in (2c)/(2d).

(2) a. Some student smokes.
b. Some x is an A who also has the B property.
c. John is a student and smokes, or Mary is a student and smokes, or ...
d. x_0 is an A with the B property, or x_1 is an A with the B property, or ...

This perspective on indefinite expressions is more than just a formal logical curiosity. The view that indefinite expressions share substantive properties with disjunction has also been championed as crucial part of accounts of indefinites in several guises from within linguistic semantics.[2] Consequently, it is important to ask whether (something like) illusory inferences from disjunction can be replicated in the domain of indefinite quantification. In Experiments 2 and 3 we show that this is the case.

[2]In particular, the alternative semantics of Kratzer and Shimoyama (2002) and the inquisitive semantics of Ciardelli (2009) and Mascarenhas (2011).

An effect of premise order

Experiment 1

The mental model matching strategy does not intrinsically invoke order. By contrast, we cannot treat something as an answer without having a question first. Thus, the explanation of the illusory inference offered by ETR, unlike the matching-based account, immediately predicts an order effect. In experiment 1, we examined whether illusory inferences from disjunction are mitigated if the order of premises is reversed.

We examined four illusory inference problems and four control problems that were not hypothesized to yield illusory inferences. Both types of problems involved two premises, where the first premise consisted of a disjunction and the second premise consisted of an atomic proposition or a negated atomic proposition. The target and control problems were variants of the following two examples:

(3) **Sample Target Problem**
There is an ace and a queen, or else a king and a ten.
There is a king.
What if anything follows?

(4) **Sample Control Problem**
There is an ace and a king, or else a queen and a jack.
There isn't an ace.
What if anything follows?

We predicted that in the target problem participants should systematically draw the illusory inference that there is a ten in the hand, and so on in similar problems. This should be the case despite the lack of co-reference in the premise statements above; since we are using indefinite expressions like "a king" and "a ten," rather than proper names like "Jane" and "Mark," co-reference is not guaranteed in our examples. We predicted moreover that the incidence of these mistakes in target problems should be far greater than the incidence of invalid inferences in the control problems. Crucially, we predicted that subjects who saw the target problems with premises in reversed order would make significantly fewer mistakes.

Participants and design Two batches of 120 members of the mTurk worker community carried out eight reasoning problems. The first group (average age 35 years, $\sigma = 12$, 62 female, 57 male, 1 declined to state) saw all target and control problems in canonical order; the second group (average age 30, $\sigma = 9$, 75 male, 46 female) saw the same materials but with premises in reverse order. Subjects served as their own controls. The order of presentation was randomized for each subject. In each problem, the premise statements were followed by the question, "what if anything follows?" and a text box to record responses.

Type	Pattern	ILL	INV
Target	aq $\vee$ kx ; k	108(90%)	-
Target	j2 $\vee$ a8 ; j	108(90%)	-
Target	q8 $\vee$ 2a ; 2	113(94%)	-
Target	xk $\vee$ qa ; x	111(93%)	-
Control	ak $\vee$ qj ; $\neg$a	-	15(13%)
Control	j8 $\vee$ ax ; $\neg$j	-	17(14%)
Control	28 $\vee$ kj ; $\neg$k	-	16(13%)
Control	8k $\vee$ qa ; $\neg$q	-	16(13%)

Table 1: Results of experiment 1, canonical order condition. *"a", "j", "k", "q", "2", "8", and "x" stand for the cards ace, jack, king, queen, two, eight, and ten, respectively.*

Procedure The experiment was carried out over the internet using Qualtrics and participants were anonymously recruited and paid through the Amazon mTurk website. Each participant was rewarded with USD 0.25 for their participation. Participants were invited to engage in a study of reasoning in which they had to say what they can conclude from a set of statements. They were asked not to make notes or use search engines while performing the task. Before the target and control questions were presented, each participant was shown two worked-out sample reasoning problems of an unrelated kind using conditionals. All statements were explained to be about a large hand of cards. They were told that the experiment would last approximately 5 minutes but were given as much time as they needed, up to 10 minutes.

Results Participants' written responses were coded as follows into binary categories. For target problems, we coded a response as a '1' in the category of illusory inference (ILL) if and only if at least one illusory inference proposition was a conjunct in the written answer and no other invalid inferences were present. For control problems, we coded a response as invalid (INV) if and only if at least one invalid inference of any sort was present, excepting responses like "nothing follows," "no," and the like. In all cases, we made allowance for the fact that some participants may interpret 'or' as inclusive and some as exclusive.

Both authors of this paper coded the free-form responses into the above categories and discrepancies were resolved by agreement. 97.5% of participants made one or more illusory inferences, while 18.3% made one or more invalid inferences in control problems. We rejected the null hypothesis that illusory inferences were as frequent as invalid inferences in control problems (Wilcoxon Matched-Pairs Test, $T = 117.5$ (sum of negative difference rank), $p < .0001$). We summarize the data for the different target and control problems in canonical order in Table 1 above.

As predicted, fewer illusory inferences were made in the reversed condition than in canonical order. The number of illusory inferences dropped by approximately 10% on average when the premises were reversed. We rejected the null hypothesis that the frequency of illusory inferences was the same for the two premise orders (Mann-Whitney, $z = 2.05477$, $p < 0.038$). The results for the target cases in the reverse-order condition are summarized in Table 2. By contrast, there was no significant effect of premise order for the control problems.

Type	Pattern	ILL
Target	k ; aq $\vee$ kx	97(81%)
Target	j ; j2 $\vee$ a8	101(84%)
Target	2 ; q8 $\vee$ 2a	98(82%)
Target	x ; xk $\vee$ qa	98(82%)

Table 2: Results of experiment 1, reversed order condition. *"a", "j", "k", "q", "2", "8", and "x" stand for the cards ace, jack, king, queen, two, eight, and ten, respectively.*

Discussion Our results show first of all that the pattern of illusory inference is not due to co-reference across premises and also cast some doubt on the idea that representations of spatial configurations are required for such inferences (assuming that card scenarios do not as readily invoke spatial imagery as scenarios of people standing and peering into gardens). The explanation offered by ETR depends on neither co-reference nor spatial models.

More interestingly, the results are consistent with the prediction of ETR that reversing the order of premises would mitigate the illusory inference pattern. They cast doubt on explanations of these kinds of inference patterns involving notions like "matching" that are not relevantly asymmetric.

One could respond on behalf of mental model theory that matching only applies if we already have a set of mental models from a disjunctive premise so that we can match the information of a categorical premise to an element of this set. If we take this avenue, matching simply does not apply in the reversed-order case. However, for reasoning steps in which we are not integrating premises by "matching," mental model theory provides a more general procedure for conjoining mental models (Johnson-Laird, 2008; Khemlani & Johnson-Laird, 2009). The first premise would generate a single mental model, which would have to be conjoined with the set of mental models generated by the second premise. However, the process for conjoining mental models is also designed to make us jump to conclusions in reasoning. As we conjoin the mental model for "there is a king" with the mental models for "there is an ace and a queen, or else a king and a ten," the procedure, as defined by Johnson-Laird and Khemlani, rules out all of those mental models in which we do not have an ace and a queen. Thus, even if we maintain that the mental model matching procedure does not hold in the case in which premises of the illusory inference problem are reversed, the mental model conjoin procedure would then apply to yield the same result. Consequently, both matching and mental model conjoin procedures fail to explain why the

illusory inference is mitigated in the reversed case.[3]

Despite a significant drop in illusory inferences in the reversed-order condition, illusory inferences did not altogether disappear. How does the erotetic theory explain the fact that a significant, though lower, quantity of illusory inferences is still present in the reversed condition? We propose, tentatively for now, that those participants that made illusory inferences in the reversed order condition *overruled* the order of premises given in the experiment, choosing to process premise 2 before premise 1. Our account's default reasoning strategy (on which more in this article's general discussion section) proceeds by taking premises in the order in which they are given, but the theory is compatible with some subjects realizing that nothing new immediately follows from the premises in reversed order, subsequently reparsing the reasoning problem in the canonical order, and thereby getting the illusory inference. We leave a detailed treatment of this issue to future work.

Illusory inferences with quantifiers

Experiments 2 and 3 — indefinites

We tested for indefinite versions of illusory inferences from disjunction with two experiments. In experiment 2, our pilot, we tested the two patterns in (5):

(5) a. Some student smokes.
John is a student.
Does it follow that John smokes?

b. Some pilot writes poems.
John is a pilot.
Does it follow that John writes poems?

In the aggregate, we found that about 30% of subjects made the fallacious conclusion, significantly more than our invalid controls (Wilcoxon matched-pairs test $p < .05$). However, a by-item analysis showed that the effect was only significant for (5b). We hypothesized that subjects' prior expectations about what is typical for the topical individuals of our materials were responsible for the variance. We suspected that subjects' expectations about how unlikely students are to be smokers played a key role in making the corresponding illusory inference seem less attractive. On the erotetic theory as sketched above, it is expected that the ease with which reasoners can conjure up alternative representations for individuals should correlate inversely with the attractiveness of the relevant illusory inferences.

In experiment 3, we factored out entirely subjects' expectations about the topical individuals, by using completely unfamiliar properties in both restrictor and nuclear-scope positions in our materials.

[3]We are open to the possibility that mental model theory could revise or expand the mental-model-building procedure so as to capture our order effect, but as far as we know, no such proposal exists in the literature at the time. We thank an anonymous reviewer for highlighting this.

Participants and design We recruited 977 workers from the mTurk crowdsourcing platform to solve twelve reasoning problems. Statements in each reasoning problem were about biological organisms and properties. We concocted the examples ourselves with no regard for the truth of the sentences or coherence of the statements as statements about biology. This was a legitimate strategy given that we presumed no knowledge of the subject matter. In (6) we give the two content variations we used for canonical targets.

(6) a. Some firmicute produces endospores.
Clostridium is a firmicute.
Does it follow that clostridium produces endospores?

b. Some thermotogum stains gram-negative.
Maritima is a thermotogum.
Does it follow that maritima stains gram-negative?

Of the twelve problems subjects saw, six constituted variations on our target illusory inferences with indefinites, and six were control inferences, three valid and three invalid. Of the six targets, three had their premises in canonical order, three were reversed. Each subject solved all twelve problems, serving as his or her own control, in a randomized order. Subjects were asked not to make use of search engines while answering the questionnaire.

Procedure Subjects were shown one inference at a time, each inference consisting of two premises and a proposed conclusion. They answered whether the conclusion followed from the premises. They were given three answers to choose from: "yes," "no," and "choose not to respond." Subjects were also asked not to make notes or use search engines while performing the task. We also asked in the general demographic questionnaire, after solving all reasoning problems, whether subjects had used any search engines. They were told that the experiment would last approximately 5 minutes but were given 10 minutes to answer the questionnaire.

Results We found that around 40% of subjects committed the predicted fallacy. This was a very significantly higher rate of reasoning error than in our controls not involving indefinites (Wilcoxon matched-pairs test $p < .01$). Contrary to our pilot experiment 2, in experiment 3 we found no effect of item content. We also found a significant order effect on the premises: when the order of the premises in stimuli as in (6) was inverted, we found significantly fewer fallacies for target stimuli (Wilcoxon matched-pairs test $p < .01$). This order effect was not significant in either our valid controls (Wilcoxon matched-pairs test $p > .61$) or our invalid controls (Wilcoxon matched-pairs test $.17 < p < .41$).

Discussion These results fit with our prediction: indefinites give rise to illusory inferences akin to those of disjunction. These fallacious inferences are novel, and crucially do not constitute syllogisms. They lie outside the scope of all extant theories of reasoning with quantifiers that we are acquainted with (see for example the comprehensive literature review by

Khemlani & Johnson-Laird, 2012). Our results also show that the order effect we found for standard illusory inferences from disjunction carries over to indefinites. As before, mental models theory is, as far as we can see, at a loss to explain this order effect, while the erotetic theory derives it immediately.

Experiment 4 — universals

Experiments 2 and 3 showed that there exist robust fallacious inferences with quantifiers beyond syllogisms. We suspected that the phenomenon was not restricted to indefinites. In experiment 4, we made a different "translation" of the standard illusory inference in (1) into the domain of quantification, building on an idea from Mascarenhas (2014). Instead of using indefinites to do the work of disjunction, we used universal quantifiers where (1) has conjunctions. Because the materials in experiment 4 contained overt disjunctions, we predicted that the number of alternative states-of-affairs to be considered by reasoners at any point would be capped at two, leading us to suspect that there would not be an effect of content as we found with indefinites in our pilot experiment 2. Thus, we used familiar expressions in our materials, rather than unfamiliar technical terms. We used five target items, of which (7a) and (7b) are representative examples:

(7) a. Every boy or every girl is coming to the party.
John is coming to the party.
Does it follow that Bill is coming to the party?

b. Mary has met every king or every queen of Europe.
Mary has met the king of Spain.
Does it follow that Mary has met the k. of Belgium?

As predicted, we found acceptance rates entirely comparable to those reported by Walsh and Johnson-Laird (2004) for the propositional counterpart, at around 82%. This result demonstrates that the range of illusory inferences with quantifiers is not limited to indefinite expressions.

Discussion — the erotetic theory of reasoning

We showed with new data that the kinds of cognitive illusions usually discussed in the mental models literature are only the tip of a large iceberg, extending from propositional reasoning with disjunctions to reasoning with quantified expressions. We argued that mental model theory does not provide an adequate account of this broader family of illusory inferences, and sketched how our erotetic theory of reasoning (Koralus & Mascarenhas, 2013) predicts and explains the full range of data.

We want to point out that we maintain the crucial insight from Johnson-Laird and his collaborators that reasoning proceeds by building mental models of premise statements. In that sense, we argue against the specific "mental model theory" we discussed above while wholly embracing the idea that reasoning is based on mental models. However, we take a different view of what these models contain and we take a different view of how mental models are updated as successive premise statements are taken into account. Specifically, we take it that mental models are updated with the aim of answering the questions they represent. In Koralus & Mascaranhas (2013) we give a complete and precise presentation and formalization of the theory for the case of propositional reasoning. We conclude presently with an informal summary of the theory's main features.

The erotetic principle

At the core of the theory is the erotetic principle, as follows.

(8) **The erotetic principle**
Part I — Our natural capacity for reasoning proceeds by treating successive premises as questions and maximally strong answers to them.
Part II — Systematically asking a certain type of question as we interpret each new premise allows us to reason in a classically valid way.

We take it that reasoning proceeds by updating an integrated mental representation of alternative possibilities in light of successive premise statements. By default, this process of updating proceeds by treating successive premises as questions and maximally strong answers to them. This is Part I of the erotetic principle (8). Statements are interpreted relative to a question that a hearer or reasoner seeks to answer in a way that goes beyond the narrow propositional contribution of the answer (Koralus, 2012).

But the theory is also concerned with the problem of success for reasoning, which is addressed by Part II of the erotetic principle. Humans are not irretrievably lost to the non-normative conclusions that their desire to find immediate strong answers leads them to. Interestingly, questions play a crucial role in leading us to normatively correct reasoning in this account. If reasoners are careful to ask polar questions (i.e. yes-no questions) about each atomic proposition that occurs in the question under consideration *before* updating with the putative answer supplied by a later premise, it can be shown that their reasoning will be classically sound in the technical sense. We prove this result as a theorem in Koralus and Mascarenhas (2013).

Key components of the theory

A theory of mental representations The first step is to specify what contribution individual premise statements make. We adopt the view that 'or' raises the question of which of the disjuncts is the case (Mascarenhas, 2009). Following standard approaches in natural language semantics, we model questions as sets of alternative answers. For example, for a premise statement like "there is an ace and a queen or a king and a jack," we obtain the set $\{a\&q, k\&j\}$. For a simple premise like "there is an ace," we obtain a singleton $\{a\}$. Indefinite expressions are interpreted as generalized disjunctions. This view on the interpretation of disjunctions and indefinites is congenial to views of content from linguistic semantics that have gained currency in the past decade.

Mental model discourses The erotetic theory is dynamic: premises are interpreted in the order that they were given, and

in principle that order can make a difference. This follows from the natural dynamics of question asking and answering.

Updating via the erotetic principle The next ingredient is an update rule that implements Part I of the erotetic principle, treating certain premises as questions and others as maximally strong answers to questions in context whenever possible. Besides treating information as questions and answers our update rule also has to allow for cases in which we simply accumulate information, as when we are given successive categorical statements.

Simple deduction rule Reasoning is not just a matter of update. Once reasoners hear and process each premise, they must then be able to perform simple transformations on the resulting mental model, to check what follows. We assume that there is a rule of disjunct simplification, validating the inference $(p \wedge q) \vee r \models p \vee r$. This rule for disjunct simplification includes conjunction elimination as a special case, as the reader can see.

Eliminating contradictions The theory takes it that reasoners do not immediately see anything wrong with contradictions. However, there must be a process allowing them to look at the representations they are entertaining and check whether they are consistent or not. This comes at a cost and is not part of default reasoning (to be defined shortly), but it must be a possibility if we want to account for the successes of our reasoning faculty. We therefore define an operation that filters the mental model in discourse, going over each alternative and eliminating all those that are contradictory.

Expanding possibilities through inquiry The mental models of the erotetic theory represent only what is minimally required to model a statement, and are therefore typically underspecified. We need an operation that expands the mental model under consideration through successive applications into one that represents every possibility with respect to some propositional atom. This is a crucial ingredient of the strategy allowing for classically sound reasoning and it implements Part II of the erotetic principle.

Default reasoning strategy Finally, we make a simple postulate describing how reasoning problems are approached by default. Namely: when given a reasoning problem with premises $P_0, \ldots, P_n$ and conclusion C, reasoners update a blank mental model discourse with each premise, in the order the premises were given. They may then apply the simple deductive rule, targeting the conclusion C. If the resulting mental model in discourse is identical to C, then the inference is deemed valid. Otherwise, it is deemed invalid. The full description of the default reasoning strategy in Koralus and Mascarenhas (2013) includes a model of how established background knowledge can influence reasoning performance.

Though we do not have the space to argue for the more general framework here, we propose the erotetic theory as a step toward a general account of reasoning and decision-making. Besides propositional reasoning, the erotetic theory has also been extended to model decision-making (Koralus, 2014), the cognitive factor in a multi-factor model of delusion (Parrott & Koralus, 2014) and aspects of moral judgment (Koralus and Alfano, in preparation).

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