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February 1991



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CATHODIC PROTECTION WITH PARALLEL CYLINDERS

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Cathodic Protection with Parallel Cylinders

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February 12, 1991

Abstract

Anodes should be placed so as to supply a uniform current density to the surface of the protected cathode to maintain it within a specified potential range relative to the adjacent electrolytic medium. Analysis of two or more parallel circular cylinders is carried out by solving Laplace's equation with the uniform current density on the cathode. The computer is used to sum terms in a solution by separation of variables, and results show the potential variation and average potential drop as functions of the distance of separation. Potential-contour plots compare the extent of significant electric fields for one and for two anodes.

Key words: Corrosion, potential distribution, Laplace's equation.

Electrochemical corrosion occurs because an oxidizing agent can react with the metal. Sometimes this can be counteracted by reducing the potential of the metal to the point where it does not react appreciably. Iron and steel can be protected in this way, while copper and gold may be sufficiently noble or nonreactive as to render such protection unnecessary or unprofitable and aluminum and magnesium are so active or reactive as to render such protection ineffective or impossible. When the metal is protected, the oxidizing agent continues to be reduced, and thus a cathodic protection current is required.

For steel, oxygen is a common oxidizing agent, having a relatively positive standard electrode potential and a moderately low solubility in aqueous media. Consequently, the current distribution is likely to be governed by the limiting rate of transport of oxygen to the metal surface, and to a first approximation this distribution is likely to be uniform over the surface of the metal. Thus, the boundary condition of a uniform cathodic current density becomes more appropriate than that of a uniform potential in the electrolytic medium adjacent to the metal.

In some situations the limiting current could be expected to be nonuniform, as perhaps for the flow of aerated sea water past a steel piling. Then the boundary condition would become one of a prescribed but nonuniform current density. However, in the absence of information on specific conditions, the uniform current density becomes a general design specification.

The solution of Laplace's equation, subject to the boundary condition of a uniform current density, yields a potential distribution whereby different parts of the metal are protected to different degrees. This leads to the second important criterion for a cathodic-protection system; the potential variation in the electrolytic medium adjacent to the protected metal must stay within a certain range. On the one hand, the potential of the metal relative to an adjacent reference electrode must be held negative enough to provide protection and prevent dissolution, and on the other hand it should not be so negative that excessive hydrogen evolution results. To be quantitative, the metal should be maintained less than -0.85 V relative to a saturated Cu/CuSO_4 reference electrode to protect the steel¹ and greater than -1.2 V to avoid excessive hydrogen evolution. Thus, the protected structure is to be maintained within a 0.35 V range.

Fontana and Greene¹ provide a general introduction to corrosion, including cathodic protection. Newman^{2,3} has outlined the approach for the design of cathodic-protection systems and given a few introductory examples. Nişancıoğlu⁴ reviewed the modeling of such systems and provides a good introduction to the literature.

Theoretical Development

Figure 1 shows the geometric arrangement of a pipeline to be protected cathodically by a parallel, cylindrical anode placed at a distance d . Numerical values of anode radius r_a , cathode radius r_c , and

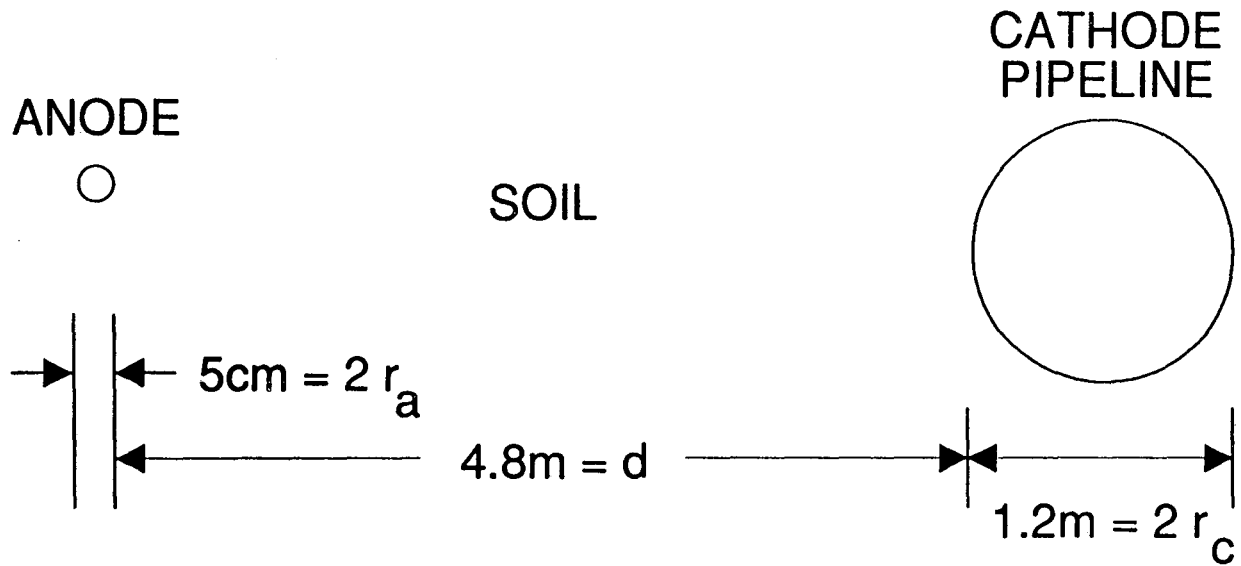


Figure 1 - Protected pipeline and parallel anode which provides the required current.

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separation distance d are used in later examples. The potential Φ satisfies Laplace's equation

$$\nabla^2 \Phi = 0 \quad (1)$$

in the electrolytic medium, here referred to as "soil," subject to the condition of a uniform current density on the cathode:

$$\frac{\partial \Phi}{\partial n} = \frac{i_{\text{avg}}}{\kappa} \quad \text{on the cathode,} \quad (2)$$

where n refers to the normal distance from a surface and κ is the conductivity of the soil. i_{avg} is the current density (taken to be a positive number) required locally for cathodic protection, after taking into account the effectiveness of any coating in reducing the transport of oxygen to the surface of the pipeline. In the soil adjacent to the anode we have taken the potential to be uniform. Because of its small size, the current distribution on the anode will frequently be nearly uniform as well. The potential in the soil far from the cylinders is taken to be zero; hence the value of the potential adjacent to the anode will be found as a result of the calculations, in its dependence on i_{avg} , κ , and the geometric lengths.

The soil is taken to have a uniform conductivity, and the ground surface and other nonidealities, such as neighboring pipelines, are ignored. An appropriate coordinate system for analytic or semi-analytic solutions is that of bipolar circles. This is a conformal mapping found described, in reasonable detail, by Moon and Spencer⁵ and in which Laplace's equation takes the form

$$\frac{\partial^2 \Phi}{\partial u^2} + \frac{\partial^2 \Phi}{\partial v^2} = 0 . \quad (3)$$

Cartesian coordinates x and y can be calculated from u and v as follows

$$x = \frac{a \sinh u}{\cosh u - \cos v} \quad \text{and} \quad y = \frac{a \sin v}{\cosh u - \cos v} . \quad (4)$$

a is a parameter of the coordinate system which must be selected to yield the particular geometric lengths as selected for figure 1. Curves of constant v are circles passing through both $x = a$, $y = 0$ and $x = -a$, $y = 0$; the curve for $v = 0$ degenerates to the line $y = 0$ extending from $x = a$ to ∞ and from $x = -a$ to $-\infty$, and the curve for $v = \pi$ degenerates to the line $y = 0$ extending from $x = -a$ to $x = a$. As v goes from 0 to π on the surface of the cathode (or the anode), one traces angular position on the electrode from the far side to the near side. Curves of constant, positive u are circles enclosing, but not quite centered on, $x = a$, $y = 0$; similarly, negative u corresponds to circles enclosing $x = -a$, $y = 0$. The curve $u = 0$ degenerates to the line $x = 0$. The cathode itself is the circle $u = u_c$ (a positive number), and the anode is the circle $u = u_a$ (a negative number).

From the geometric ratios r_c/r_a and d/r_c , one calculates u_c and u_a from the equations

$$\sinh u_a = -\frac{r_c}{r_a} \sinh u_c \quad (5)$$

and

$$\frac{d}{2r_c} = \sinh^2 \frac{u_c}{2} + \frac{r_a}{r_c} \sinh^2 \frac{u_a}{2} \quad (6)$$

by trial and error. Then one can obtain auxiliary quantities such as

$$\frac{a}{r_c} = \sinh u_c \quad \text{and} \quad \frac{a}{C} = \tanh u_c \quad (7)$$

where $x = C$, $y = 0$ is the center of the cathode. In the above discussion we have used the terms line, curve, and circle where plane, surface, and circular cylinder should be understood.

The boundary condition 2 on the cathode transforms to

$$\frac{\partial \Phi}{\partial u} = \frac{-i \text{avg } a/\kappa}{\cosh u_c - \cos v} \quad \text{at } u = u_c \quad (8)$$

The solution to equation 3 subject to the boundary conditions can be expressed by separation of variables as

$$\frac{-\kappa \Phi}{i \text{avg } a} = C_0 u + \sum_{k=1}^{\infty} C_k \frac{\sinh[k(u-u_c)]}{\cosh[k(u_c-u_c)]} \cos(kv) - \phi_{\infty} \quad (9)$$

where ϕ_{∞} is adjusted so that $\Phi = 0$ at $u = 0$, $v = 0$.

Since $\cosh u_c$ is larger than 1, the boundary condition 8 can be expanded as a power series in $\cos v$, and the coefficients C_k can be determined by comparison of the left and right sides of the equation after repeated use of the identity

$$2 \cos^2 v = \cos(2v) + 1. \quad (10)$$

For example, with $U = \cosh u_c$, the first few coefficients take the form

$$C_0 = \frac{1}{U} + \frac{1}{2U^3} + \frac{3}{8U^5} + o[U^{-7}] \quad (11)$$

$$C_1 = \frac{1}{U^2} + \frac{3}{4U^4} + \frac{10}{16U^6} + o[U^{-8}] \quad (12)$$

$$2C_2 = \frac{1}{2U^3} + \frac{4}{8U^5} + o\left[U^{-7}\right] \quad (13)$$

$$3C_3 = \frac{1}{4U^4} + \frac{5}{16U^6} + o\left[U^{-8}\right] \quad (14)$$

$$4C_4 = \frac{1}{8U^5} + o\left[U^{-7}\right] \quad (15)$$

$$5C_5 = \frac{1}{16U^6} + o\left[U^{-8}\right] \quad (16)$$

As many as 70 terms have been carried in the expansions; many terms are required when the separation distance d is small. A recursion relation can be developed for computer implementation.

Results and Discussion

A major consequence of the current flowing through the soil is that the sides of the protected pipe adjacent and opposite the anode find themselves at different potentials. The lower curve on figure 2 shows values of the potential variation $\Delta\phi$ between the soil on the near side and that on the far side, made dimensionless with the cathode current density i_{avg} and radius r_c and the soil conductivity κ . This dimensionless potential is plotted against the relative distance of separation d/r_c of the cathode and anode with the relative size of the electrodes r_c/r_a as a parameter. (The curve for $r_c/r_a = 12$ nearly coincides with that shown for $r_c/r_a = 24$.)

Table 1 gives some values shown on figure 2 as well as the individual potentials adjacent to the protected pipe on the front, side, and back and in the soil adjacent to the anode—all relative to the soil potential far from the electrodes and all made dimensionless with

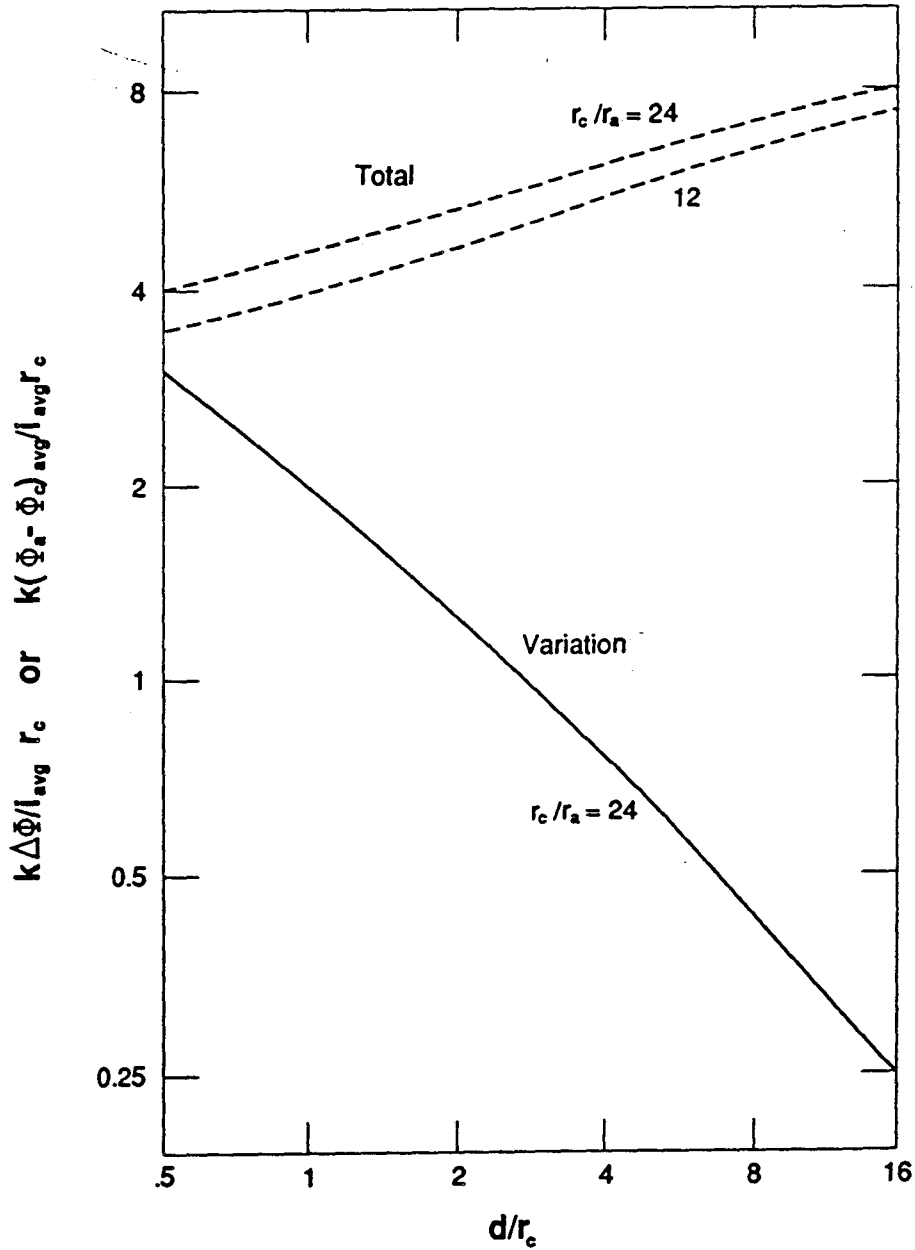


Figure 2. Variation of potential in the soil around the cathode and the average potential drop in the soil between the anode and the cathode. Note the logarithmic scales.

$$i_{\text{avg}} r_c / \kappa.$$

Figure 2 also shows the potential drop in the soil between the anode and the average of the near and far sides of the cathode. A greater sensitivity to the anode size is apparent (relative to the variation around the cathode) because a significant fraction of the overall potential drop occurs near the anode. While d/r_c must be increased to reduce $\Delta\Phi$ to a tolerable value for cathodic protection (lower curve), this gives an increasing penalty in terms of the overall potential drop (upper curves) and must ultimately be paid for in power costs and the design of the power supply and delivery system.

Figure 3 shows contours of dimensionless potential in the soil between the anode (the small dot toward the left) and the cathode. These contours are very nearly circles; those near the cathode are not

Table 1. Dimensionless potentials $\kappa\Phi/i_{\text{avg}} r_c$ at key points in the soil in the cathodic protection system. $\Phi = 0$ far away.

d/r_c	r_c/r_a	anode	----- cathode -----				near- far	total ohmic
			near side	top	far side			
0.5	24	4.1551	1.6590	-0.7839	-1.4312	3.0902	4.0412	
1	24	4.1662	0.6329	-0.9286	-1.5109	2.1439	4.6052	
2	24	4.4049	-0.3148	-1.2149	-1.6808	1.3660	5.4027	
4	24	4.8360	-1.1755	-1.6563	-1.9796	0.8041	6.4135	
8	24	5.3922	-1.9674	-2.2140	-2.4116	0.4442	7.5817	
16	24	6.0171	-2.7146	-2.8390	-2.9497	0.2350	8.8493	
0.5	12	3.4532	1.5411	-0.7948	-1.4377	2.9788	3.4015	
1	12	3.4811	0.5771	-0.9406	-1.5177	2.0947	3.9514	
2	12	3.7221	-0.3407	-1.2255	-1.6875	1.3468	4.7362	
4	12	4.1504	-1.1875	-1.6637	-1.9850	0.7975	5.7366	
8	12	4.7035	-1.9731	-2.2184	-2.4153	0.4422	6.8977	
16	12	5.3264	-2.7174	-2.8415	-2.9519	0.2344	8.1611	

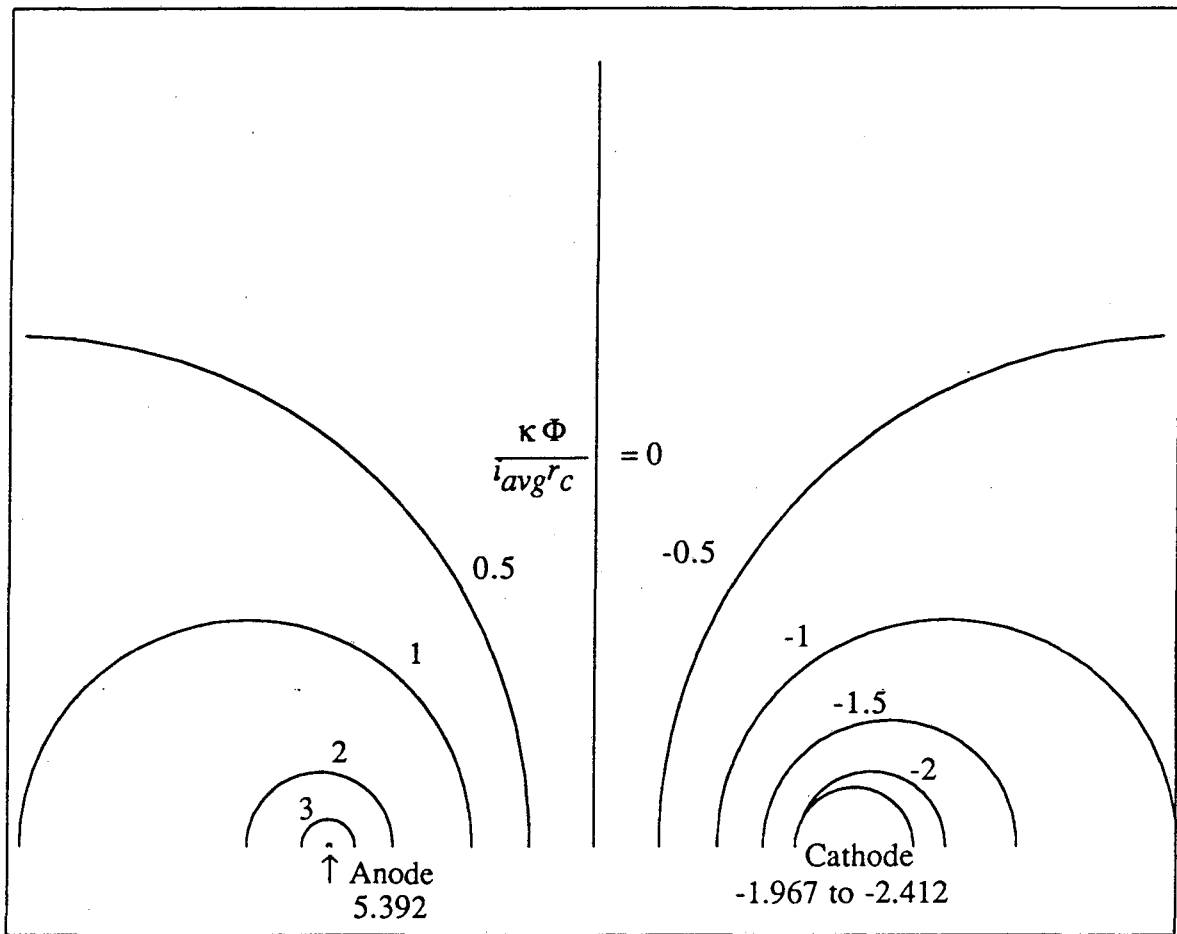


Figure 3. Equipotential contours (of the quantity $\kappa\Phi/i_{avg}r_c$) for the cathodic protection system with $r_c/r_a = 24$ and $d/r_c = 8$ and for a uniform current density on the cathode.

concentric with it, thus accounting for the potential variation on the cathode.

Cathodic protection with two anodes.—It should be possible to put the anodes much closer to the cathodically protected pipeline if two anodes are used and located symmetrically opposite each other relative to the pipeline. The mathematical analysis can be extended to this case by assuming that half of the uniformly distributed cathode current flows from each anode and superposing the two resultant potential fields. This could be done even if the anodes are not symmetrically placed if the fraction of current to be attributed to each anode were known. The only way the mathematical problem is not satisfied is that the potential distribution around an anode will not be uniform because it will be distorted by the potential fields generated by the other anodes. This problem will be relatively minor the smaller the anodes are relative to the cathode, because the nonuniform potential around an anode will then be small; besides, uniformity of potential around the anode was not the rigorously correct boundary condition anyway, and its use depended on the anode being small.

To implement the superposition for the case of two equal-sized anodes symmetrically placed merely requires that we include in Table 1 the potential in the soil at the top or bottom of the pipeline, that is, halfway between the "far" and "near" locations. Then, the superposition of the potential fields will yield the maximum variation of potential around the cathode. The resulting plot is shown in figure 4. From this figure one can see how much closer the anodes can be

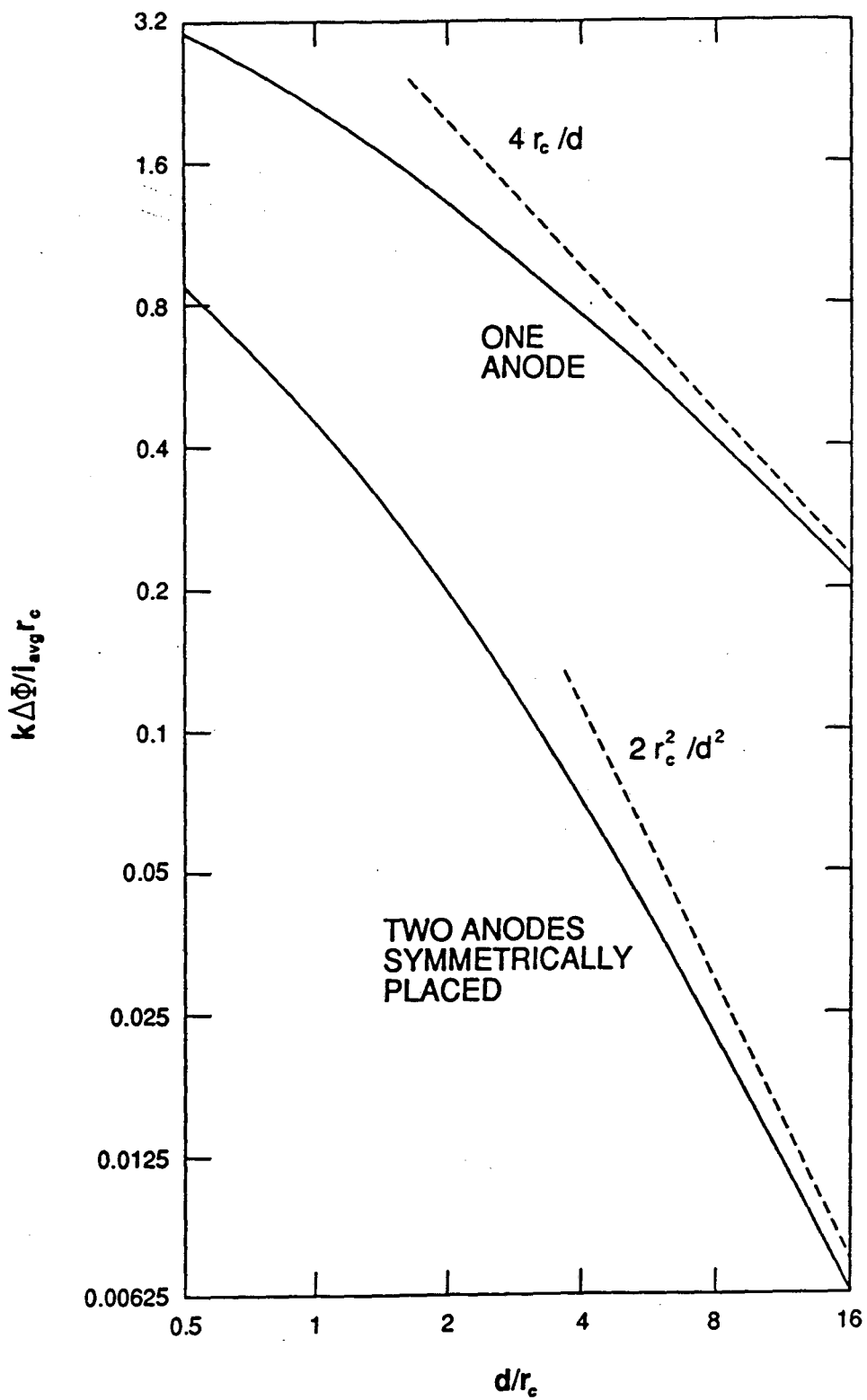


Figure 4. Variation of potential in the soil around the cathode for one or two anodes. Asymptotes for large d/r_c are also shown.

placed in this case.

The equipotential contours in figure 5 lie generally much closer to the cathode than the corresponding ones in figure 3, even though the potential variation adjacent to the cathode is similar in the two cases ($\kappa\Delta\Phi/i_{\text{avg}}r_c = 0.444$ for figure 3 and 0.490 for figure 5). Using two symmetrically placed anodes allowed the anodes to be placed at $d/r_c = 1$ instead of $d/r_c = 8$. (For $d/r_c = 1.06$, the potential variation with two anodes would have matched better that with one anode at $d/r_c = 8$.) Two close anodes reduced the overall potential variation from anode to cathode from about 7.582 to about 2.31. Furthermore, the whole system is much smaller, because the anodes are so much closer to the protected pipeline. Finally, the potential field in figure 5 drops off with large distances as a quadrapole field, while that in figure 3 behaves as a dipole field. All these factors mean that the electric field associated with cathodic protection is much smaller in figure 5 with the following consequences:

1. Interference with neighboring pipelines and other electrically conducting structures is substantially reduced.
2. The ground surface will distort the current and potential fields less.
3. The assumption of a uniform conductivity will be better for the above reason and because the anodes will lie more in the temperature zone in the soil induced by the pipeline itself.

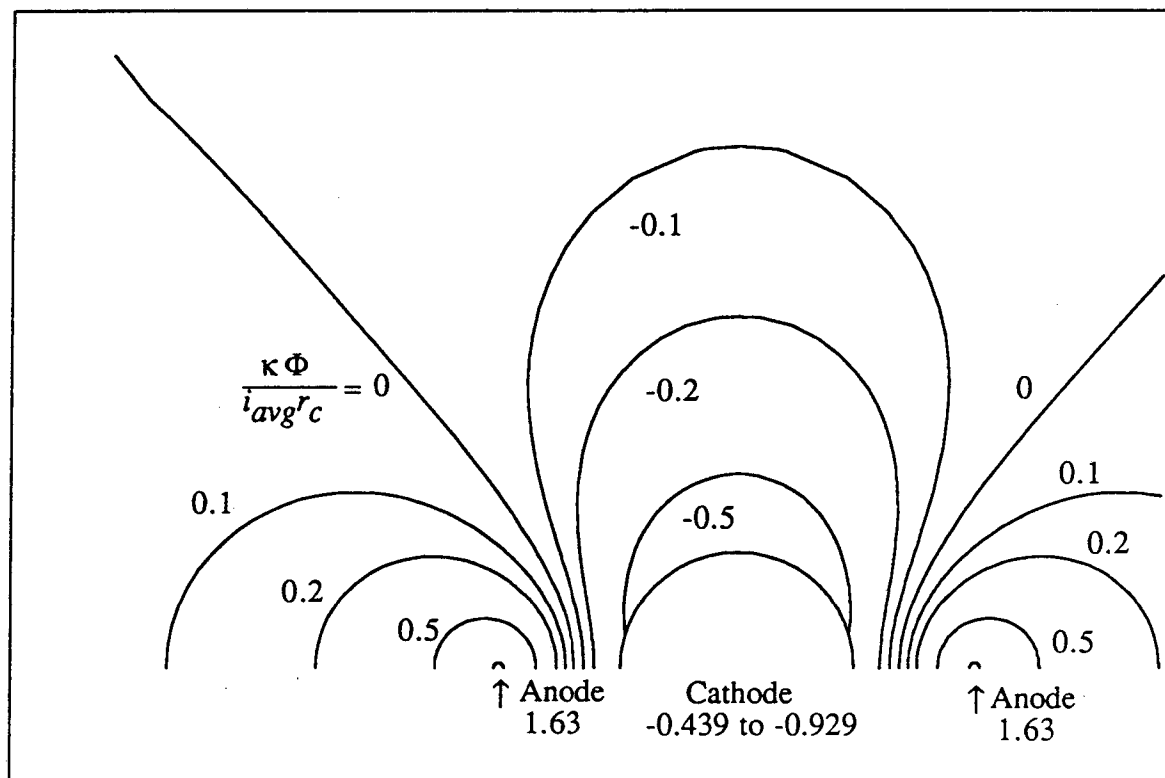


Figure 5. Equipotential contours for two symmetrically placed anodes, with $r_c/r_a = 24$ and $d/r_c = 1$.

For additional comparison, figure 6 shows the equipotential map with 2 anodes at the separation distance $d/r_c = 8$ while figure 7 shows that for one anode at $d/r_c = 1$, where table 1 shows that $\kappa\Delta\Phi/i_{\text{avg}}r_c = 4.6052$.

Examples

To get a better idea of these concepts, consider protection of a 1.2-m pipeline by one or more 5-cm cylindrical anodes. The required current density is taken here to be $1.1 \mu\text{A}/\text{cm}^2$ of bare steel or $i_{\text{avg}} = 0.11 \mu\text{A}/\text{cm}^2$ for a surface with a 90 percent effective coating—that is, a coating which already cuts the oxygen reaching the surface to 10 percent of the value in the absence of the coating. This will be used as a base case; results for 95 and 99 percent effective coatings can be inferred by appropriate reductions in the current and in the potential variation, both in the soil and in electronic conductors like the pipe and cables. For the base case, the current requirement per unit length of pipe is $2\pi r_c i_{\text{avg}} = 4.15 \text{ mA/m}$. The required current is supplied through the soil from a 5-cm diameter anode separated from the protected pipe by a distance of 4.8 m, as sketched in figure 1. The soil conductivity is taken to be $\kappa = 10^{-5} \text{ S/cm}$.

One anode.—Here $d/r_c = 8$ and $r_c/r_a = 24$. From table 1 or figure 2, $\kappa\Delta\Phi/i_{\text{avg}}r_c = 0.444$, and with the base values of κ and i_{avg} this leads to

$$\Delta\Phi = 0.444 \times 0.11 \times 10^{-6} \times 60 / 10^{-5} = 0.293 \text{ V} .$$

This is within the 0.35 V window prescribed for proper cathodic

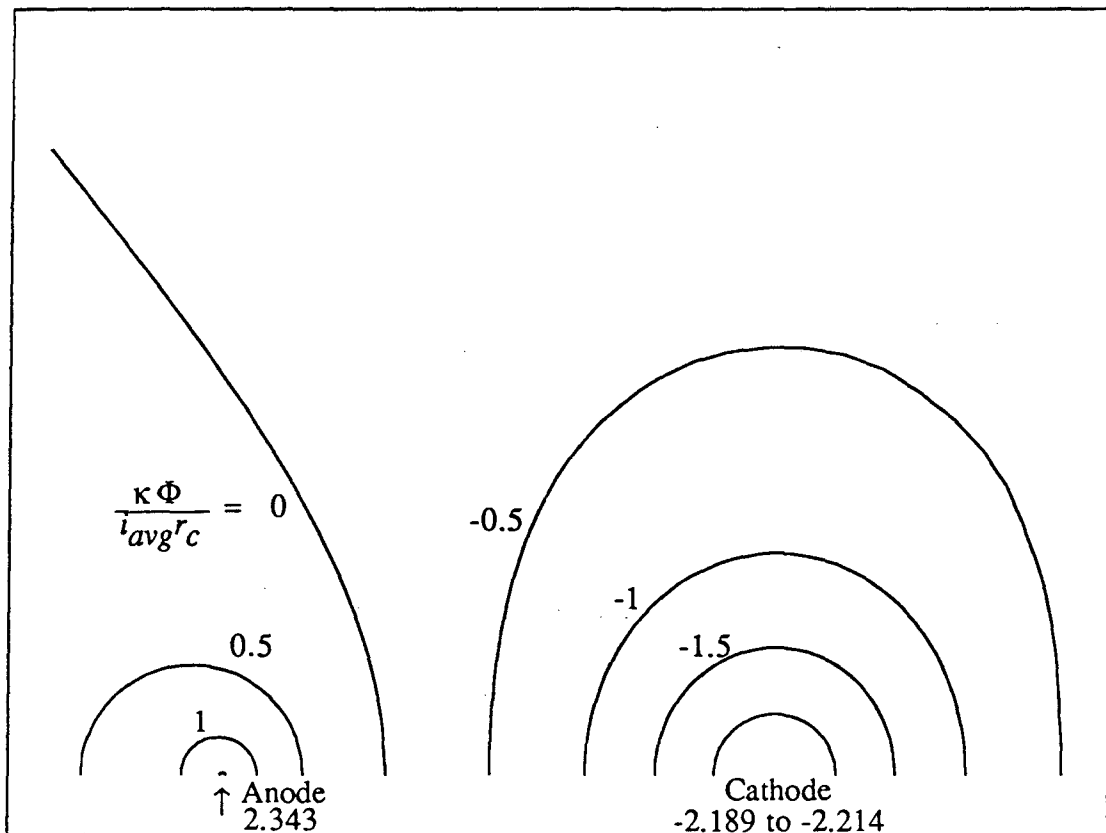


Figure 6. Equipotential contours for two anodes, $r_c/r_a = 24$ and $d/r_c = 8$.

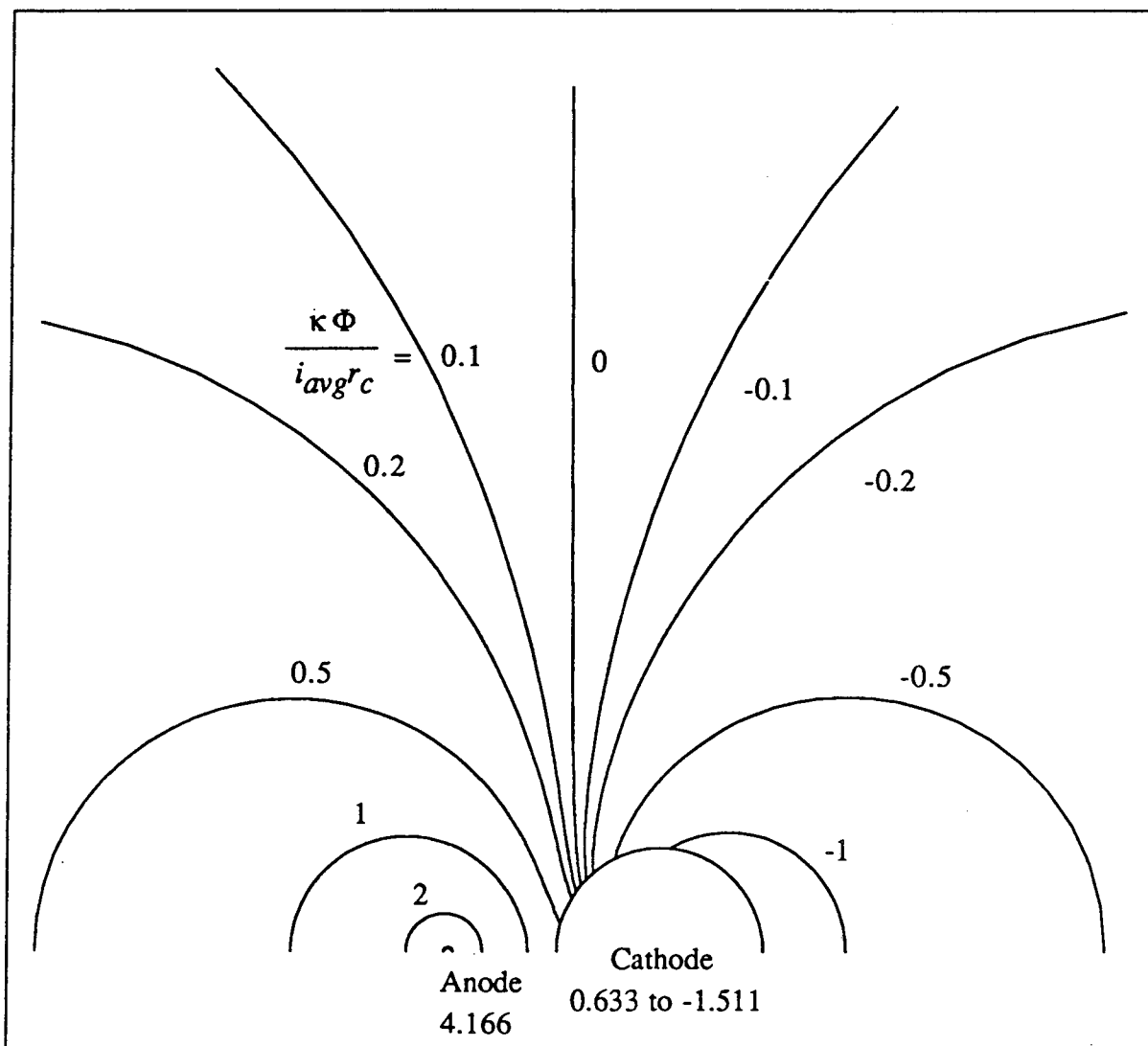


Figure 7. Equipotential contours for one anode, $r_c/r_a = 24$ and $d/r_c = 1$.

protection. At the same time, one can easily calculate the maximum potential variation for other coating efficiencies, soil conductivities, separation distances, or anode sizes. For example, with a coating effectiveness of 95 percent, the current requirement would be halved. Still with $\Delta\Phi = 0.293$ V, the dimensionless potential variation would be $\kappa\Delta\Phi/i_{\text{avg}}r_c = 0.888$, and figure 2 shows that $d/r_c = 3.48$. Thus, a single anode could protect the pipeline if it were placed 2.1 m away for a coating effectiveness of 95 percent. Similarly, for a coating effectiveness of 99 percent, we seek a value of $\kappa\Delta\Phi/i_{\text{avg}}r_c = 4.44$, for which figure 2 gives $d/r_c < 0.5$. The anode could be placed less than 30 cm from the pipeline under these conditions.

Another consequence of the current flowing through the soil is potential drop between the anode and the cathode. The means of supplying the anode with the required potential involves a power supply and rectifier and current carrying cables and will be dealt with in a later section. For the base case, table 1 or figure 2 gives a dimensionless potential drop of 7.582 resulting in

$$\Phi_{\text{anode}} - \Phi_{\text{cathode}} = 7.582 \times 0.11 \times 10^{-6} \times 60 / 10^{-5} = 5.0 \text{ V} .$$

To this must yet be added potential losses at the surface of the electrodes themselves and eventually the losses in the current delivery system. The added power cost is not overwhelming, but a large applied potential increases the possibility that the cathode itself will stray from the bounds of proper protection. The contour values in figure 3 can be multiplied by $i_{\text{avg}}r_c/\kappa = 0.66$ V to yield the potential distribution.

Two anodes.—To obtain the same protection ($\Delta\Phi = 0.293$ V, $\kappa\Delta\Phi/i_{\text{avg}}r_c = 0.444$, $d/r_c = 8$ with one anode) one can place each of two anodes as close as $d/r_c = 1.06$ or 64 cm away from the pipeline. For 95 or 99 percent coating effectiveness, where $\kappa\Delta\Phi/i_{\text{avg}}r_c = 0.888$ or 4.44, figure 4 shows that $d/r_c < 0.5$, and the two anodes can be placed within 30 cm of the pipeline and still not violate the potential-variation criterion.

The approximate ohmic drop between the cathode and the anodes could also be obtained by superposition if we had calculated and recorded in table 1 the potential at an anode due to the opposite anode. The calculations used to develop figure 5 and summarized there give an average dimensionless potential drop of 2.314. This corresponds to a reduction from 5 V for one anode at 4.8 m to 1.5 V for two anodes at 60 cm, with less danger of straying from the bounds of proper protection.

Potential variation in the anode.—Let us specify that current is to be supplied to the (single) anode every 300 m; half this distance, $L = 150$ m, must be supplied by this connection. With the assumption that the pipe is properly protected, the current per unit length passing between the anode and the cathode remains constant at $2\pi r_c i_{\text{avg}}$. The potential variation along the anode due to this current flowing along its length is

$$\Delta V = \rho' \pi r_c i_{\text{avg}} L^2, \quad (17)$$

where ρ' is the resistivity of the anode per unit length. For the base case we shall assume this to be $\rho' = 10.7 \mu\Omega/\text{cm}$, yielding

$$\Delta V = 10.7 \times 10^{-6} \times \pi \times 60 \times 0.11 \times 10^{-6} \times (150 \times 100)^2$$

$$= 0.050 \text{ V .}$$

This potential variation really adds to that calculated earlier (0.293 V) in determining the overall potential variation in the soil adjacent to the protected pipeline—use of a substantially larger value of L would mean that the pipe near the connection would tend to evolve hydrogen while the pipe at the maximum distance L from a connection would tend to be underprotected.

Potential variation in the cables.—If one power supply is used and is placed at the center of a 3.6 km length of the pipeline, then provision must be made to distribute this current to the different sections of anode of length $2L$, there being six sections on either side of the rectifier. Let us use a value $\rho'' = 42 \mu\Omega/\text{cm}$ for the cables connecting the anode sections to the rectifier. Then a similar formula can be used to estimate the potential drop within the cable:

$$\Delta V_{\text{cable}} \approx 42 \times 10^{-6} \times \pi \times 60 \times 0.11 \times 10^{-6} \times (1800 \times 100)^2$$

$$= 28.2 \text{ V .}$$

Potential distributions in the system are sketched in figure 8. Variable resistors would apparently be needed to avoid applying a large potential to the section of the pipe near the rectifier.

Understanding the Potentials

Reference electrodes.—At a distance L from station to station, reference electrodes are to be supplied to provide a measure of how

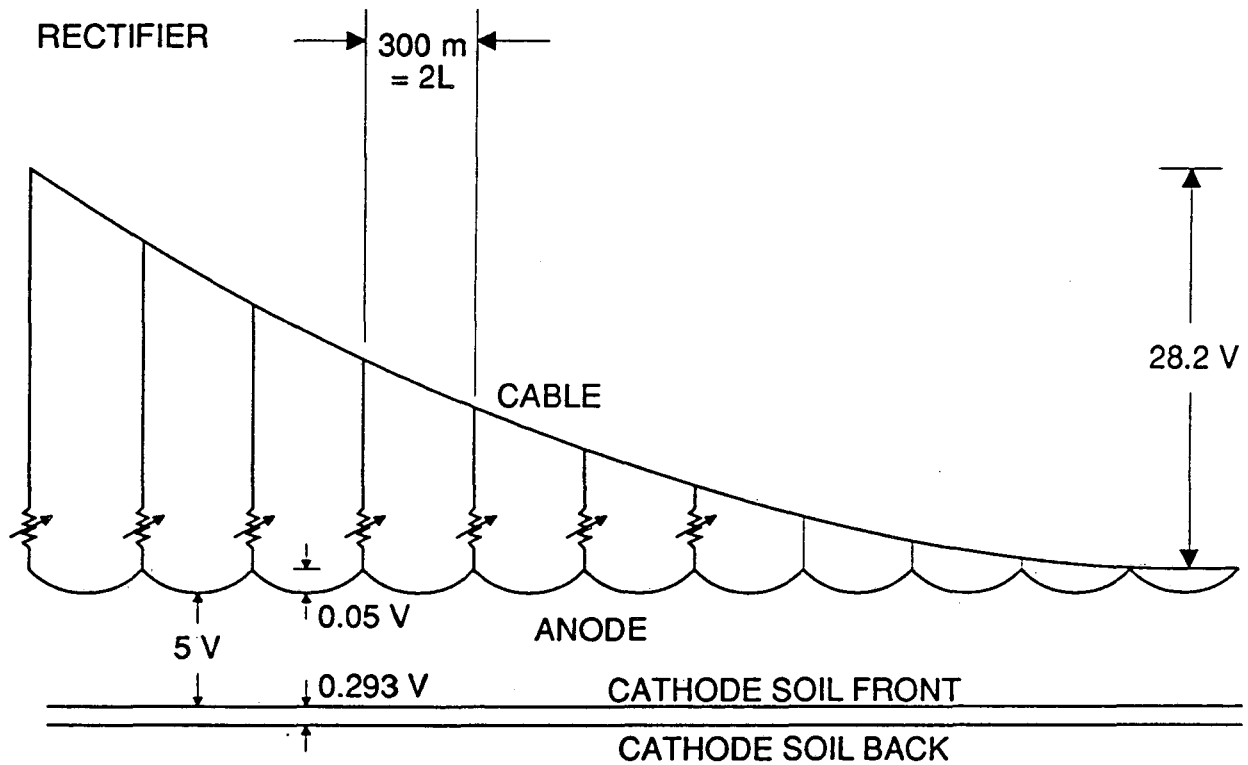


Figure 8 - Potentials in the conductors of the system. Variable resistors are suggested between the anode and the power-supply cable.

well and how uniformly the pipeline is protected. They are also used to adjust the resistors in the system, and they provide the only meaningful way of telling how effective the coating is and whether the correct current is being impressed.

While potentials in the electronic conductors can, at least in principle, all be measured relative to some common point, such as the pipeline at the rectifier, potentials in the soil can only be measured by means of reference electrodes placed in the soil at the point where the potential is to be measured, although the potential of the reference electrode itself could be referred to the same common point. Nevertheless, potentials in the soil so measured should be regarded to be different by a constant amount from a corresponding point in a metal. Thus the desired protection range (the protected metal between -0.85 and -1.2 V relative to the reference electrode) is not centered around zero, and a different range would be appropriate with a different type of reference electrode.

Soil.—On a potential map like figure 3, the potentials in the soil are being assessed relative to a reference electrode in the soil a distance away (ignoring telluric effects). The potential relative to metal parts of the system could be subject to some physical adjustment and interpretation. The potential calculated for the soil is determined by the assumed current distribution on the cathode and the potential far from the cathodic protection system. It is assumed that there aren't any stray currents impressed from other sources or any way for the current to flow to distant electrodes. Then, the poten-

tial of the cathode, relative to the adjacent soil and eventually the distant soil, is determined by the balance of electrochemical reactions for hydrogen evolution, oxygen reduction, and iron dissolution, and is apparently indeterminate in the absence of hydrogen evolution and iron dissolution. Hence, the reference electrodes are used to guide the cathodic protection process and assess its success. Similarly, the potential of the anode relative to the adjacent soil is governed by the interfacial processes at the specified impressed current. To some extent, the system is insensitive to the precise impedance of the anode as long as it is uniform and can be compensated for by a proper applied potential as assessed by the reference electrodes adjacent to the cathode.

If the reference electrode is not immediately adjacent to the metal to be protected, some allowance must be made for the ohmic drop between the reference electrode and the metal. The resistance of any coating on the pipeline must be considered here as well. An equipotential map such as figure 3 together with a knowledge of the current flowing in the soil can help in interpreting the meaning of a measurement of a pipeline relative to a reference electrode which is at some distance.

Electronic conductors.—The open-circuit potential for oxygen evolution on the anode at a pH of 7 is 0.815 V versus a normal hydrogen electrode or about 0.499 V versus Cu/CuSO_4 (see figure 9). With an additional allowance of 0.5 V for the overpotential, the anode potential relative to a distant Cu/CuSO_4 reference electrode becomes

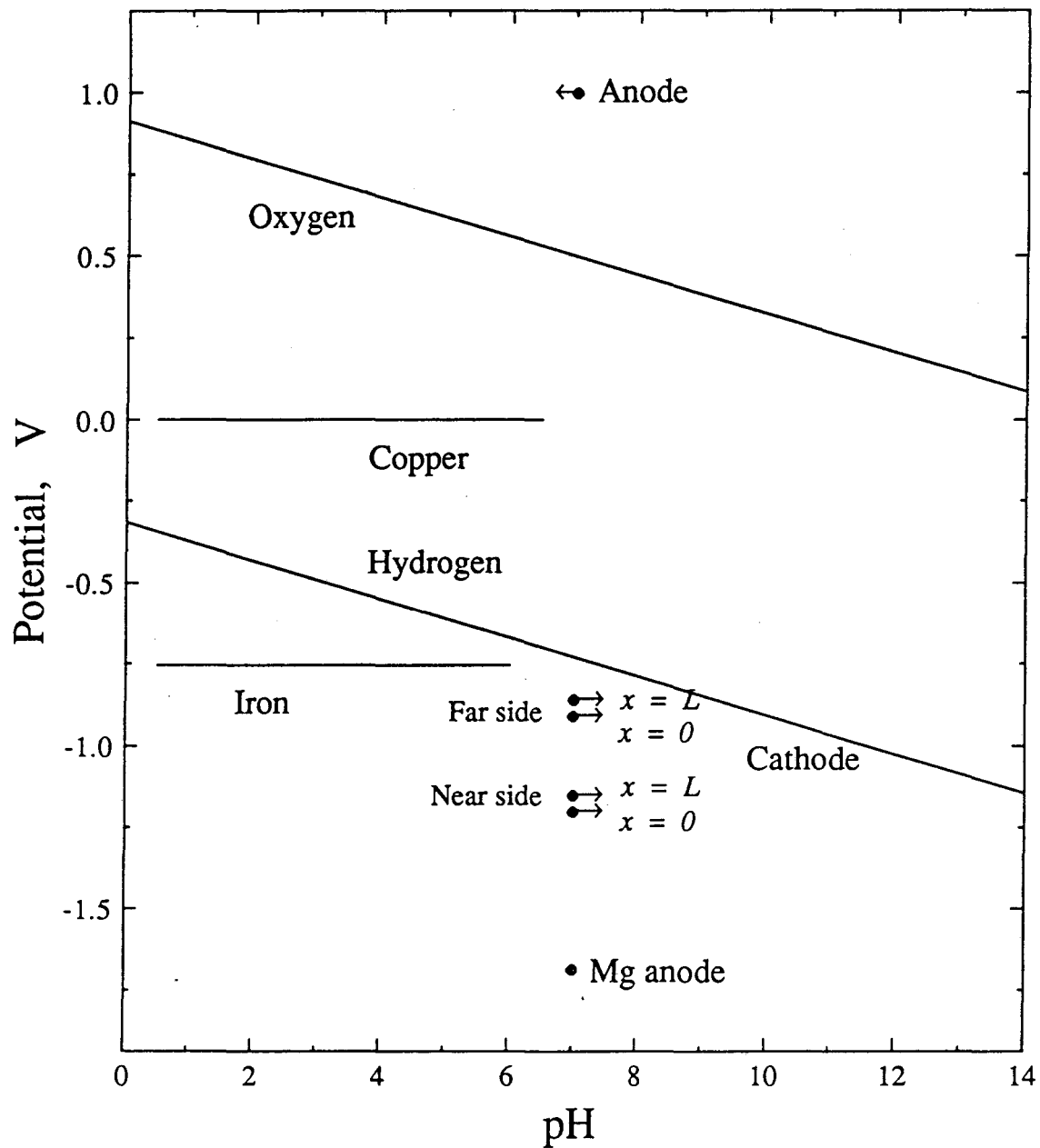


Figure 9. Potentials relative to an adjacent saturated Cu/CuSO_4 reference electrode. Arrows indicate direction of pH shift induced by the expected electrochemical reactions.

$$V_a - \Phi_\infty = 0.499 + 0.5 + 0.66 \times 5.392 = 4.558 \text{ V} .$$

Let us say that the system resistors are adjusted so that the cathode on the near side at $x = 0$ where the anode connection occurs is -1.2 V relative to an adjacent Cu/CuSO_4 reference electrode. Table 2 attempts to supplement the potential map in figure 3 by indicating values of some potentials of electrodes and soil at various locations in the system (see also figure 9). Potentials for a magnesium sacrificial anode, electronically connected to the protected pipeline are also indicated for later reference.

Table 2. Supplemental potential map for the base case.

location	metal to ∞	metal to soil	soil to ∞
Anode	4.558	0.999	3.558
Cathode, near side, $x = 0$	-2.498	-1.2	-1.298
Cathode, far side, $x = 0$	-2.498	-0.907	-1.592
Cathode, near side, $x = L$	-2.448*	-1.15	-1.298
Cathode, far side, $x = L$	-2.448*	-0.857	-1.592
Mg anode (1.0 V overpotential)	-2.498	-1.686	-0.812

Sacrificial Anodes

The above discussion deals with impressed-current anodes, where the protective current is supplied by a power system and rectifier, a possibility which becomes attractive for a large installation. It is instructive to review the design factors relevant when Zn or Mg anodes are to be used. They are assumed here to be ribbons installed

*The anode potential and the soil at infinity have changed by 50 mV from the value near the anode connector. The cathode itself is assumed to be of uniform potential.

parallel to the main pipeline and are approximated by circular cylinders.

Magnesium anode.—A single magnesium ribbon 19 by 9.6 mm could ideally provide the required base-case protective current by acting as a sacrificial anode over a period of time of

$$\tau = \frac{A_{Mg} \rho_{Mg} 2F}{M_{Mg} 2\pi r_c i_{avg}} = \frac{1.9 \times 0.96 \cdot 1.74 \times 2 \times 96487}{24.32 \times 2\pi \times 60 \times 0.11 \times 10^{-6}}$$

$$= 6.07 \times 10^8 \text{ s} = 19.2 \text{ years.}$$

This corresponds to

$$\frac{M_{Mg}}{2F} = \frac{24.32}{2 \times 96487} \frac{3600 \times 24 \times 365.25}{1000} = 3.98 \frac{\text{kg}}{\text{A}\cdot\text{yr}}$$

Fontana and Greene⁶ state that the actual consumption is 8.17 kg/A·yr, the additional consumption corresponding to hydrogen evolution on the magnesium, and the single ribbon would last only 9.35 years. Thus, 2 such ribbons might be expected to supply the base-case current for 18.7 years, and 3 ribbons might be good for 28 years. With a coating effectiveness of 95 or 99 percent, the duration could be longer.

Two such ribbons are to be placed, let us assume, 60 cm from the pipeline. This corresponds to figure 5 (except for the radius of the anode) and yields a potential variation of $0.490 \times 0.66 = 0.323$ V. With half the current coming from each magnesium ribbon and a radius $r_{Mg} = 1.5$ cm, one can estimate the required ohmic potential drop. For 5-cm anodes, figure 5 gave an average potential drop of $2.314 \times 0.66 = 1.527$ V. A correction needs to be added to account for the smaller anode:

$$\Phi_{\text{Mg}} - \Phi_c = 1.527 + \frac{i_{\text{avg}} r_c}{2\kappa} \ln \frac{5}{1.5} \approx 1.924 \text{ V.}$$

Half the cathode current density is used for this calculation to account for the two anodes.

The available potential if the magnesium ribbons are connected directly to the pipeline would be

$$V_c - \Phi_c = V_{\text{Mg}} - \Phi_c = \frac{-1.2 - 0.85}{2} = -1.025 \text{ V,}$$

$$V_{\text{Mg}} - \Phi_{\text{Mg}} = -1.686 \text{ V}$$

(assuming an overpotential of 1.0 V), and

$$\Phi_{\text{Mg}} - \Phi_c = -1.025 + 1.686 = 0.661 \text{ V.}$$

This is not enough to overcome the required ohmic potential drop of 1.924 V, and the two anode ribbons might be expected to supply $0.661/1.924 = 34$ percent of the required base-case current. Furthermore, a reduction of the size of the ribbons over the years would increase the required ohmic potential drop. Thus we can conclude that two magnesium anode ribbons will not adequately protect the pipeline under the base-case conditions.

Because of the limited negative potential of the Mg, the overall ohmic drop between the anode and the protected structure becomes comparable in importance to the potential variation along the surface of that structure. To restate the criterion, with the assumption that Mg can be found at -1.686 V relative to Cu/CuSO_4 , the ohmic potential drop from the anode to the remote point of the pipeline (the "top" for the two-anode configuration) should be less than $1.686 - 0.85 = 0.836$

V to assure adequate protection, and that to the near point of the pipeline should be greater than $1.686 - 1.2 = 0.486$ V to avoid excessive hydrogen evolution. Of course, if the coating effectiveness is 95 or 99 percent (instead of 90 percent), the current requirement is reduced, and there is a danger of overprotection. It would be prudent to have variable resistors in the connection between the anodes and the pipeline (compare figure 8). However, the elaborate cable system to the distant rectifier would be eliminated.

With a coating efficiency of 95 percent, the current requirement would be halved, and the criterion of potential variation around the pipeline allows the two anodes to be placed closer than 30 cm. The ohmic drop at this distance and current would be estimated to be

$$\frac{4.0412}{2} \frac{0.66}{2} + \frac{0.66}{4} \ln \frac{5}{1.5} = 0.865 \text{ V.}$$

(The first term is an estimate based on table 1 for a single 5-cm anode. Both terms are divided by 4 because the current to each anode is one-fourth that of the base-case cathode current.) The two anode ribbons could now be expected to supply $0.661/0.865 = 76$ percent of the required current. With a coating efficiency of 99 percent, two magnesium ribbons located 30 cm from the pipeline should provide adequate protection.

For the base case, we could estimate that $0.865 \times 4 / 0.661 = 5.2$ or 6 Mg-ribbon anodes placed symmetrically 30 cm from the pipeline would be adequate. For a coating effectiveness of 95 percent, 3 anodes at this distance should work.

Zinc anode.—Zinc is not as negative as Mg (perhaps⁶ -1.15 V versus Cu/CuSO_4). This means that there is no danger of overprotection, but at the same time there is little potential available to drive current through any significant resistance. Galvanizing steel puts the zinc anode where it can be the most effective. For a pipeline, a flame-sprayed Zn coating might be suitable, or a zinc ribbon could be wrapped around the pipe.

Summary

A procedure based on the primary current and potential distribution cannot yield the proper criteria for scale up. It will not give the proper distance to reduce the potential variation around the protected pipeline, and it cannot show correctly how this distance changes with the current requirements and the soil conductivity.

Proper design of this cathodic-protection system requires the solution of Laplace's equation in the two-cylinder geometry with the condition of uniform current density on the cathode. Quantitative results are shown on figures 2 and 4 and in table 1 and are illustrated by examples.

If the coating effectiveness is reduced nonuniformly by isolated holidays, it would be appropriate to look at the design problem by the approximation of disk electrodes in an insulating plane.⁷

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List of Symbols

a	parameter of the coordinate transformation, cm
A_{Mg}	cross-sectional area of Mg ribbon, cm^2
C	value of x at center of cathode, cm
C_k	parameters in series solution for the potential
d	distance between electrodes, cm
F	Faraday's constant, 96,487 C/equiv
i_{avg}	current density on cathode, A/cm^2
k	integer
L	half the distance between anode connections, cm
n	normal distance from electrode surface, cm
r_a	radius of the anode, cm
r_c	radius of the cathode, cm
u, v	bipolar-circle coordinates
u_a	value of u on the anode
u_c	value of u on the cathode
U	$\cosh u_c$

V	potential in an electrode or cable, V
x, y	cartesian coordinates, cm
κ	conductivity of the soil, S/cm
ρ'	resistance of the anode per unit length, Ω/cm
ρ''	resistance of the anode cable per unit length, Ω/cm
ρ_{Mg}	density of magnesium, g/cm^3
τ	time for consumption of sacrificial anode at 100 percent current efficiency, s
ϕ_{∞}	parameter in series solution for the potential
Φ	potential in the soil, V

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Figure Captions

Figure 1. Protected pipeline and parallel anode which provides the required current.

Figure 2. Variation of potential in the soil around the cathode and the average potential drop in the soil between the anode and the cathode. Note the logarithmic scales.

Figure 3. Equipotential contours (of the quantity $\kappa\Phi/i_{\text{avg}}r_c$) for the cathodic protection system with $r_c/r_a = 24$ and $d/r_c = 8$ and for a uniform current density on the cathode.

Figure 4. Variation of potential in the soil around the cathode for one or two anodes. Asymptotes for large d/r_c are also shown.

Figure 5. Equipotential contours for two symmetrically placed anodes, with $r_c/r_a = 24$ and $d/r_c = 1$.

Figure 6. Equipotential contours for two anodes, $r_c/r_a = 24$ and $d/r_c = 8$.

Figure 7. Equipotential contours for one anode, $r_c/r_a = 24$ and $d/r_c = 1$.

Figure 8. Potentials in the conductors of the system. Variable resistors are suggested between the anode and the power-supply cable.

Figure 9. Potentials relative to an adjacent saturated Cu/CuSO_4 reference electrode. Arrows indicate direction of pH shift induced by the expected electrochemical reactions.

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