ANALOGY GENERATION IN SCIENTIFIC PROBLEM SOLVING

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A number of researchers have discussed the important role of analogical reasoning in science and education [1-12]. This paper describes research on the spontaneous use of analogies in problem solving by scientifically trained subjects. This occurs when the subject first spontaneously shifts his attention to a situation B which differs in some significant way from the original problem situation A, and then tries to apply findings from B to A. This is difficult for many people to do, possibly because it involves breaking out of the assumptions built up in considering the original problem. As a result, although spontaneous analogies are a more naturalistic phenomenon to study than provoked analogies, they are difficult to capture and record. However, by intentionally focusing on subjects who are known to have done creative work in the past, a number of such cases have been documented.

Ten experienced problem solvers were interviewed on a variety of problems. Most were video-taped. The subjects were advanced doctoral students and professors in technical fields. The findings summarized here are based on detailed protocol analyses of six of the problem solutions from this group that included the most significant uses of analogies. This brief paper concentrates on examples from the protocol of a single subject.

The first finding is that: spontaneous analogies have been observed to play a significant role in the solutions of a number of scientifically trained subjects. Solutions have lasted up to 90 minutes and some include reasoning patterns that are very complex. This complexity has led to a research focus of working toward a macro-level theory of the dynamic processes by which analogies are generated, evaluated, and applied. This is an appropriate initial strategy for mapping out a complex domain of processes about which little is known. From transcript analyses the general hypothesis was formulated that the following processes are fundamental in making an inference by analogy: [2]

(1) Given the initial conception A of an incompletely understood situation, the analogous conception B is generated, or "comes to mind";

(2) the analogy relation between A and B must be "confirmed";

(3) conception B must be well understood, or at least predictive; and

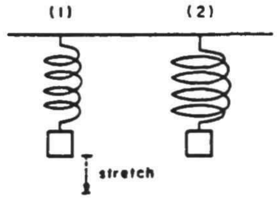
(4) the subject transfers conclusions or methods from B back to A.

This hypothesis is consistent with our observation that many successful solutions by analogy are not "instant solutions". Analogies are often proposed tentatively, and processes (2) and (3) especially can be quite time consuming. The last three processes can occur in any order, and subjects are often observed to move back and forth between them several times while gradually completing each step. This suggests that the subjects do not use a simple, well-ordered procedure for controlling their solution processes at this level. This paper focuses on steps (1) and (2). As will be shown there appear to be not one, but several ways of generating analogies, and several ways of confirming them.

EXAMPLE OF A SOLUTION CONTAINING ANALOGIES

Five subjects have generated analogies in thinking aloud about the following problem:

Fig. 1



Spring Coils Problem. A weight is hung on a spring. The original spring is replaced with a spring made of the same kind of wire, with the same number of coils, but with coils that are twice as wide in diameter. Will the spring stretch from its natural length, more, less, or the same amount under the same weight? (Assume the mass of the spring is negligible compared to the mass of the weight.) Why do you think so?

This problem was given to seven subjects. Four attempted to relate the problem to the analogy of a bending rod, as in the following verbatim, condensed transcript:

(1) S2: Um, I have one good idea to start with. It occurs to me that a spring is nothing but a rod wound up, uh, and therefore maybe I could answer the question for a rod. (Draws fig. 2)... I have a strong intuition, a physical imagistic intuition that this (rod a) will bend a lot more than that (rod b) will. In fact, the intuition is confirmed by taking it to the limiting case. It becomes intuitively obvious to me that as one moves the weight closer and closer to the fulcrum that the thing will not bend at all.

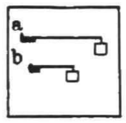


Fig. 2

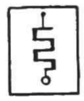


Fig. 3

S2 goes on to infer that if the rod situation is truly analogous, the wider spring will stretch farther. Here the subject is able to achieve a high degree of certainty about the behavior of the rods (process 3 above). He reports doing this on the basis of what he calls physical intuition and by thinking about an extreme case, giving us reason to suspect that he is using some type of imagistic simulation process. But he is uncertain as to whether he can confirm the idea that the spring and the rod are analogous.

(2) S2: But it occurs to me that there's something clearly wrong with that metaphor because...its slope [the bending rod's] would steadily increase as you... went away from the point of attachment, whereas in a [stretched] spring, the slope of the spiral is constant... I don't see how that could make the bow go away; just to wind it [the rod] up. Damn it! [13]

He spends a large part of his 45 minute solution trying to resolve this issue. This transcript and others indicate that processes 1 through 4 above can indeed take place separately. S2 has apparently completed processes 1, 3, and 4 so far.

METHODS FOR CONFIRMING ANALOGY RELATIONS.

Determining a match between key relationships in both situations is the first and most obvious method for confirming analogy relations (process 2 above) [5]. Thus subject S2 above is worried because he cannot obtain a match between the changing slope in a bending rod and the constant slope in a stretched spring. However, other confirmation methods are also possible.

Confirmation via bridging analogies. Rather than throwing out the rod analogy, S2 proceeded to generate a second related analogy: the "zig-zag spring" shown in fig. 3. Such subjects are observed to generate an intermediate case when they refer to a situation that has aspects in common with two previous situations A and B. It is hypothesized that S2 attempts here to form a cognitive bridging analogy which links his conceptual frameworks for the rod situation and the original spring situation.

Figure 4 shows how such a bridging analogy can be effective [2]. The link labeled 1 represents the initial tentative analogy relation conjectured to exist between conceptions A and B. Here A is the poorly understood initial problem situation and B is a well understood situation. Inadequately vs. well-understood conceptions are represented by dotted vs. solid squares, respectively, and tentative vs. confirmed analogy relations are represented by dotted vs. solid links between squares, respectively. Figure 4 shows how the subject might establish a confirmed link between A and B by bridging back to conception A via conception C. If the analogy links (2) and (3) are confirmed (with respect to the same salient relationships between variables), then A can become well understood and become analogous to B, since under the above conditions, A being analogous to C and C being analogous to B means that A is analogous to B. We call this analogical transitivity. It should be emphasized that since "confirmed" generally means "intuitively compelling" rather than "proven" in this context, analogical transitivity is considered a form of plausible reasoning which does not lead to conclusions carrying the force of a logical implication. This diagramming system also allows one to construct macro-level "maps" of hypothesized cognitive processes occurring during complex solutions involving many analogies.[3]

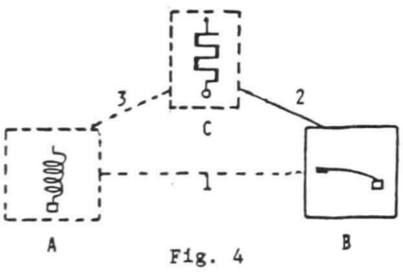


Fig. 4

A second bridge. Unfortunately, at this point the subject still could not reconcile the bending going on in the zig-zag spring with the lack of change in slope in the original helical spring (link 3 is unconfirmed), so his initial attempt at a bridge failed. However, he later generates a second, more successful attempt at a bridge in the form of an analogy to a polygonal spring. He is confident that a spring with hexagonal coils would not be essentially different from one with circular coils, and this leads him to a really new insight:

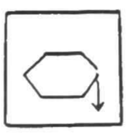


Fig. 5

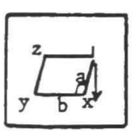


Fig. 6

(3) S2: Aha!... What if I start with a rod and bend it once (makes bending motion with hands) and...bend it again.. Clearly there can't be a hell of a lot of difference between the circle and say, a hexagon...(Draws fig. 5) Now that's interesting. Just looking at this it occurs to me that when force is applied here, you not only get a bend on this segment, but because there's a pivot here, you get a torsion effect...Aha! Maybe the behavior of the spring has something to do with twist forces... Let me accentuate the torsion force by making a square (draws fig. 6) where there's a right angle. Now...I have two forces introducing a stretch. I have the force that bends this...segment [a] and in addition I have a torsion force which twists at vertex, um, x...Now I feel I have a good model of a spring... Now making the sides longer certainly would make the [square] spring stretch more.

I: How can you tell?

S2: Physical intuition...and also recollection.. the longer the segment (moves hands apart) the more the bendability (moves hands as if bending a rod)... Now the same thing would happen to the torsion I think, because if I have a longer rod (moves hands apart), and I put a twist on it (moves hands as if twisting a rod), it seems to me--again physical intuition--that it will twist more... again, now I'm confirming that by using this method of limits. As I bring my hand up (moves right hand slowly toward left hand) closer and closer to the original place where I hold it, I realize very clearly that it will get harder and harder to twist... And my confidence is now 99% [that the wide spring stretches more]...I feel a lot better about it. [14]

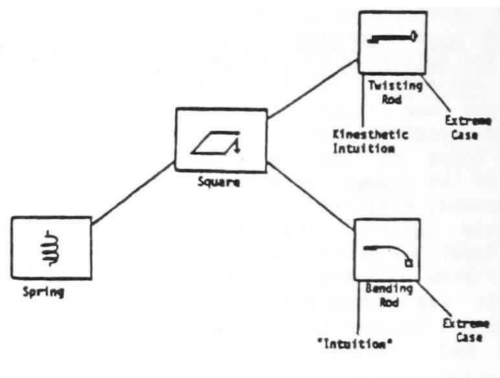


Fig. 7

Here he is able to firmly connect the original spring to the bending rod case via the bridging analogy of a polygonal spring. In addition, considering the polygonal spring triggers the recognition of a torsion effect. Thus, in the subject's final understanding of the spring, the spring is linked via the intermediate square spring case to two simpler cases, the twisted rod and the bent rod, as shown in fig.7. The torsion factor is an important insight, because not only is it true that wider springs stretch farther, but in fact the force provided by a helical spring is primarily due to torsion rather than to bending.

In summary, two major processes involved in confirming analogy relations have been identified: matching key features or relationships; and forming a bridging analogy.

ANALOGY GENERATION MECHANISMS

Analysis of transcripts has led us to propose the hypothesis that there are not one, but at least three types of analogy generation mechanisms: generation via an abstract principle, generative transformations, and associative leaps.

Generation from a principle. A plausible mechanism for generating analogies can be derived from the common situation in science where a single equation or abstract principle applies to two or more different contexts, such as a pendulum and an oscillating electrical circuit. This suggests that analogies may be formed by first recognizing that the original problem situation, A, is an example of an abstract equation or principle, P. The analogous situation, B, is then recalled or generated as a second example of principle P. However, although evidence for this pattern has been observed on occasion in interviews, little evidence of it was observed in the analogies generated for the spring problem. Instead, two other types of analogy formation processes appear to predominate, which I have called generative transformations and associative leaps.

Generative transformations. These occur when a subject modifies an aspect of problem A to create a new situation B. Examples of evidence for generative transformations from the present protocol are: (1) The subject refers to bending a rod into a polygon (protocol segment 3). (2) The subject referring to the spring as a "rod wound up" in the first line of the transcript indicates that the rod idea may have been generated by thinking about unwinding the spring. At another point he refers to the rod as an "unwound spring."

Associative leaps. In contrast to modifying the problem in a generative transformation, evidence for an associative leap occurs when the subject refers to an analogous situation B which is very different qualitatively in a number of ways from the original situation. The subject may also refer to "being reminded of" B. S2 generated evidence for several associative leaps in the middle of the protocol when he said: "I feel as though I'm reasoning in circles and I think I'll make a deliberate effort to break out of the circle somehow..., like rubber bands, molecules, polyesters...". apparently attempting to link the spring problem to other situations he knows something about. Although he was unable to use any of the associative leaps above effectively in this case, subjects have been observed to use this type of analogy generation technique successfully in other protocols. For example, one subject used an analogy to a U-tube to solve a problem about hydraulic forces in an apparatus whose shape and topology were quite different.

It is hypothesized that an associative leap takes place when an established conceptual framework for situation B in memory is activated by an association to some aspect of the original situation A, and that a generative transformation occurs when the subject focuses on an internal representation of the existing problem situation A and changes an aspect of it to create situation B. This leads to the prediction that an analogy generated via a transformation should more often be a novel invention (such as the hexagonal spring) and should more often contribute as a simpler case rather than as a more familiar case. Generative transformations and associative leaps have been the primary analogy generation methods observed by us so far [15].

METHODS FOR UNDERSTANDING A SITUATION AND FOR TRANSFERRING KNOWLEDGE FROM B BACK TO A

With regard to process 3 above, the requirement that conception B must be predictive or well understood, we note briefly that this can be achieved via factual knowledge, physical intuition, analysis in terms of a theory, or (recursively) via another analogous case C. Some methods for applying knowledge from B to A (process 4 above) are: (1) transferring a prediction directly from B to corresponding variable relationships in A; (2) transferring a partial understanding of certain variable relationships, which with further analysis can lead to a prediction in A; and (3) transferring a method of attack from B to A. [20]

EXTREME CASES AND PHYSICAL INTUITION

Minimizing or maximizing a feature of the problem sometimes makes the problem easier to analyze, and we call this using an extreme case. Extreme cases seem to be generated primarily via generative transformations or problem operators. Interestingly, the apparent function of many of the extreme cases observed so far has been to enhance the subject's use of physical intuition in the form of imagistic simulations. S2 indicates that his final understanding is based at the lowest level on such physical intuitions. This suggests that certain relationships between forces and other physical variables such as "bending" can be represented at a deep level in terms of imagistic intuitions rather than abstract principles or equations. (See ref. [3]).

CONCLUSION

Further research is needed in order to evaluate and add to the results of this exploratory study. A number of basic concepts for analyzing patterns of analogical reasoning have been proposed, including: the generation of analogies via transformations and associative leaps; the evaluation of analogy relations via the formation of bridges and the matching of key relationships; and the understanding of situations via the use of extreme cases which can enhance physical intuitions. Recursive combinations of these processes can account for many of the patterns observed in other complex solutions involving a number of linked analogies. Many solutions by analogy are not "instant solutions", but a more extended process of conjecture and testing. This gives us reason to believe that some of these processes are learnable, rather than being exclusively a product of "genius", and that developing students' abilities to use generative transformations, leaps, and bridges may be possible and desirable.

When a transformation leads to a confirmable analogy, we call it a conserving transformation since it conserves the salient relationships in the problem. In a broader sense, conserving transformations appear to play a fundamental role at different levels in physics, mathematics, technological invention, and music [16-19]. Conserving transformations appear to be an important cognitive process worthy of further investigation.

In the case of S2, the bending rod analogy served as a first order model which gave him an initial handhold on the problem. Persistent criticisms and transformations of this model during his vigorous 45 minute solution eventually led him to evolve a much better model in the form of a square spring with torsion effects. Thus, sophisticated uses of analogy in relatively difficult problems can involve a repeated conjecture, criticism, and modification process that can produce chains of successively more powerful analogies. Analogous cases can either play a temporary heuristic role in helping to generate conjectures during the solution, or they can play the more permanent role of a model in the final solution, or both. Certain parallels between these processes and processes of science described in [6-8,16], among others, suggest that further research along these lines may be of interest to those studying the processes of hypothesis formation and model construction in science.

NOTES

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[2] Clement, J., "The Role of Analogy in Scientific Thinking: Examples from a Problem Solving Interview", technical report, Department of Physics and Astronomy, University of Massachusetts at Amherst, 1977.

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[13] His meaning can be clarified here by noting that a bug would experience a slope of constant steepness in walking down a freely supported, stretched spring and a slope of increasing steepness on a bending rod. He later says that by "spiral" here he meant "helix".

[14] Two of the subjects in the sample, including S2, had heard that the author was interested in studying analogies prior to their interview. However, the subjects did not know which problems in the interview were amenable to solutions by analogy. After the interview, S2 was asked if he thought this prior knowledge had affected this solution and he felt strongly that he had simply solved the problem in the way he normally would. Most importantly, none of the subjects had communicated with the author concerning any of the theories or concepts developed in this paper, such as transformations, leaps, bridges, etc.

[15] Bridging analogies are generated under the multiple constraints of being similar to two situations. This leads us to predict that bridging analogies should ordinarily be generated via a transformation rather than a leap, and this is so far consistent with our data. In future reports we will show how non-generative transformations can also be involved in confirming an analogy relation.

[16] Cohen, I.B., The Newtonian Revolution: With Illustrations of the Transformation of Scientific Ideas. Cambridge U. Press, 1980.

[17] Krueger, T., Imagery Pattern in Creative Problem Solving. Encina Press, Las Cruces, New Mexico, 1981.

[18] Shepard, R., "Psychophysical Complementarity", in M. Kubovy and J.R. Pomerantz (Eds) Perceptual Organization, Lawrence Erlbaum Associates, Hillsdale, NJ, 1979.

[19] Rissland, E.L., "Example Generation", COINS Technical Report 80-14, University of Massachusetts at Amherst, 1980.

[20] The four criteria for a useful analogy on the first page apply to the strongest use of an analogy where a prediction is transferred. Transfer of a partial understanding or possible method of attack can occur without criteria 2 and 3 being fully satisfied.