Title
THE NATURE OF NUCLEAR DYNAMICS

Author
Swiatecki, W.J.

Publication Date
1982-04-01

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Introductory Remarks given at the Winter Workshop on Nuclear Dynamics, Granlibakken, Tahoe City, CA, February 21-26, 1982; and to be published in the Proceedings

THE NATURE OF NUCLEAR DYNAMICS

W.J. Swiatecki

April 1982

NUCLEAR SCIENCE DIVISION

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

DISCLAIMER

LBL-14073

THE NATURE OF NUCLEAR DYNAMICS

W. J. SWIATECKI

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

Introductory Remarks given at the Winter Workshop on Nuclear Dynamics, Granlibakken, Tahoe City, CA, February 21-26, 1982, excerpt from the Proceedings, LBL-14138.

THE NATURE OF NUCLEAR DYNAMICS*

W. J. SWIATECKI

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Several important advances in the description of nuclear dynamics have taken place in the past few years. There are the TDHF calculations, including recent attempts to treat residual interactions, statistical Master Equation approaches and equations of motion with dissipation. To me, one of the clarifying messages, which is coming through the multitude of different approaches that have been taken, is the following:

> "The nature of nuclear dynamics is dominated by the presence or absence of symmetries."

I have prepared a transparency in which I try to put together a number of insights into the nature of nuclear statics and nuclear dynamics and in which the presence or absence of symmetries plays a dominant role.

I will explain the word "Plastodynamics" in the title of the transparency (Fig. 1) at the end of my talk.

The transparency is in two parts, the upper referring to statics, the lower to dynamics. In each part I have isolated two limiting cases: The "Chaotic Regime" at the top and the "Ordered Regime" at the bottom, with a Transitional Regime in between. The Ordered Regime is further subdivided into cases corresponding to Fast and Slow deformations.

---

*This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

The Chaotic Regime corresponds to the limiting case in which there are no degeneracies in the single-particle spectrum of the system. Insofar as degeneracies are associated with symmetries, this implies an absence of symmetries.

The Ordered Regime is the limiting case dominated by symmetries of various kinds, leading to strong degeneracies in the single-particle spectrum. (This is actually the more familiar situation on which, for two reasons, a lot of stress has been laid in the past. First, nuclei in their ground states usually seek out symmetric configurations in order to make use of the extra stability associated with a completely filled set of degenerate levels. Second, we only know how to solve the Schrodinger equation analytically in simple situations, characterized by a high degree of symmetry.)

A typical example of a situation dominated by symmetries is the famous Hill-Wheeler Box --an infinitely deep, sharp, box-like potential well, filled with eigenfunctions given by products of three sines: $\sin k_x x \cdot \sin k_y y \cdot \sin k_z z$. (Refs. (1), (2).) It is a prototype of an independent-particle model of a nuclear system. If you fill the Hill-Wheeler potential well with A particles up to a Fermi energy E_F, and then plot the total energy,

$$V = \sum_{i=1}^{A} \varepsilon_i ,$$

against a volume-preserving stretching deformation $1+\alpha$, you find a result shown in the middle of the transparency. (The longitudinal dimension is stretched by a factor $1+\alpha$, the transverse dimensions are compressed by a factor $1+\frac{1}{2}\alpha$.) Each steep, parabola-like curve is the energy $\Sigma\alpha_i$ corresponding to the case when the particles stay in their original orbitals and the wavelengths are simply stretched in one dimension and compressed in the other, following the deformation of the box. In other words, the nodal structure of each wave function is frozen. The energy for such a constrained deformation rises very steeply and soon exceeds the energy corresponding to the optimum redistribution of particles into the lowest orbitals at the given deformation. The energy corresponding to this optimum redistribution is given by a rippled curve consisting of the bottom pieces of many separate parabolas. The envelope of the parabolas represents the ground-state energy of the system pretty well, except for magic number situations. For example, in the case of the Hill-Wheeler box, $A = 60$ is a magic number for the cube. Thus the parabola centered at $\alpha = 0$ is anomalously low if $A = 60$. (See Ref. (2)).

Note that at each deformation there are many parabolas (the different excited states of the total system) and that there are many crossings between them. Such crossings of the energy levels of a system are possible, in general, only because of the symmetries present. (The reflection symmetries in the case of the box).

Suppose now we go over to the Chaotic Regime by putting in dents and corrugations in the sides of the box in order to break down the symmetries. The energy spectrum of the system is now shown in the upper picture in the transparency. The ground-state energy as a function of shape is not very different from what it was before (except that the special stability of the unstretched shape with $\alpha = 0$ has been destroyed). But the excited states are now not allowed to cross and each will be a (somewhat ripply?) curve with a trend more or less like the ground state.

If you look at these two plots of $\Sigma\varepsilon_i$ against deformation, you realize that there are two characteristic stiffnesses in the problem of stretching a prototype nuclear system: the stiff curvature of each parabola, and the soft curvature of the ground-state envelope. The stiff curvature is made up of A contributions, each associated with stretching and squeezing the separate wave functions. The total is proportional to the number of particles in the system and has, therefore, the properties of a coefficient of elasticity of a solid body. It can also be readily shown that the second, soft curvature is proportional to the area of the system and has, therefore, the characteristics of a surface-energy coefficient of a fluid.

The elastic stiffness coefficient for stretching (i.e., quadrupole type) distortions turns out to have a beautifully simple form, viz

$$V(\alpha) = V_o + (\text{coeff})\,\alpha^2, \tag{1}$$

where $\text{coeff} = V_o = (\frac{3}{5}E_F)A = \text{Total Energy!}$ (2)

The surface-energy coefficient γ can also be calculated and the result can be written as

$$4\pi r_o^2\gamma = \frac{3\pi}{40}\left(\frac{3}{\pi}\right)^{1/3} E_F \quad , \tag{3}$$

where r_o, the radius constant, is related to the density of particles. The actual numerical value of γ, as given by eq. (3), is unimportant, since an infinitely steep wall is not a realistic representation of a nuclear surface. The important result is that the stiffness is proportional to the surface area. Thus the ground-state energy in the Chaotic Regime, and the average trend of the ground state in the Ordered Regime, both follow the potential energy of a fluid with surface tension. On the other hand, for fast deformations in the Ordered Regime (when the nodal structures remain frozen) and for magic nuclei, the system acts like an elastic solid. Thus, in the Ordered Regime, the elastic response to stretching is given by Eq. (1) and in the Chaotic Regime by

$$V(\alpha) = \text{L.D.} = -c_1(\text{Volume}) + c_2(\text{Area}) + \text{corrections.} \tag{4}$$

The nuclear potential-energy problem is fairly well understood and what I described has been known for many years. It has also been known for some time that one can do a fair job of describing phenomenologically the Transitional Regime in the potential energy by simply multiplying the shell-effect (i.e. the deviation from the Liquid Drop energy, Eq. (4)) by a damping factor

$$e^{-\theta^2} \tag{5}$$

or

$$(1-2\theta^2)\, e^{-\theta^2} \quad , \tag{6}$$

where θ is a measure of the deformation from a symmetric, magic number configuration. (Refs. (2), (3)).

In the case of dynamics, the situation is much less clear. In the lower part of the transparency (Fig. 1) I have, nevertheless, made an attempt to order some of the simplest insights in a pattern paralleling the discussion of the statics.

First, in the Chaotic Regime, where there are no symmetries or regularities and the particle motions may be assumed to be randomized, there ought to be some simple limiting form of the dynamics of a large leptodermous, dolichohodous system, based on statistical, phase-space considerations. It seems fairly certain to me that this limit is represented by combining the Liquid Drop potential energy with the Wall Formula for dissipation and disregarding inertial effects, which appear to be small compared to the dissipation. The result is an astonishingly simple equation of motion for the time evolution of a nuclear shape (Refs. (4), (5), (6)):

$$\frac{dn}{dt} = \frac{P}{\rho\bar{v}} \qquad (7)$$

Here dn/dt is the rate of normal displacement of a point on the surface, P is the excess pressure at that point due to the conservative forces (surface tension and electric) and $\rho\bar{v}$ is a characteristic constant of the one-body dissipation theory (the product of the mass density of the system and the mean particle speed, about 1×10^{-22} MeV sec fm^{-4} for a nucleus). Equation (7) is, I believe, the dynamical counterpart of the Liquid Drop potential-energy equation of nuclear statics.

By way of contrast, in the dynamics of the Ordered Regime one has a more complicated situation, with all the crossing parabola-like energy levels. The discussion of this regime has centered since the work of Hill and Wheeler on estimating the probability flow at such crossings or near-crossings. There are the two limiting cases of adiabatic (slow) and diabatic (fast) motion (see, for example, Ref. (7)), where you either stay on the lowest soft envelope level or on one of the stiff parabolas with frozen nodal structure. The introduction of a "friction kernel" seems a natural way to span the two limits and recently Nörenberg has studied a particularly simple equation of motion with such a friction kernel. (Ref. (8)). It is shown in the lower part of Fig. 1. The friction kernel, with a characteristic amplitude a and a characteristic memory time τ, collects information about the speed of the deformation, $d\alpha/dt$, from $t = o$ to $t = t$, and the force constructed in this way is balanced against an intertial force, proportional to $d^2\alpha/dt^2$. You may easily verify that if the memory is short compared to characteristic dynamical times (i.e. if the motion is slow) the equation reduces to a damped motion. If the motion is fast, the equation reduces to the harmonic oscillator equation. In this limit the system is oscillating up and down one of the stiff elastic parabolas, with the elasticity coefficient given by V_o. It is noteworthy that if this elasticity is combined with an inertia of irrotational flow one gets a resonance frequency for stretching vibrations (as given by Bertsch or Nix & Sierk, see Ref. (9)):

$$\hbar\omega = \frac{(9\pi)^{1/3}}{\sqrt{2}} \frac{\hbar^2}{mr_o^2} A^{-1/3} = 64.7 A^{-1/3} \text{ MeV},$$

where m is the nuclear mass unit, 931.5 MeV. This is in good agreement with observed giant quadrupole oscillations. (It appears that, in effect, the Bertsch-Nix-Sierk interpretation of the giant resonances is equivalent to treating each volume element of the nucleus as a little Hill-Wheeler box, oscillating elastically with effectively frozen nodal structure.)

If my assignment of the giant resonances to the Ordered Regime, and of the wall formula dynamics to the Chaotic Regime is correct, then the analogy with the statics suggests looking for a phenomenological theory encompassing both limits. Could we do that by inserting some-where a factor $e^{-\theta^2}$, telling the system about its proximity to a magic, highly symmetric configuration?

Perhaps my transparency may stimulate some attempts in this direction. In any case, I hope that it has helped to bring out the relation between the giant-resonance dynamics and the wall formula dynamics and that it will discourage conclusions along the lines that, if one is confirmed experimentally, the other must be wrong. They each have a place in the appropriate regime of shapes and motions. What the quantitative range of validity is for each regime, is a question not easy to answer from first principles, and we will have to rely heavily on experimental findings and further studies of the Transitional Regime.

It seems clear that we are still a long may from being able to describe quantitatively all the different aspects of nuclear macroscopic dynamics. But I am beginning to have the feeling that we are getting close to a qualitative understanding of how it will probably all come out. The framework needed to support the richness of nuclear dynamics will have to include the description of fluids (ordinary, superfluid and superviscid), as well as the elastic vibrations of solids and, perhaps, their plastic flow. In this connection I would like to finish with a paragraph from a note by Georg Süssmann, entitled "On the continuity between the solid and the liquid state." The note was written in Berkeley in 1973 and remains, I believe, unpublished. In the introduction Süssmann says:

"According to our usual experience there is a rather clear cut distinction between solids and liquids. The vast majority of condensed materials is, under normal conditions, either shape preserving and thus in a solid state, or quickly fluid and thus in a liquid state. (This fact is stressed by the remarkable discontinuity known as melting which seems to extend to very high pressures and temperatures.) Of course, we know of counter examples as tar or (silly) putty, but they are rare and show a rather complicated behavior. There are two phenomena that bridge the gap between the solids and the fluids: viscosity tends to make a fluid somehow like a solid, whereas relaxation tends to make a solid somehow like a fluid. In the following we will give a short account of these concepts which interpolate between elastodynamics and hydrodynamics in the much broader frame of plastodynamics."

I believe that, in addition to first-principles numerical studies on the lines of TDHF with residual interactions, it will be useful to develop a phenomenological theory of nuclear dynamics. In this endeavor the mathematical structure of Süssmann's plastodynamics should be a useful background.

References

1. D.L. Hill and J.A. Wheeler, Phys. Rev. 89 (1953), 1102, especially Fig. 12.
2. W.D. Myers and W.J. Swiatecki, Nucl. Phys. 81 (1966), 1, especially Fig. 7.
3. W.D. Myers, "Droplet Model of Atomic Nuclei", IFV Plenum, 1977.
4. W.J. Swiatecki, Progr. Particle & Nucl. Phys. 4 (1980), 383.
5. J. Randrup and W.J. Swiatecki, Ann. Phys. (N.Y.) 125 (1980), 193.
6. J.P. Blocki et al., Ann. Phys. (N.Y.) 113 (1978), 330.
7. L. Wilets, "Theories of Nuclear Fission", Clarendon Press, Oxford, 1964.
8. W. Nörenberg, Phys. Lett. 104B (1981), 107.
9. J.R. Nix and A.J. Sierk, Phys. Rev. C21 (1980), 396.

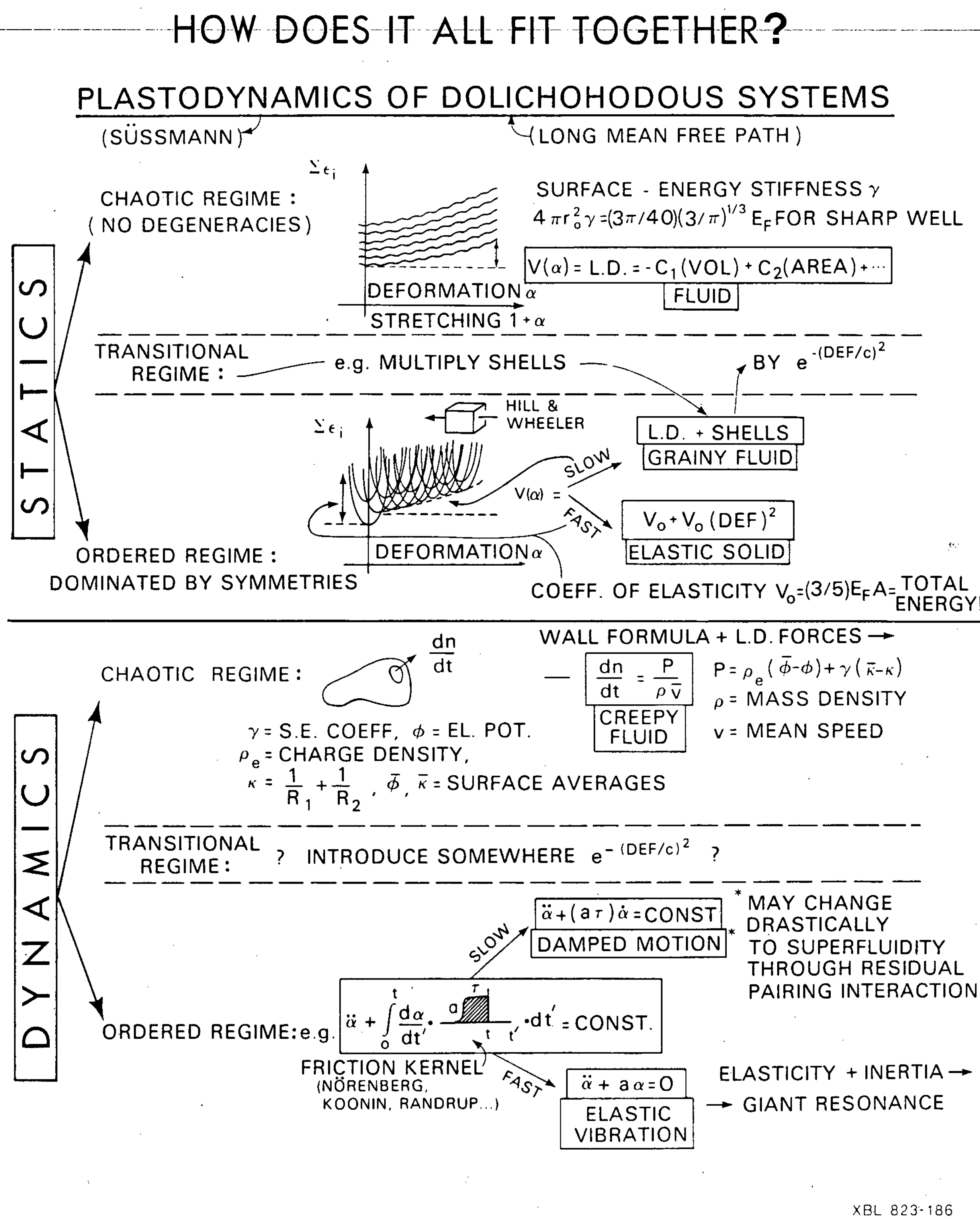

Fig. 1. An attempt to sketch the relation to each other of various limiting theories of nuclear statics and nuclear dynamics. The principal message is that the description of macroscopic nuclear dynamics may be expected to call for a rich mathematical structure, including the theory of fluids (ordinary, superfluid and superviscid) of elastic solids and of plastic flow, depending on the presence or absence of symmetries and regularities in the configurations and deformations in question.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720