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VECTOR CURRENTS AND CURRENT ALGEBRA. III. DUAL RESONANCE MODEL WITH UNIVERSALLY COUPLED VECTOR MESONS*

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ABSTRACT

We extend the model for conserved vector currents in the dual resonance model to include the infinite set of universally coupled vector mesons. One- and two-current amplitudes satisfying current algebra and factorizing on the M highest trajectories are constructed for a form factor falling like $(q^2)^{-M}$. Physically acceptable completely factorized amplitudes are not obtained in the limit $M \to \infty$, however. Complete factorization and unsubtracted dispersion relations in q^2 for single-current amplitudes are shown to indeed imply exponentially falling form factors. However, we then prove that no acceptable completely factorized two-current amplitudes can be constructed from a current coupling only to the universal vector mesons.

I. INTRODUCTION

The successful construction of a dual Reggeized resonance model (DRM) for hadronic amplitudes[1] suggests the possibility of a similar model for amplitudes involving the electromagnetic and weak currents. In the case of axial currents, however, the present dual N-particle amplitudes suffer an obvious and fatal flaw: taken as amplitudes for N pions, they fail to vanish for $p_i^{\mu} \to 0$ (except for $N = 4$). Consequently, the formulation of a model for a physically reasonable axial current with a pion-pole-dominated divergence requires a simultaneous reformulation of the hadronic model.

For vector currents, on the other hand, the situation is more promising. A fundamental requirement for the existence of a physically acceptable vector-meson-dominated conserved current is the existence of universally coupled vector mesons, since such mesons give the only contribution for soft currents $(q_i^{\mu} \to 0)$ and provide the full charge coupling. This requirement is met by the DRM, since as we have previously shown,[2] the lowest mass vector meson couples universally.[3] Furthermore, assuming the dominance of this vector meson, we have been able to construct[2] dual amplitudes for one current $[V_c^{\mu}(q)]$ and two currents $[M_{ab}^{\mu\nu}(q_1,q_2)]$ plus N spinless hadrons that obey exactly the current algebra divergence condition,

$$q_{1\mu}\, M_{ab}^{\mu\nu}(q_1,q_2) = i\, f_{abc}\, V_c^{\nu}(q_1 + q_2)\,, \tag{1.1}$$

and factorize on all leading trajectories.

In order to satisfy factorization for nonleading trajectories and obtain more rapidly falling form factors, it is natural to include poles in q^2 corresponding to higher mass vector mesons. Here we extend our previous model to include all the vector mesons in the DRM which couple universally, i.e., the lowest mass vector meson and its recurrences, one at each mass $m_\ell^2 = m^2 + 1 + \ell$ $(\ell = 0, 1, 2, \cdots)$.[4] Since the low lying trajectories in the DRM have a very large degeneracy, these vector mesons are only a small subset of the total, but, as noted above, they play a particularly vital role in models for currents.

Applying generalized vector-meson dominance for these mesons, we can construct one- and two-current amplitudes that (i) obey the current algebra condition, (ii) factorize on the M highest trajectories, and (iii) have form factors that fall like $(q^2)^{-M}$. On the other hand, if only leading trajectory factorization is required, the current algebra condition can be satisfied for arbitrary form factors, as demonstrated in Appendix B below.

We feel that these results give a good indication of the power of factorization in determining the structure of currents in zero-width models and suggest that in a full solution to the problem form factors will fall exponentially. However, the limit $M \to \infty$ of our amplitudes does not lead to a full solution. Indeed, we prove that complete factorization cannot be obtained for currents satisfying our requirements (see Sec. II and Figs. 1 and 2), if only the universally coupled vector mesons are included. If factorization of $M^{\mu\nu}$ in channels containing a single current (Fig. 2 c) is imposed, form factors are

required to be exponential, but the resultant amplitude develops unphysical poles in the two current channel which is dual to the single-current channels.[5]

Therefore approximate solutions, such as those presented here, are the most that can be obtained if only the universally coupled vector mesons are included. However, a completely factorizable solution may be obtainable if some or all of the other vector mesons are included. The major difficulty with this lies in the tremendous number of existing parameters (the current-vector-meson coupling constants, f_n--see Fig. 1 a) that are apparently arbitrary if only single-current amplitudes are considered, but are in fact severely constrained in a non-obvious manner by the connection of these amplitudes to the two-current amplitudes through factorization (Fig. 2 c). In the conclusion we discuss briefly the full problem and suggest some possible ways of formulating it in a general manner.

In Sec. II we review the properties of current amplitudes and the strong consequences duality has for them. We introduce a general operator notation for currents. The divergence (Ward) identities in the DRM which are essential to the construction of current amplitudes are discussed in detail in Appendix A. We then discuss the properties of the current algebra parameterization with M highest trajectories factorizable. The details are given in Appendices B and C. In Sec. III, we use a systematic approach to the construction of factorized current amplitudes in order to demonstrate the insufficiency of the universally coupled vector mesons: the two current amplitudes are constructed from

the single-current amplitudes by use of quadratic factorization. Finally, some general comments on dual models for currents and the work of other authors[6-10] are made in Sec. IV.

II. PARTIALLY FACTORIZABLE CURRENT ALGEBRA AMPLITUDES

We seek here, as in II, currents consistent with the simplest dual resonance model for mesons: the hadronic amplitudes are products of orbital factors, $B(p_1,\cdots p_N)$ (N-point beta functions),[1] and SU(3) internal symmetry factors,[11] $\frac{1}{2}\mathrm{Tr}(\lambda_1\lambda_2\cdots\lambda_N)$, summed over permutations of the particle momenta (p_i).[4] Of course, all the existing hadronic models are at the moment rather conjectural, and it is possible that only new models will admit consistent vector currents. In any case, we believe our methods have quite general applicability: for example, the existence of universally coupled vector mesons follows with only very natural, weak restrictions on the trajectories in the N-point beta functions.[12]

We note one feature of the hadronic model which has vital importance in the construction of current amplitudes. The hadronic spectrum[13] is in fact smaller than that exhibited explicitly by the operator formalism of FGV[14] because certain "spurious" states actually do not couple.[2,13] We will insure the absence of such states by using the modified vertex function,[15] $\hat{V}(p)$ which has no coupling to spurious states.

Before describing the partially factorizable current algebra parameterizations, we discuss briefly some of the general properties of dual current amplitudes. For more extensive discussion the reader is referred to I.

A. Properties of Dual Vector Current Amplitudes

The assumption that current amplitudes satisfy planar duality has important consequences because the divergence conditions can be applied to each term in the dual decomposition of an amplitude.[2] Thus CVC implies, for each term of the single current amplitude[16] (see Fig. 3),

$$q_\mu \, V_{i,P}{}^{\mu}(q) = 0 \, . \tag{2.1}$$

As $q_\mu \to 0$, this term has just two soft poles,

$$V_{i,P}{}^{\mu}(q) \to \left[\frac{q^\mu + 2p^\mu_{P(i-1)}}{\left(q + p_{P(i-1)}\right)^2 - m^2} - \frac{q^\mu + 2p^\mu_{P(i)}}{\left(q + p_{P(i)}\right)^2 - m^2} \right] A_{\text{hadron}} \, ,$$

and clearly satisfies (2.1).

For the two-current terms with adjacent currents there is only one soft pole and thus the divergence in q_1 is nonvanishing as $q_{1\mu} \to 0$. By use of CVC and quadratic factorization (Fig. 2 c), this can be extended to $q_1{}^2 = 0$, $q_2{}^2 = t$, and all $k_i{}^2$, implying a $J = 1$ fixed pole in the t channel. This yields the usual right-signature fixed poles in isospin antisymmetric amplitudes and in addition wrong signature fixed poles in isospin symmetric amplitudes, a stronger result than that which obtains without duality (for the case of two hadrons this actually can be shown with just the assumption of dispersion relations for signatured amplitudes).

In the remainder of this paper exotic resonances are assumed to be absent.[17] Duality and current algebra (1.1) then give the particularly simple divergence conditions (see Fig. 4)[16]

$$q_{1\mu}\, M^{\mu\nu}_{ij,P}(q_1,q_2) = 0\,, \qquad (i \neq j) \tag{2.2}$$

and

$$\begin{aligned} q_{1\mu}\, M^{\mu\nu}_{ii,P}(q_1,q_2) &= V_{i,P}{}^{\nu}(q_1 + q_2)\,, \\ M^{\mu\nu}_{ii,P}(q_1,q_2)q_{2\nu} &= -V_{i,P}{}^{\mu}(q_1 + q_2)\,. \end{aligned} \tag{2.3}$$

These actually hold independently of current algebra for $q_1^2 = 0$ and $q_2^2 = t$. We remark that it is quite natural that only the adjacent current terms have fixed poles, since they are the only ones with singularities in the two-current (t) channel where fixed poles occur. As we found in II, the nonadjacent current terms automatically satisfy (2.2) and need not be considered further.

In Sec. I we have briefly mentioned the strong consistency conditions the hadronic amplitudes place on the current amplitudes. These are shown diagramatically in Figs. 1 and 2. We have already emphasized the power of the quadratic factorization constraint (Fig. 2 c), and our explicit models also show this clearly. The linear factorization constraints (Figs. 1 b and 2 b) are weaker, but will play an important role in Sec. III. The determination of the current amplitudes by the vector-meson amplitudes through unsubtracted dispersion relations (USDR) in q^2 (Figs. 1 a and 2 a) means that they can be expected to possess many of the same properties; e.g., Regge behavior, duality (pole-dominated USDR in subenergies), etc. However, since the sum over vector mesons must in fact be infinite, the possibility of nonuniformities in

convergence must be kept in mind. Indeed, the existence of a fixed pole in the two-current amplitude which cannot be present in the vector-meson amplitudes, implies such a nonuniformity. On the other hand, there seems to be no reason why duality should be violated; the current algebra divergence condition is not in conflict with duality, i.e., it does not require that any invariant amplitude not satisfy USDR in the subenergies.[18]

B. Operator Approach to Currents

In order to implement these consistency conditions, we find it convenient to use the operator formalism. All the intermediate states in Figs 1 and 2 have the form[14]

$$|\lambda\rangle = \prod_r [a^{\mu+}_{(r)}]^{\lambda_r}/(\lambda_r!)^{\frac{1}{2}}|0\rangle$$

at mass $m^2 + n$, $n \geq R \equiv \sum r\,\lambda_r$. This rich spectrum contains many vector mesons: $a^{\mu+}_{(r)}|0\rangle$ and many more formed from contraction of higher rank tensors with $g^{\mu\nu}$, $\epsilon^{\mu\nu\lambda\sigma}$, and q^μ.

The lowest mass vector meson $(m^2 + 1)$, $a^{\mu+}_{(1)}|0\rangle$, used in the model of II, recurs at $m^2 + 2$, $m^2 + 3, \cdots$. These mesons play a unique role because their spin-one parts are exactly conserved even off the mass shell. Their amplitude to N spinless particles is thus conserved,[2,13] $q_\mu B^\mu(q) = 0$, where

$$B^\mu(q) = \langle 0|[\sqrt{2}\, a^\mu_{(1)} + q^\mu]|p\rangle \qquad (2.4)$$

with

$$|p\rangle = \hat{V}(p_1)\, D(R,k_1^2)\, \hat{V}(p_2) \cdots \hat{V}(p_N)|0\rangle$$

$$= V(p_1)\, D(R,k_1^2)\, V(p_2) \cdots V(p_N)|0\rangle \ .$$

Conservation follows from the fact that

$$q_\mu \langle 0|[\sqrt{2}\, a_{(1)}{}^\mu + q^\mu]| = \langle 0|S(q)$$

is the first spurious state generated by the spurious state operator $S(q)$[19] (see Appendix A), and thus has vanishing coupling to $|p\rangle$.

These universally coupled vector mesons can be used to construct conserved vector current amplitudes. Thus, as will be shown in Sec. III, the "current operator"

$$\mathcal{J}^\mu(q) = F(q^2)[\sqrt{2}\, a_{(1)}{}^\mu + q^\mu] \tag{2.5}$$

is conserved and generates the couplings $\langle 0|\mathcal{J}^\mu(q)\, \hat{V}(p)|\lambda\rangle$ of the current to a spin-zero particle (p) and an excited state $|\lambda\rangle$ $(p + q)$. It obeys USDR in q^2 if $F(q^2)$ falls faster than any power as $|q^2| \to \infty$. Note the crucial role the vertex $\hat{V}$ plays: it both eliminates spurious excited states $|\lambda_s\rangle$ and insures current conservation. The one-to-one relationship between current conservation and absence of spurious states is also seen clearly in the calculations of Sec. III.

More general conserved currents can be constructed from operators like $(q^2 g^{\mu\nu} - q^\mu q^\nu)\, a_{(n)}{}^\nu$, $q \cdot a_{(m)}\, a_{(n)}{}^\mu - q \cdot a_{(n)}\, a_{(m)}{}^\mu$, etc. Indeed the fundamental divergence conditions are easily put in

algebraic form. Condition (2.1) becomes

$$q_\mu \mathcal{J}^\mu(q) = \mathcal{O}(q) , \tag{2.6}$$

where $\mathcal{O}$ is some operator which, like S, satisfies $\langle 0|\mathcal{O}|p\rangle = 0$, and (2.3) becomes

$$\mathcal{O}(q_2)\, \hat{V}(p_1) \,D\, \hat{V}(p_2)\cdots D\, \hat{V}(p_N)\, \mathcal{J}(q_1)$$

$$= \hat{V}(p_1) \,D\, \hat{V}(p_2)\cdots D\, \hat{V}(p_N)\, \mathcal{J}(q_1 + q_2) , \tag{2.7}$$

to within terms which have vanishing $|0\rangle$ matrix elements. This formulation of the conditions is undoubtedly much more general than the specific N-point beta function model considered here. The difficulty with solving them is that (2.7) constrains the huge class of solutions to (2.6) very strongly but in a very nonobvious manner. For the remainder of this paper we make the approximation (2.5).

C. Partially Factorizable Solutions

With the restriction to dominance by the universally coupled vector mesons, we can construct approximate solutions to (2.1) through (2.3) [alternatively, (2.6) and (2.7)], if we allow violations of factorization on low lying trajectories. With this restriction, complete factorization cannot be obtained, as we show in Sec. III. Here we discuss the important features of the approximate solutions; the mathematical details are relegated to Appendices B and C.

In II we gave amplitudes with single vector-meson poles which satisfy the current algebra divergence conditions and factorize on leading trajectories. If only leading-trajectory factorization is

required, this can be generalized to include all universally coupled vector mesons with arbitrary coupling constants f_n (see Appendix B).

More interesting, however, are the current algebra amplitudes which factorize on all trajectories lying less than M units below the leading trajectory and have the form factor

$$F_M(q^2) = \overbrace{\prod_{\ell=0}}^{M-1} \left(1 - \frac{q^2}{m^2 + 1 + \ell}\right)^{-1} = \frac{B[1 - \alpha(q^2), M]}{B(1 - \alpha(0), M]} \underset{q^2 \to \infty}{\sim} (q^2)^{-M} \qquad (2.8)$$

(see Appendix C). This model, which satisfies essentially all the other requirements discussed above, is obtained by truncating the expansion (A.4) for $\hat{V}$ after M terms to yield $\hat{V}_M$ (see Eq. C.3). The factorization of the single-current amplitude is then violated by the presence of spurious states on trajectories that are M or more units below the leading one. We note also that the current matrix element for a spinless particle and an excited state on a trajectory k units below the leading trajectory in general behaves like

$$F_M(q^2)(q^2)^k \sim (q^2)^{-M+k} \qquad (2.9)$$

as $q^2 \to \infty$.

Factorization is violated in a more serious manner by the two-current amplitudes. These are written, following Brower and Halpern[20] and II, as the sum of three terms

$$M^{\mu\nu}(q_1,q_2) = M_H^{\mu\nu}(q_1,q_2) + M_C^{\mu\nu}(q_1,q_2) + M_{FP}^{\mu\nu}(q_1,q_2) \; . \qquad (2.10)$$

The terms $M_H^{\mu\nu}$ and $M_C^{\mu\nu}$ are purely Regge behaved; $M_H^{\mu\nu}$ contains

all the vector-meson poles and $M_C^{\mu\nu}$ cancels its unwanted Regge-behaved divergence. The exact current algebra divergence comes from $M_{FP}^{\mu\nu}$, which has fixed poles in J_t. The term $M_H^{\mu\nu}$ is constructed by using $\hat{V}_M$ and has contributions from spurious states on trajectories displaced by M units or more. The sum $M_C^{\mu\nu} + M_{FP}^{\mu\nu}$ contributes only to such trajectories and the contribution is badly nonfactorizable. Furthermore, since this piece has no poles in q^2, it corresponds to subtractions in the q^2 dispersion relations contrary to our requirements. This fact, along with (2.9), means that the current algebra sum rule is satisfied uniformly in q^2 and is saturated for large q^2 by the low lying nonfactorized poles. This hints at the failure of this parameterization as $M \to \infty$.[21] In fact, in this limit $M_C^{\mu\nu} + M_{FP}^{\mu\nu}$ would have no poles in k_i^2 (Fig. 2 c). Since it is Regge behaved for $k_i^2 \to -\infty$ and nonzero, it must violate Regge behavior for $k_i^2 \to +\infty$.

We believe that more general parameterizations with the M highest trajectories factorizing can be constructed with only the condition that $F(q^2)$ decrease at least as rapidly as $(q^2)^{-M}$. This connection between the asymptotic behavior of form factors and factorization is very suggestive--but only suggestive, due to the negative result of the following section.

III. INSUFFICIENCY OF UNIVERSALLY COUPLED VECTOR MESONS

We first construct the single-current amplitudes assuming dominance of the universally coupled vector mesons and find that factorization and unsubtracted dispersion relations in q^2 imply exponential form factors. The two-current amplitudes are then constructed from the single-current amplitudes and are found to possess unphysical singularities which violate linear factorization.

The complete determination of the two-current amplitudes by the single-current amplitudes follows from quadratic factorization and unsubtracted dispersion relations which imply[22] (see Fig. 2 c)

$$M^{\mu\nu}(q_2,p_1,\cdots,p_N,q_1) = \sum_n \frac{V_n{}^{\mu}(q_2,p_1,\cdots,p_i)\, V_n{}^{\nu}(p_{i+1},\cdots,p_N,q_1)}{k_i{}^2 - m_n{}^2}, \tag{3.1}$$

where n labels the full spectrum of internal states in the $k_i{}^2$ channel ($k_i = q_2 + p_1 + \cdots + p_i$). We remind the reader that, although $V_n{}^{\mu}$ is exactly divergenceless on-mass-shell ($k_i{}^2 = m_n{}^2$) and in the expansion

$$V_n{}^{\mu}(q,p_1,\cdots,p_i) = q^{\mu}\, V_n{}^{(0)} + p_1{}^{\mu}\, V_n{}^{(1)} + \cdots + p_i{}^{\mu}\, V_n{}^{(i)} \tag{3.2}$$

each invariant amplitude, $V_n{}^{(\ell)}$, is evaluated on-mass-shell, the 4-vectors allow nonzero divergence off-mass-shell,

$$q_{\mu}\, V_n{}^{\mu}(q,p_1,\cdots,p_i) = (k_i{}^2 - m_n{}^2)\, D_n \ . \tag{3.3}$$

Consequently, both $M^{\mu\nu}$ and its divergence are determined entirely by the on-mass-shell amplitudes $V_n{}^{\mu}$.

A. Single-Current Amplitude and Exponential Form Factors

A likely candidate for the single-current amplitude is[23]

$$V_\lambda{}^{\mu}(q,p_1,\cdots,p_i) = F(q^2)\langle 0|[\sqrt{2}\, a_{(1)}{}^{\mu} + q^{\mu}]\, \hat{V}(p_1)\, D\cdots\hat{V}(p_i)|\lambda\rangle \,, \tag{3.4}$$

since the vertex $\hat{V}$ eliminates spurious states and makes $V_\lambda{}^{\mu}$ exactly conserved (see Eqs. A.1 and A.5). Further, in order to compensate the polynomials in q^2 introduced by $\hat{V}$ (see Eq. A.4), one suspects that $F(q^2)$ must fall faster than any power. We now demonstrate that this is indeed the case.

The single-current amplitude by assumption is determined from the amplitudes for the on-mass-shell universally coupled vector mesons of mass $m_\ell{}^2$, $B_{\ell\lambda}{}^{\mu}$ by writing unsubtracted dispersion relations in q^2. Thus[22]

$$V_\lambda{}^{\mu} = \sum_{\ell=0}^{\infty} f_\ell\, F_\ell(q^2)\, B_{\ell\lambda}{}^{\mu},\quad F_\ell(q^2) = 1/[1 - q^2/(m^2 + 1 + \ell)] \,, \tag{3.5}$$

where

$$B_{\ell\lambda}{}^{\mu} = \langle 0|\sum_{r=0}^{J}\{[\sqrt{2}\, a_{(1)}{}^{\mu} + q^{\mu}] - \frac{r}{\ell - r + 1}\, q^{\mu}\}P_r(\ell)\, V(p_1)D\cdots V(p_i)|\lambda\rangle \,, \tag{3.6}$$

and

$$P_r(x) = \frac{x(x-1)\cdots(x-r+1)\, S(S-1)\cdots(S-r+1)}{r!(m^2+1)\cdots(m^2+r)} \,.$$

This expression is obtained by using (A.6) to eliminate all except one $\hat{V}$ and then letting the projection operator act on the on-mass-shell state $\langle 0|[\sqrt{2}\, a_{(1)}{}^{\mu} + q^{\mu}]$ at $q^2 = m^2 + 1 + \ell$ (see Eq. A.9). The sum in (3.6) terminates at $r = J$ since P_r annihilates all on-mass-shell states $|\lambda\rangle$ at $k^2 = m^2 + J$ for $r > J$.

The difficulty with (3.5) for general q^2 is that spurious intermediate states contribute and it is not conserved. By demanding either (i) no spurious states contribute,

$$V_{\lambda_s}{}^{\mu} = 0, \qquad \text{for } k_i^2 = m^2 + J ,$$

or (ii) current conservation,

$$q_{\mu}\, V_{\lambda}{}^{\mu} = 0, \qquad \text{for } k_i^2 = m^2 + J ,$$

we arrive at identical conditions on the f_{ℓ},

$$\sum_{\ell=0}^{\infty} f_{\ell}(m^2 + 1 + \ell)^n = 0 \text{ , for } n = 1,2,3,\cdots. \tag{3.7}$$

Writing a dispersion relation for $F(q^2)$, it is easy to see that (3.7) requires that $F(q^2)$ fall faster than any power as $|q^2| \to \infty$ ("exponential form factor").

With exponential form factors, we have

$$F(q^2)\, Q(q^2 - m^2 - 1) = \sum_{\ell=0}^{\infty} f_{\ell}\, F_{\ell}(q^2)\, Q(\ell)$$

for any finite polynomial Q, as is easily demonstrated by writing a dispersion relation for the left-hand side. Applying this result to (3.5) for on-mass-shell states $|\lambda\rangle$ we obtain

$$V_\lambda{}^\mu = F(q^2)\langle 0|\sum_{r=0}^{J}\{[\sqrt{2}\, a_{(1)}{}^\mu + q^\mu] - \frac{r}{q^2 - m^2 - r}\, q^\mu\}$$

$$\times\ P_r(q^2 - m^2 - 1)\ V(p_1)D\cdots V(p_i)|\lambda\rangle$$

$$= F(q^2)\langle 0|[\sqrt{2}\, a_{(1)}{}^\mu + q^\mu]\ \hat{V}(p_1)D\cdots\hat{V}(p_i)|\lambda\rangle\ .$$

Therefore, (3.4) with exponential form factors, is indeed required by unsubtracted dispersion relations in q^2 and factorization (correct internal spectrum without spurious states).

B. Two-Current Amplitude

Using the above single-current amplitude (3.4) and quadratic factorization (3.1), we easily obtain the two current amplitude[24]

$$M^{\mu\nu}(q_1,q_2) = F(q_1{}^2)\ \hat{B}^{\mu\nu}(q_1,q_2)\ F(q_2{}^2)\ , \tag{3.8}$$

where

$$\hat{B}^{\mu\nu}(q_1,q_2) = -\langle 0|[\sqrt{2}\, a_{(1)}{}^\nu + q_2{}^\nu]\hat{V}(p_1)D\cdots\hat{V}(p_N)[\sqrt{2}\, a_{(1)}{}^\mu + q_1{}^\mu]|0\rangle\ .$$

The structure of this amplitude is most easily studied by using its integral representation which is readily obtained from (A.4) and (A.9). Using the notation of Appendix A, we have

$$\hat{B}^{\mu\nu}(q_1,q_2) = -\langle \mathcal{B}^{\mu\nu}\rangle_{N+2}\ , \tag{3.9}$$

where

$$\mathcal{B}^{\mu\nu} = \left[(\bar{\mathcal{V}}_{(1)}{}^{\mu} + \frac{q_1{}^{\mu}}{m^2 + 1 - q_1{}^2} u' \frac{\partial}{\partial u'})(\mathcal{V}_{(1)}{}^{\nu} + \frac{q_2{}^{\nu}}{m^2 + 1 - q_2{}^2} u' \frac{\partial}{\partial u'}) \right.$$

$$\left. + 2u\, g^{\mu\nu} \right] {}_2F_1(m^2 + 1 - q_1{}^2,\ m^2 + 1 - q_2{}^2;\ m^2 + 1;\ u') \Big|_{u'=u} .$$

We examine the singularities in the two-current (t) channel. They arise from divergences of the integrand as $u \to 1$ [i.e., $I_{N+2} \propto (1 - u)^{-\alpha_t - 1}$], where the hypergeometric function has the behavior

$$ {}_2F_1(m^2 + 1 - q_1{}^2,\ m^2 + 1 - q_2{}^2;\ m^2 + 1;\ u) $$

$$= \frac{\Gamma(m^2 + 1)\ \Gamma(q_1{}^2 + q_2{}^2 - m^2 - 1)}{\Gamma(q_1{}^2)\ \Gamma(q_2{}^2)}$$

$$\times\ {}_2F_1(m^2 + 1 - q_1{}^2, m^2 + 1 - q_2{}^2;\ m^2 + 2 - q_1{}^2 - q_2{}^2;\ 1 - u)$$

$$+ (1 - u)^{q_1{}^2 + q_2{}^2 - m^2 - 1} \frac{\Gamma(m^2 + 1)\ \Gamma(m^2 + 1 - q_1{}^2 - q_2{}^2)}{\Gamma(m^2 + 1 - q_1{}^2)\ \Gamma(m^2 + 1 - q_2{}^2)}$$

$$\times\ {}_2F_1(q_1{}^2, q_2{}^2;\ q_1{}^2 + q_2{}^2 - m^2;\ 1 - u) . \quad (3.10)$$

The first term yields the usual poles on the trajectory α_t and its daughters. The second term, however, gives poles at $\alpha_t - q_1{}^2 - q_2{}^2 + m^2 + 1 = 2q_1 \cdot q_2 + 1 = 0, 1, 2, \cdots$. Such singularities are clearly unphysical, since their positions depend on the current "masses" $q_i{}^2$. The presence of these anomalous singularities in place

of the desired fixed pole can be understood, if we notice that our amplitude (3.8) has vanishing divergence, $q_{1\mu}\,\hat{B}^{\mu\nu} \equiv 0$. As we argued in I, the absence of an unphysical $J = 1$ intermediate state at $t = q_2^{\,2}$ implies a nonvanishing divergence $q_{1\mu}\,M^{\mu\nu} \to V^{\nu}$ for $q_{1\mu} \to 0$ which, when combined with quadratic factorization, implies a fixed pole. Our anomalous singularity violates the conditions of this theorem by providing just such an unphysical state.[25]

The origin of the vanishing divergence of our $M^{\mu\nu}$ can be seen clearly in (3.4). Although, if the invariant amplitudes are evaluated on-mass-shell at $k_i^{\,2} = m^2 + J$, the infinite series for $\hat{V}$ terminates and the basic equation (3.3) holds, it is clear that (3.4) as it stands represents a certain off-shell continuation which is divergenceless everywhere. Since in our case (3.1) can be rewritten in terms of this off-shell continuation,

$$M^{\mu\nu} = \sum_{\lambda\lambda'} V_{\lambda}{}^{\nu} \langle \lambda | D(R, k_i^{\,2}) | \lambda' \rangle \, V_{\lambda'}{}^{\mu} \ ,$$

it is obvious that $M^{\mu\nu}$ has vanishing divergence. We note that this off-shell continuation is never needed in our derivation of $M^{\mu\nu}$, since $M^{\mu\nu}$ obeys USDR in $k_i^{\,2}$, but unhappily it provides an equivalent formulation. This appears to be the origin of the difficulty with the universally coupled vector meson approximation.

IV. CONCLUSION

We have seen that, if only the universally coupled mesons are included, vector currents completely consistent with the present DRM cannot be constructed. However, the existence of partially factorizable amplitudes consistent with current algebra is encouraging and leads to some optimism that the inclusion of further vector mesons may allow a full solution. The difficulty with this (and part of the source of optimism) is the vast number of mesons available in the DRM. Clearly some guide to selecting the appropriate current (analogous to the minimal principle of electrodynamics) is needed. We mention two approaches that may yield this guide.

First, the algebraic approach suggested in this paper (Sec. II.B) should be developed further. By expressing the divergence conditions as conditions on N-point functions we were able to restrict our attention to the current-ground state-arbitrary resonance vertex. Of course, the fundamental object in a zero-width model is the vertex for a current and two arbitrary resonances. The current algebra divergence conditions are expressed naturally in terms of it, but, at present, the conditions that duality imposes are not well understood. Generally, one would like to be able to see directly how the singularities in dual channels (e.g., t and k_i^2) are related. This would help circumvent difficulties like those encountered in Sec. III where we satisfied factorization in one channel and then found unpermitted singularities in the dual channel. We expect that a deeper understanding of duality[26] will allow a concise vertex formulation of the conditions on currents.

A second approach is to temporarily ignore the factorization constraints and explore various dual parameterizations satisfying current conservation and having good large-q^2 behavior (e.g., Bjorken limit, electroproduction limit, etc.). As has been previously noted,[9] parameterizations like those presented here have bad large-q^2 behavior, whereas the parameterizations given by other authors[6-9] have good behavior. Furthermore, amplitudes of the form discussed in Refs. 7-10 have many properties suggested by field theory, e.g., relationships between asymptotic behavior of form factors and the spins of particles and fixed poles, electroproduction scaling, relationship between threshold behavior of electroproduction structure functions and elastic form factors, etc.[27] On the other hand, the only models which have successfully satisfied current algebra for N-point functions[2,10] have used the divergence identities and no one has yet succeeded in combining these with current amplitudes of a more general type.[28] If this could be done with just leading trajectory factorization, much could probably be learned about the role of high-mass vector mesons.

Finally, the results of this paper suggest a crucial role for exponential form factors in a factorized model for conserved currents. We suggest that the difficulty of combining the requirements of good large-q^2 behavior, factorization, and conservation may be reduced if exponential form factors are assumed initially.

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APPENDIX A. DIVERGENCE IDENTITIES

In this Appendix we review and generalize the divergence (Ward) identities of the DRM which are essential to the construction of current amplitudes.

(i) Operator Notation

The divergence identities are conveniently expressed in the FGV operator notation by using the spurious state operator[19]

$$S(k) = \sqrt{2}\, k \cdot a_{(1)} + k^2 + \sum_r [r(r+1)]^{\frac{1}{2}} a^+_{(r)} \cdot a_{(r+1)} - R \,. \tag{A.1}$$

We have followed the notation of Ref. 15. Spurious states $\langle \lambda_s |$ then have the form

$$\langle \lambda_s | = \langle \lambda | S(k) \quad , \qquad (\text{all } \langle \lambda |) \,,$$

where k is the momentum of the state directed to the right. The basic "commutation" relations of S with the vertex and propagator of FGV are

$$S(k)\, V(p) = V(p)[S(-k-p) - p^2]$$

and

$$[S(k) - m^2 - \ell]\, D(R+\ell, k^2) = D(R+\ell+1, k^2)[S(k) - \ell] \,, \tag{A.2}$$

for all ℓ.

It is also convenient to introduce the "projection" operator[15,29]

$$\mathcal{P}(k) = 1 - [S^+(-k) - m^2] \frac{1}{S(k)[S^+(-k) - m^2]} S(k) ,$$

which has the explicit form[15]

$$\mathcal{P}(k) = \sum_{\ell=0}^{\infty} \frac{(-1)^\ell}{\binom{-m^2 - 1}{\ell}} \binom{S^+(-k) - m^2}{\ell} \binom{S(k)}{\ell} \quad . \tag{A.3}$$

To eliminate spurious intermediate states one replaces the vertex $V(p)$ of FGV by[15]

$$\hat{V}(p) = \mathcal{P}(k)\, V(p)\, \mathcal{P}^+(-k - p)$$

$$= \sum_{\ell=0}^{\infty} \frac{(-1)^\ell}{\binom{-m^2 - 1}{\ell}} \binom{S^+(-k) - m^2}{\ell} V(p) \binom{S(k + p) - m^2}{\ell} . \tag{A.4}$$

This vertex has vanishing coupling to spurious states, $\langle \lambda_s | \hat{V} = \hat{V} | \lambda_s \rangle = 0$, since

$$S(k)\, \hat{V}(p) = \hat{V}(p)\, S^+(-k - p) = 0 \quad . \tag{A.5}$$

The following identities are useful:

$$\hat{V}D\hat{V} = \hat{V}DV = VD\hat{V} \quad ; \tag{A.6}$$

$$\hat{V}|0\rangle = V|0\rangle, \quad \langle 0|\hat{V} = \langle 0|V \quad , \tag{A.7}$$

where $|0\rangle$ has momentum squared of m^2;

$$[(S - m^2 - \ell)\cdots(S - m^2 - 1)(s - m^2)]S^+$$

$$= [(S^+ - \ell - 1)(S - m^2 - \ell - 1) - (\ell + 1)(m^2 + \ell + 1)]$$

$$\times (S - m^2 - \ell)\cdots(S - m^2 - 1) \; ; \qquad (A.8)$$

and

$$\langle 0|[\sqrt{2}\, a_{(1)}{}^{\mu} + q^{\mu}](S^+ - m^2)(S^+ - m^2 - 1)\cdots(S^+ - m^2 - r + 1)$$

$$= \langle 0|(q^2 - m^2 - 1)(q^2 - m^2 - 2)\cdots(q^2 - m^2 - r + 1)$$

$$\times [[\sqrt{2}\, a_{(1)}{}^{\mu} + q^{\mu}](q^2 - m^2 - r) - rq^{\mu}] \; , \qquad (A.9)$$

where $\langle 0|$ has momentum squared of q^2.

(ii) Divergence Identities

Using the operator techniques, we may easily rederive and generalize the divergence identities of II.

The amplitude

$$B^{\mu}(q) \equiv \langle 0|[\sqrt{2}\, a_{(1)}{}^{\mu} + q^{\mu}]|p\rangle \; , \qquad (A.10)$$

where

$$|p\rangle = \hat{V}(p_1)\, D(R,k_1{}^2)\, \hat{V}(p_2)\cdots\hat{V}(p_{N-1})|0\rangle$$

$$= V(p_1)\, D(R,k_1{}^2)\, V(p_2)\cdots V(p_{N-1})|0\rangle \; , \qquad (A.11)$$

for $q^2 = m^2 + 1 + k$, describes the scattering of a universal vector meson and N scalars of lowest mass. Evaluation of (A.10) yields an integral representation for B^{μ}:

$$B^{\mu}(q) = \int_0^1 du_1 \cdots du_{N-2}\ \mathscr{V}_{(1)}{}^{\mu}(u_1,\cdots,u_{N-2})\ I_{N+1}(u_1,\cdots,u_{N-2})$$

$$\equiv \langle \mathscr{V}_{(1)}{}^{\mu} \rangle_{N+1} , \tag{A.12}$$

where

$$\mathscr{V}_{(1)}{}^{\mu} = q^{\mu} + 2p_1{}^{\mu} + 2p_2{}^{\mu} + \cdots + 2p_{N-1}{}^{\mu}(u_1 \cdots u_{N-2}) \tag{A.13}$$

and I_{N+1} is the usual integrand for the $(N + 1)$-point function for all scalars. Following Fubini and Veneziano[13] we represent the integral $\int du\ I_{N+1}$ by bracketts $(\langle\ \rangle_{N+1})$.

One can easily verify[2] that B^{μ} is conserved for all q^2. We now generalize this result

$$q_{\mu}\langle u^j\ \mathscr{V}_{(1)}{}^{\mu} \rangle_{N+1}$$

$$= \langle 0|[\sqrt{2}\ q \cdot a_{(1)}{}^{\mu} + q^2]V(p_1)D(R + j,k_1{}^2)\cdots D(R + j,k_{N-2}{}^2)V(p_{N-1})|0\rangle$$

$$= \langle 0|S(q)\ V(p_1)\ D(R + j,k_1{}^2)\cdots D(R + j,k_{N-2}{}^2)\ V(p_{N-1})|0\rangle$$

$$= \langle 0|V(p_1)D(R + j + 1,k_1{}^2)\cdots D(R + j + 1,k_{N-2}{}^2)V(p_{N-1})$$

$$\times\ [S(-p_N) - m^2 - j]|0\rangle$$

$$+\ j\langle 0|V(p_1)D(R + j,k_1{}^2)\cdots D(R + j,k_{N-2}{}^2)V(p_{N-1})|0\rangle$$

$$= j\langle u^j \rangle_{N+1} + (p_N{}^2 - m^2 - j)\langle u^{j+1} \rangle_{N+1} , \tag{A.14}$$

where $u = u_1 u_2 \cdots u_{N-2}$. We have used (A.1) and (A.2) and allowed p_N^2 to be arbitrary for later applications. For $j = 0$ and $p_N^2 = m^2$ this reduces to $q_\mu B^\mu(q) = 0$.

In II we also introduced the amplitude (II.3.1)

$$B^{\mu\nu}(q_1,q_2) \equiv -\langle 0|[\sqrt{2}\, a_{(1)}{}^\nu + q_2{}^\nu]V(p_1)D(R,k_1^2)\cdots D(R,k_{N-1}^2)V(p_N) [\sqrt{2}\, a_{(1)}^{\mu+} + q_1{}^\mu]|0\rangle , \tag{A.15}$$

whose spin-one parts on the mass shell ($q_i^2 = m^2 + 1 + \ell$) are amplitudes for adjacent universal vector mesons. The corresponding integral representation is

$$\begin{aligned} B^{\mu\nu}(q_1,q_2) &= -\langle \mathcal{V}_{(1)}{}^\mu \mathcal{V}_{(1)}{}^\nu \rangle_{N+2} - 2g^{\mu\nu}\langle u\rangle_{N+2} \\ &\equiv -\langle \mathcal{B}^{\mu\nu}(q_1,q_2)\rangle_{N+2} , \end{aligned} \tag{A.16}$$

where $\mathcal{V}^\nu$ is given by an expression similar to (A.10),

$$\bar{\mathcal{V}}^\mu = q_1{}^\mu + 2p_N{}^\mu + 2p_{N-1}{}^\mu u_{N-1} + \cdots + 2p_1{}^\mu(u_1\cdots u_{N-1}) . \tag{A.17}$$

Using the operator formalism we easily verify and generalize (II.3.9),

$$q_{1\mu}\langle u^j \mathcal{B}^{\mu\nu}\rangle_{N+2}$$

$$= \langle 0|[\sqrt{2}\, a_{(1)}{}^{\nu} + q_2{}^{\nu}]V(p_1)D(R + j,k_1{}^2)\cdots D(R + j,k_{N-1}{}^2)V(p_N)S^+(q_1)|0\rangle$$

$$= \langle 0|[\sqrt{2}\, a_{(1)}{}^{\nu} + q_2{}^{\nu}](S^+(-q_2) - m^2 - j)V(p_1)D(R + j + 1,k_1{}^2)\cdots$$

$$\cdots D(R + j + 1,k_{N-1}{}^2)V(p_N)|0\rangle$$

$$+ \; j\langle 0|[\sqrt{2}\, a_{(1)}{}^{\nu} + q_2{}^{\nu}]V(p_1)D(R + j,k_1{}^2)\cdots D(R + j,k_{N-1}{}^2)V(p_N)|0\rangle$$

$$= -q_2{}^{\nu}\langle u^{j+1}\rangle_{N+2} + (q_2{}^2 - m^2 - 1 - j)\langle u^{j+1}\mathcal{V}_{(1)}{}^{\nu}\rangle_{N+2}$$

$$+ \; j\langle u^j \mathcal{V}_{(1)}{}^{\nu}\rangle_{N+2} \;, \qquad \text{(A.18)}$$

and similarly,

$$\langle u^j \mathcal{B}^{\mu\nu}\rangle_{N+2}\, q_{2\nu}$$

$$= -q_1{}^{\mu}\langle u^{j+1}\rangle_{N+2} + (q_1{}^2 - m^2 - 1 - j)\langle u^{j+1}\bar{\mathcal{V}}^{\mu}\rangle_{N+2} + j\langle u^j \bar{\mathcal{V}}^{\mu}\rangle_{N+2} \,.$$

From (A.18) and (A.14) we obtain the generalization of (II.3.11):

$$q_{1\mu}\langle u^j \mathcal{B}^{\mu\nu}\rangle_{N+2}\, q_{2\nu}$$

$$= j^2\langle u^j\rangle_{N+2} + [j(q_1{}^2 + q_2{}^2) - (2j + 1)m^2 - (j + 1)^2 - j^2]\langle u^{j+1}\rangle_{N+2}$$

$$+ \; (q_1{}^2 - m^2 - 1 - j)(q_2{}^2 - m^2 - 1 - j)\langle u^{j+2}\rangle_{N+2} \,. \qquad \text{(A.19)}$$

The identities (A.14), (A.18), and (A.19) are used extensively in constructing the current algebra parameterizations of Appendices B and C.

(iii) Current Algebra Identity

We now rederive the identity used here and in II to introduce the current algebra fixed pole. First consider

$$B_C^{\mu\nu}(q_1,q_2) \equiv -\langle[\bar{\mathcal{V}}_{(1)}{}^{\mu} + (q_1{}^{\mu} + 2q_2{}^{\mu})u][\bar{\mathcal{V}}_{(1)}{}^{\nu} + (q_2{}^{\nu} + 2q_1{}^{\nu})u]\rangle_{N+2}$$

$$\equiv -\langle \mathcal{B}_C^{\mu\nu}\rangle_{N+2} \cdot \tag{A.20}$$

From (A.18) for $j = 0$ and (A.14) for an $(N+2)$-point function, we have

$$q_{1\mu}\, B_C^{\mu\nu}(q_1,q_2)$$

$$= -(t - m^2 - 1)[\langle u\, \mathcal{V}_{(1)}{}^{\nu}\rangle_{N+2} + (q_2{}^{\nu} + 2q_1{}^{\nu})\langle u^2\rangle_{N+2}] \cdot \tag{A.21}$$

To obtain the current algebra identity we examine (A.21) as a function of $\alpha_t = t - m^2$. As $\alpha_t \to 1$ only the pole at $\alpha_t = 1$ in the quantity in brackets contributes. Its residue is an $(N+1)$-point function and is essentially the required right-hand side of (2.3). A straightforward computation using the integral representation gives

$$q_{1\mu}\, B_C^{\mu\nu}(q_1,q_2) \xrightarrow[\alpha_t \to 1]{} \langle \mathcal{V}_{(1)}{}^{\nu}(q_1 + q_2)\rangle_{N+1} - q_1{}^{\nu}\langle 1\rangle_{N+1} \cdot$$

We have explicitly indicated that the momentum of $\mathcal{V}^{\nu}$ is $(q_1 + q_2)$. In the integral representation, setting $\alpha_t = 1$ is equivalent to multiplying the usual integrand I_{N+2} by $(1 - u)^{\alpha_t - 1}$. Hence, defining

$$B_{FP}^{\mu\nu}(q_1,q_2) \equiv -\langle \mathcal{B}_C^{\mu\nu}(1-u)^{\alpha_t-1}\rangle_{N+2} + g^{\mu\nu}\langle 1\rangle_{N+1} , \qquad (A.22)$$

we have

$$q_{1\mu}\, B_{FP}^{\mu\nu}(q_1,q_2) = \langle \mathcal{V}_{(1)}{}^{\nu}(q_1+q_2)\rangle_{N+1}$$

and similarly (A.23)

$$B_{FP}^{\mu\nu}(q_1,q_2)q_{2\nu} = -\langle \mathcal{V}_{(1)}{}^{\mu}(q_1+q_2)\rangle_{N+1} .$$

APPENDIX B. CURRENT ALGEBRA PARAMETERIZATION WITH ARBITRARY FORM FACTORS

The current algebra parameterization of II is expressed in compact notation and then generalized to arbitrary form factors,

$$F(q^2) = \sum_{\ell=0}^{\infty} f_\ell \, F_\ell(q^2) \, , \tag{B.1}$$

where

$$F_\ell(q^2) = 1/[1 - q^2/(m^2 + 1 + \ell)]$$

and $\sum_\ell f_\ell = 1$.

As discussed in Sec. II.C, the two-current amplitude is written as

$$M^{\mu\nu}(q_1,q_2) = M_H^{\mu\nu}(q_1,q_2) + M_C^{\mu\nu}(q_1,q_2) + M_{FP}^{\mu\nu}(q_1,q_2) \, .$$

From II, we have for a single pole form factor $(f_0 = 1)$,

$$M_H^{\mu\nu}(q_1,q_2) = F_0(q_1^2) \, \mathcal{P}_0^{\mu}{}_{u'}(q_1) \, B^{\mu'\nu'}(q_1,q_2) \, \mathcal{P}_0^{\nu}{}_{\nu'}(q_2) \, F_0(q_2^2)$$
$$+ [2(m^2 + 1)g^{\mu\nu} + q_1^{\mu} q_2^{\nu}]\langle u^2 \rangle_{N+2} \, ,$$

$$M_C^{\mu\nu}(q_1,q_2) = -F_0(t) \, B_c^{\mu\nu}(q_1,q_2) \, , \tag{B.2}$$

$$M_{FP}^{\mu\nu}(q_1,q_2) = F_0(t) \, B_{FP}^{\mu\nu}(q_1,q_2) \, ,$$

where

$$\bar{\rho}_{\ell}{}^{\mu}{}_{\mu'}(q) = g^{\mu}{}_{\mu'} - \frac{q^{\mu}q_{\mu'}}{m^2 + 1 + \ell} \quad . \tag{B.3}$$

One easily verifies, using (A.18) and (A.19) that

$$q_{1\mu}\, M_H^{\mu\nu}(q_1,q_2) = -q_{1\mu}\, M_C^{\mu\nu}(q_1,q_2)$$

$$= (m^2 + 1)[\langle u\, \mathcal{V}_{(1)}{}^{\nu}\rangle_{N+2} + (q_2{}^{\nu} + 2q_1{}^{\nu})\langle u^2\rangle_{N+2}] ,$$

and observes from (A.23) that $M_{FP}^{\mu\nu}$ gives the required current algebra divergence so that $M^{\mu\nu}$ has the required properties.

For arbitrary form factors it is convenient to introduce a vector meson "propagator"

$$\Delta^{\mu}{}_{\mu'}(q) \equiv \sum_{\ell} f_{\ell}\, F_{\ell}(q^2)\, \bar{\rho}_{\ell}{}^{\mu}{}_{\mu'}(q) \quad , \tag{B.4}$$

which satisfies

$$q_{\mu}\, \Delta^{\mu}{}_{\mu'}(q) = q_{\mu'} \quad . \tag{B.5}$$

Our generalization can then be written as

$$M_H^{\mu\nu}(q_1,q_2) = [\Delta^{\mu}{}_{\mu'}(q_1) - g^{\mu}{}_{\mu'}]B^{\mu'\nu'}(q_1,q_2)[\Delta^{\nu}{}_{\nu'}(q_2) - g^{\nu}{}_{\nu'}]$$

$$- \sum_{\ell} f_{\ell}[F_{\ell}(q_1{}^2)\rho_{\ell}{}^{\mu}{}_{\mu'}(q_1)g^{\nu}{}_{\nu'} + g^{\mu}{}_{\mu'}\bar{\rho}_{\ell}{}^{\nu}{}_{\nu'}(q_2)\, F_{\ell}(q_2{}^2)]$$

$$\times \langle \mathcal{B}^{\mu'\nu'}(1 - u)^{\ell}\rangle_{N+2}$$

$$+ \sum_{\ell} f_{\ell}\langle[\mathcal{B}^{\mu\nu} + q_1{}^{\mu}q_2{}^{\nu}u^2 + 2(m^2 + 1 + \ell)g^{\mu\nu}\, u^2](1 - u)^{\ell}\rangle_{N+2} \quad ; \tag{B.6}$$

$$M_C^{\mu\nu}(q_1,q_2) = -\sum_\ell \frac{f_\ell(m^2 + 1 + \ell)}{\alpha_t - 1 - \ell} \langle \mathcal{B}_C^{\mu\nu}(1-u)^\ell \rangle_{N+2} ; \quad (B.7)$$

$$M_{FP}^{\mu\nu}(q_1,q_2) = \sum_\ell \frac{f_\ell(m^2 + 1 + \ell)}{\alpha_t - 1 - \ell} \langle \mathcal{B}_C^{\mu\nu}(1-u)^{\alpha_t - 1} \rangle_{N+2}$$

$$+ F(t)\, g^{\mu\nu}\langle 1 \rangle_{N+1} \quad (B.8)$$

$$= F(t)\, B_{FP}^{\mu\nu}(q_1,q_2) .$$

The third term clearly has the desired current algebra divergence with the correct form factor, $F(t)$. The divergences of the other two terms cancel as required,

$$q_{1\mu}\, M_H^{\mu\nu}(q_1,q_2) = -q_{1\mu}\, M_C^{\mu\nu}(q_1,q_2)$$

$$= \sum_\ell f_\ell(m^2 + 1 + \ell)\langle [u\, \mathcal{V}_{(1)}{}^\nu + (q_2{}^\nu + 2q_1{}^\nu)u^2](1-u)^\ell \rangle_{N+2} . \quad (B.9)$$

In obtaining this result we have first used (A.14) and (A.18) along with the trivial identity

$$\sum_{j=0}^{\ell} (-1)^j \binom{\ell}{j} j u^j = -\ell\, u(1-u)^{\ell-1}$$

to obtain

$$q_{1\mu}\langle \mathcal{B}^{\mu\nu}(1-u)^{\ell}\rangle_{N+2}$$

$$= \langle[-q_2{}^{\nu}\, u + (q_2{}^2 - m^2 - 1 - \ell)u\, \mathcal{V}_{(1)}{}^{\nu}](1-u)^{\ell}\rangle_{N+2} \qquad \text{(B.10)}$$

and

$$q_{1\mu}\langle u\, \bar{\mathcal{V}}_{(1)}{}^{\mu}(1-u)^{\ell}\rangle_{N+2}$$

$$= \langle[u + (q_2{}^2 - m^2 - 1 - \ell)u^2](1-u)^{\ell}\rangle_{N+2} .$$

This amplitude $M^{\mu\nu}$ factorizes for leading trajectories, since it differs from the factorizable function $\Delta^{\mu}{}_{\mu'}\, B^{\mu\nu'}\, \Delta^{\nu}{}_{\nu'}$ by terms of order u or higher; such terms give no contribution to leading trajectories in $k_1{}^2$ (channels dual to the t channel).

We remark that this parameterization has a structure similar to the arbitrary form factor parameterization of Brower and Halpern[20] for the double-flip amplitude for $N = 2$, although it does not reduce exactly to their result.

APPENDIX C. CURRENT ALGEBRA PARAMETERIZATION

FACTORIZING ON M LEADING TRAJECTORIES

Here we generalize the construction of II to obtain amplitudes satisfying current algebra and factorizing (without spurious states) on the M leading trajectories.[19] The parameterization given here has form factors with the specific form

$$F_M(q^2) = F_0(q^2)\, F_1(q^2)\cdots F_{M-1}(q^2) = \frac{B[1 - \alpha(q^2),M]}{B[1 - \alpha(0),M]} , \qquad (C.1)$$

but we conjecture that the asymptotic condition $F(q^2) \sim (q^2)^{-M}$ is sufficient to allow the construction of more general parameterizations with M-trajectory factorization.

In Sec. III, we considered the amplitude

$$\hat{B}^{\mu\nu}(q_1,q_2)$$

$$= -\langle 0|[\sqrt{2}\, a_{(1)}{}^{\nu} + q_2{}^{\nu}]\, V(p_1)\, D(R,k_1{}^2)\, V(p_2)\cdots D(R,k_{N-1}{}^2)\, \hat{V}(p_N)$$

$$[\sqrt{2}\, a^{\mu+}_{(1)} + q_1{}^{\mu}]|0\rangle . \qquad (C.2)$$

We observe that the unphysical singularities in t can be avoided if we replace $\hat{V}(p)$ by

$$\hat{V}_M(p) \equiv \sum_{\ell=0}^{M} \frac{(-1)^{\ell}}{\binom{-m^2 - 1}{\ell}} \binom{S^+(-k) - m^2}{\ell} V(p) \binom{S(k + p) - m^2}{\ell} , \qquad (C.3)$$

i.e., we consider

$$\hat{B}_M^{\mu\nu}(q_1,q_2) = -\langle 0|[\sqrt{2}\, a_{(1)}^{\nu} + q_2^{\nu}]V(p_1)D(R,k_1^2)\cdots D(R,k_{N-1}^2)\hat{V}_M(p_N)$$

$$\times\, [\sqrt{2}\, a_{(1)}^{\mu+} + q_1^{\mu}]|0\rangle \qquad (C.4)$$

$$= -\sum_{\ell=0}^{M} \frac{(-1)^{\ell}}{\binom{-m^2-1}{\ell}} \langle 0|[\sqrt{2}\, a_{(1)}^{\nu} + q_2^{\nu}]$$

$$\times \binom{S^{+}(-q_2) - m^2}{\ell} V(p_1)\, D(R+\ell,k_1^2)\cdots$$

$$\times\, D(R+\ell,k_{N-1}^2)\, V(p_N) \binom{S(-q_1) - m^2}{\ell}$$

$$\times\, [\sqrt{2}\, a_{(1)}^{\mu+} + q_1^{\mu}]|0\rangle\ .$$

One immediately observes that (C.4) differs from (C.2) by terms with the propagators shifted by $M + 1$ units or more. Such terms contribute only to trajectories displaced below the leading trajectory by $M + 1$ units or more. We also observe that $\hat{B}_M^{\mu\nu}$ is conserved only if q_1^2 or q_2^2 equals $m^2 + 1 + n$ for $n \leq M - 1$, i.e., only for the first M vector mesons. In fact,

$$q_{1\mu}\, \hat{B}_M^{\mu\nu}(q_1,q_2) = -\frac{(m^2+1)\cdots(m^2+M)}{M!} F_M^{-1}(q_1^2)\, F_M^{-1}(q_2^2)$$

$$\times \langle (q_2^2 - m^2 - M - 1)\, u^{M+1}\, \mathcal{V}_{(1)}^{\nu} - (M+1)\, q_2^{\nu}\, u^{M+1}\rangle_{N+2}\ . \qquad (C.5)$$

To obtain this result we have used (A.9) and the identity

$$\hat{V}_M S^+ = \frac{(S^+ - m^2)\cdots(S^+ - m^2 - M)\, V(S - m^2 - 1)\cdots(S - m^2 - M)}{M!(m^2 + 1)\cdots(m^2 + N)} ,$$

which follows from (A.8).

We follow the decomposition (2.10) and define

$$M_H^{\mu\nu}(q_1,q_2) = F_M(q_1^2)\, \hat{B}_M^{\mu\nu}(q_1,q_2)\, F_M(q_2^2)$$

$$+ \frac{(m^2 + 1)\cdots(m^2 + M)}{M!} \langle u^M \mathcal{B}^{\mu\nu} + 2Mg^{\mu\nu}\, u^{M+1} \rangle_{N+2} \quad ; \qquad (C.6)$$

$$M_C^{\mu\nu}(q_1,q_2) = -\sum_\ell \frac{f_\ell(m^2 + 1 + \ell)}{\alpha_t - 1 - \ell} \langle \mathcal{B}_C^{\mu\nu}\, (1 - u)^\ell \rangle_{N+2} \quad ; \qquad (C.7)$$

$$M_{FP}^{\mu\nu}(q_1,q_2) = \sum_\ell \frac{f_\ell(m^2 + 1 + \ell)}{\alpha_t - 1 - \ell} \langle \mathcal{B}_C^{\mu\nu}(1 - u)^{\alpha_t - 1} \rangle_{N+2}$$

$$+ F(t)\, g^{\mu\nu}\langle 1 \rangle_{N+1} \qquad (C.8)$$

$$= F(t)\, B_{FP}^{\mu\nu}(q_1,q_2) .$$

The third term clearly has the required current algebra divergence. The divergences of the other two terms cancel as required,

$$q_{1\mu}\, M_H^{\mu\nu}(q_1,q_2) = -q_{1\mu}\, M_C^{\mu\nu}(q_1,q_2)$$

$$= \frac{(m^2 + 1)\cdots(m^2 + M)}{(M - 1)!} \langle u^M \mathcal{V}_{(1)}{}^\nu + (q_2{}^\nu + 2q_1{}^\nu)u^{M+1} \rangle_{N+2} .$$

The divergence of $M_H^{\mu\nu}$ is calculated by using (C.5) and (A.18) for $j = M$. The divergence of $M_C^{\mu\nu}$ is calculated as in Appendix B to obtain (B.9). The sum over ℓ is then done by using the sum rule

$$\sum_{\ell=0}^{M-1} f_\ell(m^2 + 1 + \ell)\ell^n = 0, \quad \text{for} \quad n < M - 1 \tag{C.9}$$

and the expression and the definition of f_ℓ (B.1).

$$f_{M-1} = (-1)^{M-1} \frac{(m^2 + 1)\cdots(m^2 + M - 1)}{(M - 1)!}$$

which follow from (C.1) and the definition of f_ℓ (B.1).

This amplitude factorizes on all trajectories lying M units or less below the leading one because it differs from $\hat{B}_M^{\mu\nu}$ by terms that do not contribute to these trajectories. This is clear for $M_H^{\mu\nu}$, (C.6) since it differs from $\hat{B}_M^{\mu\nu}$ by terms with at least M powers of u. The sum $M_C^{\mu\nu} + M_{FP}^{\mu\nu}$ is proportional to

$$\sum_{\ell=0}^{M-1} \frac{f_\ell(m^2 + 1 + \ell)}{\alpha_t - 1 - \ell}\left[1 - (1 - u)^{\alpha_t - 1 - \ell}\right](1 - u)^\ell ,$$

which by (C.9) also has at least M powers of u.

We remark that for $M = 1$, this parameterization reduces to the result of II (see Eq. B.2).

FOOTNOTES AND REFERENCES

* This work was supported in part by the U.S. Atomic Energy Commission.

† National Science Foundation Predoctoral fellow.

1. For a review see D. Sivers and J. Yellin, Rev. Mod. Phys. (to be published).

2. R. C. Brower and J. H. Weis, Phys. Rev. 188, 2486 and 2495 (1969). Hereafter referred to as I and II, respectively. Equation numbers from these papers are preceded by I or II.

3. Precisely, the amplitude for this meson and N spinless hadrons, $B^{\mu}(q)$, is exactly conserved for all q^{μ}, $q_{\mu} B^{\mu}(q) = 0$.

4. For simplicity we assume here that all external and internal particles are in the same family, i.e., $p_i^2 = m^2$, $\alpha_i = s_i - m^2$.

5. These poles are located at $t = q_1^2 + q_2^2 + n$. A simple interpretation of this result is given in Sec. III.

6. M. Bander, Nucl. Phys. B13, 587 (1969).

7. H. Sugawara, Tokyo University of Education Report, 1969 (unpublished); I. Ohba, Prog. Theoret. Phys. (Kyoto) 42, 432 (1969).

8. M. Ademollo and E. del Giudice, Nuovo Cimento 63A, 639 (1969).

9. R. C. Brower, A. Rabl, and J. H. Weis, Nuovo Cimento 65A, 654 (1970).

10. D. Z. Freedman, Phys. Rev. (to be published).

11. Chan Hong-Mo and J. Paton, Nucl. Phys. B10, 516 (1969).

12. It follows if the trajectory in a channel with M hadrons and one current is the same as the trajectory in the channel with only the M hadrons. This allows soft poles to survive in factorization. See Eq. (II.2.10).

13. K. Bardakci and S. Mandelstam, Phys. Rev. 184, 1640 (1969); S. Fubini and G. Veneziano, Nuovo Cimento 64A, 811 (1969). Additional linear dependencies, such as those discovered by M. A. Virasoro (University of Wisconsin Report, 1969) for $m^2 = -1$, may play a similarly vital role.

14. S. Fubini, D. Gordon, and G. Veneziano, Phys. Letters 29B, 679 (1969), hereafter referred to as FGV. See also Y. Nambu, University of Chicago preprint (1969) and L. Susskind, Yeshiva University preprints (1969).

15. R. C. Brower and J. H. Weis, Lettera Nuovo Cimento 3, 285 (1970). See Appendix A for a summary.

16. The subscripts indicate the permutation P of the hadronic momenta and the position(s) of the current(s) just before P(i). From now on we restrict ourselves to the permutations shown in the figures. The explicit internal symmetry factors for no exotic resonances are given by (II.2.15) and (II.3.20).

17. Therefore, we are treating only the nondiffractive contribution. For $I_t = 0$ and large q^2 the pomeranchon contribution may be very important. For an example of a model for this contribution see II, Sec. IV, and Ref. 9, Sec. IV.

18. The invariant amplitudes given in II and Appendices B and C satisfy USDR with the sole exception of the amplitude multiplying $g^{\mu\nu}$. The existence of such a right-signature $J = 0$ fixed pole in this amplitude has also been conjectured by other authors and there appears to be some experimental support for it--see M. Damashek and F. C. Gilman, SLAC-PUB-697, 1969.

19. C. B. Chiu, S. Matsuda, and C. Rebbi, Phys. Rev. Letters 23, 1526 (1969); F. Gliozzi, Lettera Nuovo Cimento 2, 846 (1969); C. B. Thorn, Phys. Rev. (to be published); M. A. Virasoro, University of Wisconsin Report 1969.

20. R. C. Brower and M. B. Halpern, Phys. Rev. 182, 1779 (1969).

21. We remark that in this limit

$$F(q^2) \xrightarrow[M\to\infty]{} \frac{\Gamma[1-\alpha(q^2)]}{\Gamma[1-\alpha(0)]}\,(M)^{q^2}\,.$$

The factor $(M)^{q^2} = \exp(q^2 \log M) \approx \exp(q^2 \sum_{\ell=0}^{M} \frac{1}{\ell})$ is similar to the very singular form factors obtained by S. Fubini and G. Veneziano (private communication, 1969) and L. Susskind, Yeshiva University preprint, 1969.

22. This expression really stands for a set of unsubtracted dispersion relations, one for each invariant amplitude in the overcomplete set used here. See also Ref. 18.

23. Here we label the states by the overcomplete occupation number basis.

24. The easiest way to verify that this is the correct result is to note that (a) the residues of poles in k_i^2 are given by (3.4), and (b) for $t < t_0$ unsubtracted dispersion relations can be written in k_i^2 so the poles completely determine the function.

25. The anomalous singularity imitates the fixed pole at $q_1 \cdot q_2 = 0$, since it is then at an integer. Further we observe from (3.10) that, like a fixed pole, it does not contribute to the residues at the vector-meson poles.

26. A beginning on this has recently been made by S. Fubini and G. Veneziano, Nuovo Cimento (to be published).

27. Some of these properties have been touched upon in Ref. 9. They will be discussed in more detail in a forthcoming report: J. H. Weis, Dual Resonance Models for Vector Currents, Lawrence Radiation Laboratory Report UCRL-19780, May 1970.

28. D. Z. Freedman (Ref. 10) has given amplitudes of the form of Ref. 7 that satisfy the current algebra for one vector current and one scalar current. The scalar current indeed has good large-q^2 behavior but the vector current is introduced by the same techniques discussed here and in II and therefore has bad large-q^2 behavior. What we are suggesting here is to combine the good features into a single current.

29. M. Kaku and C. B. Thorn, Phys. Rev. (to be published).

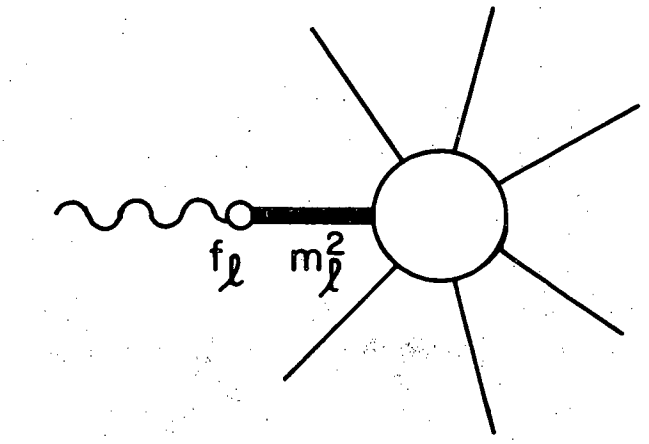
FIGURE CAPTIONS

Fig. 1. Constraints on the single-current amplitude. (a) Vector-meson dominance. (b) Factorization. The amplitude must be expressible as a sum over the poles shown by the heavy lines and these poles must correspond to states in the hadronic spectrum.

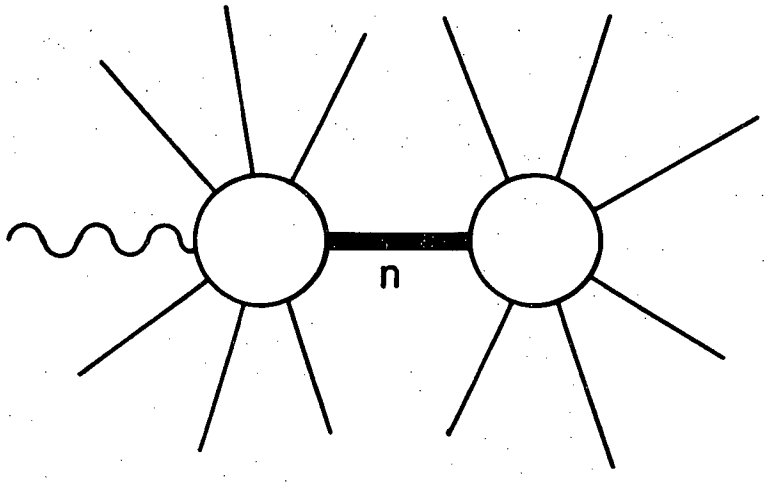
Fig. 2. Constraints on the two-current amplitude. (a) Vector-meson-dominance. (b) Linear factorization. (c) Quadratic factorization. The amplitude must be expressible as a sum over the poles shown by the heavy lines, and these poles must correspond to states in the hadronic spectrum.

Fig. 3. Divergence condition for single-current amplitude.

Fig. 4. Divergence conditions for two-current amplitudes. (a) Non-adjacent currents. (b) Adjacent currents.



(a)



(b)

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Fig. 1

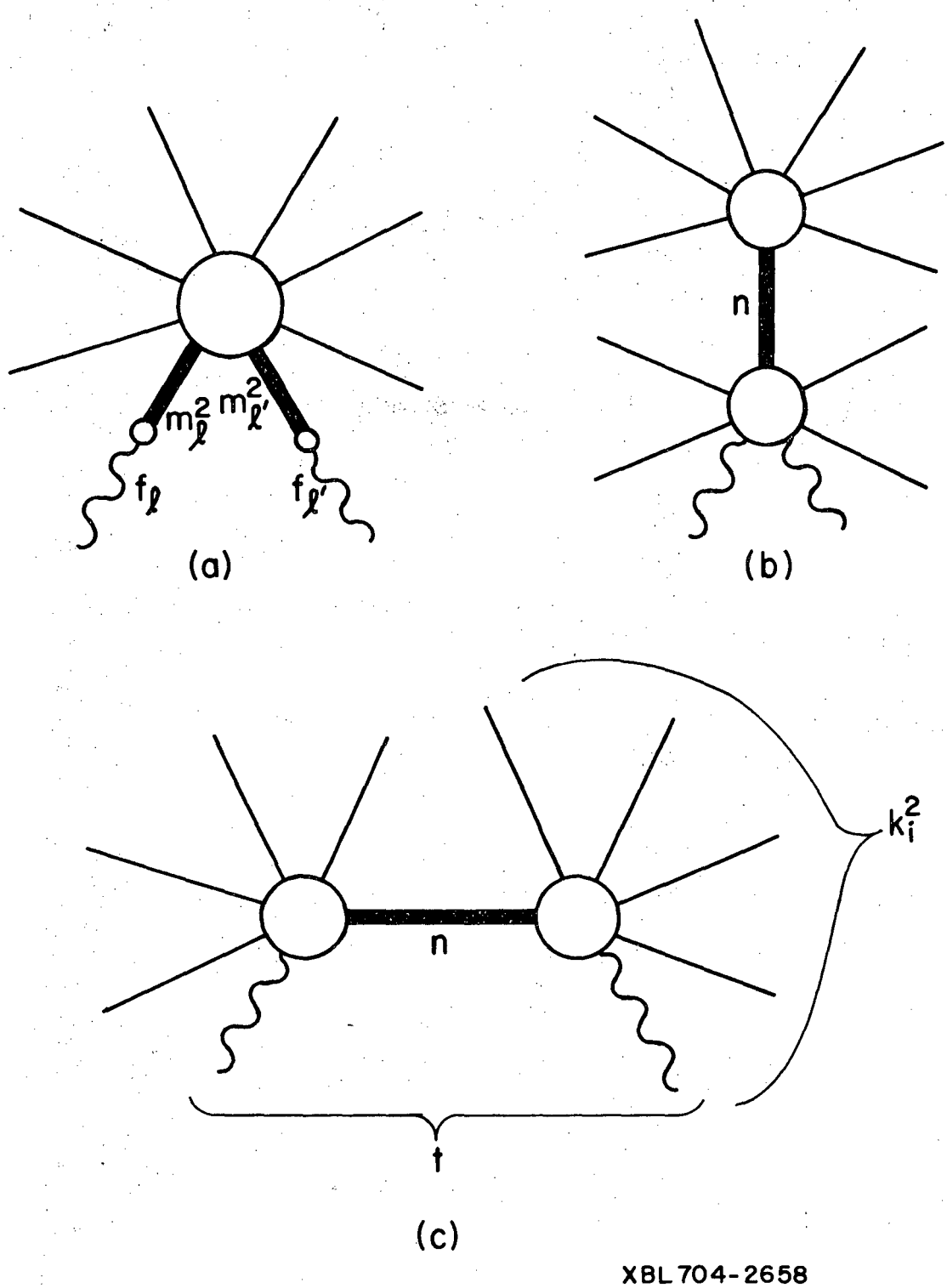


Fig. 2

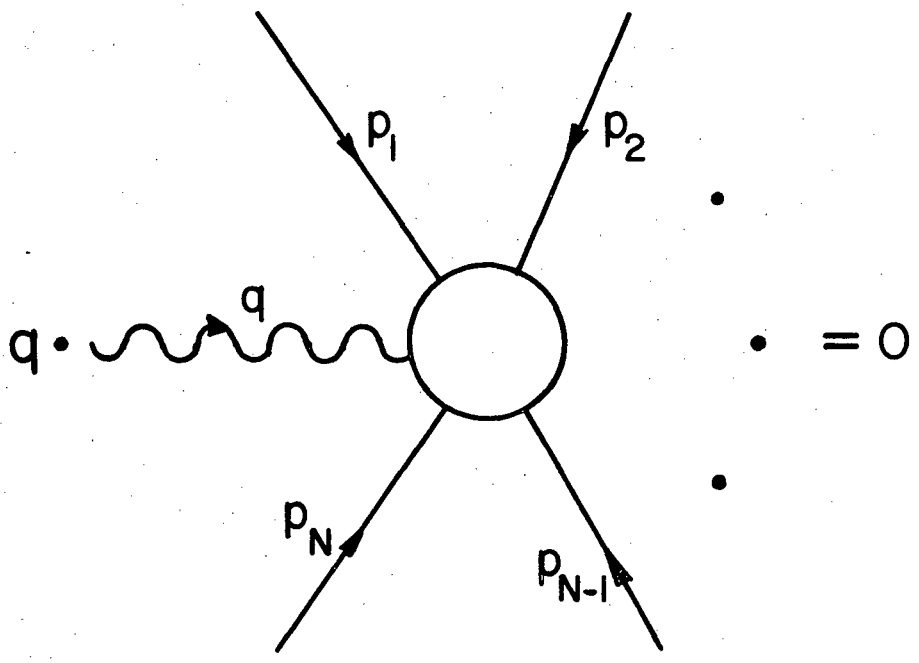
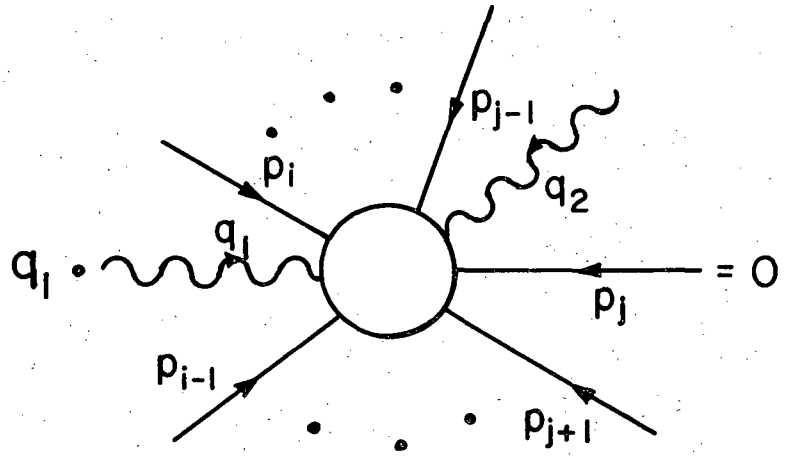
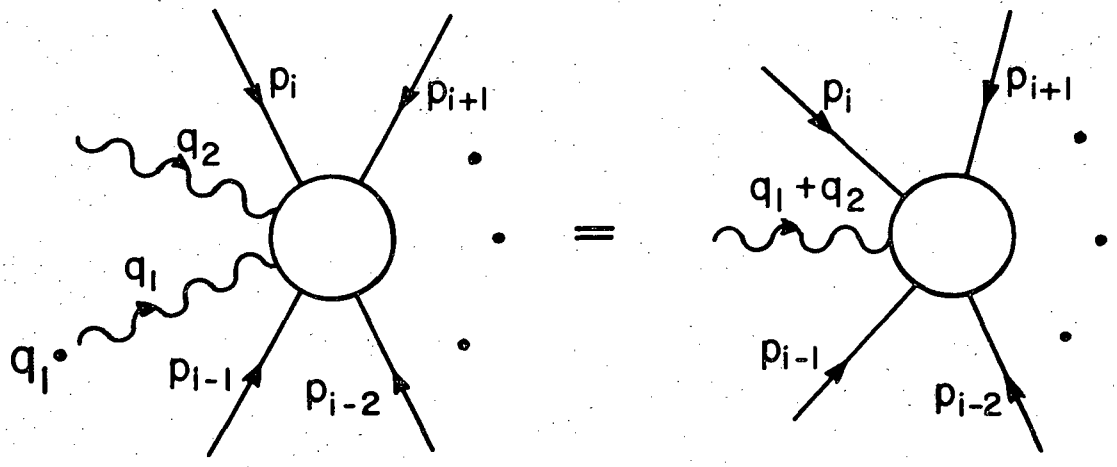


Fig. 3



(a)



(b)

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Fig. 4

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