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It has been shown[1] that, if parity and time reversal are conserved in the decay of the Σ particles, the observed lifetime ratio $Y =$ (mean life of Σ^+)/(mean life of Σ^-) and the observed branching ratio $X =$ (frequency of $\Sigma^+ \rightarrow p + \pi^0$)/(frequency of $\Sigma^+ \rightarrow n + \pi^+$) cannot be accounted for with a decay interaction transforming as a component of a spherical tensor of rank $\frac{1}{2}$ ($\Delta T = \frac{1}{2}$ rule [2,3,4,5,6]). Present experimental evidence of parity nonconservation in the β decay,[7] in the $\pi \rightarrow \mu + \nu$ decay and in the $\mu \rightarrow e + \nu + \nu$ decay,[8] and the less direct evidence from the τ and θ decays,[9] encourages the hypothesis that also in the weak decay interactions of hyperons and K mesons parity is not conserved. It was shown that, if the decay interaction satisfies $\Delta T = \frac{1}{2}$ and no other assumptions are made, the point $P \equiv (Y, X)$ in the Y-X plane is limited to a "permitted region" of the plane,[2] as reported in Fig. 1. It is seen from Fig. 1 that the experimental point lies inside the permitted region. It is well known that, if time reversal is satisfied, the ensuing symmetry condition on the S matrix limits the form of the decay matrix element by a theorem first used by Watson in the interpretation of photomeson production.[10] This limitation, in the case of parity conservation, strongly restricts

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the "permitted region" in the Y-X plane, which is reduced to a line, depending on the assumed spin and parity of the Σ. We shall show that in the case under discussion of parity nonconservation the "permitted region" in Fig. 1 is not essentially reduced by the further requirement of time-reversal invariance.

Consider first the decay of a Σ with spin $\frac{1}{2}$. The possible states for the final pion-nucleon system are $s_{\frac{1}{2}}$ with $T = 3/2$ and $1/2$, and $p_{\frac{1}{2}}$ with $T = 3/2$ and $1/2$. If time reversal is satisfied the $\Delta T = \frac{1}{2}$ rule leads to the following expressions for the total decay probabilities of the Σ particles:

$$W(\Sigma^+ | po) = \frac{2}{3}(A_3^2 + A_{31}^2) + \frac{1}{3}(A_1^2 + A_{11}^2) - \frac{2\sqrt{2}}{3}\left[A_3A_1 \cos(\alpha_3 - \alpha_1) + A_{31}A_{11} \cos(\alpha_{31} - \alpha_{11})\right], \quad (1)$$

$$W(\Sigma^+ | n+) = \frac{1}{3}(A_3^2 + A_{31}^2) + \frac{2}{3}(A_1^2 + A_{11}^2) + \frac{2\sqrt{2}}{3}\left[A_3A_1 \cos(\alpha_3 - \alpha_1) + A_{31}A_{11} \cos(\alpha_{31} - \alpha_{11})\right], \quad (1')$$

$$W(\Sigma^- | n-) = 3(A_3^2 + A_{31}^2) \quad . \quad (1'')$$

The quantities A are real numbers and the α are the appropriate pion-nucleon phase shifts at the Σ decay energy. The indices refer to the final pion-nucleon states according to the usual convention. From Eqs. (1) we can write for the measurable quantities Y and X

$$Y = \frac{3}{1 + \xi^2} \quad , \tag{2}$$

$$X = \frac{2 + \xi^2 - 2\sqrt{2}\,\xi\,\eta}{1 + 2\,\xi^2 + 2\sqrt{2}\,\xi\,\eta} \quad , \tag{2'}$$

where

$$\xi = \left(\frac{A_1^{\,2} + A_{11}^{\,2}}{A_3^{\,2} + A_{31}^{\,2}} \right)^{\frac{1}{2}} \tag{3}$$

and

$$\eta = \frac{A_3 A_1 \cos(\alpha_3 - \alpha_1) + A_{31} A_{11} \cos(\alpha_{31} - \alpha_{11})}{\sqrt{A_3^{\,2} + A_{31}^{\,2}}\;\sqrt{A_1^{\,2} + A_{11}^{\,2}}} \quad . \tag{3'}$$

From Eq. (3') it can be shown that for any value of the real quantities A_1, A_3, A_{11} and A_{31} we have

$$\eta^2 \leq \max \left[\cos(\alpha_3 - \alpha_1),\ \cos(\alpha_{31} - \alpha_{11}) \right] \quad ,$$

where $\max \left[a,\ b \right]$ means the larger of the two values a and b. At the energy corresponding to the Σ decay energy, $\cos(\alpha_3 - \alpha_1) \cong 0.95$ and $\cos(\alpha_{31} - \alpha_{11}) \cong 0.99$. However, as evident from Eq. (2), η^2 is always less than unity. Therefore the permitted region of Fig. 1 is not essentially restricted. The same conclusion is found to hold also for the higher spin values. We therefore conclude that, if parity is not conserved in the Σ decays, the experimental values of X and Y are not in disagreement with the $\Delta T = \frac{1}{2}$ rule. However it must be remarked that the conditions imposed in such case from the $\Delta T = \frac{1}{2}$ rule are not very stringent, so that the nondisagreement does not provide any conclusive evidence.

The more stringent condition imposed by the $\Delta T = \frac{1}{2}$ rule on the Λ^0 branching ratio,

$$w(\Lambda^0 \rightarrow p\pi^-) = 2w(\Lambda^0 \rightarrow n\pi^0) \quad ,$$

seems to be satisfied by the recent data of Steinberger, who finds a value of 66 ± 5% for the fraction of Λ^0 decaying tinto $p + \pi^-$.[11] Moreover, the latest data[12] for the $\frac{\tau^+ \rightarrow \pi^+ + \pi^- + \pi^+}{\tau^+ \rightarrow \pi^+ + \pi^0 + \pi^0}$ ratio, giving a value $\cong 0.39 \pm 0.09$, do not contradict the value predicted for such ratio for a spin-zero τ meson from the $\Delta T = \frac{1}{2}$ rule, namely a value $\gtrsim \frac{1}{4}$. This conclusion, however, is not stringent since it also follows if a $\Delta T = \frac{3}{2}$ contribution is present. As is well known,[2,3] the $\Delta T = \frac{1}{2}$ rule forbids the $K^+ \rightarrow \pi^+ + \pi^0$ decay mode for K^+ of even spin in the absence of electromagnetic interactions. The transition probability for such process is known experimentally to be much lower than that of $K^0 \rightarrow \pi^+ + \pi^-$. It has been pointed out by Gell-Mann that the electromagnetic corrections may be too small to account for the observed $K^+ \rightarrow \pi^+ + \pi^0$ rate.[13] These small $\Delta T = \frac{3}{2}$ and $\Delta T = \frac{5}{2}$ amplitudes are related to the observed ratios

$$f = w(K_1^0 \rightarrow 2\pi^0)/w(K_1^0 \rightarrow \pi^+ + \pi^-)$$

and

$$g = w(K^+ \longrightarrow \pi^+ + \pi^0)/w(K_1^0 \longrightarrow 2\pi)$$

by

$$f = \tfrac{1}{2}(1 + 3\sqrt{2}\,(x_3 - x_5)\cos(\alpha_2 - \alpha_0)) \qquad (4)$$

$$g = \tfrac{3}{4}(x_3 + \tfrac{2}{3}x_5)^2 \ . \qquad (4')$$

Here we denote by $x_3\, e^{i(\alpha_2 - \alpha_0)}$ the ratio of the reduced matrix element of the $\Delta T = \frac{3}{2}$ transition to the reduced matrix element of the $\Delta T = \frac{1}{2}$ transition, and by $x_5\, e^{i(\alpha_2 - \alpha_0)}$ the similar ratio of $\Delta T = \frac{5}{2}$ to $\Delta T = \frac{1}{2}$. The value of g can be determined from the data if we assume that only one K meson exists. In this case g is found to be $\sim \frac{1}{430 \pm 100}$ from the observed ratio of the K^0 and K^+ lifetimes ($\tau_{K^0} = 1.0^{0.3}_{-0.2} \times 10^{-10}$ sec,[14] $\tau_{K^+} = 1.24 \pm 0.02 \times 10^{-8}$ sec[15]),

and from the measured[12] $\frac{P(K^+ \longrightarrow \text{other final states})}{P(K^+ \longrightarrow \pi^+ + \pi^0)} = 2.5 \pm 0.4$. The most recent experimental value of f is $f \simeq 1/16$, due to Steinberger. Assuming again the symmetry of the S-matrix, α_2 and α_0 are the phase shifts of the final pion-pion system in $\ell = 0$, $T = 2$ and in $\ell = 0$, $T = 0$ respectively. The final pion energies are near the resonance proposed to explain the second pion-proton maximum.[16] According to Dyson such resonance could occur in the $T = 0$ state. If α_0 is near 90° and α_2 small, so that we have $\cos(\alpha_2 - \alpha_0) \cong 0$, it is clearly impossible to explain the observed f ratio with small $\Delta T = \frac{3}{2}$ and $\Delta T = \frac{5}{2}$ contributions. If we tentatively assume that the two phase shifts are sufficiently small, so that we have $\cos(\alpha_2 - \alpha_0) \cong 1$, we find from Eq. (10) the two sets of solutions $x_3 \cong -0.05$, $x_5 \cong 0.16$, and $x_3 \cong -0.12$, $x_5 \cong 0.09$. Such values may be too large to be accounted for as electromagnetic corrections (amplitudes of the order e^2 according to perturbation theory). Another difficulty, also mentioned by Gell-Mann, is that whereas the probability for $K^+ \rightarrow \pi^+ + \pi^0$ would be expected in this model to be proportional to e^4, that for $K^+ \rightarrow \pi^+ + \pi^0 + \gamma$ should turn out proportional to e^2. It must be remarked, however, that the phase space available to the $2\pi + \gamma$ final state is expected to be much smaller than that for 2π (the phase space for $2\pi + \gamma$ is about 5 times that for 3π). Moreover, for a zero spin K^+ the two final mesons are left in the $\ell = 1$, $P = -1$ state by an E1 (and also possibly by M1) gamma transition, and thus they have to overcome a centrifugal barrier. However, no cases at all of $K^+ \rightarrow 2\pi + \gamma$ have been reported so far.

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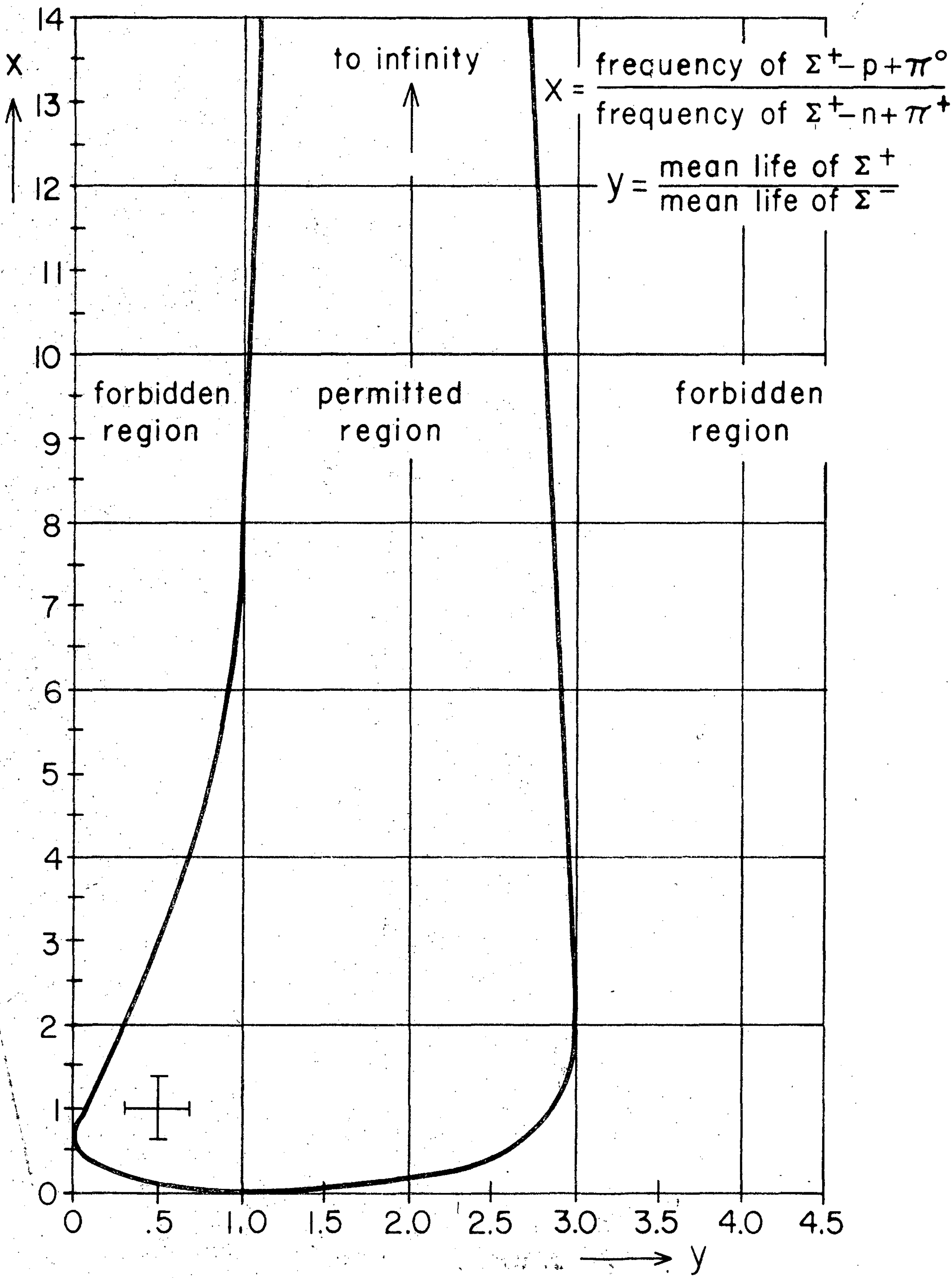
FIGURE CAPTION

<u>Figure 1:</u> The permitted region in the Y-X plane.


x
y
to infinity
x = frequency of Σ⁺ − p + π⁰ / frequency of Σ⁺ − n + π⁺
y = mean life of Σ⁺ / mean life of Σ⁻
forbidden region
permitted region
forbidden region
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
0
.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5