

UCLA

UCLA Electronic Theses and Dissertations

Title

Essays on International Economics

Permalink

<https://escholarship.org/uc/item/1c64t22g>

Author

Senel, Gonca

Publication Date

2014

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA
Los Angeles

Essays on International Economics

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Economics

by

Gonca Şenel

2014

© Copyright by

Gonca Şenel

2014

ABSTRACT OF THE DISSERTATION

Essays on International Economics

by

Gonca Şenel

Doctor of Philosophy in Economics

University of California, Los Angeles, 2014

Professor Lee E. Ohanian, Chair

In these essays, I examine (i) international finance and its effects on the economies (ii) immigration and its effects on labor markets. The first chapter explores whether the intervention of a lender of last resort (LLR) with seniority improves the welfare of the countries that are solvent but illiquid. In a model with information asymmetries and incomplete contracts I find that depending on the initial parameters, LLR intervention may or may not help these countries in overcoming this problem since an LLR intervention decision with seniority incorporates a trade-off between higher levels of intervention and lower liquidation. Results of my simulation analysis show that there exists some conditions under which LLR intervention creates lower level of welfare and is not preferable. On the other hand, if the conditions are such that LLR intervention is preferable, given some restrictions, I find that LLR intervention should be conducted without the seniority requirement. Second and third chapters analyze the effect of immigration on welfare through endogenous technological choice of firms. In the second chapter, I empirically test whether immigration of different types of labor (skilled vs unskilled) affects technology choice of firms differently. Specifically, I test whether firms change their technology in such a way that they will increase the productivity of the labor type that has become more abundant. In order to achieve this, I use census data between years 1970 and 2006 and use instrumental variable technique. Regression results show that high skilled immigration has a strong and positive association with the high-skilled intensive production technology choice of firms while low skilled immigration has a strong negative association with the high-skilled intensive production techniques. In other words, there is a strong association between immigration and endogenous technological choice of firms. In

the third chapter, I analyze how immigration affects the long-run welfare of immigration through endogenous choice of firms. Existing theoretical models predict that immigration would depress the wages. However, empirical literature finds that immigration effect on wages is either positive or insignificant. In order to match the theory with these empirical findings, I embed endogenous technological change in a model similar to Auerbauch and Kotlikoff (1987). The results show that the standard model underestimates the effect of immigration to native skilled workers by 95% while it overestimates the effect on native unskilled workers by 31%. Comparing the fiscal effects of immigration in terms of burden of an immigrant through net present discount value (NPV) calculations existing models overestimate NPV of an additional low skilled immigrant approximately by 35% and underestimate the value of an additional high skilled immigrant by 15%.

The dissertation of Gonca Şenel is approved.

Antonio E. Bernardo

Ariel T. Burstein

Mark L. J. Wright

Lee E. Ohanian, Committee Chair

University of California, Los Angeles

2014

To my mom, Mualla Senel, for her unconditional love;
To my father, Cemali Senel, for not letting me give up;
To my brother, Sinasi Senel, for being there for me at the right time.

TABLE OF CONTENTS

1	Seniority, Bailouts and the Effects of Lender of the Last Resort . . .	1
1.1	Introduction	1
1.2	Literature Review	3
1.3	The Model without LLR	4
1.3.1	Solution of the Model without LLR	6
1.4	The Model with LLR	13
1.4.1	The Model with LLR Decision of No Intervention	15
1.4.2	The Model with Possible LLR Intervention with Seniority	16
1.5	Comparison of Policies	31
1.5.1	Comparison of Total Surplus	31
1.5.2	Comparison of Expected Return to the Good Country	36
1.5.3	Simulation Exercise:	40
1.6	Conclusion	42
1.7	Appendix	43
1.7.1	Proofs	43
1.7.2	Figures	55
	References	59
2	Immigration and Endogenous Technology Choice	60
2.1	Introduction	60
2.2	Construction of Technology Choice Variable	61
2.3	Results	62
2.3.1	OLS Estimation	63
2.3.2	2SLS Estimation	63
2.4	Conclusion	66

2.5	Appendix	66
2.5.1	Data Construction	66
References		69
3	Immigration, Endogenous Technology Choice and Welfare Analysis .	71
3.1	Introduction	71
3.2	Literature Review	73
3.3	The Model	74
3.3.1	Individuals	74
3.3.2	Firms	78
3.3.3	Government	80
3.3.4	Pension Funds	80
3.3.5	Competitive Equilibrium	81
3.4	Calibration	81
3.4.1	Calibration of Parameters for Individual's Preferences	81
3.4.2	Calibration of Parameters for Production Function	81
3.4.3	Calibration of Parameters for Efficiency Units	83
3.4.4	Calibration of Parameters for Law of Motion for Population	84
3.4.5	Calibration of Parameters for Government and Social Security	86
3.5	Results	86
3.5.1	Experiment I: Increase in the High Skilled Labor	87
3.5.2	Experiment II: Increase in Low Skilled Labor	102
3.6	Conclusion	116
3.7	Appendix	118
References		128

LIST OF FIGURES

1.1	Timing of Events and Outcomes	5
1.2	Symmetric Countries: Gains from trade and entry	14
1.3	Comparison of Total Surplus	56
1.4	Comparison of Ex-ante Expected Return to the Good Country	57
1.5	Comparison of Ex-post Expected Return to the Good Country	58
3.1	Share of High and Low Skilled Labor with Technical Change	89
3.2	Unskilled and Skilled Labor Wages	90
3.3	Labor Decisions	91
3.4	Asset Holdings for Cohort 2 and Cohort 3	93
3.5	Asset Holdings for Cohort 4 and Cohort 5	94
3.6	Consumption Decisions for Cohort 2 and Cohort 3	95
3.7	Consumption Decisions for Cohort 4 and Cohort 5	95
3.8	Economy Aggregates	97
3.9	Pension Payment Share and Capital Rent	98
3.10	Equivalent Variation	100
3.11	Net Present Value Analysis-Direct Effect	101
3.12	Net Present Value Analysis- Long Run Effect	102
3.13	Share of High and Low Skilled Labor with Technical Change	104
3.14	Unskilled and Skilled Labor Wages	105
3.15	Labor Decisions	106
3.16	Asset Holdings for Cohort 2 and Cohort 3	108
3.17	Asset Holdings for Cohort 4 and Cohort 5	109
3.18	Consumption Decisions for Cohort 2 and Cohort 3	110
3.19	Consumption Decisions for Cohort 4 and Cohort 5	110
3.20	Economy Aggregates	112

3.21 Pension Payment Share and Capital Rent	113
3.22 Equivalent Variation	114
3.23 Net Present Value Analysis-Direct Effect	115
3.24 Net Present Value Analysis- Long Run Effect	116

LIST OF TABLES

2.1	OLS Regression Table	63
2.2	IV Regression Table 1	65
2.3	IV Regression Table 2	65
3.1	Efficiency Units wrt Skill, Origin and Time	83
3.2	Initial Distribution of Population	84
3.3	Survival Probabilities for Different Generations($j=m,n$)	84
3.4	Fertility Rates for different origins and different skill levels	85
3.5	Skill Heritability Matrices	85
3.6	Initial Stationary Distribution of Population	87
3.7	Final Stationary Distribution of Population	88
3.8	Final Stationary Distribution of Population	103

ACKNOWLEDGMENTS

I am greatly indebted to my committee - Lee E. Ohanian, Mark L. J. Wright, Ariel T. Burstein, Antonio E. Bernardo - for their invaluable guidance.

My deepest gratitude is to my main advisor, Prof. Lee E. Ohanian. I would like to thank him for all his patience and understanding, which made this journey more bearable and exciting. His invaluable support helped me overcome each struggle that I faced.

In addition, I would like to thank Muruvvet I. Buyukboyaci, Naime Camliyurt, Bulent Camliyurt, Umit Corut, Ahmet Faruk Coskun, I. Engin Pehlivanoglu, Emel Sen for their friendship and support. I am also grateful to Nida Cakir-who enlightened my last 7 years-for all her support and kindness.

Lastly, I would like to thank my dearest friend Ozlem Ekmekciler-who made me believe in happy coincidences-for being next to me at each milestone that I passed during my last 5 years.

VITA

- 2005 B.S. (Management), Bilkent University, Ankara, Turkey.
- 2005–2007 M.A. (Economics), Bilkent University, Ankara, Turkey.
- 2006–2008 Teaching Assistant, Bilkent University, Ankara, Turkey.
- 2009–2012 Teaching Fellow, Department of Economics, University of California, Los Angeles.
- 2010 M.A. (Economics), University of California, Los Angeles.
- 2010 C.Phil. (Economics), University of California, Los Angeles.
- 2012–2013 Special Reader, Department of Economics, University of California, Los Angeles.

CHAPTER 1

Seniority, Bailouts and the Effects of Lender of the Last Resort

1.1 Introduction

Since Germany and IMF's controversial bailout of Greece and Ireland, the question of effectiveness of bailouts have become more prevalent. Even though bailouts are continuously given to sovereign countries, researchers are far from reaching a conclusion on the efficacy of these bailouts. While some researchers claim that the bailouts were essential in order to rescue these countries from default, others think that bailouts just postponed an inevitable and even necessary default. Although researchers cannot agree on the effectiveness of bailouts, they intrinsically presume the same initial condition: seniority of the lender of the last resort (LLR). In this study, however we show that their common presumption, LLR seniority, will affect their conclusions about the success of the bailout considerably. Given a framework under which there is uncertainty about the LLR intervention and information asymmetries, we examine conditions under which financial bailouts may help countries overcome their debt overhang problem by saving them from a liquidity trap and show that the effectiveness of the LLR intervention depends heavily on the LLR seniority requirement.

In order to achieve this, this study analyzes how LLR intervention with seniority affects welfare. In many real world practices, when lender of the last resort decides on intervention, the lender usually requires seniority in case of default. At first, this may seem beneficial in the sense that seniority plays an important role in reducing the face value of the contract that the LLR will demand. In addition, because the LLR can implement the seniority requirement, it can solve the debt overhang problem unlike a private investor. Another advantage of LLR intervention is that by having enough

resources and by investing in many countries, the LLR is able to break even since some of the countries are illiquid but solvent. On the other hand, LLR intervention with seniority will reduce the expected return to the investors, creating possible early runs and higher premium requirements that may lead to lower returns for the countries. This paper describes the simulation results based on this logic and our aim is to show when seniority may be harmful and when not.

In order to account for early liquidation possibility of investors, we consider an environment where there exists information asymmetries. In other words, investors do not know the exact conditions of the countries in which they invest; but rather they receive private signals about these countries' success. When the country has a bad signal, investors cannot differentiate the underlying reason for this signal. The country might either have had a bad productivity shock recently even though in reality it has good investment opportunities, or the country does not have good investment opportunities and continuing to invest in this country is loss of resources. The underlying reason for the second scenario might be that the country is corrupted and the resources available are not used in the most efficient way. Accordingly, even though there are some signals about the results of their investments, the investors may not be sure about the "type" of the country with which they are interacting.

Due to uncertainty about the quality of projects, investors might be willing to liquidate their investments early, and the LLR intervention might be beneficial for the countries facing early liquidation. In addition, seniority may help LLR intervention because it lowers the return that LLR needs to charge and with the help of seniority, countries are able to finance their investments that are terminated early and would not be continued due to debt overhang problem. On the other hand, imposing seniority will reduce the expected returns for the investors further and this will increase their incentives to early liquidate projects creating a crowding-out effect. Given a setting where the contracts are incomplete, we find that there are some conditions under which LLR intervention may be beneficial for the country. On the contrary, we also find that LLR intervention should not be carried out with seniority requirement given some parameter restrictions. In addition, we also consider possible contracts where LLR requires seniority and we compare the outcomes of these contracts. From our analysis we find that contracts that

does not trigger early liquidation gives higher welfare as compared to the contracts that require higher returns and triggering early liquidation.

1.2 Literature Review

There are mainly two studies from which this study has benefited. In the study by Corsetti et al.(2006) the authors aim to analyze the catalytic effects of IMF through increasing the number of investors willing to continue lending to the country. This study shows that rather than increasing the moral hazard, existence of IMF decreases the probability of default. In addition, with the framework that they propose, they show that the seniority might have opposite effects. They claim that seniority increases the IMF's willingness to intervene decreasing the liquidation. On the other hand, they claim that seniority would increase the liquidation levels through increasing the cost of default. However, in their model, they presume that the cost of default is irrelevant of the payoffs. This study, however, attempts to make the connection between the costs of defaults with the payoffs. In addition, we aim to show that in some cases, the cost of LLR intervention may be higher than its benefits when it triggers early liquidation. Moreover, in Corsetti et al.(2006) they assume that IMF does not have enough resources to cover early liquidation amounts. However, in Roubini and Setser (2004) the authors claim that IMF usually has enough resources to cover the early liquidation amounts. With this statement in mind, we find that even though LLR has enough resources to cover the early liquidation amounts, intervention may have increased liquidation and reduced welfare. Different from another study, Saravia(2010), this study aims to examine the welfare implications of the LLR intervention in a setting where liquidation decision is endogenous rather than assuming liquidity shocks. ¹

The rest of this paper is as follows: In Section 2 the model is proposed and its solution under different settings have been computed. Comparison and simulation results of the model can be found in Section 3. Section 4 concludes

¹Beside these studies, there have been vast amount of studies on sovereign debt literature. Some of the other important papers on country indebtedness are Eaton and Gersovitz(1981) Krugman(1988) Bulow and Rogoff(1989a, 1989b). Borenstein(1990), Atkeson(1991), Rajan(1992), Cole and Kehoe(1998), Kletzer and Wright(2000), Reinhart, Rogoff and Savastano(2003), Rose and Spiegel(2004), Miller, Tomz and Wright (2006) and Tornell and Lane (1998).

1.3 The Model without LLR

The aim of this study is to show that the LLR intervention, if considered together with the seniority rule of LLR, may lead to inefficient outcomes. When the LLR loans have seniority over investor's loans, bailout decision may trigger the liquidation of the investments as proposed in Corsetti et al. (2006). The underlying reason is that if the country defaults even in case of LLR intervention, the investor will not be able save any portion of the non-liquidated amount because of the seniority of LLR and this will reduce the incentives of investors to continue the investment. Before getting into detail of LLR intervention let us first construct and solve the model without LLR and then discuss possible effects of LLR intervention.

In this model we have three time periods: $t=0,1,2$. Investors make their investments at $t=0$ and the final outcomes are realized at $t=2$. In the interim period which is $t=1$, investors receive some signals about the type of the country. According to these signals they make decisions whether to early liquidate the project or not.

For each country there exists an investment technology that requires I amount of initial investment and final outcomes are realized at time $t=2$. The country does not have any initial resources to finance this investment and has to borrow abroad. Investors finance these projects and the country borrows from a continuum of investors $[0,1]$ at $t=0$ and issues a debt contract with face value F_{INV} .

There are two types of countries in the economy : with probability θ the country has good investment opportunities (these countries will be called as "good type" throughout the paper) and with probability $(1 - \theta)$ the country does not have good investment opportunities (these countries will be called "bad type" throughout the paper). Depending on the type of the country, investment technology generates a random outcome R_σ , $\sigma \in \{H, L, 0\}$. If the country is of good type, it generates R_H with probability p_H and R_L with probability $(1 - p_H)$ at $t=2$. If the country is of bad type, it generates 0 at $t=2$. Countries always choose to start to project at $t=0$ and continue the project at $t=1$ since they have limited liability.

Investors are risk neutral and in competitive markets. They do not know the type of the country but they know θ . There is no time discounting for investors. For simplicity,

international interest rates are normalized to zero. Investors issue debt contracts based on their current information at $t=0$. The contracts are incomplete and the payments at $t=2$ are not contingent on the type of the country. In addition, we assume that investors are able to set the face value in such a way that they will be able to set either liquidation-proof contracts or they can embed the risk of liquidation which will be explained in detail in the next section. At $t=1$, they receive signals that are informative (but not perfect) about the type of the country and they may choose to liquidate the project. If they ask for early liquidation, they will only be able to get $I \times R_L$ (where $R_L < 1$) at $t=1$ due to inefficiencies related with early liquidation. As can be seen, we assume that early liquidation value is equal to the default value of the good country at $t=2$. This means that when the country is of good type, investors need not liquidate their investments since they would be able to get at least this amount if they have waited till $t=2$. However, since the country can also be of bad type, there is the risk of continuing a project that will yield zero return at $t=2$.

Signals also depend on the type of the country. If the country is of good type with probability p_H the investors get a signal of R_G and with $(1 - p_H)$ the investor gets a signal of R_L . If the country is of bad type, investors will always get the signal R_L .

In this context, the timing of events, signals and outcomes are as follows:

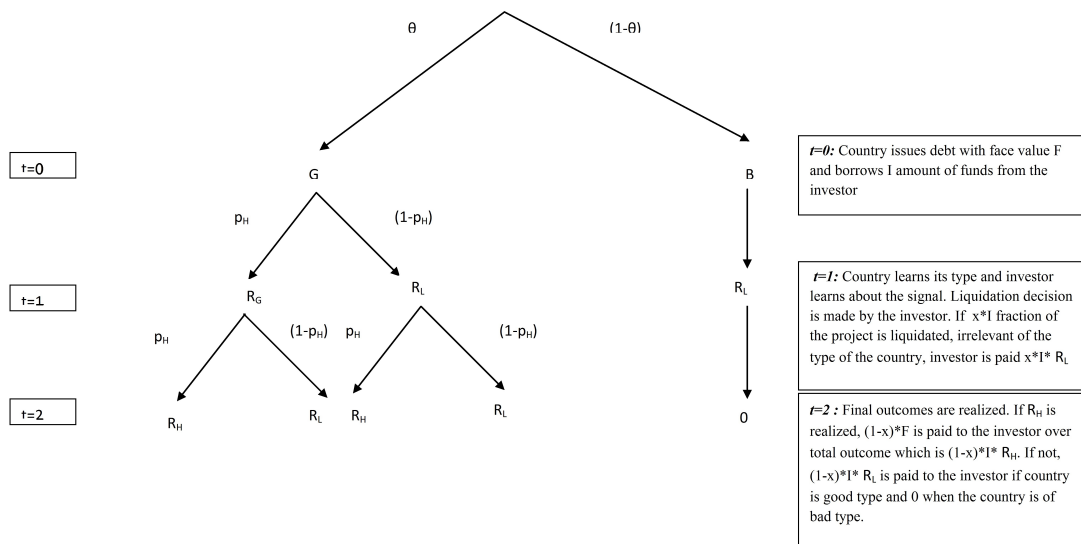


Figure 1.1: Timing of Events and Outcomes

Given the set up mentioned above , the information set of the investors are as follows:

At $t=0$, θ which is probability of investing a good country, is known by everyone :

$$P(G) = \theta$$

$$P(B) = 1 - \theta$$

At $t=1$, signal about the project is received. There are two types of signals: either R_G or R_L . Seeing one of these signals, the investor updates his/her beliefs about the quality of the country. The beliefs are updated with Bayesian updating. Ex-post beliefs are as follows:

$$P(G|R_L) = \pi = \frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)}$$

$$P(G|R_G) = 1$$

Before solving the model without LLR intervention we need to make some assumptions about the outcomes of the project. These are:

Assumption 1:

$$I < \theta \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) \quad (1.1)$$

This assumption is needed in order to have investment with positive expected return.

Assumption 2:

$$I \times R_H > F_{INV} \quad (1.2)$$

This assumption is needed in order to make sure that when the investment ends up with success, investor will be fully paid so that the investors will invest in the project at $t=0$.

1.3.1 Solution of the Model without LLR

Given this setting our aim is to show that under some conditions that will be stated below, countries might face with inefficient liquidation and this might create room for LLR intervention.

In the model that is constructed above, we assumed that investors are able to embed the risk of liquidation at $t=1$ into the contracts at $t=0$. Accordingly, we will be solving

the Subgame Perfect Bayesian Equilibria. In the first equilibrium, we will consider the case where the investor will set the face value of the contract F_{INV} such that irrespective of the signal s /he gets s /he will not be willing to liquidate the project at $t=1$. In the second equilibrium, F_{INV} will be set such that investors will liquidate the project in case of a bad signal at $t=1$. In the third equilibrium we will analyze whether there exists a face value that will create liquidation irrespective of the signal at $t=1$. In order to have subgame perfect equilibrium, we solve the model backwards and determine the face value of the contract depending on the possible liquidation decisions stated above.

1.3.1.1 Equilibrium 1: No Liquidation:

In this equilibrium face value F_{INV} will be characterized such that with their updated beliefs at $t=1$, investors will still choose to stay in the contract. In other words, in the first equilibrium face value of the contract is set such that at $t=1$ the investors will continue the project irrespective of the signal that they receive. The underlying reason is that the face value of the contract is high enough so that the investors are willing to bear the risk of waiting for the final outcome rather than early liquidation. We solve the model backwards in order to determine the face values of the contracts .

Investor's Problem

1. Investor's Problem at $t=1$:

In order to have no liquidation at $t=1$, face value F_{INV} should be such that given the face value at $t=1$ the investor will not liquidate whether it is a good signal or a bad signal. In order to have this satisfied we have to check that investors will be still willing to stay in the contract after they update their beliefs when they receive the bad or the good signal.

(a) Bad Signal:

If they receive the bad signal, then they will update their beliefs about facing a good country according to the following:

$$P(G|R_L) = \pi = \frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)}$$

Since investors will receive F_{INV} only if it is a good country and the project ends up as a success, and will receive R_L if it is a good country and the project ends up as a failure and will receive nothing if the country is of bad type, investors have the expected return stated below. Accordingly, in order to guarantee that the investor will not liquidate the project if the bad signal is received, s/he needs to have an expected payoff with his/her updated beliefs that is greater than the early liquidation value which is equal to R_L . In other words, following inequality should be satisfied in order to have no liquidation in case of receiving a bad signal:

$$\pi \times (p_H \times F_{INV} + (1 - p_H) \times I \times R_L) > I \times R_L$$

Equivalently,

$$\frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)} \times (p_H \times F_{INV} + (1 - p_H) \times I \times R_L) > I \times R_L$$

(b) Good Signal:

If they receive the good signal, since $P(G|R_G) = 1$, then in order to stay in the contract when the good signal is received F_{INV} should be such that expected payoff with his/her updated beliefs is greater than the early liquidation value :

$$(p_H \times F_{INV} + (1 - p_H) \times I \times R_L) > I \times R_L$$

This means that if the first condition holds then the second one will hold for sure since $P(G|R_L) = \pi < 1$.

2. Investor's Problem at t=0

At time t=0, investor will set the interest rate such that expected return at t=0 will be equal to I because of risk neutrality of the investor. In addition, while setting the interest rates he also will incorporate that s/he will not be liquidating the project at=1. Given this decision F_{INV} will be such that:

$$\underbrace{\theta \times p_H}_I \times \underbrace{(p_H \times F_{INV} + (1 - p_H) \times I \times R_L)}_{II} +$$

$$\underbrace{(1 - \theta \times p_H)}_{III} \times \underbrace{\left(\frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)} \times (p_H \times F_{INV} + (1 - p_H) \times I \times R_L) \right)}_{IV} = I$$

I: Probability of seeing a good signal at t=1

II: Expected return after updating beliefs given that a good signal is received

III: Probability of seeing a bad signal at t=1

IV: Expected return after updating beliefs given that a bad signal is received

If we solve this equation for F_{INV} we get:

$$F_{INV} = \frac{I - \theta \times (1 - p_H) \times I \times R_L}{\theta \times p_H}$$

In order to have the equilibrium F_{INV} should still satisfy the conditions that we stated in the investor's problem at t=1. In other words, $\frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)} \times (p_H \times F_{INV} + (1 - p_H) \times I \times R_L) > I \times R_L$ must be satisfied. If it is not satisfied, then the LLR will be liquidating when the bad signal is received and this will not be an equilibrium.

Total Surplus: Since there is no early liquidation, whole project will continue irrespective of the type of the signals and the type of the country. Accordingly total surplus will be:

$$S^{NL} = \theta \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

Expected Return to the Good Country: Given this face value of debt, if we consider the expected return to the good country rather than the total surplus, since none of the investors will liquidate, expected return to the good country will be:

$$U^{NL} = (p_H \times I \times R_H - p_H \times F_{INV})$$

1.3.1.2 Equilibrium 2: Partial Liquidation

In this equilibrium, investor knows that s/he will liquidate when the bad signal is received. Having this risk, investor will require for a higher return in order to be compensated for

a risk of early liquidation.

Investor's Problem

1. Investor's Problem at t=1:

In order to have liquidation at t=1 contingent on receiving a bad signal, face value F_{INV}^{NLLR} should be such that given that value, at t=1 the investor will liquidate the investment because the expected return with the updated beliefs will be less than the liquidation value. On the other hand, investor would be willing to continue the project if the signal is good. Accordingly, F_{INV}^{NLLR} should be such that the conditions below are satisfied:

(a) Bad Signal:

If bad signal is received, investor would no longer be willing to stay in the contract. Same as the previous case, investor will update his/her beliefs using Bayesian updating:

$$P(G|R_L) = \pi = \frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)}$$

In order to have the investor liquidate the project, F_{INV}^{NLLR} should be such that expected return with the updated beliefs is less than the liquidation value:

$$\pi \times (p_H \times F_{INV}^{NLLR} + (1 - p_H) \times I \times R_L) < I \times R_L \quad (1.3)$$

which means

$$\frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)} \times (p_H \times F_{INV}^{NLLR} + (1 - p_H) \times I \times R_L) < I \times R_L$$

So that F_{INV}^{NLLR} will be such that the investor will choose to liquidate and get R_L .

(b) Good Signal:

If good signal is received by the investor then he knows that $P(G|R_G) = 1$ and accordingly will continue with the project if the expected returns with the

updated beliefs is greater than the liquidation value:

$$(p_H \times F_{INV}^{NLLR} + (1 - p_H) \times R_L) > I \times R_L \quad (1.4)$$

2. Investor's Problem at t=0

At t=0 knowing that s/he will liquidate when the bad signal is received and will stick with the project only if the good signal is received, s/he will incorporate this risk in to the contract and because of risk neutrality, the face value F_{INV}^{NLLR} will be such that expected return will be equal to I: $\underbrace{\theta \times p_H}_I \times \underbrace{(p_H \times F_{INV}^{NLLR} + (1 - p_H) \times R_L)}_{II} + \underbrace{(1 - \theta \times p_H)}_{III} \times \underbrace{I \times R_L}_{IV} = I$

I: Probability of seeing a good signal at t=1

II: Expected return after updating beliefs after seeing a good signal

III: Probability of seeing a bad signal at t=1

IV: Expected return after updating beliefs after seeing a bad signal

Note that, different from the previous case where the investors would always stay in the contract, they are expecting to get $I \times R_L$ and early liquidate the project in case of a bad signal which will occur with probability $(1 - \theta \times p_H)$. If we solve the equality further:

$$\theta \times p_H^2 \times F_{INV}^{NLLR} + (1 - \theta \times p_H^2) \times I \times R_L = I$$

In other words,

$$F_{INV}^{NLLR} = \frac{I - (1 - \theta \times p_H^2) \times I \times R_L}{\theta \times p_H^2}$$

In order to have F_{INV}^{NLLR} as the face value of the contract in this equilibrium, F_{INV}^{NLLR} should satisfy the conditions that are stated in investor's problem at t=1. Accordingly, the parameters should be such that the following conditions are satisfied :

$$\frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)} \times (p_H \times \frac{I - (1 - \theta \times p_H^2) \times R_L}{\theta \times p_H^2} + (1 - p_H) \times R_L) < I \times R_L$$

and

$$(p_H \times \frac{I - (1 - \theta \times p_H^2) \times R_L}{\theta \times p_H^2} + (1 - p_H) \times I \times R_L) > I \times R_L$$

Total Surplus: Since there is early liquidation in case of a bad signal, only some fraction (p_H) of the project will continue. Investors who get the good signal will continue the project and their fraction will be $\theta \times p_H$. On the other hand, since investors who get the bad signal will be early liquidating, that fraction of the project will not continue. Accordingly total surplus will be:

$$S^{NLLR} = \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

Expected Return to the Good Country: Since investors who receive the bad signal will early liquidate, only p_H fraction of the investment will continue and will yield the following expected return to the good country:

$$U^{NLLR} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NLLR})$$

Remark 1 *In this equilibrium there is room for welfare improvement. The underlying reason is if a social planner with full information existed then only good countries would get invested and the total surplus would be equal to the total surplus in case of no liquidation:*

$$S^{SP} = \theta \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

Accordingly, by having an LLR, good countries may be able to finalize a higher fraction of the investment and this may increase the total surplus.

1.3.1.3 Equilibrium 3: Full Liquidation

This cannot be an equilibrium because if the investor knows that s/he will liquidate and get $I \times R_L$ irrespective of the signal, then s/he will choose not to invest at $t=0$ since $I \times R_L < I$.

From these analyses we can state that if equilibrium with partial liquidation occurs, then there will be room for LLR intervention since some of the investment has been early

liquidated and this can be compensated by LLR intervention.

1.4 The Model with LLR

Now let's assume that there is an LLR that has the necessary resources that can be used to continue the project in case of early liquidation of the investors. This means that if fraction $f \times I$ of the investment has been liquidated by the investors then LLR can pay this amount to the country and the project can continue fully. However, we assume that in case of an LLR intervention, even though the project continues, the return of the portion of the project which is carried out by LLR will yield a return denoted by R_M that is lower than the return of the project in case of success if it had been carried out by the investor which is given as R_H . The underlying reason of this assumption is that investors would be more effective in monitoring these projects and this will lead to less moral hazard and increase the returns.

In this model, in case of early liquidation at $t=1$, LLR will commit to pay the amount that has been liquidated by the investors. This means that there is room for LLR action if there exists an early liquidation. This means that LLR will play a role only if some (or all) of the investors early liquidate their investment. Accordingly, we would be considering the equilibria where at least the investors who get the bad signal liquidate the project and LLR will finance the amount that has been liquidated. In return, LLR will sign a debt contract with face value F_{LLR} at time $t=1$ with the country which will be paid at $t=2$ when the final outcome has been realized. In our set up we assume that LLR does not have any information superior to the investor. This means that while giving the bailout decision, LLR is not able to differentiate the good and the bad countries from each other and LLR only knows the probability distribution of the type of the countries which is θ . In addition, we assume that the LLR intervention decision is given at $t=1$ before realization of investor's liquidation decisions and signals. The underlying reason of this assumption is that LLR has to commit to pay the amount that is requested by the country and cannot make the payments contingent on the liquidation information (in this case, LLR would pay to the good country and would refuse to pay to the bad country depending on the fraction of the liquidated amount). Accordingly, LLR intervention

decision is independent of the signals and the liquidation decisions. Given this setting we will have the following time line.

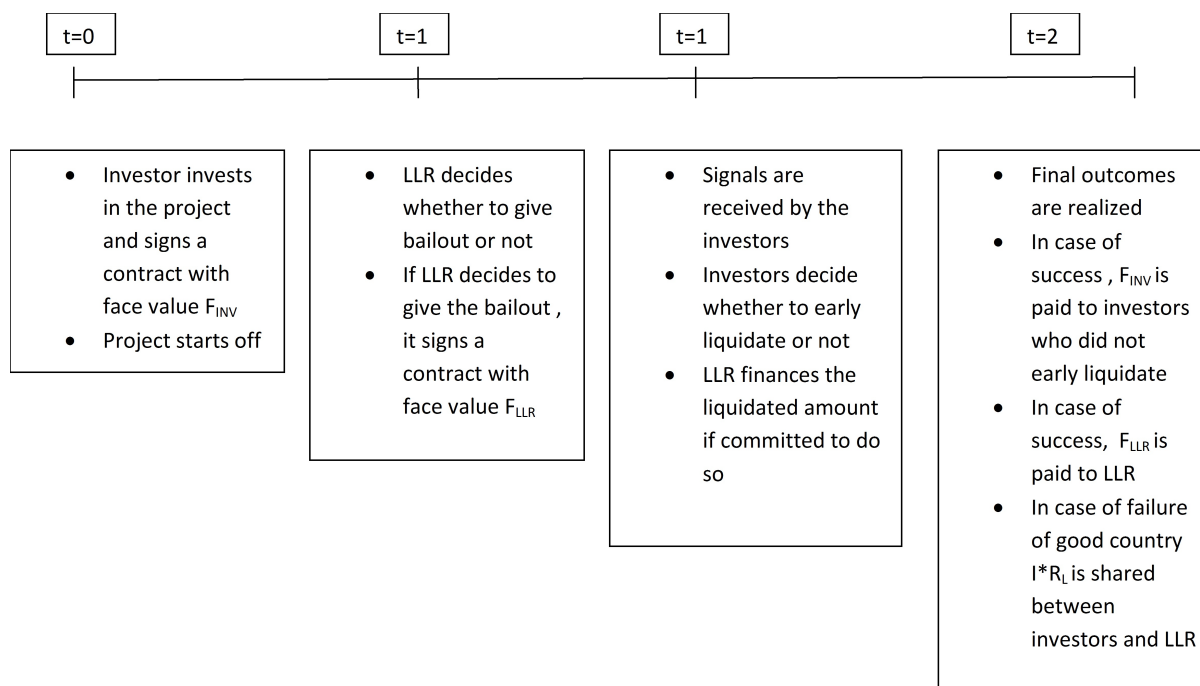


Figure 1.2: Symmetric Countries: Gains from trade and entry

In this section, we assume that at $t=0$ there is an ex-ante probability of intervention of LLR that is known by the countries and the investors and denoted by α . Knowing this probability and their possible actions in case of intervention, investors will incorporate this possibility into the contracts at $t=0$.

In addition, LLR might choose to intervene with or without seniority requirement. This means that at $t=1$ while constructing the debt contract, LLR might require that it will be paid prior to the investors in case of default. On the other hand, LLR might also choose to be paid pro-rata in case of default. Since the seniority requirement affects the payment to both investors and LLR and since investors can incorporate this possibility into their contracts at $t=0$, this will indirectly affect the welfare of the countries and also the total surplus. We consider the cases where LLR has seniority and it does not have seniority in two different subcases.

1.4.1 The Model with LLR Decision of No Intervention

In this section, our aim is to analyze the consequences of no LLR intervention. We will assume that even though there exists an LLR and with probability α LLR will intervene, if LLR is allowed to intervene, it will choose not to do so. Accordingly, our results will be the same with the findings when there is no LLR. Even though the conditions for each equilibrium is the same and have to hold, for the sake of gaining space we are just reporting the outcomes that we are interested in. However, it has to be noted that all conditions that are needed to be satisfied that are stated in case of no LLR are still valid and should be holding.

1.4.1.1 Equilibrium with No Early Liquidation:

In this equilibrium investors will still choose to stay in the contract irrespective of the signal that they receive. Accordingly, in this equilibrium we do not have room for LLR intervention.

1.4.1.2 Equilibrium with Partial Liquidation:

In this equilibrium, investor knows that s/he will liquidate when the bad signal is received. Having this risk, investor will require for a higher return in order to be compensated for a risk of early liquidation.

Ex-Ante and Ex-post Total Surplus: Since there is early liquidation in case of a bad signal, only some fraction of the project will continue. Investors who get the good signal will continue the project and their fraction will be $\theta \times p_H$. On the other hand, since investors who get the bad signal will be early liquidating, that fraction of the project will not continue. Since there is no policy that has been implemented, ex-ante and ex-post total surplus will be the same and will be equal to:

$$S^{EP,NLLR} = S^{EA,NLLR} = \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

Ex-Ante and Ex-post Expected Return to the Good Country: Since investors who receive the bad signal will early liquidate, only p_H fraction of the investment will continue. Since there is no policy that has been implemented, ex-ante and ex-post expected return to the good country will be the same and will be equal to:

$$U^{EP,NLLR} = U^{EA,NLLR} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NLLR})$$

where F_{INV}^{NLLR} was found to be:

$$F_{INV}^{NLLR} = \frac{I - (1 - \theta \times p_H^2) \times I \times R_L}{\theta \times p_H^2}$$

1.4.1.3 Equilibrium with Full Liquidation:

This cannot be an equilibrium because if the investor knows that s/he will liquidate and get $I \times R_L$ irrespective of the signal, then s/he will choose not to invest at $t=0$ since $I \times R_L < I$.

1.4.2 The Model with Possible LLR Intervention with Seniority

In this case, we assume that with probability α LLR will intervene and require seniority in case of default at $t=2$. This means that at $t=2$, if the good country fails to have a good outcome and gets a return of $I \times R_L$, then LLR will be paid $\min\{F_{LLR}^S, I \times R_L\}$ and the investors will be paid the remaining which will be equal to $\max\{0, I \times R_L - F_{LLR}^S\}$. Since we assume that the possible LLR intervention can be incorporated into the contracts that the investor writes, then seniority decision will affect the investors in two aspects. Firstly, at $t=0$, knowing that there is a possible LLR intervention at $t=1$, they will require to be paid a higher return in case of success since they will be paid lower in case of default because of the seniority decision of the LLR. Secondly, at $t=1$, if LLR intervenes, the liquidation decision of the investors will also be affected. Investors, who get the good signal and were initially willing to stay in the contract might choose to early liquidate the project since their expected return from the investment has reduced after learning that LLR will certainly intervene. In that case LLR would finance the whole project

which will reduce the return of the project even though the project has been carried out fully. Keeping these interactions in mind, we consider the Subgame Perfect Bayesian Equilibria where the intervention may or may not trigger early liquidation.

1.4.2.1 Equilibrium with Possible LLR Intervention with Seniority without Triggering

In this equilibrium, only the investors who get the bad signal will liquidate the project. On the contrary, investors who get the good signal will still be willing to stay in the contract even if LLR intervenes with seniority (so that the intervention will not create any triggering). In order to get this equilibrium, given the face value that LLR requires which is denoted by $F_{LLR}^{S,NT}$, we solve the investor's problem backwards and get the face value that the investor will want at $t=0$ denoted by $F_{INV}^{S,NT}$ while taking into consideration the probability of LLR intervention at $t=1$ denoted by α . Then, we calculate $F_{LLR}^{S,NT}$ such that the expected return to the LLR will be equal to the amount that has been financed by LLR.

Investor's Problem:

1. Investor's Problem at $t=1$

In this equilibrium, investors know that if they receive a good signal irrespective of LLR intervention (which will be with seniority if it occurs), they will not liquidate the project. On the other hand they will be liquidating the project if they receive a bad signal irrespective of the LLR intervention. Then $F_{INV}^{S,NT}$ should satisfy the conditions explained below:

(a)

$$(p_H \times F_{INV}^{S,NT} + (1 - p_H) \times I \times R_L) > I \times R_L \quad (1.5)$$

The condition above is needed in order to guarantee that investor would not liquidate in case of a good signal and no LLR intervention. Since there is no LLR intervention in the first place, in case of a good signal, $P(G|R_G) = 1$ and expected return is $(p_H \times F_{INV}^{S,NT} + (1 - p_H) \times I \times R_L)$. In order to guarantee

that the investor will not early liquidate, this value should be greater than the liquidation value which is equal to $I \times R_L$.

(b)

$$(p_H \times F_{INV}^{S,NT} + (1 - p_H) \times \left(\frac{Max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H} \right)) > I \times R_L \quad (1.6)$$

This condition needs to hold in order to guarantee that investor would not liquidate in case of a good signal and with LLR intervention with seniority. If the LLR intervenes investors know that project will yield $Max\{0, I \times R_L - F_{LLR}^{S,NT}\}$ to them in case of default. Since only investors with a mass of p_H will not liquidate the project, they will share this equally and get $\frac{Max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H}$ each. Accordingly, in order to have them stick with the project even in case of LLR intervention with seniority, investors should be expecting to get a return that is higher than the early liquidation value.

(c)

$$\pi \times (p_H \times F_{INV}^{S,NT} + (1 - p_H) \times R_L) < I \times R_L \quad (1.7)$$

The above condition is needed for the investor to liquidate the project in case of a bad signal. It states that expected return of the investor in case of continuing the project should be less than the liquidation value. However, we need to keep in mind that existence of LLR does not effect the returns of the investors who get the bad signal. The underlying reason is that if bad signal receivers decide to stay in the contract then there is no room for LLR intervention because good signal receivers will also stay in the project since their expected returns are higher. On the other hand, if investors who receive the bad signal decide to early liquidate, then LLR intervention decision will not affect them since by early liquidating they accept to get $I \times R_L$.

2. Investor's Problem at t=0

The equilibrium will be such that with the face value $F_{INV}^{S,NT}$ for the investors, LLR intervention with seniority will not create any triggering. Then the $F_{INV}^{S,NT}$ will be such that expected return at t=0 should be equal to I:

$$\underbrace{(1 - \alpha) \times \theta \times p_H \times p_H \times F_{INV}^{S,NT}}_I + \underbrace{(1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L}_{II}$$

$$\begin{aligned}
& + \underbrace{(1 - \alpha) \times (1 - \theta \times p_H) \times I \times R_L}_{III} + \underbrace{\alpha \times \theta \times p_H \times p_H \times F_{INV}^{S,NT}}_{IV} \\
& + \underbrace{\alpha \times \theta \times p_H \times (1 - p_H) \times \left(\frac{\max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H} \right)}_V + \underbrace{\alpha \times (1 - \theta \times p_H) \times I \times R_L}_{VI} = I
\end{aligned}$$

I : Expected return if LLR does not intervene, if the investor gets a good signal and if the project ends up as success

II : Expected return if LLR does not intervene, if the investor gets a good signal and if the project ends up as failure

III : Expected return if LLR does not intervene, if the investor gets a bad signal and early liquidates the project

IV : Expected return if LLR intervenes, if the investor gets a good signal and if the project ends up as success

V : Expected return if LLR intervenes, if the investor gets a good signal and if the project ends up as failure

VI : Expected return if LLR intervenes, if the investor gets a bad signal and early liquidates the project

If we solve it further:

$$\begin{aligned}
F_{INV}^{S,NT} &= \frac{I - \alpha \times \theta \times p_H \times (1 - p_H) \times \left(\frac{\max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H} \right)}{\theta \times p_H^2} \\
&\quad - \frac{(1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L - (1 - \theta \times p_H) \times I \times R_L}{\theta \times p_H^2}
\end{aligned}$$

LLR's Problem: At $t=1$, LLR will set the face value of the contract such that the expected return of the project to the LLR will be equal to expected payment of LLR in case of early liquidation of the investors. Then at $t=1$ before the signals have been realized and the early liquidation decisions have been made LLR will set $F_{LLR}^{S,NT}$ such that :

$$\underbrace{\theta \times p_H \times F_{LLR}^{S,NT}}_I + \underbrace{\theta \times (1 - p_H) \times \text{Min}\{F_{LLR}^{S,NT}, I \times R_L\}}_{II} = \underbrace{\theta \times (1 - p_H) \times I}_{III} + \underbrace{(1 - \theta) \times I}_{IV}$$

I : Expected return of the project for LLR if the country is good and the project is a success

II : Expected return of the project for LLR if the country is good and the project is a failure

III : Expected payment to the country for early liquidation if the country is good

IV : Expected payment to the country for early liquidation if the country is bad

1.4.2.2 Equilibrium Outcomes with Possible LLR Intervention with Seniority without Triggering:

In this section our aim is to analyze the outcomes of LLR intervention. We calculate these both ex-ante and ex-post in order to see whether there is time-inconsistency problem. In addition, we also calculate the expected return to the good country together with the total surplus in order to see whether the LLR intervention policies improve the conditions for illiquid but solvent countries.

Ex-Ante Expected Total Surplus: Firstly, we should note that the project will yield a positive outcome only if the country is of good type. In addition, if the country is good, in case of no triggering p_H fraction of the project will be continued with the investors which will yield outcome of $I \times R_H$ with probability p_H and $I \times R_L$ with probability $(1 - p_H)$. Accordingly, at $t=0$ expected total surplus will be equal to:

$$S^{EA,S,NT} = \underbrace{\theta \times \alpha \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L))}_I + \underbrace{\theta \times \alpha \times ((1 - p_H) \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L))}_{II} + \underbrace{\theta \times (1 - \alpha) \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L))}_{III} - I$$

I : Expected surplus if the country is good, if LLR intervenes and p_H fraction of the investors stay in the contract

II : Expected surplus if the country is good, if LLR intervenes and $(1 - p_H)$ fraction of the investment is financed by the LLR

III : Expected surplus if the country is good and if LLR does not intervene

Ex-Post Total Surplus if Intervention Occurs: If LLR is allowed to intervene and if LLR intervenes with seniority in such a way that it will not trigger liquidation we have the following total surplus at t=1:

$$S^{EP,S,NT,I} = \theta \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)) \\ + \theta \times ((1 - p_H) \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)) - I$$

Ex-Post Total Surplus if Intervention Does Not Occur: If LLR is allowed to intervene but if LLR does not intervene, we have the following surplus at t=1:

$$S^{EP,S,NT,NI} = \theta \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)) - I$$

Ex-Ante Expected Return to the Good Country: If the country is good, $(1 - p_H)$ fraction of the investors will get signal L and liquidate early and the amount that is liquidated will be covered by LLR in return for $F_{LLR}^{S,NT}$. Then at t=0, the expected net income of the good country will be equal to :

$$U^{EA,S,NT} = \underbrace{\alpha \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT})}_I + \underbrace{\alpha \times p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{S,NT})}_{II} \\ + \underbrace{(1 - \alpha) \times (p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT}))}_{III}$$

I : Expected return to the good country if LLR intervenes, if p_H fraction of the investors stay in the contract and if the project is a success

II : Expected return to the good country if LLR intervenes, if $(1 - p_H)$ fraction of the investment is carried out by the LLR and if the project is a success

III : Expected return to the good country if LLR does not intervene, if p_H fraction of the investors stay in the contract and if the project is a success

Ex-Post Expected Return to the Good Country if Intervention Occurs: If LLR is allowed to intervene and if LLR intervenes with seniority in such a way that it will not trigger liquidation we have the following expected return to the good country:

$$U^{EP,S,NT,I} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT}) + p_H \times ((1 - p_H) \times I \times R_M) - p_H \times F_{LLR}^{S,NT}$$

Ex-Post Expected Return to the Good Country if Intervention does not Occur: If LLR is allowed to intervene but if LLR does not intervene, we have the following expected return to the good country:

$$U^{EP,S,NT,NI} = (p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT}))$$

1.4.2.3 Equilibrium with Possible LLR intervention with Seniority with Triggering

In this equilibrium, if LLR decides to intervene with seniority at time $t=1$, investors will liquidate the project regardless of the signal they receive. At $t=1$, LLR might choose to intervene with seniority and may choose to sign a contract that has a face value denoted by $F_{LLR}^{S,T}$. Then, in the equilibrium this value has the possibility of being high enough to force investors to liquidate the project early since the expected return of the investment in case of default might have decreased considerably and this may make the investment unattractive to continue. In this case investors will early liquidate the project even though they receive the good signal and know that the country is of good type. On that aspect, LLR intervention might create crowding out and since LLR will not be as successful as the investor in carrying out the projects, LLR intervention may lead to inefficient outcomes. Similar to previous case, investors will incorporate the possibility that LLR can intervene with probability α in which case they will choose to liquidate the project regardless of the signal that they receive. In order to calculate the face value of the contract for the investor which is denoted by $F_{INV}^{S,T}$ we need to solve the model backwards as we have done before.

Investor's Problem

1. Investor's Problem at $t=1$

Since in this equilibrium investor will liquidate the project regardless of the signal s/he receives in case of LLR intervention with seniority and will not liquidate the project if LLR does not intervene and if s/he gets a good signal. Accordingly, the face value of the contract, $F_{INV}^{S,T}$, should be such that the following conditions are satisfied:

(a)

$$(p_H \times F_{INV}^{S,T} + (1 - p_H) \times (\frac{Max\{0, I \times R_L - F_{LLR}^{S,T}\}}{p_H})) < I \times R_L \quad (1.8)$$

The condition stated above guarantees that the investor will choose to early liquidate the project if LLR intervenes with seniority and if the good signal is received.

(b)

$$\pi \times (p_H \times F_{INV}^{S,T} + (1 - p_H) \times I \times R_L) < I \times R_L \quad (1.9)$$

This condition is needed so that the investors who get the bad signal will also liquidate their projects in case of an LLR intervention with seniority.

(c)

$$(p_H \times F_{INV}^{S,T} + (1 - p_H) \times I \times R_L) \geq I \times R_L \quad (1.10)$$

Above condition states that even though the investors who get the good signal will liquidate the project if LLR intervenes with seniority, in case of no LLR intervention they would still be willing to stay in the contract since the expected return of the project in case of no intervention is higher than the early liquidation value which is $I \times R_L$ so that continuation of the project is still justified.

2. Investor's Problem at t=0

At t=0, investors will set $F_{INV}^{S,T}$ such that they incorporate the probability of LLR intervention and resulting early liquidation into their contracts:

$$\underbrace{\alpha \times I \times R_L}_I + \underbrace{(1 - \alpha) \times (\theta \times p_H \times (p_H \times F_{INV}^{S,T} + (1 - p_H) \times I \times R_L))}_{II} + \underbrace{(1 - \theta \times p_H) \times I \times R_L}_{III} =$$

I : Expected return in case of LLR intervention creating early liquidation

II : Expected return if LLR does not intervene and if the investor receives a good signal

III : Expected return if LLR does not intervene and if the investor receives and bad signal and early liquidates

This equation incorporates the condition that if LLR intervenes, the investor foresees that s/he will certainly liquidate the project and get $I \times R_L$. On the other hand at $t=0$, investor still has to incorporate the possibility that LLR will not intervene in which case s/he still may continue the project of s/he receives a good signal.

If we solve the equation further:

$$F_{INV}^{S,T} = \frac{I - (1 - \alpha) \times (1 - \theta \times p_H^2) \times I \times R_L - \alpha \times I \times R_L}{(1 - \alpha) \times \theta \times p_H^2} = \frac{I - I \times R_L}{(1 - \alpha) \times \theta \times p_H^2} + I \times R_L$$

LLR's Problem: At $t=1$, LLR will set the face value of the contract such that the expected return of the project to the LLR will be equal to expected payment of LLR in case of early liquidation of the investors. Then at $t=1$ before the signals have been realized and the early liquidation decisions have been made LLR will set $F_{LLR}^{S,T}$ such that :

$$\theta \times (p_H \times (F_{LLR}^{S,T}) + (1 - p_H) \times \text{Min}\{F_{LLR}^{S,T}, I \times R_L\}) = I$$

The equation above states that the expected return of LLR should be equal to I . The underlying reason is if LLR intervenes all investors will liquidate irrespective of the type of the signal that they receive. Accordingly, LLR should finance the whole investment which is equal to I .

Lemma 1 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1) (1.2) are satisfied, if (1.8) (1.9) and (1.10) hold, then there exists an equilibrium for "LLR seniority with triggering early liquidation" policy. In this equilibrium, $F_{LLR}^{S,T} \geq I \times R_L$ holds.*

Proof. Conditions stated above are necessary and sufficient conditions for existence of equilibrium. For the second part, let's assume that condition $F_{LLR}^{S,T} \geq I \times R_L$ does not hold. Then $F_{LLR}^{S,T} < I \times R_L$. In this case, equation for $F_{LLR}^{S,T}$ becomes:

$$\theta \times (p_H \times (F_{LLR}^{S,T}) + (1 - p_H) \times F_{LLR}^{S,T}) = I$$

$F_{LLR}^{S,T} = \frac{I}{\theta}$ and it has to satisfy $F_{LLR}^{S,T} = \frac{I}{\theta} < I \times R_L$. However since $R_L \leq 1$ and $\theta \leq 1$ this condition contradicts with initial assumption which is $F_{LLR}^{S,T} < I \times R_L$. Accordingly, $F_{LLR}^{S,T} \geq I \times R_L$. ■

1.4.2.4 Equilibrium Outcomes with Possible LLR intervention with Seniority with Triggering:

Ex-ante Expected Total Surplus: In this equilibrium, we would see full liquidation of the project by the investors in case of LLR intervention which means that if the country is of good type and if the project ends up as a success the return of the project will be equal to $I \times R_M$. On the other hand, if LLR does not intervene, then the project will be carried out only by the investors who get the good signal. Accordingly, expected total surplus will be equal to:

$$S^{EA,S,T} = \underbrace{\alpha \times \theta \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)}_I + \underbrace{(1 - \alpha) \times \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)}_{II} - I$$

I : Expected return of the project if LLR intervenes, if country is of good type and if the project is a success

II : Expected return of the project if LLR does not intervene, if country is of good type and if the project is a success

Ex-Post Total Surplus if Intervention Occurs:

$$S^{EP,S,T,I} = \theta \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L) - I$$

Ex-Post Total Surplus if Intervention does not Occur:

$$S^{EP,S,T,NI} = \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

Ex-ante Expected Return to the Good Country: In case of possible LLR intervention all investors will liquidate early and the amount that is liquidated will be covered by LLR in return for $F_{LLR}^{S,T}$. Then the expected net income of the good country will be:

$$U^{EA,S,T} = \alpha \times (p_H \times (I \times R_M - F_{LLR}^{S,T})) + (1 - \alpha) \times (p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,T}))$$

Ex-Post Expected Return to the Good Country if Intervention Occurs:

$$U^{EP,S,T,I} = (p_H \times (I \times R_M - F_{LLR}^{S,T}))$$

Ex-Post Expected Return to the Good Country if Intervention does not Occur:

$$U^{EP,S,T,NI} = (p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,T}))$$

1.4.2.5 Equilibrium with Possible LLR intervention without Seniority

Let's assume that with probability α LLR will give the bailout in case of early liquidation without requiring the seniority. This means that at $t=1$ LLR will commit to pay the early liquidation amount and in case of a good country with default it will be paid proportional to the early liquidation payment. In this context we claim that existence of LLR intervention without seniority will have no effect on the liquidation decision of the investors.

Investor's Problem Like in previous cases, we will solve the model backwards in order to get the face value of the contract denoted by F_{INV}^{NS} .

1. Investor's Problem at $t=1$

Similar to the other cases, in order to have room for LLR intervention, we want to have the partial liquidation equilibrium when there is no LLR intervention. Accordingly, F_{INV}^{NS} should be such that it satisfies following conditions at $t=1$

(a)

$$(p_H \times F_{INV}^{NS} + (1 - p_H) \times (I \times R_L)) \geq I \times R_L \quad (1.11)$$

This condition guarantees that the investors who get the good signal will not liquidate the project in case of no LLR intervention.

(b)

$$\pi \times (p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) < I \times R_L \quad (1.12)$$

This condition is needed so that the investors who get the bad signal will liquidate the project.

Even though the above conditions only guarantees that there will be partial liquidation equilibrium when there is no LLR, in the following proposition, we prove that these conditions are sufficient to have an equilibrium where there is LLR intervention without seniority and it never triggers early liquidation.

Proposition 1 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if conditions (1.1) (1.2) (1.5) (1.6) and (1.7) hold for F_{INV}^{NS} , so that will be partial liquidation equilibrium without LLR intervention, then in this equilibrium, at $t=1$ if LLR decides to intervene without seniority, this policy will not affect the liquidation decisions of investors.*

Proof. If investors are in partial liquidation equilibrium, this means that investors who get the good signal will stay in the contract while the investors who get the bad signal will liquidate. Then the following conditions must be holding if there is no LLR intervention:

1) If the bad signal occurs the investor will update his/her beliefs using Bayesian updating:

$$P(G|R_L) = \pi = \frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)}$$

Since in this equilibrium F_{INV}^{NS} should be such that expected return with the updated beliefs is less than the liquidation value:

$$\pi \times (p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) < I \times R_L$$

which means

$$\frac{\theta \times (1 - p_H)}{\theta \times (1 - p_H) + (1 - \theta)} \times (p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) < I \times R_L$$

2) If the good signal is received by the investor then he knows that $P(G|R_G) = 1$ and accordingly F_{INV}^{NS} should be such that the investor receiving the good signal will always continue the project :

$$(p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) > I \times R_L$$

Then if LLR decides to intervene without seniority, expected returns to the investor will change as follows:

1) Investors who early liquidate the project will get the same expected return since they are not affected from LLR decision. This means that above condition still holds and they will liquidate the project:

$$\pi \times (p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) < I \times R_L$$

2) On the other hand, investors who stay in the contract will be able to get least $I \times R_L$ in case of default because of non-seniority of the LLR. Accordingly, expected return of the contract in case of LLR intervention with seniority will be equal to:

$$(p_H \times F_{INV}^{NS} + (1 - p_H) \times (I \times R_L + \frac{Max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\}}{p_H}))$$

If we compare it with expected return in case of no LLR intervention:

$$\begin{aligned} (p_H \times F_{INV}^{NS} + (1 - p_H) \times (I \times R_L + \frac{Max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\}}{p_H})) \\ > (p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) \\ > I \times R_L \end{aligned}$$

Accordingly, LLR intervention without seniority will not affect the actions of the investors.

■

This proposition is important since it proves that LLR intervention can trigger liquidation only when the seniority requirement is imposed by LLR. Accordingly, possible inefficiencies related with early liquidation can only be observed when there is seniority.

2. Investor's Problem at t=0

In this equilibrium, investors will set the face value of the contract F_{INV}^{NS} while taking into consideration the possible intervention of LLR summarized by α . Then F_{INV}^{NS} will be such that:

$$\underbrace{(1 - \alpha) \times (\theta \times p_H \times (p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) + (1 - \theta \times p_H) \times I \times R_L)}_I$$

$$\begin{aligned}
& + \alpha \times \underbrace{\left((\theta \times p_H \times (p_H \times F_{INV}^{NS} + (1 - p_H) \times (I \times R_L + \frac{\max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\}}{p_H})) \right)}_{II} \\
& + \underbrace{(1 - \theta \times p_H) \times I \times R_L}_{III} = I
\end{aligned}$$

I : Expected return of the project if LLR does not intervene

II : Expected return of the project if LLR intervenes without seniority and if the good signal is received.

III : Expected return of the project if LLR intervenes without seniority and if the bad signal is received.

We should note that since the investors who get the bad signal will liquidate they will accept to be paid $I \times R_L$.

If we solve it further

$$F_{INV}^{NS} = \frac{I - (1 - \theta \times p_H) \times I \times R_L - \alpha \times \theta \times p_H \times (1 - p_H) \times \left(\frac{\max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\}}{p_H} \right)}{\theta \times p_H} - (1 - p_H) \times I \times R_L$$

LLR's Problem In this equilibrium LLR will set the face value of the contract F_{LLR}^{NS} such that the expected return will be equal to expected payment:

$$\theta \times (p_H \times F_{LLR}^{NS} + (1 - p_H) \times \text{Min}\{(1 - p_H) \times I \times R_L, F_{LLR}^{NS}\}) = \theta \times (1 - p_H) \times I + (1 - \theta) \times I$$

1.4.2.6 Equilibrium Outcomes with Possible LLR intervention without Seniority:

Ex-ante Expected Total Surplus: The project will continue as a whole with investors and LLR together if the LLR decides to intervene without seniority. However, we also need to note that the fraction of the project that is carried out by LLR will yield a lower return denoted by $I \times R_M$. In case of no LLR intervention, only some fraction of the project will be carried out by the investors who get the good signal. Accordingly, at $t=0$ ex-ante expected total surplus will be equal to:

$$\begin{aligned}
S^{EA,NS} &= \underbrace{\theta \times \alpha \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L))}_I \\
&\quad + \underbrace{\theta \times \alpha \times ((1 - p_H) \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L))}_{II} \\
&\quad + \underbrace{\theta \times (1 - \alpha) \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L))}_{III} - I
\end{aligned}$$

I : Expected surplus if the country is good, if LLR intervenes and p_H fraction of the investors stay in the contract

II : Expected surplus if the country is good, if LLR intervenes and $(1 - p_H)$ fraction of the investment is financed by the LLR

III : Expected surplus if the country is good and if LLR does not intervene

Ex-Post Total Surplus if Intervention Occurs:

$$\begin{aligned}
S^{EP,NS,I} &= \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) \\
&\quad + \theta \times (1 - p_H) \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L) - I
\end{aligned}$$

Ex-Post Total Surplus if Intervention does not Occur:

$$S^{EP,NS,NI} = \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

Ex-ante Expected Return to the Good Country: In the case of possible LLR intervention, investors who get the bad signal will liquidate and the amount that is liquidated will be covered by LLR in return for F_{LLR}^{NS} . Then the expected net income of the good country will be:

$$\begin{aligned}
U^{EA,NS} &= \underbrace{\alpha \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_M - p_H \times F_{INV}^{NS} - F_{LLR}^{NS})}_I \\
&\quad + \underbrace{(1 - \alpha) \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NS})}_{II}
\end{aligned}$$

I : Expected return to the good country if LLR intervenes without seniority

II : Expected return to the good country if there is no LLR intervention

Ex-Post Expected Return to the Good Country if Intervention Occurs:

$$U^{EP,NS,I} = (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_M - p_H \times F_{INV}^{NS} - F_{LLR}^{NS}))$$

Ex-Post Expected Return to the Good Country if Intervention does not Occur:

$$U^{EP,NS,NI} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NS})$$

1.5 Comparison of Policies

1.5.1 Comparison of Total Surplus

In this section we will compare the total surplus of different policies in order to determine whether LLR intervention is welfare improving. Also in order to check whether LLR intervention is time consistent we also look at payoff of policies at $t=1$ after the intervention.

1.5.1.1 Comparison of Ex-Ante Expected Total Surplus:

In this section, we will be comparing the following ex-ante expected surplus figures : $S^{EA,NLLR}$, $S^{EA,NS}$, $S^{EA,S,NT}$, $S^{EA,S,T}$. However, we have to keep in mind that for some parameter values there may not exist equilibrium for a given policy. In these cases, we compare the policies under which the equilibrium exists. If all policies are possible, we find that LLR intervention without seniority and seniority without triggering early liquidation will generate the same highest ex-ante expected total surplus.

Proposition 2 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1) (1.2) are satisfied, if conditions (1.5) (1.6) and (1.7) hold for both F_{INV}^{NS} and F_{INV}^{NLLR} , so that equilibria for "LLR intervention without seniority" and "no LLR intervention" policies exist, then "LLR intervention without seniority" gives the higher expected total surplus.*

Proof. If we compare the ex-ante expected total surplus in both cases:

$$\begin{aligned}
S^{EA,NS} &= \theta \times \alpha \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)) \\
&+ \theta \times (1 - \alpha) \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)) \\
&+ \theta \times \alpha \times ((1 - p_H) \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)) \\
S^{EA,NLLR} &= \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I
\end{aligned}$$

Then it is clear that $S^{EA,NS} \geq S^{EA,NLLR}$. ■

This proposition shows that LLR intervention will always be welfare improving when it is imposed without seniority. The underlying reason is when there exists an LLR and it decides to intervene without seniority, then investors will still be in the contract. In addition, the fraction that has been early liquidated will be financed by LLR and accordingly, the project will continue fully. In this equilibrium even though the fraction that has been covered by LLR will have a lower return as compared to the fraction that has been carried out by initial investors, LLR intervention will still improve the total expected welfare.

Proposition 3 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.5), (1.6), (1.7) hold for $F_{INV}^{S,NT}$ and conditions (1.11) (1.12)hold for F_{INV}^{NS} , so that equilibria for both "LLR intervention without seniority" and "LLR intervention with seniority without triggering early liquidation" exist, then these policies will give the same expected total surplus.*

Proof. Please refer to the definitions of ex-ante expected total surplus under these policies. ■

The intuition of this proposition is that, when seniority does not create early liquidation, then LLR will finance the same fraction of the investment as in the case of no seniority and accordingly, this will create the same ex-ante surplus. Accordingly, this shows that intervention without seniority will be enough to improve the welfare.

Proposition 4 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.5), (1.6), (1.7) hold for $F_{INV}^{S,NT}$ and conditions (1.8) (1.9) (1.10)hold for $F_{INV}^{S,T}$, so that equilibria for both "LLR intervention with seniority without triggering early liquidation" and "LLR intervention with seniority with*

triggering early liquidation” exist, then ex-ante expected total surplus will be higher with “LLR intervention with seniority without triggering early liquidation” policy

Proof. From the equations we have:

$$\begin{aligned}
S^{EA,S,NT} &= \theta \times \alpha \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)) \\
&\quad + \theta \times \alpha \times ((1 - p_H) \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)) \\
&\quad + \theta \times (1 - \alpha) \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)) - I
\end{aligned}$$

and

$$\begin{aligned}
S^{EA,S,T} &= \alpha \times \theta \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L) \\
&\quad + (1 - \alpha) \times \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I
\end{aligned}$$

Under the assumption that $R_M \leq R_H$, as can be seen $S^{EA,S,NT} \geq S^{EA,S,T}$ ■

Since the fraction of the project that has been carried by the investors will yield a higher surplus as compared to part that has been financed by LLR and since we will see full financing of LLR in case of triggering equilibrium, seniority without triggering early liquidation will create a higher expected surplus. Accordingly, in order to maximize the ex-ante total surplus, LLR should set the face value of the contract so that investors are still willing to continue the project.

Proposition 5 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.3),(1.4) are satisfied for F_{INV}^{NLLR} and conditions (1.8) (1.9) (1.10)hold for $F_{INV}^{S,T}$ but if there exists at least one of the conditions (1.5), (1.6), (1.7) that does not hold for $F_{INV}^{S,NT}$ so that LLR is allowed to intervene with seniority only with triggering early liquidation, then no intervention may create higher ex-ante expected total surplus if the following condition holds:*

$$p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) > (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)$$

Proof.

$$S^{EA,S,T} = \alpha \times \theta \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L) + (1 - \alpha) \times \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

$$S^{EA,NLLR} = \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

If subtract one from the other we find that $S^{EA,NLLR} - S^{EA,S,T} = \alpha \times \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - \alpha \times \theta \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)$

No intervention will generate a higher ex-ante total surplus if $p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - (p_H \times I \times R_M + (1 - p_H) \times I \times R_L) > 0$ ■

This proposition summarizes that when the parameters are such that seniority requirement is possible in an equilibrium where it triggers early liquidation, then if the return R_M in case of LLR intervention is low, then continuation of the project partially with the investors might yield a higher total surplus as compared to full continuation of the project with LLR.

According to the propositions that are stated above, when all contracts are possible under the given parameter values, no seniority requirement and seniority requirement without triggering early liquidation will give the highest ex-ante expected total surplus. This is an important finding in the sense that if the main aim of LLR is to maximize the ex-ante expected total surplus, it should choose to intervene and give the bailout. However, the results show that if seniority is a necessary condition for LLR intervention, it should be implemented in such a way that it does not trigger early liquidation. In order to do that LLR should keep the face value of the contract low and accordingly, investors who were initially willing to stay in the contract (which are good signal receivers) will continue to stay.

On the other hand, if LLR is only allowed to intervene with seniority in such a way that it will trigger liquidation which means it will keep the face value of the debt contract high creating incentives for investors to early liquidate, then, we find that under some conditions no LLR intervention may create a higher ex-ante expected total surplus. This means that if LLR can only intervene with seniority and demanding high face value of debt contract, then under the condition stated in the previous proposition, it would rather choose not to intervene. On the other hand, if seniority without triggering early

liquidation is not possible and if the contract without seniority requirement is available then, "no seniority" policy will yield the highest ex-ante total surplus.

1.5.1.2 Comparison of Ex-Post Total Surplus:

Now let's assume that $t=1$ and with exogenous probability α LLR is allowed to intervene. In order to determine which policy gives the highest ex-post total surplus we compare : $S^{EP,NLLR}$, $S^{EP,NS,I}$, $S^{EP,S,NT,I}$, $S^{EP,S,T,I}$. Like in the previous case, we compare the policies when there exists a policy with an equilibrium. Results are summarized in the following proposition.

Proposition 6 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1)-(10) hold for face values for respective contracts, so that equilibria for all contracts exist at the same time, then LLR intervention without seniority and LLR intervention with seniority without triggering early liquidation will give the highest ex-post total surplus. In addition under the condition below, no LLR intervention is a better option as compared to LLR intervention with seniority that will trigger liquidation:*

$$p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) > (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)$$

Proof. If we compare the $S^{EP,NLLR}$, $S^{EP,NS,I}$, $S^{EP,S,NT,I}$, $S^{EP,S,T,I}$:

$$S^{EP,NLLR} = S^{EA,NLLR} = \theta \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) - I$$

$$S^{EP,NS,I} = \theta \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)) + \theta \times ((1 - p_H) \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)) - I$$

$$S^{EP,S,NT,I} = \theta \times (p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L)) + \theta \times ((1 - p_H) \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)) - I$$

$$S^{EP,S,T,I} = \theta \times (p_H \times I \times R_M + (1 - p_H) \times I \times R_L) - I$$

Accordingly, in order to maximize both ex-ante and ex-post expected surplus, LLR should intervene either without seniority or with seniority requirement in such a way that the face value of the contract is low enough to prevent early liquidation. In addition when $p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_L) > (p_H \times I \times R_M + (1 - p_H) \times I \times R_L)$ holds, if LLR only

has option to intervene with seniority while triggering early liquidation, then it should choose not to intervene. ■

1.5.2 Comparison of Expected Return to the Good Country

In the previous section we considered cases where LLR's main aim is to maximize the total surplus. However, in real world the main aim of LLR may be to help countries that are solvent but illiquid rather than increasing the total surplus. Accordingly, in this section we compare the expected return of the good country under different policies. In addition, in order to check for time consistency of these policies, we also consider ex-post return to the good country after LLR intervention decision at $t=1$.

1.5.2.1 Comparison of Ex-Ante Expected Return to the Good Country:

Proposition 7 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1) (1.2) are satisfied, if conditions (1.5) (1.6) and (1.7) hold for both F_{INV}^{NS} and F_{INV}^{NLLR} , so that equilibria for "LLR intervention without seniority" and "no LLR intervention" policies exist, then $(1 - p_H) \times I \times R_L - F_{LLR}^{NS} \leq 0$.*

Proof. Please see Appendix. ■

Proposition 8 *For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1) (1.2) are satisfied, if conditions (1.5) (1.6) and (1.7) hold for both F_{INV}^{NS} and F_{INV}^{NLLR} , so that equilibria for "LLR intervention without seniority" and "no LLR intervention" policies exist and if the following condition holds, "no LLR intervention" policy gives higher return to the good country as compared to the "LLR intervention without seniority" policy:*

$$(1 - p_H) \times I \times R_M < \frac{\frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I}{\theta} - (1 - p_H) \times (1 - p_H) \times I \times R_L}{p_H}$$

Proof. Please see Appendix. ■

This result is striking in the sense that even though "LLR intervention without seniority" policy was creating a higher ex-ante total surplus as compared to "no LLR

intervention”, the ex-ante expected return for the good country. The underlying reason is, if the return from the project that has been financed by LLR is not high enough to cover F_{LLR}^{NS} , then LLR might be making the good country worse off by intervening.

Proposition 9 *For given values of parameters $I, \theta, p_H, \alpha, R_H, R_M,$ and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.5), (1.6), (1.7) hold for $F_{INV}^{S,NT}$ and conditions (1.11) (1.12)hold for F_{INV}^{NS} , so that equilibria for both ”LLR intervention without seniority” and ” LLR intervention with seniority without triggering early liquidation” exist, then they will give the same ex-ante expected return to the good country.*

Proof. Please see Appendix ■

Proposition 10 *For given values of parameters $I, \theta, p_H, \alpha, R_H, R_M,$ and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.5), (1.6), (1.7) hold for $F_{INV}^{S,NT}$ and conditions (1.8) (1.9) (1.10)hold for $F_{INV}^{S,T}$, so that equilibria for both ”LLR intervention with seniority without triggering early liquidation” and ”LLR intervention with seniority with triggering early liquidation” exist, then LLR intervention with seniority without triggering liquidation will always generate higher expected return to the good country.*

Proof. Please see Appendix. ■

According to these propositions, under some conditions, even though LLR might be increasing ex-ante expected total surplus by intervening (without seniority or with seniority without triggering early liquidation) these policies may reduce ex-ante expected return to the good country

1.5.2.2 Comparison of Ex-Post Expected Return to the Good Country:

We compare the policies with each other under the scenario that LLR is allowed to intervene. Accordingly, in this section we will be comparing $U^{EP,NLLR}, U^{EP,NS,I}, U^{EP,S,NT,I}, U^{EP,S,T,I}$.

Proposition 11 *For given values of parameters $I, \theta, p_H, \alpha, R_H, R_M,$ and R_L if assumptions (1.1) (1.2) are satisfied, if conditions (1.5) (1.6) and (1.7) hold for both F_{INV}^{NS} and F_{INV}^{NLLR} , so that equilibria for ”LLR intervention without seniority” and ”no LLR intervention” policies exist and if the following condition holds, ”LLR intervention without*

seniority” policy gives higher ex-post expected return to the good country as compared to the”no LLR intervention” policy:

$$\theta \times \alpha \times p_H \times (1 - p_H) \times I \times R_M > (1 - \theta \times p_H) \times I - (2 \times \theta \times p_H - \theta - \theta \times p_H^2) \times I \times R_L$$

Proof. Please see Appendix. ■

Proposition 12 *or given values of parameters $I, \theta, p_H, \alpha, R_H, R_M,$ and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.5), (1.6), (1.7) hold for $F_{INV}^{S,NT}$ and conditions (1.11) (1.12)hold for F_{INV}^{NS} , so that equilibria for both ”LLR intervention without seniority” and ” LLR intervention with seniority without triggering early liquidation” exist, then,”LLR intervention with seniority without triggering liquidation” policy will generate a higher ex-post expected return to the good country.*

Proof. Please see Appendix. ■

Proposition 13 *For given values of parameters $I, \theta, p_H, \alpha, R_H, R_M,$ and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.5), (1.6), (1.7) hold for $F_{INV}^{S,NT}$ and conditions (1.8) (1.9) (1.10)hold for $F_{INV}^{S,T}$, so that equilibria for both ”LLR intervention with seniority without triggering early liquidation” and ”LLR intervention with seniority with triggering early liquidation” exist, then, if the following condition holds, ”LLR intervention with seniority without triggering liquidation” will generate a higher ex-post expected return to the good country: $p_H^2 \times I \times (R_H - R_M)$*

$$\begin{aligned} &+ \frac{-(-2-\alpha+(1+\alpha) \times p_H)(-1+\theta \times p_H) \times I}{\theta} \\ &+ \frac{(1+(-1+\alpha) \times \theta + \theta \times p_H(1-2 \times \alpha + (-1+\alpha) \times p_H)) \times I \times R_L}{\theta} > 0 \end{aligned}$$

Proof. Please see Appendix. ■

Proposition 14 *For given values of parameters $I, \theta, p_H, \alpha, R_H, R_M,$ and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.5), (1.6), (1.7) hold for $F_{INV}^{S,NT}$ and conditions (1.3) (1.4) hold for F_{INV}^{NLLR} , so that equilibria for both ”LLR intervention with seniority without triggering early liquidation” and ”no LLR intervention” exist, and if one of*

the two cases stated below holds, "no LLR intervention" policy gives higher return to the good country as compared to the "LLR intervention with seniority without triggering early liquidation" policy:

$$1) \frac{(1-\theta \times p_H) \times I}{\theta} \leq I \times R_L$$

and

$$p_H \times (1 - p_H) \times I \times R_M + \frac{I \times (-\alpha + (-1 + \alpha) \times p_H) \times (1 + \theta \times p_H \times (-1 + R_L) - \theta \times R_L) - \theta \times R_L \times I \times p_H \times (1 - p_H)}{\theta} < 0$$

$$2) \frac{(1-\theta \times p_H) \times I}{\theta} > I \times R_L$$

and

$$p_H \times (1 - p_H) \times I \times R_M + \frac{I \times (-1 + \theta \times (R_L + p_H \times (1 + (-\alpha + (-1 + \alpha) \times p_H) \times R_L) - R_L) \times I \times p_H \times (1 - p_H))}{\theta} < 0$$

Proof. Please see Appendix. ■

Proposition 15 For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.5), (1.6), (1.7) hold for $F_{INV}^{S,NT}$ and conditions (1.3) (1.4) hold for F_{INV}^{NLLR} so that equilibria for both "LLR intervention with seniority without triggering early liquidation" and "no LLR intervention" exist, and if the following condition holds, "no LLR intervention" policy gives higher return to the good country as compared to the "LLR intervention with seniority with triggering early liquidation" policy:

$$p_H \times I \times R_M < p_H^2 \times I \times R_H + \frac{(-\theta \times (1 - p_H) + (1 - \theta \times p_H^2)) \times I \times R_L}{\theta}$$

Proof. Please see Appendix. ■

Proposition 16 For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if assumptions (1.1),(1.2) are satisfied,if conditions (1.8), (1.9), (1.10) hold for $F_{INV}^{S,NT}$ and conditions (1.11) (1.12) hold for F_{INV}^{NS} , so that equilibria for both "LLR intervention without seniority" and "LLR intervention with seniority with triggering early liquidation" exist, and if the following condition holds, "LLR intervention without seniority" policy gives higher return to the good country as compared to the "LLR intervention with seniority with triggering early liquidation" policy:

$$p_H^2 \times I \times (R_H - R_M) > \frac{(1 - \theta \times p_H) \times (1 - R_L)}{\theta}$$

Proof. Please see Appendix. ■

From these propositions, we see that ex-post success of the policies in terms of the expected return to the good country will depend on the parameter values. This means that if aim of LLR is to maximize the expected return to the good country a policy that has a higher expected ex-ante return might make the country worse off at t=1 when the LLR is allowed to conduct this policy.

1.5.3 Simulation Exercise:

In this part, we illustrate the optimal policies with given parameter values. We give the initial parameters such that there exist equilibria for all policies and we compare their outcomes. While determining the existence of equilibria and the face values of the contracts, we use the routine described below:

- 1) We first check whether the necessary assumptions hold for the given parameter values.
- 2) If they hold, we calculate F_{LLR} for given parameter values.
- 3) Given F_{LLR} , we calculate F_{INV} using investor's problem at t=0.
- 4) Given F_{INV} , we check whether the equilibrium liquidation conditions that solve investor's problem at t=1 are satisfied
- 5) If conditions that are given by the investor's problem at t=1 are satisfied, parameters are such that the equilibrium we consider exists
- 6) If an equilibrium exists then we calculate the corresponding ex-ante and ex-post total surplus and expected returns to the good country
- 7) If an equilibrium does not exist, we state that there is not an equilibrium with stated specifications and given parameters
- 8) We compare the effect of LLR intervention policies that are possible in equilibrium. In addition, in order to examine the consistency of LLR intervention policies, we compare ex-ante and ex-post figures separately.

Parameter Values	Definitions
$I : 1$	Initial Investment
$\theta : [0 : 0.01 : 1]$	Prior probability of being matched with a good country
$p_H : [0 : 0.01 : 1]$	Probability of having a good signal at t=1 / Probability of success for the good country
$\alpha : 0.5$	Probability of LLR Intervention
$R_H : 1.9$	Return on investment when the good country continues with investor and succeeds
$R_M : 1.8$	Return on investment when the good country continues with LLR and succeeds
$R_L : 0.95$	Return on investment when it is a failure

1.5.3.1 Comparison of Ex-Ante and Ex-post Total Surplus:

From our comparison in the previous section, since the results and conditions are the same for ex-ante and ex-post total surplus, we are reporting the simulation analysis only for ex-ante total surplus.

From these figures in 1.3, we see that LLR should choose to intervene without triggering early liquidation in order to maximize the ex-ante expected total surplus, which can be done either by no seniority requirement or by requiring seniority but demanding a lower face value. In addition, we see that intervention is always a better option. In addition, we see that ex-ante total surplus increases when θ and p_H increase since the probability of a good outcome increases with these two parameters.

1.5.3.2 Comparison of Ex-Ante and Ex-post Expected Return to the Good Country:

From these Figure 1.4 and Figure 1.5 we see that for given values of parameters, given θ , for high values of p_H , no intervention might be a better option in order to increase the ex-ante expected return to the good country. However, if p_H is low, no seniority or seniority without triggering early liquidation will give the higher ex-ante expected return

to the good country. On the contrary, if LLR aims to increase the ex-ante total surplus for given θ , then irrespective of p_H , LLR would intervene either without seniority or with seniority without triggering liquidation and this would make the good country worse off for higher levels of p_H .

Similarly, for given p_H even though for lower values of θ if LLR intervenes either without seniority or with seniority without triggering liquidation, this intervention will make the good countries worse off since no intervention policy gives higher ex-ante expected return.

If we compare the ex-ante and ex-post expected return to the good countries under different policies for given θ , we see that for some interval of p_H , even though LLR intervention gives the higher ex-ante expected return, ex-post no intervention is better for the good country. This also holds when we look at the behavior for given p_H with respect to θ . Accordingly, intervention decisions and policy requirements should be conducted carefully in order not to create any inconsistencies.

1.6 Conclusion

From our analysis, we find that depending on the policies and the objective of LLR, LLR intervention may create higher welfare by financing a project that would not be carried out otherwise. However, we also find that while increasing the total welfare, LLR might make the good countries worse off. The underlying reason is under some parameter values, even though ex-ante expected total surplus is higher when there is intervention, the face values of the contracts are such that this surplus is shared between investors, LLR and the bad countries so that the good countries have lower ex-ante expected return as compared to the no intervention case.

In addition, we find that LLR intervention with no seniority will yield the same ex-ante expected total surplus and ex-ante expected return to the good country as compared to the outcomes when LLR intervenes with seniority without triggering liquidation. This means that seniority is not needed if objective of LLR is to maximize outcomes ex-ante.

On the other hand, we find that, in order to have the highest outcomes ex-post, seniority requirement might be a better option under some parameter values. In this

case, LLR should require seniority in such a way that investors who were willing to stay in the contract will still continue to finance the project. This can be done only if LLR sets the face value of the contract such that even though investors do not get what they would get when there is no intervention, they are still able to save some of their investment in case of default and continuation value is higher than the liquidation value. This can be achieved only if LLR sets the face value of its contract low enough so that it is still able to break even without crowding out the initial investment. Otherwise, LLR intervention with seniority will always create lower outcomes.

Consequently, this study shows that LLR intervention may be beneficial or harmful depending on the initial conditions. Under some conditions, intervention may improve the welfare while making the illiquid but solvent countries worse off. In addition, if intervention is a favorable act, then LLR should conduct it in such a way that it does not create crowding out of the investment by harming the existing investors.

1.7 Appendix

1.7.1 Proofs

Proof of Proposition 8. If there exists an equilibrium with "no LLR intervention" then in order to have existence $\pi \times (p_H \times F_{INV}^{NLLR} + (1 - p_H) \times I \times R_L) < I \times R_L$ must hold. If we write it in an open form we get $\frac{\frac{1-p_H}{-1+\theta p_H} + I \times R_L}{p_H} > 0$ implying that $I \times R_L > \frac{1-p_H}{1-\theta p_H}$ or $I \times R_L < \frac{1-\theta p_H}{1-p_H}$. On the other hand, if $(1 - p_H) \times I \times R_L - F_{LLR}^{NS} > 0$, F_{LLR}^{NS} should be such that the following condition is satisfied: $\frac{1-\theta p_H}{\theta} = F_{LLR}^{NS} < (1 - p_H) \times I \times R_L$ indicating that $\frac{1-\theta p_H}{(1-p_H) \times \theta} < I \times R_L$. However since we have found $I \times R_L < \frac{1-\theta p_H}{1-p_H}$, this contradicts $\frac{1-\theta p_H}{(1-p_H)} < \frac{1-\theta p_H}{(1-p_H) \times \theta} < I \times R_L$.

We need to compare:

$$\begin{aligned} U^{EA,NS} &= \alpha \times p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_M - p_H \times F_{INV}^{NS} - F_{LLR}^{NS}) \\ &\quad + (1 - \alpha) \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NS}) \end{aligned}$$

and

$$\begin{aligned}
U^{NLLR} &= p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NLLR}) \\
&= \alpha \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NLLR}) + (1 - \alpha) \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NLLR})
\end{aligned}$$

where

$$\begin{aligned}
U^{EA,NS} - U^{NLLR} &= \alpha \times p_H \times (1 - p_H) \times I \times R_M \\
&\quad + \underbrace{p_H^2 \times F_{INV}^{NLLR}}_I - \underbrace{(p_H \times p_H \times F_{INV}^{NS} + \alpha \times p_H \times F_{LLR}^{NS})}_{II}
\end{aligned}$$

$$\begin{aligned}
I : p_H^2 \times F_{INV}^{NLLR} &= p_H^2 \times \frac{\frac{I - (1 - \theta \times p_H) \times I \times R_L - (1 - p_H) \times R_L}{\theta \times p_H}}{p_H} \\
&= p_H^2 \times \frac{(I - (1 - \theta \times p_H) \times I \times R_L) - \theta \times p_H (1 - p_H) \times R_L}{\theta \times p_H \times p_H} \\
&= \frac{(I - (1 - \theta \times p_H) \times I \times R_L) - \theta \times p_H (1 - p_H) \times R_L}{\theta}
\end{aligned}$$

$$\begin{aligned}
II : &\underbrace{(p_H \times p_H \times F_{INV}^{NS} + \alpha \times p_H \times F_{LLR}^{NS})}_{II} \\
&= \frac{I - (1 - \theta \times p_H) \times I \times R_L - \alpha \times \theta \times (1 - p_H) \times (1 - p_H) \times I \times R_L - \theta \times p_H \times (1 - p_H) \times I \times R_L + \alpha \times \theta \times (1 - p_H) \times I + \alpha \times (1 - \theta) \times I}{\theta} \\
&= \frac{(1 + \alpha \times \theta \times (1 - p_H) + \alpha \times (1 - \theta)) \times I - (1 - \theta \times p_H + \alpha \times \theta \times (1 - p_H) \times (1 - p_H) + \theta \times p_H \times (1 - p_H)) \times I \times R_L}{\theta}
\end{aligned}$$

then

$$U^{EA,NS} - U^{NLLR} = \frac{\theta \times \alpha \times p_H \times (1 - p_H) \times I \times R_M - (\alpha \times \theta \times (1 - p_H) + \alpha \times (1 - \theta)) \times I + \alpha \times \theta \times (1 - p_H) \times (1 - p_H) \times I \times R_L}{\theta}$$

Then,

$$U^{EA,NS} - U^{NLLR} > 0 \text{ iff } (1 - p_H) \times I \times R_M > \frac{\frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I}{\theta} - (1 - p_H) \times (1 - p_H) \times I \times R_L}{p_H}$$

Proof 2: ■

Proof.

$$\begin{aligned}
&\theta \times p_H \times (p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) + \\
&\alpha \times \theta \times p_H \times (1 - p_H) \times \left(\frac{\max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\}}{p_H} \right) + \\
&(1 - \theta \times p_H) \times I \times R_L \\
&= I
\end{aligned}$$

becomes

$$\begin{aligned}
\theta \times p_H \times (p_H \times F_{INV}^{NS} + (1 - p_H) \times I \times R_L) + (1 - \theta \times p_H) \times I \times R_L &= I \\
\theta \times p_H \times (p_H \times F_{INV}^{NLLR} + (1 - p_H) \times R_L) + (1 - \theta \times p_H) \times I \times R_L &= I
\end{aligned}$$

$$F_{INV}^{NLLR} = \frac{I - (1 - \theta \times p_H) \times I \times R_L - (1 - p_H) \times R_L}{\theta \times p_H \times p_H}$$

$$\text{then } F_{INV}^{NS} = F_{INV}^{NLLR}$$

$$U^{EA,NS} - U^{NLLR} = \alpha \times p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{NS})$$

$$\text{where } F_{LLR}^{NS} = \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - (1 - p_H) \times (1 - p_H) \times I \times R_L}{p_H}$$

$$\text{Then } U^{EA,NS} - U^{NLLR} > 0 \text{ iff } (1 - p_H) \times I \times R_M > \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - (1 - p_H) \times (1 - p_H) \times I \times R_L}{p_H}$$

■

Proof of Proposition 9. We need to compare:

$$U^{EA,S,NT} = \alpha \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT}) + \alpha \times p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{S,NT}) + (1 - \alpha) \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT})$$

$$U^{EA,NS} = \alpha \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NS}) + \alpha \times p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{NS}) + (1 - \alpha) \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NS})$$

$$\text{Then } U^{EA,S,NT} - U^{EA,NS} = \alpha \times p_H \times p_H \times F_{INV}^{NS} + \alpha \times p_H \times F_{LLR}^{NS} + (1 - \alpha) \times p_H \times p_H \times F_{INV}^{NS} - \alpha \times p_H \times p_H \times F_{INV}^{S,NT} - \alpha \times p_H \times F_{LLR}^{S,NT} - (1 - \alpha) \times p_H \times p_H \times F_{INV}^{S,NT}$$

$$= \underbrace{p_H \times p_H \times F_{INV}^{NS} + \alpha \times p_H \times F_{LLR}^{NS}}_I - \underbrace{(p_H \times p_H \times F_{INV}^{S,NT} + \alpha \times p_H \times F_{LLR}^{S,NT})}_{II}$$

$$F_{LLR}^{NS} = \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - (1 - p_H) \times \text{Min}\{(1 - p_H) \times I \times R_L, F_{LLR}^{NS}\}}{p_H}$$

$$= \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - \theta \times (1 - p_H) \times \text{Min}\{(1 - p_H) \times I \times R_L, F_{LLR}^{NS}\}}{\theta \times p_H}$$

$$F_{INV}^{NS} = \frac{I - (1 - \theta \times p_H) \times I \times R_L - \alpha \times \theta \times (1 - p_H) \times \max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\} - \theta \times p_H \times (1 - p_H) \times I \times R_L}{\theta \times p_H^2}$$

$$I : p_H^2 \times \frac{I - (1 - \theta \times p_H) \times I \times R_L - \alpha \times \theta \times (1 - p_H) \times \max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\} - \theta \times p_H \times (1 - p_H) \times I \times R_L}{\theta \times p_H^2}$$

$$+ \alpha \times p_H \times \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - \theta \times (1 - p_H) \times \text{Min}\{(1 - p_H) \times I \times R_L, F_{LLR}^{NS}\}}{\theta \times p_H}$$

$$= \frac{I - (1 - \theta \times p_H) \times I \times R_L - \alpha \times \theta \times (1 - p_H) \times \max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\} - \theta \times p_H \times (1 - p_H) \times I \times R_L}{\theta}$$

$$+ \frac{\alpha \times \theta \times (1 - p_H) \times I + \alpha \times (1 - \theta) \times I - \alpha \times \theta \times (1 - p_H) \times \text{Min}\{(1 - p_H) \times I \times R_L, F_{LLR}^{NS}\}}{\theta}$$

$$= \frac{(1 + \alpha \times \theta \times (1 - p_H) + \alpha \times (1 - \theta)) \times I - (1 - \theta \times p_H + \alpha \times \theta \times (1 - p_H) \times (1 - p_H) + \theta \times p_H \times (1 - p_H)) \times I \times R_L}{\theta}$$

$$F_{INV}^{S,NT} = \frac{I - \alpha \times \theta \times p_H \times (1 - p_H) \times (\frac{\max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H}) - (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L - (1 - \theta \times p_H) \times I \times R_L}{\theta \times p_H^2}$$

$$F_{INV}^{NLLR} = \frac{I - (1 - \theta \times p_H) \times I \times R_L - (1 - p_H) \times R_L}{\theta \times p_H \times p_H}$$

$$F_{LLR}^{S,NT} = \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - (1 - p_H) \times \text{Min}\{F_{LLR}^{S,NT}, I \times R_L\}}{p_H}$$

$$F_{LLR}^{NS} = \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - (1 - p_H) \times \text{Min}\{(1 - p_H) \times I \times R_L, F_{LLR}^{NS}\}}{p_H}$$

$$\begin{aligned}
II &: p_H^2 \times \frac{I - \alpha \times \theta \times p_H \times (1 - p_H) \times \left(\frac{\max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H} \right) - (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L - (1 - \theta \times p_H) \times I \times R_L}{\theta \times p_H^2} \\
&+ \alpha \times p_H \times \frac{\frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I}{\theta} - (1 - p_H) \times \text{Min}\{F_{LLR}^{S,NT}, I \times R_L\}}{p_H} \\
&= \frac{I - \alpha \times \theta \times (1 - p_H) \times \max\{0, I \times R_L - F_{LLR}^{S,NT}\} - (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L - (1 - \theta \times p_H) \times I \times R_L}{\theta} \\
&+ \frac{\alpha \times \theta \times (1 - p_H) \times I + \alpha \times (1 - \theta) \times I - \alpha \times \theta \times (1 - p_H) \times \text{Min}\{F_{LLR}^{S,NT}, I \times R_L\}}{\theta} \\
&= \frac{I - \alpha \times \theta \times (1 - p_H) \times I \times R_L - (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L - (1 - \theta \times p_H) \times I \times R_L + \alpha \times \theta \times (1 - p_H) \times I + \alpha \times (1 - \theta) \times I}{\theta} \\
&= \frac{(1 + \alpha \times \theta \times (1 - p_H) + \alpha \times (1 - \theta)) \times I - (\alpha \times \theta \times (1 - p_H) + (1 - \alpha) \times \theta \times p_H \times (1 - p_H) + (1 - \theta \times p_H)) \times I \times R_L}{\theta} \\
I - II &= \frac{(\alpha \times \theta \times (1 - p_H) \times p_H - \alpha \times \theta \times p_H \times (1 - p_H)) \times I \times R_L}{\theta} = 0 \quad \blacksquare
\end{aligned}$$

Proof of Proposition 10. We need to compare:

$$\begin{aligned}
U^{EA,S,NT} &= \alpha \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT}) + \alpha \times p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{S,NT}) + \\
(1 - \alpha) \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT})
\end{aligned}$$

and

$$U^{EA,S,T} = \alpha \times p_H \times (I \times R_M - F_{LLR}^{S,T}) + (1 - \alpha) \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,T})$$

or

$$\begin{aligned}
U^{EA,S,T} &= \alpha \times p_H \times (p_H \times I \times R_M) + \\
&\alpha \times p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{S,T}) + (1 - \alpha) \times p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,T})
\end{aligned}$$

Then $U^{EA,S,NT} - U^{EA,S,T}$

$$= \alpha \times p_H \times I \times (1 - R_L + p_H \times (R_H - R_M))$$

Then we have $U^{EA,S,NT} - U^{EA,S,T} > 0 \quad \blacksquare$

Lemma 2 For given values of parameters I , θ , p_H , α , R_H , R_M , and R_L if there exist equilibria for both policies "no LLR intervention" and "LLR intervention without seniority", then $(1 - p_H) \times I \times R_L - F_{LLR}^{NS} \leq 0$.

Proof. If there exists an equilibrium with "no LLR intervention" then in order to have existence $\pi \times (p_H \times F_{INV}^{NLLR} + (1 - p_H) \times I \times R_L) < I \times R_L$ must hold. If we write it in an open form we get $\frac{\frac{1 - p_H}{-1 + \theta p_H} + I \times R_L}{p_H} > 0$ implying that $I \times R_L > \frac{1 - p_H}{1 - \theta p_H}$ or $I \times R_L < \frac{I}{R_L} < \frac{1 - \theta p_H}{1 - p_H}$. On

the other hand, if $(1 - p_H) \times I \times R_L - F_{LLR}^{NS} > 0$, F_{LLR}^{NS} should be such that the following condition is satisfied: $\frac{1-\theta p_H}{\theta} = F_{LLR}^{NS} < (1 - p_H) \times I \times R_L$ indicating that $\frac{1-\theta p_H}{(1-p_H)\theta} < I \times R_L$. However since we have found $I \times R_L < \frac{1-\theta p_H}{1-p_H}$, this contradicts $\frac{1-\theta p_H}{(1-p_H)} < \frac{1-\theta p_H}{(1-p_H)\theta} < I \times R_L$.

■

Proof of Proposition 11. We need to compare:

$$U^{EP,NS,I} = p_H \times (p_H \times I \times R_H + (1 - p_H) \times I \times R_M - p_H \times F_{INV}^{NS} - F_{LLR}^{NS})$$

and

$$U^{EP,NLLR} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NLLR})$$

where

$$U^{EP,NS,I} - U^{NLLR} = p_H \times (1 - p_H) \times I \times R_M + \underbrace{p_H^2 \times F_{INV}^{NLLR}}_I - \underbrace{(p_H \times p_H \times F_{INV}^{NS} + p_H \times F_{LLR}^{NS})}_{II}$$

$$I : p_H^2 \times F_{INV}^{NLLR} = p_H^2 \times \frac{\frac{I - (1 - \theta \times p_H) \times I \times R_L - (1 - p_H) \times R_L}{\theta \times p_H} - (1 - p_H) \times R_L}{p_H} = p_H^2 \times \frac{(I - (1 - \theta \times p_H) \times I \times R_L) - \theta \times p_H (1 - p_H) \times R_L}{\theta \times p_H \times p_H}$$

$$= \frac{I - (1 - \theta \times p_H) \times I \times R_L - \theta \times p_H (1 - p_H) \times R_L}{\theta}$$

$$II : p_H^2 \times \frac{1}{\theta \times p_H^2} \times (I - (1 - \theta \times p_H) \times I \times R_L - \alpha \times \theta \times (1 - p_H) \times \max\{0, (1 - p_H) \times I \times R_L - F_{LLR}^{NS}\} - \theta \times p_H \times (1 - p_H) \times I \times R_L) + p_H \times \frac{1}{\theta \times p_H} \times (\theta \times (1 - p_H) \times I + (1 - \theta) \times I - \theta \times (1 - p_H) \times \text{Min}\{(1 - p_H) \times I \times R_L, F_{LLR}^{NS}\})$$

From the previous lemma we know that $(1 - p_H) \times I \times R_L - F_{LLR}^{NS} \leq 0$. Accordingly,

$$II := \frac{(2 - \theta \times p_H) \times I - (1 - 2 \times \theta \times p_H + \theta) \times I \times R_L}{\theta}$$

then

$$U^{EP,NS,I} - U^{NLLR} = \frac{\theta \times \alpha \times p_H \times (1 - p_H) \times I \times R_M - (1 - \theta \times p_H) \times I - (2 \times \theta \times p_H - \theta - \theta \times p_H^2) \times I \times R_L}{\theta}$$

Then,

$$U^{EP,NS} - U^{NLLR} > 0 \text{ iff } \theta \times \alpha \times p_H \times (1 - p_H) \times I \times R_M > (1 - \theta \times p_H) \times I - (2 \times \theta \times p_H - \theta - \theta \times p_H^2) \times I \times R_L \quad \blacksquare$$

Proof of Proposition 12. We need to compare:

$$U^{EP,S,NT,I} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT}) + p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{S,NT})$$

$$U^{EP,NS,I} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NS}) + p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{NS})$$

$$\text{Then } U^{EP,S,NT,I} - U^{EP,NS,I} = p_H \times p_H \times F_{INV}^{NS} - p_H \times p_H \times F_{INV}^{S,NT} + p_H \times F_{LLR}^{NS} - p_H \times F_{LLR}^{S,NT}$$

$$= \underbrace{p_H^2 (F_{INV}^{NS} - F_{INV}^{S,NT})}_I + \underbrace{p_H \times (F_{LLR}^{NS} - F_{LLR}^{S,NT})}_{II}$$

$$I : p_H^2 (F_{INV}^{NS} - F_{INV}^{S,NT})$$

$$= \frac{\alpha \times \theta \times (1-p_H) \times (-p_H \times I \times R_L - \max\{0, (1-p_H) \times I \times R_L - F_{LLR}^{NS}\}) + \max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{\theta}$$

$$II : p_H \times (F_{LLR}^{NS} - F_{LLR}^{S,NT})$$

$$= \frac{\theta \times (1-p_H) \times (\min\{F_{LLR}^{S,NT}, I \times R_L\} - \min\{(1-p_H) \times I \times R_L, F_{LLR}^{NS}\})}{\theta}$$

Now we need to compare the relationship between $(1-p_H) \times I \times R_L$, F_{LLR}^{NS} and $I \times R_L$, $F_{LLR}^{S,NT}$

Case 1: $(1-p_H) \times I \times R_L \leq F_{LLR}^{NS}$ and $I \times R_L \leq F_{LLR}^{S,NT}$

Then we need to have $F_{LLR}^{NS} > F_{LLR}^{S,NT} > I \times R_L$

Then,

$$U^{EP,S,NT,I} - U^{EP,NS,I}$$

$$= \frac{\alpha \times \theta \times (1-p_H) \times (-p_H \times I \times R)}{\theta} + \frac{\theta \times (1-p_H) \times p_H \times I \times R_L}{\theta} = \frac{(1-\alpha) \times \theta \times (1-p_H) \times p_H \times I \times R_L}{\theta} > 0$$

Case 2: $(1-p_H) \times I \times R_L \leq F_{LLR}^{NS}$ and $I \times R_L > F_{LLR}^{S,NT}$

$$U^{EP,S,NT,I} - U^{EP,NS,I}$$

$$= \frac{(1-\alpha) \times \theta \times (1-p_H) \times (F_{LLR}^{S,NT} - (1-p_H) \times I \times R_L)}{\theta}$$

Claim: $F_{LLR}^{S,NT} \geq (1-p_H) \times I \times R_L$

Assume this is not true then, $F_{LLR}^{S,NT} < (1-p_H) \times I \times R_L$

Then $F_{LLR}^{S,NT}$ becomes:

$$F_{LLR}^{S,NT} = \frac{\theta \times (1-p_H) \times I + (1-\theta) \times I}{\theta} < (1-p_H) \times I \times R_L$$

Since we are given $(1-p_H) \times I \times R_L < F_{LLR}^{NS}$

F_{LLR}^{NS} becomes:

$$F_{LLR}^{NS} = \frac{\theta \times (1-p_H) \times I + (1-\theta) \times I - \theta \times (1-p_H) \times (1-p_H) \times I \times R_L}{\theta \times p_H} > (1-p_H) \times I \times R_L$$

Then we need to have the following condition hold:

$$\theta \times (1-p_H) \times I + (1-\theta) \times I > \theta \times (1-p_H) \times I \times R_L$$

implying that

$$\frac{\theta \times (1-p_H) \times I + (1-\theta) \times I}{\theta} > (1-p_H) \times I \times R_L$$

which contradicts with the necessary condition for $F_{LLR}^{S,NT} < (1-p_H) \times I \times R_L$ to hold.

This means that $F_{LLR}^{S,NT} \geq (1-p_H) \times I \times R_L$ implying that $U^{EP,S,NT,I} - U^{EP,NS,I} \geq 0$

Case 3: $(1-p_H) \times I \times R_L \geq F_{LLR}^{NS}$ and $I \times R_L < F_{LLR}^{S,NT}$

Then

$$U^{EP,S,NT,I} - U^{EP,NS,I} = \frac{(1-\alpha) \times \theta \times (1-p_H) \times (I \times R_L - F_{LLR}^{NS})}{\theta}$$

Since $(1-p_H) \times I \times R_L > F_{LLR}^{NS}$ then $U^{EP,S,NT,I} - U^{EP,NS,I} > 0$

Case 4: $(1-p_H) \times I \times R_L \geq F_{LLR}^{NS}$ and $I \times R_L \geq F_{LLR}^{S,NT}$

$$U^{EP,S,NT,I} - U^{EP,NS,I} = \frac{(1-\alpha) \times \theta \times (1-p_H) \times (F_{LLR}^{S,NT} - F_{LLR}^{NS})}{\theta}$$

Then

$$F_{LLR}^{S,NT} = \frac{\theta \times (1-p_H) \times I + (1-\theta) \times I}{\theta}$$

$$F_{LLR}^{NS} = \frac{\theta \times (1-p_H) \times I + (1-\theta) \times I}{\theta}$$

This implies that $U^{EP,S,NT,I} - U^{EP,NS,I} = 0$

Accordingly, we can say that $U^{EP,S,NT,I} \geq U^{EP,NS,I}$. ■

Lemma 3 *If there exist equilibria for both "LLR intervention with seniority without triggering liquidation" and "LLR intervention with seniority with triggering liquidation" policies then, $F_{LLR}^{S,NT} \leq I \times R_L$.*

Proof. By contradiction. Assume $F_{LLR}^{S,NT} > I \times R_L$.

$$F_{LLR}^{S,NT} = \frac{\theta \times (1-p_H) \times I + (1-\theta) \times I}{\theta} - (1-p_H) \times I \times R_L$$

$$F_{LLR}^{S,NT} = \frac{\theta \times (1-p_H) \times I + (1-\theta) \times I - \theta \times (1-p_H) \times I \times R_L}{\theta \times p_H} = \frac{(1-\theta \times p_H) \times I - \theta \times (1-p_H) \times I \times R_L}{\theta \times p_H} > I \times R_L$$

$$F_{INV}^{S,NT} = \frac{I - (1-\alpha) \times \theta \times p_H \times (1-p_H) \times I \times R_L - (1-\theta \times p_H) \times I \times R_L}{\theta \times p_H^2}$$

In order to have the equilibrium exist we need the following condition to hold:

$$(p_H \times F_{INV}^{S,NT} + (1-p_H) \times \left(\frac{\text{Max}\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H} \right)) > I \times R_L$$

In other words:

$$\left(\frac{I - (1-\alpha) \times \theta \times p_H \times (1-p_H) \times I \times R_L - (1-\theta \times p_H) \times I \times R_L}{\theta \times p_H} + (1-p_H) \times \left(\frac{\text{Max}\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H} \right) \right) > I \times R_L$$

since $I \times R_L > F_{LLR}^{S,NT}$

$$\left(\frac{I - (1-\alpha) \times \theta \times p_H \times (1-p_H) \times I \times R_L - (1-\theta \times p_H) \times I \times R_L}{\theta \times p_H} \right) > I \times R_L$$

$$I - (1-\alpha) \times \theta \times p_H \times (1-p_H) \times I \times R_L - (1-\theta \times p_H) \times I \times R_L > \theta \times p_H \times I \times R_L$$

we end up having the following condition

$$I > (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L + (1 - \theta \times p_H) \times I \times R_L + \theta \times p_H \times I \times R_L$$

$$I > (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L + I \times R_L$$

If there exists an equilibrium with "LLR intervention with seniority with triggering early liquidation" then the face values of the contracts with the investor will be:

$$F_{INV}^{S,T} = \frac{I - (1 - \alpha) \times (1 - \theta \times p_H^2) \times I \times R_L - \alpha \times I \times R_L}{(1 - \alpha) \times \theta \times p_H^2}$$

and the following condition should be satisfied:

$$(p_H \times F_{INV}^{S,T} + (1 - p_H) \times \left(\frac{\text{Max}\{0, I \times R_L - F_{LLR}^{S,T}\}}{p_H} \right)) < I \times R_L$$

Since we have already proven that $I \times R_L - F_{LLR}^{S,T}$, condition becomes:

$$(p_H \times F_{INV}^{S,T}) < I \times R_L$$

$$(p_H \times \frac{I - (1 - \alpha) \times (1 - \theta \times p_H^2) \times I \times R_L - \alpha \times I \times R_L}{(1 - \alpha) \times \theta \times p_H^2}) < I \times R_L$$

If we write it in a more open form:

$$\left(\frac{I - (1 - \alpha) \times (1 - \theta \times p_H^2) \times I \times R_L - \alpha \times I \times R_L}{(1 - \alpha) \times \theta \times p_H} \right) < I \times R_L$$

$$\left(\frac{I + (1 - \alpha) \times \theta \times p_H^2 \times I \times R_L - I \times R_L}{(1 - \alpha) \times \theta \times p_H} \right) < I \times R_L$$

$$\left(\frac{I + (1 - \alpha) \times \theta \times p_H^2 \times I \times R_L - I \times R_L}{(1 - \alpha) \times \theta \times p_H} \right) < I \times R_L$$

$$I < (1 - \alpha) \times \theta \times p_H \times I \times R_L - (1 - \alpha) \times \theta \times p_H^2 \times I \times R_L + I \times R_L = (1 - \alpha) \times \theta \times (1 - p_H) \times I \times R_L + I \times R_L$$

If we write this condition together with the condition above:

$$(1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L + I \times R_L < I < (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L + I \times R_L$$

Contradiction. So $I \times R_L \geq F_{LLR}^{S,NT}$ ■

Proof of Proposition 13. We need to compare:

$$U^{EP,S,NT,I} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT}) + p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{S,NT})$$

$$U^{EP,S,T,I} = p_H \times (I \times R_M - F_{LLR}^{S,T})$$

$$U^{EP,S,T,I} = p_H \times (p_H \times I \times R_M) + p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{S,T})$$

$$U^{EP,S,NT,I} - U^{EP,S,T,I} = p_H \times p_H \times I \times (R_H - R_M) + p_H \times (F_{LLR}^{S,T} - F_{LLR}^{S,NT} - p_H \times F_{INV}^{S,NT})$$

$$= p_H \times p_H \times I \times (R_H - R_M) +$$

$$\frac{1}{\theta} \times (I - \theta \times (1 - p_H) \times I \times R_L - \theta \times (1 - p_H) \times I - (1 - \theta) \times I + \theta \times (1 - p_H) \times \text{Min}\{F_{LLR}^{S,NT}, I \times R_L\} - I + \alpha \times \theta \times (1 - p_H) \times \max\{0, I \times R_L - F_{LLR}^{S,NT}\} + (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L + (1 - \theta \times p_H) \times I \times R_L)$$

From the previous Lemma, we know that $F_{LLR}^{S,NT} \leq I \times R_L$

$$U^{EP,S,NT,I} - U^{EP,S,T,I}$$

$$= p_H \times p_H \times I \times (R_H - R_M) + \frac{1}{\theta} \times (-(-2 - \alpha + (1 + \alpha) \times p_H) \times (-1 + \theta \times p_H) \times I + (1 + (-1 + \alpha) \times \theta + \theta \times p_H \times (1 - 2 \times \alpha + (-1 + \alpha) \times p_H)) \times I \times R_L)$$

$$U^{EP,S,NT,I} - U^{EP,S,T,I} > 0 \text{ iff}$$

$$p_H^2 \times I \times (R_H - R_M) + \frac{1}{\theta} \times (-(-2 - \alpha + (1 + \alpha) \times p_H) \times (-1 + \theta \times p_H) \times I + (1 + (-1 + \alpha) \times \theta + \theta \times p_H \times (1 - 2 \times \alpha + (-1 + \alpha) \times p_H)) \times I \times R_L) > 0 \quad \blacksquare$$

Proof of Proposition 14. We need to compare:

$$U^{EP,S,NT,I} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{S,NT}) + p_H \times ((1 - p_H) \times I \times R_M - F_{LLR}^{S,NT})$$

and

$$U^{EP,NLLR} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NLLR})$$

where

$$U^{EP,S,NT,I} - U^{NLLR} = p_H \times (1 - p_H) \times I \times R_M + \underbrace{p_H^2 \times F_{INV}^{NLLR}}_I - \underbrace{(p_H \times p_H \times F_{INV}^{S,NT,I} + p_H \times F_{LLR}^{S,NT,I})}_{II}$$

$$I : p_H^2 \times F_{INV}^{NLLR} = \frac{I - (1 - \theta \times p_H) \times I \times R_L - \theta \times p_H (1 - p_H) \times R_L}{\theta}$$

$$\begin{aligned} II : p_H^2 \times \frac{1}{\theta \times p_H^2} (I - \alpha \times \theta \times p_H \times (1 - p_H) \times (\frac{\max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H}) - (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L - (1 - \theta \times p_H) \times I \times R_L) + p_H \times \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - (1 - p_H) \times \text{Min}\{F_{LLR}^{S,NT}, I \times R_L\}}{p_H} \\ = \frac{I - \alpha \times \theta \times p_H \times (1 - p_H) \times (\frac{\max\{0, I \times R_L - F_{LLR}^{S,NT}\}}{p_H}) - (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L - (1 - \theta \times p_H) \times I \times R_L}{\theta} \\ + \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - \theta \times (1 - p_H) \times \text{Min}\{F_{LLR}^{S,NT}, I \times R_L\}}{\theta} \end{aligned}$$

$$\text{where } F_{LLR}^{S,NT} = \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - (1 - p_H) \times \text{Min}\{F_{LLR}^{S,NT}, I \times R_L\}}{p_H}$$

Case 1: $I \times R_L - F_{LLR}^{S,NT} \geq 0$ (which means $\frac{(1 - \theta \times p_H) \times I}{\theta} \leq I \times R_L$)

$$\begin{aligned}
U^{EP,S,NT,I} - U^{NLLR} &= p_H \times (1 - p_H) \times I \times R_M \\
&+ \frac{I - (1 - \theta \times p_H) \times I \times R_L - \theta \times p_H \times (1 - p_H) \times R_L}{\theta} \\
&- \frac{1}{\theta} (I - \alpha \times \theta \times (1 - p_H) \times (I \times R_L - (\frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I}{\theta}))) \\
&- ((1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L \\
&- (1 - \theta \times p_H) \times I \times R_L) + \theta \times (1 - p_H) \times I + (1 - \theta) \times I \\
&- \theta \times (1 - p_H) \times (\frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I}{\theta}) \\
&= p_H \times (1 - p_H) \times I \times R_M + \frac{I \times (-\alpha + (-1 + \alpha) \times p_H) \times (1 + \theta \times p_H \times (-1 + R_L) - \theta \times R_L) - \theta \times R_L \times I \times p_H \times (1 - p_H)}{\theta}
\end{aligned}$$

Case 2: $I \times R_L - F_{LLR}^{S,NT} < 0$ (which means $\frac{(1 - \theta \times p_H) \times I}{\theta} > I \times R_L$)

$$\begin{aligned}
U^{EP,S,NT,I} - U^{NLLR} &= p_H \times (1 - p_H) \times I \times R_M + \frac{I - (1 - \theta \times p_H) \times I \times R_L - \theta \times p_H \times (1 - p_H) \times R_L}{\theta} \\
&- (\frac{I - (1 - \alpha) \times \theta \times p_H \times (1 - p_H) \times I \times R_L - (1 - \theta \times p_H) \times I \times R_L}{\theta} + \frac{\theta \times (1 - p_H) \times I + (1 - \theta) \times I - \theta \times (1 - p_H) \times I \times R_L}{\theta})
\end{aligned}$$

$$\begin{aligned}
U^{EP,S,NT,I} - U^{NLLR} &= p_H \times (1 - p_H) \times I \times R_M \\
&+ \frac{I \times (-1 + \theta \times (R_L + p_H \times (1 + (-\alpha + (-1 + \alpha) \times p_H) \times R_L)) - R_L \times I \times p_H \times (1 - p_H))}{\theta}
\end{aligned}$$

■

Proof of Proposition 15. We need to compare:

$$U^{EP,S,T,I} = (p_H \times (I \times R_M - F_{LLR}^{S,T})) = p_H \times I \times R_M - p_H \times F_{LLR}^{S,T}$$

and

$$U^{EP,NLLR} = p_H \times (p_H \times I \times R_H - p_H \times F_{INV}^{NLLR}) = p_H^2 \times I \times R_H - p_H^2 \times F_{INV}^{NLLR}$$

$$U^{EP,S,T,I} - U^{NLLR} = p_H \times I \times R_M - p_H \times F_{LLR}^{S,T} - p_H^2 \times I \times R_H + p_H^2 \times F_{INV}^{NLLR}$$

$$= p_H \times I \times R_M - p_H^2 \times I \times R_H - (p_H \times F_{LLR}^{S,T} - p_H^2 \times F_{INV}^{NLLR})$$

$$= p_H \times I \times R_M - p_H^2 \times I \times R_H - (p_H \times \frac{I - \theta \times (1 - p_H) \times I \times R_L}{\theta \times p_H} - p_H^2 \times \frac{I - (1 - \theta \times p_H^2) \times I \times R_L}{\theta \times p_H^2})$$

$$= p_H \times I \times R_M - p_H^2 \times I \times R_H - (\frac{I - \theta \times (1 - p_H) \times I \times R_L - I + (1 - \theta \times p_H^2) \times I \times R_L}{\theta})$$

$$= p_H \times I \times R_M - p_H^2 \times I \times R_H - \frac{(-\theta \times (1 - p_H) + (1 - \theta \times p_H^2)) \times I \times R_L}{\theta}$$

$$U^{EP,S,T,I} - U^{NLLR} > 0 \text{ iff } p_H \times I \times R_M > p_H^2 \times I \times R_H + \frac{(-\theta \times (1 - p_H) + (1 - \theta \times p_H^2)) \times I \times R_L}{\theta} \quad \blacksquare$$

Lemma 4 *If there exists an equilibrium with "LLR intervention with seniority with triggering early liquidation" policy, then there also exists equilibrium with "no LLR intervention" with given parameter values*

Proof. The first condition necessitates the following conditions to hold given $F_{INV}^{S,T} = \frac{I-(1-\alpha)\times(1-\theta\times p_H^2)\times I\times R_L-\alpha\times I\times R_L}{(1-\alpha)\times\theta\times p_H^2}$:

$$1)(p_H\times F_{INV}^{S,T} + (1-p_H)\times(\frac{Max\{0, I\times R_L-F_{LLR}^{S,T}\}}{p_H})) < I\times R_L$$

$$2) \pi\times(p_H\times F_{INV}^{S,T} + (1-p_H)\times I\times R_L) < I\times R_L$$

$$3)(p_H\times F_{INV}^{S,T} + (1-p_H)\times I\times R_L) \geq I\times R_L$$

$$\text{Then } F_{INV}^{NLLR} = \frac{I-(1-\theta\times p_H^2)\times I\times R_L}{\theta\times p_H^2} = \frac{I-I\times R_L}{\theta\times p_H^2} + I\times R_L \leq \frac{I-I\times R_L}{(1-\alpha)\times\theta\times p_H^2} + I\times R_L = F_{INV}^{S,T}$$

In order to have equilibrium with "no LLR intervention" we need the following conditions to hold:

$$1)\pi\times(p_H\times F_{INV}^{NLLR} + (1-p_H)\times I\times R_L) < I\times R_L$$

$$2)(p_H\times F_{INV}^{NLLR} + (1-p_H)\times R_L) > I\times R_L$$

since $\pi\times(p_H\times F_{INV}^{S,T} + (1-p_H)\times I\times R_L) < I\times R_L$ holds then $\pi\times(p_H\times F_{INV}^{NLLR} + (1-p_H)\times I\times R_L) \leq \pi\times(p_H\times F_{INV}^{S,T} + (1-p_H)\times I\times R_L) < I\times R_L$ which indicates that the first condition holds.

$(p_H\times F_{INV}^{NLLR} + (1-p_H)\times R_L) = (p_H\times(\frac{I-I\times R_L}{\theta\times p_H^2} + I\times R_L) + (1-p_H)\times R_L) = \frac{I-I\times R_L}{\theta\times p_H} + I\times R_L > I\times R_L$ which indicates that second condition also holds.

Accordingly, there exists equilibrium with "no LLR intervention" with the given parameters. ■

Proof of Proposition 16. We need to compare:

$$U^{EP,S,T,I} = (p_H\times(I\times R_M - F_{LLR}^{S,T})) = p_H\times I\times R_M - p_H\times F_{LLR}^{S,T}$$

and

$$U^{EP,NS,I} = p_H\times(p_H\times I\times R_H + (1-p_H)\times I\times R_M - p_H\times F_{INV}^{NS} - F_{LLR}^{NS})$$

$$U^{EP,NS,I} - U^{EP,S,T,I} = p_H^2\times I\times(R_H - R_M) - p_H\times F_{LLR}^{NS} - p_H^2\times F_{INV}^{NS} + p_H\times F_{LLR}^{S,T}$$

$$U^{EP,NS,I} - U^{EP,S,T,I} = p_H^2\times I\times(R_H - R_M) - p_H\times(F_{LLR}^{NS} + p_H\times F_{INV}^{NS} - F_{LLR}^{S,T})$$

$$U^{EP,NS,I} - U^{EP,S,T,I} = p_H^2\times I\times(R_H - R_M) - p_H\times(F_{LLR}^{NS} + p_H\times F_{INV}^{NS} - F_{LLR}^{S,T})$$

From Lemma 4, we know that there exists equilibrium with "no LLR intervention" policy. In addition, from Lemma 2 we know that if there exist equilibria for both policies "no LLR intervention" and "LLR intervention without seniority", then $(1 - p_H) \times I \times R_L - F_{LLR}^{NS} \leq 0$. Using these, the difference between ex-post returns to the good countries under "LLR intervention without seniority" and "LLR intervention with seniority with triggering early liquidation" policies will be:

$$\begin{aligned}
U^{EP,NS,I} - U^{EP,S,T,I} &= p_H^2 \times I \times (R_H - R_M) - (p_H \times F_{LLR}^{NS} + p_H^2 \times F_{INV}^{NS} - p_H \times F_{LLR}^{S,T}) \\
F_{LLR}^{NS} &= \frac{\frac{\theta \times (1-p_H) \times I + (1-\theta) \times I}{\theta} - (1-p_H) \times \text{Min}\{(1-p_H) \times I \times R_L, F_{LLR}^{NS}\}}{p_H} \\
&= \frac{\theta \times (1-p_H) \times I + (1-\theta) \times I - \theta \times (1-p_H) \times \text{Min}\{(1-p_H) \times I \times R_L, F_{LLR}^{NS}\}}{\theta \times p_H} \\
&= \frac{\theta \times (1-p_H) \times I + (1-\theta) \times I - \theta \times (1-p_H) \times (1-p_H) \times I \times R_L}{\theta \times p_H} \\
F_{INV}^{NS} &= \frac{I - (1-\theta \times p_H) \times I \times R_L - \alpha \times \theta \times (1-p_H) \times \max\{0, (1-p_H) \times I \times R_L - F_{LLR}^{NS}\} - \theta \times p_H \times (1-p_H) \times I \times R_L}{\theta \times p_H^2} \\
&= \frac{I - (1-\theta \times p_H) \times I \times R_L - \theta \times p_H \times (1-p_H) \times I \times R_L}{\theta \times p_H^2} \\
F_{LLR}^{S,T} &= \frac{I - \theta \times (1-p_H) \times I \times R_L}{\theta \times p_H} \quad U^{EP,NS,I} - U^{EP,S,T,I} = p_H^2 \times I \times (R_H - R_M) - \frac{(1-\theta \times p_H)(1-R_L)}{\theta} \\
\text{Then } U^{EP,NS,I} &> U^{EP,S,T,I} \text{ iff } p_H^2 \times I \times (R_H - R_M) > \frac{(1-\theta \times p_H)(1-R_L)}{\theta} \quad \blacksquare
\end{aligned}$$

1.7.2 Figures

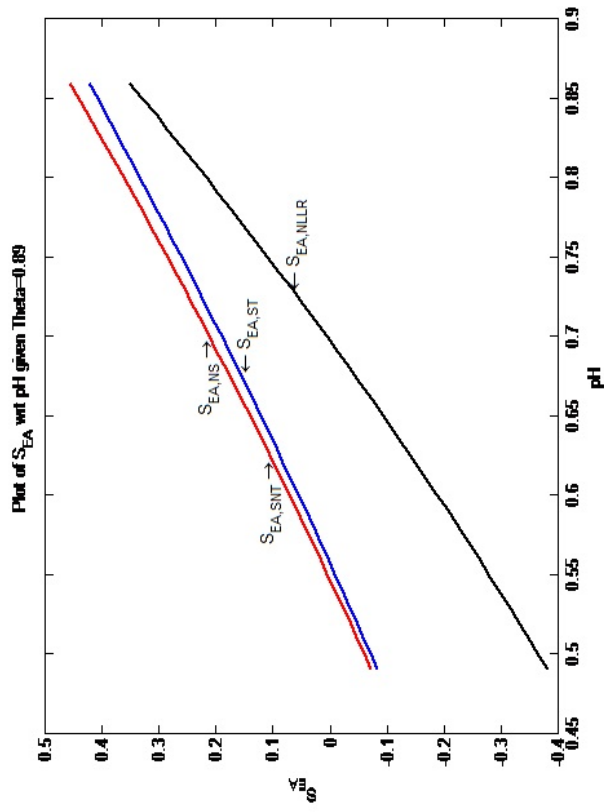
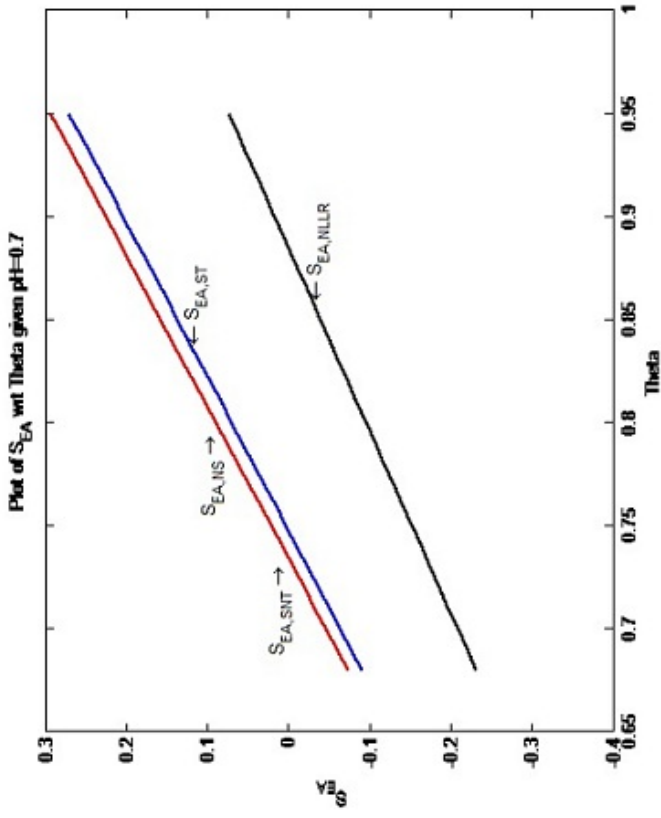


Figure 1.3: Comparison of Total Surplus

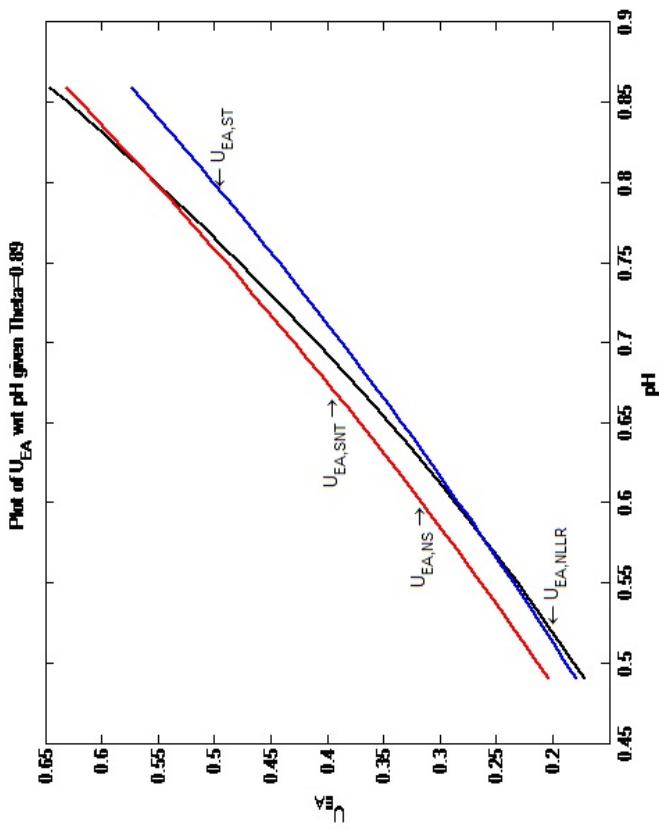
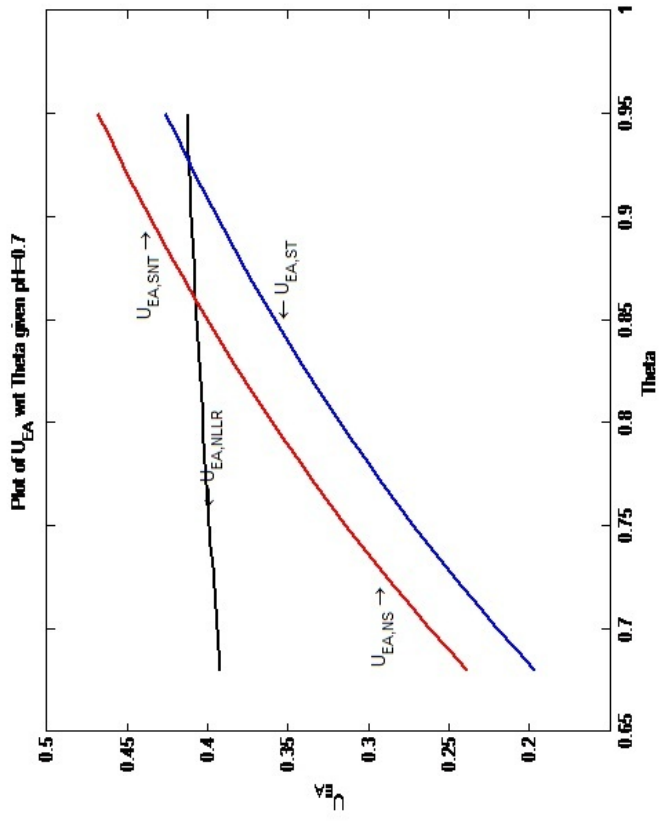


Figure 1.4: Comparison of Ex-ante Expected Return to the Good Country

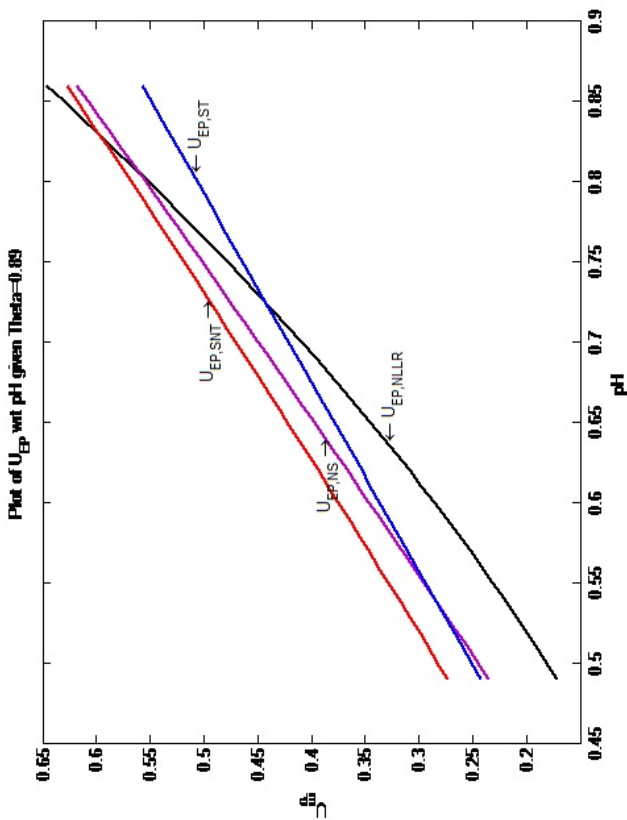
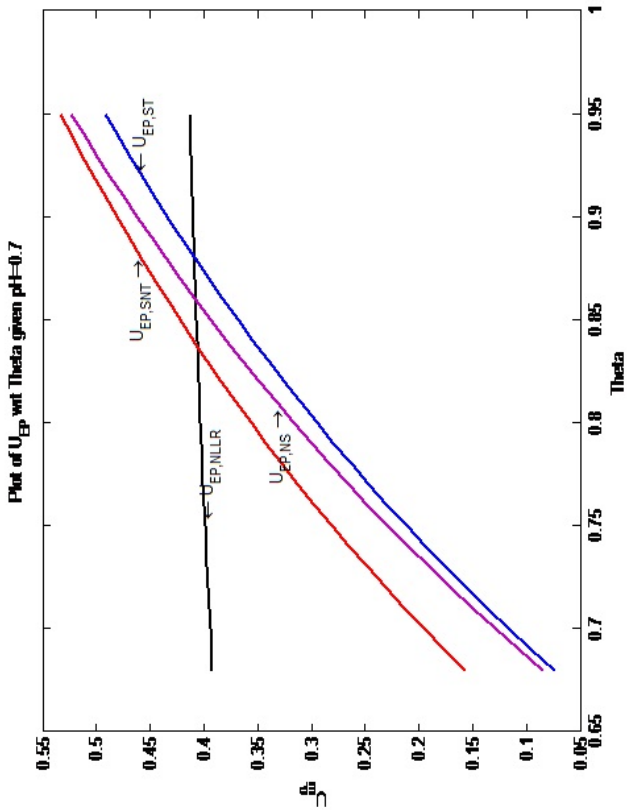


Figure 1.5: Comparison of Ex-post Expected Return to the Good Country

REFERENCES

- [1] Atkeson, A. (1991). 'International lending with moral hazard and risk of repudiation'. *Econometrica*, vol. 59, pp. 465-90.
- [2] Borenstein, E., (1989), "Debt overhang, credit rationing and investment" (processed, International Monetary Fund, Washington, DC).
- [3] Bulow, J. and Rogoff, K. (1989), "Sovereign debt: Is to forgive to forget?", *American Economic Review*, 79, 43-50.
- [4] Bulow, J., and Rogoff, K. 1989, "A constant recontracting model of sovereign debt", *Journal of Political Economy* 97, 155-178.
- [5] Cole, H. L. and Kehoe, P. J. (1998), "Models of Sovereign Debt: Practical vs. General Reputations", *International Economic Review*, 29, 55-70.
- [6] Corsetti, G., Guimaraes, B., and Roubini, N., (2006), "International Lending of Last Resort and Moral Hazard: A Model of IMF's Catalytic Finance." *Journal of Monetary Economics*, 53(3): 441-71.
- [7] Eaton, J. and Gersovitz, M. (1981), "Debt with potential repudiation: Theoretical and empirical analysis", *Review of Economic Studies* 48, 289-309.
- [8] Kletzer, K. M., and Wright B.D. (2000): "Sovereign Debt as Intertemporal Barter," *American Economic Review*, 90, 621-639.
- [9] Krugman, P. (1988), 'Financing vs. Forgiving a Debt Overhang', *Journal of Development Economics*, 29, 253-268.
- [10] Miller, David, Michael Tomz, and Mark Wright (2005), "Sovereign Debt, Default, and Bailouts," Manuscript, Federal Reserve Bank of Minneapolis
- [11] Rajan, R.G., "Insiders and Outsiders: The Choice Between Informed and Arm-length Debt," *Journal of Finance*, Sept. 1992, 47(4), pp. 1367-1400.
- [12] Reinhart, C. M., Rogoff, K. S. and Savastano, M.A. (2003). "Debt intolerance". In *Brookings Papers on Economic Activity*, William Brainard and George Perry(eds.), Vol. 1, pp. 1-74.
- [13] Rose, A. K., & Spiegel, M. M. (2004). "A gravity model of sovereign lending: Trade, default, and credit." *IMF Staff Papers*, 51, 50-63.
- [14] Roubini, N., Setser, B., 2004. *Bailouts or Bailins? Responding to Financial Crises in Emerging Economics*. Institute for International Economics, Washington, DC.
- [15] Saravia, Diego(2010). "On the role and effects of IMF seniority", *Journal of International Money and Finance*, Volume 29, Issue 6,
- [16] Tornell, A., Lane, P.R., (1998). "Are windfalls a curse? A non-representative agent model of the current account". *Journal of International Economics* 44, 83-112.

CHAPTER 2

Immigration and Endogenous Technology Choice

2.1 Introduction

Immigration is an important concept in the history of developed countries, especially US. Increase in the foreign-born workers with different skill levels has always been an important aspect that has been investigated by the researchers. While there is a vast amount of research that analyzes the effect of immigration on employment and wages¹ some recent work has focused more on the effect of immigration on technology choice of firms like Beaudry, Doms and Lewis(2010) , Doms and Lewis(2006) , Lewis (2011) and Peri(2012) . These studies have found that immigration has an effect on the technology choice of firms. Specifically Peri(2012) finds that immigration increases the use of low-skilled efficient technologies.

In this paper, my aim is to take one step further of work of Peri(2012). To be more specific, rather than analyzing the total immigration effect, I differentiate labor with respect to their skill type and test existence of endogenous technology choice. This is an important aspect to be analyzed because the results in Peri (2012) might be found as a result of dominating effect of low skilled immigrants. By differentiating immigrants with respect to skill type, we will be able see what the individual effect of each type of skill on the technology choice.

In order to analyze the effect of different types of labor, I generalize the production technologies within states. Because of endogeneity problem and omitted variable bias , as in the case of Peri (2012) I use two sets of instruments; the first one is distance to Mexican border and the second one is the imputed number of immigrants inferred from existence of ethnic enclaves prior to 1970. However, rather than the aggregate immigration, I consider

¹Just to name a few: Antolji and Card (1991), Borjas (1987, 1994, 2003, 2006, 2009), Borjas and Hanson (2008), Borjas and Katz (2007), Card(1990, 2001, 2009), Card and Lewis (2007), Cortes (2008), Friedberg (2001), Ottaviano and Peri (2006, 2012)

skilled and unskilled labor immigration separately. These instruments are less correlated with the productivity but good estimator for the inflow of immigrants.

Based on the analysis, I find that immigration has significant effect on the technology choice. More importantly, I find that immigration of different skill types have different and opposite effects on the technology choice. The results suggests that increase in high skilled immigration leads to in high skilled biased technology adoption and increase in the low skilled immigrants leads to increase in the low skill biased technologies. In other words, we find evidence that immigration may effect endogenous technology change.

The rest of the paper is as follows: Section 2 explains the method that we use in order to create the technology choice variable. Section 3 summarizes the results and Section 4 concludes.

2.2 Construction of Technology Choice Variable

In this section, we represent the economies of each US state with individual production function and construct a variable that will represent their technology choice. As in Peri(2012), I consider a production technology for each US state s in year t for a perfectly tradeable good using the following production function:

$$Y_{st} = A_{st}K_{st}^{\alpha}[(\beta_{st}H_{st})^{\frac{\sigma-1}{\sigma}} + ((1 - \beta_{st})L_{st})^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}(1-\alpha)} \quad (2.1)$$

where Y_{st} is the total production of the good. K_{st} is the aggregate physical capital; H_{st} is the total number of hours worked by high skilled workers ; L_{st} is the total number of hours worked by low skilled workers; A_{st} is the total factor productivity; β_{st} is the degree of skill bias of the productivity used in state s and year t and σ is the elasticity of substitution between two types of labor.

In this study, our only aim is to explore the effect of immigrants with different types of skill on the intensity of the respective labor in the production function by estimating the following relationship:

$$\widehat{\beta}_{st} = \alpha_{st} + \eta_1 \frac{\Delta N_{st}^{F,H}}{N_{st}^H} + \eta_2 \frac{\Delta N_{st}^{F,L}}{N_{st}^L} + \epsilon_{st} \quad (2.2)$$

where $\widehat{\beta}_{st}$ is the change in the share of the skilled labor in the production function, α_{st} is the fixed effects, $\frac{\Delta N_{st}^{F,H}}{N_{st}^H}$ is the change in skilled labor due to immigration and $\frac{\Delta N_{st}^{F,L}}{N_{st}^L}$ is the change in the low skilled labor due to immigration. In our analysis, in order to conduct the above experiment we need to get the growth rate of β_{st} . In order to calculate growth rate of the skilled labor share in the production function, first we use the following first order conditions of the producer:

$$\left(\frac{\beta_{st}}{1-\beta_{st}}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{H_{st}}{L_{st}}\right)^{\frac{-1}{\sigma}} = \frac{w_{st}(H)}{w_{st}(L)}$$

which can be written in the following form:

$$\beta_{st} = \frac{(w_{st}^H)^{\frac{\sigma}{\sigma-1}} H_{st}^{\frac{1}{\sigma-1}}}{(w_{st}^H)^{\frac{\sigma}{\sigma-1}} H_{st}^{\frac{1}{\sigma-1}} + (w_{st}^L)^{\frac{\sigma}{\sigma-1}} L_{st}^{\frac{1}{\sigma-1}}}$$

where w_{st}^H , w_{st}^L are the labor wages with respect to different types of labor. From this equation, given the wages and total labor supply for each type of labor at each point in time and for each state, we can calculate β_{st} for each state and time and analyze the effects of immigration on skill intensity β_{st} .

In order to calculate β_{st} , we need to construct variables H_{st} , L_{st} , $w_{st}(H)$ and $w_{st}(L)$. I use public use micro-data samples (IPUMS) of US Decennial Census and of the American Community Survey for fifty US states for years 1970, 1980, 1990, 2000 and 2006. The details of the variables and the construction of the data can be found in the Appendix.

2.3 Results

In our regression analysis, we estimate Equation (2.2) in order to get an idea about the signs of η_1 and η_2 so that we can make an inference about the existence of endogenous technology choice of firms. In theory, if endogenous technology choice of firms exists, then we will expect that sign of η_1 is positive and sign of η_2 is negative. The underlying reason is, since β_{st} represents the intensity of high skilled labor in the production technology, if there is endogenous technology choice, inflow of high skilled immigrants will increase the skill intensity (indicating a positive η_1). Similarly, we would predict that inflow of low skilled immigrants would affect the skill intensity of production negatively so that the

technology will be more unskilled biased.

2.3.1 OLS Estimation

In this part of the analysis, we report simple OLS estimates of Equation (2.2) using the changes between 1970 and 2000 and 2000-2006 change. Each column reports coefficients η_1 and η_2 . For the first column we do the estimation using all data between 1970-2006 while in the second column we use data before 2006 and for the third column we use the lagged value of $\widehat{\beta}_{st}$ as an explanatory variable in order to check for the consistency. In all of the regressions we use time and state fixed effects in order to account for the state specific effects and the cycles. Results of OLS estimation show that the signs of

	All Obs.		Obs. before 2006		Lagged value of $\widehat{\beta}_{st}$	
	Coef.	SE	Coef.	SE	Coef.	SE
$\frac{\Delta N_{st}^{F,L}}{N_{st}^L}$	-0.431	*	-0.496		-0.435	*
$\frac{\Delta N_{st}^{F,H}}{N_{st}^H}$	0.146		0.440		0.226	

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.1: OLS Regression Table

the coefficients are consistent with our expectation indicating that correlation between high skilled immigration growth and the growth of high skilled technology bias is positive while it is negative in case of low skilled immigration. However, we see that only the low skilled immigration effect is significant. This may be due to endogeneity problem and omitted variable bias so in the next section we use instrumental variable technique in order to address this issue.

2.3.2 2SLS Estimation

Since there are potential endogeneity problems, we use instrumental variables. We use the ethnic enclaves within US states as in the case of Card(2001) in order to calculate the imputed growth rates. In addition, we use the instrumental variables in Peri (2012) that determine the distance to US-Mexico border. We use two different methodologies in order to calculate the imputed growth rate instruments.

2.3.2.1 Construction of the Instruments

In the regression analysis I construct the instrumental variables based on the approach by Peri(2010) who combines the instruments based on the past settlement of immigrants augmented by their national rate of growth, as used in Card 2001, with the geographical instruments based on the distance from the border between US and Mexico. In order to calculate the growth rate of immigrants based on past settlement, I use 3 approaches.

Approach 1: In this approach while calculating the growth rates, rather than using all 10 nationalities for both skilled and unskilled labor, we divide nationalities into two subgroups: low skilled labor supplier nationalities and the high skilled labor supplying nationalities by sorting nationalities with respect to total high skilled and low skilled immigrants coming from these nationalities getting the ones that supply the highest fraction. Based on the analysis, I define the high skilled labor supplying nationalities (Group 1) as Eastern Europe and Russia, China, India, Rest of Asia, and low skilled labor supplying countries as (Group 2) Mexico and Rest of Latin America.² In order to calculate the instrumental variable we use a procedure that is similar to Approach 1. Below is the explanation for the imputed growth rate of high skilled labor and it is the same for the low skilled labor. For each nationality in Group 1 I calculate the growth rate of the total working age population which is : $g_{n,1960-t} = (Pop_{n,t} - Pop_{n,1960})/Pop_{n,1960}$. I multiply the growth rate of each nationality of origin with the initial population in 1960 and this would give the imputed immigrant population: $\widehat{Pop}_{n,i,t} = Pop_{n,i,1960} * (1 + g_{n,1960-t})$. State level total imputed working age population is calculated by adding over nationalities in Group 1 : imputed population with respect to states: $\widehat{Pop}_{i,t} = \sum_{n \in Group1} \widehat{Pop}_{n,i,t}$. Imputed growth rate is calculated as follows: $(\widehat{Pop}_{i,t+10} - \widehat{Pop}_{i,t}) / (\widehat{Pop}_{i,t} + Pop_{US,i,s,t})$ where $Pop_{US,i,s,t}$ is native working age population with skill level s. We use base year as 1960 and 1970 in different calculations.

Approach 2: This approach is very similar to Approach 1. Only difference is we construct our groups with respect to countries, not the nationalities. We construct two groups of countries: high skilled labor supplier countries and low skilled labor supplier

²I sort nationalities with respect to their share in the total supply, $Pop_{n,s}/Pop_s$ where $Pop_{n,s} = \sum_{i,t} Pop_{n,i,s,t}$ and $Pop_s = \sum_{n,i,t} Pop_{n,i,s,t}$, and the share of the specific type of labor in the total supply of the specific nationality : $Pop_{n,s}/Pop_n$ where $Pop_n = \sum_{i,s,t} Pop_{n,i,s,t}$. If both of these values are high, I define it as a supplier of labor with skill level s

countries. Using the same procedure in Approach 2 we get the following country groups: Based on the analysis, I define the high skilled labor supplying nationalities (Group 1) as Canada, China, Korea, Phillipines, India, Africa, and low skilled labor supplying countries as (Group 2) Mexico, Central America, West Indies and Italy. Procedures to calculate the growth is the same with Approach 1.

2.3.2.2 2SLS Estimation Results

Table 2.2 shows the results when the instrumental variable has been constructed based on the country groupings (Approach 2) and Table 2.2 shows the results when the instrumental variable is constructed based on the nation groupings (Approach 1) with base year 1970. I do the same regression with different state, border dummies in order to check robustness. I only report the coefficients for the variables that are of our interest. Results show that as expected the coefficient for the high skilled immigration growth is positive and significant while it is negative and significant for the low skilled immigration growth. In other words, the regression analysis show that firms indeed change their production technology in such a way that they will increase the productivity of the more abundant factor.

	All Obs.		Obs. before 2006		Obs. before 2006 after 1970	
	Coef.	SE	Coef.	SE	Coef.	SE
$\frac{\Delta N_{st}^{F,L}}{N_{st}^L}$	-1.019	***	-1.235	***	-0.259	**
$\frac{\Delta N_{st}^{F,H}}{N_{st}^H}$	0.782		1.125	*	1.309	***

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.2: IV Regression Table 1

	All Obs.		Obs. before 2006		Control for Initial Values	
	Coef.	SE	Coef.	SE	Coef.	SE
$\frac{\Delta N_{st}^{F,L}}{N_{st}^L}$	-1.107	***	-1.274	**	-0.378	***
$\frac{\Delta N_{st}^{F,H}}{N_{st}^H}$	0.973	**	1.328	*	0.555	**

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table 2.3: IV Regression Table 2

2.4 Conclusion

This paper analyzes the effect of immigration of different types of labor on the technology choice of firms. In order to do that we use state level data and construct instruments based on the ethnic enclaves within US states using census data between year 1970 and 2006.

Based on our analysis, we find that immigration has significant effect on the technology choice. Specifically, we find that immigration of different skill types have different and opposite effects on the technology choice. The results suggests that increase in high skilled immigration leads to increase in the intensity of the high skilled labor in the production. Conversely, increase in the low skilled immigration creates a decline in the intensity of high skilled labor in the production. This supports that immigration might create endogenous technology change of the firms in such a way that the increase the intensity of the more abundant factor in production.

2.5 Appendix

2.5.1 Data Construction

I follow the procedure in Peri(2010) in order to construct the data. I use 1% Sample for Census 1960, the 1% State Sample Form 1 for Census 1970, the 1% Metropolitan Sample for the Censuses 1980 and 1990, the 1% Sample for Census 2000, and the 1% Sample of the American Community Survey(ACS) for 2006. I generate two data series for working age population and total population. Details will be described below.

2.5.1.1 Working Age Population

- Eliminate people who are not civilians (those with gq equal to 0, 3, or 4)
- Eliminate people younger than 17 and older than 65
- Eliminate people who have not worked last year. We define these people as the ones who have worked 0 weeks last year.(those who have $wkswork2=0$ for the years 1960 and 1970 and $wkswork1=0$ for datasets including years 1980-2006.)

- Eliminate people with invalid salary reported (those with $\text{incwage} =$ or $\text{incwage} = 999999$)
- Eliminate people who have experience < 1 and > 40 ($(\text{experience}) = (\text{age}) - (\text{time first worked})$). Latter Variable (time first worked) is 16 for workers with no HS degree, 19 for HS graduates, 21 for people with some college education and 23 for college graduates.)
- Eliminate people who are self employed. ($\text{classwkrd} < 20$ or $\text{classwkrd} > 28$)

2.5.1.2 Total Population

- Eliminate people who are not civilians (those with gq equal to 0, 3, or 4)
- Eliminate people younger than 17 and older than 65

2.5.1.3 Individual Variables

Hours Worked and Employment: In order to calculate the total hours of worked for each group that is of interest (nativity, skill level etc) for each person in the group we multiply hours worked with the personal weight (perwt) and add over all members of the group.

Average Hourly Wage: In order to calculate the average hourly wage for each group that is of interest (nativity, skill level etc) for each person in the group we weight the his/her hourly wages by the hours worked by the individual.

Education: In the analysis we define two education (skill) levels. A person is define low-skilled if s/he has a high school degree or less ($\text{educd} \leq 64$) and high-skilled if s/he has more than high school degree ($\text{educd} > 64$).

Experience: We define experience as follows: $(\text{experience}) = (\text{age}) - (\text{time first worked})$. (time first worked) is 16 for workers with no HS degree, 19 for HS graduates, 21 for people with some college education and 23 for college graduates.

Immigration Status: Immigrants are defined as the people who are not citizens or who are naturalized citizens. For 1960 this corresponds to $\text{bpld} \geq 15000$ except for the codes 90011 and 90021. For the years 1970-2006 we use the variable citizen in order to determine immigration status. A person is immigrant if $\text{citizen} = 2$ or $\text{citizen} = 3$.

Weeks Worked in a Year: For the years 1960 and 1970, weeks worked in the last year is represented by the variable `wkswork2` which is given in intervals. For each interval, median has been used in order to obtain an approximate value. Median values are follows: 6.5 weeks if `wkswork2=1`; 20 weeks if `wkswork2=2`; 33 weeks if `wkswork2=3`; 43.5 weeks if `wkswork2=4`; 48.5 weeks if `wkswork2=5`; 51 weeks if `wkswork2=6`. For the censuses 1980, 1990, 2000 and ACS 2006 exact number of weeks worked is represented by the variable `wkswork1`.

Hours Worked in a Week: For the years 1960 and 1970, hours worked in the last year is represented by the variable `hrswork2` which is given in intervals. For each interval, median has been used in order to obtain an approximate value. Median values are follows: 7.5 hours if `hrswork2=1`; 22 hours if `hrswork2=2`; 32 hours if `hrswork2=3`; 37 hours if `hrswork2=4`; 40 hours if `hrswork2=5`; 44.5 hours if `hrswork2=6`; 54 hours if `hrswork2=7`; 70 hours if `hrswork2=8` For the censuses 1980, 1990, 2000 and ACS 2006 exact number of hours worked is represented by the variable `uhrswork`.

Hours Worked in a Year: Hours worked in a year is calculated by multiplying the hours worked in a week by the weeks worked in a year.

Yearly Wages: In order to calculate the yearly wages in constant 1999 US dollars we multiply `incwage` by the price deflator. Deflators that have been used are the following:

Top Codes for Yearly Wages: Following Peri(2010) I multiply the topcodes for

Year	1959	1969	1979	1989	1999	2005
Deflator	5.65	4.51	2.30	1.29	1	0.82

yearly wages in 1960, 1970 and 1980 by 1.5.

Hourly Wages: For each individual hourly wage is constructed by dividing the yearly wage by the hours worked in a year.

REFERENCES

- [1] Altonji, Joseph G., and David Card. "The Effects of Immigration on the Labor Market Outcomes of LessSkilled Natives." In . John M. Abowd and Richard B. Freeman, eds., *Immigration, Trade, and the Labor Market*. Chicago: University of Chicago Press, 1991, pp. 20134.
- [2] Beaudry P, Doms M, Lewis E. 2010 "Should the PC be Considered a Technological Revolution? Evidence from US Metropolitan Areas." *Journal of Political Economy* 118(5): 9881036
- [3] Borjas GJ. 1987 "Immigrants, Minorities, and Labor Market Competition." *Industrial and Labor Relations Review* 40(3): 382392
- [4] Borjas GJ. 1994 "The Economics of Immigration." *Journal of Economic Literature* 32(4): 16671717.
- [5] Borjas GJ. 2003 "The Labor Demand Curve is Downward Sloping: Reexamining the Impact of Immigration on the Labor Market." *The Quarterly Journal of Economics* 118(4): 13351374
- [6] Borjas GJ. 2006 "Native Internal Migration and the Labor Market Impact of Immigration." *Journal of Human Resources* 41(2): 221257 32
- [7] Borjas GJ. 2009 "The Analytics of the Wage Effect of Immigration." Mimeo, Harvard University
- [8] Borjas GJ, Grogger J, Hanson GH. 2008 "Imperfect Substitution Between Immigrants and Natives: A Reappraisal." NBER Working Paper 13887
- [9] Borjas GJ, Katz LF. 2007 "The Evolution of the MexicanBorn Workforce in the United States" in Borjas GJ, ed., *Mexican Immigration to the United States*. Chicago: University of Chicago Press, 1326
- [10] Card D. 1990 "The Impact of the Mariel Boatlift on the Miami Labor Market." *Industrial and Labor Relations Review* 43(2): 245257
- [11] Card D. 2001 "Immigrant Inflows, Native Outflows, and the Local Labor Market Impacts of Higher Immigration." *Journal of Labor Economics* 19(1): 2264
- [12] Card D. 2009 "Immigration and Inequality," *The American Economic Review* 99(2): 121
- [13] Card D, Lewis E. 2007 "The Diffusion of Mexican Immigrants During the 1990s: Explanations and Impacts" in Borjas, George J., ed., *Mexican Immigrant to the United States*. Chicago and London: The University of Chicago Press, 193228
- [14] Cortes P. 2008 "The Effect of LowSkilled Immigration on US Prices: Evidence from CPI Data." *Journal of Political Economy* 116(3): 381422
- [15] Doms M, Lewis E. 2006 "Labor Supply and Personal Computer Adoption." Federal Reserve Bank of San Francisco Working Paper 200618

- [16] Friedberg RM. 2001 “The Impact of Mass Migration on the Israeli Labor Market.” *The Quarterly Journal of Economics* 116(4): 1373-1408.
- [17] Lewis E. 2011a “Immigration, Skill Mix, and Capital-Skill Complementarity.” *Quarterly Journal of Economics* 126(2): 1029-1069
- [18] Ottaviano G, Peri G. 2006 “Rethinking the Effect of Immigration on Wages.” *National Bureau of Economic Research Working Paper* 12497
- [19] Peri G. 2012 “The Effect of Immigration on Productivity: Evidence from U.S. States.” *Review of Economics and Statistics* 94(1): 348-358

CHAPTER 3

Immigration, Endogenous Technology Choice and Welfare Analysis

3.1 Introduction

Even though immigration has a long history in U.S., it has been a hot topic again due to significant inflows of immigrants in recent decades. In 1970 only 4.7 % of the population was foreign-born, and by 2000, immigrants has increased to 11.1% of total U.S. population and policy makers are debating on the overall impact of immigration on welfare and social security. Some policy makers perceive immigration as a remedy for the problems related with public finance, while the others are more cautious about its potential negative effect on the labor market.

Beside the policy makers, the topic of immigration and its effects attract academic attention, too. One strand of the literature empirically investigates the effect of immigration on the labor market and prices. Interestingly, from the analysis in countries with high immigration rates, they find that the effect of immigration on wages is insignificant. Another strand of the literature focuses on the theoretical analysis of immigration effects with models built on Auerbauch and Kotlikoff (1987). For example, Storesletten (2000) explores the fiscal sustainability and burden of each immigrant for the government while Akin (2011) analyzes the welfare effects of immigration. Even though these and other various studies look at different dimensions of immigration effects, they share one structural feature that is very important for their analysis. All of these papers assume a standard production function which predicts that the wages will decline when there is immigration.¹ This creates a discrepancy between the results of the theoretical papers and the

¹They use a production function of the form $Y_t = K_t^\alpha [(\beta H_t)^{\frac{\sigma-1}{\sigma}} + ((1-\beta)L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}(1-\alpha)}$ and its first order conditions give the following relationship which predicts that immigration of a specific type of labor will decrease its relative wage : $(\frac{\beta}{1-\beta})^{\frac{\sigma-1}{\sigma}} (\frac{H_t}{L_t})^{\frac{-1}{\sigma}} = \frac{w_t(H)}{w_t(L)}$

empirical findings which might lead to incomplete results for the theoretical models.

In this paper, my aim is to address this problem. In this paper, my aim is to address this issue and reconcile the theory with the empirical finding that immigration might actually have a positive effect on wages. I will draw upon endogenous technology change literature pioneered by Acemoglu (1998) and Caselli Coleman (2006).² This literature suggests that firms are able to change their production techniques in order to increase the productivity of the abundant type of labor which leads to increases in wages even in case of increase in supply. In this paper, I suggest that endogenous technical change is a good candidate to explain the empirical findings. In order to validate this, first I empirically show the existence of such firm behavior. Next, I embed the directed technical change in a model that is similar to Auerbauch and Kotlikoff. Based on this modified model, I analyze both the fiscal and welfare effects of immigration. I find that models found in those papers might be over/under estimating the results. Specifically, I explore the effect of increase in high skilled immigrants from 1% to 4%. The results show that if we ignore the technical change and its wage effects, the standard model underestimates the effect of immigration to skilled natives by 95% while it over estimates the effect on unskilled natives by 31%. One explanation can be, because firms are able to change their technology in order to use the more abundant factor more effectively, in this case high skilled labor as a result of immigration, productivity of high skilled labor increases. This results in increase in relative wages for the high skilled natives and immigrants creating a higher welfare. On the other hand, because relative wages will go down for low skilled natives and immigrants, the welfare will be lower than the case with standard constant technology.

In addition, comparing the fiscal burden of an immigrant through net present discounted value of future tax payments, transfers, social security benefits and payments I find that, the standard model overestimates NPV of one additional low skilled immigrant approximately by 35% and underestimate the value of one additional high skilled immigrant by 15%. In addition, capital accumulation is 2% lower and low skilled labor supply is 1% higher if the technical change is not taken into account. In other words, analysis of a standard model might be incomplete in welfare analysis.

²While the former analyzes the change in the technology supply, the latter investigates the change in technology demand

The rest of this paper is structured as follows. Section 2 summarizes the related literature. Section 3 constructs the empirical background of the model where we will show the evidence for technical change. Section 4 will explain the model. Section 5 will summarize the calibration of the model. Section 6 will explore the results. Section 7 will conclude

3.2 Literature Review

This paper draws upon both theoretical and empirical studies on immigration as well as the endogenous technology change literature. The first strand of empirical studies focus on the labor market effects of immigration. They analyze the effect of immigration on the wages and unemployment. There is tremendous amount of existing research and in depth literature review can be found in Okkerse (2008) and Lewis (2012)³. Even though they use different types of data (area vs country-wide) and they conduct their analysis based on different structural forms on production technologies (perfect vs imperfect substitution between natives and immigrants) based on their findings they conclude that immigration has a slight effect on labor markets in terms of wage changes and unemployment.

The second strand of literature aims at explaining the underlying reasons of wage stagnancy. In the literature following channels have been commonly proposed : 1) Change in Product Mix; 2) Change in Production Techniques. Studies that focus on the first explanation claims that there are multiple sectors/products and skill mix changes can be absorbed by the change in share of each product in production (Leamer (1995)). The second explanation is based on the assumption that firms are able to change their technologies in direction of the more abundant factor so that their demand moves in the same direction with immigration. Some of the studies that examined the effect of these two factors on wages are : Dustmann and Glitz (2012), Hanson and Slaughter (2002), Lewis (2003) and Gonzales and Ortega (2011). These papers find that the effect of change in the production techniques dominates the skill mix channel. In addition, there are studies that investigate labor technology choice alone. Some of the important papers that analyze this relationship are: Beaudry, Doms and Lewis(2010) , Doms and Lewis(2006)

³Just to name a few: Antolji and Card (1991), Borjas (1987, 1994, 2003, 2006, 2009), Borjas and Hanson (2008), Borjas and Katz (2007), Card(1990, 2001, 2009), Card and Lewis (2007), Cortes (2008), Friedberg (2001), Ottaviano and Peri (2006, 2012)

, Lewis(2011), Peri (2012). They find that the local labor supply affects the technology choice of the firms positively in the direction of the more abundant factor.

Based on the empirical analysis, technology choice channel might play an important role in explaining wage movements. In this paper, my aim is to construct a model that will generate results that are similar to empirical findings. In order to do that, I use technology choice mechanism similar to Caselli and Coleman(2006)and I embed this setting in to a model where we can analyze the long run intergenerational effects as in the case of Akin(2012), Auerbach and Kotlikoff (1987) and Storesletten(2000).

3.3 The Model

Based on the evidence presented in the previous section, the aim of this paper is to introduce a setting where firms are allowed to choose their technologies (endogenous choice of technique) that will mimic the wage stagnancy after immigration. This feature of the model will help us understand the long-term effects of immigration on welfare in a framework that is more close to the findings in the data. I combine two lines of thought in order to analyze the welfare effects of immigration. As in Auerbauch and Kotlikoff(1987) I use an overlapping generations (OLG) model with government and pension funds. Different from Auerbauch and Kotlikoff (1987) and other studies⁴ that study the effect of immigration using conventional production function, I embed a production technology that is similar to Caselli and Coleman(2006) which will allow firms to choose their optimal technologies. In the model I will use the following agents and analyze their effects: a) Heterogenous Firms b) Firms c) Government d) Pension Funds

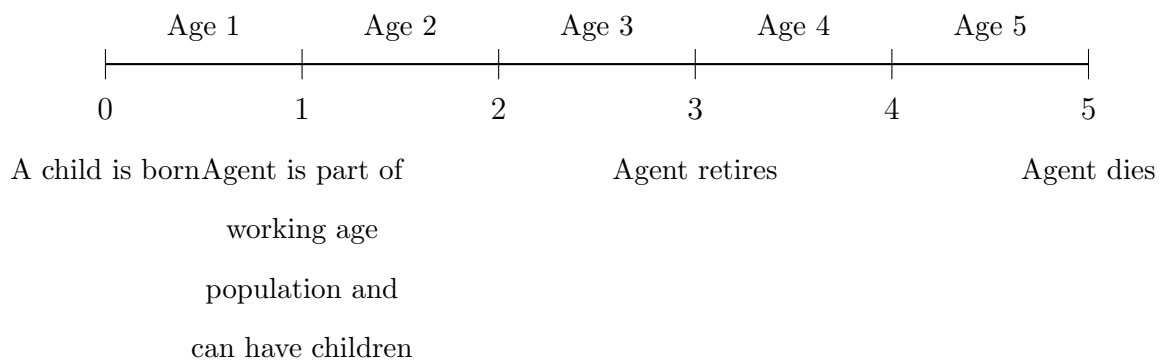
3.3.1 Individuals

Individuals live in a OLG framework with 5 periods - child for one period, working age for two periods and retired for the last two periods. Each period is assumed to be 20 years. Number of periods is set in such a way that enables us to analyze the immigration effect on population dynamics, labor decisions of working age population, as well as the asset holding behaviors of both working age and retired population which will be explained in

⁴some examples are: Storesletten (2000), Kitao (2012)

detail below. There is uncertainty about the survival of individuals from one period to the next. Survival probabilities change with age and are exogenously given. Individuals are heterogeneous with respect to their nativity (origin) and educational attainment (skill). Even though life span of an individual is uncertain due to unexpected death, each period a constant fraction of the agents die which means there is no aggregate uncertainty. There is perfect foresight in the future path of the population and accordingly all economic aggregates (like prices, output etc) are known with certainty.

3.3.1.1 Life Span of an Individual



There is no bequests in the model and all agents are born with 0 initial wealth. At age 1 individuals are assumed to be children, they do not work or save. Government transfers denoted by χ is the only financial resource at their disposal and it is a constant fraction of total production. At age 2, agents become adults and they start working and supply labor for the next two periods (during age 2 and 3). Since they are not allowed to save in their childhood they start age 2 with zero initial wealth.

Individuals are allowed to have children only at age 2. Population dynamics are dependent on the following factors: the fertility of individuals at age 2, skill distribution of their new-borns and survival probabilities of each cohort. Further discussion on demographics and skill transition can be found in the next sections.

After working for two periods during age 2 and 3 individuals retire at age 4 and live for two more periods. Since there is no bequests, they die with 0 assets. Between ages 2-5 individuals also decide on their consumption c and asset holdings a .

Individuals are heterogeneous and heterogeneity comes from the differences in origin and educational attainment. There are two types of individuals with respect to origin: native

and immigrant where natives are denoted by n and immigrants are denoted by m . In terms educational attainment there are two types of individuals: high skilled and low skilled where high skilled are denoted by h and low skilled are denoted by l . An agent with age $i \in \{1, 2, 3, 4, 5\}$ origin $j \in \{m, n\}$ and skill level $s \in \{h, l\}$ is denoted by (i, j, s) . Productivity of individuals are age, origin and skill dependent and is characterized by the efficiency levels denoted by $e(i, j, s)$. We can further decompose the age effect on efficiency in the following way: $e(i, j, s) = \tilde{e}(j, s)\hat{e}(i)$ where $\tilde{e}(j, s)$ is skill and origin specific productivity and $\hat{e}(i)$ is time specific productivity.

3.3.1.2 Individual's Problem

Heterogeneous agents maximize their expected lifetime utility by choosing the sequence of assets $a_{i+1}(i, j, s)$, labor $l_i(i, j, s)$ and $c_i(i, j, s)$ for $i \in \{2, 3, 4, 5\}$ by solving the following problem:

$$\max \frac{(c_t^\gamma(1, j, s)(1 - l_t(1, j, s))^{(1-\gamma)})^{1-\eta}}{1 - \eta} + \sum_{i=2}^5 \left(\prod_{k=1}^{i-1} \lambda(k) \right) \beta^{i-1} \frac{(c_t^\gamma(i, j, s)(1 - l_t(i, j, s))^{(1-\gamma)})^{1-\eta}}{1 - \eta} \quad (3.1)$$

subject to the following constraints:

$$c_t(1, j, s) = \chi_t(1, j, s), l_t(1, j, s) = 0$$

for every $i \geq 2$

$$b_t(i, j, s) + (1 - \tau_w - \tau_b)w_t(s)e(i, j, s)l_t(i, j, s) + (1 + (1 - \tau_r)r_t)a_t(i, j, s) + \chi_t(i, j, s) = c_t(i, j, s) + a_{t+1}(i, j, s)$$

where if $2 \leq i \leq 3$, $b_t(i, j, s) = 0$ and if $i > 3$, $l_t(i, j, s) = 0$

There is uncertainty in the lifespan of each individual and $\lambda(i)$ is the conditional probability of survival of an agent at age i . τ_w is the wage income tax and τ_b is the contribution rate of each individual to the pension fund. Beside labor income individuals get capital income net of taxes denoted by τ_r . $b_t(i, j, s)$ is the pension payments paid to retirees at age 4 and 5. $\chi_t(i, j, s)$ is the amount of transfers from government. In the first period, agents consume government transfers. In period 2 and 3 they supply $l_t(i, j, s)$ units of labor and get $(1 - \tau_w - \tau_b)w_t(s)e(i, j, s)l_t(i, j, s)$ where $w_t(s)e(i, j, s)l_t(i, j, s)$ is total labor earnings.

3.3.1.3 Population Dynamics

Immigration affects the long-run population distribution directly and indirectly. The direct effect of immigration is through increasing certain type of labor in the economy. The indirect effect of immigration comes from the change in the population distribution as a result of different immigrant fertility rates and skill heredity probabilities (probability of transferring parent's skill level to the descendant) Accordingly, following factors need to be analyzed in order to understand the distribution of population with respect to age and origin: a) Initial distribution of the population b) Skill Heredity between individuals and their children c) Number of children each individual has d) Immigration Policy.

Skill Transmission to Children

In order to keep the evolution of population more simple, I assume that both immigrants and natives are fertile only at age 2. All children regardless of the origin of their parents are assumed to be native. Transmission of skills from parents to children follows a Markov process. Let $\varphi(j, s)$ denote the number of children per person with origin j and skill s . Let $\mu(2, j, s)$ denote the number of parents for each origin and skill at time t . Let $\pi(j, s)$ be the probability that a parent of origin j and skill s will have a high skilled child. Then the number of newborns of each skill level $s \in \{h, l\}$:

$$\mu(1, n, h) = \sum_{j,s} \varphi(j, s) \mu(2, j, s) \pi(j, s)$$
$$\mu(1, n, l) = \sum_{j,s} \varphi(j, s) \mu(2, j, s) (1 - \pi(j, s))$$

Immigration Policy

Immigration policy $\psi = \psi(2, m, h), \psi(2, m, l)$ determines the size of the immigrant population at age 2 (immigrants of age 20-39) of each skill levels. It is given as a fixed fraction of the total population in the current period.

Law of Motion for Population

Let μ denote the total population in the economy at time t . Given the immigration

policy ψ , children per person $\varphi(j, s)$, skill transition probabilities $\pi(j, s)$ and survival probabilities $\lambda(i)$, population evolves according to :

$$\begin{aligned}
\mu(1, n, h) &= \sum_{j,s} \varphi(j, s) \mu(2, j, s) \pi(j, s) \\
\mu(1, n, l) &= \sum_{j,s} \varphi(j, s) \mu(2, j, s) (1 - \pi(j, s)) \\
\mu(i + 1, n, j) &= \lambda(i) \mu(i, n, s) ; \quad i \in \{1, 2, 3, 4\} , \quad s \in \{l, h\} \\
\mu(2, m, s) &= \psi(2, m, s) \mu ; \quad s \in \{l, h\} \\
\mu(i + 1, m, j) &= \lambda(i) \mu(i, m, s) ; \quad i \in \{2, 3, 4\} , \quad s \in \{l, h\} \\
\mu &= \sum_{i,j,s} \mu(i, j, s) \tag{3.2}
\end{aligned}$$

3.3.2 Firms

In this paper, different from the exiting literature, I will allow firms to choose from a set of production technologies. The production function is similar to Caselli and Coleman (2006) where firms choose their intensity of skilled and unskilled labor from a menu of production techniques that is called "technology frontier":

$$Y_t = K_t^\alpha [\Phi_1(A_t H_t)^{\frac{\sigma-1}{\sigma}} + \Phi_2(A_t L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}(1-\alpha)} \tag{3.3}$$

Competitive firms hire two types of labor H_t and L_t and capital K_t to produce output with a constant elasticity of substitution (CES) technology where A_t is an exogenous labor augmenting productivity process with deterministic growth rate g . σ is the elasticity of substitution between high skilled and low skilled labor. Φ_i can either be interpreted as the efficiency or the intensity of each type of labor in production of the the final good. Labor output L_t and H_t are calculated in terms of the efficiency units which are a function of age and the type specific productivity:

$$L_t = \sum_{i \in \{2,3\}} \sum_{j \in \{n,m\}} l_t(i, j, l) e(i, j, l) \mu_t(i, j, l) \tag{3.4}$$

where $e(i,j,l)$ is the productivity of low skilled labor with type (i,j) and $\mu_t(i,j,l)$ is the mass of low skilled workers with type (i,j) . Similarly,

$$H_t = \sum_{i \in (2,3)} \sum_{j \in (n,m)} l_t(i,j,h) e(i,j,h) \mu_t(i,j,h) \quad (3.5)$$

Aggregate capital is the sum of total individual wealth:

$$K_t = \sum_{i \in (2,3)} \sum_{j \in (n,m)} \sum_{s \in (l,h)} a_t(i,j,s) \mu_t(i,j,s) \quad (3.6)$$

The law of motion for capital determines the aggregate investment:

$$I_t = K_{t+1} - (1 - \delta)K_t \quad (3.7)$$

3.3.2.1 Firm's Problem

Firm's problem is to maximize the profits given the rental rate of capital r_t , wage rates $w_t(l)$, $w_t(h)$ and the depreciation rate δ and the production frontier :

$$\max_{\Phi_{1,t}, \Phi_{2,t}, K_t, L_t, H_t} \{ K_t^\alpha [\Phi_{1,t} (A_t H_t)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t} (A_t L_t)^{\frac{\sigma-1}{\sigma}}]^{(\frac{\sigma}{\sigma-1})(1-\alpha)} - (r_t + \delta)K_t - w_t(l)L_t - w_t(h)H_t \} \quad (3.8)$$

subject to

$$\Phi_{1,t}^\omega + \kappa \Phi_{2,t}^\omega \leq B \quad (3.9)$$

where above constraint specifies that on the technology frontier there is a trade-off between low skill and high skill intensity. Parameters κ and ω determine the degree of the trade-off while parameter B specifies the height of the technology frontier.

In order to ensure that there is an interior solution for $\Phi_{1,t}$ and $\Phi_{2,t}$ meaning that firms employ both types of labor, we assume $\omega > \sigma - 1$ ⁵

⁵For proof please see Caselli and Coleman (2006)

3.3.3 Government

Government collects taxes (T_t) in order to finance its expenditures on government consumption (G_t) and transfers (Tr_t). In addition, accidental bequests (T_{Bt}), defined as capital returns arising from unexpectedly deceased agents, will be collected and redistributed by the government. Taxes are collected in forms of labor income and capital income taxes:

$$T_t = \tau_w w_t(l)L_t + \tau_w w_t(h)H_t + \tau_r r_t K_t \quad (3.10)$$

where L_t is the aggregate low-skilled labor and H_t is the aggregate high skill labor and K_t is the aggregate capital.

Since the economy is growing at rate g , I assume that transfers grow at the same rate

$$Tr_t = (1 + g)^t \sum_{i,j,s} \chi(i, j, s) \mu_t(i, j, s) \quad (3.11)$$

Government spending is a constant fraction \bar{y} of the aggregate output:

$$G_t = \bar{y} Y_t \quad (3.12)$$

In equilibrium transfers will be set such that government keeps a balanced budget each period:

$$T_t + T_{Bt} = G_t + Tr_t \quad (3.13)$$

3.3.4 Pension Funds

The social security system is pay-as-you-go. All social security contributions are collected by the social security authority and redistributed to the retirees. Pensions are a constant fraction of net labor income of the productivity type (i, j, s) ⁶

$$b_t(i, j, s) = \begin{cases} 0 & i \leq 3 \\ \zeta(1 - \tau_w - \tau_b)w_t(s)e(i, j, s) & i > 3 \end{cases}$$

⁶Pension funds system setting where social security benefits are related to a retiree's averaged indexed monthly earnings (AIME) can be found in the Appendix and the analysis are available upon request.

3.3.5 Competitive Equilibrium

Given the initial distribution of assets a_0 , population μ_0 , government transfers $\chi(i, j, s)$ and government expenditures g_i ; tax rates τ_b, τ_w, τ_r , fertility rates $\varphi(i, j, s)$, skill heredity probabilities $\pi(i, j, s)$, survival probabilities λ_i and immigration policy ψ , a competitive equilibrium for this economy is a sequence for

$$\{w(s), r, H, L, K, T, Pen, G, Tr, \tau_b, l(i, j, s), c(i, j, s), a(i, j, s)\mu(i, j, s)\}$$

such that for each t :

- $l(i, j, s), c(i, j, s), a(i, j, s)$ solve the individual's problem
- K, H, L, ϕ_1, ϕ_2 solve the firm's problem
- The goods market clears : $Y = I + G + \sum_{i,j,s} \mu(i, j, s)c(i, j, s)$
- The labor market clears(Eqn(3.4) and Eqn(3.5) hold)
- Aggregate capital equals aggregate private wealth (Eq.(3.6) holds)
- Transfers balance government's budget (Eqn(3.13) holds)
- Pension tax rate balances the social security balance (Eqn(3.64) holds)
- Population evolves according to Eqn(3.2)

3.4 Calibration

3.4.1 Calibration of Parameters for Individual's Preferences

Coefficient of relative risk aversion η is assumed to be 2. Share of consumption in utility function (γ) is assumed to be 0.32 so that the average labor supply is calibrated approximately to be 0.3. Time discount β is assumed to be 0.67.

3.4.2 Calibration of Parameters for Production Function

Following Katz and Murphy (1992) elasticity of substitution between skilled and unskilled labor denoted by σ is set at 1.4. In addition, share of capital in production is set at 1/3

following the standard convention in order to match the U.S. historical value.

Since our aim is to understand the movements on the technology frontier as a result of immigration, first I determine the following technology frontier for U.S:

$$\Phi_{1,t}^\omega + \kappa \Phi_{2,t}^\omega \leq B$$

The same methodology as in Caselli and Coleman (2006) has been used in order to find an estimate for the parameters κ , ω and B using the data they have provided in their paper.

In order to get the estimates for the US technology frontier first I generate the series for the relative efficiency levels for each country i at time t using the following equation⁷

$$\frac{\phi_1^i}{\phi_2^i} = (\kappa (\frac{H_t^i}{L_t^i})^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\omega-1}} \quad (3.14)$$

This equation explains how efficiency ratio that firms choose is related to the relative skill supply. If we take the logarithm of this equation and assume all countries have their own production technology choice we get the following relationship:

$$\begin{aligned} \log\left(\frac{\phi_1^i}{\phi_2^i}\right) &= \frac{1}{\omega^i - 1} (\log(\kappa^i) + \log\left(\left(\frac{H_t^i}{L_t^i}\right)^{\frac{\sigma-1}{\sigma}}\right)) \\ \log\left(\frac{\phi_1^i}{\phi_2^i}\right) &= \frac{1}{\omega^i - 1} \left(\frac{\sigma - 1}{\sigma}\right) \log\left(\frac{H_t^i}{L_t^i}\right) + \frac{1}{\omega^i - 1} \log(\kappa^i) \end{aligned}$$

If we assume that κ^i is uncorrelated with $\left(\frac{H_t^i}{L_t^i}\right)$ and regress relative efficiency on relative skill supply, we can back out an estimate of ω . In addition, from the regression residual Caselli and Coleman (2006) backs out γ^i for each country including US. Instead, I use Eqn(3.14) in order to backout κ^i . If we place these values in to the following equation we can back out an estimate for B which will be the technology frontier for the US:

$$\Phi_{1,t}^{US \omega} + \kappa^{US} \Phi_{2,t}^{US \omega} = B^{US}$$

Based on the regression analysis we find that technology frontier for US economy is such that ω is equal to 1.43 and κ is equal to 1.31 and B is equal to 6.54.

⁷Please see Technical Appendix for the solution of the problem

3.4.3 Calibration of Parameters for Efficiency Units

Efficiency units have been calibrated in order to match the hourly wages of the individuals with different age, skill and nativity. For comparison purposes we have used ACS 2004 and CPS 2004 as well as Census 2000 5% Sample. Hourly wages are reported based on Census 2000 since we get similar results for other data sources mentioned. In order to calculate the hourly wages with respect to each age, skill and nativity group first we divide the sample in to 5 age groups based on our model. We define a worker as unskilled if s/he has a higher degree or lower. We define a worker as immigrant if s/he is recorded as "Foreign Born". In order to calculate the hourly wages for each group, we use the wage/salary income for the past year weighted for each individual. We calculate total hours worked by multiplying hours worked within a week and total number of working weeks weighted. We exclude individuals with working hours less than 1250. In addition, we calculate individual hourly wages (unweighted) and exclude individuals with hourly wages less than \$3.5 and more than \$500 . Based on our calculations, we construct hourly wage tables that we will use as an approximation for the efficiency units that we will use in our analysis. We normalize the unskilled immigrant wage at time 2 equal to 1 and calculate the relative efficiency of other agent as well as the time specific productivities so that relative wages will replicate the relative efficiencies.

		Skill	
		L	H
Origin	N	1.1716	1.7502
	M	1	1.9202

(a) Efficiency Units wrt Skill and Origin

		Efficiency Units
		Time
2	1	
3	1.2621	

(b) Efficiency Units wrt Time

Table 3.1: Efficiency Units wrt Skill, Origin and Time

3.4.4 Calibration of Parameters for Law of Motion for Population

I calibrate the following parameters in order to determine the effect of immigration on the population distribution:

- In order to determine the distribution of population with respect to age, nativity and skill we use the CPS 2009 sample. Based on this sample first we calculate age and skill distribution for both natives and the immigrants. Based on these calculations I find the percentage of each group in the total population.

	Native		Immigrant	
Generation	Unskilled	Skilled	Unskilled	Skilled
1	0.1007	0.1604	0.0067	0.0060
2	0.0846	0.1346	0.0271	0.0242
3	0.0893	0.1422	0.0248	0.0222
4	0.0480	0.0763	0.0100	0.0090
5	0.0113	0.0180	0.0020	0.0018

Table 3.2: Initial Distribution of Population

- I get the survival probabilities from National Vital Statistics Reports, United States Life Tables, 2006. I assume that survival probabilities are the same for both immigrants and the natives. Results can be found in the Data Appendix.

		Survival Probability
Generation(i)	1	0.98747
	2	0.97719
	3	0.91256
	4	0.61552
	5	0

Table 3.3: Survival Probabilities for Different Generations(j=m,n)

- In order to calculate the growth of the population together with its distribution the fertility rates of cohort 2 is needed. In our model each individual is a parent

of its own children, there is no households. Fertility is reported in terms of the average number of children during lifetime. We use the education specific fertility rates that are found in Camarota (2005). In order to calculate the average fertility rates with respect to the skill levels that is defined in this paper, I calculate the skill distribution of the immigrants and natives from ACS 2002 and find the average fertilities. Results can be found in the Data Appendix.

		Skill Level(s)	
		L	H
Origin(j)	n	2.3070	1.8007
	m	3.3290	1.9657

Table 3.4: Fertility Rates for different origins and different skill levels

- Skill transition probabilities between parent and the child is needed in order to calculate the skill distribution of the newborns. In order to calibrate the inter-generational transition of skills, we I use General Social Survey for 2008 where respondents are asked schooling level of respondent's parents as well as his/her own schooling. We consider individuals who are between 25 and 55 years old. Results can be found in the Data Appendix.

		Child	
		L	H
Parent	L	0.7519	0.2480
	H	0.3503	0.6496

(a) Skill Heritability for Natives

		Child	
		L	H
Parent	L	0.7059	0.2940
	H	0.2667	0.7332

(b) Skill Heritability for Immigrants

Table 3.5: Skill Heritability Matrices

3.4.5 Calibration of Parameters for Government and Social Security

In order to analyze the effect of immigration on the government budget as well as the social security pensions, we need to calibrate the tax rates and the structure of the social security pensions.

3.4.5.1 Government and Social Security

Government puts taxes on the labor income and the capital income. Parameters in the government budget have been taken from Heer and Irmen (2008). In their analysis, the government share is set equal to the average ratio of government consumption in GDP in the US during 1959-93 according to the Economic Report of the President (1994). The tax rates τ_w and τ_r are computed as the average values of the effective US tax rates over the time period 1965-88 that are reported by Mendoza, Razin, and Tesar (1994). Government transfers, tr , will adjust according to the equilibrium condition of the government budget. Government expenditures to children have been taken from Isaacs(2009) and reported as 2.4% of GDP.

τ_w	0.24
τ_r	0.42
G as % of GDP	0.195
Transfers to cohort 1 as % of GDP	0.024
ζ	0.5
τ_b	will adjust wrt SS balance
tr	will adjust wrt Govt balance

3.5 Results

Based on our calibration, initial stationary distribution of the population is as follows:

Generation	Native		Immigrant	
	Unskilled	Skilled	Unskilled	Skilled
1	0.2588	0.1477	0	0
2	0.1508	0.086	0.05	0.01
3	0.0869	0.0496	0.0288	0.0058
4	0.0468	0.0267	0.0155	0.0031
5	0.017	0.0097	0.0056	0.0011

Table 3.6: Initial Stationary Distribution of Population

In our further analysis we explore the steady state and transition effects of increase in the immigration. In the first part of our analysis we analyze high skilled immigration effects and in the second part we repeat the same exercise with an increase in the number of low skilled immigrants. Specifically, we assume that initially low skilled and high skilled immigration as percentage of population is $(\Psi_1, \Psi_2) = (0.01, 0.05)$ and investigate the immigration policy change by 0.03 with respect to either high skilled or low skilled labor.

3.5.1 Experiment I: Increase in the High Skilled Labor

In this experiment, high skilled immigration is raised to 0.04 of total population from 0.01 of the population. Below is shown new stationary distribution of the population. In the new steady state the ratio of high skilled immigrants is higher for all generations as expected. In addition, there is an indirect positive effect of high skilled immigration on the population size of the high skilled natives. The underlying reason is because high skilled immigrants are more fertile than the natives and they have high probability of having a high skilled child then this raises the high skilled young native generation (assuming the children of immigrants will be born as natives) Increased high skilled immigration also reduced the share of the low skilled labor in the economy for all cohorts.

	Native		Immigrant	
Generation	Unskilled	Skilled	Unskilled	Skilled
1	0.2383	0.1611	0	0
2	0.1322	0.0894	0.05	0.04
3	0.0726	0.0491	0.0274	0.022
4	0.0372	0.0251	0.0141	0.0113
5	0.0129	0.0087	0.0049	0.0039

Table 3.7: Final Stationary Distribution of Population

3.5.1.1 Effect of High Skilled Immigration on Skill Shares

When firms are allowed to choose their optimal technology on the technology frontier after an increase in the skill immigration, firms will choose to increase the productivity of the production factor that has become more abundant which is in this case high skilled labor. As shown in Figure 3.1, firms will increase the productivity of their high skilled labor while decreasing the productivity of their low skilled labor. On the other hand, when firms are not allowed to choose their technology, their productivity of different type of labor will be constant at the before policy change levels.

Productivity of High and Low Skilled Labor with Technical Change

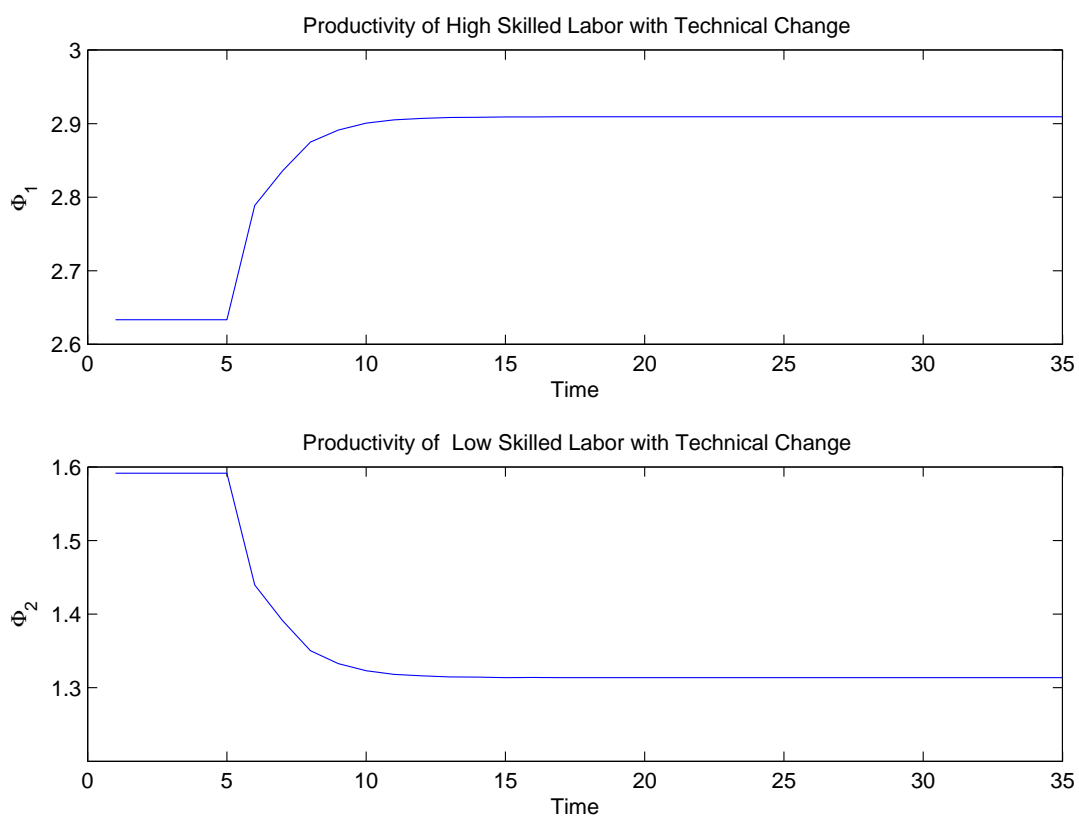


Figure 3.1: Share of High and Low Skilled Labor with Technical Change

3.5.1.2 Effect of High Skilled Immigration on Wages

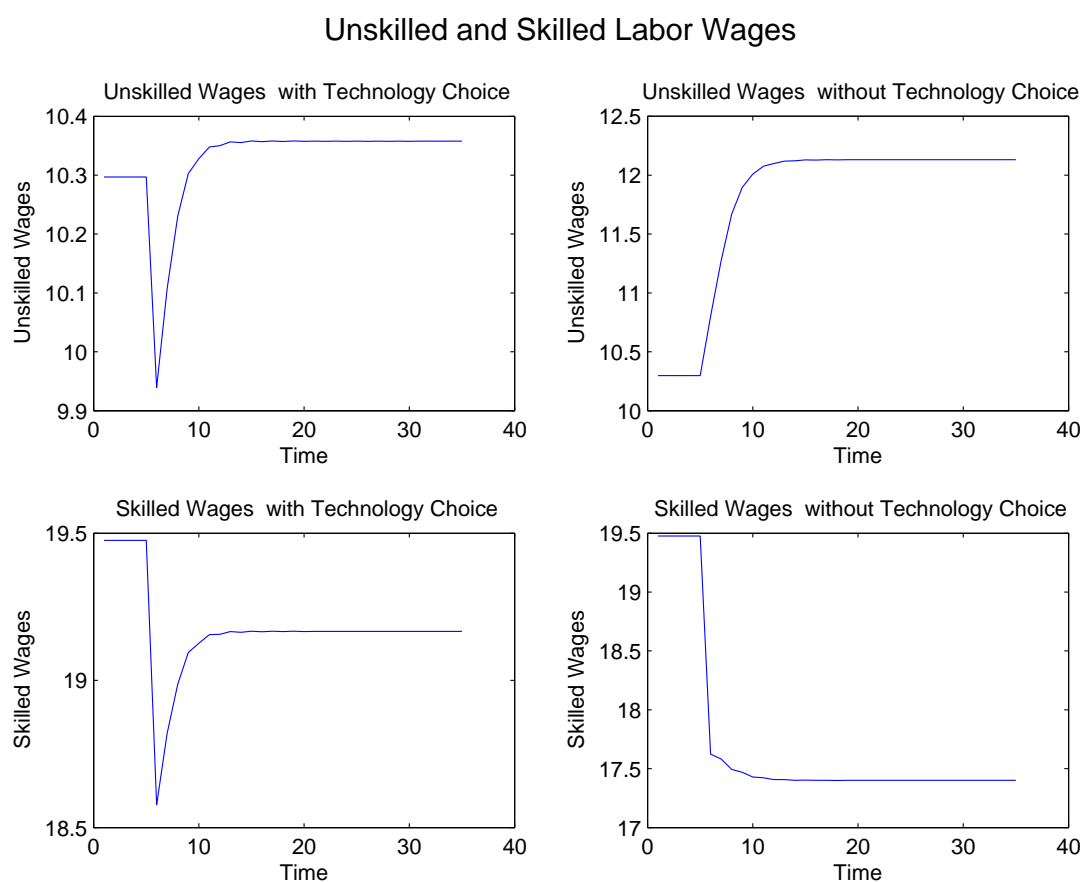


Figure 3.2: Unskilled and Skilled Labor Wages

Figure 3.2 shows the wage movement during transition to new steady state. Right panel shows the case when the firms have fixed production technology while left panel shows the wages when the firms are allowed to choose their optimal technology.

When firms are not allowed to change their production technology, increase in high skill immigration leads to increase in the low skilled wages and decrease in the high skilled wages. The underlying reason is, because there is more skilled labor in the economy, increase in supply will lower the wages for the skilled labor. On the other hand, because relative supply of the low skilled labor has declined because of the change in the population distribution wages of unskilled labor goes up.

On the other hand, when firms are allowed to change their technology, opposite wage movements of different type of labor is mitigated. As shown in the lower left panel of Figure 3.2, when high skill immigration increases the wages initially fall. However, the

magnitude of the fall in the high skilled wages are not as high as the case of no technology change. The underlying reason is, because firms increase the productivity of their high skilled labor, then their demand for the high skilled labor shifts up leading to a smaller decrease in the high skilled labor wages. In addition, because firms continue to increase the productivity of their skilled labor, wages start to increase to a new steady state that is lower than the initial one. Considering the low skilled labor, since firms are able to change their technology, productivity of the low skilled labor decreases as a result of high skilled immigration creating a decrease in low skilled wages. However, because of the population size effects of immigration, low skilled labor becomes less abundant creating a rebound in low skilled wages. However, because of the change in the technology, low skilled wages do not increase as much as they do in case of no technical change .

3.5.1.3 Effect of High Skilled Immigration on Labor Choices

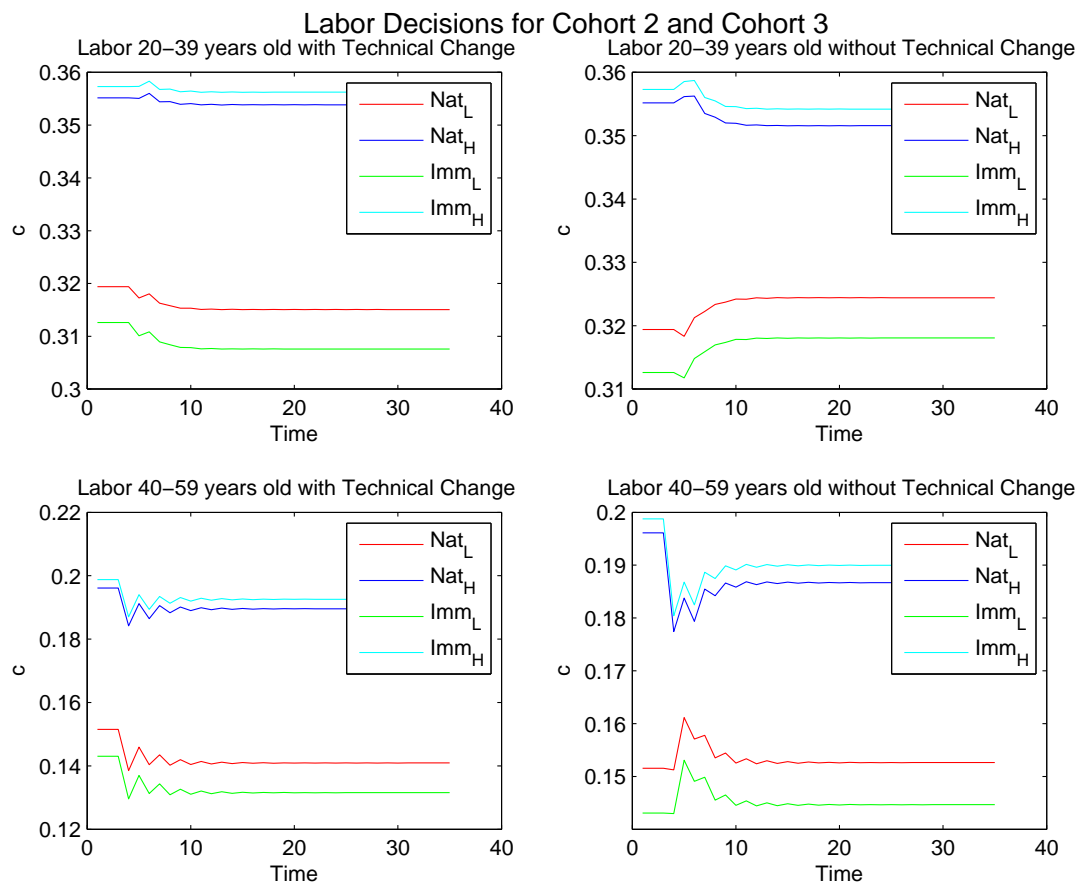


Figure 3.3: Labor Decisions

Figure 3.3 shows labor choices for different types of labor at different ages. If we compare the steady state labor choices under no technology choice (right panel) we see that high skilled workers reduce their labor as a result of decrease in the wages while low skilled workers increase their labor as a result of increase in wages. However given that the relative mass of low skilled workers decline, aggregate low skilled labor in effective units decline while the aggregate effective high skilled labor in effective units go up.

Considering the case with technology choice, we see that both high skilled and low skilled workers reduce their labor. Both high skilled and low skilled workers reduce the amount of labor they supply because of the decline in wages.

3.5.1.4 Effect of High Skilled Immigration on Asset Holdings and Consumption

Since individuals at age 1 are required to consume all transfers that they get, all agents start the second period with zero assets. Accordingly, individuals decide on their asset for ages 3, 4 and 5. Asset holding decisions with and without technological change are shown in Figure 3.4 and Figure 3.5. Asset holding decisions of individuals depend on their wages. In case of no technology choice, high skilled workers reduce their asset holdings while low skilled workers save more. However, when firms are allowed to choose their technology, because the wage for high skilled workers do not decline as much as they do in case of no technology choice, both types of workers increase their asset levels.

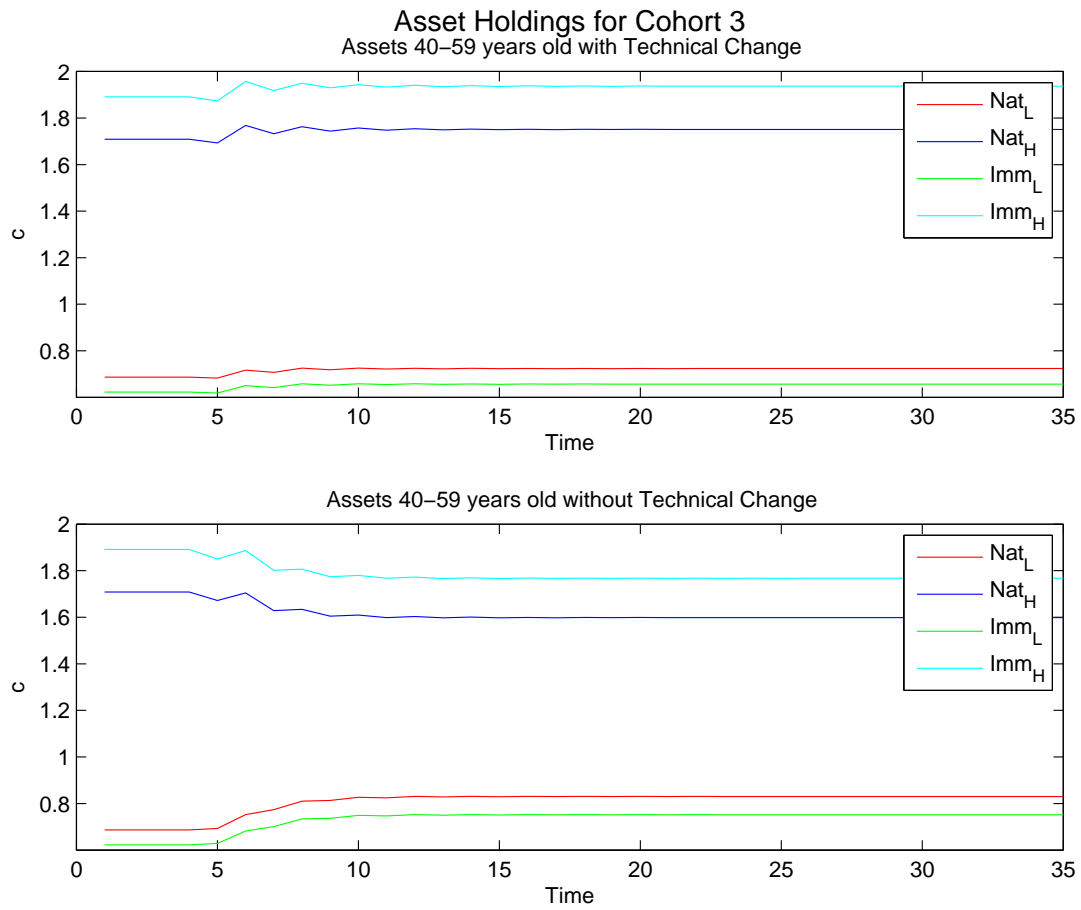


Figure 3.4: Asset Holdings for Cohort 2 and Cohort 3

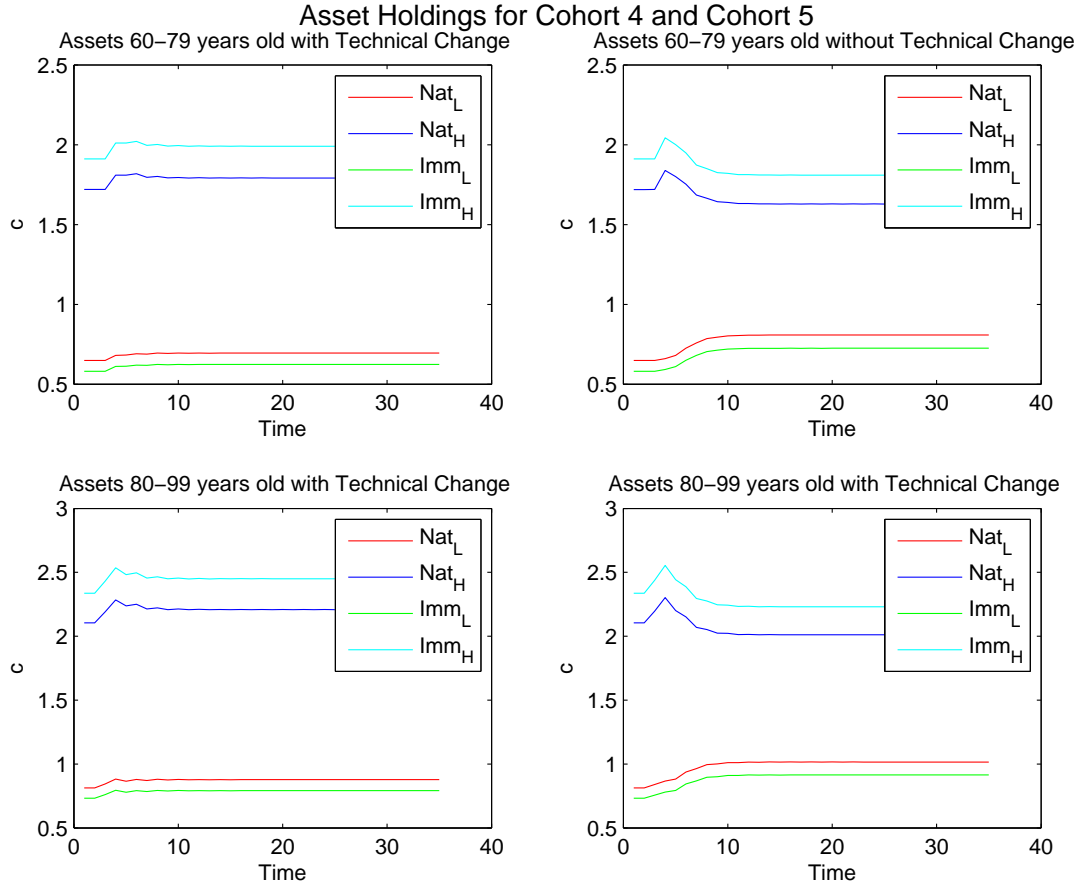


Figure 3.5: Asset Holdings for Cohort 4 and Cohort 5

3.5.1.5 Effect of High Skilled Immigration on Consumption Decisions

Consumption decisions with and without technological change are shown in Figure 3.6 and Figure 3.7. Similar to asset holding decisions of individuals, consumption choices of individuals depend on their wages. In case of no technology choice, high skilled workers reduce their consumption while low skilled workers consume more. However, when firms are allowed to choose their technology, because the wage for high skilled workers do not decline as much as they do in case of no technology choice, both types of workers increase their consumption levels.

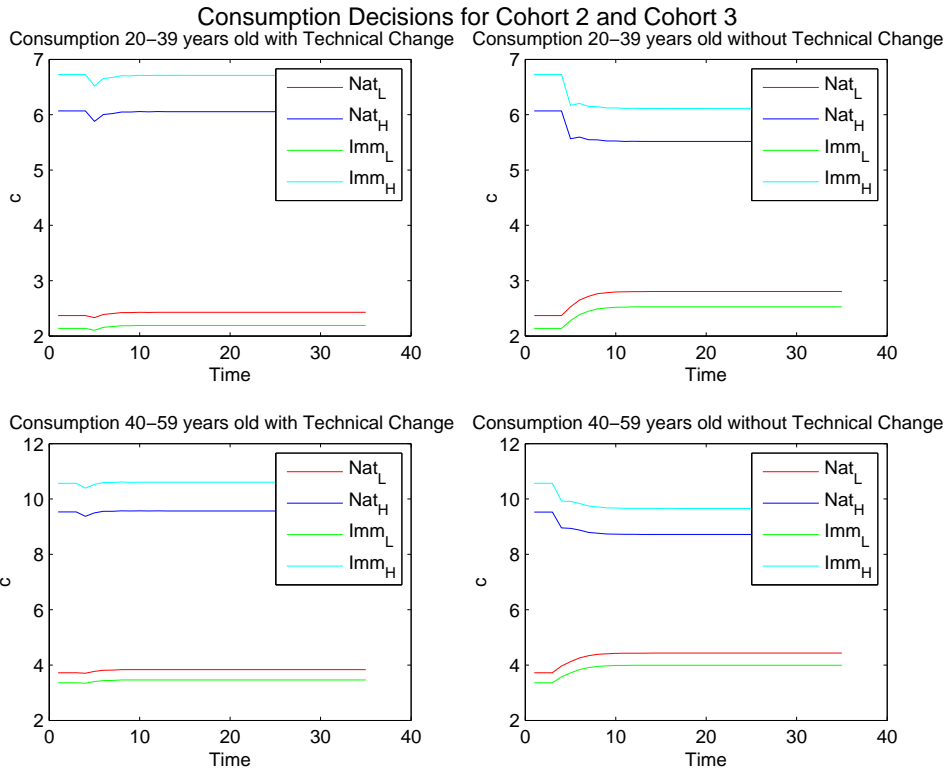


Figure 3.6: Consumption Decisions for Cohort 2 and Cohort 3

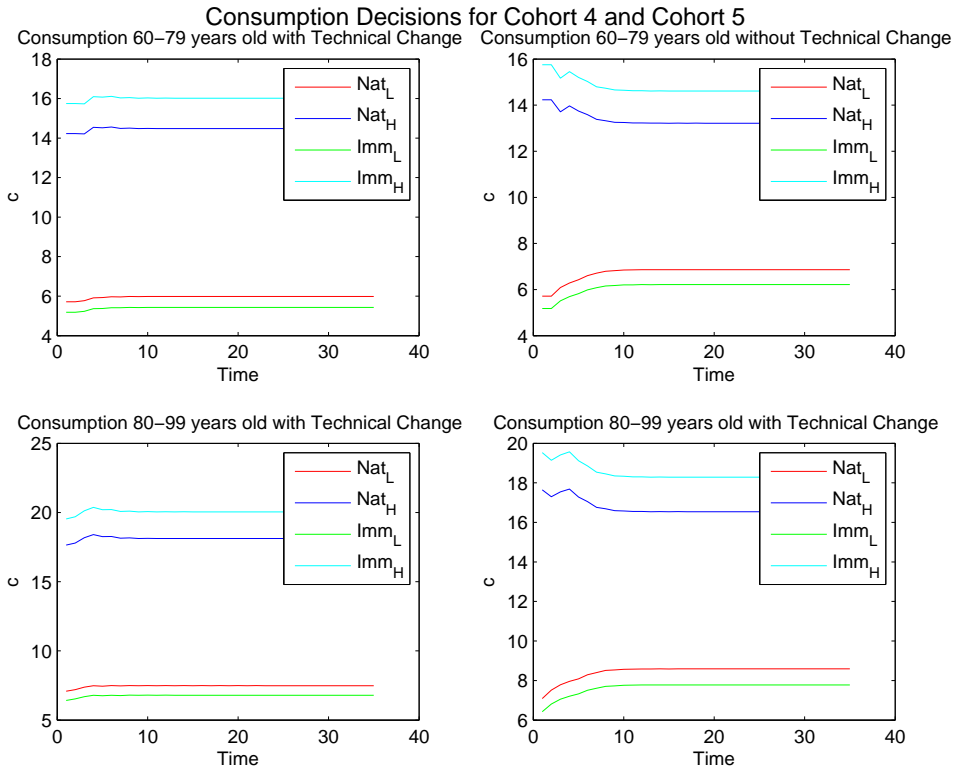


Figure 3.7: Consumption Decisions for Cohort 4 and Cohort 5

3.5.1.6 Effect of High Skilled Immigration on Economy Aggregates

Given individual asset and labor decisions of individuals and the evolution of the distribution of the aggregate population, aggregate labor and capital values are shown in Figure 3.8. When firms are not allowed to choose their technology, low skilled workers increase their labor. However, since the relative mass of the unskilled workers go down, then population effect dominates the individual labor choice effect and aggregate low skilled labor goes down. When firms are allowed to choose their technology, because of the wage decline, low skilled workers reduce their labor supply which pushes down the aggregate labor supply even further.

On the other hand, when technology is allowed to change, high skilled wages do not decline as much as it does in case of no technology choice case. Accordingly, high skilled workers do not reduce their labor as much as they do in case of no technology case. In addition, because of the increase in the high skilled worker population, aggregate high skilled labor goes up more.

Initial effect of immigration on the aggregate capital is negative. Because immigrants are assumed to enter the workforce without any initial capital, capital per effective labor declines. Then because of increase in wages, asset holdings increase creating increase in the total capital. However, when technology choice is allowed, high skilled workers start to save more than the low skilled workers and together with increase in the high skilled population, aggregate capital is higher than the no technology choice case.

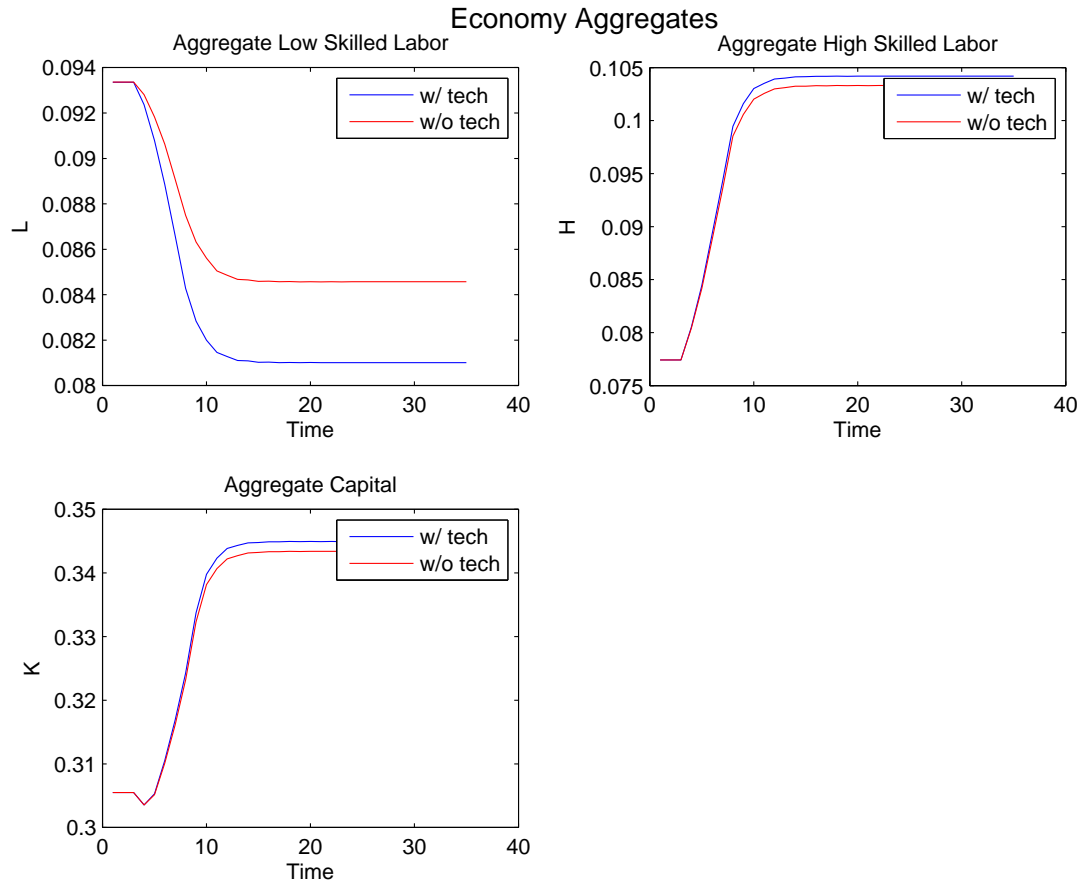


Figure 3.8: Economy Aggregates

3.5.1.7 Effect of High Skilled Immigration on Pension Payment Share and Interest Rates

Figure 3.9 shows effects of high skilled immigration on the replacement rates and interest rates. Results show that technology choice do not have significant effect on the results. Interest rate is lower in case of endogenous technology case because of higher capital supply. In addition, pension payment share is the same in both cases because even though wage allocation changes, total labor income is constant in the long run leading to same pension payment share in case of constant replacement rate. In addition, because the relative population of old people declined, pension payment share goes down.

Pension Payment Share and Capital Rent

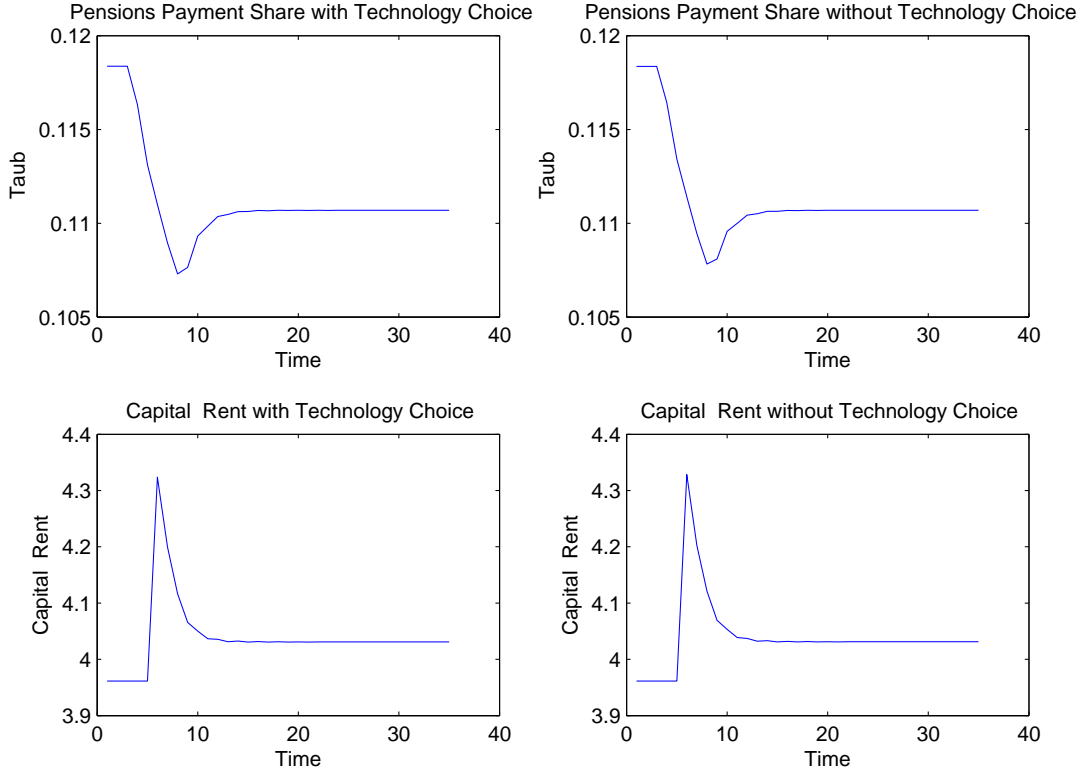


Figure 3.9: Pension Payment Share and Capital Rent

3.5.1.8 Effect of High Skilled Immigration on Welfare and Government Budget

In order to analyze the welfare effects of immigration policy with and without technical change we consider the initial assets individuals needed to have in order to achieve the lifetime utility under the new immigration policy given the initial levels of prices. In other words, given life time utility function $v(p_{ss}, w_{ss}) = u_0$ at the initial steady state with price p_{ss} and initial wealth w_{ss} , equivalent variation (EV) is the initial wealth required (in terms of the consumption good) in order to acquire the utility after the immigration policy change $v(p_1, w_1) = u_1$ with initial prices:

$$v(p_{ss}, w_{ss} + EV) = u_1$$

Figure 3.10 summarizes the welfare effect of high skilled immigration with and without technology change allowed. Firstly, in case of no technology choice, low skilled workers

increase their consumption while increasing their labor which have opposite effects on utility. However, positive effect of consumption dominates the negative effect of increase in the working hours and lifetime utility increases. Accordingly, the equivalent variation rises meaning that high skilled immigration increases the welfare of the low skilled labor when technology choice is not taken in to consideration. On the other hand, when technology choice is allowed, low skilled workers increase their consumption less than the previous case and reduce working hours. In that case, high skill immigration effect on the low skilled worker is still positive but less significant. To be more specific EV results show that no technology choice case over estimates the welfare effect of high skilled immigration on unskilled natives by 31%.

Considering the immigration effects on the high skilled labor, the positive effect of high skilled immigration is still present for both cases. However, the reason is different than the case of low skilled workers. Specifically, in case of no technology choice high skilled workers reduce their consumption and working hours and the aggregate effect is positive. When firms are allowed to change their technology, high skilled workers will increase their consumption and reduce their labor. The aggregate effect is greater than the case with no technology case. Specifically, EV results shown in Figure 3.10 show that the standard model underestimates the welfare effect of immigration to skilled natives by 95% while it over estimates the effect on unskilled natives by 31%.

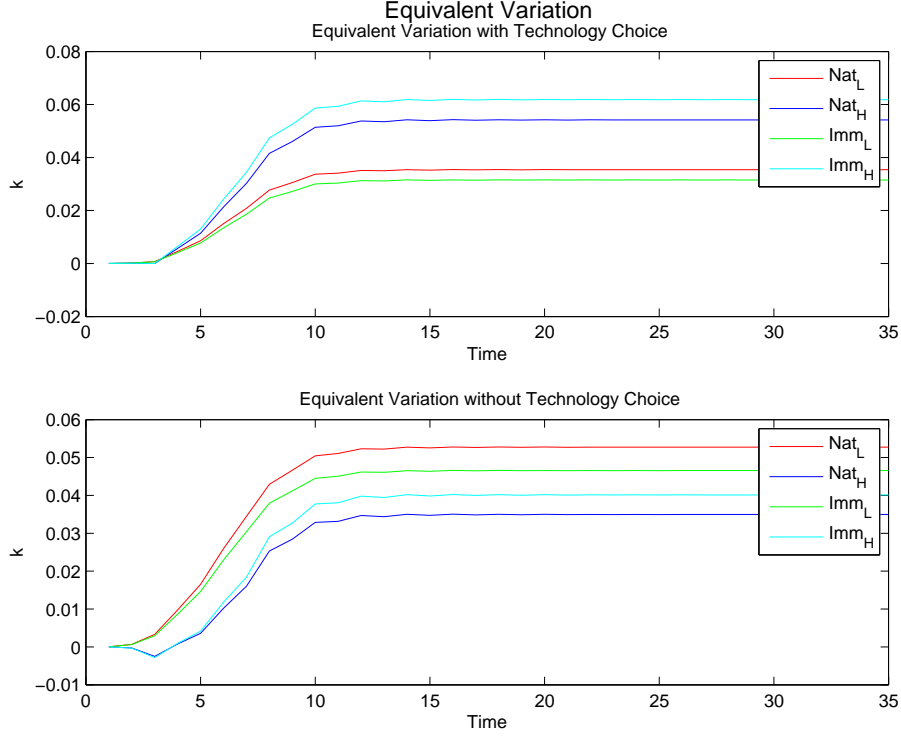


Figure 3.10: Equivalent Variation

In addition to welfare analysis, as in Storesletten (2000) we calculate the net discounted gain to government of admitting one extra immigrant for each skill type. These calculations are partial equilibrium analysis since I ignore that entrance of one additional immigrant will change the prices. In addition, in these calculations I combine the effect of the immigrant on the government and social security budget and calculate the total effect of immigrant on the economy. Based on this explanation, I denote $J(i,m,s,t)$ as the tax revenues plus pension funds payments of the immigrant minus the pension benefits and transfers given to the immigrant of type (i,m,s) at time t . I calculate the lifetime benefits based on the assumption that the agent will live till s/he is old (exogenous death shocks are not allowed) Then net discounted value of an immigrant who entered the country at time t at age 2 (which means s/he was born at age $t-1$) is calculated as :

$$\begin{aligned}
 NPV(m, s, t - 1) = & \sum_{i=2}^5 \frac{(1 + R_t)}{\prod_{k=0}^{i-2} (1 + R_{t+k})} J(i, m, s, t + i - 2) \\
 & + [\varphi(j, s)\pi(j, s)NPV(n, h, t) + \varphi(j, s)(1 - \pi(j, s))NPV(n, l, t)]
 \end{aligned}$$

We assume that the children of the immigrants are born as natives. Accordingly, $NPV(n, h, t)$ is the net present value of the new born child of the immigrant at time t and $\pi(j, s)$ probability that the immigrant will have a high skilled child and $\varphi(j, s)$ is the number of children the immigrant will have.

In Figure 3.11 I calculate the direct effect of individuals (NPV1): net present discounted value of future tax payments, transfers, social security benefits and payments. Results show that immigrants have a higher NPV1 since they do not receive the transfer payments that have been paid to natives at age 1. In addition, high skilled workers have a higher NPV1 because they earn more and pay higher taxes. Comparing technology effect, when technology choice is allowed NPV1 of low skilled labor is lower because of lower wages and it is higher for the high skilled labor as a result of higher wages.

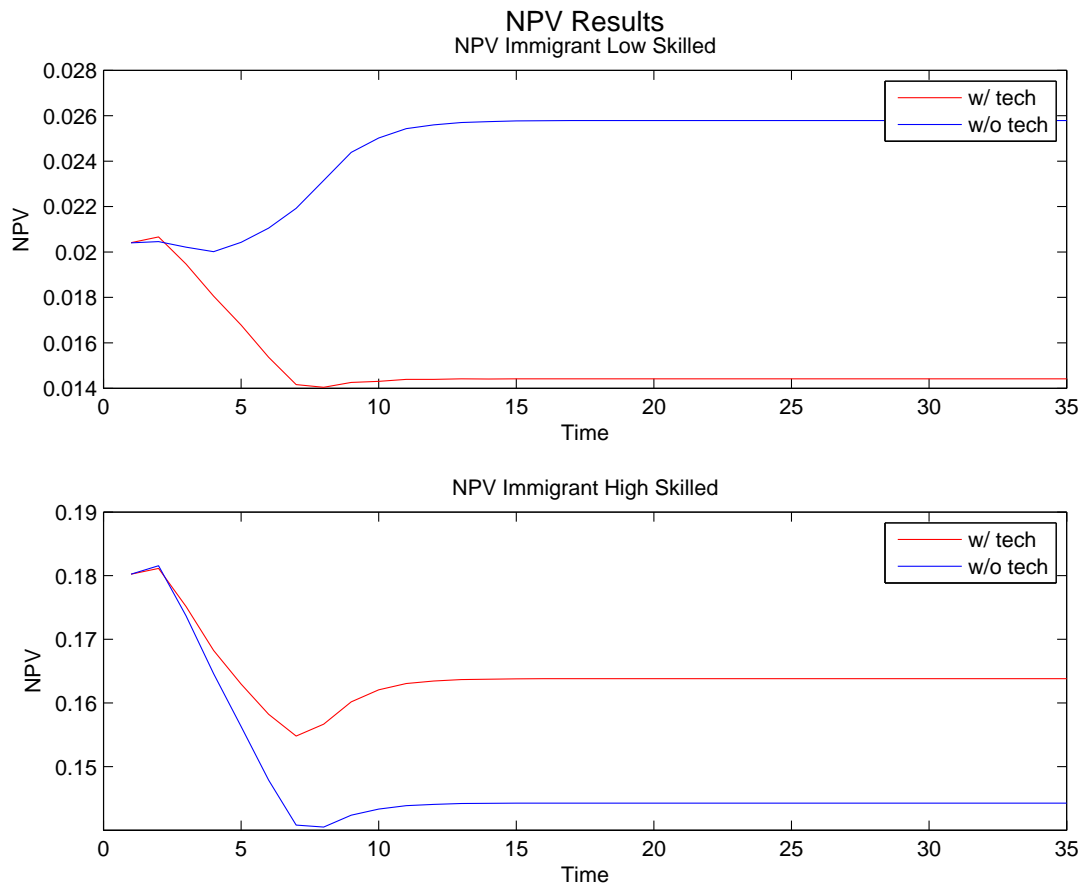


Figure 3.11: Net Present Value Analysis-Direct Effect

If we consider the long run effect of one additional labor through their children, in Figure 3.12 we see that in case of technology choice, its effect is higher for the high

skilled immigration while it is lower for the low skilled labor due to change in wages. Specifically, the paper finds that, immigration overestimate NPV of one additional low skilled immigrant approximately by 35% and underestimate the value of one additional high skilled immigrant by 15%.

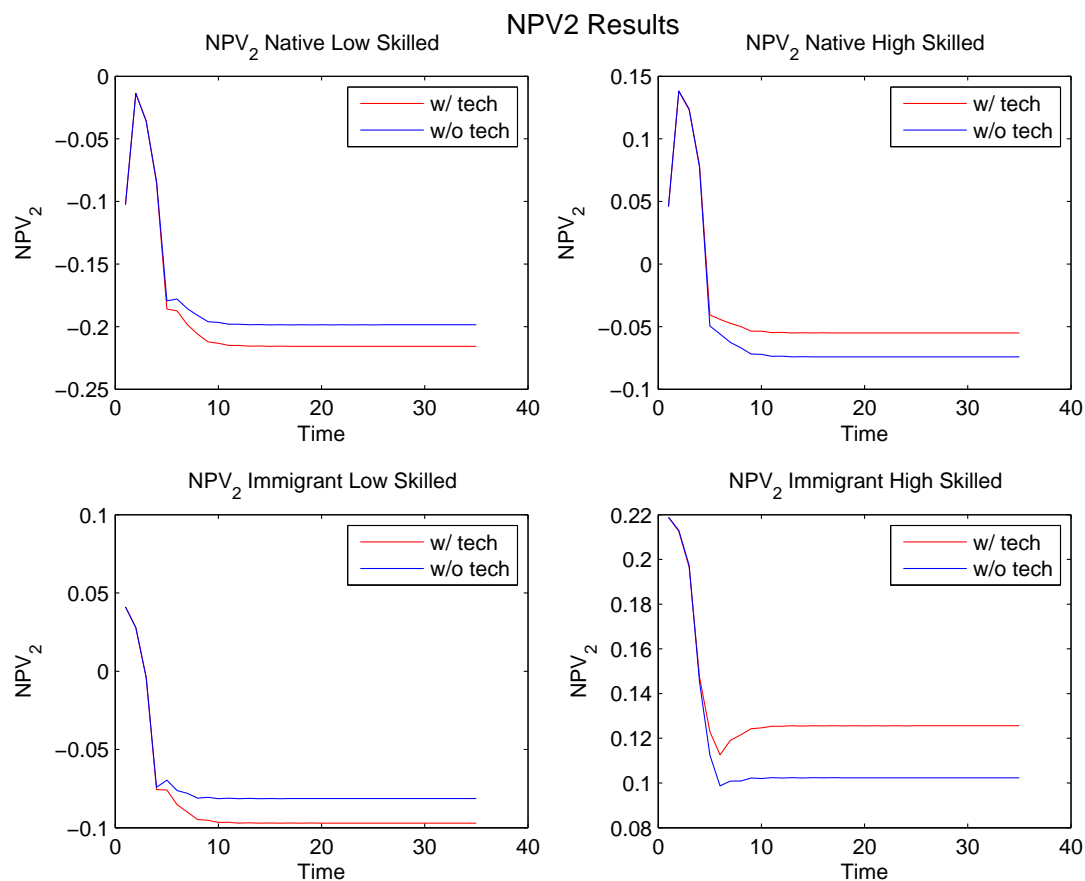


Figure 3.12: Net Present Value Analysis- Long Run Effect

3.5.2 Experiment II: Increase in Low Skilled Labor

In this experiment, low skilled immigration is raised to 0.08 of total population from 0.05 of the population. Below is shown the new stationary distribution of the population. In the new steady state, the ratio of high skilled immigrants is higher for all generations. In addition, there is an indirect positive effect of low skilled immigration on the population size of the high skilled natives. The underlying reason is since low skilled immigrants are more fertile and their probability of having low skilled child is higher than the natives, low skilled native generation goes up. Besides, increase in low skilled immigration reduces the ratio of high skilled workers in the population

	Native		Immigrant	
Generation	Unskilled	Skilled	Unskilled	Skilled
1	0.2678	0.1448	0	0
2	0.1439	0.0778	0.080	0.0100
3	0.0765	0.0414	0.0425	0.0053
4	0.0380	0.0206	0.0211	0.0026
5	0.0127	0.0069	0.0071	0.0009

Table 3.8: Final Stationary Distribution of Population

3.5.2.1 Effect of Low Skilled Immigration on Skill Shares

When firms are allowed to choose their optimal technology on the technology frontier after an increase in the low skilled immigration, firms will choose to increase the productivity of the production factor that has become more abundant which is in this case low skilled labor. As shown in Figure 3.13, firms will increase the productivity of their low skilled labor while decreasing the productivity of their high skilled labor. On the other hand, when firms are not allowed to choose their technology, their productivity of different type of labor will be constant at the before policy change levels.

Share of High and Low Skilled Labor with Technical Change

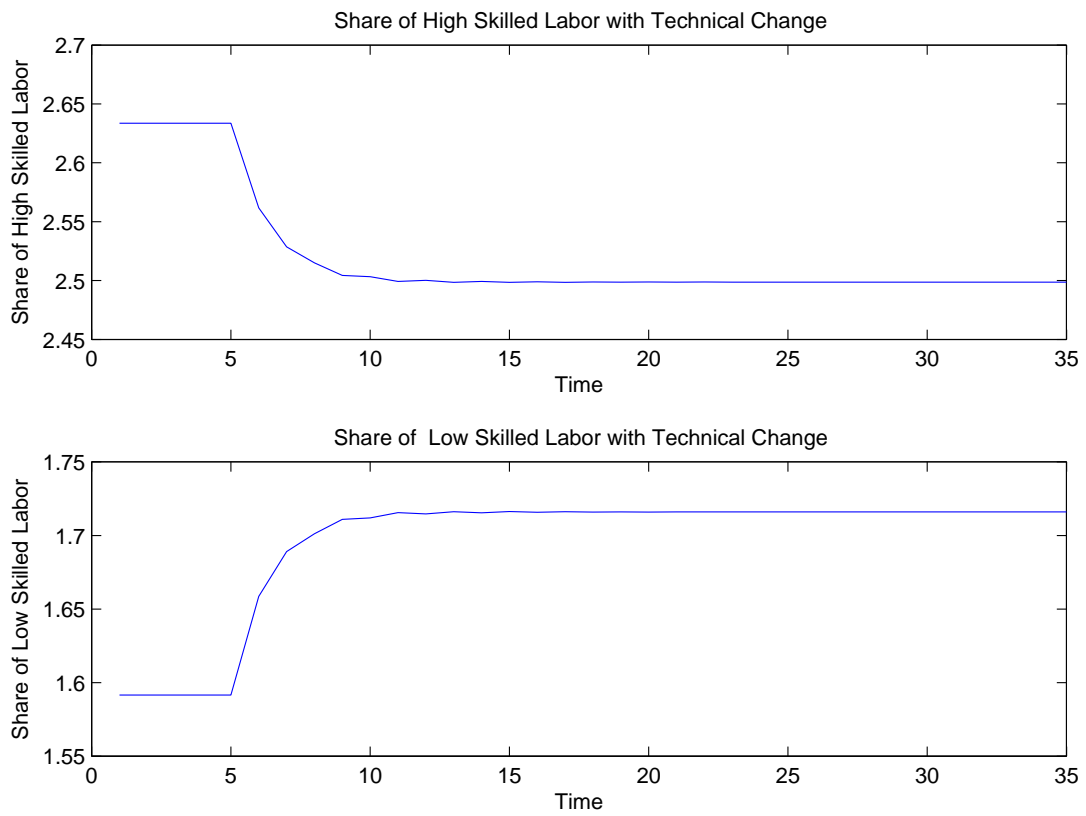


Figure 3.13: Share of High and Low Skilled Labor with Technical Change

3.5.2.2 Effect of High Skilled Immigration on Wages

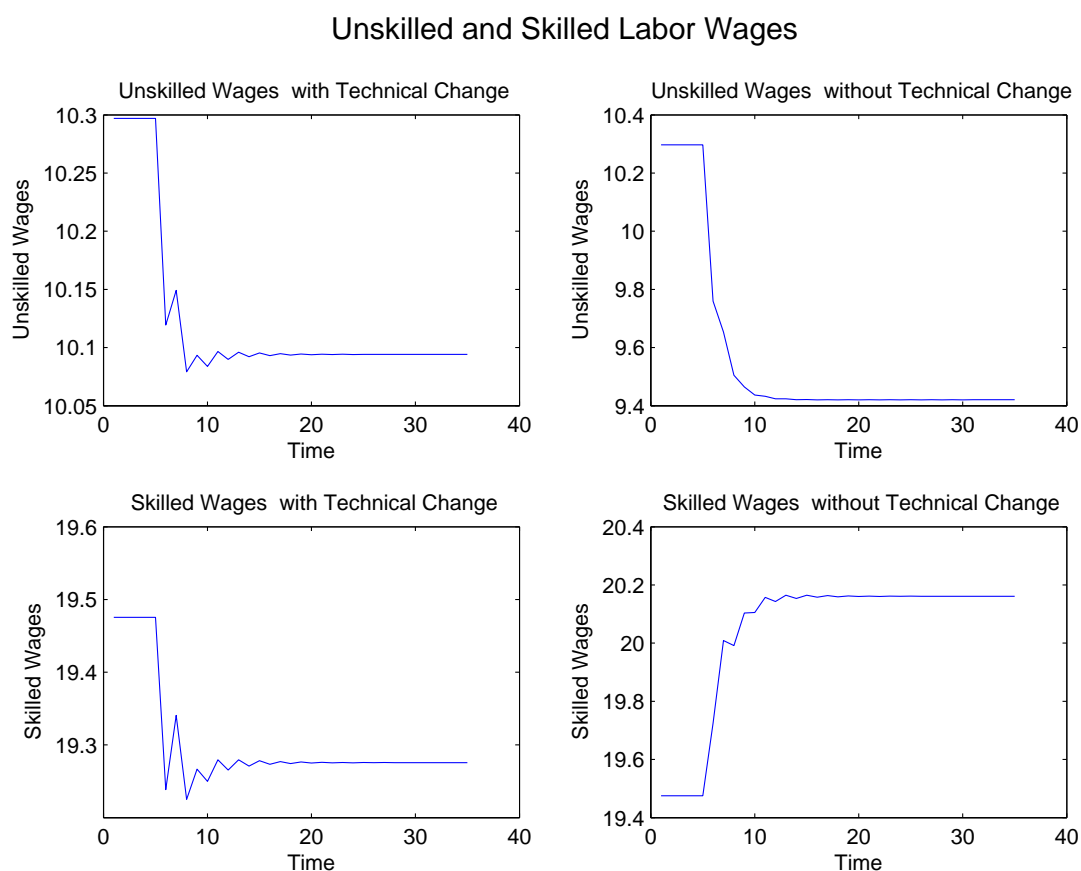


Figure 3.14: Unskilled and Skilled Labor Wages

Figure 3.14 shows the wage movement during transition to new steady state. Right panel shows the case when the firms have fixed production technology while left panel shows the wages when the firms are allowed to choose their optimal technology.

When firms are not allowed to change their production technology, increase in low skill immigration leads to increase in the low skilled wages and decrease in the high skilled wages. The underlying reason is, because there is more skilled labor in the economy, increase in supply will lower the wages for the unskilled labor. On the other hand, because relative supply of the high skilled labor has declined because of the change in the population distribution, wages of skilled labor goes up.

On the other hand, when firms are allowed to change their technology, opposite wage movements of different type of labor is mitigated. As shown in the lower left panel of Figure 3.14, when low skill immigration increases, the wages initially fall. However, the

magnitude of the fall in the low skilled wages are not as high as the case of no technology change. The underlying reason is, because firms increase the productivity of their high skilled labor, then their demand for the low skilled labor shifts up leading to a smaller decrease in the low skilled labor wages. In addition, because firms continue to increase the productivity of their low skilled labor, wages start to increase to a new steady state that is lower than the initial one. Considering the high skilled labor, since firms are able to change their technology, productivity of the high skilled labor decreases as a result of low skilled immigration creating a decrease in high skilled wages. However, because of the population size effects of immigration, high skilled labor becomes less abundant creating a rebound in high skilled wages. However, because of the change in the technology, high skilled wages do not increase as much as they do in case of no technical change and stay lower than the initial steady state .

3.5.2.3 Effect of Low Skilled Immigration on Labor Choices

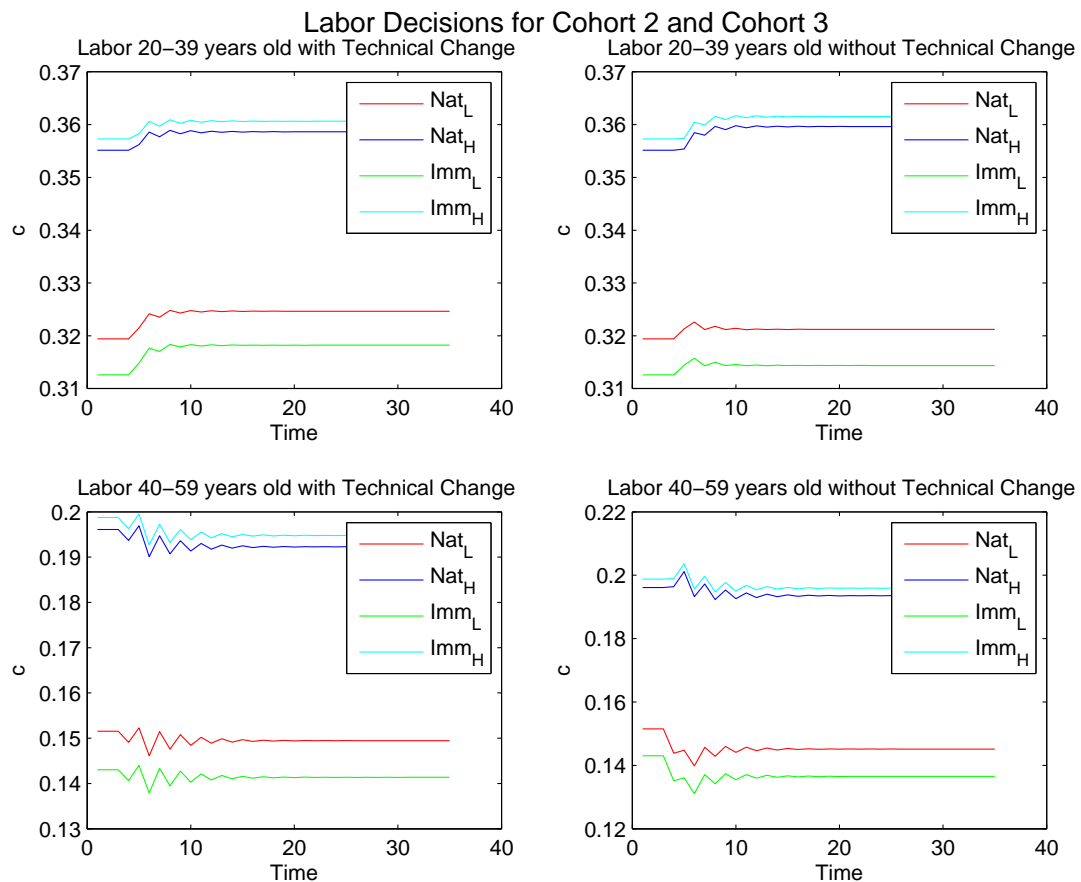


Figure 3.15: Labor Decisions

Figure 3.15 shows labor choices for different types of labor at different ages. If we compare the steady state labor choices under no technology choice (right panel) we see that low skilled workers reduce their labor as a result of decrease in the wages while high skilled workers increase their labor as a result of increase in wages. However given that the relative mass of high skilled workers decline, aggregate high skilled labor in effective units decline while the aggregate effective low skilled labor in effective units go up. Considering the case with technology choice, we see that for low skilled labor we see a slight increase in the labor supply due to relative increase in wages.

3.5.2.4 Effect of Low Skilled Immigration on Asset Holdings and Consumption

Since individuals at age 1 are required to consume all transfers that they get, all agents start the second period with zero assets. Accordingly, individuals decide on their asset for ages 3, 4 and 5. Asset holding decisions with and without technological change are shown in Figure 3.16 and Figure 3.17. Asset holding decisions of individuals depend on their wages. In case of no technology choice, low skilled workers reduce their asset holdings while high skilled workers save more. However, when firms are allowed to choose their technology, because the wage for the low skilled workers do not decline as much as they do in case of no technology choice, both types of workers increase their asset levels.

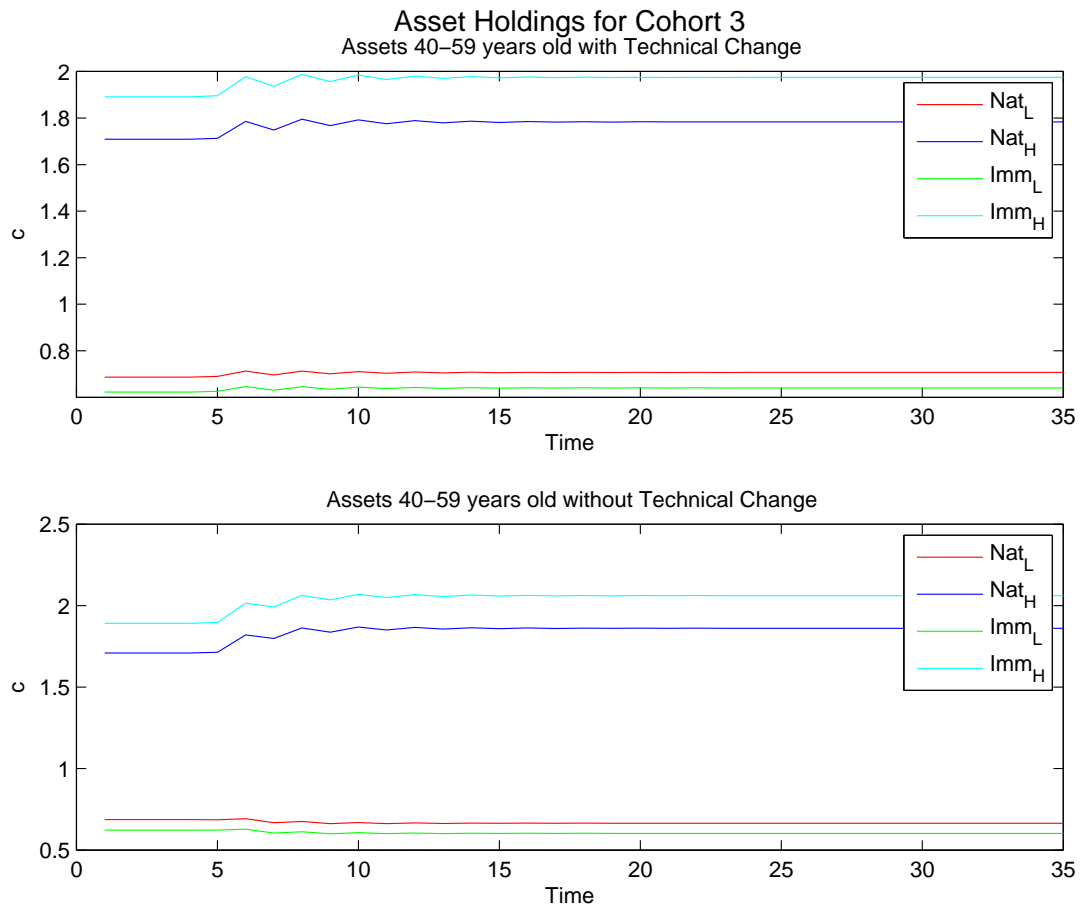


Figure 3.16: Asset Holdings for Cohort 2 and Cohort 3

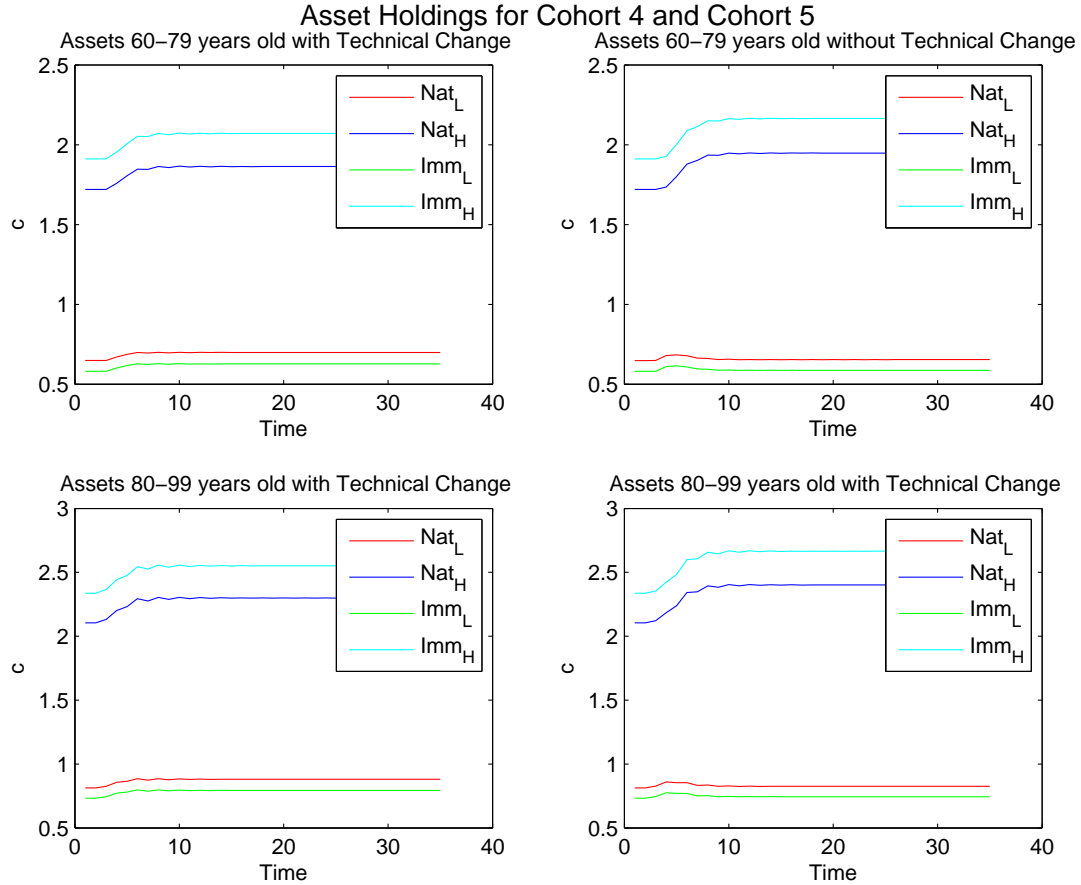


Figure 3.17: Asset Holdings for Cohort 4 and Cohort 5

3.5.2.5 Effect of Low Skilled Immigration on Consumption Decisions

Consumption decisions with and without technological change are shown in Figure 3.18 and Figure 3.19. Similar to asset holding decisions of individuals, consumption choices of individuals depend on their wages. In case of no technology choice, low skilled workers reduce their consumption while high skilled workers consume more. However, when firms are allowed to choose their technology, because the wage for low skilled workers do not decline as much as they do in case of no technology choice, both types of workers increase their consumption levels.

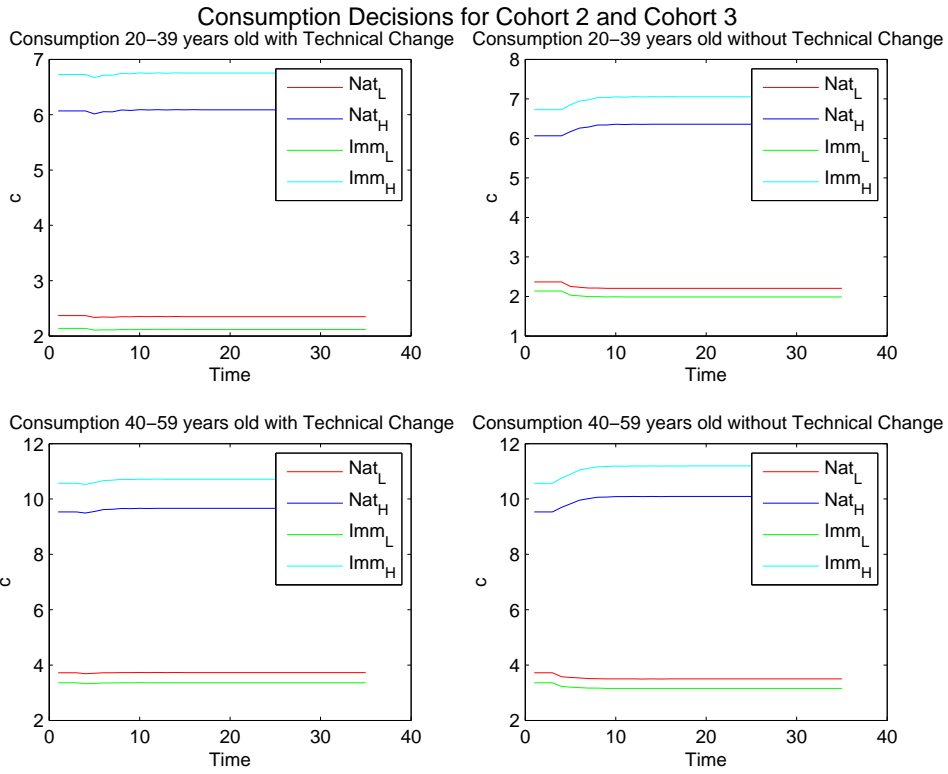


Figure 3.18: Consumption Decisions for Cohort 2 and Cohort 3

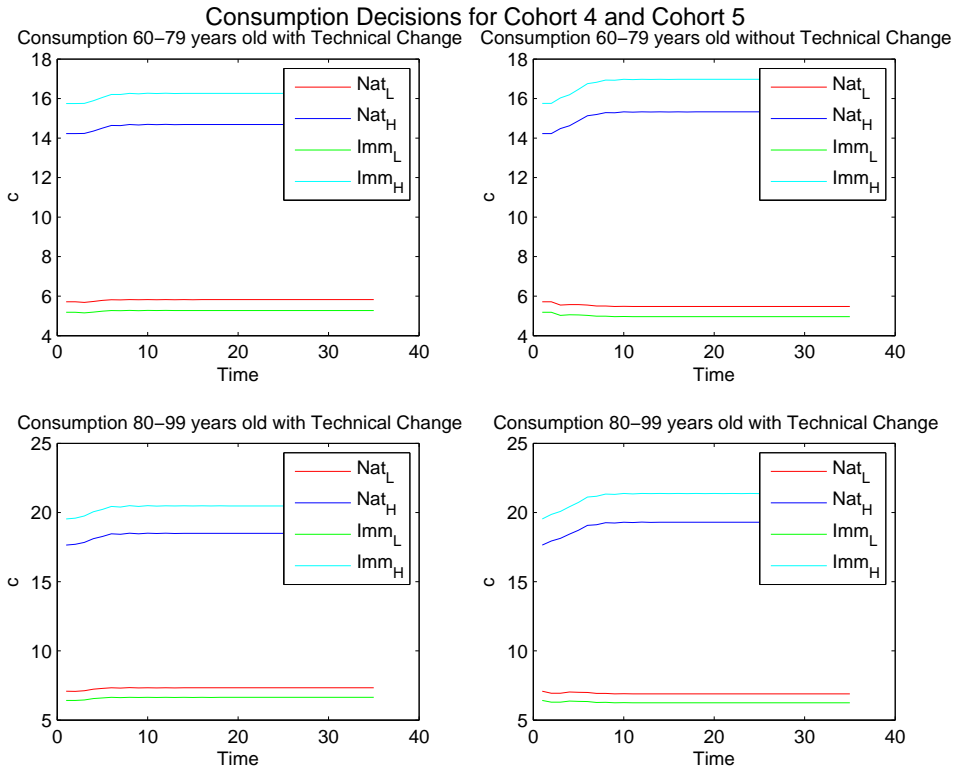


Figure 3.19: Consumption Decisions for Cohort 4 and Cohort 5

3.5.2.6 Effect of Low Skilled Immigration on Economy Aggregates

Given individual asset and labor decisions of individuals and the evolution of the distribution of the aggregate population, aggregate labor and capital values are shown in Figure 3.20. When firms are not allowed to choose their technology, high skilled workers increase their labor. However, since the relative mass of the high skilled workers go down, then population effect dominates the individual labor choice effect and aggregate high skilled labor goes down. When firms are allowed to choose their technology, because of the wage decline, high skilled workers reduce their labor supply which pushes down the aggregate labor supply even further.

On the other hand, when technology is allowed to change, low skilled wages do not decline as much as it does in case of no technology choice case. Accordingly, low skilled workers do not reduce their labor as much as they do in case of no technology case. In addition, because of the increase in the low skilled worker population, aggregate low skilled labor goes up more.

Initial effect of immigration on the aggregate capital is negative. Because immigrants are assumed to enter the workforce without any initial capital, capital per effective labor declines. Then because of increase in wages, asset holdings increase creating increase in the total capital. However, when technology choice is allowed, low skilled workers start to save more than the high skilled workers and together with increase in the low skilled population, aggregate capital is higher than the no technology choice case.

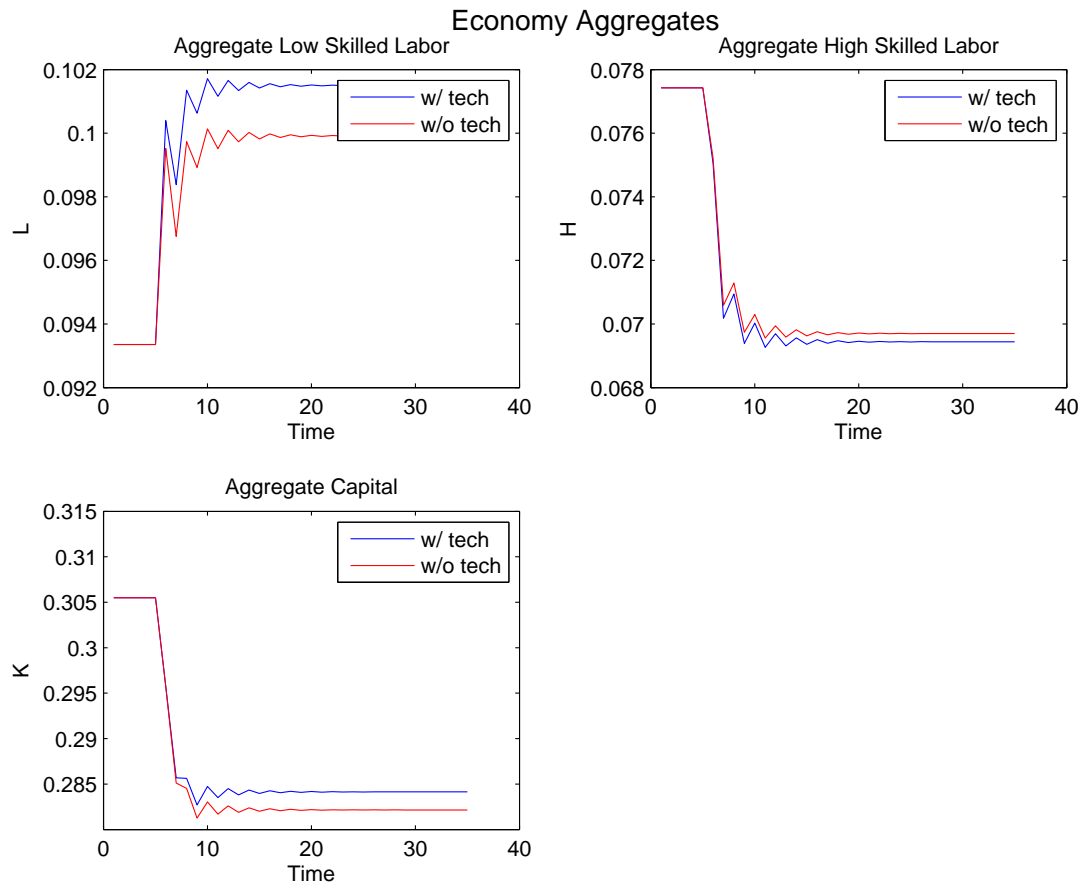


Figure 3.20: Economy Aggregates

3.5.2.7 Effect of Low Skilled Immigration on Pension Payment Share and Interest Rates

Figure 3.21 shows effects of high skilled immigration on the replacement rates and interest rates. Results show that technology choice do not have significant effect on the results. Interest rate is lower in case of endogenous technology case because of higher capital supply. In addition, pension payment share is the same in both cases because even though wage allocation changes, total labor income is constant in the long run leading to same pension payment share in case of constant replacement rate. In addition, because the relative population of old people declined, pension payment share goes down.

Pension Payment Share and Capital Rent

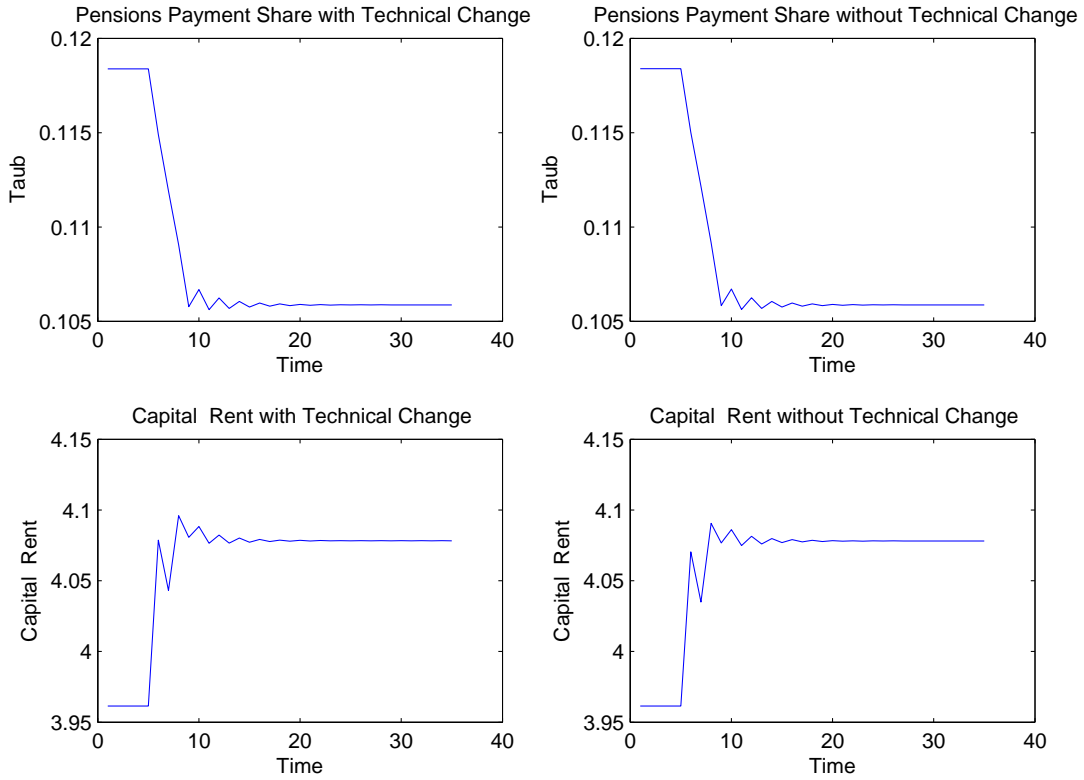


Figure 3.21: Pension Payment Share and Capital Rent

3.5.2.8 Effect of Low Skilled Immigration on Welfare and Government Budget

Figure 3.22 summarizes the welfare effect of high skilled immigration with and without technology change allowed. Firstly, in case of no technology choice, high skilled workers increase their consumption while increasing their labor which have opposite effects on utility. However, positive effect of consumption is dominated by the negative effect of increase in the working hours and lifetime utility decreases. Accordingly, the equivalent variation goes down meaning that low skilled immigration decreases the welfare of the high skilled labor when technology choice is not taken in to consideration. On the other hand, when technology choice is allowed, high skilled workers increase their consumption less than the previous case and reduce working hours. In that case, low skill immigration effect on the high skilled worker is still negative and more significant. To be more specific EV results show that no technology choice case underestimates the welfare effect of low

skilled immigration on high skilled natives by 25%.

Considering the immigration effects on the low skilled labor, the negative effect of low skilled immigration is still present for both cases. However, the reason is different than the case of high skilled workers. Specifically, in case of no technology choice low skilled workers reduce their consumption and working hours and the aggregate effect is negative. When firms are allowed to change their technology, low skilled workers will increase their consumption and reduce their labor. The aggregate effect is greater than the case with no technology case but still negative. Specifically, EV results shown in Figure 3.22 show that the standard model overestimates the negative welfare effect of immigration to unskilled natives by 50%.

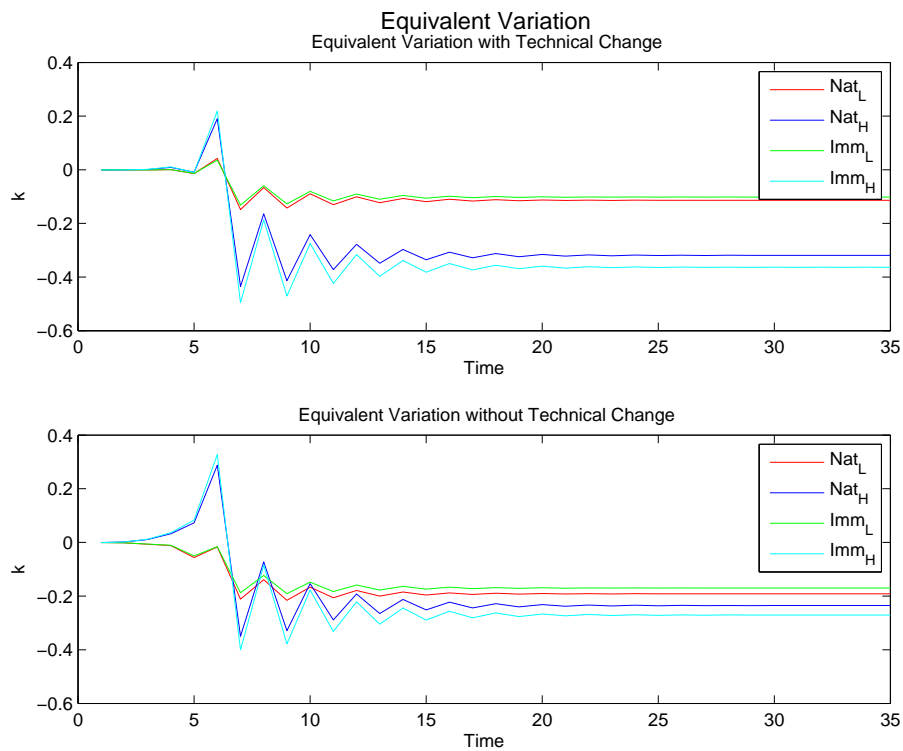


Figure 3.22: Equivalent Variation

In Figure 3.23 I calculate the direct effect of individuals (NPV1): net present discounted value of future tax payments, transfers, social security benefits and payments. Results show that immigrants have a higher NPV1 since they do not receive the transfer payments that have been paid to natives at age 1. In addition, high skilled workers have a higher NPV1 because they earn more and pay higher taxes. Comparing technology effect, when technology choice is allowed NPV1 of low skilled labor is higher because of

higher wages and it is lower for the high skilled labor as a result of higher wages.

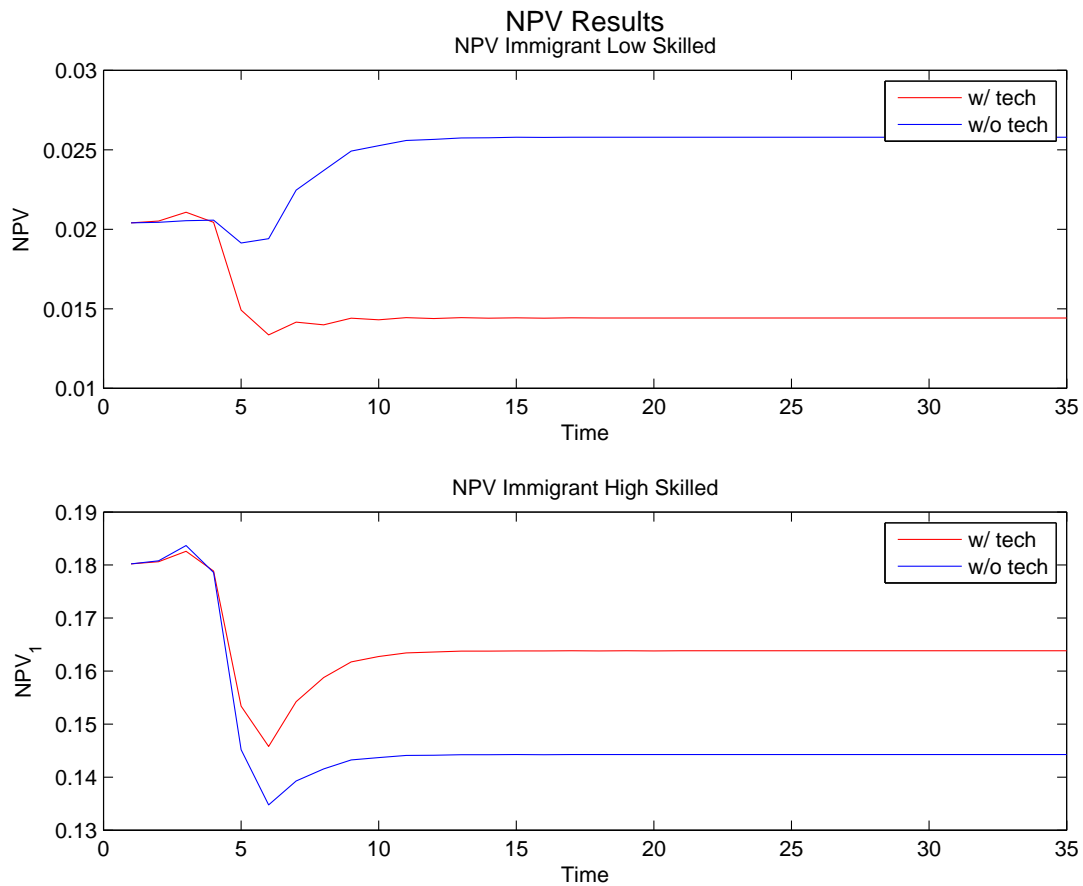


Figure 3.23: Net Present Value Analysis-Direct Effect

If we consider the long run effect of one additional labor through their children, in Figure 3.24 we see that in case of technology choice, its effect is lower for the high skilled immigration while it is higher for the low skilled labor due to change in wages. Specifically, the paper finds that, standard model underestimate NPV of one additional low skilled immigrant approximately by 1% and overestimate the value of one additional high skilled immigrant by 15%.

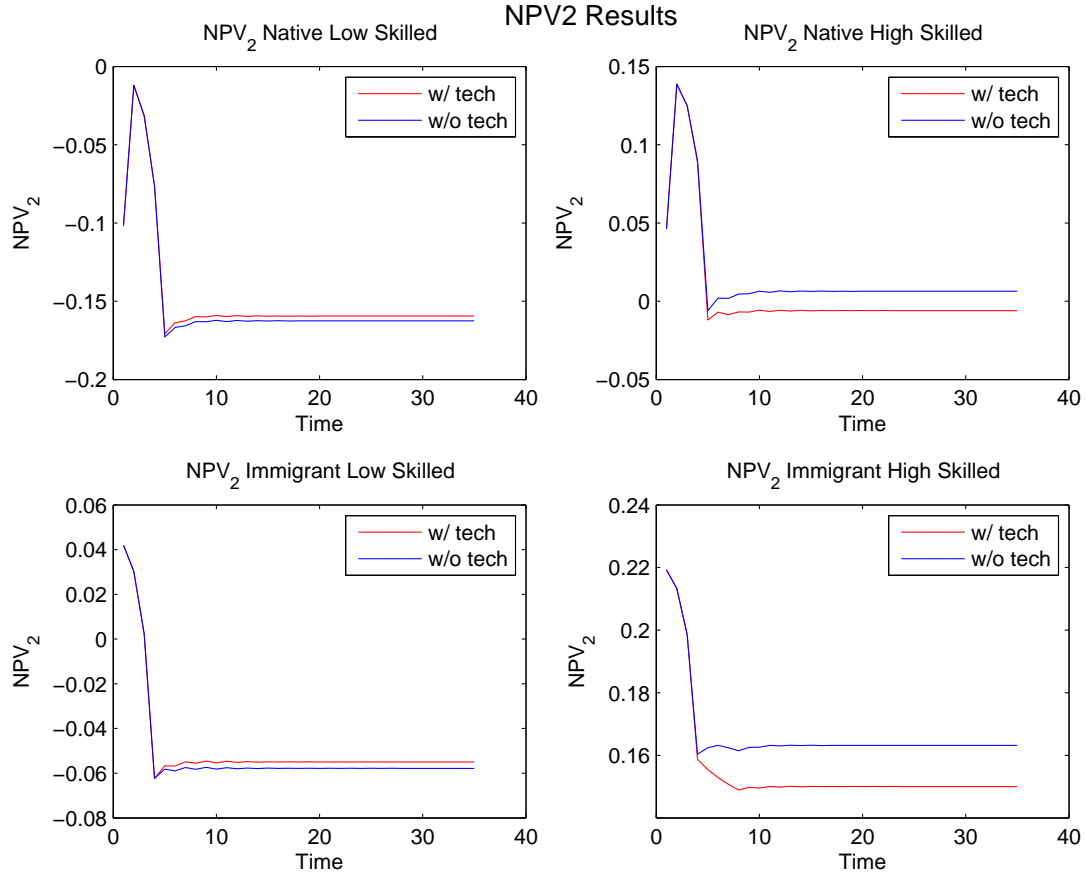


Figure 3.24: Net Present Value Analysis- Long Run Effect

3.6 Conclusion

This paper has three contributions to the existing immigration literature. First it points out a discrepancy between the empirical findings and the predictions of the theoretical models. In order to make the model closer to the data, this paper proposes a mechanism (endogenous technology change) and empirically check the validity of the proposal. Following this, the paper modifies the standard model by embedding the endogenous choice of technology and compares the results with and without allowing for the technical change. The results show that the standard model underestimates the effect of high skilled immigration to skilled natives by 95% while it over estimates the effect on unskilled natives by 31%. Comparing the fiscal effects of immigration in terms of burden of an immigrant through net present discount value of future tax payments, transfers, social security benefits and payments the paper finds that, immigration overestimate NPV of

one additional low skilled immigrant approximately by 35% and underestimate the value of one additional high skilled immigrant by 15%. In addition, capital accumulation is 2% lower and low skilled labor supply is 1% higher if the technical change is not taken into account. Considering the unskilled immigration, the model shows that low skilled immigration has negative effects on the natives regardless of the skill type. Besides, results show that negative effect is overpredicted for the low skilled natives while it is underpredicted for the high skilled natives. Based on these results, analysis of a standard model might be incomplete in welfare analysis. For further research, more counterfactuals are needed in order to analyze the sensitivity of results for different parameters. Besides, as in Storesletten (2000), fiscal sustainability through immigration should be re-evaluated in order to get more accurate results. In addition, we may also want to analyze the effect of other potential channels like skill biased technical change.

3.7 Appendix

Solution of the Individual's Problem

$$\max E \left\{ \frac{(c_t^\gamma(1, j, s)(1 - l_t(1, j, s))^{(1-\gamma)})^{1-\eta}}{1 - \eta} + \sum_{i=2}^5 \left(\prod_{k=1}^{i-1} \lambda_k \right) \beta^{i-1} \frac{(c_t^\gamma(i, j, s)(1 - l_t(i, j, s))^{(1-\gamma)})^{1-\eta}}{1 - \eta} \right\} \quad (3.15)$$

subject to the following constraints:

$$c_t(1, j, s) = \chi_t(1, j, s), \quad l_t(1, j, s) = 0$$

for every $i \geq 2$

$$b_t(i, j, s) + (1 - \tau_w - \tau_b)w_t(s)e(i, j, l)l_t(i, j, s) + (1 + (1 - \tau_r)r_t)a_t(i, j, s) + \chi_t(i, j, s) = c_t(i, j, s) + a_{t+1}(i, j, s)$$

where if $2 \leq i \leq 3$, $b_t(i, j, s) = 0$

if $i > 3$, $l_t(i, j, s) = 0$

Solution:

$$1. \quad c_t(1, j, s) = \Psi_t(1, j, s), \quad l_t(1, j, s) = 0$$

$$2. \quad \text{for } 2 \leq i \leq 3$$

$$\beta^t \gamma (1 - \eta) c_t^{(1-\eta)\gamma-1} (1 - l_t)^{(1-\gamma)(1-\eta)} = \Omega_t \quad (3.16)$$

$$\beta^t (1 - \gamma) (1 - \eta) c_t^{(1-\eta)\gamma} (1 - l_t)^{(1-\gamma)(1-\eta)-1} = \lambda_t (1 - \tau_w - \tau_b) w_t(s) e(i, j, s) \quad (3.17)$$

$$(1 + (1 - \tau_r)r_{t+1})\Omega_{t+1} = \Omega_t \quad (3.18)$$

$$3. \quad \text{for } i \geq 4$$

$$l_t(i, j, s) = 0 \quad (3.19)$$

$$\beta^t \gamma (1 - \eta) c_t^{(1-\eta)\gamma-1} = \Omega_t \quad (3.20)$$

- if $t=4$

$$(1 + (1 - \tau_r)r_{t+1})\Omega_{t+1} = \Omega_t \quad (3.21)$$

- if $t=5$

$$b_t(i, j, s) + (1 + (1 - \tau_r)r_t)a_t(i, j, s) + \chi_t(i, j, s) = c_t(i, j, s) \quad (3.22)$$

Accordingly, the solution will be:

1. $c_t(1, j, s) = \chi_t(1, j, s)$, $l_t(1, j, s) = 0$

2. for $2 \leq i \leq 3$

$$\frac{(1 - \gamma)}{\gamma} \frac{c_t}{(1 - l_t)} = (1 - \tau_w - \tau_b)w_t(s)e(i, j, s) \quad (3.23)$$

$$\lambda_t \beta (1 + (1 - \tau_r)r_{t+1})c_{t+1}^{(1-\eta)\gamma-1} (1 - l_{t+1})^{(1-\gamma)(1-\eta)} = c_t^{(1-\eta)\gamma-1} (1 - l_t)^{(1-\gamma)(1-\eta)} \quad (3.24)$$

3. for $i \geq 4$

$$l_t(i, j, s) = 0 \quad (3.25)$$

$$\beta^t \gamma (1 - \eta) c_t^{(1-\eta)\gamma-1} = \Omega_t \quad (3.26)$$

- if $t=4$

$$\lambda_t (1 + (1 - \tau_r)r_{t+1}) \beta c_{t+1}^{(1-\eta)\gamma-1} = c_t^{(1-\eta)\gamma-1} \quad (3.27)$$

- if $t=5$

$$b_t(i, j, s) + (1 + (1 - \tau_r)r_t)a_t(i, j, s) + \chi_t(i, j, s) = c_t(i, j, s) \quad (3.28)$$

Solution of the Firm's Problem

$$\max_{\Phi_{1,t}, \Phi_{2,t}, K_t, L_t, H_t} \{ K_t^\alpha [\Phi_{1,t} (A_t H_t)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t} (A_t L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} (1-\alpha) - (r_t + \delta) K_t - w_t(l) L_t - w_t(h) H_t \} \quad (3.29)$$

subject to

$$\Phi_{1,t}^\omega + \kappa \Phi_{2,t}^\omega \leq B \quad (3.30)$$

1. FOC wrt K_t :

$$\alpha K_t^{\alpha-1} [\Phi_{1,t}(A_t H_t)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t}(A_t L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}(1-\alpha)} - \delta = r_t \quad (3.31)$$

2. FOC wrt L_t :

$$K_t^\alpha (1-\alpha) \Phi_{2,t} A_t^{\frac{\sigma-1}{\sigma}} L_t^{\frac{-1}{\sigma}} [\Phi_{1,t}(A_t H_t)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t}(A_t L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{1-\alpha\sigma}{\sigma-1}} = w_t(l) \quad (3.32)$$

3. FOC wrt H_t :

$$K_t^\alpha (1-\alpha) \Phi_{1,t} A_t^{\frac{\sigma-1}{\sigma}} H_t^{\frac{-1}{\sigma}} [\Phi_{1,t}(A_t H_t)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t}(A_t L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{1-\alpha\sigma}{\sigma-1}} = w_t(h) \quad (3.33)$$

4. FOC wrt $\Phi_{1,t}$:

$$K_t^\alpha (1-\alpha) \frac{\sigma}{\sigma-1} (A_t H_t)^{\frac{\sigma-1}{\sigma}} [\Phi_{1,t}(A_t H_t)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t}(A_t L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{1-\alpha\sigma}{\sigma-1}} = \Omega_t \omega \Phi_{1,t}^{\omega-1} \quad (3.34)$$

5. FOC wrt $\Phi_{2,t}$:

$$K_t^\alpha (1-\alpha) \frac{\sigma}{\sigma-1} (A_t L_t)^{\frac{\sigma-1}{\sigma}} [\Phi_{1,t}(A_t H_t)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t}(A_t L_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{1-\alpha\sigma}{\sigma-1}} = \Omega_t \kappa \omega \Phi_{2,t}^{\omega-1} \quad (3.35)$$

Second and third expressions give the following relationship:

$$\frac{\phi_1}{\phi_2} = \left(\frac{H_t}{L_t}\right)^{\frac{1}{\sigma}} \frac{w_t(h)}{w_t(l)} \quad (3.36)$$

Last two expressions give the following relationship:

$$\frac{1}{\kappa} \left(\frac{\phi_1}{\phi_2}\right)^{\omega-1} = \left(\frac{H_t}{L_t}\right)^{\frac{\sigma-1}{\sigma}} \quad (3.37)$$

$$\frac{\phi_1}{\phi_2} = \left(\kappa \left(\frac{H_t}{L_t}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\omega-1}} \quad (3.38)$$

$$\phi_1 = \phi_2 \left(\kappa \left(\frac{H_t}{L_t}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\omega-1}} \quad (3.39)$$

If we put this expression in to the technology constraint:

$$(\phi_2(\kappa(\frac{H_t}{L_t})^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\omega-1}})^{\omega} + \kappa\phi_2^{\omega} = B \quad (3.40)$$

$$\phi_2^{\omega}[(\kappa(\frac{H_t}{L_t})^{\frac{\sigma-1}{\sigma}})^{\frac{\omega}{\omega-1}} + \kappa] = B \quad (3.41)$$

$$\phi_2 = \left(\frac{B}{[(\kappa(\frac{H_t}{L_t})^{\frac{\sigma-1}{\sigma}})^{\frac{\omega}{\omega-1}} + \kappa]} \right)^{\frac{1}{\omega}} \quad (3.42)$$

$$\phi_1 = (B - \kappa\phi_2^{\omega})^{\frac{1}{\omega}} \quad (3.43)$$

Stationary Version of the Solutions

Since exogenous growth rate is g and because of immigration population rate is η , in order to solve the model we will detrend the model both in the aggregate and the individual state. Aggregates grow at the rate of $(1+g)(1+\eta) - 1$ and the individual choice variables a_t and c_t grow at the rate of $(1+g)$. In addition, tr_t and b_t variables also grow with the rate of $(1+g)$. Therefore, we will divide aggregate variables K_t, Y_t with $(A_t(H_t + L_t))$ and tr_t, b_t, a_t, c_t by A_t . In addition, while r_t is constant in the stationary equilibrium, w_t is increasing with rate $(1+g)$.

Stationary aggregate variables are defined as:

$$\begin{aligned} \tilde{k}_t &\equiv \frac{K_t}{A_t(H_t+L_t)}, \quad \tilde{T}_t \equiv \frac{T_t}{A_t(H_t+L_t)}, \quad \tilde{G}_t \equiv \frac{G_t}{A_t(H_t+L_t)} \\ \tilde{C}_t &\equiv \frac{C_t}{A_t(H_t+L_t)}, \quad \tilde{Y}_t \equiv \frac{Y_t}{A_t(H_t+L_t)}, \quad \tilde{Beqt} \equiv \frac{Beqt}{A_t(H_t+L_t)} \end{aligned}$$

Stationary individual variables:

$$\tilde{c}_t \equiv \frac{c_t}{A_t}, \quad \tilde{a}_t \equiv \frac{a_t}{A_t}, \quad \tilde{b}_t \equiv \frac{b_t}{A_t}, \quad \tilde{w}_t \equiv \frac{w_t}{A_t}, \quad \tilde{\chi}_t \equiv \frac{\chi_t}{A_t}$$

Stationary Version of Individual's Problem

Accordingly, the solution will be:

$$1. \quad \tilde{c}_t(1, j, s) = \tilde{\chi}_t(1, j, s), \quad l_t(1, j, s) = 0$$

2. for $2 \leq i \leq 3$

$$\frac{(1-\gamma)}{\gamma} \frac{\tilde{c}_t}{(1-l_t)} = (1-\tau_w - \tau_b)\tilde{w}_t(s)e(i, j, s) \quad (3.44)$$

$$\lambda_t \beta (1 + (1-\tau_r)r_{t+1})c_{t+1}^{\tilde{c}_t^{(1-\eta)\gamma-1}(1+g)^{(1-\eta)\gamma-1}(1-l_{t+1})^{(1-\gamma)(1-\eta)}} = \tilde{c}_t^{(1-\eta)\gamma-1}(1-l_t)^{(1-\gamma)(1-\eta)} \quad (3.45)$$

$$\lambda_t \beta (1 + (1-\tau_r)r_{t+1})c_{t+1}^{\tilde{c}_t^{(1-\eta)\gamma-1}(1+g)^{(1-\eta)\gamma-1}(1-l_{t+1})^{(1-\gamma)(1-\eta)}} \quad (3.46)$$

$$= \tilde{c}_t^{(1-\eta)\gamma-1} \left[\frac{\tilde{c}_t}{(1-\tau_w - \tau_b)\tilde{w}_t(s)e(i, j, s)} \right]^{(1-\gamma)(1-\eta)} \quad (3.47)$$

$$\lambda_t \beta (1 + (1-\tau_r)r_{t+1})c_{t+1}^{\tilde{c}_t^{(1-\eta)\gamma-1}(1+g)^{(1-\eta)\gamma-1}(1-l_{t+1})^{(1-\gamma)(1-\eta)}} \quad (3.48)$$

$$= \tilde{c}_t^{(-\eta)} \left[\frac{1}{(1-\tau_w - \tau_b)\tilde{w}_t(s)e(i, j, s)} \right]^{(1-\gamma)(1-\eta)} \quad (3.49)$$

3. for $i \geq 4$

$$l_t(i, j, s) = 0 \quad (3.50)$$

- if $t=4$

$$\lambda_t (1 + (1-\tau_r)r_{t+1})\beta c_{t+1}^{\tilde{c}_t^{(1-\eta)\gamma-1}(1+g)^{(1-\eta)\gamma-1}} = \tilde{c}_t^{(1-\eta)\gamma-1} \quad (3.51)$$

- if $t=5$

$$\tilde{b}_t(i, j, s) + (1 + (1-\tau_r)r_t)\tilde{a}_t(i, j, s) + \tilde{\chi}_t(i, j, s) = \tilde{c}_t(i, j, s) \quad (3.52)$$

Stationary Version of Firm's Problem

FOC wrt K_t :

$$\alpha(\tilde{k}_t)^{\alpha-1} \left[\Phi_{1,t} \left(\frac{H_t}{H_t + L_t} \right)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t} \left(\frac{L_t}{H_t + L_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{(\frac{\sigma}{\sigma-1})(1-\alpha)} - \delta = r_t \quad (3.53)$$

$$(\tilde{k}_t)^\alpha (1 - \alpha) \Phi_{2,t} \left(\frac{L_t}{H_t + L_t} \right)^{\frac{-1}{\sigma}} \left[\Phi_{1,t} \left(\frac{H_t}{H_t + L_t} \right)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t} \left(\frac{L_t}{H_t + L_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\alpha\sigma}{\sigma-1}} \quad (3.54)$$

$$= \tilde{w}_t(l) \quad (3.55)$$

$$(\tilde{k}_t)^\alpha (1 - \alpha) \Phi_{1,t} \left(\frac{H_t}{H_t + L_t} \right)^{\frac{-1}{\sigma}} \left[\Phi_{1,t} \left(\frac{H_t}{H_t + L_t} \right)^{\frac{\sigma-1}{\sigma}} + \Phi_{2,t} \left(\frac{L_t}{H_t + L_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1-\alpha\sigma}{\sigma-1}} \quad (3.56)$$

$$= \tilde{w}_t(h) \quad (3.57)$$

Stationary Version of Government's Problem

$$\tilde{T}_t = \tau_w \tilde{w}_{l,t} \frac{L_t}{A_t H_t + A_t L_t} + \tau_w \tilde{w}_{h,t} \frac{H_t}{A_t H_t + A_t L_t} + \frac{\tau_r r_t K_t}{A_t H_t + A_t L_t} \quad (3.58)$$

$$\tilde{T}r_t = \sum_{i,j,s} \tilde{\chi}(i, j, s) \tilde{\mu}_t(i, j, s) \quad (3.59)$$

$$\tilde{T}_t + \tilde{B}eq_t = \tilde{G}_t + \tilde{T}r_t \quad (3.60)$$

$$\tilde{G}_t = \bar{y} \tilde{Y}_t \quad (3.61)$$

Stationary Version of Balanced Social Security

$$\sum_{i \in 4,5,j,s} \zeta (1 - \tau_w - \tau_b) \tilde{w}_t(s) e(i, j, s) \tilde{\mu}_t(i, j, s) = \sum_{i \in 2,3,j,s} \tau_b \tilde{w}_t(s) e(i, j, l) l_t(i, j, s) \tilde{\mu}_t(i, j, s) \quad (3.62)$$

Pension Funds as Functions of AIME

The social security system is pay-as-you-go. All social security contributions are collected by the social security authority and redistributed to the retirees. In the United States social security benefits are related to a retiree's averaged indexed monthly earnings (AIME). We define AIME as follows:

$$AIME(j, s) = \frac{1}{2} \left\{ \frac{\bar{w}_t}{\bar{w}_{t-1}} w_{t-1}(s) e(3, j, s) l_{(t-1)}(3, j, s) + \frac{\bar{w}_t}{\bar{w}_{t-2}} w_{t-2}(s) e(2, j, s) l_{(t-2)}(2, j, s) \right\} \quad (3.63)$$

Pensions of a retiree with type (j,s) are a fraction of the lifetime earnings (which is called replacement rate) indexed by the overall average economy-wide labor income in order to correct for the aggregate growth. In US, replacement rate is piece-wise linear function of AIME:

$$b_t(i, j, s) = \begin{cases} 0.9AIME(j, s) & AIME(j, s) \leq 0.2\bar{w}_t \\ 0.32AIME(j, s) & 0.2\bar{w}_t < AIME(j, s) \leq 1.24\bar{w}_t \\ 0.15AIME(j, s) & AIME(j, s) > 1.24\bar{w}_t \end{cases}$$

The formula can be translated in the following way: An individual can get 90% of the first part of his/her AIME below 20% of the economy-wide average wage earnings (\bar{w}_t), 32% of the next part of his/her AIME that is above 20% of \bar{w}_t and below 124% of \bar{w}_t and 15% of the next part of his/her AIME that is above 124% of \bar{w}_t .

Each period social security budget is balanced. In this system τ_b - the contribution rate of the workers- will adjust in order to keep the social security budget balanced:

$$P_t = \sum_{i \in 4,5,j,s} b_t(i, j, s)\mu_t(i, j, s) = \sum_{i \in 2,3,j,s} \tau_b w_t(s)e(i, j, s)l_t(i, j, s)\mu_t(i, j, s) \quad (3.64)$$

Individuals' Problem if Social Security is a function of Lifetime Earnings

$$\max E \left\{ \frac{(c_t^\gamma(1, j, s)(1 - l_t(1, j, s))^{(1-\gamma)})^{1-\eta}}{1 - \eta} + \sum_{i=2}^5 \left(\prod_{k=1}^{i-1} \lambda_k \right) \beta^{i-1} \frac{(c_t^\gamma(i, j, s)(1 - l_t(i, j, s))^{(1-\gamma)})^{1-\eta}}{1 - \eta} \right\} \quad (3.65)$$

subject to the following constraints:

$$c_t(1, j, s) = \chi_t(1, j, s), l_t(1, j, s) = 0$$

for every $i \geq 2$

$$b_t(i, j, s) + (1 - \tau_w - \tau_b)w_t(s)e(i, j, l)l_t(i, j, s) + (1 + (1 - \tau_r)r_t)a_t(i, j, s) + \chi_t(i, j, s) = c_t(i, j, s) + a_{t+1}(i, j, s)$$

where if $2 \leq i \leq 3$, $b_t(i, j, s) = 0$

if $i > 3$, $l_t(i, j, s) = 0$

$$b_t(i, j, s) = \begin{cases} 0.9E(s) & E(s) \leq 0.2\bar{w}_t \\ 0.32E(s) & 0.2\bar{w}_t < E(s) \leq 1.24\bar{w}_t \\ 0.15E(s) & E(s) > 1.24\bar{w}_t \end{cases}$$

$$E(s) = \frac{1}{2} \left\{ \frac{\bar{w}_t}{w_{t-1}} w_{t-1}(s) e(3, j, s) l_{(t-1)}(3, j, s) + \frac{\bar{w}_t}{w_{t-2}} w_{t-2}(s) e(2, j, s) l_{(t-2)}(2, j, s) \right\}$$

1. $c_t(1, j, s) = \Psi_t(1, j, s)$, $l_t(1, j, s) = 0$

2. for $2 \leq i \leq 3$

$$\beta^{i-1} \left(\prod_{k=1}^{i-1} \lambda_k \right) \gamma c_{t+1}^{(1-\eta)\gamma-1} (1 - l_{t+1})^{(1-\gamma)(1-\eta)} = \Omega_{t+1} \quad (3.66)$$

$$\begin{aligned} & \beta \lambda_1 (1 - \gamma) c_{t+1}(2, j, s)^{(1-\eta)\gamma} (1 - l_{t+1}(2, j, s))^{(1-\gamma)(1-\eta)-1} \\ & = \Omega_{t+1} (1 - \tau_w - \tau_{bt+1}) w_{t+1}(s) e(2, j, s) \\ & + (\Omega_{t+3} + \Omega_{t+4}) \zeta \frac{1}{2} \frac{\bar{w}_{t+3}}{w_{t+1}} w_{t+1}(s) e(2, j, s) \end{aligned}$$

$$\begin{aligned} & \beta^2 \lambda_1 \lambda_2 (1 - \gamma) c_{t+2}(3, j, s)^{(1-\eta)\gamma} (1 - l_{t+1}(3, j, s))^{(1-\gamma)(1-\eta)-1} \\ & = \Omega_{t+2} (1 - \tau_w - \tau_{bt+2}) w_{t+2}(s) e(3, j, s) \\ & + (\Omega_{t+3} + \Omega_{t+4}) \zeta \frac{1}{2} \frac{\bar{w}_{t+3}}{w_{t+2}} w_{t+2}(s) e(3, j, s) \\ & (1 + (1 - \tau_r) r_{t+1}) \Omega_{t+1} = \Omega_t \end{aligned} \quad (3.67)$$

3. for $i \geq 4$

$$l_t(i, j, s) = 0 \quad (3.68)$$

$$\beta^t \gamma (1 - \eta) c_t^{(1-\eta)\gamma-1} = \Omega_t \quad (3.69)$$

- if $t=4$

$$(1 + (1 - \tau_r) r_{t+1}) \Omega_{t+1} = \Omega_t \quad (3.70)$$

- if $t=5$

$$b_t(i, j, s) + (1 + (1 - \tau_r)r_t)a_t(i, j, s) + \chi_t(i, j, s) = c_t(i, j, s) \quad (3.71)$$

Accordingly, the solution will be:

1. $c_t(1, j, s) = \chi_t(1, j, s)$, $l_t(1, j, s) = 0$

2. for $2 \leq i \leq 3$

$$\begin{aligned} & \frac{(1 - \gamma)}{\gamma} \frac{c_{t+1}(2, j, s)}{(1 - l_{t+1}(2, j, s))} = w_{t+1}(s)e(2, j, s) \\ \left[(1 - \tau_w - \tau_{bt+1}) + \left(\frac{1}{1 + (1 - \tau_r)r_{t+4}} + 1 \right) \frac{1}{(1 + (1 - \tau_r)r_{t+3})(1 + (1 - \tau_r)r_{t+2})} \zeta \frac{1}{2} \frac{\overline{w_{t+3}}}{\overline{w_{t+1}}} \right] \end{aligned}$$

$$\begin{aligned} & \frac{(1 - \gamma)}{\gamma} \frac{c_{t+2}(3, j, s)}{(1 - l_{t+2}(3, j, s))} = w_{t+2}(s)e(3, j, s) \\ \left[(1 - \tau_w - \tau_{bt+2}) + \left(\frac{1}{1 + (1 - \tau_r)r_{t+4}} + 1 \right) \frac{1}{(1 + (1 - \tau_r)r_{t+3})} \zeta \frac{1}{2} \frac{\overline{w_{t+3}}}{\overline{w_{t+2}}} \right] \end{aligned}$$

$$\begin{aligned} & \lambda_2 \beta (1 + (1 - \tau_r)r_{t+2}) c_{t+2}(3, j, s)^{(1-\eta)\gamma-1} (1 - l_{t+2}(3, j, s))^{(1-\gamma)(1-\eta)} \\ & = c_{t+1}(2, j, s)^{(1-\eta)\gamma-1} (1 - l_{t+1}(2, j, s))^{(1-\gamma)(1-\eta)} \end{aligned}$$

$$\begin{aligned} & \lambda_3 \beta (1 + (1 - \tau_r)r_{t+3}) c_{t+3}(4, j, s)^{(1-\eta)\gamma-1} \\ & = c_{t+2}(3, j, s)^{(1-\eta)\gamma-1} (1 - l_{t+2}(3, j, s))^{(1-\gamma)(1-\eta)} \end{aligned}$$

3. for $i \geq 4$

$$l_t(i, j, s) = 0 \quad (3.72)$$

$$\beta^t \gamma (1 - \eta) c_t^{(1-\eta)\gamma-1} = \Omega_t \quad (3.73)$$

- if $t=4$

$$\lambda_t(1 + (1 - \tau_r)r_{t+1})\beta c_{t+1}^{(1-\eta)\gamma-1} = c_t^{(1-\eta)\gamma-1} \quad (3.74)$$

- if $t=5$

$$b_t(i, j, s) + (1 + (1 - \tau_r)r_t)a_t(i, j, s) + \chi_t(i, j, s) = c_t(i, j, s) \quad (3.75)$$

REFERENCES

- [1] Akin, S., 2012. "Immigration, Fiscal policy, and welfare in an aging population." *The B.E. Journal of Macroeconomics (advances)* 12 (1).
- [2] Altonji, Joseph G., and David Card. "The Effects of Immigration on the Labor Market Outcomes of Less Skilled Natives." In . John M. Abowd and Richard B. Freeman, eds., *Immigration, Trade, and the Labor Market*. Chicago: University of Chicago Press, 1991, pp. 20134.
- [3] Auerbach, Alan J., and Laurence J. Kotlikoff, "Dynamic Fiscal Policy " Cambridge, MA: Cambridge University Press, 1987.
- [4] Beaudry P, Doms M, Lewis E. 2010 "Should the PC be Considered a Technological Revolution? Evidence from US Metropolitan Areas." *Journal of Political Economy* 118(5): 9881036
- [5] Borjas GJ. 1987 "Immigrants, Minorities, and Labor Market Competition." *Industrial and Labor Relations Review* 40(3): 382392
- [6] Borjas GJ. 1994 "The Economics of Immigration." *Journal of Economic Literature* 32(4): 16671717.
- [7] Borjas GJ. 2003 "The Labor Demand Curve is Downward Sloping: Reexamining the Impact of Immigration on the Labor Market." *The Quarterly Journal of Economics* 118(4): 13351374
- [8] Borjas GJ. 2006 "Native Internal Migration and the Labor Market Impact of Immigration." *Journal of Human Resources* 41(2): 221257 32
- [9] Borjas GJ. 2009 "The Analytics of the Wage Effect of Immigration." Mimeo, Harvard University
- [10] Borjas GJ, Grogger J, Hanson GH. 2008 "Imperfect Substitution Between Immigrants and Natives: A Reappraisal." NBER Working Paper 13887
- [11] Borjas GJ, Katz LF. 2007 "The Evolution of the Mexican Born Workforce in the United States" in Borjas GJ, ed., *Mexican Immigration to the United States*. Chicago: University of Chicago Press, 1326
- [12] Card D. 1990 "The Impact of the Mariel Boatlift on the Miami Labor Market." *Industrial and Labor Relations Review* 43(2): 245257
- [13] Card D. 2001 "Immigrant Inflows, Native Outflows, and the Local Labor Market Impacts of Higher Immigration." *Journal of Labor Economics* 19(1): 2264
- [14] Card D. 2009 "Immigration and Inequality," *The American Economic Review* 99(2): 121
- [15] Card D, Lewis E. 2007 "The Diffusion of Mexican Immigrants During the 1990s: Explanations and Impacts" in Borjas, George J., ed., *Mexican Immigrant to the United States*. Chicago and London: The University of Chicago Press, 193228

- [16] Caselli F, Coleman II WJ. 2006 “The World Technology Frontier.” *American Economic Review* 96(3): 499-522
- [17] Cortes P. 2008 “The Effect of Low Skilled Immigration on US Prices: Evidence from CPI Data.” *Journal of Political Economy* 116(3): 381-422
- [18] Dustmann C, Glitz A. 2012 “How Do Industries and Firms Respond to Changes in Local Labor Supply?” Mimeo, University College London.
- [19] Doms M, Lewis E. 2006 “Labor Supply and Personal Computer Adoption.” Federal Reserve Bank of San Francisco Working Paper 200618
- [20] Friedberg RM. 2001 “The Impact of Mass Migration on the Israeli Labor Market.” *The Quarterly Journal of Economics* 116(4): 1373-1408.
- [21] Gonzales L, Ortega F. 2011 “How Do Very Open Economies Absorb Large Immigration Flows? Recent Evidence from Spanish Regions.” *Labour Economics* 18(1): 57-70
- [22] Hanson GH, Slaughter MJ. 2002 “Labor Market Adjustments in Open Economies: Evidence from US States.” *Journal of International Economics* 57(1): 3-29
- [23] Lewis E. 2003 “Local, Open Economies Within the U.S.: How Do Industries Respond to Immigration?” Federal Reserve Bank of Philadelphia Working Paper 0401
- [24] Lewis E. 2011a “Immigration, Skill Mix, and Capital Skill Complementarity.” *Quarterly Journal of Economics* 126(2): 1029-1069
- [25] Ottaviano G, Peri G. 2006 “Rethinking the Effect of Immigration on Wages.” National Bureau of Economic Research Working Paper 12497
- [26] Peri G. 2012 “The Effect of Immigration on Productivity: Evidence from U.S. States.” *Review of Economics and Statistics* 94(1): 348-358
- [27] Storesletten, Kjetil, “Sustaining Fiscal Policy through Immigration,” *Journal of Political Economy*, CVIII (2000), 300-323.