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CHEMICAL EVOLUTION OF IRREGULAR GALAXIES AND THE PRIMORDIAL ^{4}He ABUNDANCE

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ABSTRACT

We study several models for the origin and chemical evolution of compact irregular galaxies in order to determine the primordial ^{4}He abundance, Y_p, from the zero metallicity intercept of the observed Y versus Z correlations. This study confirms the suggestion that a straight-line fit to the observations does not necessarily give the correct primordial ^{4}He abundance. This is especially true for the extrapolation of the Y versus N/H data which depends upon the relative contributions from secondary and primary nitrogen in low metallicity stars. The extrapolation of the O/H data is also slightly nonlinear even for primary oxygen in a closed-box model with instantaneous recycling due to the time dependence of the hydrogen mass fraction, the breakdown of the instantaneous recycling approximation, the production of helium, nitrogen, and oxygen in stars of different mass, and ejection of metal-poor material from low-mass stars. Our best fits to the data, even after excluding possible contamination of H II regions from neighboring Wolf-Rayet stars, gives $Y_p = 0.228 \pm 0.005$ for O/H and $Y_p = 0.223 \pm 0.006$ for N/H. These primordial helium abundances are as much as 2 σ below the minimum helium abundance which can be produced in the standard homogeneous big bang model with three light neutrino flavors. This discrepancy may be due to shortcomings of the chemical evolution models, additional systematic errors in the determination of the helium and/or metal abundances in extragalactic H II regions, or effects of nonstandard primordial nucleosynthesis.

Subject headings: galaxies: evolution — galaxies: abundances — galaxies: irregular — nuclear reactions, nucleosynthesis, abundances

1. INTRODUCTION

All models of big bang nucleosynthesis are tightly constrained by the observed primordial helium abundance, Y_p. Its value is usually inferred from the zero intercept of a straight-line fit to the observed helium abundance, Y, as a function of heavier element abundance in metal-poor compact irregular galaxies (see Peimbert & Torres-Peimbert 1976; Pagel 1982, 1987, 1991). In this paper we study the correlations of Y with N/H and O/H in the context of various models for the galactic chemical evolution of these species in irregular galaxies. We find that, independent of the many uncertainties in the chemical evolution of irregular galaxies, the predictions from the most realistic models can deviate significantly from a straight line. In particular, optimum fits to Y versus N/H exhibit significant curvature (Pagel et al. 1992) primarily due to the contribution from secondary sources (Torres-Peimbert, Peimbert, & Fierro 1989), although the fit to oxygen data even with these corrections is nearly indistinguishable from that of other recent studies, e.g., Pagel (1991) and Pagel et al. (1992). The optimum fits to the data yield a value for Y_p which is somewhat below that predicted by the standard homogeneous big bang model with three light neutrino flavors. The resolution of this dilemma may require a modification to either our chemical evolution models, the method of determination of the helium and/or metallicity in extragalactic H II regions, or the standard homogeneous big bang.

The standard big bang model (Wagoner, Fowler, & Hoyle 1967; Schramm & Wagoner 1977; Yang et al. 1984; Boesgaard & Steigman 1985; Olive et al. 1990) is parameterized by the entropy per baryon, the universal expansion rate during nucleosynthesis (which depends on the energy density in photons, electrons, baryons, and light neutrinos), and the neutron lifetime. With the best current values of available input parameters, Y_p is constrained to be $0.236 \leq Y_p \leq 0.244$ (Walker et al. 1991) in standard big bang nucleosynthesis. An observed Y_p less than 0.236 could thus present a serious problem for the standard big bang or the method of extrapolating to find Y_p; a recent study (Fuller, Boyd, & Kalen 1991, hereafter FBK) suggests that such a problem may indeed exist.

There are variants of the standard big bang model which could produce $Y_p < 0.236$. Such modifications include baryon inhomogeneities (Alcock, Fuller, & Mathews 1987; Applegate, Hogan, & Scherrer 1988; Fuller, Mathews, & Alcock 1988; Kurki-Suonio et al. 1988; Malaney & Fowler 1988; Boyd & Kajino 1989; Alcock et al. 1990; Kajino & Boyd 1990; Mathews et al. 1990b; Kawano et al. 1991; Matthews, Schramm, & Meyer 1993) degenerate or nonthermal neutrinos (Yahil & Beaudet 1976; Beaudet & Yahil 1977; Olive et al. 1991; Kang & Steigman 1992), late decaying particles (Fukugita, Kawasaki, & Yanagida 1989; Dimopoulos et al. 1988), cosmic strings (Vilenkin 1985; Malaney & Butler 1989), or time-dependent physical constants (Kolb, Perry & Walker

1986). A review of such scenarios has recently been made (Malaney & Mathews 1993). In any of these models, however, the observed value of Y_p provides an important test and constraint.

The derivation of Y_p from an extrapolation to zero metallicity of the best linear fit to the observed Y versus metallicity correlation (see Pagel 1982, 1991; Pagel & Simonson 1989; Pagel et al. 1992) can be justified by the closed-box model with instantaneous recycling and a constant nucleosynthesis yield (Lynden-Bell 1975; Tinsley 1980). If a linear fit is made to all existing Y-versus-metallicity data, including those for the Sun, Orion, and other high-metallicity objects, then $Y_p \sim 0.24 \pm 0.01$ is obtained (Pagel 1991; Baldwin et al. 1991). Pagel (1991), however, has noted that a fit restricted to low-metallicity extragalactic H II regions produces a line with a steeper slope and a smaller value for Y_p. He finds $Y_p = 0.225 \pm 0.005$ from a fit to Y versus O/H when the data are restricted to metallicity less than 0.25 of solar. The slope of the Y-versus-N/H data was also found (FBK) to increase at low N/H values thus further decreasing the primordial helium abundance. The statistical analyses of Olive Steigman, & Walker (1991) and Walker et al. (1991) found that an increased slope at low metallicity did result in a smaller χ^2. However, they did not find an overwhelming statistical preference for this or any other approach to extrapolate to the primordial helium abundance.

In addition to these problems associated with the statistical uncertainties in the data, it is important to have a theoretical framework (e.g. Lequex et al. 1979) in which to model the early chemical evolution of compact irregular galaxies. The lack of such a framework has made it difficult to reach quantitative conclusions about the best way to fit the Y versus Z correlation. Therefore, in the present work we attempt to develop a general chemical evolution framework in which the sensitivity to various aspects of irregular-galaxy evolution can be explicitly highlighted. We show that, for a quite general set of models, simple analytic relations can be derived which improve the extrapolation of the Y versus Z correlation. We also supplement these results with numerical calculations which highlight some nonanalytic effects.

Section 2 presents the formalism for the chemical evolution models. Section 3 presents the results of calculations, compares them to the Y versus O/H, Y versus N/H, and N/H versus O/H data, and discusses them in the context of the uncertainties of both the chemical evolution models and the observations. Our conclusions are given in § 4.

2. IRREGULAR GALAXY EVOLUTION

2.1. *General Considerations*

The extrapolation of the observed helium abundance to zero metallicity involves data from a set of irregular galaxies each of which could have had a different star-formation history along with differing amounts of inflow, outflow, and mixing of interstellar gas. Ideally, one might construct detailed models for each irregular galaxy separately and somehow infer the primordial helium abundance from the resulting ensemble. However, within the context of the instantaneous recycling approximation, it is possible to show that the closed-box model for galaxy evolution with constant yields of synthesized helium and metals (e.g., Tinsley 1977, 1980) gives a Y versus Z correlation which does not depend upon the star formation history. We show below that this conclusion is a good approximation in the context of much more general models than the closed-box model with instantaneous recycling. Thus, it is quite reasonable to consider single fits to the ensemble of data even though these data represent galaxies with different histories.

On the other hand, a strict linear relation between Y and Z is only true for a primary element like oxygen. We show below that for an element like nitrogen, which may also be produced by a secondary process, the relationship between Y and Z can be quite nonlinear. Also, even for a primary element like oxygen, there are effects both from the evolution of the hydrogen mass fraction and from inclusion of finite stellar lifetimes which cause a deviation from linearity in the Y versus O/H correlation at low metallicity. These effects can decrease the primordial helium abundance below that inferred from a simple linear fit to Y versus O/H or N/H.

In the present work we construct two general models for the formation and evolution of irregular galaxies. These will be used to construct the Y versus Z correlations for primary and secondary elements. One model assumes that irregular galaxies form by hierarchical merging of protogalactic clouds (Peebles 1965; Press & Schechter 1974; Carlberg 1988; 1990; Carlberg & Couchman 1989; Mathews & Schramm 1993). A second model assumes that irregular galaxies grow by the continuous (perhaps stochastic) accretion of primordial gas (e.g., Chiosi & Matteucci 1982; Matteucci & Chiosi 1983). For the purposes of deriving an analytic Y versus. Z correlation for secondary elements in the second model, and for relaxing the instantaneous recycling approximation, it will be assumed that star formation is regulated by the accretion rate and/or vice versa.

2.2. *Formalism*

We begin by defining quantities $f_{\rm in}$, and $f_{\rm out}$ as the fraction of the total mass of an evolving irregular galaxy which is flowing in or out per unit. Thus, we write the rate of change of the total mass, $M_{\rm tot}$ as

$$\frac{dM_{\rm tot}}{dt} = (f_{\rm in} - f_{\rm out})M_{\rm tot} . \qquad (1)$$

The rate of change of the total mass of gas, m_g, can then be written in the instantaneous recycling approximation (IRA) as

$$\frac{dm_g}{dt} = \left[-(1 - R)\psi(t) + \frac{(\mu_{\rm in} f_{\rm in} - \mu_{\rm out} f_{\rm out})}{\mu} \right] m_g , \qquad (2)$$

where R is the fraction of the progenitor mass of a star which is returned to the interstellar medium at the end of its lifetime, $\psi(t)$ is the fraction of the interstellar gas mass going into new star formation per unit time, and μ is the fraction of the mass of an irregular galaxy in the form of gas.

Using the chain rule and these definitions it is possible to write (Tinsley 1977; Clayton 1987) the general rate of change of the mass fraction, Z (where in this case Z can represent Y, $Z_{\rm N}$, or $Z_{\rm O}$), as

$$\frac{dZ}{dt} = R y_Z \psi(t) + \frac{\mu_{\rm in} f_{\rm in}}{\mu} (Z_{\rm in} - Z) - \frac{\mu_{\rm out} f_{\rm out}}{\mu} (Z_{\rm out} - Z) , \quad (3)$$

where, y_Z is the average mass fraction of newly synthesized nuclide Z in the ejecta of stars. In principle, y_Z should contain an implicit factor of $(1 - Z)$ to account for the abundance of available synthesizable material and to guarantee that (dZ/dt) goes to zero as Z approaches unity (Tinsley 1980). For example, one might write for the mass fraction of newly synthesized helium, $y_{He} = (1 - Y)p_Z$, where p_Z is a constant. However, over the range of Z considered here, this factor changes only slightly for helium ($\sim 5\%$) and is completely insignificant for heavier elements. Therefore, to a good approximation we can take y_Z to be time independent in the present work even for helium. This simplifies the analytic expressions and is consistent with the level of uncertainty inherent in separable models for galactic chemical evolution. The effect of this approximation has been checked by us and also independently (Balbes 1992) using our models but including the factor of $(1 - Y)$ for helium. Those fits lead to a primordial helium abundance which is indistinguishable from those obtained in the present work with y_Z constant.

For nitrogen, however, which could also have a secondary contribution, we write

$$y_N = y_{N_1} + y_{N_2} Z_O . \quad (4)$$

The secondary production of nitrogen in the CNO cycle is almost entirely from carbon (Audouze, Lequeux, & Vigroux 1975; Vigroux, Audouze, & Lequeux 1976; Dearborn, Tinsley, & Schramm 1978). However, oxygen and carbon are expected to remain in about the same abundance ratio. Therefore, we scale the secondary nitrogen production with oxygen in our calculations.

Equation (3) is all that is needed to derive the desired Y versus Z correlations. The general analytic solutions to equation (3) for helium, oxygen and nitrogen respectively are

$$Y = e^{-\nu(t)}\left\{ Y_p + \int_0^t \left[R y_{He} \psi(t) + \frac{\mu_{in} f_{in}}{\mu} Y_{in} - \frac{\mu_{out} f_{out}}{\mu} Y_{out} \right] e^{+\nu(t')} dt' \right\} , \quad (5)$$

$$Z_O = e^{-\nu(t)} \int_0^t \left[R y_O \psi(t) + \frac{\mu_{in} f_{in}}{\mu} Z_{in} - \frac{\mu_{out} f_{out}}{\mu} Z_{out} \right] e^{+\nu(t')} dt' , \quad (6)$$

$$Z_N = e^{-\nu(t)} \int_0^t \left[R y_N(Z_O) \psi(t) + \frac{\mu_{in} f_{in}}{\mu} Z_{in} - \frac{\mu_{out} f_{out}}{\mu} Z_{out} \right] e^{+\nu(t')} dt' , \quad (7)$$

where

$$\nu(t) = \int_0^t \left(\frac{\mu_{in} f_{in} + \mu_{out} f_{out}}{\mu} \right) dt' . \quad (8)$$

In general, explicit infall and outflow terms prohibit a simple analytic relation between these elements. However, within the context of some rather general models simple analytic relations can be constructed as shown below.

We also study scenarios in which the IRA is not imposed. If the IRA is relaxed equation (2) must be rewritten as

$$\frac{dm_g}{dt} = \left[-B(t) + \frac{(\mu_{in} f_{in} - \mu_{out} f_{out})}{\mu} + E(t) \right] m_g , \quad (9)$$

where $B(t)$ is the fraction of interstellar gas converted into stars per unit time:

$$B(t) = \int m\phi(m)\psi(t) dm , \quad (10)$$

and $E(t)$ is the fraction of the interstellar medium returned from stars per unit time:

$$E(t) = \int m\phi(m)\xi_m \psi(t - \tau_m) m_g(t - \tau_m) dm/m_g(t) . \quad (11)$$

Here, $\phi(m)$ is the initial mass function (IMF) normalized such that $\int m\phi(m)dm = 1$, and ξ_m is the fraction of the initial progenitor mass which is ejected into the interstellar medium by the end of a star's lifetime.

The new equation for the evolution of the helium or metal mass fraction becomes

$$\frac{d(Zm_g)}{dt} = \left[P_z(t) - ZB(t) + \int m\phi(m)\xi_m \psi(t - \tau_m) \times Z(t - \tau_m) m_g(t - \tau_m) dm/m_g(t) + \frac{\mu_{in} f_{in}}{\mu}(Z_{in} - Z) - \frac{\mu_{out} f_{out}}{\mu}(Z_{out} - Z) \right] m_g , \quad (12)$$

where $P_z(t)$, is the rate of ejection of newly synthesized material. We compute these rates for helium and oxygen using the ejected mass fractions, $X_Z^{ej}(m)$, as a function of progenitor mass from Chiosi & Maeder (1986):

$$P_Z(t) = \int X_z^{ej}(m) m\phi(m)\psi(t - \tau_m) m_g(t - \tau_m) dm/m_g(t) . \quad (13)$$

The yields of Chiosi & Maeder (1986) are for progenitor stars with $m > 5\ M_\odot$. There could also be some contribution to helium production from stars of lower mass. By choosing this higher mass cutoff we maximize the possible nonlinear effects due to the difference between helium and nitrogen evolution since nitrogen can be produced in lower mass stars. We also minimize the difference between oxygen and helium evolution since oxygen production occurs exclusively in high-mass stars. However, as we shall see, the nonlinear effects from the lower mass cutoff for helium production have very little effect on the extrapolated primordial helium abundance.

For secondary nitrogen, $X_Z^{ej}(m)$ is taken to be proportional to the initial oxygen abundance for stars with $m > 1.4\ M_\odot$ which burn hydrogen via the CNO cycle during the main sequence. The primary nitrogen component is taken as proportional to the primary oxygen production.

In equations (9)–(13) m_g refers to the total gas mass in the hierarchical clustering model or the gas mass of the central condensation for the accretion model.

Since $Z(t - \tau_m) < Z(t)$, one can immediately see that the corrections for finite stellar lifetimes in equation (12) will have the effect of decreasing the metallicity relative to that given by the IRA for times in which the stellar lifetimes are comparable to the metallicity enrichment time. Also, since helium is produced by longer lived stars than oxygen, and similarly, nitrogen is produced by longer lived stars than helium, there can be slight delays in the appearance of helium relative to oxygen and nitrogen relative to helium.

2.3. *Hierarchical Clustering Model and Closed-Box Model*

These models give the simplest analytic solutions for Y versus Z. In the hierarchical clustering scenario, compact irregular galaxies are presumed to grow by a sequence of binary mergers of clouds of ever increasing mass (e.g., Mathews & Schramm 1993). On average, merging clouds will have experienced the same amount of star formation. Therefore, $\mu_{in} = \mu_{out} = \mu$, and $Z_{in} = Z_{out} = Z$, and the infall and outflow terms drop out of equation (3). This model allows for infall and outflow and hence is more general than a closed-box model. However, this model produces the same Y versus/Z relation as the closed-box model. In this case, the helium and oxygen production terms, equations (5) and (6), are identical to within a constant of proportionality. Similarly, the nitrogen mass fraction can be simply related to the oxygen mass fraction (Lynden-Bell 1975; Tinsley 1980). Thus, we write

$$Y = Y_p + \frac{y_{\rm He}}{y_{\rm O}} Z_{\rm O} , \quad (14a)$$

$$Z_{\rm N} = \frac{y_{\rm N_1}}{y_{\rm O}} Z_{\rm O} + \frac{y_{\rm N_2}}{y_{\rm O}} \frac{Z_{\rm O}^2}{2} , \quad (14b)$$

so that

$$Y = Y_p + y_{\rm He} \frac{Z_{\rm N}}{(y_{\rm N_1} + y_{\rm N_2}(Z_{\rm O}/2))} . \quad (14c)$$

Clearly, if there is a secondary contribution to nitrogen ($y_2 \neq 0$), then the rate of increase of the helium mass fraction decreases with increasing metallicity, and in the limit of no primary contribution to nitrogen, Y would increase as $Z_{\rm N}^{1/2}$. This will cause the intercept of the correlation line to occur for lower values of Y_p than that inferred from a simple straight-line fit.

A fit to Y as a function of O/H or N/H also requires a conversion from mass fraction to abundance relative to hydrogen,

$$n_Z/n_{\rm H} = \frac{Z}{XA} , \quad (15)$$

where X is the hydrogen mass fraction and A is the atomic mass relative to hydrogen. For our purpose, the hydrogen mass fraction can be written:

$$X \approx 1.0 - Y - 1.3 Z_{\rm O} - Z_{\rm N} , \quad (16)$$

where the factor of 1.3 accounts for the added mass fraction of carbon and elements heavier than oxygen (Anders & Grevasse 1989).

2.4. *Accretion Model*

For this model we assume that irregular galaxies grow by the accretion of metal-poor primordial material. Thus, we have $Y_{\rm in} = Y_p$, $Z_{\rm in} = 0$ (for oxygen and nitrogen), and $\mu_{\rm in} = 1$. It is further assumed that star formation is regulated by the accretion rate (e.g., Matteucci & Chiosi 1983) or vice versa (e.g., Burkert, Truran, & Hensler 1992) so that $\psi(t) m_g(t) = \gamma f_{\rm in} m_g/\mu$, where γ is a constant of proportionality. With this assumption Y will again depend linearly on $Z_{\rm O}$ as in equation (14a). The presence of the infall term, however, now causes the dependence of nitrogen on oxygen to grow as

$$Z_{\rm N} = \frac{y_{\rm N_1}}{y_{\rm O}} Z_{\rm O} + \frac{y_{\rm N_2}}{y_{\rm O}} Z_\infty \times \left[Z_{\rm O} + Z_\infty \left(1 - \frac{Z_{\rm O}}{Z_\infty}\right) \ln\left(1 - \frac{Z_{\rm O}}{Z_\infty}\right) \right] , \quad (17a)$$

so that

$$Y = Y_p + y_{\rm He} \frac{Z_{\rm N}}{(y_{\rm N_1} + y_{\rm N_2}(Z_\infty/Z_{\rm O}))} \times \left[Z_{\rm O} + Z_\infty \left(1 - \frac{Z_{\rm O}}{Z_\infty}\right) \ln\left(1 - \frac{Z_{\rm O}}{Z_\infty}\right) \right] . \quad (17b)$$

Here, Z_∞, is the asymptotic oxygen abundance obtained by the equilibrium between enrichment from star formation and dilution from metal-poor infall. This asymptotic behavior means that at late times the nitrogen abundance is not as large as in the hierarchical clustering or closed-box model. By an expansion of the logarithmic term in equation (17b) it is possible to show that for small $Z_{\rm O}$, it reduces to equation (14c). That is, the behavior of both models near the intercept is identical. Thus, our results for Y_p are insensitive to the value of Z_∞. For the present purposes we take $Z_\infty = 0.01$. In the actual fits to data it was found that an increase in Z_∞ was compensated by a decrease in $y_{\rm He}/y_{\rm O}$ such that the optimum curve through the data was always a curve similar to that from the hierarchical clustering model independently of the value for Z_∞.

2.5. *Numerical Model*

To study the effects of the above assumptions of instantaneous recycling and constant average yields as a function of time we have also numerically evaluated equations (9)–(13) in the context of the hierarchical clustering model. That is, the inflowing and outflowing matter are taken to have the same metallicity as the interstellar gas. For the IMF we used the three-segment power law of Miller & Scalo (1979) normalized to unity over the mass-weighted integral from 0.1 to 62 $M_\odot$. The fraction of initial mass returned to the interstellar medium, ξ_m, is taken from Iben & Renzini (1983; 1984). We take the stellar lifetimes, τ_m, from Rana (1987). For the infall rate we assumed an exponentially decaying rate with a decay constant of 5 Gyr and a time scale to build up to the mass of the LMC of 10^{10} yr. Such an exponential rate is chosen to maximize the effects of relaxing the IRA at early times and low metallicity.

The star formation rate, $\psi(t)$, was taken as proportional to the infall rate, with a constant of proportionality adjusted such that 10% of the total mass is in gas after 10^{10} yr. The total surface density throughout the evolution was fixed at 120 $M_\odot$ pc^{-2}. These parameters are typical of observed compact irregular galaxies (Matteucci & Chiosi 1983; Gallagher & Hunter 1984).

3. RESULTS

Note that the analytic derivations of the Y versus Z correlations in the previous section did not require specification of the star formation rate or the infall or outflow rates. Furthermore, as we shall see, in the numerical model in which these quantities are specified the correlations have little dependence upon these quantities. We are therefore justified in presuming that such models could apply equivalently to galaxies with vastly different histories of star formation and infall. The only important parameters of the models are Y_p and the relative yields, $y_{\rm He}/y_{\rm O}$, $y_{\rm N_1}/y_{\rm O}$, and $y_{\rm N_2}/y_{\rm O}$. Various fits to the data and the associated reduced chi-squared, χ_r^2, are summarized in Table 1 and in Figures 1 and 2.

Fits to these correlations are usually based upon the minimization of χ_r^2 weighted by the uncertainty in the observed helium abundance, σ_Y. However, there is also an uncertainty, σ_Z, in the observed metallicity which should be included in the estimate of the error in Y_p. To account for the latter uncertainty we introduce the standard total weighting factor, $\sigma_{\rm tot}$, (Bevington 1969) for errors in the dependent variables,

$$\sigma_{\rm tot}^2 = \sigma_Y^2 + \sigma_Z^2 (dY/dZ)^2 . \quad (18)$$

This weighting was used to determine the minimum in χ^2. However, excluding the errors in O/H or N/H makes little

TABLE 1

SUMMARY OF FITS TO Y VERSUS Z DATA

Model	Data	Y_p	y_{He}/y_O	y_{N_1}/y_O	y_{N_2}/y_O	Z_∞	χ_r^2
			Pagel (1991) Data Set				
Linear	O/H	0.2253 (42)	169 (34)	...	...	...	0.47
	N/H	0.2336 (30)	2060 (480)	...	...	...	0.56
Merger	O/H	0.2246 (42)	14.6 (2.9)	0.00016	47	...	0.46
	N/H	0.2209 (47)	16.4 (3.5)	0.00016 (0.00756)	47 (9)	...	0.50
Accretion	O/H	0.2246 (42)	14.5 (2.9)	0.0011	46	0.01	0.46
	N/H	0.2206 (48)	17.2 (3.6)	0.0011 (0.0062)	46 (12)	0.01	0.50
Numerical	O/H	0.2248 (42)	3.71 (67)	0.0045	0.51	...	0.46
	N/H	0.2202 (42)	4.26 (58)	0.0011 (0.0184)	0.51 (29)	...	0.49
			Pagel et al. 1992 Data Set				
Linear	O/H	0.2279 (53)	115 (40)	...	...	...	0.25
	N/H	0.2328 (40)	1415 (51)	...	...	...	0.24
Merger	O/H	0.2276 (54)	9.7 (3.4)	5.7×10^{-7}	64	...	0.25
	N/H	0.2334 (82)	13.9 (3.8)	5.7×10^{-7} (4.3×10^{-5})	64 (8)	...	0.22
Accretion	O/H	0.2276 (54)	9.8 (3.4)	0.000082	41	0.01	0.25
	N/H	0.2230 (62)	11.9 (4.2)	0.000082 (0.013)	41 (3)	0.01	0.22
Numerical	O/H	0.2276 (55)	2.60 (82)	0.000064	0.71	...	0.25
	N/H	0.2214 (50)	3.76 (99)	0.000064 (0.025)	0.71 (16)	...	0.22

NOTES.—Numbers in parentheses are the $\pm 1\ \sigma$ uncertainties in the last digits of the table entries. Note that the relative yields, y_Z/y_O, are in units of abundance relative to hydrogen for the linear fits, but in units of relative mass fraction in the ejecta for the analytic chemical evolution models. For the numerical models, the yields given are factors by which the ejected mass fractions of Chiosi & Maeder 1986 must be multiplied to obtain an optimum fit.

difference in the fit or the uncertainty. The quoted χ_r^2 values take into account the number of free parameters in the fits as listed in Table 1.

Optimum fits were made to the data sets using the method of parabolic extrapolation (Bevington 1969). The root sum errors for the various parameters,

$$\sigma_a = \left\{ \sum \left[\sigma_i^2 \left(\frac{\partial a_j}{\partial y_i} \right)^2 \right] \right\}^{1/2} , \quad (19)$$

were determined from the inverse of the curvature matrix (Bevington 1969).

3.1. O/H *Extrapolation*

The dashed line on Figure 1 shows our best straight-line fit to the Y versus O/H data of Pagel (1991). The solid line shows the result of the hierarchical clustering, closed-box, or accretion models. The dotted line shows the result of relaxing the instantaneous recycling approximation. In Table 1 we summarize the results of these fits along with results from fits to the

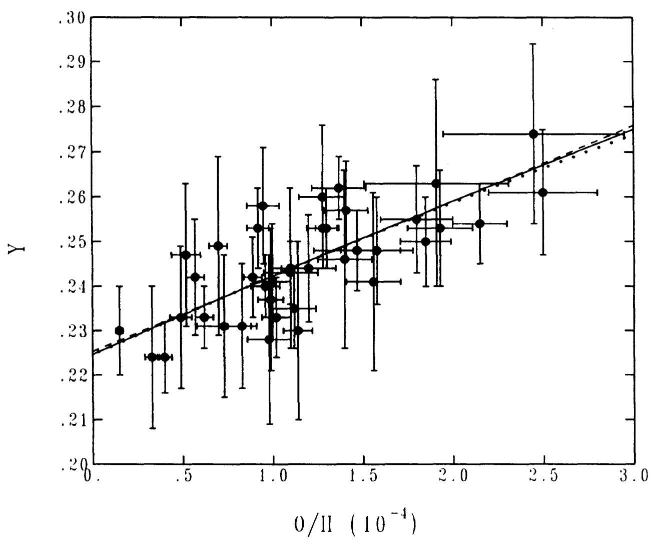

FIG. 1.—Y vs. O/H data of Pagel (1991) compared to the best straight-line fit (*dashed line*), the hierarchical clustering, closed box, or accretion models (*solid line*), and the best numerical fit (*dotted line*) for the hierarchical clustering model with the instantaneous recycling approximation relaxed and with an assumed exponential star formation rate.

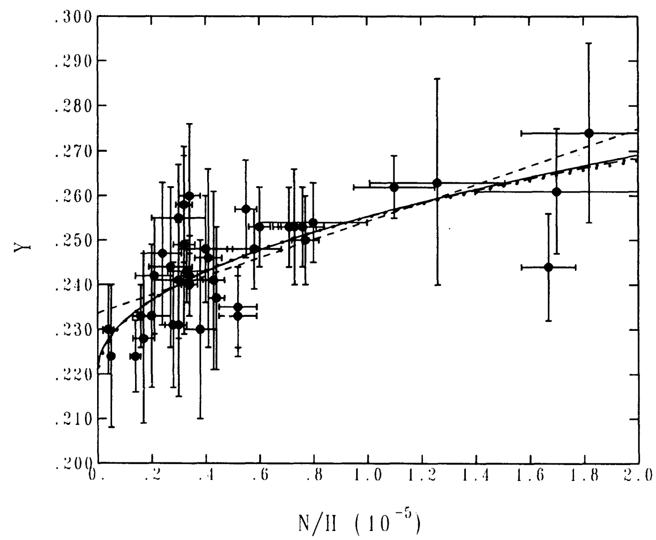

FIG. 2.—Y vs. N/H data of Pagel (1991) compared to the best straight-line fit (*dashed line*), the hierarchical clustering or closed-box model (*solid line*), the accretion model (*dot-dashed line*), and the numerical model with the instantaneous recycling approximation relaxed (*dotted line*).

data set of Pagel et al. (1992) which include a number of newer observations and from which H II regions in the vicinity of Wolf-Rayet stars have been removed.

The Y versus O/H correlations given by the analytic and numerical models deviate slightly below the straight-line fits at low and high metallicity. The reason for the deviation of the analytic models can be traced to the evolution of the hydrogen mass fraction (eq. [16]). When the IRA is relaxed, a further deviation from linearity occurs which is most manifest at the highest oxygen abundances in the fit. This deviation results from the dilution of oxygen by the ejection of metal-poor material from low-mass stellar envelopes. It does not affect helium since the mass fraction of helium in the metal-poor ejecta differs only by a few percent from that of the interstellar abundance. For oxygen, however, that is not the case. There is also a slight delay in the appearance of helium relative to oxygen at the lowest metallicities due to the fact that some helium is ejected from stars of lower mass than those which produce oxygen (Chiosi & Maeder 1986). This tends to cancel the effect of oxygen dilution by metal-poor ejection at low metallicity so the net effect is to produce about the same (even slightly greater) inferred primordial helium abundance in the numerical model than in the models for which the IRA is invoked.

From the above discussion it is clear that, although a straight line is not a bad approximation to the Y versus O/H correlation, its use can cause one to overestimate Y_p at the level of ~ 0.0007. While this is a small correction, it is significant in that it reduces the primordial helium abundance even further below that required by the standard homogeneous big bang.

Our best fit straight line to the Y versus O/H correlation of Pagel (1991) is for $Y_p = 0.2253 \pm 0.0042$, and $dY/d(\mathrm{O/H}) = 169 \pm 34$. For the Wolf-Rayet excluded data set of Pagel et al. (1992) we obtain $Y_p = 0.2279 \pm 0.0053$, and $dY/d(\mathrm{O/H}) = 115 \pm 40$. These results are slightly above these obtained for the hierarchical clustering model for which $Y_p = 0.2246 \pm 0.0042$, and $dY/dZ = y_{\mathrm{He}}/y_{\mathrm{O}} = 14.6 \pm 2.9$ or $Y_p = 0.2276 \pm 0.0054$, and $dY/dZ = y_{\mathrm{He}}/y_{\mathrm{O}} = 9.7 \pm 3.4$ for the Pagel (1991) and Pagel et al. (1982) data sets, respectively. The fits using the accretion and numerical models are quite similar to the fit from the hierarchical clustering model. There was a slight increase in Y_p (~ 0.0002) when fitting the numerical model to the Pagel (1991) data set. This increase could be traced to the difference between the average lifetime of helium producing and nitrogen producing stars as mentioned above. There was overall, however, only a slight improvement in χ_r^2 when going from a straight-line fit to the chemical evolution models.

In the chemical evolution models there was a slight dependence of the fits on the parameters affecting nitrogen due to the contribution of the nitrogen mass fraction to the determination of the hydrogen mass fraction. The parameters for nitrogen in these fits were, therefore, fixed at the optimum values obtained from the fits to the Y versus N/H data described in the next section. This procedure, however, had almost no effect on the primordial helium abundance inferred from the oxygen data.

3.2. N/H *Extrapolation*

It has been argued (e.g., Steigman, Gallagher, & Schramm 1989) that since both helium and secondary nitrogen can be produced in intermediate-mass stars whereas oxygen is only produced in massive stars, nitrogen should correlate better with helium than oxygen does. The dashed line on Figure 2 shows the best fit straight line to the Y versus N/H data set of Pagel (1991). The solid line is for the hierarchical clustering model. The dot-dashed line is for the accretion model and is barely distinguishable from the hierarchical clustering model as mentioned previously. The dotted line is for the numerical model. These results and the fits to the data of Pagel et al. (1992) are summarized in Table 1. To make these fits we optimized Y_p, $y_{\mathrm{He}}/y_{\mathrm{O}}$, $y_{\mathrm{N}_1}/y_{\mathrm{O}}$ and $y_{\mathrm{N}_2}/y_{\mathrm{O}}$ to minimize the χ^2. We did not introduce an a priori prejudice that nitrogen should be secondary or primary.

The best fit for the hierarchical clustering and accretion models was for $Y_p = 0.221 \pm 0.005$ for the Pagel (1991) data, and $Y_p = 0.223 \pm 0.006$ for the Pagel et al. (1992) data. The other parameters and χ_r^2 are summarized in Table 1. For nitrogen there was an appreciable improvement in χ_r^2 when going from a straight-line fit to the chemical evolution models.

The best fit for the nitrogen data in the numerical model was slightly lower, $Y_p = 0.220 \pm 0.005$ for the Pagel (1991) data, and $Y_p = 0.221 \pm 0.006$ for the Pagel et al. (1992) data. The reason that relaxing the IRA has a stronger effect on nitrogen than oxygen is in part due to the fact that nitrogen is produced by lower mass stars than helium in our adopted yields. Hence, the appearance of nitrogen is delayed relative to helium. Also, dilution of synthesized metals by the ejection of metal-poor material into the interstellar medium contributes to a steepening of $\Delta Y/\Delta Z$ at low metallicity. Therefore, a lower primordial helium abundance is inferred when the IRA is relaxed.

As can be seen in Table 1, the best fit to the low-metallicity data is consistent with only a small contribution from primary nitrogen. In the limit of no primary nitrogen, then the relationship between Y and N/H has the particularly simple form for the hierarchical clustering model:

$$Y = Y_p + \frac{y_{\mathrm{He}}}{y_{\mathrm{O}}}\left(\frac{2y_{\mathrm{O}}}{y_{\mathrm{N}_2}} Z_{\mathrm{N}}\right)^{1/2} . \tag{20}$$

Figure 3 also shows the N/H versus O/H correlation for the straight-line and analytic model fits. For the analytic models, the best fits to the helium versus nitrogen or oxygen data determine nitrogen versus oxygen as a function of time. For the straight-line fits the nitrogen versus oxygen relation can be defined for equivalent helium values. Plotting N/H versus O/H

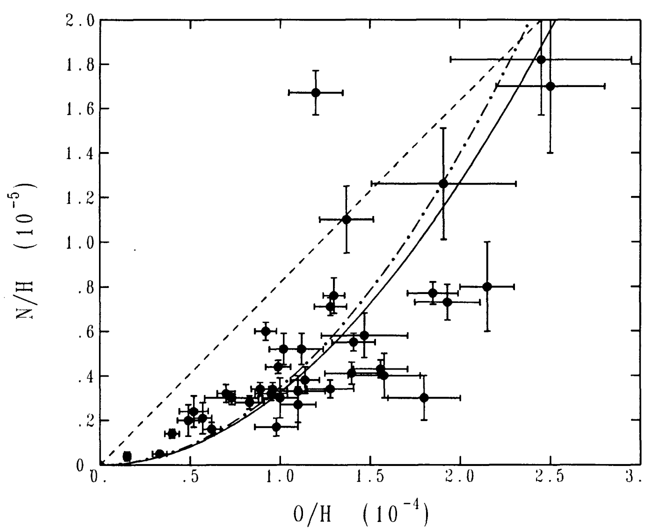

FIG. 3.—N/H vs. O/H correlations inferred from the best fits to the Y vs. Z data using the hierarchical clustering model (*solid line*), accretion model (*dot-dashed line*), and straight-line fit (*dashed line*). The data are from Pagel (1991).

in this way is a check on the consistency of the data. Although the uncertainties in the data are large, and there is considerable scatter in the points, the model curves are at least consistent with the trends in the data whereas the straight-line fits imply that nitrogen is a bit too abundant relative to oxygen.

As a final remark we note that a recent paper (Pagel & Kazlauskas 1992) has pointed out that plotting the data as N/O versus O/H indicates a nonzero intercept which precludes a purely secondary nucleosynthesis for nitrogen. They in fact have used a straight-line fit to the N/O ratios to infer a slightly higher Y_p from the nitrogen abundance than that derived here. In the present work we have included some contribution to primary nitrogen, as indicated in Table 1, so that we also obtain a nonzero intercept for the N/O ratio. Our relative primary nitrogen yields range from $y_{N_1}/y_O \sim 10^{-6}$ to 10^{-3}. For a typical oxygen yield of 10 $Z_O^{\odot}$, this would imply a primary nitrogen yield of up to 10% $Z_N^{\odot}$. Pagel & Kazlauskas (1992), however, infer $y_{N_1}/y_O \sim 10^{-2}$ from the N/O ratios. We note that recent preliminary calculations (Weaver & Woosley 1992) of primary nitrogen production in low-metallicity massive stars give $y_{N_1}/y_O \sim 10^{-4}$ to 10^{-2} from the mixing of hydrogen into the helium burning shell. This range is the result of sensitivity to uncertainties in the treatment of convection. Since the range of calculated yields is consistent with either our yield or that derived by Pagel & Kazlauskas (1992), there is presently no compelling stellar evolution argument to prefer one value over the other.

Although the approach of Pagel & Kazlauskas (1992) is an alternative way of analyzing the data, there are also large uncertainties in the derived N/O ratios and there is a large scatter in these data (for which many points fall several standard deviations away from the correlation). Therefore, we have opted in the present work to consider direct fits to Y versus Z data. We believe that this provides the best indicator of the primordial helium abundance.

4. CONCLUSIONS

Several conclusions can be drawn from the present analysis. First, the Y versus N/H data prefer a nonlinear fit consistent with a significant contribution from secondary sources. This lowers the inferred primordial helium abundance below that obtained from a simple linear fit to the Y versus N/H data. In addition, the correction for the evolution of the hydrogen mass fraction slightly reduces the inferred primordial helium abundance from both N/H and O/H correlations below that of a straight-line fit. Furthermore, relaxing the IRA also reduces the primordial helium abundance derived from the correlation with nitrogen by delaying the ejection of secondary nitrogen relative to helium. The opposite is true for oxygen where it is helium that is delayed relative to oxygen so that relaxing the IRA can actually result in a slight increase in the inferred primordial helium abundance. For both the Pagel (1991) and Pagel et al. (1992) data sets, the model fits give values for Y_p which are in slightly better agreement with each other than the simple straight-line fits, although this improvement is not particularly statistically significant.

The weighted average of the three model fits to the four data sets considered in this work is $Y_p = 0.224$. Since the errors are not purely statistical and the different determinations cannot be treated as independent, it is appropriate to quote the mean error which is ± 0.005. The straight-line fits give a value for Y_p of 0.231 ± 0.004. However, the average of the model fits (in which linearity is not assumed) to the Pagel et al. (1992) set alone gives $Y_p = 0.225 \pm 0.005$, whereas for the Pagel (1991) data set $Y_p = 0.223 \pm 0.004$. It is clear that no matter how the different fits are averaged (except in the case of the purely straight-line fit which we and others [e.g., Steigman, Gallagher, & Schramm 1989] have argued is inappropriate), the inferred primordial helium abundance tends to be $\sim 2\,\sigma$ below the lower limit, $Y_p > 0.236$, allowed in the standard homogeneous big bang with three neutrino flavors. Clearly there is a potential problem.

If this discrepancy should persist as the data are improved, one can envision three possible solutions of this dilemma. One is that there are significant chemical evolution effects not included in these models. Such effects might arise from time dependence of the yields or the IMF, or from the ejection of metal-enriched material from irregular galaxies by supernovae. Also, we have implicitly assumed instantaneous mixing within the H II regions. Incomplete mixing of stellar ejecta would also modify the extrapolation to zero metallicity (Clayton 1992). Another possibility is that there are larger systematic errors in the determinations of helium and/or metal abundances in the H II regions of compact irregular galaxies than that reflected in the quoted uncertainties in the data. A third possibility is that some modification to the standard homogeneous big bang with three light neutrinos is required.

Regarding the second solution, systematic uncertainties in the data may indeed be larger than those reflected in the quoted errors. Pagel (1982, 1991) and Pagel et al. (1992) note that several such sources may exist in the H II regions from which the data are derived. These include corrections for neutral ^{4}He, uncertainties in the ionizing UV-flux and in effective recombination coefficients, processes involving grains, interstellar reddening, and absorption lines of metals. These error sources are reviewed in Davidson & Kinman (1985). Some of these can be minimized by restricting the data to only those extragalactic H II regions in irregular and compact blue galaxies where the metallicity is globally low and the ionizing UV-flux is believed to be large enough that the correction for neutral ^{4}He is small (Lequeux et al. 1979; Kunth & Sargent 1983; Peimbert & Torres-Peimbert 1976; Pagel & Simonson 1989). Corrections for collisional excitation of He I lines have been made (Pagel 1991), but are also uncertain (Clegg, Lambert, & Tompkin 1981; Ferland 1986; Pagel 1987; Berrington & Kingston 1987; Clegg 1987; Aller 1990). Such effects could increase the uncertainty to of order 0.01 (Davidson & Kinman 1985). However, Pagel (1991) has deduced that the total systematic uncertainties may only be as large as 0.005. These effects may account for some of the discrepancy, if the systematic errors are in the direction to cause the helium mass fraction to be consistently underestimated.

Regarding the third possible solution, one can envision several fixes for the standard homogeneous big bang model, e.g., an unstable ν_τ; (Kolb & Scherrer 1982; Scherrer & Turner 1988), neutrino degeneracy, or nonthermal neutrino distributions (Kang & Steigman 1991; Olive et al. 1991) either of which might allow for a lower value of Y_p while satisfying the constraints from other light-element abundances. In addition, baryon inhomogeneous models can produce a value for Y_p which is lower than the standard value (e.g., Mathews, Alcock, & Fuller 1990a; Mathews et al. 1993). For a baryon density $\sim 20\%$ of the closure density, and if a mechanism, such as late time expansion (Alcock et al. 1990) or stellar processing (Mathews et al. 1990b; Delyannis et al. 1989), is invoked to destroy the excess ^{7}Li production. (In neither the standard model nor the inhomogeneous models is a closure density in baryons consistent with the presently derived upper limit to Y_p.) Late decaying particle models could also produce a low primordial helium abundance, but one must find a way to diminish the overproduction of ^{6}Li (Dimopoulos et al. 1988).

Finally, the present analysis resolves a question which has troubled recent analyses of the Y versus Z data, that of truncation of the data set. While there has been general agreement that the data set used for the linear fits of past analyses should be truncated above some metallicity, the data are not yet of sufficient quality to provide a signature of where the truncation should be made (Walker et al. 1991). Furthermore, the values obtained for Y_p depend on the subset of the data chosen (FBK; Walker et al. 1991). However, the present study suggests that linear analyses using even the truncated nitrogen data set of Pagel et al. (1992) would not be likely to give the correct value for Y_p. This is because most of the observable curvature manifested by the $Z_N^{1/2}$ dependence of Y versus N/H would appear only within the first few data points with (N/H) $< 4 \times 10^{-6}$.

Interestingly, when the chemical evolution models are used, the higher metallicity data define the coefficient of $Z^{1/2}$ in equation (20) which, in turn, defines Y_p. Clearly, however, more data at the very lowest metallicity values would aid enormously in determining the extent to which nitrogen is secondary, and hence, defining the value for Y_p.

We conclude with a suggestion of how, perhaps, to best determine the primordial helium abundance in analogy with the determination of the primordial lithium abundance on extremely metal-poor stars (e.g., Rebolo, Molara, & Beckman 1988). Figure 4 shows a plot of Y as a function of log (O/H). If even a single well-defined helium abundance determination could be made in an object with (N/H) $\lesssim 10^{-7}$, or (O/H) $\lesssim 10^{-6}$, it would firmly fix the helium abundance at the time of galaxy formation which, if a first generation of pregalactic stars did not contribute to helium production (Carr, Bond, & Arnold 1984), would determine the primordial helium abundance.

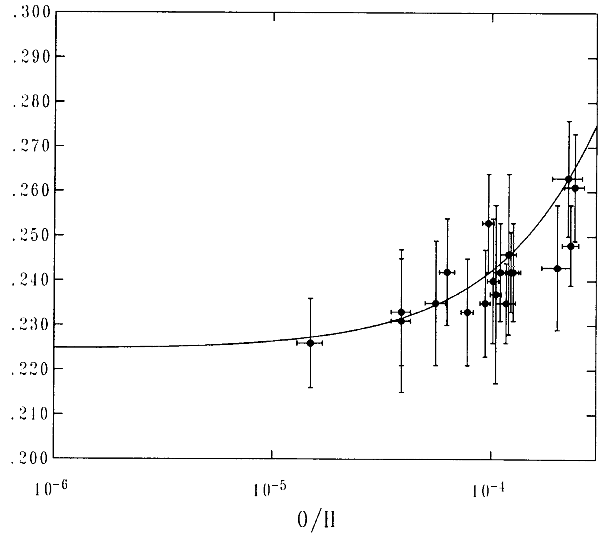

FIG. 4.—Y versus log (O/H) for the hierarchical clustering model (*solid line*) compared with data from Pagel (1991).

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