Title
A NEW APPROACH TO MODELING AND INTERPRETING ELECTROMAGNETIC SOUNDING DATA

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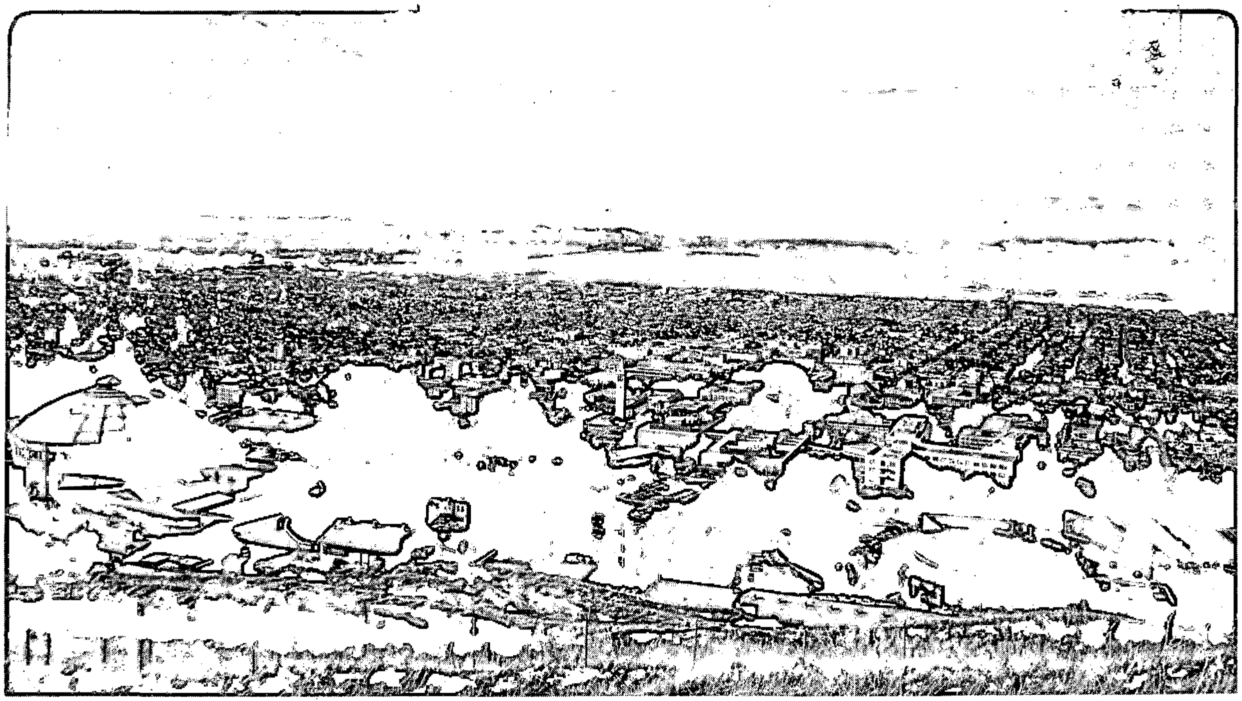

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Introduction

Electromagnetic (em) sounding has for many years been a standard tool for detecting subsurface electrical conductors and for mapping electrical conductivity in horizontally layered structures, with applications to mineral, groundwater, and petroleum prospecting. Although instrumentation and field techniques have been developed for more general applications to deep crustal investigations (e.g., Morrison *et al.,* 1978; Wilt *et al.,* 1983; Zollinger and Morrison, 1987), a major impediment to the application and acceptance of controlled-source electromagnetics has been the difficulty of processing, displaying, and interpreting the data. At the low frequencies needed to penetrate the earth, the fields are governed by a diffusion equation, and this simple fact has thwarted efforts to display the data as reflections from subsurface interfaces, as is done in seismic methods. Further, the solutions to a vector-diffusion equation in an inhomogeneous conductor must be obtained numerically, and for anything other than elementary models the computation time is much too great. Because the high cost of forward computer modeling has precluded even simple generalized inversion of observed data, many fundamental questions remain about uniqueness, resolution, and the most appropriate method of measurement in specific situations.

For the past several years we have devoted an increasing amount of our effort in crustal electromagnetic sounding to the search for a new approach to modeling and interpreting em methods. Our overall goal in this work has been to develop a method for imaging the subsurface conductivity distribution. Rather than describe the results of a forward model by profiles or contour plots of electric or magnetic fields observed on the surface, we have sought a representation that would provide some direct visualization of the subsurface conductivity contrasts that give rise to the observed or calculated fields. An ultimate goal of the work has been to develop an inverse method that can automatically generate a model for the subsurface conductivity distribution from the field data. As a result of this research we have discovered a new method for calculating the electromagnetic response of inhomogeneous conducting media that is not only computationally efficient but promises to provide the inverse solutions we have long sought. At the present time the method can be used to:

1) model em problems (the forward solution) with a speed at least **two orders of magnitude** faster than conventional methods,
2) provide reflectivity images of subsurface conductivity contrasts similar to those obtained in reflection seismic methods.

Theory and Applications

a) Theory

A two-dimensional low-frequency electromagnetic field (up to 1.0 MHz) obeys the diffusion equation

$$(\nabla^2 - \sigma\mu \frac{\partial}{\partial t})u(t) = f(t). \tag{1}$$

Here $u(t)$ is either the electric or the magnetic field, and $f(t)$ is an external source. The theory applies to the 3-D vector fields as well, but we have chosen a scalar problem for simplicity. Now let us imagine a field $v(q)$ that is completely independent from the diffusion field *u(t)* and which satisfies the wave equation

$$(\nabla^2 - \sigma\mu \frac{\partial^2}{\partial q^2}) v(q) = g(q). \tag{2}$$

The independent variable q is something like "time" but has the dimension of square root of time (expressed as the unit q). It can be shown that under the proper initial conditions, *u(t)* and *v(q)* have the following unique relationship (Lavrent'ev *et al.,* 1980):

$$u(t) = \frac{1}{2\sqrt{\pi t^3}} \int_0^\infty q e^{-q^2/4t} \; v(q)\, dq. \tag{3}$$

The necessary condition for this equation to be valid is that the source functions *f(t)* and *g(q)* satisfy the same relationship. Since the transformation involves only t and q, we omit the spatial variables x, y, and z in the above equations.

b) Applications

The implication of the transformation is significant in both the modeling and the interpretation of electromagnetic data. In the case of em modeling, it is known that the computing speed associated with an explicit time-stepping method (Oristaglio and Hohmann, 1984) is dictated by the "grid-diffusion time." For a typical conductivity distribution the range of the grid-diffusion time, 0.1 μsec to 1.0 μsec, is used for the time-stepping interval. If we were to solve the same problem in the transformed q-domain, the computing speed would be dictated by the Courant-Friedrichs-Lewy (CFL) stability condition for the wave equation. The CFL condition in this case would give us a range of q-stepping of approximately 0.1 mq to 1.0 mq. It is therefore immediately clear that modeling in the q-domain would be **a few orders of magnitude faster** than in the t-domain. Once the solution is obtained in the q-domain, the t-domain field may be computed using the integral given by Equation (3). To see if the integral indeed generates valid solutions in the t-domain, we have tested a model consisting of a conductive cylinder in a conducting whole-space (Fig. 1). The medium is excited by an infinite line source. We first calculate the total field *v(q)* (Fig. 2) and then use the integral to transform *v(q)* into *u(t)* (Fig. 3), which is actually the total electric field *e(t)*. Now, from the original diffusion equation, we compute the total electric field, shown in Fig. 4. The agreement between two independent solutions is excellent, and it numerically proves that the integral transform is valid.

In basic terms the foregoing exercise can be summarized as follows: A new field *v(q)* has been defined that is a function of pseudotime, q, and which satisfies a wave equation in which the "velocity" is $(\sigma\mu)^{-1/2}$. The new field *v(q)* propagates like a dispersionless field and behaves exactly like a seismic wavelet passing through a medium with velocity discontinuities. Numerical solutions for this wave equation are well understood and require orders of magnitude fewer time steps in a finite-difference or finite-element approximation. When a solution is obtained for *v(q)*, the real field quantity *u(t)*, which is the solution of the actual diffusion equation, can be uniquely computed using the integral given by Equation (3).

The other, and perhaps more important, application of the integral is in em data processing. In this processing technique one tries to solve for the "wave-like" field *v(q)* from a set of data *u(t)*. This is essentially an inverse problem for *v(q)*, for which the integral equation has to be solved numerically. However, if we modify the integral equation slightly, we can obtain the wave field *v(q)* directly without having to solve a system of linear equations. The equation is first Fourier transformed from t to ω. Then, taking only the imaginary parts of both sides of the equation, we find

$$\mathrm{Im}(\tilde{u}\ (\omega)) = - \int_0^\infty e^{-(w/2)^{1/2} q} \sin\left[(w/2)^{1/2} q\right] v(q)\, dq. \tag{4}$$

This equation can be cast into a convolution integral by changing variables, $\omega = 2e^{2\xi}$ and $q = e^{-\eta}$. From the property of the convolution, the solution for the wave field can be formally written as an inverse Fourier transform,

$$v(e^{-\eta}) = - \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\tilde{D}(k)}{\tilde{H}(k)}\, e^{ik\eta}\, dk\,, \tag{5}$$

where $\tilde{D}(k)$ and $\tilde{H}(k)$ are Fourier transforms ($\eta \rightarrow k$) of data $e^{\eta}\,\mathrm{Im}(\tilde{u}\,(2e^{2\eta}))$ and modified kernel $e^{\eta}\, e^{-e^{\eta}}\sin(e^{\eta})$, respectively. The solution $v\,(e^{-\eta})$ is nothing but the $v(q)$ in logarithmic q scale. The same solution (Eq. 5) was independently obtained by Lee and Morrison (1987) from Equations (1) and (2). The inverse Fourier transform, however, is numerically unstable, because the function $\tilde{H}(k)$ decreases exponentially for large k. In order to stabilize the inverse transform, especially when the data $\tilde{D}(k)$ are contaminated by noise, an optimum inverse filter will be designed and incorporated.

At this point a simple numerical test has been made for the inverse Fourier transform. The data used are $-\partial B_z/\partial t$ ($-i\,\omega\mu H_z$ in frequency domain, Fig. 5); we generate these data by shutting off a steady current through a line source. Instead of using an inverse filter the inverse Fourier transform is convolved with a Gaussian window, which is essentially a low-pass filter. The result is shown in Fig. 6 in logarithmic q scale. The wavelet propagates with a velocity of $(\sigma\mu)^{-1/2}$. Therefore, the "travel time" is proportional to the distance between the source and the receiver. This means that an interpreter can easily estimate the location of the source (or reflectors) in a manner similar to what is being routinely done to interpret seismic traces.

Acknowledgment

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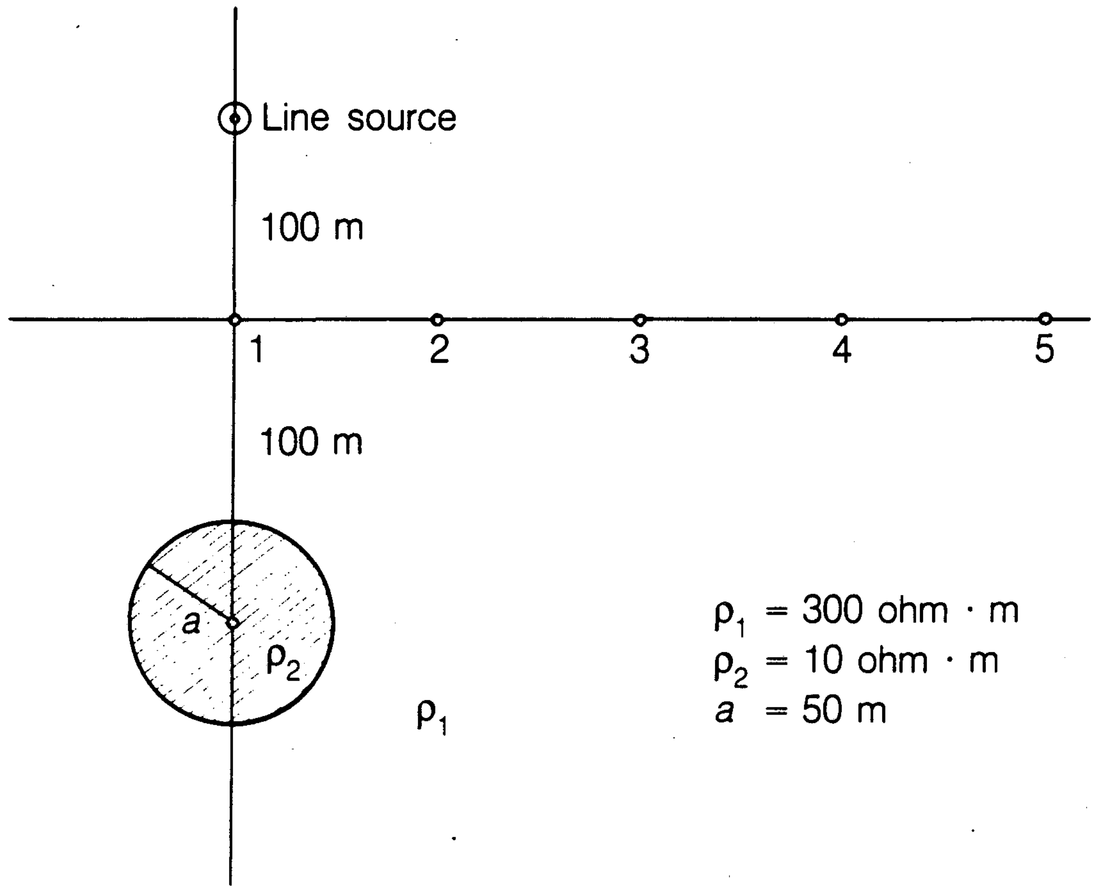

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Figure 1. A conductive circular cylinder in a uniform whole-space. The medium is excited by a line source of current. Fields are computed at positions 1, 2, 3, 4, and 5, separated by 100 m.

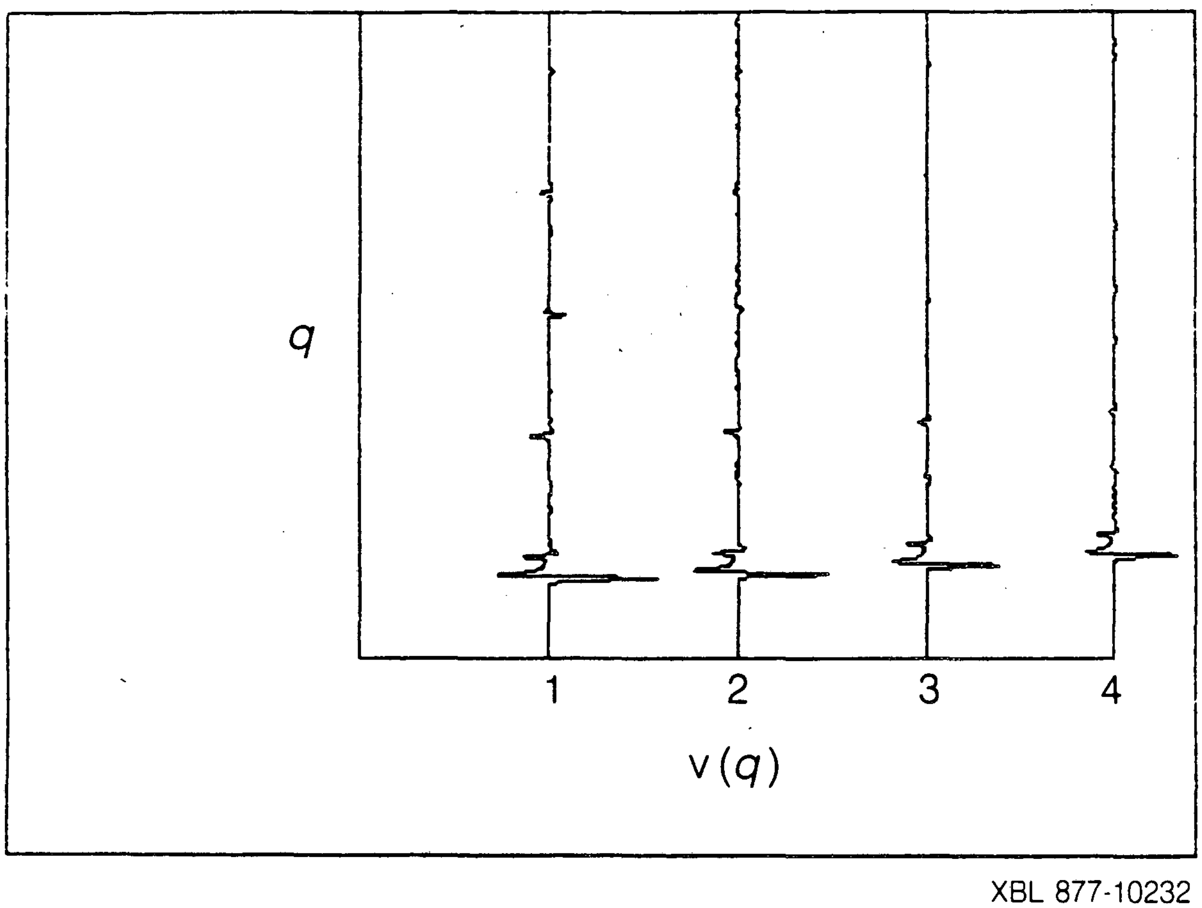

Figure 2. Total wave field $v(q)$ in the presence of a cylinder. The incident and numerous multiple reflections are normalized by the maximum of all traces.

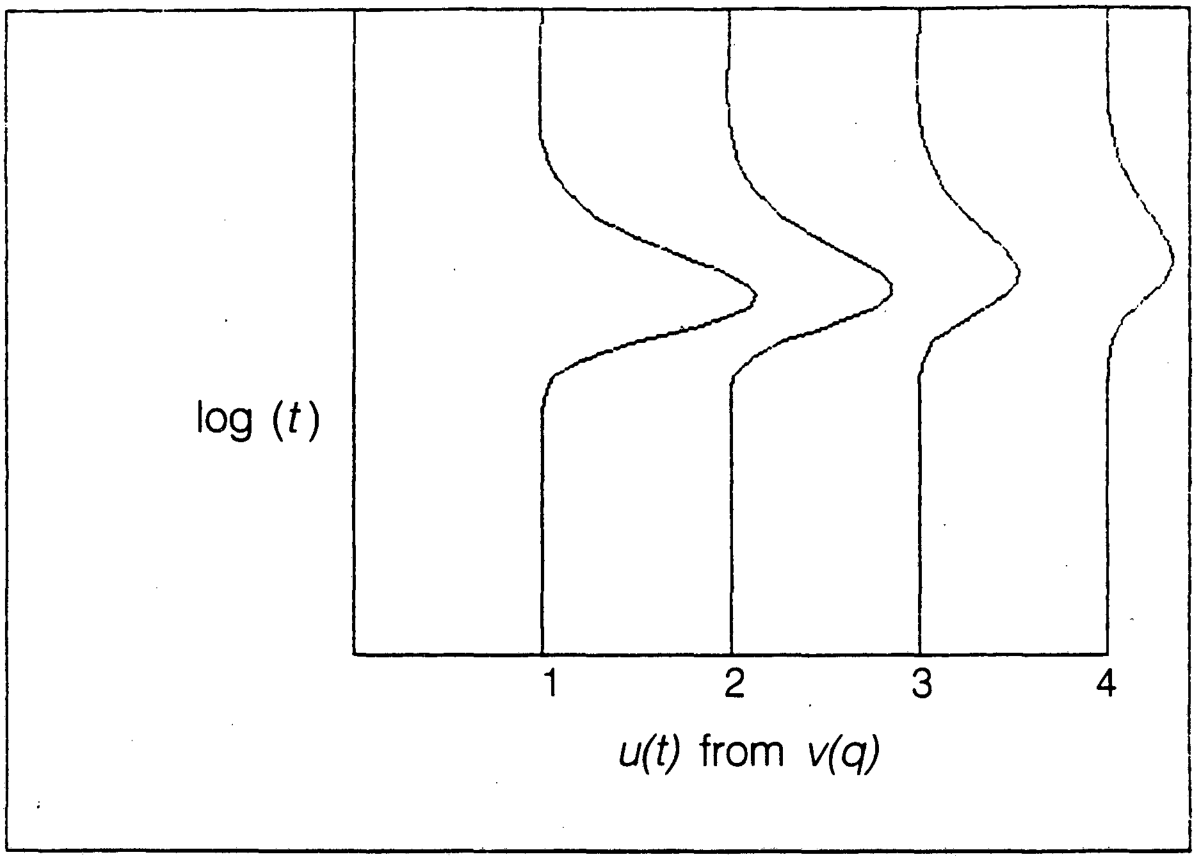

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Figure 3. Transformed total electric field $u(t)$ ($= e(t)$). $v(q)$ in Figure 2 is used to produce this result. All traces are normalized by 1.0020×10^{-5} and plotted in log (t) scale.

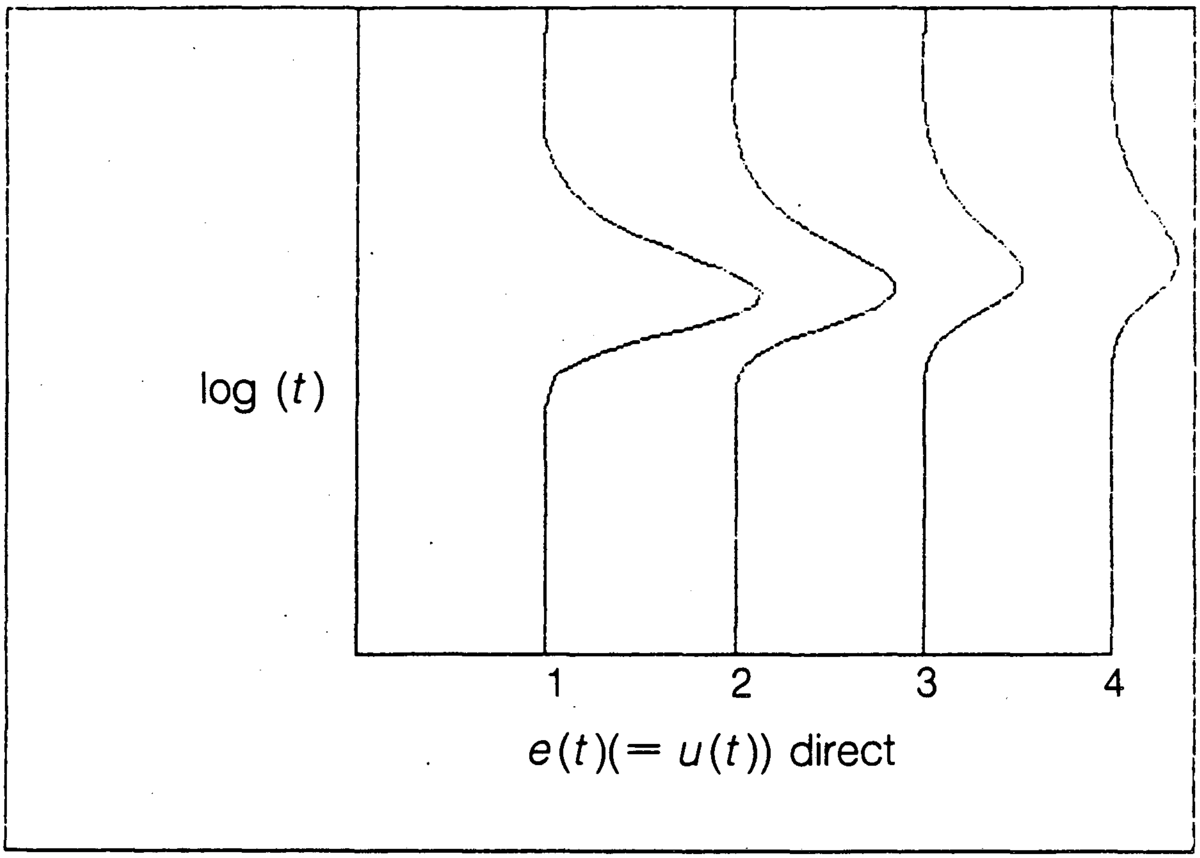

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Figure 4. Conventional time-domain solutions for the total electric field $e(t)$. All traces are normalized by 1.0147×10^{-5} and displayed in log (t) scale.

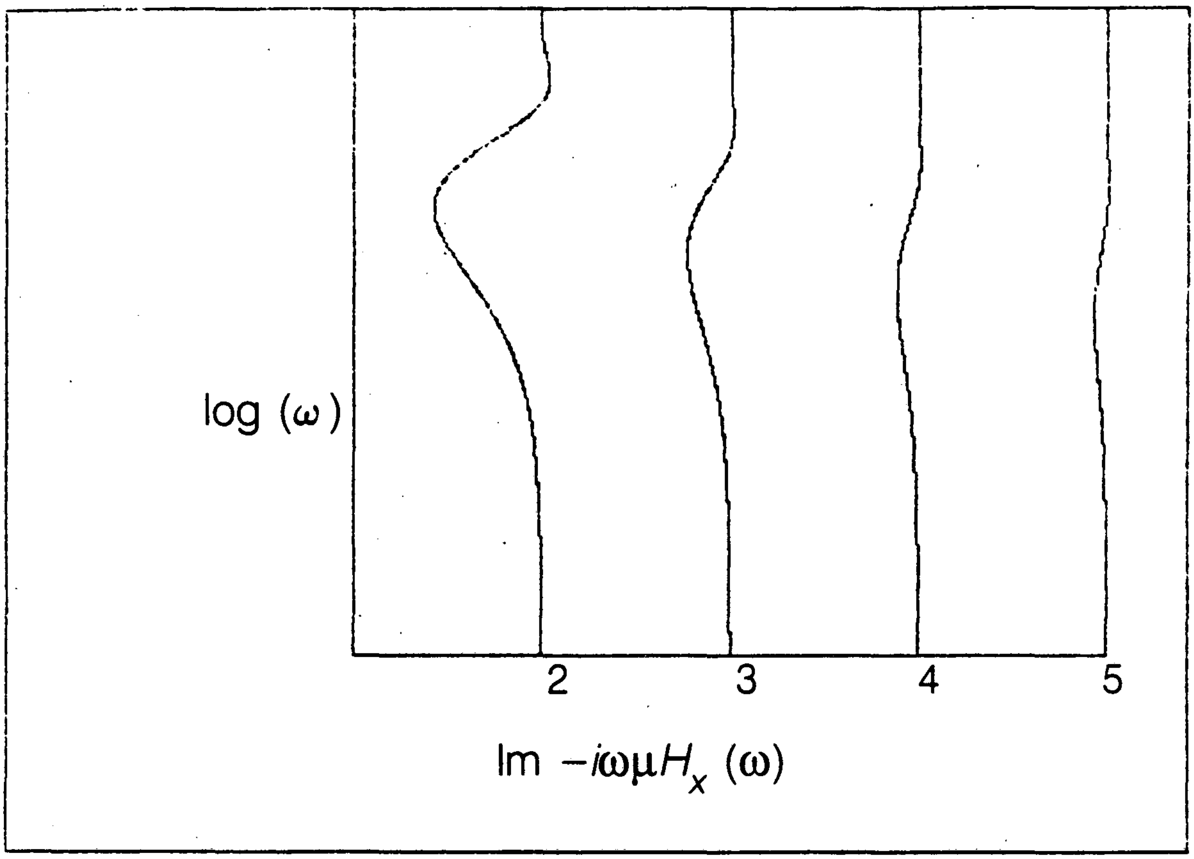

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Figure 5. Imaginary part of $-i\,\omega\mu H_x\,(\omega)$ due to a line source plotted in log (ω) scale. Amplitudes are normalized by 4.5188×10^{-10}.

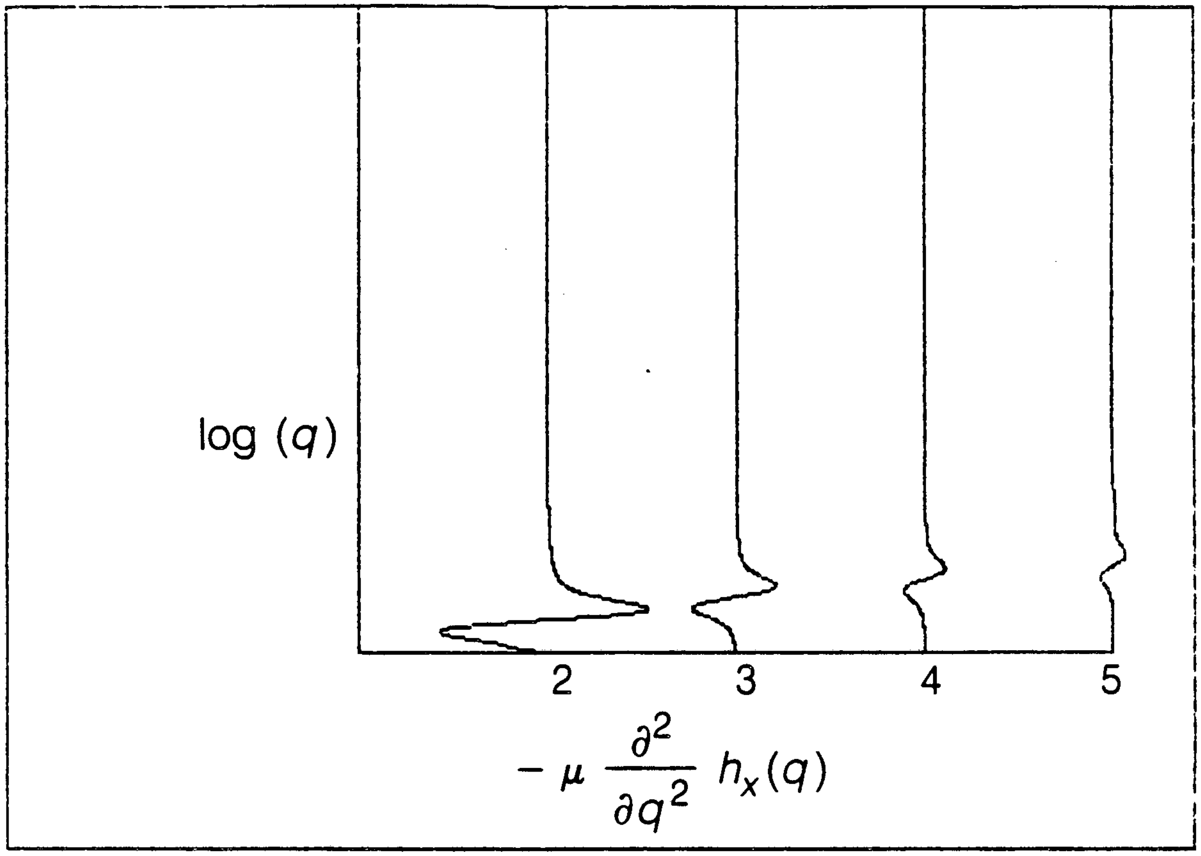

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Figure 6. Inversely Fourier-transformed wave field associated with the frequency-domain (ω) data shown in Figure 5. All traces are normalized by 8.6271 $\times 10^{-8}$. A Gaussian window was used to produce this result. Note that the operator $\partial/\partial t$ transforms into $\partial^2/\partial q^2$.

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