LBL-20269
Preprint c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Physics Division

Submitted for publication

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September 1985

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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Higgs Boson Triplets with $M_W = M_Z \cos\theta_w$*

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Abstract

We construct a potential for Higgs doublets and triplets that preserves $\rho = M_W^2/M_Z^2 \cos^2\theta_w = 1$, allowing the triplets to make the dominant contribution to W and Z boson masses.

*This work was supported by the Director, Office of Energy Research, Office of High Energy Physics and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

At present we know precious little about the spontaneous breaking of the $SU(2) \times U(1)$ gauge symmetry of the electroweak interactions. Perhaps our most important clue is the approximate equality of the rho parameter to unity, $\rho \equiv M_W^2/M_Z^2 \cos^2\theta_w = 1$, satisfied experimentally to within a few percent. This relationship follows if the symmetry breaking gives equal masses to $W^\pm$ and W^3. If the symmetry breaking is due to strong interactions — either of strongly coupled Higgs scalars or strong gauge interactions as in technicolor models — then the equality of $W^\pm$ and W^3 masses must be maintained to all orders in these strong interactions. This can be ensured if the strong interactions obey a global "custodial" SU(2) symmetry.[1] In the standard model with a complex Higgs doublet[2] there is a custodial SU(2) which corresponds precisely to the isospin of the SU(2) sigma model:[3] that is, it is the diagonal SU(2) subgroup which survives the spontaneous symmetry breaking of the global $SU(2)_L \times SU(2)_R$ symmetry of the scalar interactions.

Most other irreducible representations of $SU(2)_L$ do not give $\rho = 1$ even in tree approximation.* For instance, the complex triplet representation, $(t, y) = (1, -1)$, which can generate a Majorana mass for the neutrino while breaking lepton number spontaneously,[5,6] would by itself give $\rho = 2$. The real triplet, $(t, y) = (1, 0)$, would give $\rho = \infty$, as would any real representation. However, it has been noted that one complex and one real triplet taken together (or equivalently three real representations) would give $\rho = 1$ in tree approximation if they have equal vacuum expectation values.[7,8] In general this appears to be an unnatural condition, both aesthetically and in the technical sense that it need not survive quantum corrections from a strongly interacting Higgs sector.

In this paper we exhibit a potential in which this equality of vacuum expectation values is naturally preserved by the interactions of the Higgs potential. It is guaranteed by a custodial $SU(2)$ which survives spontaneous breaking of a global $SU(2)_L \times SU(2)_R$ symmetry, precisely as in the standard model. The complex and real triplet together form a (1,1) representation of $SU(2)_L \times SU(2)_R$, and the model may be understood as a straightforward generalization of the standard model in which the complex doublet forms a $(\frac{1}{2}, \frac{1}{2})$ representation. The extension to $SU(2)_L \times SU(2)_R$ invariant potentials for all representations (t, t)

*The requirement that an irreducible representation of $SU(2)_L$ give $\rho = 1$ in tree approximation yields[4] a Diophantine equation in the isospin t and hypercharge y, $t^2 + t - 3y^2 = 0$, which has 11 solutions for $t < 1,000,000$, the largest being $t, y = 489060\frac{1}{2}, 282359\frac{1}{2}$. We are offering a prise for the most original model based on this representation.

is straightforward.* (However when more than one representation is present, verification of symmetry breaking to a physically acceptable vacuum requires more work.)

While the potential naturally preserves $\rho = 1$, the model *in toto* is no more natural than other models with elementary scalars. Gauge interactions contribute quadratic divergences to scalar self-energies, of order $g^2\Lambda^2$ where g is a gauge coupling constant and Λ a cutoff parameter, giving rise for instance to the GUT hierarchy problem. Because the hypercharge interactions break the custodial SU(2), the model is afflicted not only with the problem of controlling the overall scale of Higgs boson masses but also with quadratically divergent contributions to $\rho - 1$. We are looking into whether the proposed solutions already in the marketplace — supersymmetry and dynamical symmetry breaking — are applicable. In this paper we have nothing new to say about this most serious naturalness problem and will not discuss it further.

In previous work[5,6] using the complex triplet to generate a Majorana mass for the neutrino, the constraint $\rho = 1$ forces the triplet vev to be much smaller than the doublet vev, $v_3 << v_2$. The model contains a true Goldstone boson, the "Majoron", which leads to severe phenomenological constraints, many cosmological in origin. In our model, because of the global $SU(2)_L \times SU(2)_R$, we have no Goldstone boson and $\rho = 1$ is automatic, whether v_2/v_3 is large or small. One interesting new possibility is that the triplets make the dominant contribution to the W mass, $v_3 \geq v_2$ or even $v_3 >> v_2$. The doublet vev v_2 could then be much smaller than the 250 GeV value of the standard model, so that quark and charged lepton masses could be obtained with larger Yukawa coupling constants than the very small values needed in the standard model. Lepton number will be conserved unless we choose to break it explicitly by introducing a Majorana coupling of the complex triplet to the leptons.** The model has very different phenomenological implications than the triplet Majoron models,[5,6] both because v_3 can be large and because of the absence of a Goldstone boson. In this paper we confine ourselves to describing the bosonic sector of the model.

The scalar fields are the usual complex doublet, written in the 2×2 matrix notation which best displays the $SU(2)_L \times SU(2)_R$ symmetry of the potential,

*This generalization of the three triplet ansatz is also given (without specifying a potential) by Robinett.[9]

**This breaks the custodial $SU(2)$ but the contribution to $\rho - 1$ is acceptably small.

$$\Phi = i \begin{pmatrix} -i\phi_0 & i\phi_+ \\ -i\phi_- & i\bar{\phi}_0 \end{pmatrix} \tag{1}$$

and the complex triplet χ and real triplet ς, written as an analogous 3×3 matrix,

$$\chi = \begin{pmatrix} -i\chi_0 & \varsigma_+ & i\chi_{++} \\ -i\chi_- & \varsigma_0 & i\chi_+ \\ -i\chi_{--} & \varsigma_- & i\bar{\chi}_0 \end{pmatrix}. \tag{2}$$

Our phase conventions are such that $\phi_0^* = \bar{\phi}_0, \phi_-^* = -\phi_+, \chi_0^* = \bar{\chi}_0, \chi_-^* = -\chi_+, \chi_{--}^* = \chi_{++}, \varsigma_-^* = -\varsigma_+$, and $\varsigma_0^* = \varsigma_0$. The action of $SU(2)_L \times SU(2)_R$ rotations is then $\Phi \to U_L \Phi U_R^\dagger$ and $\chi \to U_L \chi U_R^+$ where $U_{L,R} = e^{-i\theta_{L,R}\hat{n}_{L,R}\cdot\vec{T}_{L,R}}$ is a rotation of magnitude $\theta_{L,R}$ about the axis $\hat{n}_{L,R}$ and $\vec{T}_{L,R}$ are the appropriate representations of the $SU(2)$ generators. The generators $\vec{T}_L$ and T_R^3 are just the gauged generators of $SU(2)_L \times U(1)_Y$ and therefore must be invariances of the potential. We further require the potential to be symmetric under the full global $SU(2)_L \times SU(2)_R$.

For simplicity in this paper we also impose a discrete symmetry, $\chi \to -\chi$, to eliminate cubic vertices from the potential. (This does not qualitatively effect the physics except in one instance noted below.) Then the most general $SU(2)_L \times SU(2)_R$ symmetric potential may be written in the convenient form (inspired by the form of V in ref. (6))

$$\begin{aligned} V(\Phi,\chi) &= \lambda_1(\mathrm{Tr}\,\Phi^+\Phi - v_2^2)^2 + \lambda_2(\mathrm{Tr}\,\chi^+\chi - 3v_3^2)^2 \\ &+ \lambda_3(\mathrm{Tr}\,\Phi^+\Phi - v_2^2 + \mathrm{Tr}\,\chi^+\chi - 3v_3^2)^2 \\ &+ \lambda_4(\mathrm{Tr}\,\Phi^+\Phi\,\mathrm{Tr}\,\chi^+\chi - 2\,\mathrm{Tr}\,\Phi^+T^i\Phi T^j \cdot \mathrm{Tr}\,\chi^+T^i\chi T^j) \\ &+ \lambda_5\left(3\,\mathrm{Tr}\,\chi^+\chi\chi^+\chi - (\mathrm{Tr}\,\chi^+\chi)^2\right). \end{aligned} \tag{3}$$

We impose the conditions $\lambda_1 + \lambda_2 + 2\lambda_3 > 0$, $\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3 > 0$, $\lambda_4 > 0$, $\lambda_5 > 0$ so that V is positive semidefinite. To minimize V it is convenient to choose an $SU(2)_L$ gauge such that Φ is proportional to the unit matrix, $\Phi = \frac{1}{\sqrt{2}}h_\phi I$. Then the λ_4 term, which assures proper alignment of the two vevs, is

$$\frac{1}{2}\lambda_4 h_\phi^2 \left[(\mathrm{Im}\,\chi_0 - \varsigma_0)^2 + (Re\chi_0)^2 + \chi_{--}^*\chi_{--} + \chi_-^*\chi_- + \varsigma_-^*\varsigma_-\right]$$

For $\lambda_4 > 0$ this has its minimum at $\mathrm{Im}\,\chi^0 = \varsigma^0$ with the other components of χ and ς vanishing. The entire potential is then minimized by $\langle h_\phi \rangle_0 = v_2$ and $\langle \mathrm{Im}\,\chi_0 \rangle_0 = \langle \varsigma_0 \rangle_0 = v_3$. In the matrix notation the minimum is at $\langle \Phi \rangle_0 = \frac{1}{\sqrt{2}} v_2 I$ and $\langle \chi \rangle_0 = v_3 I$, so that $SU(2)_L \times SU(2)_R$ is spontaneously broken to the diagonal $SU(2)$ subgroup, the custodial $SU(2)_C$.

The gauge invariant kinetic energy terms are

$$\mathcal{L}_{KE} = \frac{1}{2} \mathrm{Tr}\left[(D^\mu \Phi)^\dagger (D_\mu \Phi) \right] + \frac{1}{2} \mathrm{Tr}\left[(D^\mu \chi)^\dagger (D_\mu \chi) \right] \tag{4}$$

where $D^\mu = \partial^\mu \Phi - ig\vec{T} \cdot \vec{W} \Phi + ig' \Phi T_3 B$ and $D^\mu \chi$ is defined similarly. Shifting scalar fields to have vanishing vevs we find that the mixture of scalar fields which mix with W_- and W_3, B are respectively

$$\begin{aligned} w_+ &= \frac{1}{v}(v_2 \phi_+ + 2 v_3 (\chi_+ + i\varsigma_+)) \\ z &= \frac{1}{v}(v_2 \phi_3 + 2\sqrt{2} v_3 \chi_5) \end{aligned} \tag{5}$$

where

$$v \equiv \sqrt{v_2^2 + 8 v_3^2} \tag{6}$$

and ϕ_3 and χ_5 are the real parts of ϕ_0 and χ_0, defined $\phi_0 = \frac{1}{\sqrt{2}}(\phi_3 + i(h_\phi + v_2))$ and $\chi_0 = \frac{1}{\sqrt{2}}(\chi_5 + i(h_\chi + \sqrt{2} v_3))$. The third neutral field is $\varsigma_0 = h_\varsigma + v_3$. From eq. (4) the W_+ mass is $M_W = \frac{1}{2} g v$ and $M_Z = M_W / \cos\theta_w$.

The scalar mass spectrum is obtained from the quadratic terms in the potential eq. (3). There are three massless particles, precisely the swallowed Higgs bosons of eq. (5). The remaining ten massive particles form a 5, a 3, and two 1's of $SU(2)_C$. The masses of the 5 and 3 are

$$m_5^2 = 3\lambda_4 v_2^2 + 24 \lambda_5 v_3^2 \tag{7}$$

$$m_3^2 = \lambda_4 (v_2^2 + 8 v_3^2) \tag{8}$$

while the two 1's are the eigenstates of the mass matrix

$$M^2_{h_\phi, h_1} = \begin{pmatrix} 8(\lambda_1 + \lambda_3) v_2^2 & 8\sqrt{3} \lambda_3 v_2 v_3 \\ 8\sqrt{3} \lambda_3 v_2 v_3 & 24(\lambda_2 + \lambda_3) v_3^2 \end{pmatrix} \tag{9}$$

where h_ϕ was defined above and $h_1 = \frac{1}{\sqrt{3}}(\sqrt{2} h_\chi + h_\varsigma)$. The composition of the 5 and 3 in terms of components of Φ and χ are as given in ref. (8), where they were deduced group theoretically assuming the existence of the $SU(2)_C$ symmetry. The mixing of the singlets h_ϕ and h_1 cannot be determined group theoretically.

Our model has no Majoron because the corresponding "lepton" $U(1)$ is broken explicitly by the λ_4 interaction. This $U(1)$ rephases the complex triplet $(\chi^0, \chi^-, \chi^{--})$, but not ς or ϕ, so that it is broken by terms in the λ_4 interaction proportional to $\varsigma^0 \chi^0$ and $\varsigma^+ \chi^-$. These terms are dictated by the $SU(2)_L \times SU(2)_R$ symmetry and the necessity of a physically acceptable vacuum. Were we to take $\lambda_4 = 0$, a condition which could be naturally maintained by the "lepton" $U(1)$ symmetry, we would in fact find an additional triplet of Goldstone bosons, eq. (8), reflecting the larger initial symmetry of V with $\lambda_4 = 0$. But with $\lambda_4 = 0$ the potential does not align the vevs of Φ and χ and prevent the photon from acquiring a mass.*

An interesting regime of the model has the five λ_i of the same order of magnitude and $v_3 > v_2$. In this case the triplets make the dominant contribution to the W and Z masses. Diagonalizing the mass matrix eq. (9) to leading order in the small parameter $v_2^2 / 3 v_3^2$ we find that one of the eigenstates has a mass proportional to v_2^2,

$$m^2_{h_L} = 8 \frac{\lambda_1 \lambda_2 + \lambda_1 \lambda_3 + \lambda_2 \lambda_3}{\lambda_2 + \lambda_3} v_2^2 \tag{10}$$

substantially lighter than the other surviving scalars with masses $m_5^2 \sim 24 \lambda_5 v_3^2$, $m_3^2 \sim 8 \lambda_4 v_3^2$, and $m^2_{h_H} \sim 24(\lambda_2 + \lambda_3) v_3^2$. This light boson h_L has couplings to quarks and charged leptons that are enhanced by v/v_2 relative to the couplings of the standard model Higgs. We have investigated whether it might be the $\xi(2220)$, the possibly narrow state seen in $\psi \to \gamma\xi \to \gamma \bar{K} K$ by the Mark III collaboration.[10] This hypothesis is apparently excluded by the bound[11] on $\Upsilon \to \gamma\xi$.

The potential discussed here, with no cubic interactions, has two gauge inequivalent degenerate minima, distinguished by the sign of the vev $\langle \chi \rangle_0 = \pm v_3 I$. Such a degeneracy might have cosmological implications. The degeneracy is lifted by allowing cubic interactions, which do not otherwise qualitatively change the principal results.

*The cubic interaction which aligns the vevs also breaks the "lepton" $U(1)$.

Note added: After completion of this manuscript we became aware of a paper by P. Galison (Nucl. Phys. B232, 26 (1984)) in which $\rho = 1$ is achieved with an $SU(n)_L \times SU(n)_R$ symmetric potential for $\Phi\epsilon(n, \bar{n})$. This is a stronger requirement than ours (we only impose $SU(2)_L \times SU(2)_R$), and the three real triplet[7] construction is not a special case (e.g., for $n = 3$ Galison's construction gives three complex triplets).

Acknowledgment

We thank Michael Dugan and Graciella Gelmini for helpful discussions. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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