Title
Contract, Mechanism Design, and Technological Detail

Author
Watson, Joel

Publication Date
2002

2002-04

UNIVERSITY OF CALIFORNIA, SAN DIEGO

DEPARTMENT OF ECONOMICS

CONTRACT, MECHANISM DESIGN, AND TECHNOLOGICAL DETAIL

BY

JOEL WATSON

DISCUSSION PAPER 2002-04
FEBRUARY 2002

Contract, Mechanism Design, and Technological Detail

Joel Watson*

Preliminary version, May 2001
Revised January 2002

Abstract

This paper develops a theoretical framework for studying contract and enforcement in settings of complete, but unverifiable, information. The main point of the paper is that the consideration of renegotiation *necessitates* formal examination of other technological constraints, especially those having to do with the timing and nature of inalienable productive decisions. The main technical contributions include (a) results that characterize of the sets of implementable state-contingent payoffs under various assumptions about renegotiation opportunities, and (b) a result establishing conditions under which, when trading opportunities are durable and trade decisions are reversible, stationary contracts are optimal. The analysis refutes the validity of the "mechanism design with ex post renegotiation" program, it demonstrates the validity of other mechanism design models in dynamic environments, and it highlights the need for a more structured game-theoretic framework. *JEL Classification: C70, D74, K10.*

*University of California, San Diego. Internet: http://weber.ucsd.edu/~jwatson/. I thank Jesse Bull, Vince Crawford, Aaron Edlin, Oliver Hart, W. Bentley MacLeod, John Moore, Joel Sobel, and my colleagues at UC San Diego for their comments. The research reported herein is supported by NSF grant SES-0095207.

Economic models of contract have yielded important insights regarding the nature of contractual imperfections, optimal contractual form, and contractual scope. Many of the insights derive from mechanism design analysis, which strips away institutional details to focus on just a few fundamental ingredients. Mechanism design is an abstract and elegant methodology. The mathematical elegance comes at a cost, however, to the extent that real institutions and technology limit the formation and enforcement of contracts.

Researchers have rightly turned their attention to the actual constraints that institutions impose, in an attempt to clarify our understanding of contractual imperfections and to inform the design of enforcement systems. One issue that has received a great deal of attention is the possibility that parties can *renegotiate* in the midst of a contractual relationship. Hart and Moore's (1988) seminal article shows how renegotiation following specific investments can greatly inhibit the parties' ability to attain an efficient outcome. Recently, theorists have attempted to incorporate renegotiation constraints into otherwise standard mechanism design analysis. Maskin and Moore (1999) developed the basic *mechanism design with ex post renegotiation* (MDER) program, which assumes that parties can renegotiate the contractually-specified outcome after sending messages to an external enforcer. Maskin and Moore's methodology and characterization results have been widely accepted and employed.[1]

In this paper, I study how renegotiation opportunities interact with the productive technology of contractual relationships. Specifically, I do two things. First, I show that, in order to adequately address renegotiation, we *must* account for the *technology of trade*. In other words, our models must explicitly describe the timing and nature of productive actions. Second, I develop a general analytical framework and I characterize the set of implementable outcomes under a variety of assumptions about when renegotiation can take place. My main technical result is that, for most environments in which trade decisions can be reversed, stationary contracts are optimal and implementability does not depend on the degree to which trading opportunities are durable.

My results do not challenge the legitimacy of mechanism design theory for the study of contract. However, my results reveal that the *application* of mechanism design theory can be seriously flawed if one does not incorporate the proper technological constraints. Indeed, I show that a segment of the contract theory literature has such a flaw. Specifically, I find that the popular MDER program never accurately describes the scope of contracting; this program can only be justified on the basis of unreasonable incompleteness assumptions that are hidden in the current literature.

[1]The MDER program builds from Maskin's (1999) seminal contribution on Nash implementation (without renegotiation). The literature contains numerous, high-profile papers that adopt the MDER program, including the important work of Che and Hausch (1999), Edlin and Reichelstein (1996), Segal (1999), and Segal and Whinston (2000).

On the other hand, I show that the "mechanism design with interim renegotiation" (MDIR) program, which assumes that renegotiation occurs only before parties send messages to an external enforcer, is justified in some settings.

To see the main idea behind my research, consider the mechanism design ideal. The mechanism design program starts with a set of players, a set of *states* (describing the informational setting), a set of *outcomes* (describing those things players care about that can be controlled publicly), and a specification of preferences over states and outcomes. For a contractual setting, the outcome is usually assumed to describe verifiable trade decisions, which an external enforcement authority can compel. Then the scope of enforceable contracts is given by the set of mechanisms (game forms). The goal of contracting is to implement the players' preferred state-contingent value function, which either helps them share risk or gives them the incentive to make investments that influence the state.

Within this mechanism design framework, institutional constraints are best modelled as a limit on the class of mechanisms. To represent the "ex post renegotiation" constraint, for example, theorists have focused on game forms with a final outcome-renegotiation stage, where the players always negotiate—with fixed bargaining weights—to an ex post efficient outcome, subject to some contractually-specified default outcome. Ex post renegotiation is a convenient modeling component, because the resulting constrained mechanism design problem can be easily transformed into a standard unconstrained problem. To get the unconstrained representation, one simply redefines the players' payoffs over outcomes to be the post-renegotiation payoffs.[2]

The problem with abstract assumptions such as "ex post renegotiation" is that, because they have no direct institutional foundation, they are difficult to interpret. Questions such as "Does renegotiation occur before or after verifiable trade decisions?" and "How does an external enforcer compel a particular trade decision when players can renegotiate ex post?" have no clear answers. To provide answers to these questions, and to understand the effect of renegotiation opportunities, one must sort out the timing of trade, enforcement, and renegotiation. This requires dispensing with the treatment of verifiable trade decisions as "public."

My framework has a more detailed theoretical foundation than is common in the literature. I develop a structured, game-theoretic model that explicitly accounts for the following essential elements: (a) the timing and nature of individual, inalienable decisions; (b) the manner in which the external enforcer compels behavior; and (c) at what times the parties have the opportunity to renegotiate their contract.[3]

[2]On a technical level, ex post renegotiation also creates useful theoretical regularities. Segal and Whinston's (2000) differential analysis takes advantage of these.

[3]My approach is thus allied with that of Hart and Moore (1988), MacLeod and Malcomson (1993), and Nöldeke and Schmidt (1995), who model individual trade decisions. I elaborate on this

With these elements in place, one can clearly describe and interpret renegotiation constraints. Regarding (a), observe that, in reality, most trade decisions are inalienable; for example, a buyer and a seller cannot instruct the court to deliver the good that they wish to exchange.[4] Thus, on (b), courts do not enforce trading outcomes by actually making the trade decisions for the contracting parties. Rather, enforcement occurs via transfers and penalties that courts compel conditional on the parties' verifiable, individual trade decisions.[5]

The key observation—which is obvious once trade decisions are formally modelled—is that there are two ways of building *options* into contracts. First, a contract can trigger externally enforced transfers on the basis of whether a party sends a particular verifiable message before the trading opportunity. Second, the trade decisions *themselves* may serve as options, with the externally enforced transfer simply a function of the parties' behavior.[6] There is no practical difference between using these two types of options in the absence of renegotiation opportunities. However, when renegotiation is possible, its effect depends on whether it can occur (i) before the parties send messages or (ii) between the message phase and when the parties make trade decisions. In the former case, renegotiation affects implementability exactly as characterized by the MDIR program. In the latter case, message-based options are subject to renegotiation. Parties can avoid the detrimental consequences of renegotiation by using trade decisions as options, but the technology of trade limits the scope of this scheme. Implementability in this case is not characterized by any of the literature's standard mechanism design programs.

The following section contains the analysis of an extended example, in which the contracting parties face a nondurable trading opportunity. The example introduces my modeling framework and it illustrates my basic points. I examine three different settings that are distinguished by when, if ever, the contracting parties can renegotiate their contract. I characterize the set of implementable state-contingent payoff vectors

in the next section.

[4]Or, at least, the court would not honor such a request. In a mechanism design model, inalienable trade decisions imply a constraint on the class of mechanisms. For example, suppose that, in a particular trading relationship, the seller must decide whether to deliver intermediate good A, good B, or nothing. Then, the mechanism must be a game form in which the seller makes this decision at some point. Because the decision is inalienable, we cannot have a mechanism that specifies only, say, the "good A" and "nothing" outcomes (so that the seller is not allowed to choose "good A").

[5]Specific performance, for instance, is a court order to act (under penalty of contempt) rather than the court taking the action on behalf of the contracting party.

[6]Mechanism design models study options of the first type; more structured models, such as Nöldeke and Schmidt's (1995), focus on the second type. The law treats option generally, as a limit on a parties "power to revoke an offer" (Section 25, *Restatement (Second) of Contracts*; see Barnett 1999). In addition to more conventional forms, options are implicitly created by liquidated damage provisions and standard breach remedies. (A party has the option of breaching and then paying the damage amount.)

for each setting and I relate them to the set identified by the standard mechanism design programs from the literature.

Sections 2-5 define and analyze my general modeling framework. Section 2 describes a "period" of interaction, in which the contracting parties form and renegotiate their contract, make trade decisions, and interact with the external enforcer. In Section 3, I derive necessary conditions for the implementation of state-contingent payoff vectors under various assumptions about when the parties can renegotiate.

In Section 4, I explain how to model settings in which the trading opportunities are durable and trade decisions are reversible. In such settings, the parties interact over an infinite number of periods. The parties can make any particular trade decision in one period and then reverse it in a future period. Parties can write long-term contracts and they can renegotiate at various dates within a period.

Section 5 contains my technical results. I provide a theorem that ranks by inclusion the sets of implementable state-contingent payoffs under various assumptions about renegotiation. This result formalizes for a very general environment what the example of Section 1 demonstrates. I then present the main result—that for most of the settings studied here, *stationary* contracts are always optimal. This result implies that the parties' long-term contracting problem can be reduced to a simple static problem, which has important positive and normative implications and clarifies the proper use of mechanism design theory. The optimality of stationary contracts further implies that the extent of durability is irrelevant to the contracting problem, which gives a useful benchmark for future theoretical work.

Section 6 comprises two examples that further clarify the shortcomings of the MDER program while confirming, with qualifications, that hold-up is indeed a problem in some contractual relationships. Section 7 contains concluding remarks.

Numerous authors have argued for the kind of research reported herein. Hurwicz (1994) speaks of the importance of incorporating institutional constraints into design problems—a step that, for the most part, has yet to be taken in any general, compelling way. He suggests that institutional constraints should be represented as limiting design to a class of game forms, whereby the " 'desired' game form [is embedded in what he calls] the 'natural' game form" (p.12). My framework may be interpreted as this natural game form. Anderlini, Felli, and Postlewaite (2001), Segal and Whinston (2000), and others recognize the need to study technological and institutional constraints in contracting environments. Furthermore, the contract theory literature has seen several debates regarding renegotiation and its relation to messages and productive actions.[7] Against this backdrop, my message should be clearly

[7]For example, Nöldeke and Schmidt (1995) point out that Hart and Moore's (1988) underinvestment problem disappears if the parties' individual trade decisions are verifiable, rather than just partially verifiable. Edlin and Hermalin (2000) argue that Nöldeke and Schmidt (1998) and Bernheim and Whinston (1998) incorrectly model the timing of options, investment, and renegotiation—that,

understood: To make sense of modeling choices and to have an instructive debate, we must use a theoretical framework that explicitly accounts for the technology of trade and enforcement.

1 An Example

Consider a simple contractual setting, featuring a buyer, a seller, and an external enforcement authority (the court). To be concrete, imagine that the buyer is a masonry supply company that hopes to gain new customers at a regional trade show. The seller is an advertisement agency. The buyer wishes to hire the seller to develop an advertising package for the trade show.

The parties' payoffs depend on (a) specific investments, (b) whether the buyer adopts the advertisement package that is developed by the seller, and (c) monetary transfers. The parties' investments determine the *state of the relationship* θ. Suppose that $\theta \in \{\mathrm{H}, \mathrm{L}\}$, where H indicates the "high" state—meaning the advertising package will be successful—and L denotes the "low" state—where the advertisement will not be successful. The buyer's decision to adopt the advertisement can also be described as "the buyer consummates the trade" or "the buyer accepts delivery."

Above any instantaneous investment costs, the payoffs are defined as follows. In state H, if the buyer adopts the advertisement package and makes a monetary transfer p to the seller, then the buyer gets $5 - p$ and the seller gets $3 + p$. The buyer's value of 5 here is the profit generated by a successful advertisement. The seller's value of 3 reflects the extra profit the advertising agency will receive from future clients due to its public success with the masonry firm. In state H, if the buyer decides not to adopt the advertisement package yet transfers p to the seller, then the buyer gets $-p$ and the seller gets p. In state L, the advertisement package is worthless to both the buyer and the seller; in this case, regardless of whether the buyer adopts the advertisement, the buyer obtains $-p$ and the seller obtains p.

Assume that the state is jointly observed by the contracting parties but that it cannot be verified to the external enforcement authority. On the other hand, the *trade decision* and any *messages* sent by the parties are verifiable. The trade decision in this simple example is the individual choice of the buyer as to whether to adopt the advertisement for the trade show. The trading opportunity is *nondurable*—that is, the buyer's decision of whether to adopt the advertisement cannot be reversed.[8]

because of ex post renegotiation following opportunities to exercise an option, hold-up is a severe problem. I comment on this in the Conclusion. In work that criticizes Hart and Moore's analysis of incomplete contracts, Lyon and Rasmusen (2001) argue that parties should, in reality, be able to rescind and change option orders after an opportunity for renegotiation expires. See also MacLeod (2001) on renegotiation and the timing of the resolution of uncertainty.

[8]The buyer must make his trade decisions just before the trade show begins. Once the trade

Date	1	Buyer and seller establish a contract.
	2	Unverifiable investments determine the state.
	3	[Possible renegotiation of the contract.]
	4	Parties send verifiable messages..
	5	[Possible renegotiation of the contract.]
	6	Buyer makes a verifiable decision whether to adopt.
	7	External enforcer compels a transfer.

Figure 1: Stages of the relationship in the example.

Finally, the external enforcer's role is to compel transfers between the parties on the basis of verifiable information, as specified by the parties' contract.

Modeling the Contractual Relationship

To model this contractual setting, I examine a class of game forms that explicitly account for (a) the timing and nature of individual, inalienable decisions, (b) the manner in which the external enforcer compels behavior, and (c) the opportunities parties' have to renegotiate their contract.[9] Specifically, I suppose that the parties interact over seven dates, as shown in Figure 1.

At Date 1, the buyer and the seller form a contract, which specifies a court-enforced transfer p that is to be compelled at Date 7, conditional on verifiable events. At Date 2, investments are made and the state θ is realized. I do not bother modeling the actual investment decisions, or the investment costs, because the analysis will only concern how future payoffs (after Date 2) can be conditioned on the state.

At Date 4, the parties send verifiable messages. The trading opportunity occurs at Date 6. Here, the buyer individually decides whether or not to adopt the advertisement. Finally, at Date 7, the court automatically obtains the parties' contract, observes the messages (from Date 4) and the buyer's trade decisions (from Date 6), and compels a transfer p from the buyer to the seller as directed by the contract.[10]

show ends, there is no use for the advertisement and there is no way to undo the advertisement if it was adopted.

[9]Thus, like Hart and Moore (1988), I explicitly model the trade decisions and the external enforcer's role. In this example and in my general model, the trade decisions are fully verifiable, as is commonly assumed in the "complete contract" theory literature. Hart and Moore (1988) and MacLeod and Malcomson (1993) study settings in which trade decisions are only partially verifiable. Nöldeke and Schmidt (1995) study the variant of Hart and Moore's model in which the trade decisions are fully verifiable.

[10]I assume that the contract cannot direct the court to burn the parties' money by, say, transferring it to a third party. This assumption is justified by both an institutional reality—courts do not enforce penalties—and a renegotiation opportunity—the contracting parties would recontract just before the court acts. Regarding the assumption that verifiable information is automatically transmitted to the external enforcer, see Bull and Watson (2001) and Bull (2001).

The payoffs are then as described above.

The contracting parties may have the opportunity to renegotiate their contract at Date 3 and/or Date 5. In a renegotiation phase, the parties replace their existing contract with a new one. I assume that the new contract maximizes their joint payoff in the current state (over all feasible contracts) and that it divides the surplus of renegotiation equally between them. In other words, renegotiation is resolved according to standard bargaining solutions such as the Nash solution. The disagreement point is defined by the payoffs the parties would have received were they to continue under their original contract.[11] If the parties renegotiate, then their new contract is the one submitted to the external enforcer at Date 7.

To have a successful relationship, the parties must design a contract at Date 1 that will align their incentives to invest at Date 2. This critically depends on how the payoffs from Date 3 can be made contingent on the state. For example, suppose that only the seller makes an investment at Date 2. Suppose the seller chooses between a large investment, at an immediate monetary cost of 5, and a small investment, which costs 0. In this case, efficiency requires that the seller make the large investment at Date 2 and that the buyer adopt the advertisement at Date 6, for a total joint payoff of $(5 + 3) - 5 = 3$. However, the seller will not have the incentive to invest unless the difference between his transfer in state H and his transfer in state L is at least 2. That is, from the beginning of Date 3, his payoff in state H must be at least 5 greater than is his payoff in state L.

A *state-contingent value function* is a function $v\colon \{\mathrm{H}, \mathrm{L}\} \to \mathbf{R}^2$ that gives the parties' payoffs from the beginning of Date 3. The essential contract theory analysis involves determining the set of implementable state-contingent value functions. The results depend, of course, on our theory of behavior at Dates 3 through 6 and whether renegotiation is possible. Using the example, I next review how analysis is normally done by contract theorists and I demonstrate why we must pay closer attention to the technology of trade.

Forcing Contracts and Mechanism Design

In the example, the buyer's trade decision is verifiable, so we can think of the external enforcer as forcing the buyer to take the particular action that the contract directs. This can be done using a *forcing contract*, which specifies a large transfer from the buyer to the seller in the event that the buyer does not take the contractually-specified action. For instance, the buyer can be forced to adopt the advertisement

[11]Fixed bargaining weights capture the idea that renegotiation activity is non-contractible, so that the parties can exercise bargaining power and hold each other up during the relationship. This assumption is realistic for many applied settings and it is a key ingredient of most recent contract models in the literature. It is standard in the literature to model renegotiation using a cooperative game solution, although in some papers, such as Hart and Moore (1988) and MacLeod and Malcomson (1993), theorists analyze a non-cooperative model of bargaining.

by a contract that specifies a transfer of $\hat{p}$ if the buyer adopts and $\hat{p}+6$ if the buyer does not adopt, for any given $\hat{p}$. Regardless of the state, the buyer then has a strict incentive to adopt the advertisement. With this contract, the outcome of Dates 6 and 7 will be adoption of the advertisement and a transfer of $\hat{p}$.

Forcing contracts lie implicitly behind the literature's treatment of verifiable decisions as public (essentially alienable). The traditional view is that, because the buyer's trade decision is verifiable and can therefore be forced, we might as well assume—for modeling simplicity and elegance—that it can be taken out of the buyer's hands. In the mechanism design framework, the buyer's trade decision thus becomes an element of the abstract *outcome*, which the mechanism compels as a function of the parties' messages. Then one can perform standard mechanism design analysis, justified by the fact that the full scope of implementable values can be achieved by conditioning the outcome on the Date 4 messages.[12] The mechanism design problem is defined by the set of states (here H and L), the set of outcomes (here monetary transfers and whether the advertisement is adopted), and the specification of payoffs. Implementation means that a state-contingent value function arises from equilibrium play in the mechanism.[13]

Mechanism Design with Interim Renegotiation (MDIR)

I first perform the analysis of the example for the case in which renegotiation occurs only at Date 3, which is after parties observe the state but before the contractual mechanism is played. I call this the *interim renegotiation* case. To calculate the set of implementable state-contingent value functions in this setting, we can focus on forcing contracts. Thus, the external enforcer can impose any transfer p from the buyer to the seller, and the enforcer can also force the buyer to adopt or not adopt the advertisement. If the enforcer compels adoption, then the payoff vector is $(5-p, 3+p)$ in the high state and $(-p, p)$ in the low state. If the enforcer compels the buyer not to adopt, then the payoff vector is $(-p, p)$ regardless of the state.

By the revelation principle, we can constrain attention to direct-revelation mechanisms. These are game forms in which the players individually submit reports of the state (at Date 4) and then the external enforcer compels the prescription of a mapping from the space of message profiles to the space of outcomes.

Because renegotiation only occurs before the message phase, the contract may

[12]Most models in the mechanism design and contract theory literature implicitly associate verifiability with forcing contracts. Some game theory models, such as that of Bernheim and Whinston (1998), also take this view.

[13]Throughout this paper, I focus on "implementation" in the weak sense of not requiring uniqueness of equilibrium in each state. I find this a reasonable notion for contractual settings. Regardless of your view about this, however, much of my analysis concerns settings with "ex post renegotiation," in which multiplicity is not a problem. Furthermore the multiplicity issue should be tackled with a theory of the self-enforced component of contract. See the Conclusion on this point.

lead to an ex post inefficient outcome in some state, for some message profiles. However, renegotiation at Date 3 implies that, in both states, an ex post efficient outcome occurs *in equilibrium*. To incorporate renegotiation, we constrain attention to mechanisms that specify adoption when (H, H) is the message profile. We can further limit attention to mechanisms that specify no adoption when message profiles (H, L) and (L, H) are sent, because this makes for the most relaxed incentive constraints. Let $p^{\theta\theta'}$ denote the transfer specified by the mechanism for message profile (θ, θ'), for $\theta, \theta' \in \Theta$.

The game form implies a message game for each state, as pictured below.

B \ S	H	L
H	$-p^{HH},\ p^{HH}$	$-p^{HL},\ p^{HL}$
L	$-p^{LH},\ p^{LH}$	$-p^{LL},\ p^{LL}$

Message game in state L

B \ S	H	L
H	$5-p^{HH},\ 3+p^{HH}$	$-p^{HL},\ p^{HL}$
L	$-p^{LH},\ p^{LH}$	$5-p^{LL},\ 3+p^{LL}$

Message game in state H

We look for equilibria with truthful reporting. For truthful reporting to be a Nash equilibrium in each state, it must be that $p^{LH} \le p^{LL} \le p^{HL}$, $5 - p^{HH} \ge -p^{LH}$, and $3 + p^{HH} \ge p^{HL}$. Combining these inequalities yields

$$p^{LL} + 5 \ge p^{HH} \ge p^{LL} - 3,$$

which implies that the set of implementable state-contingent value functions in the MDIR setting is:

$$\left\{ v\colon \{\mathrm{H}, \mathrm{L}\} \to \mathbf{R}^2 \;\middle|\; v(\mathrm{L}) = (\alpha, -\alpha),\ v(\mathrm{H}) = (5 + \alpha - \beta, 3 - \alpha + \beta), \right.$$
$$\left. \text{for any } \alpha \in \mathbf{R} \text{ and } \beta \in [-3, 5] \right\}. \tag{1}$$

In each payoff vector, I list the buyer's payoff first.

Mechanism Design with Ex Post Renegotiation (MDER)

I next turn to the case in which renegotiation occurs at Date 5. That is, parties can renegotiate after sending messages (playing their part of the mechanism) but before the external enforcer imposes the outcome. This is the *ex post renegotiation* case. I first calculate how standard mechanism design theory is used to characterize the set of implementable values—the *mechanism design with ex post renegotiation* program. The basic methodology is ascribed to Maskin and Moore (1999) and has been applied widely by many theorists. Incidentally, it is worth noting that, if renegotiation is possible at Date 5, then implementability is not affected by whether renegotiation can also occur at Date 3.[14]

The simplest way of performing the MDER analysis is to redefine the mechanism design problem by internalizing the renegotiation activity. Specifically, maintaining the set of states and outcomes, we can respecify payoffs to incorporate the renegotiation. For example, consider the outcome in which no transfer is made and the advertisement is not adopted. In the standard model without renegotiation, this outcome would yield the payoff vector $(0, 0)$ in both states. However, if renegotiation can occur at Date 5, the parties would change the contract in state H; they would specify that the advertisement be adopted and that the surplus be divided equally (by selecting a transfer of $p = 1$). Thus, with ex post renegotiation, the payoff vector of the "no transfer, no adoption" outcome in state H is $(4, 4)$.

As with the interim renegotiation case, we look at direct-revelation mechanisms and truthful-reporting equilibria. We can assume that the mechanism specifies "adoption of the advertisement" when the report profile is (H, H) and when is it (L, L).[15] Furthermore, it is easy to verify that the incentive constraints are most relaxed if "no adoption" is specified for report profile (L, H) and "adoption" is specified for profile (H, L). Note that the mechanism would be renegotiated in state H in the off-equilibrium case in which the buyer reports L while the seller reports H. Internalizing the renegotiation activity, a game form implies the following message games in the two states.

B \ S	H	L
H	$-p^{HH}$, p^{HH}	$-p^{HL}$, p^{HL}
L	$-p^{LH}$, p^{LH}	$-p^{LL}$, p^{LL}

Message game in state L

B \ S	H	L
H	$5-p^{HH}$, $3+p^{HH}$	$5-p^{HL}$, $3+p^{HL}$
L	$4-p^{LH}$, $4+p^{LH}$	$5-p^{LL}$, $3+p^{LL}$

Message game in state H

As in the previous subsection, $p^{\theta\theta'}$ denotes the transfer specified by the mechanism for message profile (θ, θ').

For truthful reporting to be a Nash equilibrium in each state, it must be that $p^{LH} \le p^{LL} \le p^{HL}$, $5 - p^{HH} \ge 4 - p^{LH}$, and $3 + p^{HH} \ge 3 + p^{HL}$. Combining these

[14]Renegotiation at Date 5 implies ex post efficiency in both states, which means there is no surplus to be obtained from earlier renegotiation.

[15]Any incentive-compatible mechanism that specifies "no adoption" when the report profile is (H, H) will be renegotiated in the H state. One can alter the mechanism so that the renegotiated outcome is specified for (H, H), without affecting the incentive conditions. This is the "renegotiation-proofness principle" in action (see Brennan and Watson, 2001).

inequalities yields

$$p^{LL} + 1 \geq p^{HH} \geq p^{LL}.$$

The set of implementable state-contingent value functions for the MDER program is thus:

$$\left\{ v\colon \{\mathrm{H}, \mathrm{L}\} \to \mathbf{R}^2 \;\middle|\; v(\mathrm{L}) = (\alpha, -\alpha),\ v(\mathrm{H}) = (5 + \alpha - \beta, 3 - \alpha + \beta), \text{ for any } \alpha \in \mathbf{R} \text{ and } \beta \in [0, 1] \right\}. \tag{2}$$

Note that the opportunity to renegotiate at Date 5, specifically following out-of-equilibrium message profiles, causes a refinement in the set of implementable values relative to the case of interim renegotiation.

Trade Decisions as Options

Everything that the mechanism design program identifies to be implementable can be achieved, in practice, with forcing contracts. However, there is *no* reason to expect that parties would *limit themselves* to forcing contracts and, therefore, there is no reason to limit our theoretical analysis to such contractual forms. I next show that, when there is ex post renegotiation, the set of implementable values significantly expands when parties depart from forcing contracts and, instead, use trade decisions as options.

Suppose that at Date 1 the parties write the following contract: If the buyer adopts the advertisement, then he must pay $p' + \beta$ to the seller; if the buyer does not adopt, then he pays p'; further, the external enforcer is instructed to ignore messages sent at Date 4. For $\beta \in (0, 5)$, this is not a forcing contract—that is, it neither compels the buyer to adopt the advertisement in both states, nor compels the buyer to *not* adopt the advertisement in both states. Instead, this is an option contract, but one that uses the buyer's *trade decision*, rather than the buyer's message, as the way to exercise the option. With $\beta \in [0, 5]$, the buyer has the incentive to adopt the advertisement in state H and not to adopt in state L. From Date 6, this contract yields a payoff vector of $(5 - p' - \beta, 3 + p' + \beta)$ in state H and $(-p', p')$ in state L. Because the contract leads to the efficient trade decision in each state, it would not be renegotiated at either Date 5 or Date 3. The contract thus implements value $(5 - p' - \beta, 3 + p' + \beta)$ in state H and $(-p', p')$ in state L.

Clearly, by using the trade decision as an option, the parties are able to reduce the detrimental effect of renegotiation at Date 5. Because the trading opportunity is nondurable, there is no way for the parties to reverse it through renegotiation after Date 6. The parties could use a more complicated contract that involves transfers contingent on both trade decisions and messages. However, in this example, more complicated contracts cannot improve on the scope of the simple option scheme described in the preceding paragraph. Thus, the set of implementable state-contingent value functions in the case of ex post renegotiation is:

$$\Big\{ v\colon\{\mathrm{H},\mathrm{L}\} \to \mathbf{R}^2 \;\Big|\; v(\mathrm{L}) = (\alpha, -\alpha),\ v(\mathrm{H}) = (5+\alpha-\beta, 3-\alpha+\beta), \text{ for any } \alpha \in \mathbf{R} \text{ and } \beta \in [0,5] \Big\}. \tag{3}$$

Preliminary Comments on Ex Post Renegotiation

With the results of the example in hand, it is worthwhile to take stock and reflect a bit before proceeding with the general analysis.

Note that, in the case of ex post renegotiation, there is a discrepancy between the set of implementable value functions and the *strictly smaller* set identified by the MDER program. The MDER program misses how trade decisions can be used as options, precisely because the MDER program treats trade decisions as part of the abstract, public "outcome."[16] For this reason, we should reject the MDER program as incorporating implicit assumptions about contractual incompleteness. We should instead focus on structured models that incorporate the technology of trade and enforcement, and, where appropriate, on the MDIR program.

In response to my assertion, a mechanism design theorist might be inclined to conclude that I am mis-applying the MDER program. The theorist would argue that, if we think the trading opportunity is nondurable (and so trade decisions can be used as options without being reversed by renegotiation), then the set of implementable values is actually characterized by the *MDIR* program. Segal and Whinston (2000) and others take this position. However, the argument is flawed in two respects.

First, the MDIR program actually does *not* characterize the set of implementable value functions for the case of renegotiation at Date 5. Comparing Expressions 1 and 3, the MDIR program supports β numbers between -3 and 5, while only numbers between 0 and 5 can be supported with ex post renegotiation. The problem is that, in designing option contracts, the fixed technology of trade is not as flexible as are messages. Thus, neither the MDER nor the MDIR programs accurately model the masonry example with ex post renegotiation. The renegotiation opportunity, in a sense, occurs "in the middle of the mechanism." To analyze this renegotiation opportunity, one must examine the structured model that explicitly accounts for the technology of trade and external enforcement. In fact, mechanism design methodology is applicable, but it relies on precise modeling of the technology of trade. In particular, one cannot view the "outcome" as merely a specification of the transfer

[16] This suggests a more nuanced view of Bernheim and Whinston's (1998) "strategic ambiguity," whereby external enforcement can mold incentives without forcing any particular action profile.

and the trade decision. Rather, an "outcome" must indicate the trade decisions and transfers that result in the different states, given the technology of trade and the instructions for the external enforcer. My general model makes this formal.

Second, if one argues that the MDER program is inappropriate for settings with nondurable trading opportunities, one still must justify applying the program to settings with durable trading opportunities. You might think that the MDER program characterizes settings in which trade decisions are reversible—for example, a good delivered at time t can be returned at some future date t'—and where renegotiation can occur between times at which trade decisions can be made and reversed. However, if trade decisions can be made at times t and t' then the parties should be able to write a long-term contract that covers both times. Thus, we still have a setting of "renegotiation within the mechanism" and we still need a structured model to account for the timing of trade decisions and renegotiation opportunities. My general model incorporates these dynamic features and, as I prove in Section 5, it confirms in general the failure of the MDER program to describe reality.

2 The Main Ingredients of the General Framework

In this section, I describe in detail the main components of my general model of contract. Relative to the example of Section 1, the general model has an arbitrary trading technology and it also adds the following elements: (i) a verifiable, public random variable, (ii) additional renegotiation opportunities, and (iii) an externally enforced "continuation value," which I use to model durability of the trading opportunity. The analysis in this section and the next assumes a fixed set of continuation value functions. This set is treated as endogenous in Sections 4 and 5.

There are two contracting parties, whom I call "players." Unverifiable events are captured by the state θ, which I assume is an element of some set Θ. In the relationship's *trading and enforcement phase*, the players make verifiable trade decisions and the external enforcer compels transfers and a state-contingent continuation value. The trade decisions are represented as $a \in A$. The externally enforced transfer is denoted $y = (y_1, y_2)$, where y_i is the monetary transfer to player i. I assume $y \in \mathbf{R}^2_0$, where

$$\mathbf{R}^2_0 \equiv \{y \in \mathbf{R}^2 \mid y_1 + y_2 = 0\}.$$

Regarding the justification for such "balanced transfers," recall footnote 10. The *continuation value function* is given by $x : \Theta \to \mathbf{R}^2$. I assume that x is an element of some set X and that, for every $\theta \in \Theta$, $\max\{x_1(\theta) + x_2(\theta) \mid x \in X\}$ exists.

The players receive payoffs as a function of the state and the outcome of the trading and enforcement phase. I assume that the payoffs are additive in money,

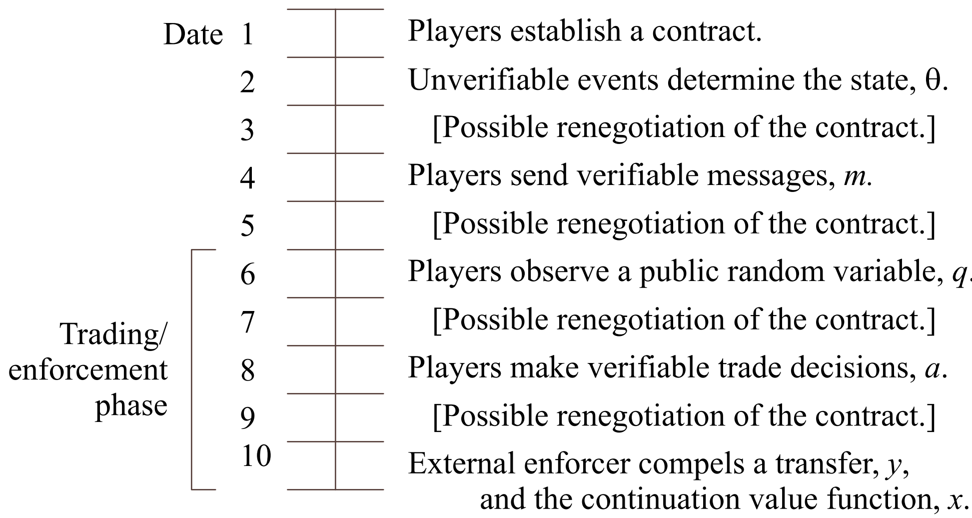

Figure 2: A contractual relationship.

with the non-monetary component given by a function $u: A \times \Theta \to \mathbf{R}^2$. I also include a discount factor δ, which measures the players' relative preferences over immediate and future gains. In state θ, with decision a, transfer y, and continuation value function x, the payoff vector is

$$(1 - \delta)\,[u(a \mid \theta) + y] + \delta x(\theta).$$

I assume that u is bounded.

The contractual relationship runs over ten dates, as shown in Figure 2. At odd-numbered dates, the players make joint contracting decisions—establishing a contract at Date 1 and possibly renegotiating it later. At even-numbered dates through Date 8, the players make joint observations and they make individual decisions—jointly observing the state at Date 2, sending verifiable messages at Date 4, jointly observing a random variable at Date 6, and making trade decisions at Date 8. At Date 10, the external enforcer compels transfers and the continuation value function.

Trading and Enforcement Phase

Dates 6 through 10 compose the trading and enforcement phase, which I now describe in more detail. At Date 6, the players jointly observe the draw q of a public random variable μ, which has support Q. The draw is verifiable. It will be sufficient to assume that $Q = [0, 1]$ and that μ is the uniform distribution. The players' behavior and external enforcement may be conditioned on q; otherwise, the draw has no direct affect on payoffs.

At Date 8, the players make inalienable trade decisions, which I assume are simultaneous and independent. Player i selects $a_i \in A_i$. Thus, $A = A_1 \times A_2$. I assume A is finite. The trade decisions are verifiable.

At Date 10, the external enforcer compels the transfer y and the continuation value function x. These are conditioned on the verifiable draw q, on the verifiable trade decision a, and on the message profile m, as specified by the players' contract. To focus on interaction in the trading and enforcement phase, at this point I take the message profile as fixed. Thus, I write $y = \hat{y}(q, a)$ and $x = \hat{x}(q, a)$ as the transfer and the continuation value function that are specified by the contract for the contingency in which q is the draw and a is the trade profile. The functions $\hat{y}$ and $\hat{x}$ are called the *externally enforced components* of the players' contract.

I shall, without loss of generality, ignore the possibility of renegotiation at Dates 7 and 9. Because players are risk-neutral, the effects of allowing renegotiation at Date 7 (just after draw q) are identical to the effects of allowing it only at Date 5 (just before draw q), which I study in the next section. The justification behind ignoring renegotiation at Date 9 at this point is that it will be covered by the analysis in Sections 4 and 5.[17]

Given the state θ and the draw q, the externally enforced components of the contract define a *trading game*, where the space of action profiles is A and the payoffs are given by

$$(1 - \delta)\,[u(\cdot \mid \theta) + \hat{y}(q, \cdot)] + \delta \hat{x}(q, \cdot)(\theta).$$

I focus on pure strategy Nash equilibria of the trading game. Let $\hat{a}(q, \theta)$ denote the equilibrium action profile that is chosen by the players in state θ following draw q. Taking the expectation over the random draw, the players' expected payoff vector in state θ is given by

$$w(\theta) \equiv \int \{(1 - \delta)\,[u(\hat{a}(q, \theta) \mid \theta) + \hat{y}(q, \hat{a}(q, \theta))] + \delta \hat{x}(q, \hat{a}(q, \theta))(\theta)\}\, d\mu(q). \tag{4}$$

The function w thus gives the state-contingent payoff vectors that can be achieved by the appropriate choice of the externally enforced components $\hat{y}$ and $\hat{x}$ and equilibrium selection $\hat{a}$.

I use the term *outcome* for any function from Θ to $\mathbf{R}^2$. Think of an outcome, therefore, as a state-contingent payoff that results from interaction in the trading and enforcement phase; this should be differentiated from the "trade outcome," which only describes the physical trade decision and monetary transfer. The function w that is defined above is an outcome.

Forcing contracts work in this general model just as they did in the example. For instance, suppose the players want to have continuation value function x^* and they want to force themselves to play action profile a^*, regardless of the state. This can

[17] See the last paragraph in Section 4.

accomplish by specifying $\hat{x}$ and $\hat{y}$ as follows. First, let $\hat{x}(q,a) \equiv x^*$ for all q and a. Second, let L be an upper bound on $|u_i(a \mid \theta)|$, for $i = 1, 2$. Then, to induce a^*, one can define $\hat{y}$ so that (i) for each $a = (a_i, a_j^*)$ for which $a_i \neq a_i^*$, we have $\hat{y}_i(q,a) = -L$ and $\hat{y}_j(q,a) = L$; and (ii) $\hat{y}(q,a) = (0,0)$ for every other profile a, for all q. That is, each player is punished for not taking his part of the prescribed trade action. Obviously, then, a^* is the only Nash equilibrium of the trading game, in every state.

Definition 1: *Externally enforced components $\hat{y}$ and $\hat{x}$ are called* **forcing** *if, for every q, there is a unique Nash equilibrium of the trading game $\langle A, (1-\delta)[u(\cdot \mid \theta) + \hat{y}(q,\cdot)] + \delta\hat{x}(q,\cdot)(\theta)\rangle$ and this equilibrium is independent of the state.*

Let $W(X,\delta)$ be defined as the set of achievable outcomes, given X and δ, and let $W^F(X,\delta)$ be the subset of W that can be supported using externally enforced components that are forcing. That is,

Definition 2: *The set $W(X,\delta)$ contains w if and only if there are contracted transfer and continuation value functions $\hat{y}$ and $\hat{x}$, and there is a function $\hat{a} : Q \times \Theta \to A$, such that*

(i) Equation 4 is satisfied and

(ii) $\hat{a}(q,\theta)$ is a Nash equilibrium of $\langle A, (1-\delta)[u(\cdot \mid \theta) + \hat{y}(q,\cdot)] + \delta\hat{x}(q,\cdot)(\theta)\rangle$, for every $\theta \in \Theta$ and $q \in Q$.

The set $W^F(X,\delta)$ contains w if and only if there are forcing components $\hat{y}$, and $\hat{x}$ and a function $\hat{a} : Q \times \Theta \to A$, such that (i) and (ii) are satisfied for every $\theta \in \Theta$ and $q \in Q$.

The following lemma identifies a useful property of the sets $W(X,\delta)$ and $W^F(X,\delta)$.

Lemma 1: *$W(X,\delta)$ and $W^F(X,\delta)$ are closed under constant transfers. For example, if $w \in W(X,\delta)$ and if $\alpha \in \mathbf{R}_0^2$ is a constant transfer, then $w + \alpha \in W(X,\delta)$ as well.*

Proof: The result follows from the fact that one can add a constant transfer $\alpha \in \mathbf{R}_0^2$ to any given function $\hat{y}$ without altering the players' incentives in the trading phase in any state. *Q.E.D.*

Contracted Mechanisms

The players' contract specifies a mechanism, which maps messages sent at Date 4 to outcomes induced in the trading and enforcement phase.[18] The revelation principle applies in the following sense. We can restrict attention to direct-revelation

[18] One way of describing a mechanism is to explicitly write y as a function of m, q, and a. Equivalently, one can isolate the trading and enforcement phase by writing y as a function of q and a, noting how y induces an outcome in W, and then thinking of the contract as a mapping from the messages to W. I adopt the latter characterization because it minimizes the amount of notation needed for the analysis.

mechanisms, each of which is defined by a message space $M \equiv \Theta^2$ and a function $f : M \rightarrow W(X, \delta)$. With such a mechanism, at Date 4 the parties simultaneously and independently report the state. For any report profile m, the mechanism specifies an element $f(m) \in W(X, \delta)$, which then determines the payoffs conditional on the state. We can concentrate on equilibria of the mechanism in which the parties report truthfully.[19]

Renegotiation

Renegotiation can occur at Dates 3, 5, 7, and 9, depending on what one assumes. I have already addressed renegotiation at Dates 7 and 9, so now I focus on renegotiation at Dates 3 and 5. We can think of these times as possible opportunities for the players to discard their originally specified f mapping and replace it with another mapping f'. I model renegotiation by supposing that the players divide the renegotiation surplus according to fixed bargaining weights π_1 and π_2, which are nonnegative and sum to one. The generalized Nash bargaining solution and several other common bargaining solutions have this representation.

To be more precise, define

$$\gamma(\theta, X, \delta) \equiv \max_{w \in W(X,\delta)} w_1(\theta) + w_2(\theta),$$

which is the maximal joint payoff that can be obtained in state θ. This joint payoff can also be written

$$\gamma(\theta, X, \delta) = (1 - \delta) \max_{a \in A} [u_1(a \mid \theta) + u_2(a \mid \theta)] + \delta \max_{x \in X} [x_1(\theta) + x_2(\theta)],$$

because it can be achieved by using a forcing contract. An outcome w is called *efficient in state* θ if $w_1(\theta) + w_2(\theta) = \gamma(\theta, X, \delta)$; otherwise, the outcome is inefficient in state θ.

Suppose the original mechanism (M, f) would lead to outcome w in state θ. If w is inefficient in state θ, then the players have a joint incentive to renegotiate the mechanism. The *renegotiation surplus* is

$$r(w, \theta, X, \delta) \equiv \gamma(\theta, X, \delta) - w_1(\theta) - w_2(\theta).$$

The players will select a new mapping f' that induces an efficient outcome. Further, the surplus will be divided according to the players' bargaining weights, so that player i obtains $w_i(\theta) + \pi_i r(w, \theta, X, \delta)$.

[19]Regarding multiplicity of equilibria, recall footnote 13.

3 Implementation Conditions Given X

A *state-contingent value function* is a function $v : \Theta \to \mathbf{R}^2$ that gives the players' expected payoff vector from the start of Date 3, as a function of the state. The players' contractual objective is to implement the particular function v of their choice. In this section, I define and characterize the set of implementable value functions, given a fixed set of continuation value functions X. I group the analysis into three categories, distinguished by whether the players have the opportunity to renegotiate at Dates 3 and 5. The characterization lemmas in this section are all straightforward variations of well-known theorems from the contract theory literature—in particular, due to Maskin (1999), Maskin and Moore (1999), and Moore and Repullo (1988). I provide the proofs of the first two lemmas; the others are proved similarly.

No Renegotiation

First consider the setting in which the players cannot renegotiate. A mechanism (M, f) implies, for each state θ, a *message game* in which the players engage at Date 4. The message game has action profiles given by M and payoffs defined by $f(\cdot)(\theta)$. For this setting, implementability is defined as follows.

Definition 3: *A mechanism (M, f) is said to* **implement** *value function v if, for each state θ, there is an equilibrium of the message game that leads to the payoff vector $v(\theta)$. Value function v is said to be* **implementable** *if there is a mechanism that implements it.*

Let $\Lambda^N(X, \delta)$ be the set of implementable value functions, under the assumption that the players cannot renegotiate.

Lemma 2: $v \in \Lambda^N(X, \delta)$ *if and only if (i) for every $\theta \in \Theta$, there is an outcome $w^{\theta\theta} \in W(X, \delta)$ such that $w^{\theta\theta}(\theta) = v(\theta)$; and (ii) for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $w^{\theta\theta'} \in W$ such that $v_1(\theta') \geq w_1^{\theta\theta'}$ and $v_2(\theta) \geq w_2^{\theta\theta'}$.*

Proof: For any direct-revelation mechanism (Θ^2, f), define $w^{\theta_1\theta_2} \equiv f(\theta_1, \theta_2)$ for all $\theta_1, \theta_2 \in \Theta$. Observe that truthful reporting is a Nash equilibrium if and only if $w_1^{\theta\theta} \geq w_1^{\theta_1\theta}$ and $w_2^{\theta\theta} \geq w_2^{\theta\theta_2}$, for every $\theta \in \Theta$ and all $\theta_1, \theta_2 \in \Theta$. Combining this fact with the definition of implementability produces the result. *Q.E.D.*

Interim Renegotiation

Next consider the setting in which renegotiation is possible at Date 3 but not at Date 5. In other words, the players can renegotiate between the time that they jointly learn the state and when the message game is played. In this setting, implementability requires an additional condition—that the equilibrium of the message game is efficient in every state. Let $\Lambda^I(X, \delta)$ denote the set of implementable value functions when there is interim renegotiation.

Lemma 3: *$v \in \Lambda^I(X,\delta)$ if and only if (i) $v_1(\theta)+v_2(\theta)=\gamma(\theta,X,\delta)$ for every $\theta \in \Theta$; and (ii) for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $w^{\theta\theta'} \in W(X,\delta)$ such that $v_1(\theta') \geq w_1^{\theta\theta'}$ and $v_2(\theta) \geq w_2^{\theta\theta'}$.*

Proof: First note that $v \in \Lambda^I(X,\delta)$ implies $v_1(\theta)+v_2(\theta)=\gamma(\theta,X,\delta)$ for every $\theta \in \Theta$. This is because if, at Date 3, the players anticipate getting an inefficient outcome in a given state, then they would renegotiate the mechanism to obtain an efficient outcome. Next note that, if a state-contingent value function v satisfies $v_1(\theta)+v_2(\theta)=\gamma(\theta,X,\delta)$, then there is an outcome $w^{\theta\theta} \in W(X\ \delta)$ such that $w_1^{\theta\theta} = v_1(\theta)$ and $w_2^{\theta\theta} = v_2(\theta)$. This follows from Lemma 1. Furthermore, observe that the players cannot gain from renegotiating at Date 3 in state θ if they anticipate that their messages will lead to an efficient outcome. Given these facts, the proof follows the same steps used to prove Lemma 2. *Q.E.D.*

Ex Post Renegotiation

Finally, consider the case in which renegotiation is possible at Date 5—between the time the players send messages and the beginning of the trading and enforcement phase. The idea is that the players interact in the contracted mechanism, which leads to an outcome w. But then, just before the outcome would be induced, the players can renegotiate to obtain a different outcome. In this setting, renegotiation implies efficient outcomes in every state and after every message profile in the mechanism.

To characterize implementability for this setting, we must incorporate renegotiation into the definition of an outcome. The set of *ex post renegotiation outcomes* is defined as

$$Z(X,\delta) \equiv \left\{ z:\Theta \to \mathbf{R}^2 \;\middle|\; \text{there is an outcome } w \in W(X,\delta) \text{ such that } z(\theta) = w(\theta) + \pi r(w,\theta,X,\delta) \text{ for every } \theta \in \Theta \right\}.$$

An ex post renegotiation outcome is a state-contingent payoff vector that results when, in every state, the players renegotiate from an outcome in $W(X,\delta)$. One can analyze mechanism design in the setting of ex post renegotiation by simply replacing $W(X,\delta)$ with $Z(X,\delta)$—that is, by thinking of the mechanism as a mapping from M to $Z(X,\delta)$, rather than a mapping from M to $W(X,\delta)$. If we constrain attention to forcing contracts, then the set of ex post renegotiation outcomes is

$$Z^F(X,\delta) \equiv \left\{ z:\Theta \to \mathbf{R}^2 \;\middle|\; \text{there is an outcome } w \in W^F(X,\delta) \text{ such that } z(\theta) = w(\theta) + \pi r(w,\theta,X,\delta) \text{ for every } \theta \in \Theta \right\}.$$

Note that all elements of Z and Z^F are efficient in every state.

Let $\Lambda^{EP}(X, \delta)$ be the set of implementable value functions when there is ex post renegotiation and let $\Lambda^{EPF}(X, \delta)$ be the subset of value functions that are supported using forcing contracts. The popular MDER program studies precisely the set $\Lambda^{EPF}(X, \delta)$. As discussed in Section 1, this program identifies $\Lambda^{EPF}(X, \delta)$ rather than $\Lambda^{EP}(X, \delta)$ since it treats trading decisions as alienable and public.

Lemma 4: *$v \in \Lambda^{EP}(X, \delta)$ if and only if (i) for every $\theta \in \Theta$, there is an outcome $z^{\theta\theta} \in Z(X, \delta)$ such that $z^{\theta\theta}(\theta) = v(\theta)$; and (ii) for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $z^{\theta\theta'} \in Z(X, \delta)$ such that $v_1(\theta') \geq z_1^{\theta\theta'}$ and $v_2(\theta) \geq z_2^{\theta\theta'}$.*

Lemma 5: *$v \in \Lambda^{EPF}(X, \delta)$ if and only if (i) for every $\theta \in \Theta$, there is an outcome $z^{\theta\theta} \in Z^F(X, \delta)$ such that $z^{\theta\theta}(\theta) = v(\theta)$; and (ii) for every pair of states $\theta, \theta' \in \Theta$, there is an outcome $z^{\theta\theta'} \in Z^F(X, \delta)$ such that $v_1(\theta') \geq z_1^{\theta\theta'}$ and $v_2(\theta) \geq z_2^{\theta\theta'}$.*

4 Durability of the Trading Opportunity

Many contractual relationships have *durable* trading opportunities. For example, a retail firm may contract with a computer software company to design and install specialized software for inventory control. The software will generate for the retailer a flow of value over time, starting as soon as the software is installed. Suppose the software can be installed as early as in January; furthermore, if the seller fails to install the software in January, it can still be installed in February, or March, or later. However, if it is installed in, say, March, then the buyer will not obtain the value of the software in January or February.

Durability implies that the trade decisions are at least partially *reversible*. For example, the seller's decision *not* to install the software in January can be reversed by installing the software in February. Sometimes, trade decisions are *fully reversible*. For instance, if the seller installs the software in January, then perhaps the software can be uninstalled at any future time.

This section concerns contractual settings with durable, fully reversible trading opportunities. To model durability and reversibility, I suppose that the players interact over an infinite number of discrete "periods," starting in Period 1.[20] In each

[20] Jackson and Palfrey (2001) analyze a dynamic model that is, on first glance, similar to the one I study here. Their model differs in important respects, however. It assumes that players can unilaterally impose a default decision in each period, which gives players more power than would be appropriate for a model of most contractual settings. It also abstracts from inalienable trade decisions, durability, and reversibility. Interestingly, though, the proof of their result for nondiscounted environments has a stationarity feature that foretells of the importance of stationarity in my model. For another interesting, but less related, modeling exercise, see Kalai and Ledyard (1997). MacLeod and Malcomson's (1993) basic model also has a dynamic trading opportunity but they focus on stationary contracts; these authors also examine a multi-period contracting environment with a state variable that follows a Markov process, and again impose a restriction on the class of contracts.

period, play proceeds as in Figure 2: the players send messages as prescribed by a contractual mechanism, the players make trade decisions, and the external enforcer compels the transfers that the players' contract prescribes. The state, which is realized at Date 2 of Period 1, is fixed for the entire game. Thus, Dates 1 and 2 occur only in Period 1; these dates are skipped in all other periods.

Preferences are given by the discounted and normalized sum of per-period payoffs, calculated using discount factor δ. Thus, in state θ, the payoff vector from Date 3 in Period 1 is

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}\left[u(a^t \mid \theta)+y^t\right],$$

where $\{a^t, y^t\}_{t=1}^{\infty}$ denotes the sequence of trade decisions and monetary transfers. I represent reversibility by assuming that the trade decision that is taken in one period does not constrain trade decisions taken in succeeding periods and it has no direct effect on the payoffs in future periods. For example, if the players make trade decision a in the current period, then they can reverse it by selecting any other trade decision a' in the next period. An infinite sequence of trade decision a means that a is never reversed. The discount factor measures the degree of durability; setting δ close to 1 means a highly durable trading opportunity, whereas the opposite is captured with δ close to 0.

The players can write long-term contracts that condition the transfer in a given period on the *entire* verifiable history—messages, trade decisions, and draws in the current period and in previous periods. From a given period, a contract's implications for the future can be summarized by the implied continuation value function. Thus, instead of keeping track of the transfers a contract prescribes for future periods (as well as the induced behavior), one can simply specify the continuation value function. Because the contracting environment is stationary (due to reversibility), the set of feasible continuation value functions is independent of the reference period and is equal to the set of implementable value functions. That is, X will give the set of possible state-contingent payoffs from Date 3 in *any* period; for Period 1 in particular, X is the set of implementable value functions. To analyze long-term contracts, I therefore adopt a recursive formulation, whereby X is endogenously determined.

To identify the set of implementable value functions, a fixed point condition is added to the analysis from preceeding sections. Borrowing a term from Abreu, Pearce, and Stacchetti (1986), I say that a set X of value functions has the *self-generation property with respect to* Λ^N if $X \subset \Lambda^N(X,\delta)$. Self-generation with respect to Λ^I, Λ^{EP}, and Λ^{EPF} are defined analogously. Let $V^N(\delta)$, $V^I(\delta)$, $V^{EP}(\delta)$, and $V^{EPF}(\delta)$ be the sets of implementable value functions for the settings of, respectively, no renegotiation, interim renegotiation, ex post renegotiation, and ex post renegotiation with forcing contracts.

Definition 4: *$V^N(\delta)$ is defined as the maximal set of value functions that has the self-generation property with respect to Λ^N. $V^I(\delta)$, $V^{EP}(\delta)$, and $V^{EPF}(\delta)$ are defined as the maximal sets of value functions that have the self-generation property with respect to Λ^I, Λ^{EP}, and Λ^{EPF}, respectively.*

This definition completes the construction of my general modeling framework.

I conclude this section with a two comments on the general model. First, note that any setting with a nondurable trading opportunity, including the example in Section 1, can be analyzed as a special case of the general model with $\delta = 0$.

Second, I wish to expound on the possibility of renegotiation at Date 9, an issue of which I earlier deferred discussion. Having renegotiation at Date 9 is equivalent to assuming that renegotiation can occur at Date 3. This is because, given balanced transfers, the only way for the players to realize a joint gain through renegotiation at Date 9 is to alter the continuation value function. The continuation value in one period is just the discounted value from the start of the next period, so renegotiation at the end of one period has the same effect as does renegotiation at the start of the next period. Note that my analysis does leave out one case: that in which non-balanced transfers are allowed and the players never can renegotiate. I have not included this case because it seems unrealistic and uninteresting and, furthermore, it is simple to analyze.

5 Characterization Results

In this section, I partially characterize and compare the sets $V^N(\delta)$, $V^I(\delta)$, $V^{EP}(\delta)$, and $V^{EPF}(\delta)$. The results presented in this section are proved in the Appendix.

Existence and Inclusion

I begin with an existence result.

Theorem 1: *$V^{EPF}(\delta)$, $V^{EP}(\delta)$, $V^I(\delta)$, and $V^N(\delta)$ are well-defined and non-empty.*

Existence is proved by finding a set that has the self-generation property and then recognizing that the Λ mappings are monotone.

To get an idea of what the implementable sets contain, note that they are bounded in that

$$v_1(\theta) + v_2(\theta) \leq \max_{a \in A} [u_1(a \mid \theta) + u_2(a \mid \theta)]$$

for every $v \in V^N(\delta) \cup V^I(\delta) \cup V^{EP}(\delta) \cup V^{EPF}(\delta)$. This is because the joint value $v_1(\theta) + v_2(\theta)$ is achieved by a discounted sequence of trade decisions and, furthermore, balanced transfers do not affect the joint value. In fact, the upper bound is attained.

For example, consider the case of ex post renegotiation with forcing contracts and suppose the players agree to a contract that forces them to select some arbitrary trade decision a' in each period. Then if a' is inefficient in the actual state, the players will renegotiate their contract so that an efficient trade decision is selected. Thus, if $v \in V^{EPF}(\delta)$ then

$$v_1(\theta) + v_2(\theta) = \max_{a \in A}[u_1(a \mid \theta) + u_2(a \mid \theta)]$$

for every state θ. This result also holds for the sets $V^{EP}(\delta)$ and $V^I(\delta)$. In the Appendix, I formalize this analysis in terms of the Λ mappings.

The following extension of Lemma 1 shows that the players can freely divide value in ways that are constant across states.

Lemma 6: *The sets $V^N(\delta)$, $V^I(\delta)$, $V^{EP}(\delta)$, and $V^{EPF}(\delta)$ are all closed under constant transfers.*

For example, take any $v \in V^I(\delta)$ and a constant $\alpha \in \mathbf{R}^2_0$, and define $v' : \Theta \to \mathbf{R}^2$ so that $v'(\theta) = v(\theta) + \alpha$ for each $\theta \in \Theta$. Then $v' \in V^I(\delta)$ as well.

The next result ranks the sets of implementable value functions by inclusion. The result confirms in general what the example in Section 1 demonstrated.

Theorem 2: $V^{EPF}(\delta) \subset V^{EP}(\delta) \subset V^I(\delta) \subset V^N(\delta)$ *and, in general, none of these sets are equal.*

In words, interim renegotiation restricts the set of implementable state-contingent value functions. Ex post renegotiation implies a further restriction. Limiting attention to forcing contracts implies still a smaller set.

Stationarity and Durability

I next report my main results, which establish the efficacy of stationary contracts and the relation between durability and implementation. For the definition of stationary contracts, let $\mathcal{P}(\mathbf{R}^2)$ denote the set of subsets of $\mathbf{R}^2$.

Definition 5: *Take as given a function $\Lambda^* : \mathcal{P}(\mathbf{R}^2) \times [0,1) \to \mathcal{P}(\mathbf{R}^2)$. A state-contingent value function v is called* **supported by a stationary contract with respect to** Λ^* *and δ, if $v \in \Lambda^*(\{v\}, \delta)$.*

In other words, stationarity means v can be supported by a contract that specifies the same continuation value function (that is, itself) in the following period, regardless of behavior in the current period. If this is the case, the transfer function y can be chosen to condition only on the verifiable events of the current period. Stationary contracts are thus simple in that they can be expressed as the repeated application of a short-term contract.

An issue of both practical and theoretical interest is the extent to which nonstationary contracts can improve on the scope of stationary contracts. For example, by making the continuation value function depend on the players' behavior, it may be possible to support a wider range of state-contingent values than can be done otherwise. Surprisingly, for most of the settings I analyze, nonstationary contracts offer no advantage.

Theorem 3: *In all settings, except possibly in the case of ex post renegotiation, all implementable value functions are supported by stationary contracts. More precisely, for each* $k \in \{EPF, I, N\}$ *and every* $\delta \in [0, 1)$*, every value function* $v \in V^k$ *is supported by a stationary contract with respect to* Λ^k *and* δ*.*

The intuition behind this theorem runs as follows. Constrain attention to forcing transfers (which is justified when $k = EPF$, $k = I$, or $k = N$) and ex post or interim renegotiation. Suppose we have a long-term contract that implements value function v. Contingent on message profile m sent in the first period, the contract specifies a distribution over trade outcomes in the first period and continuation values from the start of the second period. Because of renegotiation, the continuation values are efficient in every state. Thus, renegotiation in the first period can be viewed as adjusting the first-period trade outcome, while keeping the continuation values unchanged.

Applying this logic to characterize the continuation values from Period 2, Period 3, and so on, we can write v in terms of a random sequence of trade outcomes over time, from which the players renegotiate. The sequence of trade outcomes depends only on the message profile m, while renegotiation also depends on the actual state θ. A stationary contract can achieve the same state- and message-contingent payoffs by specifying a random trade outcome that, repeated each period, matches the random sequence of trade outcomes. The construction relies on the random draw q. Such a construction may not be possible in the case of ex post renegotiation without the constraint to forcing transfers because, when using trade decisions as options, the players' incentives at Date 8 are sensitive to the continuation values.

On the applied side, this theorem establishes that optimal contracts can always take a very simple, stationary form, whereby the players interact in the same way in each period. On the technical side, the theorem shows that the analysis of long-term contracting reduces to selecting a one-period mechanism that is repeated over time. That is, players choose a long-term contract that requires them to play the same short-term mechanism in each period.

Theorem 3 leads to

Theorem 4: $V^{EPF}(\delta)$*,* $V^I(\delta)$*, and* $V^N(\delta)$ *are all constant in* δ*.*

This result states that, except possibly for the case of ex post renegotiation, the set of implementable value functions does not depend on the degree to which the trading opportunity is durable. Thus, we can write V^{EPF}, V^I, and V^N without the δ argument. The powerful implication for contract analysis is that, for settings with no renegotiation, interim renegotiation, and ex post renegotiation with forcing contracts, the dynamic contracting problem reduces to a standard "static" problem (with $\delta = 0$).

Summary

The results of this section can be summarized as follows. Take as given a contractual setting with reversible trade decisions. Note that reversibility is vacuous if the trading opportunity is nondurable ($\delta = 0$).

(a) **Ex post renegotiation:** If the players can renegotiate just after sending messages (at Date 5 in each period), then the set of implementable state-contingent value functions is $V^{EP}(\delta)$. This set generally depends on δ.

(b) **Interim renegotiation:** If, in each period, the players can renegotiate before sending messages but not after (that is, at Date 3 but not Date 5), then the set of implementable value functions is V^I.

(c) **No renegotiation:** If the players cannot renegotiate, then the set of implementable value functions is V^N.

Note that, when there is ex post renegotiation, the implementable set depends on the technology of trade and on the discount factor.

The results lead to two important conclusions. First, the standard MDIR program and the no-renegotiation mechanism-design program are valid to study a wide range of contractual settings. Second, to understand contractual imperfection and implementability in settings where players can renegotiate ex post, it is critical that we explicitly address the technology of trade. The popular MDER program does not accurately characterize contractual scope. Instead, one must evaluate $V^{EP}(\delta)$, which depends on the technology of trade and the degree of durability.

6 Two More Examples

To further demonstrate how the proper accounting of the technology of trade improves our understanding of contractual imperfections, I present two more examples. The first involves a "cross investment," whereas the second features a pure "self investment."

Efficient Cross-Investments

Consider a contractual setting with *cross investments*—which Che and Hausch (1999) call "cooperative investments."[21] A buyer (player 1) and a seller (player 2) contract to trade one unit of an intermediate good; the parties interact over time in a setting of durability and reversibility. At Date 2 in Period 1, the seller makes an investment $\theta \geq 0$, at immediate cost θ, which enhances the buyer's value of trade. At Date 8 in each period, the buyer can accept or reject delivery of the good.[22] If he accepts then he obtains $(1-\delta)\sigma(\theta)$ in the current period, minus any transfer p made to the seller. In this event, the seller gets p. (I assume the seller's investment does not affect his delivery cost, which I normalize to zero.) If the buyer rejects delivery then both parties obtain zero in the current period, except for any transfer made between them.

I assume that σ is strictly increasing and $\sigma(0) > 0$, which means accepting delivery is always ex post efficient. The efficient investment θ^* solves $\max_\theta \sigma(\theta) - \theta$. I assume the maximum exists and is positive. I also assume that the parties can renegotiate ex post (at Date 5) in each period, and that they have equal bargaining weights.

This model is a special case of Che and Hausch's (1999) model of cooperative investments and ex post renegotiation. Che and Hausch study the forcing contract set V^{EPF} and they show that the hold-up problem severely restricts implementability. In fact, for the model here, they prove that the "null contract"—specifying no trade—is best; further, the parties are doomed to a situation in which the seller invests less than θ^*.

A very different picture emerges when trade decisions are used as options. In fact, as shown below, V^{EP} contains the value function v^* defined by $v_2^*(\theta) = \sigma(\theta^*)$ for all $\theta \geq \theta^*$ and $v_2^*(\theta) = \sigma(\theta)/2$ for $\theta < \theta^*$. Remember that this state-contingent value function gives the payoff vector from Date 3 and does not include the sunk investment cost. With the contract that implements v^*, the seller's Period 1/Date 2 investment decision is to maximize $v_2^*(\theta) - \theta$. Clearly, the seller optimally selects investment θ^* and efficiency is achieved.

Here is a stationary contract that implements v^*. The parties direct the external enforcer to, in each period, compel a transfer of $(1-\delta)\sigma(\theta^*)$ from the buyer to the seller if and only if the buyer accepts delivery; otherwise, there is no transfer. The external enforcer ignores the parties' messages and all behavior in previous periods. Note that, under this transfer function, the buyer will accept delivery in a given period if and only if $\theta \geq \theta^*$.

If the seller chooses $\theta \geq \theta^*$ then the parties never renegotiate the contract and the

[21] I avoid using the term "cooperative" here because I think it can easily be confused with "cooperative behavior" and can be misleading.

[22] If the buyer accepts delivery in one period and then rejects it in the next, this is interpreted as the buyer returning the good.

buyer accepts delivery in each period. In this case, the seller obtains $\sigma(\theta^*)$ from Date 3 of Period 1. On the other hand, if the seller chooses some $\theta < \theta^*$ then, anticipating rejection, the parties will renegotiate. The disagreement point of renegotiation is the contract's continuation value v^*, which satisfies $v_1^*(\theta) + v_2^*(\theta) = \sigma(\theta)$. Thus, the renegotiation surplus is the gain from trade in the current period, or $(1-\delta)\sigma(\theta)$, which the parties divide equally. This division implies that $v_2^*(\theta) = \sigma(\theta)/2$.

Efficient investment incentives in this example are due to technology of trade. If some other technology existed, such as if the seller makes the trade decision, then efficiency may not result and Che and Hausch's conclusions may, at least partially, reemerge. However, in the least, this example shows that cross investment may not be as problematic as the literature suggests.

Complexity and Hold-up

Next consider an example with pure *self investment*, along the lines of Segal (1999) and Hart and Moore (1999). This example will reiterate these authors' main point—that hold-up problems can exist even in cases of pure self investment—and show that the insight is still valid when one properly accounts for the technology of trade. The degree of the hold-up problem is sensitive to the technology of trade.

A buyer (player 1) and a seller (player 2) contract on a nondurable trading opportunity, so $\delta = 0$. As with the previous example, the state represents the seller's Date 2 investment, which is observed by the buyer but is not verifiable. The seller can either invest "high," which yields state H, or invest "low," yielding state L. The high investment entails a cost c, which is immediately paid by the seller and is not included in the u specification below. Low investment is costless. Clearly, the seller will have an incentive to invest high only if the difference between what he expects to obtain in states H and L weakly exceeds c.

The buyer makes the trade decision at Date 8, which is either a^h or a^l. The utility function u is defined by: $u(a^h \mid H) = (10, 0)$, $u(a^l \mid H) = (22, -22)$, $u(a^h \mid L) = (0, 0)$, and $u(a^l \mid L) = (10, -8)$. Note that this is an example of self investment, because, if the optimal trade decision is made (a^h in state H, a^l in state L), the seller's investment only affects his own cost (0 in state H, 8 in state L).[23] Assume that $c \in (0, 8)$, which means that high investment is efficient. Also assume that the players can renegotiate ex post and that the buyer has all of the bargaining power during renegotiation.

The analysis of forcing contracts runs as follows. By the revelation principle, one can focus on contracts that force a^h at price p^H when the message profile is (H,H), and force a^l at price p^L when the message profile is (L,L). The contract specifies either

[23] I use the term "self investment" as the literature does, although I would not say that it is an accurate description of the contracting environment. Although the *optimal* trade decision has the "self investment" flavor, the seller's investment affects the buyer's payoff of the suboptimal trade decision. Segal (1999) uses the term "complex" to describe such trading environments.

a^h or a^l when the message profile is (L,H), that is, when the buyer reports L and the seller reports H. Consider these cases separately.

First, suppose that the contract forces a^h at price $\hat{p}$ when the message profile is (L,H). For truthful reporting to be an equilibrium of the message game in both states, it must be that player 1 has no incentive to send message L in state H and player 2 has no incentive to send message H in state L. If player 1 deviates in state H, then there would be no renegotiation (because a^h is still specified). Player 1 thus reports truthfully in state H if and only if

$$10 - p^H \geq 10 - \hat{p}.$$

If player 2 deviates in state L, then the players would renegotiate the contractually-specified trade decision, but player 1 would get all of the surplus. Player 2 thus reports truthfully in state L if and only if

$$p^L - 8 \geq \hat{p}.$$

Combining the two inequalities yields the constraint $p^L \geq p^H + 8$.

Second, suppose that the contract forces a^l at price $\hat{p}$ when the message profile is (L,H). In this case, renegotiation would occur if player 1 reports L in state H, but renegotiation would not occur if player 2 reports H in state L. Equilibrium conditions for the message game are

$$10 - p^H \geq 12 - \hat{p} + 10$$

and

$$p^L - 8 \geq \hat{p} - 8,$$

which simplify to $p^L \geq p^H + 22$.

Clearly, then, player 2's payoff in state L must be at least as high as is his payoff in state H, which means no forcing contract induces the seller to invest high. In fact, with ex post renegotiation, *no* contract (forcing or non-forcing) can induce high investment.

To see why non-forcing contracts cannot improve on forcing contracts in this example, consider the scope of non-forcing contracts. Suppose that, given a particular message profile, the contract specifies a price of $\overline{p}^h$ if the buyer chooses trade decision a^h and a price of $\overline{p}^l$ if the buyer chooses trade decision a^l. This is necessarily a forcing contract either if $\overline{p}^l - \overline{p}^h > 12$ (in which case player 1 has the incentive to choose a^h in both states) or if $\overline{p}^l - \overline{p}^h < 10$ (in which case player 1 has the incentive to choose a^l in both states). One can easily confirm that if $\overline{p}^l - \overline{p}^h \in [10, 12]$ then player 1 has the incentive to select a^h in state L and to select a^l in state H. There are no values of $\overline{p}^l$ and $\overline{p}^h$ that give player 1 the incentive to select a^h in state H and a^l in state L.

Thus, there is only one type of non-forcing contractual provision: that which has $\overline{p}^l - \overline{p}^h \in [10, 12]$. This leads to payoff vector $(22 - \overline{p}^l, \overline{p}^l - 22)$ in state H and $(\overline{p}^h, \overline{p}^h)$

in state L. Suppose that this contractual provision is specified for the message profile (L,H). Then, if (L,H) is sent, the players would renegotiate to obtain the efficient trade decision. This implies the payoff vector $(22 - \overline{p}^l + 10, \overline{p}^l - 22)$ in state H and $(\overline{p}^h + 2, \overline{p}^h)$ in state L. The equilibrium conditions for the message game are thus

$$10 - p^H \geq 22 - \overline{p}^l + 10$$

and

$$p^L - 8 \geq \overline{p}^h.$$

Combining these inequalities with $\overline{p}^l - \overline{p}^h \in [10, 12]$, we obtain $p^L \geq p^H + 18$. Again, player 2's payoff in state L must be higher than it is in state H, implying that he does not have the incentive to invest high.

7 Conclusion

I have demonstrated that, to appropriately study institutional constraints, the analysis of contract must start with an understanding of the technology of trade. When parties can renegotiate just before making trade decisions, this technology greatly affects implementability. Thus, researchers should take a structured, game-theoretic approach to studying contract and enforcement. Researchers must be clear about exactly what is verifiable, the nature and timing of inalienable decisions, and how external enforcement occurs.

My analysis here has implications for the applicability of popular mechanism design models. Some theorists, including Segal and Whinston (2000), have stated that future work in applied contract theory will be geared toward discovering whether it is the MDER or MDIR program that is the "right" model of any particular setting. I find that this objective is in error. The MDER program under-represents the true scope of implementability. Contracting parties can often overcome contractual imperfections—to motivate efficient cross investment, for example—even when they have the opportunity to renegotiate just before making trade decisions. In settings with durable trading opportunities and reversibility, parties can effectively deal with the specter of renegotiation by writing long-term contracts. Finally, I emphasize that the mechanism design methodology is still applicable and useful, as long as one correctly defines the outcome set. Unfortunately, differential methods may be less applicable than is currently thought.[24]

I conclude that the MDER program makes hidden assumptions of contractual incompleteness. It can be justified on the basis of a restriction to forcing contracts or, possibly, by assuming that trade decisions are only partially verifiable. It can also be

[24]When trade decisions are used as options, we lose the constant-sum condition that underlies the analysis of Segal and Whinston (2001).

justified, in a durability setting, on the basis of a restriction to short-term contracts. By "short-term contracts," I mean contracts that specify a transfer function for the current period, but specify *nothing* for future periods (not even that the message game and transfer of the current period is repeated in future periods). I can think of no ready defense of these incompleteness assumptions, just as I see no reason to believe that parties would limit themselves to forcing or short-term contracts.[25]

I believe that theorists who idealize the "complete contract" view ought to understand the contractual incompleteness inherent in their own models. I have nothing against the complete contract ideal, in theory, although I find it is not widely applicable. After all, real contracting parties usually face all sorts of technological and institutional constraints. Indeed, an opportunity for parties to renegotiate—one that cannot be controlled by external enforcement—is an incompleteness restriction that many scholars find important and that lies at the heart of most recent contract theory. I simply point out that understanding this incompleteness relies on understanding the technology of trade.

My analysis also has practical relevance. In the least, it should remind us that, to some extent, messages are a theoretical construct. While we sometimes do observe contracts that require parties to send verifiable messages (in real estate transactions, for example), we also often see option contracts that merely specify transfers on the basis of productive decisions (in some procurement settings, for example). Where trading opportunities are durable, we often observe contracts with stationary terms.[26]

There are myriad promising opportunities for fruitful research on contracting with institutional constraints. Given the importance of the technology of trade, the analy-

[25]My message can be further illustrated in the context of Edlin and Hermalin's (2000) debate with Nöldeke, G. and K. Schmidt (1998). In their discussion of whether a party could let an option expire and then renegotiate from scratch, Edlin and Hermalin appeal to the "outside option principle," whereby the outside option implies an inequality constraint on the outcome of negotiation rather than serving as the disagreement point. While there are bargaining models that justify treating outside options in this way, these models blur the distinction between verifiable trade decisions and noncontractible renegotiation opportunities. If parties can exercise trade-based options in the process of renegotiating, then either trade decisions are really not fully verifiable or the opportunity to renegotiate can be partially controlled by the external enforcer (because he can observe a party's actions whenever an option can be exercised—which, for example, would be in every round of an alternating-offer bargaining game). Edlin and Hermalin may have one of these justifications in mind, or they may be thinking of an institutional constraint that limits the time in which an option may be exercised. In any case, I assert that one must model the timing of renegotiation and trade in order to understand exactly what is being assumed. Note that, in my framework, the non-contractible renegotiation opportunity is *separated* in time from the verifiable trade decisions, so that a party cannot delay the trading opportunity by refusing to make an agreement at the time of renegotiation. Other modeling approaches may be useful for comparison.

[26]For example, many construction contracts include a term such as "The contractor's fee is reduced by $\$X$ for every day following date Y that he fails to complete the project." The law recognizes long-term contracts just as it does shorter-term ones.

sis of specific contractual settings seems in order. Further, the notions of durability and reversibility deserve extended study. My work on durability reported here only addresses the specialized environment in which trade decisions are fully reversible. We should develop a better understanding of how limited reversibility influences contractual scope. Overall, it may be worthwhile to revisit some of the literature's basic concepts from a more detailed institutional foundation.[27] In conjunction with Watson (2001), which seeks to clarify the notion of contract in games, my general framework may be a good starting point for future work.

[27]Bull and Watson (2001), for example, study the notion of verifiability as it relates to evidence disclosure in an enforcement system. Schwartz and Watson (2001) study contract and enforcement when contracts are costly to form and renegotiate; they relate the costs to actual legal rules.

A Appendix: Proofs

In this appendix, I restate and prove the theorems found in Section 5. I begin with Theorems 1 and 2, which I prove together.

Theorem 1: $V^{EPF}(\delta)$, $V^{EP}(\delta)$, $V^{I}(\delta)$, and $V^{N}(\delta)$ are well-defined and non-empty.

Theorem 2: $V^{EPF}(\delta) \subset V^{EP}(\delta) \subset V^{I}(\delta) \subset V^{N}(\delta)$ and, in general, none of these sets are equal.

Proof: First note that Λ^k is monotone, for all $k \in \{EPF, EP, I, N\}$. That is, $X' \subset X$ implies $\Lambda^k(X', \delta) \subset \Lambda^k(X, \delta)$. This implies that $V^k(\delta)$ is well-defined, for all $k \in \{EPF, EP, I, N\}$.

Next note that $\Lambda^{EPF}(X, \delta) \subset \Lambda^{EP}(X, \delta)$, which follows from $Z^F(X, \delta) \subset Z(X, \delta)$. From Lemmas 2 and 3, we have $\Lambda^I(X, \delta) \subset \Lambda^N(X, \delta)$. The renegotiation-proofness principle (see Brennan and Watson 2001) implies that $\Lambda^{EP}(X, \delta) \subset \Lambda^I(X, \delta)$. Using the monotone property and the definitions of $V^{EPF}(\delta)$, $V^{EP}(\delta)$, $V^I(\delta)$, and $V^N(\delta)$, we have the inclusion statement of Theorem 2. The example analyzed in Section 1 proves the claim of unequal sets.

To prove that the implementable sets are non-empty in general, let a' be an arbitrary member of A and define v' by

$$v'(\theta) \equiv u(a' \mid \theta) + \pi \left\{ \max_{a \in A} [u_1(a \mid \theta) + u_2(a \mid \theta)] - u_1(a' \mid \theta) - u_2(a' \mid \theta) \right\}$$

for every θ. This is the value function that would result if the players renegotiated over the infinite sequence of trade decisions, with the disagreement point given by decision a' chosen in each period. One can easily confirm (with a contract that forces a' and no transfer in the current period, regardless of messages) that $v' \in \Lambda^{EPF}(\{v'\}, \delta)$, regardless of δ. Thus, $\{v'\}$ has the self-generation property with respect to Λ^{EPF}, and so $V^{EPF}(\delta) \neq \emptyset$. The inclusion property implies that the other sets are also non-empty. *Q.E.D.*

This proof and the discussion in Section 5 make it clear that, for each $k \in \{EPF, EP, I, N\}$,

$$\gamma(\theta, V^k(\delta), \delta) \equiv \max_{a \in A} [u_1(a \mid \theta) + u_2(a \mid \theta)].$$

This justifies redefining γ and r as

$$\gamma(\theta) \equiv \max_{a \in A} [u_1(a \mid \theta) + u_2(a \mid \theta)]$$

and

$$r(w, \theta) \equiv \gamma(\theta) - w_1(\theta) - w_2(\theta).$$

I use these definitions for the remainder of this appendix. I also state the following lemma for reference.

Lemma 7: *If* $v \in V^{EPF}(\delta) \bigcup V^{EP}(\delta) \bigcup V^{I}(\delta)$, *then* $v_1(\theta) + v_2(\theta) = \gamma(\theta)$, *for all* $\theta \in \Theta$.

To prove Theorems 3 and 4, I use the following four results.

Lemma 8: *Suppose* $v \in V^{EPF}(\delta)$ *and consider the direct-revelation mechanism that implements* v *in the current period—this mechanism maps message profiles into elements of* $Z^F(V^{EPF}(\delta), \delta)$. *Then for every message profile* $m \in \Theta^2$, *there exist distributions* $\alpha \in \Delta A$ *and* $\phi \in \Delta V^{EPF}(\delta)$ *such that, for each state* θ, *the payoff vector contingent on* m *in state* θ *is*

$$(1-\delta) \sum_{a \in A} [u(a \mid \theta) + \pi r(u(a \mid \cdot), \theta)] \, \alpha(a) + \delta \int v' d\phi(v').$$

In the case of interim renegotiation, the payoff vector can be written

$$(1-\delta) \sum_{a \in A} u(a \mid \theta) \alpha(a) + \delta \int v' d\phi(v'),$$

for some $\alpha \in \Delta A$ *and* $\phi \in \Delta V^{I}(\delta)$.

Proof: Consider any $v \in V^{EPF}(\delta)$. Contingent on m, in state θ the players obtain a payoff given by $w(\theta) + \pi r(w, \theta)$, where $w \in W^F(V^{EPF}(\delta), \delta)$ depends on m. Recall that $w(\theta)$ is given by Equation 4 on page 16, for an appropriately defined transfer function $\hat{y}$, a continuation value function $\hat{x}$, and a behavior function $\hat{a}$. Let ϕ' be the distribution over $V^{EPF}(\delta)$ implied by $\hat{x}(\cdot, \hat{a}(\cdot, \theta))$ and μ.

From Lemma 7, every v' in the support of ϕ' is efficient. Thus, the renegotiation surplus derives entirely from a gain achieved in the current period. Since the players divide this surplus according to their bargaining weights, relative to the disagreement point given by w, we can express the payoff vector as:

$$\begin{aligned} w(\theta) + \pi r(w, \theta) &= (1-\delta) \int [u(\hat{a}(q, \theta) \mid \theta) + \hat{y}(q, \hat{a}(q, \theta)) + \pi r(u(\hat{a}(q, \theta) \mid \cdot), \theta)] \, d\mu(q) \\ &\quad + \delta \int v' d\phi'(v'). \end{aligned} \tag{5}$$

The first and last terms in the first integral can be written

$$(1-\delta) \sum_{a \in A} [u(a \mid \theta) + \pi r(u(a \mid \cdot), \theta)] \, \alpha(a),$$

for some $\alpha \in \Delta A$ (since the trade decision may be conditioned on q). Regarding the second term, define

$$\overline{y} \equiv (1-\delta) \int \hat{y}(q, \hat{a}(q, \theta)) d\mu(q).$$

Note that $\overline{y}$ is balanced.

From Lemma 6, we know that $v' + \overline{y} \in V^{EPF}(\delta)$ for every $v' \in V^{EPF}(\delta)$. Thus, there is a distribution $\phi \in \Delta V^{EPF}(\delta)$ such that

$$\delta \int v'' d\phi(v'') = \delta \int v' d\phi'(v') + \overline{y}.$$

Substituting this into Equation (5) yields the conclusion of the theorem.

For the case of interim renegotiation, where $v \in V^I(\delta)$, the proof is simpler. Lemma 6 allows the externally enforced transfer to be merged into the continuation payoff. *Q.E.D.*

Lemma 9: *There is a number L such that, for both $i = 1, 2$, every δ, every two states $\theta, \theta' \in \Theta$, and every $v \in V^I(\delta)$, $|v_i(\theta) - v_i(\theta')| < L$.*

Proof: Recalling the definition W from Section 2, and recalling that u is bounded, we see that there is a number L' such that, for both $i = 1, 2$, every two states $\theta, \theta' \in \Theta$, and every $w \in W$, $|w_i(\theta) - w_i(\theta')| < L'$. Combining this with the implementation conditions of Lemma 3, it is not difficult to confirm the result. *Q.E.D.*

Lemma 10: *Consider any $v \in V^{EPF}(\delta) \bigcup V^I(\delta)$. For each state θ, there is a sequence $\{\alpha^t\}_{t=1}^{\infty} \subset \Delta A$ and a balanced constant transfer $\overline{y}^\theta$ such that*

$$v(\theta) = \sum_{t=1}^{\infty} \delta^{t-1}(1-\delta) \sum_{a \in A} \left[u(a \mid \theta) + \pi r(u(a \mid \cdot), \theta)\right] \alpha^t(a) + \overline{y}^\theta.$$

Proof: First take the case of ex post renegotiation. Recursive application of Lemma 8, using message profile (θ, θ), establishes the following. There are sequences $\{\alpha^t\}_{t=1}^{\infty} \subset \Delta A$ and $\{\phi^t\}_{t=2}^{\infty} \subset \Delta V^I(\delta)$ such that, for every positive integer T,

$$v(\theta) = \sum_{t=1}^{T} \delta^{t-1}(1-\delta) \sum_{a \in A} \left[u(a \mid \theta) + \pi r(u(a \mid \cdot), \theta)\right] \alpha^t(a) + \delta^T \int v' d\phi^{T+1}.$$

As $T \to \infty$, the summation term converges. Thus, the integral term also converges. Further, because of discounting and bounded utilities, the limit of the integral term, which defines $\overline{y}^\theta$, is balanced.

The same argument can be used for the case of interim renegotiation, which has efficient outcomes in equilibrium. In this case, one can focus on forcing transfers because there is no renegotiation at Date 5 in each period. *Q.E.D.*

Lemma 11: *Consider any $v \in V^{EPF}(\delta)$. For any two states $\theta, \theta' \in \Theta$ there is a sequence $\{\alpha^t\}_{t=1}^{\infty} \subset \Delta A$ and a balanced constant transfer $\overline{y}^{\theta\theta'}$ such that*

$$v_1(\theta') \geq \sum_{t=1}^{\infty} \delta^{t-1}(1-\delta) \sum_{a \in A} \left[u_1(a \mid \theta) + \pi_1 r(u(a \mid \cdot), \theta)\right] \alpha^t(a) + \overline{y}_1^{\theta\theta'}$$

and

$$v_2(\theta) \geq \sum_{t=1}^{\infty} \delta^{t-1}(1-\delta) \sum_{a \in A} \left[u_2(a \mid \theta) + \pi_2 r(u(a \mid \cdot), \theta)\right] \alpha^t(a) + \overline{y}_2^{\theta\theta'}.$$

A similar result holds in the case of interim renegotiation. For $v \in V^I(\delta)$, the conditions are

$$v_1(\theta') \geq \sum_{t=1}^{\infty} \delta^{t-1}(1-\delta) \sum_{a \in A} u_1(a \mid \theta)\alpha^t(a) + \overline{y}_1^{\theta\theta'}$$

and

$$v_2(\theta) \geq \sum_{t=1}^{\infty} \delta^{t-1}(1-\delta) \sum_{a \in A} u_2(a \mid \theta)\alpha^t(a) + \overline{y}_2^{\theta\theta'}.$$

Proof: This lemma is also proved by recursive application of Lemma 8, using message profile (θ, θ'). First consider the case of ex post renegotiation. From Lemma 8 and that the players report truthfully in the equilibrium of the message game, there exist $\alpha^1 \in \Delta A$ and $\phi^2 \in \Delta V^{EPF}$ such that

$$v_1(\theta') \geq (1-\delta) \sum_{a \in A} \left[u_1(a \mid \theta) + \pi_1 r(u(a \mid \cdot), \theta)\right] \alpha^1(a) + \delta \int v_1' d\phi^2(v')$$

and

$$v_2(\theta) \geq (1-\delta) \sum_{a \in A} \left[u_2(a \mid \theta) + \pi_2 r(u(a \mid \cdot), \theta)\right] \alpha^1(a) + \delta \int v_2' d\phi^2(v').$$

Applying the same argument for each v', again for message profile (θ, θ'), establishes the following. There are sequences $\{\alpha^t\}_{t=1}^{\infty} \subset \Delta A$ and $\{\phi^t\}_{t=2}^{\infty} \subset \Delta V^{EPF}(\delta)$ such that, for every positive integer T,

$$v_1(\theta') \geq \sum_{t=1}^{T} \delta^{t-1}(1-\delta) \sum_{a \in A} \left[u_1(a \mid \theta) + \pi_1 r(u(a \mid \cdot), \theta)\right] \alpha^t(a) + \delta^T \int v_1' d\phi^{T+1}$$

and

$$v_2(\theta) \geq \sum_{t=1}^{T} \delta^{t-1}(1-\delta) \sum_{a \in A} \left[u_2(a \mid \theta) + \pi_2 r(u(a \mid \cdot), \theta)\right] \alpha^t(a) + \delta^T \int v_2' d\phi^{T+1}.$$

As $T \to \infty$, the summation terms converge. Lemma 9 implies that

$$\delta^T \int v' d\phi^{T+1}$$

converges and its limit is balanced. Define $\overline{y}^{\theta\theta'}$ to be the limit of this term.

The same steps can be used in the interim renegotiation case; in this case, one can focus on forcing transfers because there is no renegotiation at Date 5 in each period. *Q.E.D.*

With these results in hand, I restate and prove the stationarity theorem.

Theorem 3: In all settings, except possibly in the case of ex post renegotiation, all implementable value functions are supported by stationary contracts. More precisely, for each $h \in \{EPF, I, N\}$ and every $\delta \in [0, 1)$, every $v \in V^h$ is supported by a stationary contract with respect to Λ^h and δ.

Proof: Consider any $v \in V^{EPF}$. Define a stationary contract as follows. In each period, let the continuation value function be v, regardless of the players' behavior in the period. Specify a forcing transfer function that duplicates the payoffs given by Lemmas 10 and 11, multiplied by $(1 - \delta)$, in the *current period.* Specifically, for message profile (θ, θ), the mechanism specifies a transfer of $(1-\delta)\overline{y}^{\theta}$ and a randomized trade decision defined by the distribution $\alpha^{\theta\theta}$, where

$$\alpha^{\theta\theta} \equiv (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} \alpha^t,$$

with $\{\alpha^t\}_{t=1}^{\infty}$ given by Lemma 10. Likewise, for any message profile (θ, θ') with $\theta \neq \theta'$, the mechanism specifies a transfer of $(1 - \delta)\overline{y}^{\theta\theta'}$ and a randomized trade decision defined by Lemma 11. The randomization is achieved using the public draw q.

One can readily verify that reporting truthfully is a Nash equilibrium of the implied message game. Note that renegotiation occurs at Date 5, so the actual payoffs are given by the expression in Lemma 10.

The proof for the case of interim renegotiation is similar. The above steps are not necessary for the proof of the no renegotiation case. It is obvious that, without renegotiation, $V^N(\delta) \subset V^N(0)$. Further, any value function in $V^N(0)$ can be implemented in an environment of $\delta > 0$ using a "scaled down" stationary contract. *Q.E.D.*

The final theorem of Section 4 is

Theorem 4: $V^{EPF}(\delta)$, $V^I(\delta)$, and $V^N(\delta)$ are all constant in δ.

Proof: Consider any $v \in V^{EPF}(\delta)$ and any $\delta' \in [0, 1)$. To support v under discount factor δ', we can use the same stationary contract that implements v under δ. To see this, note that we can ignore continuation payoffs in the calculation of renegotiation outcomes in a given period. Since the continuation payoff is a constant function of the state, this cancels out in the analysis of incentives in the message phase. The current-period payoff terms are scaled up or down (depending on whether $\delta' > \delta$), but the incentives to report truthfully do not change. The same method of proof establishes the result for any $v \in V^I(\delta)$; as with the preceding results, we can focus on forcing transfers. That $V^N(\delta)$ is constant in δ follows from the statements at the end of the proof of Theorem 3. *Q.E.D.*

References

Abreu, D., D. Pearce, and E. Stacchetti, "Optimal Cartel Equilibrium with Imperfect Monitoring," *Journal of Economic Theory* 39 (1986): 251-269.

Anderlini, L., L. Felli, and A. Postlewaite, "Courts of Law and Unforeseen Contingencies," University of Pennsylvania Law School Research Paper 01-05, March 2001.

Barnett, R. E., *Contracts: Cases and Doctrine*, second edition, Gaithersburgh: Aspen Publishers, 1999.

Bernheim, B. D. and M. Whinston, "Incomplete Contracts and Strategic Ambiguity," *American Economic Review* 88 (1998): 902-932.

Brennan, J. and J. Watson, "The Renegotiation-Proofness Principle and Costly Renegotiation," manuscript, June 2001.

Bull, J., "Costly Evidence Production and the Limits of Verifiability," unpublished manuscript, May 2001.

Bull, J. and J. Watson, "Evidence Disclosure and Verifiability," UC San Diego Economics Working Paper 2000-16, June 2000.

Che, Y.-K. and D. Hausch, "Cooperative Investments and the Value of Contracting," *American Economic Review* 89 (1999): 125-147.

Edlin, A. and B. Hermalin, "Contract Renegotiation and Options in Agency Problems," *Journal of Law, Economics, and Organization* 16 (2000): 395-423.

Edlin, A. and S. Reichelstein, "Holdups, Standard Breach Remedies, and Optimal Investment," *American Economic Review* 86 (1996): 478-501.

Hart, O. and J. Moore, "Incomplete Contracts and Renegotiation," *Econometrica* 56 (1988): 755-785.

Hart, O. and J. Moore, "Foundations of Incomplete Contracts," *Review of Economic Studies* 66 (1999): 115-138.

Hurwicz, L., "Economic Design, Adjustment Processes, Mechanisms, and Institutions," *Economic Design* 1 (1994): 1-14.

Jackson, M. and T. Palfrey, "Voluntary Implementation," forthcoming, *Journal of Economic Theory*; manuscript dated August 2000.

Kalai, E. and J. Ledyard, "Repeated Implementation," Northwestern University MEDS working paper, April 1997.

Lyon, T. and E. Rasmusen, "Option Contracts and Renegotiation in Complex Environments," Indiana University Working Paper, June 2001.

MacLeod, W. B., "Complexity and Contract," forthcoming in *The Economics of Contract in Prospect and Retrospect*, edited by E. Brousseau and J.-M. Glachant, Cambridge University Press (2001).

MacLeod, W. B. and J. M. Malcomson, "Investments, Holdup, and the Form of Market Contracts," *American Economic Review* 83 (1993): 811-837.

Maskin, E., "Nash Equilibrium and Welfare Optimality," *Review of Economic Studies* 66 (1999): 23-38.

Maskin, E. and J. Moore, "Implementation and Renegotiation," *Review of Economic Studies* 66 (1999): 39-56.

Moore, J. and R. Repullo, "Subgame Perfect Implementation," *Econometrica* 56 (1988): 1191-1220.

Nöldeke, G. and K. Schmidt, "Option Contracts and Renegotiation: A Solution to the Hold-Up Problem," *RAND Journal of Economics* 26 (1995): 163-179.

Nöldeke, G. and K. Schmidt, "Sequential Investments and Options to Own," *RAND Journal of Economics* 29 (1998): 633-653.

Segal, I., "Complexity and Renegotiation: A Foundation for Incomplete Contracts," *Review of Economic Studies* 66 (1999): 57-82.

Segal, I. and M. Whinston, "The Mirrlees Approach to Mechanism Design with Renegotiation (with Applications to Hold-Up and Risk-Sharing)," forthcoming, *Econometrica*; manuscript dated 2000.

Schwartz, A. and J. Watson, "The Law and Economics of Costly Contracting," manuscript, December 2001.

Watson, J., "Contract and Game Theory: Basic Concepts for Settings of Complete Information," manuscript, June 2001.