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Constructing internal diagrammatic proofs from external logic diagrams

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Abstract

Internal syntactic operations on diagrams play a key role in accounting for efficacy of diagram use in reasoning. However, it is often held that in the case of complex deductive reasoning, diagrams can serve merely as an auxiliary source of information in interpreting sentences or constructing models. Based on experiments comparing subjects' performances in syllogism solving where logic diagrams of several different forms are used, we argue that internal manipulations of diagrams, or what we call internal constructions of diagrammatic proofs, actually exist, and that such constructions are naturally triggered even for users without explicit prior knowledge of their inference rules or strategies.
Keywords: External representation; Diagrammatic reasoning; Logic diagram; Deductive reasoning

Introduction

People have tried to enhance reasoning abilities by the use of artificial devices since ancient times. In particular, symbol manipulation is a distinctive tool-use of human beings. Certainly, symbolic logic may be considered to be a tool for deductive reasoning. However, it should be noted that symbolic logic (e.g., first-order logic) is not always a usable system for untrained people. By contrast, visual-spatial representations are considered to be much more intuitive and effective for novices' actual reasoning. Consequently, over the past few decades, many researchers have shown an interest in the efficacy of diagrammatic reasoning (e.g. Allwein & Barwise, 1996; Glasgow, Narayanan, & Chandrasekaran, 1995).

An important assumption in the study of diagrammatic reasoning is that diagrams are syntactic objects to be manipulated in certain ways; we make an inference about a diagram itself, transforming it into another form or combining it with other diagrams. Such syntactic manipulations of diagrams play a crucial role in accounting for their efficacy in deductive problem solving. For example, consider the following process of checking the validity of a syllogism using Euler diagrams.

D_1 : *All A are B*

D_2 : *No C are B*

Therefore, *No C are A*

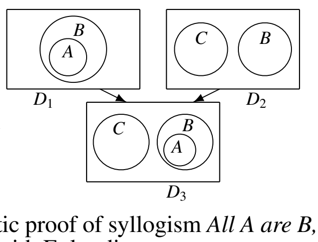


Figure 1: A diagrammatic proof of syllogism *All A are B, No C are B; therefore No C are A* with Euler diagrams.

The premise *All A are B* is represented by D_1, and the premise *No C are B* by D_2. By unifying D_1 with D_2, we can obtain diagram D_3. Here the exclusion relation holds between circles *A* and *C*, from which we can extract the correct conclusion "No *C* are *A*". In what follows, we call such a syntactic manipulation of diagrams to derive a conclusion of deductive reasoning a *construction of a diagrammatic proof.* The point here is that by unifying two diagrams in premises and observing the topological relationship between the circles, one can *automatically read off* the correct conclusion. Shimojima (1996) calls this a "free ride" property, and shows that it can be seen to exist in other kinds of diagram use in reasoning and problem solving.

In general, a deductive reasoning task would be easy if it could be replaced with a task of constructing a concrete diagrammatic proof. Typically, such a construction is supposed to be triggered by external diagrams and carried out internally, without actual drawing or movement of physical objects. However, the existence of such internal manipulations of diagrams has been the subject of controversy (see, e.g. Schwartz, 1995). Indeed, it is widely held that diagrams can serve merely as a memory-aid or an auxiliary source of information in deductive problem solving. Thus, Larkin and Simon (1987) argue that reasoning is largely independent of ways of representing information, hence diagrams are less beneficial in reasoning. Bauer and Johnson-Laird (1993) discuss efficacy of diagrams in deductive reasoning with double disjunction and argue that diagrams are used to keep track of alternative models, as postulated in mental model theory. Also in many logic textbooks, diagrams are used to depict models and aid understanding of logical representations, rather than as objects of syntactic manipulations.

In view of this situation, it is of central importance to investigate whether internal manipulations of diagrams really exist in actual reasoning with diagrams. Trafton and Trickett (2001) argue that there are mental processes of "spatial transformations" to extract information from graphs or visualization, based on an analysis of how expert scientists collect data in their researches. Shimojima and Fukaya (2003) and Shimojima and Katagiri (2008) argue for the existence of "inference by hypothetical drawing", internal transformations of external diagrams, based on eye-tracking data of subjects working with position diagrams in transitive inferential tasks. In this paper, we focus on more complex deductive reasoning tasks, namely, syllogistic reasoning tasks, and on the effects of logic diagrams externally given therein. We present evidence for the existence of internal constructions of diagrammatic proofs, on the basis of experiments comparing subjects' performances in syllogism solving where logic diagrams of several different forms are given. Our claim is consistent with the influential view in the study of external representations in general, namely, that (a) external representations can be used without being interpreted, and that (b) they can change the nature of tasks, namely, tasks with and without external representations are completely different from users' point of view (see Zhang & Norman, 1994; Scaife & Rogers, 1996).

The efficacy of logic diagrams has been investigated in the context of the studies of logic teaching method (Stenning, 1999; 2002; Dobson, 1999). In these studies, subjects are provided with substantial training in ways of manipulating diagrams. In contrast to this, our interest is in the question whether diagrams can be useful for those who are not trained in rules or strategies of diagrammatic deductive reasoning. This question is important because, in contrast to logical formulas in symbolic logic, logic diagrams in general have been expected to be much more intuitive and effective for novices' reasoning, not for experts' nor for machine reasoning. In view of the complexities of solving processes of deductive reasoning (e.g. Levesque, 1988), it is interesting to ask whether logic diagrams can have this surprising property.

Logic diagrams have also been studied in the field of formal diagrammatic logic since the 1990s (e.g. Shin, 1994; Howse, Stapleton & Taylor, 2005), and inference systems for various diagrams such as Euler and Venn diagrams have been developed. Currently, however, there are few empirical researches to investigate their cognitive foundations. Our study is also intended to provide a bridge between logical and cognitive studies of diagrammatic reasoning.

Cognitive model for reasoning with diagrams

Typical examples of deductive reasoning problems with external logic diagrams are shown in Figure 2.

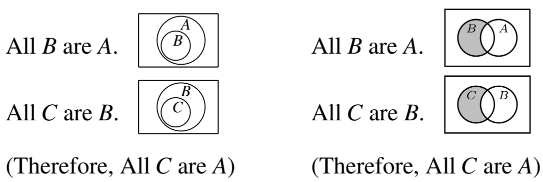


Figure 2: Examples of syllogistic reasoning tasks with diagrams

Here a syllogism is presented with logic diagrams (Euler and Venn diagrams). How can such diagrams contribute to checking the validity of a deductive argument? Let us first hypothesize a cognitive model of deductive problem solving with diagrams. The model is shown in Figure 3. This model highlights two roles of diagrams in deductive reasoning.

Regarding sentential reasoning, we assume a standard two-staged framework in natural language semantics (see, e.g. Blackburn & Bos, 2005), according to which sentences are first associated with semantic information, and then the validity of the argument is checked using some inferential mechanisms (such as model-theoretical or proof-theoretical ones). The details and precise nature of such linguistic comprehension and inference are not our concern here.

Diagrams are also associated with semantic information, but at the same time they are syntactic objects to be manipulated in reasoning processes. We distinguish two ways in which diagrams can be effective in deductive reasoning.

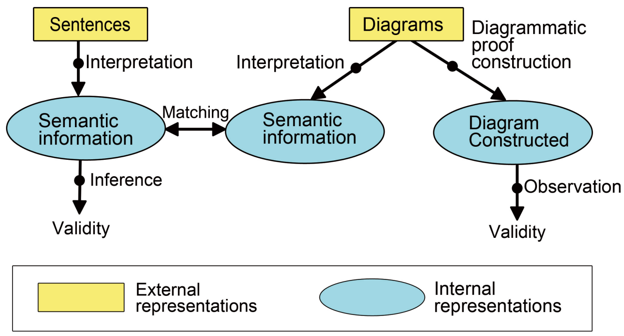


Figure 3: Cognitive model for diagrammatic reasoning

Interpretational efficacy Firstly, diagrams can help fix the correct interpretations of sentences and thereby avoid deductive reasoning errors due to misinterpretation. For example, a sentence "All *A* are *B*" is sometimes misinterpreted as equivalent to "All *B* are *A*". This is known as *illicit conversion error* in the literature (e.g. Newstead & Griggs, 1983). Subjects presented with diagrams such as the ones in Figure 2 could immediately see that the diagrams corresponding to these two sentences are topologically different, and hence deliver different semantic information. In our model, such processes are formulated as processes of matching the semantic information obtained from diagrams with the one obtained from sentences. In this case, the validity of an argument is checked based on the same kind of process as the one in linguistic reasoning. Here diagrams are used in a static way, merely as a record of information (Barwise & Etchemendy, 1991).

Inferential efficacy Secondly, and more importantly, diagrams can play a crucial role in reasoning processes themselves. More specifically, the solving processes of deductive reasoning tasks can be replaced with internal manipulations of diagrams. In other words, one can check the validity of a deductive argument by means of constructions of diagrammatic proofs. The above model assumes that such constructions are conducted through a proof-theoretical component of diagrammatic reasoning. If a task of constructing diagrammatic proofs consists of simple and intuitive steps, it is expected to be more tractable than usual linguistic inferences.

It seems to be generally agreed that logic diagrams have interpretational efficacy. For example, Stenning (2002) argues that (a)symmetricity of diagrams can aid processing of the meaning of quantified sentences in syllogisms. Mineshima, Okada, Sato, and Takemura (2008) presented experimental evidence for such interpretational effects, based on a comparison of the performances of syllogistic solving tasks with and without Euler diagrams. In what follows, we assume that logic diagrams can have interpretational efficacy, and investigate whether they can have inferential efficacy as well.

General Hypothesis

Based on the above model, we propose the following general hypothesis: (1) logic diagrams can have inferential efficacy, that is, internal constructions of diagrammatic proofs occurs in deductive reasoning with external logic diagrams, and (2) certain diagrams would naturally trigger such constructions so that even users without explicit prior knowledge of inference rules or strategies could correctly manipulate diagrams.

One way to test our hypothesis is to compare performances of deductive problem solving with several distinct diagrams which are equivalent in semantic information but are of different forms, namely, ones that have a form suitable for diagrammatic proof constructions and ones that do not. A basic assumption here is that the existence of internal constructions depends on the forms of diagrams given, and on the simplicity or naturalness of the required diagrammatic proofs. If subjects' performance with diagrams of a form suitable for diagrammatic proof constructions would be significantly better, it could count as evidence for the existence of such constructions in subjects' reasoning.

To test the claim in (2), subjects in our experiments were presented with instructions on the meaning of categorical sentences and diagrams used, but not with any instruction on rules or strategies of constructing diagrammatic proofs. We expect that if certain diagrams have inferential efficacy, it would be exploitable based on their natural properties or constraints, rather than extra conventions. In other words, processes of constructing diagrammatic proofs as postulated in our cognitive model could be conducted without explicit knowledge of the underlying rule or strategies; such internal constructions could be naturally triggered based on the correct understanding of the meaning of diagrams.

Task Analysis

Conventional devices in Euler and Venn diagrams

In our experiment, we use the following three types of diagrams: Euler diagrams, Venn diagrams having two circles, which we call "2-Venn diagrams", and Venn diagrams having three circles, which we call "3-Venn diagrams". Typical examples are shown in Figure 4.

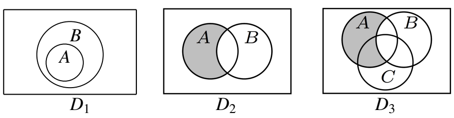


Figure 4: Representations of *All A are B* in Euler diagram (D_1), 2-Venn diagram (D_2), and 3-Venn diagram (D_3).

Euler diagrams used in our experiment are the ones introduced in Mineshima et al. (2008). Our system has the following features: (i) it uses a named *point* 'x' to indicate the existence of objects; (ii) it adopts a convention of *crossing*, according to which two circles which are indeterminate with respect of their relationship are put to partially overlap each other. Consequently, a single categorical statement can be represented by just a single diagram (see D_7 in Figure 5). This contrasts with another version of Euler system, which requires more than one diagrams to represent some categorical sentences, and hence has the well-known problem of combinatorial complexities (see chapter 4 of Johnson-Laird, 1983).

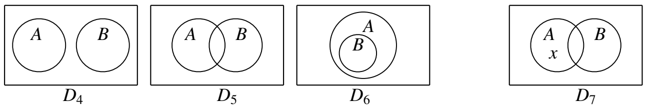


Figure 5: The diagrams corresponding to "Some A are not B" in traditional Euler system (D_4, D_5 and D_6) and the one in our Euler representation system (D_7)

In Venn diagrams, every circle partially overlaps each other, as in D_2 and D_3 in Figure 4 and D_5 in Figure 5. Such diagrams do not convey any specific information about circles, hence are subject to the convention of crossing. Meaningful relations among circles are then expressed using a novel device, *shading*, by the convention that a shaded region denotes an empty set. For example, the statement "All A are B" is represented as D_2 or D_3 in Figure 4. Note that the same information can also be conveyed by the Euler diagram D_1. Furthermore, 3-Venn diagrams use a link to connect points, which represent the disjunctive information about a point (see D_4^v and D_5^v in Figure 7 below).

Constructions of diagrammatic proofs

We compare syllogism solving tasks using these three types of diagrams in terms of difficulties in constructing the corresponding diagrammatic proofs. Deductive reasoning generally requires combining information in premises. Such a task could naturally be replaced by a task of combining presented diagrams. We expect that an inference process of combining diagrams is relatively easy to access, and accordingly, that an internal construction of a diagrammatic proof is naturally triggered if it consists only of such combining processes.

Reasoning with Euler diagrams As an illustration, consider a syllogism *All B are A, Some C are B; therefore Some C are A*. A solving process of this syllogism using Euler diagrams is shown in Figure 6.

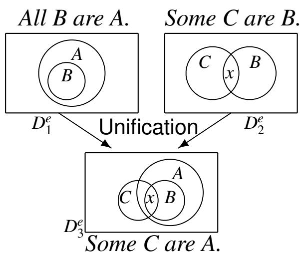


Figure 6: A proof of syllogism using Euler diagrams.

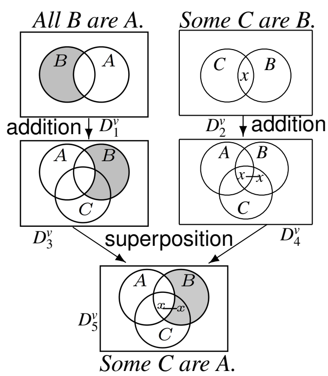


Figure 7: A proof of syllogism using Venn diagrams.

In general, a diagrammatic proof of a syllogism with Euler diagrams consists of a step of combining premise diagrams, which we call a *unification* step. It is expected that unification steps are relatively easy to access, so that such constructions of diagrammatic proofs are naturally triggered.

Reasoning with 2-Venn diagrams A solving process in 2-Venn diagrams is illustrated in Figure 7, where the premise *All B are A* is represented by D_1^v, and the premise *Some C are B* by D_2^v. Since these two diagrams contain a different circle, in order to combining them while preserving the syntax of Venn diagrams, one needs to accommodate their circles. In this case, circle C is added to D_1^v, and A to D_2^v. Then, by superposing the shaded region of D_3^v on D_4^v, one obtains diagram D_5^v, from which the conclusion "Some C are A" can correctly be read off. Here we can see that a solving process in 2-Venn diagrams consists of two steps, which we call *addition* and *superposition*. Note that a process of combining Euler diagrams, namely *unification*, can exploit the movements of circles as in Figure 6, whereas a process of combining Venn diagrams, namely *superposition*, operates on premise diagrams with the same number of circles, and hence does not involve any movement of circles.

Reasoning with 3-Venn diagrams If we start the proof with the 3-Venn diagrams D_3^v and D_4^v in Figure 7, we can skip the steps of adding a circle as required in the case of 2-Venn diagrams. The only step needed is the *superposition* step, which is expected to be relatively easy to access.

Predictions

Intuitively, 2-Venn diagrams seem to be relatively difficult to handle in solving syllogisms. For in order to construct diagrammatic proofs from 2-Venn diagrams, one has to know the relevant inference rules and strategies in advance. More specifically one has to know the successive processes of adding a circle and superposing two diagrams, as indicated in Figure 7. We expect that those who are ignorant of such a solving strategy could not appeal to concrete manipulations of the diagrams. They seem to have to draw a conclusion solely based on usual linguistic inference, with the help of semantic information obtained from 2-Venn diagrams.

To test this point, we introduce set-theoretical expressions corresponding to Venn diagrams, such as $A \cap \overline{B} = \emptyset$ for "All A are B" and $A \cap B \neq \emptyset$ for "Some A are B", as a control condition. We assume that they could only contribute to interpreting premises of syllogisms. Thus, they are used to check whether the effects of Euler, 3-Venn, and 2-Venn diagrams are interpretational or not.

In contrast to 2-Venn diagrams, both Euler diagrams and 3-Venn diagrams seem to be relatively easy to handle even for those users who are not trained to manipulate them in syllogism solving. The essential steps involved are unification and superposition steps. Given the fact that deductive reasoning generally requires combining the information in premises, these processes seem to be natural enough so that they would be immediately accessible to users. We expect that users could exploit natural constraints of diagrams and extract the right rules to draw a conclusion from Euler diagrams and 3-Venn diagrams themselves.

We will say that diagrammatic representations are *self-guiding* if the constructions of diagrammatic proofs are automatically triggered even for subjects without explicit prior knowledge of inferential rules or strategies. Then, our hypothesis amounts to saying that in syllogistic reasoning tasks, Euler diagrams and 3-Venn diagrams are self-guiding, whereas 2-Venn diagrams are not.

Based on these considerations, we predict that the performance in syllogism solving would be better when subjects use Euler diagrams or 3-Venn diagrams than when they use symbolic (set-theoretical) representations. We also predict that there would be little difference between the performance with 2-Venn diagrams and with the symbolic representations.

Method

Subjects are provided with instructions on the meanings of diagrams and then required to solve syllogistic reasoning tasks with diagrams. We conducted a pretest to check whether subjects understood the instructions correctly. The pretest was designed mainly to see whether subjects correctly understood the conventional devices of each diagram, in particular, the convention of crossing in both Euler and Venn diagrams and shading and linking in Venn diagrams.

Participants

365 undergraduates (mean age 19.78 $\pm$ 2.69 SD) in six introductive philosophy classes took part in the experiments. They gave a consent to their cooperation in the experiments, and were given small reward after the experiments. The subjects were native speakers of Japanese. The sentences and instructions were given in Japanese. The subjects were divided into four groups: Symbolic, 2-Venn, 3-Venn, and Euler groups. The four groups in this order consisted of 90, 95, 114, and 66 students, respectively. From each we excluded 26, 27, 35, 3 students (those who gave up before the end), respectively. It is notable that fewer students in the Euler group gave up compared to the other three groups.

Materials

The experiment was conducted in the booklet form.

Pretest The subjects of all groups were presented with ten representations (ten diagrams or ten set-theoretical expressions). They were asked to choose, from a list of five possibilities, all sentences which correspond to a given representation. The highest possible score on the pretest of the Symbolic group was ten and the cutoff point was set to be five. The highest possible score on the pretests of the 2-Venn, 3-Venn, and Euler groups was twelve, because there were two correct answers in two of the ten problems. Their cutoff point was set to be eight. These cutoff points were chosen carefully, based upon the results of our pilot experiments. The total time in Symbolic, 2-Venn and Euler groups was 5 minutes. The total time in the 3-Venn group was 6 minutes, since the instruction was longer than those of the other three groups. Before the pretest, the subjects in each group were presented with three examples.

Syllogistic reasoning tasks The subjects in the Symbolic group were given syllogisms with set-theoretical representations (such as the one in Figure 8). The subjects in the 2-Venn group were given syllogisms with Venn diagrams having two circles in premises (such as the one in Figure 9). The subjects in the 3-Venn group were given syllogisms with Venn diagrams having three circles in premises (such as the one in Figure 10). The subjects in the Euler group were given syllogisms with Euler diagrams (such as the one in Figure 11). We gave 31 syllogisms in total, out of which 14 syllogisms had a valid conclusion and 17 syllogisms had no valid conclusion. The subjects were presented with two premises and were asked to choose, from a list of five possibilities, a sentence corresponding to the valid conclusion. The list consists of *All-*, *No-*, *Some-*, *Some-not*, and *NoValid*. The subject-predicate order of each conclusion was *CA*. The test was a 20-minute power test, and each task was presented in random order (10 patterns were prepared). Before the test, the examples in Figure 8, 9, 10, and 11 were presented to each group.

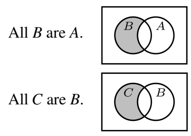
All *B* are *A*. $B \cap \overline{A} = \emptyset$

All *C* are *B*. $C \cap \overline{B} = \emptyset$

1. All *C* are *A*.
2. No *C* are *A*.
3. Some *C* are *A*.
4. Some *C* are not *A*.
5. None of them.

Correct answer: 1

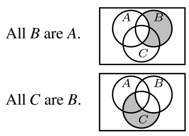
Figure 8: Example of reasoning task of Symbolic group



1. All *C* are *A*.
2. No *C* are *A*.
3. Some *C* are *A*.
4. Some *C* are not *A*.
5. None of them.

Correct answer: 1

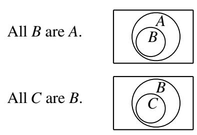
Figure 9: Example of reasoning task of 2-Venn group



1. All *C* are *A*.
2. No *C* are *A*.
3. Some *C* are *A*.
4. Some *C* are not *A*.
5. None of them.

Correct answer: 1

Figure 10: Example of reasoning task of 3-Venn group



1. All *C* are *A*.
2. No *C* are *A*.
3. Some *C* are *A*.
4. Some *C* are not *A*.
5. None of them.

Correct answer: 1

Figure 11: Example of reasoning task of Euler group

Procedure

All four groups were first given 1 minute 30 seconds to read one page instructions on the meaning of categorical sentences. In addition, the Symbolic group was given 2 minutes to read two pages instructions on the meaning of set-theoretical representations. The 2-Venn and Euler groups were given 2 minutes to read two pages instructions on the meaning of diagrams. The 3-Venn group was given 3 minutes to read two pages instructions on the meaning of diagrams. Before the pretest, all groups were given 1 minute 30 seconds to read two pages instructions on the pretest. Finally, before the syllogistic reasoning test, all four groups were given 1 minute 30 seconds to read two pages instructions, in which the subjects were warned to choose only one sentence as answer and not to take a note. These time limits were set based upon the results of our pilot experiments.

Results and Discussion

Pretest

In the Symbolic group, 39 students scored less than 5 on the pretest. In the 2-Venn group, 38 students scored less than 8 on the pretest. In the 3-Venn group, 41 students scored less than 8 on the pretest. In the Euler group, 18 students scored less than 8 on the pretest. These students are excluded from the following analysis.

Syllogistic reasoning tasks

Figure 12 shows the average accuracy rates of the total 31 syllogisms in each group. The rate for the Euler group was 85.2%, the rate for the 3-Venn group was 75.2%, the rate for the Venn group was 66.6%, and the rate for the Symbolic group was 58.7%.

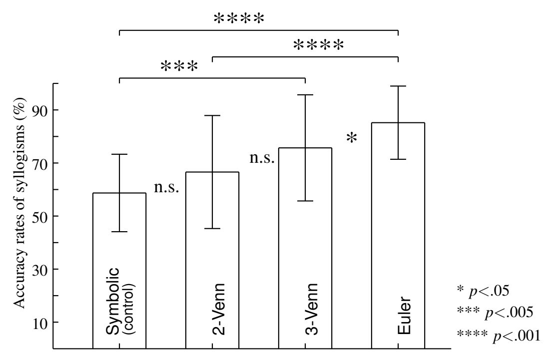


Figure 12: The average accuracy rates of 31 total syllogisms in the Symbolic, 2-Venn, 3-Venn, and Euler groups (error-bar refers to SD).

These data were subjected to a one-way Analysis of Variance (ANOVA). There was a significant main effect, F (3, 134) = 13.680. $p < .001$. Multiple comparison tests by Ryan's procedure yield the following results: (i) There was a significant difference between the Symbolic group and the Euler group, F (1, 68) = 5.935, $p < .001$. (ii) There was a significant difference between the Symbolic group and the 3-Venn group, F (1, 61) = 3.578, $p < .005$. (iii) There was no significant difference between the Symbolic group and the 2-Venn group. (iv) There was a significant difference between the 2-Venn group and the Euler group, F (1, 68) = 4.397, $p < .001$. (v) There was no significant difference between the 2-Venn group and the 3-Venn group. (vi) There was a significant difference between the 3-Venn group and the Euler group, F (1, 81) = 2.537, $p < .05$. It should be noted that if we include those subjects who failed the pretest, we still obtain similar results in each comparison: for (i), (iv) and (v), there were significant differences, $p < .001$; for (ii), there was a significant difference, $p < .01$; for (iii) and (v), there were no significant differences.

The results show that the performances of the Euler group and the 3-Venn group were better than that of the Symbolic group. This provides evidence for our hypothesis that Euler and 3-Venn diagrams have inferential efficacy and are self-guiding in the sense specified above. This means that as far as these diagrams are concerned, the internal constructions of diagrammatic proofs exist, and they can be naturally triggered for subjects without prior knowledge of inference rules or strategies. By contrast, there was little difference between the performance of the 2-Venn group and that of the Symbolic group. This suggests that 2-Venn diagrams have only interpretational efficacy, and are not self-guiding in our sense.

The results shown in Figure 12 indicate that the performance of Euler group was better than that of 3-Venn group. In particular, there was a significant difference with respect to a particular type of syllogism, namely, invalid syllogisms having an existential sentence as one of their premises. The data was subjected to a 4×2 ANOVA. As a main result, (i) there was a significant difference between this type of syllogisms in the 3-Venn group (57.8%) and the other types in the same group (83.4%), F (1, 134) = 29.434. $p < .001$. (ii) there was a significant difference between this type of syllogism in the 3-Venn group (57.8%) and that in the Euler group (84.2%), F (1, 81) = 4.926. $p < .005$. (iii) there was no significant difference between this type of syllogisms in the 3-Venn group (57.8%) and that in the Symbolic group (48.8%).

The relative difficulty in syllogism solving with 3-Venn diagrams could seem to be attributed to the difficulty in the process of drawing a conclusion from an internally constructed diagram. Such a process of extracting information may be formulated as a process of *deletion*. A deletion step in an Euler diagrammatic proof (as illustrated to the left in Figure 13) is simple in that it only requires to remove a circle without adjusting any other part of the diagram.

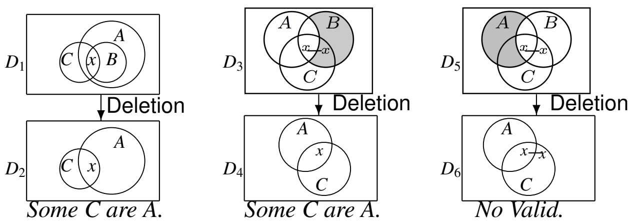


Figure 13: Deletion steps in Euler and Venn diagrams.

By contrast, deletion steps in 3-Venn diagrammatic proofs are somewhat complicated. Especially, in the step from D_5 to D_6 in Figure 13, which is an instance of invalid syllogisms having an existential sentence as one of its premise, one has to remove a circle and shading and to leave a linking point at the same region. Such complexities in deletion steps seem to reflect complexities of the processes of observing conclusions, and hence cause the difficulty in this type of syllogisms.

If our analysis is correct, the complexity of diagrams could make difficult the processes of *extracting* information. On the other hand, our results of the total 31 syllogisms suggest that 3-Venn diagrams have inferential efficacy, while 2-Venn diagrams do not. This in turn suggests that conventional devices such as shading and linking points could facilitate the processes of *combining* information by means of superposition of two diagrams. Thus, we could say that the availability of the process of combining information in diagrams depends on the complexity of the inference processes involved, whereas the availability of the process of extracting information in diagrams depends on the complexity of the conventional devices involved. Stenning and Oberlander (1995) point out that efficacy of diagrams can be ascribed to "specificity" of diagrammatic representations, and argue that diagrams could be effective because of their limited expressive power, in particular of the inability to express indeterminate or disjunctive information. In view of this, our findings are particularly interesting since they show that conventional devices to deal with indeterminacy sometimes facilitate internal manipulations of diagrams, hence contribute to their efficacy.

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