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Los Angeles

Essays on Industrial Organization and Finance

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy in Economics

by

Menghan Xu

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ABSTRACT OF THE DISSERTATION

Essays on Industrial Organization and Finance

by

Menghan Xu

Doctor of Philosophy in Economics University of California, Los Angeles, 2016 Professor Hugo Andres Hopenhayn, Chair

The dissertation consists of three essays on industrial organization with a particular focus on the structures of financial markets.

The first chapter theoretically studies how search friction affects competition and resource allocation in crowdfunding loan markets, which are described using a many-to-one matching framework. Namely, competitive fundraisers must accumulate multiple investors to complete a transaction. I develop a dynamic matching model with a fixed sample search, à la [BJ83], in which fundraisers compete in interest rates while investors look for good investment targets. I highlight two important economic forces in the model. First, investors can only observe a limited number of quotes. Second, a surplus cannot be created until a fundraiser attracts contributions from enough investors. I show that in the presence of search friction, fundraisers implement mixed strategies to set interest rates in a unique stationary equilibrium, which results in rate dispersion even if the goods are homogeneous. Regarding resource allocation, I show that in the many-to-one market, rate dispersion creates an endogenous coordination mechanism among anonymous and independent investors, thereby making it easy for them to concentrate their investments. In other words, search friction improves allocation efficiency in a crowdfunding market compared with its perfect competition counterpart.

Based on the theoretical framework constructed in the first chapter, the second chapter

empirically studies the market structure of the crowdfunding market. I construct a novel data set based on a large panel of fundraisers' behaviors. Using reduced form analysis, I find evidence of persistent rate dispersion and funding mismatches, which are consistent with the theoretical predictions of the search model. I also show that the model is identifiable and can be estimated using a non-parametric approach, which allows me to measure the allocation efficiency. Regarding methodology, I demonstrate that it is sufficient to use projects' ranks to recover search friction primitives, which reduces the computational burden and increases the precision. The estimation shows that the coordination mechanism can improve the probability of aggregate funding by 28% compared with a random matching context.

The third chapter studies how the combination of adverse selection and moral hazard affects the design of financial contracts. Specifically, the chapter studies an optimal mechanism design problem, à la [MR78], in the presence of limited enforcement. In the study, the bank (principal) designs loan contracts to screen firms (agents) with unobserved productivities. Meanwhile, the bank cannot prevent the firm from consuming acquired funds without producing anything. The impediment of forming contracts creates an endogenous outside option for all borrowers. I show that in the optimal mechanism, loan sizes for higher types are decreased by ironing, i.e., by pooling on the top. In addition, the lower types produce at the second-best level. Moreover, I show that firms' participation is independent of the enforcement level.

The dissertation of Menghan Xu is approved.

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Andrea Lynn Eisfeldt

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University of California, Los Angeles 2016

To the world, for the unknowns I will discover! To my parents, for the love I will protect!

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Time delays and flies. But it never decays or lies. I dedicate this dissertation to all my loved ones and I wish to make you proud of me.

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CHAPTER 1

Search Frictions in the Crowdfunding Market A Search Theory with Many-to-One Matching

1.1 Introduction

Growing evidence suggests that consumer search and other informational frictions soften competition and cause welfare loss in online markets, even if such markets are populated by a large number of seemingly undifferentiated competitors. Although considerable attention has been devoted to the problem in the traditional retail sector (e.g., [HS06], [DISHW12]), in which each trade involves one buyer and one seller, this paper attempts to study frictions in crowdfunding markets, in which a fundraiser has to be matched with many investors.

The paper answers two questions inspired by the combination of investors' frictional search and the many-to-one matching structure featured in crowdfunding markets. First, in the presence of search frictions on the investor side, how do fundraisers on the other side compete for multiple units of support instead of only one unit? Second, when investors, who are on "many" side of the relationship, have noisy or no information concerning peer investors' decisions, how do search frictions influence welfare in the market? I show that in an equilibrium without search frictions, borrowers will engage in Bertrand competition and investors will randomly select the project in which to invest. However, with search frictions, borrowers' mixed strategies will induce the rate dispersion, which can act as a coordination mechanism among independent decision-making investors. Thus, from a social efficiency perspective, the proportion of successful matches between the two sides will increase.

My model builds on the [BJ83] model of fixed-sample search. The market consists of two sides. Ex ante homogeneous borrowers and investors flow into the market instantaneously. Each borrower's project requires N units of fund. Upon entry, borrowers post their loan requests and commit to interest rates. On the other side, investors visit the market and make investment decisions based on their observations. Each investor is endowed with just one unit of fund. As long as a listing is matched with a sufficient number of investors, borrowers and matched investors will leave the market with their respective revenues. Otherwise, they may leave the market by encountering a Poisson death shock¹. Regarding the search technology, investors enter the market, randomly sample a finite number of listings, and select the project in which to invest. By contrast, I model the meeting process on the borrower side as a Poisson process whose intensity is endogenously determined in equilibrium. Conditional on a meeting, the matching probability depends on the listing's ranking in the investor's sample.

The model highlights several features and imperfections in the market. First, investors can only sample a limited number of listings. As observed in the data², this feature generates the possibility that all listings fail to secure funding perfectly. Moreover, it incentivizes borrowers to charge lower interest rates. Second, to study the mixed effect of frictional search and noisy information disclosure, I assume that investors make independent decisions without observing the listings' investment history. Instead, the investors form beliefs of observed listings' funding status.

The model produces two main results. First, I show that stationary equilibrium exists and is unique. In that equilibrium, borrowers employ mixed strategies in setting interest rates. The result is consistent with those of [BJ83] and [BM98]. Intuitively, when search friction does not exist, all the borrowers will set prices at a competitive level. When search is extremely expensive and every investor is captive to one counterparty, the borrower will charge monopoly price. In the intermediate cases, the competition effect will drive price

¹In most online crowdfunding markets, a fixed deadline is required. Because this is not my main focus, I simplify the model by incorporating the Poisson death shock, which allows me to derive comparative statics in a stationary environment.

²The detailed empirical analysis is covered in Chapter 2.

down, while the search cost will drive price up. In equilibrium, the borrowers will be indifferent among all prices and implement the same mixed strategy. The result explains how search frictions induce rate dispersion in the market even if the projects are of homogeneous quality.

Second, regarding matching efficiency, I show that in a one-to-one matching environment, rate dispersion simply determines how the surplus is divided between the two sides of the market and thereby has no effect on social welfare. However, in the many-to-one matching environment, when investors make their decisions independently, the rate dispersion caused by search frictions endogenously creates ex post heterogeneity among the listings. The differentiation will act as a mechanism that facilitates investors' efforts to sort the observed listings. Comparatively, absent frictional search, borrowers will engage in Bertrand competition such that the sorting mechanism disappears, as interest rates are identical. Therefore, the proportion of successful matches in the Bertrand competition scenario will be same as that in a random matching environment. In this sense, a certain level of search friction is beneficial to the allocation efficiency of the market.

The model can be naturally extended to study the case in which loan size is another dimension of borrowers' choice. The modification allows me to compare rate distributions across different sizes of loans. Consistent with the data analysis, I show larger loans have higher interest rates. Intuitively, prior to their investment decision, investors will also take the "crowd" effect into consideration and therefore require compensation for the potential risk of investment failure. To be specific, in the presence of search frictions, borrowers with both big and small projects implement a mixed strategy in pricing, but the strategy distributions will be on two disjoint sets. Moreover, I show that in some circumstances, borrowers are indifferent between issuing larger and smaller loans, facing the trade-off between funding probability and revenue earned from borrowing.

Related Literature

The paper is related to several strands of literature. First, the theoretical framework dates to seminal papers on static search models, such as [Sti61], [Var80], and [BJ83]. Our model shares the common assumption that price investigation is costly or frictional for buyers, which gives sellers some market power to charge higher prices. In the equilibria of such frameworks, sellers will employ mixed strategies, thereby generating price dispersion, even if the goods are homogeneous. Dynamic versions of [BJ83] include works such as the [BM98] on-job-search model and [HLMW12] monetary search model. My work alters the framework from a bilateral matching setup to a many-to-one matching environment.

The research regarding crowdfunding is also related to studies on network economics (e.g. [KS85], [RT03]), which consider the externality created by peers within a group or network. In my environment, the externality in each project is endogenously determined by loan size and interest rates, but agents cannot negotiate with each other to form a network. Among other papers, [BLS14] discusses the network effect in crowdfunding markets with different market structures (pre-order versus profit sharing), but there is only one project that seeks external finance, and investors' behaviors are binary. My work considers a loan market with infinitely many competitors on both sizes.

The paper is also related to studies regarding multilateral contracts. For instance, [SZ96] considers contracts between a principal (employer) and many agents (workers) under limited commitments. [Mic12] embeds matching friction into the environment. My model simplifies the contracting problem and directly studies the effects in a multilateral matching environment. The theory can shed lights on labor markets with the following features. First, each employer requires multiple employees; second, workers cannot negotiate wages; and third, production exhibits increasing returns to scale with respect to labor, which is common in start-up firms.

The remainder of this paper is organized as follows. To illustrate the basic mechanism of the theory, section 1.2 discusses a simple example with only two stages. Section 1.3 intro-

duces the dynamic model and characterizes the equilibrium. Section 1.4 discusses the main result of the model regarding allocation the efficiency of the market. Section 1.6 discusses an extension of the model with endogenous size. Section 1.7 concludes and summaries. All technical proofs, tables and figures are attached in appendices.

1.2 A Two-stage Game

Consider a simple two-stage game. There are two investors x and y, each has one unit of money and searches for investment opportunity with outside option 0. And there are two borrowers A and B, each is endowed with a project with identical value 2v > 0 and requires two units of investment. Obviously, the fund supply in the market can only support one of the two projects.

At stage one, A and B simultaneously post interest rates, denoted as r_A and r_B , respectively. At stage two, x and y independently visit the market and choose which project to invest in. Regarding search friction, assume that each investor has only a probability $\lambda \in [0, 1]$ to see both r_A and r_B and compare. With probability $1 - \lambda$, one will be randomly captive to one of the two projects. After investments, if x and y fund the same project, say A w.l.o.g., both will get r_A and A retains profit $2(v - r_A)$. Otherwise, all players get zero.

Since both sides are respectively homogeneous, it is natural that we only consider symmetric equilibrium of the game. I consider three cases.

When $\lambda = 0$, both x and y will be randomly matched with A and B. So borrowers hold all the market power and can set the rate as low as possible. In equilibrium, $r_A = r_B = 0$. The only event in which value is created is that both investors are matched with the same project, whose probability is $\frac{1}{2}$. Hence, the expected value created is *v*.

When $\lambda = 1$, investors can always compare. So $r_A = r_B = v$ in equilibrium. Facing the same interest rates, independent investors x and y will randomly choose one of the projects. If so, the probability of funding is still $\frac{1}{2}$, which implies that the expected value created is

still v.

When $\lambda \in (0, 1)$, both x and y have limited chance to observe both rates. If investors prefer higher rate, then it is easy to show that pure strategy equilibrium does not exist. The logic behind is simple. If $r_A = r_B = v$ in equilibrium, then both A and B earns zero profit anyway. But by deviating to r = 0, there is a positive probability $(1 - \lambda)^2$ that both investors are captive, so the expected profit is positive. If $r_A = r_B < v$, then by increasing the interest rate by a tiny amount ε , a borrower's winning probability can be increased by λ^2 . Accordingly, no pure strategy profile can be sustained in equilibrium. Alternatively, we consider the case that both both borrowers employ a continuous mixed strategies with support $[\underline{r}, \overline{r}]$. If so, there is zero probability that $r_A = r_B$. To solve for the mixed strategy, first, it is easy to show that $\underline{r} = 0$, otherwise each borrower can make higher profit by decreasing the lower bound. Denote the rate distribution by F(r), which can be pinned down by the indifference condition

$$\frac{1}{4}(1-\lambda)^2 \cdot v = \left[\frac{1}{4}(1-\lambda)^2 + \frac{1}{2}\lambda(1-\lambda)F(r) + \lambda^2 F(r)\right](v-r)$$

The left hand side is the expected profit of quoting r = 0. The right hand side represents the expected profit of quoting any $r \in [0, \bar{r}]$. So the equilibrium mixed strategy is unique

$$F(r) = \frac{1}{2} \frac{1-\lambda}{\lambda} \frac{r}{v-r}$$

Regarding the expected value creation, in mixed strategy equilibrium, the probability that x and y contribute to the same project is larger than 1/2, the perfect competition case.

$$(1-\lambda)^2 \times \frac{1}{2} + 2\lambda(1-\lambda) \times \frac{1}{2} + \lambda^2 = \frac{1}{2} + \frac{\lambda^2}{2}$$

The first term on LHS is corresponding to the event that x and y are randomly assigned to the same project, either A or B. The second term represents the situation that one is randomly assigned, while the other can see both rates. If so, say x sees r_A , while y sees both, then the

probability that $r_A > r_B$ is $1/2^3$. Therefore, the expected value created is $(1 + \lambda^2)v$ which is increasing in $\lambda \in (0, 1)$ and is greater than v in the cases $\lambda \in \{0, 1\}$. The simple illustrative example shows that when there is certain level of search friction, the dispersed rates can help them to coordinate to the same project, which enhances allocation efficiency of the market.

1.3 Model

In this section, we generalize our framework and introduce a dynamic search model with a many-to-one matching environment, assuming that each borrower requires many units of funding but each investor has only one unit to provide. In essence, borrowers enter, post an interest rate and wait to be funded. Investors visit the market, sample a finite number of listings and choose the most preferred one in which to invest. After a borrower and enough investors are matched, investors receive a committed return while the borrower retains the profit, and both parties leave the market.

1.3.1 Model Ingredients

Time is continuous, and the market consists of two sides - borrowers (who are referred to using the feminine pronoun below) and investors (who are referred to using the masculine pronoun below).

Borrowers At each instant, a measure $L \cdot dt$ of new borrowers enters the market. Each borrower requires *N* units of external financing, and the projects are of the same value $N \cdot v$. Upon entry, borrowers post an interest rate *r* and then wait to be funded. During the funding process, a borrower may exit the market due to a Poisson death shock with intensity $\delta > 0$.

$$\int_{r_0}^{\bar{r}} F(r)dF(r) = \frac{1}{2}$$

³Since both A and B are draw from the same distribution, denoted as F(r), then the probability that $r_A > r_B$ is equal to

Conditional on being funded, a borrower pays each investor *r* and leaves the market with profit $N \cdot (v - r)$.

Investors The instantaneous investor inflow is $I \cdot dt$. Each investor has one unit of money to invest. Assume that investors have the same reservation rate $r_0 < v$. In terms of investors' payoff, in the one-to-one market, an investor's utility from investment is simply r, while in the many-to-one environment, due to the risk of failure, his payoff could be represented as expected utility, which will be described shortly. For simplicity, we assume that investors are short-lived and invest only once. If the listing invested in encounters a death shock, all matched investors will leave the market with r_0 permanently.⁴.

Search and Matching Upon entry, an investor randomly observes $\ell \in \mathbb{N}_+$ listed projects but can invest in a maximum of one. For each listing in an investor's sample, I assume that

Assumption 1.1. *Investors can observe the rate r and size* N *of the* ℓ *listings in the sample.*

To guarantee that there are always listed projects, whose stock is denoted \mathcal{L} , I assume that

Assumption 1.2. $N \cdot L > I.^5$

I have yet to impose any structure on the distribution of ℓ . I will discuss how different search technologies affect market equilibrium in the characterization section below. On the other side of the market, after being posted, a listing waits passively for investments. Assume that each listed project can meet an investor with a Poisson intensity η , which will be endogenously determined by

$$\mathbb{E}(\ell) \cdot I = \eta \cdot \mathscr{L}$$

⁴Alternatively, the model can easily be extended by assuming that an investor can re-sample by paying a cost c > 0. If so, the only difference is that the reservation rate r_0 will be endogenously determined. Specifically, when c = 0, this is equivalent to investors having perfect information on the market's rate distribution

⁵If $I > N \cdot L$, then at each instant the market will be cleared immediately. We rule out the uninteresting case and only focus on L > I.

In the expression, the LHS is the total number of views by investors at each instant, while the RHS describes the same number but from the borrowers' perspective. In principle, the larger the investor inflow *I*, the higher the expected value of ℓ , and a lower the number of listed projects \mathscr{L} will result in more views of a given project. In terms of matching, assume that conditional on being met, a listing with rate *r* has a probability p(r) of being matched.⁶ Therefore, $\eta p(r)$ is the effective matching intensity. In brief, a borrower meets investors sequentially and can be fully funded only if matched with *N* of them.

1.3.2 Strategy and Equilibrium

In this section, I formulate agents' problems and describe aggregate variables. Then, I define the equilibrium.

Agents' Problem

To characterize the borrower's problem, denote P(r) as the probability of being funded if the interest rate is r. Thus, borrower's problem is to choose an interest rate to maximize expected profits

$$\hat{\pi} = \max_{r} P(r) \cdot N(v - r) \tag{1.1}$$

Here, I do not impose any assumption on the borrower's strategy, which could either be pure strategy or mixed strategies. In general, denote the strategy as a rate distribution F(r).

An investor's problem, however, is to choose the project with largest expected utility in his sample. Because a project's progress is invisible to investors and N is the same across all listings, the observed interest rates are the only information available to investors. Therefore, one needs to form beliefs on the funding probability of a listing with rate r conditional on his own contribution, denoted $\tilde{Q}(r)$. Thus, an investor's expected payoff can be represented

⁶I will determine the function p(r) in equilibrium.

as

$$u(r) = \tilde{Q}(r)r + (1 - \tilde{Q}(r))r_0$$

Furthermore, denote Q(r) as the actual funding probability. In equilibrium, the belief has to be consistent for all *r*, i.e.,

$$Q(r) = \tilde{Q}(r) \tag{1.2}$$

In terms of the equilibrium concept, because both sides of the market are homogeneous, I search for a symmetric equilibrium. That is, borrowers implement the same strategy to set interest rates, while investors follow the same rule to invest.

Aggregate Variables

In addition to symmetry, I focus on the stationary environment in which the market's aggregate variables and distributions are invariant over time. First, in equilibrium, the number of meetings has to be balanced. As mentioned above, we have

$$\eta \cdot \mathscr{L} = \mathbb{E}(\ell) \cdot I \tag{1.3}$$

Next, the distribution of listings should be invariant with respect to time. In the model, each listing has two attributes - rate *r* and progress $n \in \{0, 1, 2...N - 1\}$. Denote the density as g(r, n), then in equilibrium

$$\mathscr{L}g(r,n-1)\eta p(r) = \mathscr{L}g(r,n)(\eta p(r) + \delta)$$

$$Lf(r) = \mathscr{L}g(r,0)(\eta p(r) + \delta)$$
(1.4)

The LHS of the first equation is the measure of projects that are advanced from n - 1 to n, while the RHS is the outflow of projects with (r, n) due to making one unit of progress and death. The second equation describes the same balance but for new listings. Note that f(r) is not only the borrowers' strategy but also the probability distribution of new entrants. In

addition, define $g(r) \equiv \sum_{n=0}^{N-1} g(r,n)$ as the density of listings with rate r and G(r) defined as the CDF of g(r). The last condition required for stationary equilibrium is that the number of matches should be balanced. Because in equilibrium, $r \ge r_0$ for all listings, each investor invests in exactly one in his sample. However, for each listing with rate r, its matching intensity is $\eta p(r)$; therefore,

$$I = \mathscr{L} \int_{r} g(r) \eta p(r) dr$$
(1.5)

Thus far, I have demonstrated all conditions required for a stationary equilibrium. In summary,

Definition 1.1 (SE). Given state variables $\{L, I, N\}$, parameters $\{\delta, r_0, v\}$, and search technology $\{\Pr(\ell), \ell \in \mathbb{N}_+\}$, a **stationary equilibrium** consists of

- (i) Borrower's pricing strategy F(r).
- (ii) Investor's decision rule and belief \tilde{Q} ;
- (iii) Intensity of meeting η ;
- (iv) Total measure of listings \mathscr{L} ; and
- (v) Rate distribution G(r),

such that

• Borrower's profit is maximized	Equation (1.1);
• Investors' decision rule is belief-consistent	Equation (1.2);
• Balance of meetings	Equation (1.3);
• Rate distribution is invariant of time	Equation (1.4);
• Balance of matches	Equation (1.5).

1.3.3 Existence and Uniqueness

Because investors' decisions are made based on their belief $\tilde{Q}(r)$, an equilibrium might not be unique. For the present, suppose that all investors are endowed with the belief that $\tilde{Q}(r) > 0$ if and only if $r = r^*$ for some arbitrary $r^* > r_0$; then, there might exist equilibria in which all borrowers will choose r^* while investors invest up to r^* . In the remaining analysis of the paper, I exclude such unrealistic equilibria and restrict attention to monotonic decision rule. Namely, when listings' funding progress is invisible to investors and N is the same across all listings, an investor will choose the largest r in his sample. I will show below that the decision rule it is belief-consistent.

Because the aim of the paper is to study the impacts of search frictions, the properties of ℓ 's distribution play a significant role in determining the equilibrium. For instance, there are two special cases.

Proposition 1.1. When $Pr(\ell > 1) = 1$, r = v (Bertrand Competition). When $Pr(\ell = 1) = 1$, $r = r_0$ (Monopoly).

According to the proposition, if investors can always sample more than one listing and compare them, then borrowers are engaging in Bertrand competition. However, if each investor can see only one listing in his sample, then borrowers have all of the market power, and the interest rate will degenerate to $r = r_0$, which is the famous result shown in [Dia71]. In the rest of the paper, we will maintain the following assumption:

Assumption 1.3. $Pr(\ell = 1) \in (0, 1)$.

There are several possible interpretations of Assumption 1.3. At the individual level, randomness may emerge because the number of listings in the market is stochastic; although in a stationary environment, it is constant on average. Therefore, some investors can observe more while others can only observe one. From the perspective of the whole population, it might be the case that some investors treat searching and sorting as a costly behavior and

tend to invest in the first project they meet in the market. Thus, the borrowers are facing a probability distribution of different types of investors. Therefore, I do not impose any specific structure on the distribution of ℓ , except for Assumption 1.3.⁷ Given Assumption 1.3, following [BJ83] and [Var80], it is easy to show that in any symmetric equilibrium, borrowers never choose pure strategy or any point of masses.

Lemma 1.1. Under Assumption 1.3, there is no symmetric equilibrium in which all borrowers set the same rate or choose some rate with strictly positive probability.

Proof. The logic of the proof of Lemma 1.1 is simple. Suppose that borrowers implement pure strategy r < v; then, a small upward deviation will significantly increase the probability of success while the loss in payoff is small. If r = v, then according to Assumption 1.3, deviating to r_0 will generate strictly positive profits.

Based on Lemma 1.1, we focus on characterizing a mixed strategy equilibrium with a continuous distribution function $\{F(r), f(r)\}$ with support $\mathbf{r} = [\underline{r}, \overline{r}]$. Then, we have an immediate result as follows.

Lemma 1.2. *In equilibrium,* $\underline{r} = r_0$ *.*

In words, the lower bound of price dispersion is always the investors' reservation rate in equilibrium. The logic behind Lemma 1.2 is that, if $\underline{r} > r_0$, for all investors, deviating to r_0 will maintain the same probability of success because the only way in which the listing is funded is if the counterpart investor has $\ell = 1$. However, reducing the interest rate to r_0 will increase the return conditional on being funded. Hence, choosing r_0 is a profitable deviation. In equilibrium, by the indifference principle of mixed strategies, all rates $r \in \mathbf{r}$ will give borrowers the same return, which can be represented as

$$\hat{\pi} \equiv P(r)(v - r) = P(r_0)(v - r_0)$$
(1.6)

⁷In the literature, there are many alternatives to define the distribution of ℓ . For example, in [BM13], ℓ takes value 1 (captive) or 2 (non-captive). Alternatively, [HS06] assume that investors are heterogeneous in unit search costs. The lower the cost, the larger the sample one could draw. In equilibrium, the unit cost is equal to the marginal benefit from drawing one more sample.

Note that P(r) is the probability that a listing generates contributions from N investors before experiencing a death shock. Define V(r,n) as the value function of a listing with rate r and progress n, then

$$V(r,n) = (1 - \delta dt - \eta p(r)dt)V(r,n) + \eta p(r)V(r,n+1)$$

Rearrange the above dynamics and let $dt \rightarrow 0$; we have

$$V(r,n) = q(r)V(r,n+1)$$

where

$$q(r) \equiv \frac{\eta p(r)}{\eta p(r) + \delta}$$
(1.7)

is the probability that a listing with rate *r* generates one more match without encountering a death shock. Specifically, because V(r,N) = N(v-r) and $V(r,0) = \hat{\pi}$, we have

$$P(r) = q(r)^{N} = \left(\frac{\eta p(r)}{\delta + \eta p(r)}\right)^{N}$$
(1.8)

In terms of the expression of p(r), according to the setup, an investor randomly draws ℓ listings, each of which follows an i.i.d. distribution G(r). Therefore, for any distribution of ℓ , we have

Lemma 1.3. In the stationary equilibrium, given G(r), p(r) takes the form

$$p(r) = \frac{\sum_{\ell} \ell \cdot \Pr(\ell) \cdot G(r)^{\ell-1}}{\mathbb{E}(\ell)}$$
(1.9)

which is increasing in r.

For the above result, note that while the unconditional probability of drawing ℓ listings is $Pr(\ell)$, from a borrower's perspective, the probability that the met investor has ℓ listings in his sample is equal to $Pr(\ell) \cdot \ell / \mathbb{E}(\ell)$, which places greater weight on larger realizations of ℓ .

Based on the assumption that investors will choose the higher interest rate r, I show that p(r) is increasing in r. By Equation (1.8), P(r) is also increasing in r. To show that the decision rule is belief-consistent, I derive the analytic expression of Q(r) in terms of P(r) and p(r) and show that it is also increasing in r.

Lemma 1.4. Q(r) takes the form

$$Q(r) = \frac{1}{1 - P(r)} \frac{N\delta}{\delta + \eta p(r)} \left(\frac{\eta p(r)}{\delta + \eta p(r)}\right)^{N-1}$$
(1.10)

which is increasing in r.

Hence, investor's expected utility

$$u(r) = Q(r)r + (1 - Q(r))r_0$$

is also increasing in r. Therefore:

Proposition 1.2. That investors choose the highest rate in their sample is belief-consistent.

To demonstrate the existence and uniqueness of a stationary equilibrium with mixed strategies under the belief-consistent assumption that investors always choose to invest in the listing with the highest rate, several intermediate results need to be shown. First, because the matching intensity $\eta p(r)$ is independent of progress *n*, the stationary equilibrium condition (Equation (1.4)) can be simplified to

Lemma 1.5.

$$\delta \mathscr{L}g(r) = Lf(r)(1 - P(r)) \tag{1.11}$$

Intuitively, the LHS of the equation is the measure of dead projects in a unit of time, while the RHS of the equation describes the expected number of new inflows that will ultimately fail. In the stationary system, the equation must hold for all $r \in \mathbf{r}$. In addition, the above equation also implies that g and f have a monotone likelihood ratio, as P(r) is increasing in *r*. That is, for any distribution inflow f, listings with a higher interest rate will be funded faster. By rearranging Equation (1.11) and integrating both sides with respect to r, I have

$$L = \delta \mathscr{L} \int_{r} \frac{g(r)}{1 - P(r)} dr$$

By Equation (1.8), this becomes

$$L = \delta \mathscr{L} \int_{r} \frac{1}{1 - \left(\frac{\eta p(r)}{\eta p(r) + \delta}\right)^{N}} dG(r)$$

According to Lemma 1.3, p(r) is a function of G(r); therefore, we can map the interest rate distribution $r \in [r_0, \bar{r}]$ to a measure with $x \in [0, 1]$. Together with Equation (1.3), I derive the condition to specify equilibrium meeting intensity η .

Lemma 1.6. In the stationary equilibrium, η satisfies

$$\frac{\delta}{\eta} \int_0^1 \frac{1}{1 - \left(\frac{\eta \tilde{p}(x)}{\eta \tilde{p}(x) + \delta}\right)^N} dx = \frac{L}{I \mathbb{E}(\ell)}$$
(1.12)

where

$$\tilde{p}(x) \equiv \frac{\sum_{\ell} \ell \cdot \Pr(\ell) \cdot x^{\ell-1}}{\mathbb{E}(\ell)}$$
(1.13)

Moreover, $\eta > 0$ *exists and is uniquely determined.*

The above result significantly simplifies our analysis because η is uniquely determined, which allows me to straightforwardly prove the existence and uniqueness of the stationary equilibrium in the many-to-one matching framework.

Theorem 1.1. There is a unique stationary equilibrium in which investors choose the highest interest rate in their sample and borrowers implement identical mixed strategies.

Proof. The logic behind the proof is also the algorithm for solving the problem numerically. Beginning from the unique η , we can solve all of the equilibrium elements according to the

following steps.

$$(\boldsymbol{\eta},\mathscr{L}) \Rightarrow \hat{\boldsymbol{\pi}} \Rightarrow (P(r), p(r)) \Rightarrow \{G(r), g(r)\} \Rightarrow \{F(r), f(r)\}$$

For the details of the proof, see the Appendix.

1.3.4 Supply and Demand

Given existence and uniqueness, in this section, I discuss how equilibrium conditions respond to the market's state variables such as I, L and N. I will report several testable results that can be verified in the data. First result investigates how the meeting intensity η and market depth \mathcal{L} are affected by the supply/demand ratio L/I and loan size N.

Proposition 1.3. In the stationary equilibrium, (i) η is decreasing in L/I and N while \mathscr{L} is increasing in L/I and N. (ii) η converges to $\delta I\mathbb{E}(\ell)/L$ as $N \to \infty$.

Proof. First, by Lemma 1.6, we have

$$\rho \int_0^1 \frac{1}{1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^N} dx = \frac{L}{I\mathbb{E}(\ell)}$$

where $\rho \equiv \delta/\eta$. Then, monotonicity can be proven by the Implicit Function Theorem. The limit of η can be derived by applying Bernoulli's Inequality and the Squeeze Theorem. For details, see the Appendix.

In the statement above, (i) shows that, as the L/I ratio increases, given the same death rate and search technology, more projects will accumulate in the market, implying a larger \mathscr{L} , which directly indicates a lower meeting intensity η . Similarly, when N increases, more time is required for a project to be funded. Therefore, \mathscr{L} will be increasing in N while η is decreasing in N. (ii) implies that when N is increasing, the market depth \mathscr{L} will never diverge, and hence η will not diminish to zero. Based on Proposition 1.3, the next proposition shows how rate distribution is affected by market demand and supply.

Proposition 1.4. Denote the stationary distribution by G(r; I, L, N). For two pairs of market supply and demand $(I_1, L_1), (I_2, L_2)$, if $L_1/I_1 > L_2/I_2$

$$G(r; I_1, L_1, N) \succeq_{FOSD} G(r; I_2, L_2, N)$$

Given L/I, if $N_1 > N_2$ then

$$G(r;I,L,N_1) \succeq_{FOSD} G(r;I,L,N_2)$$

and \bar{r} is increasing in L/I and N.

Intuitively, both L/I and N measure the competitiveness on the borrower side. As the ratio or average loan size increase, borrowers face severer competition and hence will commit to overall higher interest rates, which gives us the FOSD result. Correspondingly, \bar{r} will be increasing with L/I and N as well. Moreover, the proposition also implies that, as a second order effect, when $N \cdot L/I$ is small, nearly all listings can be funded, and hence the interest rate will be close to r_0 . However, as the demand side expands, this rate will converge to v. Therefore, the standard deviation of the interest rate and demand/supply ratio should have an inverse U-shaped relationship. A numerical computation is reported in Figure 2.3. In addition, an immediate result of Proposition 1.3 is that that the death rate δ plays an insignificant role in determining rate distribution and profitability.

Corollary 1.1. G(r), P(r) and $\hat{\pi}$ are independent of δ .

Intuitively, when δ is large, η will be increased by the same magnitude. Although each listing's life-span become shorter, as the market becomes less competitive, the funding likelihood can compensate for the death risk and thereby hold the funding probability unchanged. In a stationary equilibrium, the borrower's strategy and profit will be remain the same and free of δ .

1.4 Welfare

In this section, I discuss the main finding of the paper, which concerns the efficiency problem in a many-to-one matching market caused by coordination. First, I state the main theoretical result. Then, I apply the model to the data and estimate the change in welfare difference between a dispersed price and random matching environment.

1.4.1 Rate Dispersion and Coordination

To begin, let us consider the special case in which N = 1. In this environment, each investor can finance a project alone. Therefore, the measure of successful listings at each unit of time is simply *I*. That is, the interest rate, regardless of being unified or dispersed, is simply a monetary transfer between the two sides. However, when N > 1, a listing cannot be fully funded by a single investor. Moreover, investors who have made an investment will be locked in with the listing and face a risk of failure; therefore, market welfare of the market could be affected. Market welfare can be expressed as

$$\mathscr{W} = L \int_{r} P(r) f(r) dr$$

which represents the expected number of listings that will ultimately be fully funded. By Equation (1.11), we have

$$\mathscr{W} = L - \delta \mathscr{L}$$

which is the difference between new inflows and dead listings at each moment. Hence, to compare efficiency levels across different search friction technologies, it is sufficient to compare equilibrium market sizes \mathcal{L} .

Theorem 1.2 (Efficiency). *Given* (I,L,N), *a market with* $Pr(\ell = 1) \in (0,1)$ *has a larger \mathscr{W} than that with* $Pr(\ell = 1) \in \{0,1\}$.

1.4.2 Discussion

Theorem 1.2 shows that, although a certain level of friction prevents investors from viewing all listings in the market, from a social perspective, under a many-to-one matching framework, friction improves welfare, measured by expected number of successfully funded projects, compared with the perfect competition case. The novel effect is induced by a mixture of dispersed price generated by search frictions and the lack of a coordination mechanism in the many-to-one environment. As discussed above, in a one-to-one matching market, there is no problem regarding coordination, hence interest rates simply determines how to divide surplus. However, in the many-to-one environment, if investors cannot observe listings' funding status and make their decisions independently, the interest rate is the unique information that helps them to make decisions. If there is no differentiation among projects, investors tend to chose which to invest in at random, which is no better than the case in which each investor can observe only one listing (the case $\ell \equiv 1$). However, under a certain level of search frictions, the most important feature generated here is the persistent rate dispersion that induces a mechanism for investors to sort the listings in their samples. In principle, listings with higher rates not only provide investors a potentially higher return but also serve as a signal that helps investors to coordinate. Such biased funding dynamics increase the overall number of fully funded projects, indicating enhanced social welfare. The general implications of the analysis shed light on mechanism design and information disclosure in such many-to-one matching environments in which the side of "one" posts the price, while the "many" side searches. It seems that in such a decentralized matching market, if there is no other effective coordination mechanism, price regulation or unifying the quality of the goods is harmful to market efficiency and perfect information disclosure might also be inefficient. Recall the simple example mentioned in the Introduction: suppose that there are two valuable projects, each of which requires two units of contribution. If there is a mechanism to differentiate the two projects, it would be easier for the two investors to contribute to the same one. In my model, rate dispersion represents such an endogenous property that improves coordination.

In general, the model applies to any two-sided market with increasing returns to scale. For instance, in the labor market for start-up companies, new firms enjoy increasing returns to scale in labor during their early stages. However, in general, adjusting hiring information such as wages is costly in the short run, and it is impossible for a firm to contract with workers based on how many employees it has hired. Therefore, if employers have perfect competition, then all of them will grow at the same pace and will reach optimal scale after a longer period. However, when the competition is imperfect and features wage dispersion, the endogenous sorting mechanism will cause some firms to reach optimal size earlier, which from the social perspective, is beneficial.

As a final remark on the theoretical analysis, one may be interested in the case in which, if there is exogenous coordination in the market, would price dispersion be beneficial? For instance, if listing progress is perfectly observable, will rate dispersion coordinate or distort it? Intuitively, the answer would be the latter. Intuitively, given rate dispersion, one possible situation is that one investor will invest his money in a project with a high rate and low progress, bypassing other projects with low rates and high progress in his sample, which violates the socially optimal allocation rule, under which more advanced projects should always have priority to be matched. If so, eliminating product differentiation, such as by unifying the interest rate, could act as a effective regulation to improve welfare.

1.5 Endogenous Sizes

In this section, we study a simple extension of the model such that loan size is another choice variable. For simplicity, I assume that in addition to choosing interest rates, borrowers can choose different loan sizes. Without loss of generality, I assume that the choice of size can be either 1 or 2. All of the elements such as search technology and the random death rate will remain the same. In terms of investors' information set, assume that they are able to observe

a project's rate and size. However, for a project of size 2, progress is still unobservable.

1.5.1 Return Equivalence

Note that because the sizes of the loans in an investor's sample could differ, it is necessary to characterize investors' return equivalent conditions among listings with different sizes. Denote $p_n(r)$ as the probability that a listing of size *n* and rate *r* will be invested in conditional on being viewed. Correspondingly, define $q_n(r) \equiv \frac{\eta p_n(r)}{\eta p_n(r) + \delta}$. By Equation (1.10), if an investor invests in a project with $(r_1, 1)$, the return is simply r_1 . For a project with $(r_2, 2)$, then the expected return will be

$$\frac{2q_2(r_2)}{1+q_2(r_2)}r + \frac{1-q_2(r_2)}{1+q_2(r_2)}r_0$$

Hence, the return equivalence relationship between $(r_1, 1)$ and $(r_2, 2)$ is as follows

$$r_1(r_2) = \frac{2q_2(r_2)}{1+q_2(r_2)}r_2 + \frac{1-q_2(r_2)}{1+q_2(r_2)}r_0$$
(1.14a)

Intuitively, given a project of size two, investors require a higher rate to compensate for the risk of failure. Moreover, note that when the two projects are return-equivalent to investors, this implies that $p_1(r_1) = p_2(r_2)$, so do $q_i(r_i)$. Therefore, the above equation can be also represented by

$$r_2(r_1) = \frac{1 + q_1(r_1)}{2q_1(r_1)} r_1 - \frac{1 - q_1(r_1)}{2q_1(r_1)} r_0$$
(1.14b)

1.5.2 Equilibrium Characterization

To begin, let us consider two extreme cases. First, when $Pr(\ell = 1) = 1$, the unique equilibrium is that all borrowers will choose $r = r_0$. Second, when $Pr(\ell > 1) = 1$, the result is that all listings are of identical size and rate (r, n) = (v, 1). When Assumption 1.3 holds, the
following lemma describes the conditions for two listing sizes to co-exist in the market.

Lemma 1.7. If $\frac{L}{I} \ge 1 + \Pr(\ell = 1)$, all borrowers will choose to issue loans of size one. If $\frac{L}{I} < z^*$ for some $z^* \in (0, 1 + \Pr(\ell = 1))$, then all borrowers will choose to issue loans of size two.

Intuitively, there are two forces that determine borrowers' loan sizes. First, when the borrower/investor ratio is higher, the borrower side will be more competitive. Then, borrowers will tend to request smaller loans to increase the likelihood of being funded. Another force is the probability that $\ell = 1$, which represents the level of search friction. When friction is high, it is more likely to generate support for all projects, and hence the larger projects are more easily funded. As stated in Lemma 1.7, when the first effect dominates the latter, only smaller loans exist in the market. However, if L/I is small enough, all the borrowers will issue larger loans because there are enough investors flowing into the market. In an extreme case, when L/I = 0.5, then even if all borrowers choose size 2, the funding probability is always one. In the intermediate cases, the two sizes may co-exist in the market and all borrowers would then generate the same expected profit.

For the remainder of the section, we assume that $\frac{L}{I} \in (z^*, 1 + \Pr(\ell = 1))$, which guarantees the co-existence of the two types of projects. Denote the rate distribution chosen by borrowers by

$${F_1(r), f_1(r)}, {F_2(r), f_2(r)}$$

and stationary rate distribution by

$$\{G_1(r), g_1(r)\}, \{G_2(r), g_2(r)\}$$

First, it is trivial that the lowest rate in equilibrium would be r_0 . In addition, denote \mathscr{L}_1 and \mathscr{L}_2 as the measure of listings of size 1 and 2, respectively. Further note that in equilibrium, it must be the case that two types of the projects are of equal profit $\hat{\pi}_2 = \hat{\pi}_1$. Denote \mathbf{r}_i as the support of listings with size *i*. The first property we will describe below is that the strategy

set is disjoint between two types in any equilibrium. Formally,

Lemma 1.8. In any equilibrium in which $\mathbf{r}_i \neq \mathbf{0}$, there exists only one pair of (r_1, r_2) that satisfies the following two conditions simultaneously.

- *Return Equivalence:* $(r_1, 1) \sim (r_2, 2)$
- *On-Path:* $r_i \in \mathbf{r}_i$ for i = 1, 2

As there is only one pair of rates (r_1, r_2) that makes investors indifferent in equilibrium, it seems natural to ask which type is relatively more preferable to them. The upper bound of the less-preferred type's strategy set should be indifferent to the lower bound of the morepreferred type's strategy set. The less preferred type has the lowest rate at r_0 . The following proposition claims that larger projects should be preferred to smaller projects in equilibrium because larger borrowers are willing to offer much higher interest rates to investors.

Proposition 1.5. In equilibrium, larger projects have higher interest rates. Specifically, $r_1 \in [r_0, \bar{r}_1]$, while $r_2 \in [\underline{r}_2, \bar{r}_2]$ such that

$$(\bar{r}_1,1) \sim (\underline{r}_2,2)$$

Proof. As indicated in Lemma 1.8, there exists only one pair of (r_1, r_2) that satisfies *Return Equivalence* and *On-Path*, which implies that \mathbf{r}_1 and \mathbf{r}_2 are two disjoint intervals. Furthermore, it is obvious that there is no utility gap for investors. Otherwise, all borrowers would have an incentive to fill the gap, which is a profitable deviation. Hence, in equilibrium, there are only two possible cases.

- Case 1. Larger projects have higher rates
- Case 2. Smaller projects have higher rates

The detail of the proof is shown in the Appendix

Proposition 1.5 implies that, comparatively, when loans could be of heterogeneous sizes, borrowers tend to offer higher interest rates for larger loans to compensate for the risk of failure. Specifically, when loans are of different sizes, the interest rates are distributed in disjoint ranges. Even if investors are concerned about both the return r and expected funding probability, in equilibrium, monotonicity guarantees that the interest rate is the dominant factor taken into consideration.

1.6 Conclusion

The chapter theoretically studies how search frictions affect price competition and allocative efficiency in a many-to-one matching market. Motivated by the significant mismatch and price dispersion observed in an online crowdfunding market, I argue that search frictions faced by investors play a significant role in affecting borrowers' pricing behavior.

By analyzing a dynamic fixed-sample search model, I show that, in a unique stationary equilibrium with search frictions, borrowers employ mixed strategies in setting interest rates, which causes search rate dispersion. Further, the model generates several predictions that are consistent with empirical facts generated from my data, such as the price and dispersion levels being positively correlated with the borrower/investor ratio of the market.

More important, the model shows that in such a many-to-one framework, the price dispersion caused by search frictions will facilitate coordination among investors who decide independently. Given a fixed inflow of external funding, the number of funded projects will increase due to frictions.

By extending the model to heterogeneous sizes, I show not only that borrowers with larger loans set higher interest rates to compensate for the risk of failure, but also that the supports of the interest rate across sizes are disjoint.

1.7 Appendices

1.7.1 Proof of Lemma 1.3

For each rate *r*, by the definition of p(r), the measure of projects with rate *r* that are funded is $\eta \mathscr{L}$

$$I \cdot \Pr(r_{invest} \le r) = \eta \mathscr{L} \cdot \Pr(r_{fund} \le r)$$

The LHS is the total measure of investors whose invested project is below r, while the lefthand side represents the funded listings with rate no larger than r. The LHS can be represented as

$$I \cdot \Pr(r_{invest} \le r) = I \cdot \sum_{l} \Pr(\ell) \cdot G(r)^{l}$$

For an investor, for each realization of l, the probability that the best rate is less than r is $G(r)^{l}$. However, the measure of funded projects with rate r can also be represented as

$$\Pr(r_{fund} \le r) = \int_{r_0}^r g(r)p(r)dr$$

Take the derivative of the above two expressions w.r.t. r, then we have

$$I \cdot g(r) \sum_{l} l \cdot \Pr(\ell) \cdot G(r)^{l-1} = \eta \mathscr{L} \cdot g(r) p(r)$$

Moreover, by Equation (1.3), we have

$$I \cdot \mathbb{E}(\ell) = \eta \cdot \mathscr{L}$$

Hence we have the desired result.

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1.7.2 Proof of Lemma 1.4

When an investor observes a project with rate r, he has to infer the distribution of the project's progress distribution. First, provided that the project is still listed, its age distribution is exponential with parameter δ . Hence, the probability that

$$\int_0^\infty \delta e^{-\delta t} \cdot \frac{e^{-\eta p(r)t} (\eta p(r)t)^n}{n!} dt$$

which is equal to

$$\frac{1}{1-P(r)}\frac{\delta(\eta p(r))^n}{(\delta+\eta p(R))^{n+1}}$$

Given n, the probability that the project can reach N before death is

$$\left(\frac{\eta p(R)}{\delta + \eta p(R)}\right)^{N-n-1}$$

Therefore, the expected probability is equal to

$$\frac{1}{1-P(R)}\sum_{n=0}^{N-1}\frac{\delta(\eta p(R))^n}{(\delta+\eta p(R))^{n+1}}\left(\frac{\eta p(R)}{\delta+\eta p(R)}\right)^{N-n-1}$$

which gives us

$$Q(r) = \frac{1}{1 - P(r)} \frac{N\delta}{\delta + \eta p(r)} \left(\frac{\eta p(r)}{\delta + \eta p(r)} \right)^{N-1}$$

To show that Q(r) is increasing in r, by the expression of P(r), we have

$$Q(r) = \left(\sum_{n=0}^{N-1} \left(\frac{\eta p(r)}{\delta + \eta p(r)}\right)^n\right)^{-1} \left(\frac{\eta p(r)}{\delta + \eta p(r)}\right)^{N-1} = \sum_{n=1}^{N-1} \left(\frac{1}{q(r)^n}\right)^{-1}$$

As q(r) is increasing in r, Q(r) is also increasing.

1.7.3 Proof of Lemma 1.5

By stationary conditions (Equation (1.4)), we have

$$g(r,n) = g(r,n-1) \cdot \frac{\eta p(r)}{\eta p(r) + \delta}$$

for all $n \in \{1, 2...N - 1\}$. In addition, we know that

$$g(r) = \sum_{n=0}^{N-1} g(r,n)$$

hence we have

$$\mathscr{L}g(r,N-1) = \mathscr{L}g(r,0) \cdot \left(\frac{\eta p(r)}{\eta p(r) + \delta}\right)^{N-1} = \frac{Lf(r)}{\eta p(r) + \delta} \cdot \left(\frac{\eta p(r)}{\eta p(r) + \delta}\right)^{N-1}$$

and by

$$Lf(r) = \mathscr{L} \cdot (\delta \cdot g(r) + \eta p(r) \cdot g(r, N-1))$$

and Equation (1.8), we have

$$Lf(r) = \mathscr{L} \cdot \delta g(r) + Lf(r)P(r)$$

1.7.4 Proof of Lemma 1.6

By Equation (1.11) and Equation (1.8), we have

$$L = \delta \mathscr{L} \int_{r} \frac{1}{1 - \left(\frac{\eta p(r)}{\eta p(r) + \delta}\right)^{N}} dG(r)$$

Note that G(r) is continuous with domain [0,1]. Moreover, by Lemma 1.3, p(r) can be represented as p(G(r)), which implies that p is a function of G. Therefore, we can define

 $\tilde{p}(x)$ as in Equation (1.13) and obtain Equation (1.12). To show that there exists a unique η that solves the problem, define

$$\mathscr{F}(\rho) = \rho \int_0^1 \frac{1}{1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^N} dx - \frac{L}{I\mathbb{E}(\ell)}$$

where $\rho \equiv \delta/\eta$. First, we show that \mathscr{F} is monotone in ρ .

$$\frac{\partial \mathscr{F}}{\partial \rho} = \int_0^1 \frac{1}{1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^N} dx - \rho \int_0^1 \frac{N\left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^{N-1}}{\left(1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^N\right)^2} \frac{\tilde{p}(x)}{(\tilde{p}(x) + \rho)^2} dx$$

For simplicity, denote $\tilde{q}(x) \equiv \frac{p(x)}{(\tilde{p}(x)+\rho)}$, then the above partial derivative equals

$$\int_0^1 \frac{1}{1 - \tilde{q}(x)^N} dx - \int_0^1 \frac{N \tilde{q}(x)^{N-1}}{\left(1 - \tilde{q}(x)^N\right)^2} \tilde{q}(x) (1 - \tilde{q}(x)) dx$$

To show that the term is positive, it is sufficient to show that the integrand is always positive, or equivalently

$$1 - \tilde{q}(x)^N - N\tilde{q}(x)^N (1 - \tilde{q}(x)) > 0$$

It is easy to show that, first, the term is decreasing in q(x); second, when $\tilde{q}(x) = 0, 1$, the above term equals 1, 0, respectively, as $\tilde{q}(x) \in (0, 1)$, the above term is positive. Therefore, \mathscr{F} is increasing in ρ and hence is decreasing in η . To demonstrate the existence and uniqueness of η , I rewrite the above equation as

$$\mathscr{F}(\rho) = \int_0^1 \frac{\tilde{p}(x) + \rho}{\sum_{n=0}^N \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^n} dx - \frac{L}{I\mathbb{E}(\ell)}$$

When $\rho = 0$, the above equation becomes

$$\mathscr{F}(0) = \frac{1}{(N+1)\mathbb{E}(\ell)} - \frac{L}{I\mathbb{E}(\ell)}$$

By Assumption 1.2, the above term is negative. When ρ goes to infinity, $\mathscr{F}(0)$ will also go to infinity. By the Intermediate Value Theorem, there is a ρ such that $\mathscr{F}(\rho) = 0$. Because $\mathscr{F}(\rho)$ is increasing in ρ , the solution is unique. Therefore, η is uniquely determined. \Box

1.7.5 Proof of Theorem 1.1

Because η is uniquely determined, \mathscr{L} is also uniquely determined. Because the lower bound of the borrowers' mixed strategy is always r_0 , in equilibrium, the indifference condition implies that

$$\hat{\pi} = P(r_0) \cdot N(v - r_0)$$

where

$$P(r_0) = \left(\frac{\eta p(r_0)}{\delta + \eta p(r_0)}\right)^N$$

Note that $G(r_0) = 0$; therefore, $p(r_0) = \tilde{p}(0)$. Therefore, $P(r_0)$ is uniquely specified, as is $\hat{\pi}$. In the next step, provided that the equilibrium profit is uniquely determined, we have

$$P(r) = \frac{\hat{\pi}}{v - r}$$

So

$$p(r) = \frac{\delta}{\eta} \frac{\left(\frac{\hat{\pi}}{v-r}\right)^{1/N}}{1 - \left(\frac{\hat{\pi}}{v-r}\right)^{1/N}}$$

is uniquely determined. By Lemma 1.3, I find unique G(r) and g(r). Finally, $f(r) = \frac{\delta \mathscr{L}g(r)}{L(1-P(r))}$ is unique. In summary, beginning from the existence and uniqueness of η , all equilibrium elements of the equilibrium can be derived step by step.

1.7.6 Proof of Proposition 1.3

The monotonicity can be proven by the Implicit Function Theorem. Thus, the limit of η can be derived by applying Bernoulli's Inequality and, immediately, the Squeeze Theorem.

(i) Monotonicity

By Lemma 1.6, we have

$$\frac{\delta}{\eta} \int_0^1 \frac{1}{1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \frac{\delta}{\eta}}\right)^N} dx = \frac{L}{I\mathbb{E}(\ell)}$$

Define $\rho \equiv \delta/\eta$ and $z \equiv L/I$, and hence

$$\mathscr{F}(\rho, N, z) \equiv \rho \int_0^1 \frac{1}{1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^N} dx - \frac{z}{\mathbb{E}(\ell)}$$

Obviously, both $\partial \mathscr{F} / \partial N$ and $\partial \mathscr{F} / \partial z$ are negative. For $\partial F / \partial \rho$, we have shown in the proof of Lemma 1.6 that $\partial \mathscr{F} / \partial \rho$. By the Implicit Function Theorem, we have

$$\frac{d\rho}{dN} = -\frac{\partial \mathscr{F}/\partial N}{\partial \mathscr{F}/\partial \rho} > 0$$

and

$$\frac{d\rho}{dz} = -\frac{\partial \mathscr{F}/\partial z}{\partial \mathscr{F}/\partial \rho} > 0$$

Therefore, ρ is increasing in both *N* and *z*, which implies that η is decreasing in *N* and *z*. Moreover, by Equation (1.3), \mathcal{L} is increasing in *N* and *z*.

By $\rho = \delta/\eta$, we conclude that η is decreasing in *N* and \mathscr{L} is increasing in *N*.

(ii) Limit

To find the limit of η , first, it is obvious that

$$\frac{L}{I\mathbb{E}(\ell)} \le \rho$$

However,

$$\left(\frac{\tilde{p}(x)+\rho}{p(x)}\right)^{N} = \left(1+\frac{\rho}{\tilde{p}(x)}\right)^{N} > 1+\frac{N\rho}{\tilde{p}(x)}$$

which is guaranteed by Bernoulli's Inequality. Therefore,

$$\frac{L}{I\mathbb{E}(\ell)} = \rho \int_0^1 \frac{1}{1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^N} dx$$
$$< \rho \int_0^1 \frac{1}{\frac{N\rho}{N\rho + \tilde{p}(x)}}$$
$$= \frac{1}{N} \int_0^1 \tilde{p}(x) dx + \rho$$

Therefore, we have

$$\frac{L}{I\mathbb{E}(\ell)} > \rho > \frac{L}{I\mathbb{E}(\ell)} - \frac{1}{N} \int_0^1 \tilde{p}(x) dx$$

By the Squeeze Theorem, ρ converges to $\frac{L}{I\mathbb{E}(\ell)}$ as *N* goes to infinity. Thus, η converges to $\delta I\mathbb{E}(\ell)/L$.

1.7.7 Proof of Proposition 1.4

By Lemma 1.3, as p(r) and G(r) are bijective, comparing G(r; L, I, N) is equivalent to comparing p(r) under different (N, I, L). Recall that

$$p(r) = \frac{\delta}{\eta} \frac{\left(\frac{\hat{\pi}}{v-r}\right)^{1/N}}{1 - \left(\frac{\hat{\pi}}{v-r}\right)^{1/N}}$$

Let $\rho \equiv \delta/\eta$ and write $\hat{\pi} = P(r_0)(v - r_0)$. Then, we have

$$p(r) = \rho \cdot \frac{\frac{p(r_0)}{\rho + p(r_0)} \left(\frac{v - r_0}{v - r}\right)^{1/N}}{1 - \frac{p(r_0)}{\rho + p(r_0)} \left(\frac{v - r_0}{v - r}\right)^{1/N}}$$

By Proposition 1.3, it is easy to show that p(r) is decreasing in L/I and N, as is G(r). In terms of \bar{r} , because p(r) is decreasing in L/I and N, while $\tilde{p}(x)$ is fixed with $\tilde{p}(1) = 1$, then

we have that \bar{r} is increasing in L/I and N.

1.7.8 **Proof of Theorem 1.2**

The proof consists of two steps. In step one, we compare two cases by fixing the search technology. In the first case, we assume that fundraisers are free to set their rates. In the second case, we assume that the market imposes a price regulation such that all listings have to set a rate of r^* . In step two, we will show that for any search technology, a unique price will result in same level of social welfare.

Step One When borrowers are free to set their prices, when $Pr(\ell = 1) \in (0,1)$, as mentioned in the proof of Proposition 1.3

$$\int_0^1 \frac{\rho}{1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^N} dx = \frac{L}{I\mathbb{E}(\ell)}$$

where $\rho = \delta/\eta$. However, when the price regulation is imposed, then for each meeting, the probability that the project will be invested in is

$$p^* = \frac{\sum_{\ell} \Pr(\ell) \cdot \ell \cdot \frac{1}{\ell}}{\mathbb{E}(\ell)} = \frac{1}{\mathbb{E}(\ell)}$$

Further, note that in this case, all projects' funding probability is

$$P^* = \left(\frac{p^*}{p^* + \rho^*}\right)^N$$

where $ho^* = \delta/\eta^*$. Hence, we have

$$\frac{\rho^*}{1 - \left(\frac{p^*}{p^* + \rho^*}\right)^N} = \frac{L}{I\mathbb{E}(\ell)}$$

By Equation (1.9),

$$\int_0^1 \tilde{p}(x)dx = \int_r p(r)dG(r) = \frac{1}{\mathbb{E}(\ell)}$$

Therefore, for any $\rho > 0$, Function

$$h(z, \boldsymbol{\rho}) \equiv \frac{\boldsymbol{\rho}}{1 - \left(\frac{z}{z + \boldsymbol{\rho}}\right)^{N}}$$

is convex in z, and strictly convex when N > 1, by Jensen's Inequality, we have

$$\int_0^1 \frac{\rho}{1 - \left(\frac{p(x)}{p(x) + \rho}\right)^N} dx \le \frac{\rho}{1 - \left(\frac{p^*}{p^* + \rho}\right)^N}$$

Moreover, " \leq " takes "=" if and only if N = 1. As has already been shown, $h(z, \rho)$ is increasing in ρ , to have

$$\int_0^1 \frac{\rho}{1 - \left(\frac{\tilde{p}(x)}{\tilde{p}(x) + \rho}\right)^N} dx = \frac{\rho^*}{1 - \left(\frac{p^*}{p^* + \rho^*}\right)^N} = \frac{1}{\mathbb{E}(\ell)}$$

We have $\rho^* \ge \rho$ or $\eta^* \le \eta$, which indicates that

$$\mathscr{L}^* \geq \mathscr{L}$$

Step Two To show that for any search technology, when there is no rate dispersion, \mathscr{L} remains constant. Suppose that there is no rate dispersion, we have

$$p^* = \frac{1}{\mathbb{E}(\ell)}$$

Moreover, $\eta \mathscr{L} = \mathbb{E}(\ell)I$, a project's success rate is always

$$P^* = \left(\frac{\eta p^*}{\eta p^* + \delta}\right)^N = \left(\frac{I}{I + \delta \mathscr{L}}\right)^N$$

In the market, we have

$$\delta \mathscr{L} = L(1 - P^*) = L - L \cdot \left(\frac{I}{I + \delta \mathscr{L}}\right)^N$$

which indicates that \mathscr{L} is uniquely determined regardless of search technology. \Box

1.7.9 Proof of Lemma 1.7

Given that all borrowers choose a loan of size one, we need to show that no one has an incentive to deviate to size two under any interest rate. Assume the opposite; suppose that there exists a profitable deviation (r', 2). Then, by Equation (1.14b), it has to be the case that

$$r' \in [r_0, r_2(\bar{r})]$$

Moreover, denote $r = r_1(r')$, and the profit is equal to

$$2q_2(r')^2(v-r') = 2q_1(r)^2(v-r_2(r))$$

by Equation (1.14b), and hence profitable deviation implies that

$$q_1(r)(v-r) - (1-q_1(r))(v-r_0) > 0$$

By the indifference condition, we have

$$1 - \frac{1 - q_1(r)}{q_1(r_0)} > 0$$

Because $q_1(\cdot)$ is increasing, profitable deviation exists if and only if

$$1 - \frac{1 - q_1(r_0)}{q_1(r_0)} > 0 \Leftrightarrow q_1(r_0) > \frac{1}{2}$$

According to the analysis of the one-to-one model, the above inequality implies that all borrowers will issue loans of size one if and only if

$$\frac{L}{I} \ge 1 + \Pr(\ell = 0)$$

1.7.10 Proof of Lemma 1.8

For Return Equivalence, as

$$r_1 = \frac{2q_2(r_2)}{1+q_2(r_2)}r_2 + \frac{1-q_2(r_2)}{1+q_2(r_2)}r_0$$

and

$$q_2(r) = \frac{\eta p_2(r)}{\eta p_2(r) + \delta}$$

we have

$$p_2(r_2) = \frac{\delta}{\eta} \frac{r_1 - r_0}{2(r_2 - r_1)} \tag{1.15}$$

Moreover, if (r_n, n) is *On-Path*, it must be the case that

$$\hat{\pi}_n = nq_n(r_n)^n(v-r_n) = \left(\frac{\eta p_n(r_n)}{\eta p_n(r_n) + \delta}\right)^n n(v-r_n)$$

for n = 1, 2. In addition, as $p_1(r_1) = p_2(r_2)$, denoted $p^*(r_1, r_2)$, we have

$$p^*(r_1, r_2) = \frac{\delta}{\eta} \frac{r_1 - r_0}{2(r_2 - r_1)}$$
$$= \frac{\delta}{\eta} \frac{\hat{\pi}_1}{v - r_1 - \hat{\pi}_1}$$
$$= \frac{\delta}{\eta} \frac{(\hat{\pi}_2/2)^{1/2}}{(v - r_2)^{1/2} - (\hat{\pi}_2/2)^{1/2}}$$

Eliminating $p^*(r_1, r_2)$, the first two equations above yield

$$2\hat{\pi}_1(v-r_2) = (v-r_1)^2 + (\hat{\pi}_1 - v + r_0)(v-r_1) + \hat{\pi}_1(v-r_0)$$

while the last two equations indicate that

$$2\hat{\pi}_1(v-r_2) = \frac{\hat{\pi}_2}{\hat{\pi}_1}(v-r_1)^2$$

Note that the above two equations hold for all pairs of (r_1, r_2) that satisfy *Return Equiv*alence and *On-Path*. However, obviously, at most two pairs of (r_1, r_2) could satisfy them simultaneously, as both terms $\hat{\pi}_1 - v + r_0$ and $\hat{\pi}_1$ are non-zero in equilibrium. Equate them and eliminate of r_2 , and we have

$$\frac{\hat{\pi}_2}{\hat{\pi}_1}(v-r_1)^2 = (v-r_1)^2 + (\hat{\pi}_1 - v + r_0)(v-r_1) + \hat{\pi}_1(v-r_0)$$

Because $\hat{\pi}_2 \ge \hat{\pi}_1$, the above quadratic equation has only one positive root. Therefore, the pair (r_1, r_2) is uniquely determined.

1.7.11 **Proof of Proposition 1.5**

Case 1. To show that case 1 is satisfies the equilibrium conditions, we need to show that neither type has an incentive to deviate. First consider a size-two borrower who deviates by choosing interest rate $r_2 < \underline{r}_2$. In this case, his profit can be represented as

$$\tilde{\pi}_2(r_2) = 2\tilde{q}_2(r_2)^2(v-r_2)$$

Because there is no gap in investors' utility, there exists r_1 such that $q_1(r_1) = \tilde{q}_2(r_2)$. Therefore, the profit can be represented as

$$\tilde{\pi}_2(r_2) = 2q_1(r_1)^2(v-r_2)$$

By Equation (1.14a), we have

$$\frac{2q_1(r_1)}{1+q_1(r_1)}(v-r_2) = v - r_1 + \frac{1-q_1(r_1)}{1+q_1(r_1)}(v-r_0)$$

Multiply both sides by $(1+q_1(r_1))q_1(r_1)$; we have

$$\tilde{\pi}_2(r_2) = (1+q_1(r_1))\hat{\pi}_1 - (1-q_1(r_1))q_1(r_1)(v-r_0)$$

Hence, deviating to a rate $r_2 \in [r_0, \underline{r_2}]$ is equivalent to selecting a $q \in [q_1(r_0), q_1(\overline{r_1})]$, i.e.,

$$\tilde{\pi}_2(q) = (1+q)\hat{\pi}_1 - (1-q)q(v-r_0)$$

Take the derivative w.r.t. q; we have

$$rac{\partial ilde{\pi}_2(q)}{\partial q} = \hat{\pi}_1 - (v - r_0) + 2q(v - r_0)$$

Because $\hat{\pi}_1 = q_0(v - r_0)$, verifying the sign of above derivative is equivalent to checking that of $q_0 - 1 + 2q$. As $\hat{\pi}_2 > \hat{\pi}_1$, it is easy to show that $q^* \equiv q_1(\bar{r}_1) > 1/2$ (A direct result from Lemma 1.7). Therefore, there exists $\tilde{q} \ge q_0$ such that $q_0 - 1 + 2q > 0$ for all $q \in [\tilde{q}, q^*]$. If $\tilde{q} = q_0$, then $\tilde{\pi}_2(q)$ will be increasing on the whole support. Otherwise, within $q \in [q_0, \tilde{q}]$, the term is decreasing. That is, within $q \in [\tilde{q}, q^*]$, $\tilde{\pi}_2(q)$ is increasing. Then, it is sufficient to show that $\tilde{\pi}_2(q_0) \le \hat{\pi}_2$. Assume that $\tilde{\pi}_2(q_0) > \hat{\pi}_2$, then $2q_0^2(v - r_0) > 2q^{*2}(v - \underline{r}_2) >$ $\hat{\pi}_1 = q_0(v - r_0)$, which indicates that $q_0 > 1/2$, so $q_0 - 1 + 2q$ is greater than zero for all $q \in [q_0, q^*]$, contradiction. Therefore, there is no profitable deviation for a size-2 borrower.

Next, consider a size-1 borrower's incentive to deviate by choosing $r > \bar{r}_1$. Again, by *Return Equivalence*, we have

$$\tilde{\pi}_1(r_1) = \frac{\hat{\pi}_2}{1 + q_2(r_2)} + \frac{1 - q_2(r_2)}{1 + q_2(r_2)} q_2(r_2)(v - r_0)$$

Similarly, the problem is equivalent to deviating to $q \in [q_2(\underline{r}_2), q_2(\overline{r}_2)]$,

$$\tilde{\pi}_1(q) = \frac{\hat{\pi}_2}{1+q} + \frac{(1-q)q}{1+q}(v-r_0)$$

Again, taking derivative w.r.t. q, we have

$$\frac{\partial \tilde{\pi}_1(q)}{\partial q} = -\frac{\hat{\pi}_2}{(1+q)^2} - \frac{q^2 + 2q - 1}{(1+q)^2}(v - r_0) < -\frac{\hat{\pi}_1}{(1+q)^2} - \frac{q^2 + 2q - 1}{(1+q)^2}(v - r_0)$$

The inequality is guaranteed by $\hat{\pi}_2 > \hat{\pi}_1$. Further, the sign of the last term is equivalent to

$$-q_0 - q^2 - 2q + 1$$

As $q > q^* > 1/2$, the above term is negative. Therefore, $\frac{\partial \tilde{\pi}_1(q)}{\partial q} < 0$, which is a sufficient condition for there to be no profitable deviation for investors with size 1.

Case 2. Let us consider the case in which smaller projects have higher average rates. Specifically, $r_2 \in [r_0, \bar{r}_2]$ and $r_1 \in [\underline{r}_1, \bar{r}_1]$. As a size-2 borrower has no incentive to deviate to size 1, we have

$$2q_0^2(v - r_0) \ge q_0(v - r_0)$$

which indicates that $q_0 > 1/2$. Now, consider a size-2 borrower who deviates as above; then, we have

$$\tilde{\pi}_2(q) = (1+q)\hat{\pi}_1 - (1-q)q(v-r_0)$$

The derivative w.r.t. q takes the form $q_0 - 1 + 2q$, which is strictly positive because $q > q_0 > 1/2$.

CHAPTER 2

Search Frictions in the Crowdfunding Market Evidence and Estimation

2.1 Introduction

Search models, starting with [Sti61], have been playing a critical role in economics. Search frictions resulting in consumers' limited information regarding sellers' prices have been used to explain many empirical facts and economic phenomena. While the literature of search models has been developing for a half century since [Sti61], the methodologies for measuring search costs and their welfare impacts are few. Meanwhile, with the rapid development of e-commerce and online transactions, there has been a debate whether search frictions exist in different online markets.

In the spirit of the seminal works by [Sti61] and [BJ83], Chapter 1 constructs a theoretical framework to analyze the market structure of crowdfunding markets under friction, and provides predictions on agents' behavior and efficiency impacts. Based on the theoretical framework and motivated by the ongoing literature, this chapter attempts to answer two empirical questions regarding online crowdfunding markets. First, does search friction exist in the market? Second, if the cost does exist, how can we estimate it and evaluate its impacts?

To answer the first question, I construct a novel data set that includes a large panel of borrowers' and investors' transaction information. I perform a set of reduced form analyses and present a list of stylized facts, which is testable and confirmed to be consistent with the predictions of my theoretical model. Practically, fundraisers in the market preset a financial goal that entails a certain processing period to attract a sufficient number of backers; if this goal is not met, the funding procedure fails. The data allows me to observe, for example, each listing's life cycle, including instantaneous funding progress, number of matched investors, and whether the project is being fully funded or not. Besides, by investors' and borrowers' unique IDs, I can track their repetitive trading behaviors during the sample period.

Using the reduced form analysis, I find that even if the listed loans are of identical quality in terms of duration and risk, (1) rate dispersion is always present in the market. In addition, by observing each listing's funding process, I find that (2) a large number of high-rate projects were not ultimately funded, while some low-rate loans that were posted during the same period and exhibited the same quality were successfully financed. These facts seem to suggest that frictions such as investors' search patterns play a significant role in the market, even though competing fundraisers are only a click away. Moreover, by tracking listings' funding dynamics, I find that (3) for most of the listings, funding speed, as measured by the time between two successive bids, is independent of the funding progress. The third finding is puzzling because the platform discloses each listing's funding progress, which intuitively should act as an exogenous coordination mechanism that enhances a listing's likelihood of being funded. This finding also contradicts the herding effect discussed by [ZL12] and [KB14]. To resolve this puzzle, by taking high frequency screenshots of the website, I find that because the market has a relatively high trading speed, and processing and updating a large number of investments is time-consuming, the dynamic information disclosure regarding listings' funding status always has a delay or even a malfunction, thereby preventing investors from learning others' behavior from this information. The empirical patterns observed refer to the main question motivating the paper: If the many-to-one market lacks a functional coordination channel, is there any endogenous mechanism that could improve matching efficiency?

The evidence suggests that search frictions do play a significant role in fundraisers' competition and investors' behaviors. However, search frictions, as a special case of information imperfection, are hard to identify solely by reduced form analysis. Identifying search frictions requires implementing a structural quantitative method to evaluate the effect. Based on the theoretical framework developed in Chapter 1, I show that the primitive of search friction, described as each investor's sample size, can be uniquely identified by price dispersion data. In addition to the evidence presented above, I demonstrate a set of additional stylized facts that support the predictions of the model.

To estimate the primitives regarding search friction, I show that it can be uniquely recovered by the probability of one single matching given state variables and interest rates, and the probability is also directly related to a project's overall funding likelihood, which is observable in the data. More importantly, to evaluate the welfare impact induced by search friction, I show that it is sufficient to identify and estimate the relative terms instead of the absolute terms. For instance, rather than estimating one's funding probability at a particular interest rate and certain market conditions, the funding probability according to the listing's rank in the market can be simply computed by transforming the rate distribution to propensity score distribution. The method simplifies my estimation from two perspectives. First, it reduces the number of parameters to be identified in the model. For instance, I need not to identify the support of interest rate distribution, denoted by [r, \bar{r}], as the transformed distribution always has the support $x \in [0, 1]$. Second, the estimation efficiency can be enhanced significantly, as many observations whose transformed distributions are identical can be combined to reduce standard errors.

Thanks to the scale of data and neatness of the model structure, I implement a nonparametric estimation method to measure the funding likelihood, which can uniquely pin down the search friction primitives. My estimation reports that the coordination mechanism induced by search friction can improve allocation efficiency by about 28%.

2.1.1 Related Literature

The paper is related to recent empirical studies regarding price dispersion in online markets following the rapid development of e-commerce.

For example, [BS00], [CKW01] and [Goo01] attempt to understand the nature of search costs by comparing the degree of price dispersion between online and traditional retail markets. The most relevant work is [HS06], who develops a method to estimate a [BJ83] non-sequential search model using price data from online book retailers. In essence, they show that the price distribution can be used to identify and estimate the search cost distribution when the market is static and goods are homogeneous. My work consider a dynamic environment with stationary equilibrium. Other studies include [DISHW12], who uses consumers' web search histories to study their search behavior and concludes that a non-sequential search model better describes buyers' behavior in online markets. A similar methodology is applied by [HS04], who studies the rate dispersion in S&P500 funds. All of the markets studied in the above works feature long-lived sellers and bilateral trading, whereas my work focuses on a market with frequent entry and exit in which transactions take the form of many-to-one matching. The most important feature that I try to measure is the coordination effect induced by price dispersion and its effect on welfare.

Moreover, the paper contributes to the increasing number of empirical studies regarding crowdfunding markets. As mentioned above, [KB14] and [ZL12] attempt to empirically understand investor dynamics over project funding cycles. They conclude that contributions by peer investors will exacerbate network externalities and hence affect others' decisions. My work studies the problem of investors who have no information about their peer investors' decisions and that form their beliefs based on others' behavior; consequently, price dispersion can help them to coordinate. Moreover, my work considers not only the dynamics of a project's funding process but also competition among fundraisers. Other studies about crowdfunding include [WL13], who studies auctions in crowdfunding markets ¹, and [KOU14], who identifies and estimates a signaling model using data from a US crowdfunding credit platform. All of these works assume that a fundraiser is a monopolist on the borrower side. Some census data regarding the crowdfunding market are surveyed in [ACG13].

The rest of the paper is organized as follows. Section 2.2 describes the market mechanism and data. Section 2.3 demonstrates the stylized facts observed from the data. Section 2.4 reviews the model described in Chapter 1, and 2.5 provides more empirical facts that support the model. Section 2.6 describes my estimation strategy and reports results. Section 2.7 summarizes. All the technical proofs, tables and graphs are attached in appendices.

2.2 Institutional Background and Data

I collected data from Hongling Capital, Inc. (or my089.com), one of China's largest online crowdfunding loan platforms, based in Shenzhen. In this section, I describe how the platform operates. To demonstrate that rate dispersion exists among nearly homogeneous loans, I devote particular emphasis to two types of listings, called "primary" and "secondary" loans. Then, I report summary statistics and empirical evidence.

The trading mechanism in the platform works as follows. (1) Borrowers enter the platform, post information such as the amount requested, purposes and a brief personal introduction, and commit to interest rates and the duration of the loan. (2) Investors view the listings and choose which to invest in. (3) If the requested amount is raised before the deadline², borrowers receive the money and repay the debt according to committed terms. (4) Otherwise, the loan will be canceled, and the money will immediately be returned to the investors.

¹All principal platforms have since abandoned this mechanism.

²At my089.com, the deadline is 2 days, but most of the projects are delisted before then.

2.2.1 Data Overview

The dataset has a horizon of 4 months (Jan 1st - May 1st, 2015) and includes information on 508,888 listings, of which 216,402 are fully funded. The average interest rate is approximately 14.03%, and the average maturity is approximately 130.747 days (weighted by loan size). The average daily trading volume is 155 million Chinese Yuan(CNY) (approximately 25 million US Dollar(USD)). The data consist of three parts

- Loan Information records basic information regarding each listing, such as the borrower's ID, loan size, interest rate, and duration.
- **Bidding Record** has information on a listing's bidding history, which records the time and amount invested for each participating investor. One issue to note is that the information only includes loans that were fully funded.
- **Snapshot** records all listings' funding dynamics on the platform every 10 minutes. It allows us to continuously track the platform's characteristics, such as the pace of funding and new postings.

2.2.2 Loan Classification and Dealership

The loans are classified into several categories, but 99.57% of trading volume is generated by two types. I name them "primary" and "secondary" loans due to their features. A detailed description is provided in Table 2.1.

Primary Loans The first type is called "primary" loans because they are listed by the platform itself. Moreover, the platform also monitors these projects and guarantees investors' returns. In this sense, the platform is not only an informational intermediary but also works like a bank, which finances and monitors the borrowing side while absorbing deposits and

providing insurance to the other side.³.

The primary loans account for approximately 31% of total trading volume. They have large sizes, high returns and long maturities. In my sample, all primary loans have an annualized interest rate of 18%. The average size is 3.8 million CNY, and the average duration is approximately 611 days. Moreover, all primary loans were fully funded in my sample.

Secondary Loan In addition to primary loans, the platform allows its clients to issue shortterm loans using their assets (either cash deposited or unexpired investments) within the platform as collateral. The most commonly used collateral is unexpired primary loans. To some extent, the short-term loan is a financial derivative with primary loans as an underlying asset, and I thus term such loans "secondary". As mentioned above, primary loans are return guaranteed, and hence, secondary loans are also secured. If a secondary loan's payment is delayed, the platform will freeze the debt holder's asset and prepay the investors.

Compared with primary loans, the average maturity and rate of secondary loans are smaller. On average, the maturity is 27.98 days, while the interest rate is 12.81%. Such loans represent approximately 68% percent of the market. In the sample, secondary loans' funding probability is approximately 40%.

In terms of market risk, because primary loans are return guaranteed and secondary loans backed by within-platform collateral, all of the listings in the market share the same systematic risk.

Dealership through Secondary Loans As illustrated in Table 2.1, there is an approximately 5% difference between the returns on primary and secondary loans. This creates an opportunity for the platform's clients to make a profit from investing in long-run primary loans by issuing short-term secondary loans. Figure 2.1 illustrates the daily transaction

³Due to the lack of a national credit system, most of China's P2P lending platforms guarantee investors' returns by monitoring the projects issued by the platforms. If default occurs, a platform uses its reserve fund to repay investors. [WXM] provides a detailed introduction to China's online lending market.

volumes of primary loans and one-month net value loans, which are highly correlated. In addition, as the user ID of each borrower and bidder on a listing can be observed, the data allow us to track every registered customer's lending and borrowing behavior simultaneously. There were 7,746 of a total of 109,828 users who engaged in both borrowing and lending. Using this information, I identified dealer behaviors within the market. As illustrated in Figure 2.2 below, overall 63.04% of the primary loans are contributed to by "dealers" using money borrowed through secondary loans.

2.3 Stylized Facts

The dealers' behavior creates an ideal natural laboratory to study competition in crowdfunding markets, as it unifies the risk and value of each borrower's project, which helps us to rule out other unobservable heterogeneity that may also cause price dispersion. In this section, I report key empirical evidence that motivates the main questions of the paper. I focus only on empirical patterns regarding 1-month secondary loans made for the purpose of investing in primary loans. The selection procedure generally ensures that the listings in our sample are of the same quality in terms of their

- (a) Duration: 1 month
- (b) Value: primary loans' return
- (c) Risk: platform's systematic risk

Accordingly, our restricted sample has a total of 297,685 listings, of which 126,121 were fully funded. The average size and rate are 31,090 CNY and 14.32%, respectively. On average, the number of investors per listing is 6.36, conditional on the listing being fully funded. The summarized statistics are reported in Table 2.2.

Funding Probability

Our first evidence for the existence of market friction is the funding probability of the listings in the market.

Fact 1. While funding probability is increasing in the interest rate, there is a significant failure rate for listings of all rate ranges.

According to Table 2.3, funding probability is increasing in the interest rate. However, even for listings with the highest interest rate level ($\geq 16\%$), the funding probability is only approximately 60%, while listings with a lower rate still have an approximately 20% chance of being funded. Further, to avoid time-dependent variation such as market demand and supply, I compute the funding probability by percentile during each hour and observe a similar pattern. According to Table 2.4, for listings at the 80th percentile, the funding probability is only approximately 65%, while listings at the 20th percentile have a success rate of approximately 30%.

Interest Rate Variation

The second set of evidence concerns interest rate dispersion. As shown in Table 2.3, the interest rates for 1-month loans range from 10% to 17%. As predicted in many theoretical works (e.g., [BJ83]), when search frictions play a role, every seller employs identical mixed strategies such that the market will have a non-degenerate price distribution. Alternatively, price dispersion can simply be described by unobservable individual-level heterogeneity. For instance, it is possible that fundraisers with high-quality projects might be willing to offer higher interest rates. Although I have argued that the borrowers have nearly identical returns (primary loans) on their projects in our data, to demonstrate that search friction is the main cause of rate dispersion, it is necessary to compare individual-level rate dispersion and market-level rate dispersion. Suppose at individual level, the dispersion level is significantly smaller, then it might be the case that rate differentiation is induced by some unobservable

heterogeneity in stead of mixed strategy induced by search friction. However, in most empirical works (e.g., [HS06], [DISHW12]), the price distribution is generated from a static snapshot of the market. Hence, those works generally assume that price dispersion results from the use of mixed strategies. Fortunately, in our data, I am able to observe repeated listing behavior by the same borrower, which allows us to compare rate distributions at the individual and aggregate levels.

To measure rate dispersion, I employ two measures for robustness. First, following [JNS11], I define dispersion as follows

$$d_t = \sqrt{\frac{1}{\sum_{k=1}^{K_t} w_{kt}} \sum_{k}^{K_t} (r_{kt} - \bar{r}_t)^2 \cdot w_{kt}}$$

In the formula, r_{kt} is the interest rate for listing k in period t. \bar{r}_t is the weighted average rate in period t. Further, w_{kt} represents the weight imposed on listing kt. If we weight each listing equally, then $w_{kt} = 1$. Alternatively, we can weight each listing by its size. In brief, this measure represents the root mean squared difference between the traded rates and the respective valuation based on a weighted calculation of the difference. In our sample, on average, each borrower posts 4.29 listings per day. I compare dispersion at the individual level with aggregate dispersion on daily and weekly bases. For comparison, I also compute the daily and weekly dispersion for all listings in our sample.

According to Table 2.5, the daily and overall levels of dispersion are slightly smaller than that at the aggregate level, while dispersion at the weekly level is slightly larger. This suggests that individual-level dispersion plays a prominent role in market-level dispersion. Second, I perform a simple comparison of the minimum and maximum rates quoted by different borrowers. As indicated in Table 2.6, there is no significant difference between dispersion at the aggregate and individual levels. ⁴ In summary,

⁴For the min/max measure, as an individual's number of listings is much smaller than the total level, I consider the min/max for individuals but 10th percentile/90th percentile for the aggregate-level data. Hence, in principle, aggregate-level statistics should be somewhat smaller.

Fact 2. Individual-level interest rate dispersion is similar to that that at the aggregate level.

Progress Effect

The third motivation for the paper is the puzzle that investors do not respond to listings' funding progress even if the information is disclosed by the platform. Intuitively, a listing with more advanced progress will be filled faster than those that have just been listed. According to [ZL12] and [KB14], herding behavior plays an significant role in listings' funding dynamics. I implement a similar empirical strategy to theirs but observe that this effect is weak. Specifically, to determine whether funding is accelerated by funding progress, I regress the time interval between two successive bids on the listing's lagged progress, controlling for other variables⁵.

$$DT_{it} = \alpha PROGRESS \ CUMULATED_{it-1} + X_{it}\beta_1 + X_i\beta_2 + X_t\beta_3 + \varepsilon_{it}$$

The model (Panel 1 in Table 2.8) predicts that when the progress is increased by 10%, the funding speed can be increased by approximately 37 seconds. The impact is very small compared with average funding duration, which is approximately 65 minutes in the sample. Then, I separate the sample into two groups. In the first group, the funding duration is less than 90 minutes. It contains 90% of the listings in the sample, while listings in the second group have longer funding times. According to panel 2, the majority of the listings' funding processes are independent of the amount of funding secured to date. However, even for listings that required more time to be funded, the increase in the funding rate attributable to funding progress is quite limited (10% progress difference equates to an approximately 6-minute decrease in funding time). The results of the reduced-form analysis suggest that funding progress plays an insignificant role in listings' funding dynamics, especially for the

⁵In their analysis, because both successful and failed listings' funding dynamics are observable, they measure the correlation between lagged progress and the probability of generating one more investor. However, in my data, because the platform erases bidding records for failed loans, I can only track funding dynamics for successful listings. Therefore, I use funding speed as the proxy to text herding behavior

trading environment with relatively high frequency.

Quality of Progress Bar Another finding indicating that progress may not be a useful tool to coordinate investors is that there is a significant malfunction in the progress bar shown on the website. I take snapshots of the website every ten minutes, which includes the progress of each listing. Moreover, I collect the investment records of all the successful loans that contain investors' investing time and amount. We found that the website has certain level of delay or malfunction in reporting listings' funding progress across all listings. Figure 2.4 shows the delay measured by percentage. The investigation consists of 124,191 listings, of which 75,860 has a progress delay at different levels. Moreover, 33,257 listings' progress bars always stopped at 0% during the funding process.

In summary, the imperfect funding probability together with persistent rate dispersion at both the aggregate and individual levels provide clear evidence of market friction and imperfect competition among borrowers. Moreover, the errors in the information regarding funding progress prevents investors from effectively coordinating among themselves. Motivated by these facts, I introduce the structural model in the next section.

2.4 Model - Revisited

This section provides a review of the search model discussed in Chapter 1. Briefly speaking, the environment describes a dynamic crowdfunding market with constant flows of homogeneous borrowers and investors. Each borrower requires many units of funds but each investor has only one unit of money. Payoffs cannot be realized unless a borrower is matched with enough investors. Borrowers compete by setting interest rates; investors can only observe limited access to interest rate information.

The key ingredients and notations are summarized as follows.

Time is continuous. Borrowers and investors flow into the market instantaneously with

measure $L \cdot dt$ and $I \cdot dt$, respectively. Each borrower requires N units of investment and each investor has only one unit of fund. The total value can be created by the financial project is $N \cdot v$. Borrowers enter the market, post interest rate r and wait for investors. During the funding process, she may exit the market due to death shock with intensity δ . Denote \mathscr{L} as the stock of listings. Investors has reserved value r_0 for their investments. Upon entry, an investor can randomly observe ℓ listed projects. Since interest rate r is the only attribute that differentiates the projects, we assume that the investor will invest in the project with highest rate. As the expected value of ℓ is analogous to investor's search efficiency, the borrower's chance of being met is positively correlated with $\mathbb{E}(\ell)$ and I/L. Besides Assumption 1.3, I do not impose any parametric structure on the distribution of $\Pr(\ell)$.

Regarding the market equilibrium, as summarized in Definition 1.1 in Chapter 1, I define the stationary equilibrium as follows. Given state variables $\{L,I,N\}$, parameters $\{\delta,r_0,v\}$ and investor's search technology $Pr(\ell)$, the equilibrium consists of (i) borrower's pricing strategy F(r), (ii) intensity of meeting η , (iii) total measure of listings \mathcal{L} and (iv) stationary rate distribution G(r). In equilibrium, agents maximizes payoffs, and both meeting and matching process have to be balanced and invariant with respect to time. In addition to the elements mentioned above, denote P(r) as the funding probability of a project with rate r, q(r)be the probability that a project accumulates one additional contribution. Thus, borrower's equilibrium profit is equal to

$$\hat{\pi} = \cdot P(r) \cdot (v - r) = q(r)^N (v - r)$$

Besides, I define p(r) as the probability of being matched conditional being met, so

$$q(r) = \frac{\eta p(r)}{\eta p(r) + \delta}$$

2.4.1 Main Results

The model exhibits several key theoretical results that are essential to the identification and estimation. First, by Theorem 1.1, the stationary equilibrium exist and unique. Moreover, all the borrowers implement identical mixed strategies in quoting prices such that price dispersion emerges. It also implies that rate dispersion at individual level and aggregate dispersion are of the same magnitude, which is consistent with Fact 2 - although there might be certain degree of heterogeneity among borrowers, the individual level dispersion, as predicted in the model, is the dominant factor of interest rate dispersion.

In addition, as an intermediate step of showing uniqueness, Lemma 1.6 provides an important result, which is also helpful regarding estimation strategy mentioned later. In the statement, we define (Equation (1.13))

$$\tilde{p}(x) \equiv \frac{\sum_{\ell} \ell \cdot \Pr(\ell) \cdot x^{\ell-1}}{\mathbb{E}(\ell)}$$

where $x \in [0, 1]$. The lemma transforms the probability w.r.t. $r \in [r_0, \bar{r}]$ to percentile representation $x \in [0, 1]$. As will be mentioned shortly, the transformation has two advantages. (i) It reduces the structure and requires estimating fewer parameters of the model; (ii) It decreases computational burden and increases estimation precision. The most important result, which is also our main target of estimation, is Theorem 1.2, which states that the differentiation created by search frictions can coordinate investors' behavior and enhance social efficiency. In next section, I will derive the estimators of efficiency loss of the market.

The last set of results demonstrates some testable features regarding matching efficiency, rate distribution and demand, supply and loan sizes with respect. For instance, Proposition 1.4 describes the relationships demand, supply and rate dispersion. In the extension with endogenous loan size, I show that larger projects have higher rates. Besides estimating welfare impact of search friction, in Section 2.6, I will provide some additional facts which support the predictions of the model.

2.5 Estimation

2.5.1 Estimation Strategy

The theoretical analysis of the model (Theorem) shows that when a many-to-one matching market lacks a coordination mechanism, price dispersion induced by search frictions could reduce welfare loss. The expected number of fully funded projects, which measures matching efficiency, will increase. In this section, we attempt to quantitatively estimate the efficiency gain due to rate dispersion by introducing a straightforward method to measure search cost primitives. Specifically, we compute the ratio of welfare under random matching to that under dispersed rates, denoted as

$$w \equiv \frac{\mathscr{W}_{\text{Random Matching}}}{\mathscr{W}_{\text{Dispersed Price}}}$$

As shown in Theorem 1.2, the ratio has to be no greater than one. To estimate the welfare difference, first, we show that it is sufficient to identify and estimate the following function

Proposition 2.1. Define

$$H(x) \equiv \tilde{p}(x) \cdot \mathbb{E}(\ell)$$

where $x \in [0, 1]$ is a sufficient statistic to estimate w.

The proposition implies that to estimate the welfare ratio, I need not know the absolute level of rate dispersion. Instead, an estimation based on percentile data is sufficient. The result is directly implied by

according to Lemma 1.6. In the expression, the only unknown is η ; *L*,*I*,*N* are observable in the data. Suppose that H(x) is known, as $\tilde{p}(1) = 1$, $\mathbb{E}(\ell)$ is achieved. Further, the whole function of $\tilde{p}(x)$ is known. Accordingly, η can be uniquely solved, by \mathscr{L} . By that

$$\mathscr{W} = L - \delta \mathscr{L}$$

I am able to compute welfare directly. To identify and estimate *H*, we have the following result.

Proposition 2.2. *Given* (N, I, L)*,* H(x) *can be computed by*

$$H(x) = \frac{P(x;z,N)^{1/N}}{1 - P(x;z,N)^{1/N}} \frac{z}{\int_0^1 \frac{dx}{1 - P(x,z;N)}}$$
(2.1)

where $z \equiv L/I$ and P(x; z, N) is the funding probability of project at the x percentile.

In the RHS of the expression, z = L/I and N are observable, while P(x; z, N) can be nonparametrically estimated by computing the average probability of funding at each percentile. Moreover, the proposition simplifies the estimation by avoiding the need to identify of r_0 and v, which might be determined by some unobservable elements such as the change in the platform's systematic risk and variables external to the market. Given H(x), I describe my estimation strategy as follows.

- 1. Separate the data into four-hour intervals.⁶
- 2. Compute $z_t = L_t/I_t$
- 3. Compute average number of investments per listing, denoted N_t .
- 4. For each *t* with the same I_t , L_t and N_t , compute the average funding probability P(x;z,N) by choosing proper grid for x.⁷
- 5. Compute H(x) according to Equation (2.1)

⁶The selection of 4 hours reflects a trade-off between sample size and market volatility. In principle, the longer the time intervals, the larger the sample in each group, meaning that I can compute P more precisely. However, as supply and demand may vary over time, we need to keep the time interval shorter to maintain the stationary assumption.

⁷Although we have approximately 250,000 observations in the sample, conducting kernel estimation requires a high dimension of x; therefore, in our result, I only use the 10th and 25th percentiles.

2.5.2 Result

Table 2.13 and 2.14 report the estimated result of H(x) based on grids with the 10th and 25 percentile, respectively. Figure 2.5 graphically illustrates the function. Note that in the expression of H(x), H(0) is equal to $Pr(\ell = 1)$, while H(1) is the estimate of $\mathbb{E}(\ell)$. According to the estimates, on average, approximately 35% of investors enter the market by only looking at one listing. On average, the number of listings observed by each investor is approximately 1.76. The reported number implies that, although the number of listings in the platform seems to be large, each investor's effective observation is quite limited. In a behavioral interpretation, there could be a significant level of idiosyncratic randomness associated with investors' decision making, which gives borrowers some market power and leads to dispersed interest rates. Next, I use the estimated H(x) to compute the welfare ratio w under different combinations of N and L/I, and the result is shown in Figure 2.6. According to the graph, when NL < I, there will be no welfare loss between the dispersed price and random matching cases because $\mathcal{L} = 0$ in either case. Moreover, when N = 1, the ratio is equal to 1. According to our data, the $N \cdot L/I$ ratio is approximately 1.8 on average, while the average N is approximately equal to 6. According to the calibration, w = 0.7807. That is, with a dispersed rate as a coordination mechanism, the approximate welfare gain could be approximately 28.09%

2.6 Additional Empirical Evidence

To assess the validity of the model, I document several additional empirical facts. Specifically, I focus on the average interest rate level, dispersion level and their correlation across observable state variables.

First, our model predicts that given constant (L, I, N), aggregate rate dispersion is the result of borrowers' strategies. However in the long run, as demand and supply are fluctuating over time, rate dispersion should correspond to variations in the state variables. I compare rate dispersion across different time horizons. According to Table 2.7, I find

Fact 3. *Rate dispersion is increasing in the time horizon.*

According to Table 2.7, the overall dispersion (0.6756%) is twice that of hourly dispersion (0.3403%). Moreover, I compute the same statistics for successful listings alone. The table reports that the rate dispersion of successful loans is relatively low compared with statistics corresponding to all listings. However, the two numbers converge as the time horizon increases.

Fact 4. As the time horizon increases, the difference in dispersion between all and successful listings declines.

While the hourly dispersion is of approximately 3 times lower (0.3403% vs 0.1268%), the overall dispersion is nearly identical (0.6756% vs 0.6218%) between whole sample and successful listings. The facts confirm our model prediction, which suggests that in the short run, price dispersion is due to individual-level dispersion. However in the long run, dispersion would correspond to fluctuations in the market's state variables such as supply and demand.

The next set of tests attempts to compare the average rate and dispersion under different levels of demand and supply. According to Proposition 1.4, when L/I increases, the rate distribution will FOSD the distribution with a lower L/I ratio. This implies that the mean of interest rates should be increasing in L and decreasing in I. As reported in Table 2.9, we have

Fact 5. *The average rate level is positively correlated with funding demand and negatively correlated with funding supply.*

As for rate dispersion, when L/I is small, the rate will degenerate to r_0 ; when L/I increases, rate will converge to v as the market becomes more competitive. Therefore, the second moment of interest rates should demonstrate an inverse U-shaped relationship with

z = L/I. Therefore, I run a regression w.r.t the JNS measure of the rate dispersion level, again z and z^2 , controlling for other observable variables. As reported in Table 2.10, under the hourly measure, the pattern is significant. However, in the regression using daily averages, as demand and supply fluctuate within a day in the market, the pattern disappears.

Fact 6. There is an inverse U-shaped relationship between rate dispersion and the supply/demand ratio (as measured by z = L/I).

The last two facts concern the size effect in the crowdfunding market. According to the analysis in Section 6, when L/I increases, N decreases on average. Consistent with the model prediction, according to Table 2.12,

Fact 7. Average loan size is negatively correlated with the number of listings and positively correlated with the number of investors.

Finally, I compare interest rates across listings with different sizes during the same period by estimating a random effects model controlling for aggregate variables during that period. As reported in Table 2.11, there is a significant and positive correlation between sizes and rates. Consistent with our theoretical analysis, when size is another choice variable available to fundraisers, they will offer a higher rate to compensate for the size of their requests.

Fact 8. Rate and size are positively correlated.

2.7 Conclusion

This chapter empirically investigates the impacts of search frictions in online crowdfunding markets. The essay accomplishes two main tasks.

First, by using a hand-collected data set, I provide evidence of the existence of search frictions in crowdfunding markets. As predicted by theory in Chapter 1, I show that the majority of price dispersion is contributed by individual fund-raisers' dispersion by controlling
a loan's attributes such as risk, maturity and return (Fact 2). In addition, there exists persistent mismatch between borrowers and lenders, which implies that information aggregation is not perfect in such platforms (Fact 1). I provide a list of stylized facts that are consistent with model implications in Chapter 1. Motivated by the fact that the progress bar might be malfunctioning in such market, I attempt to use price dispersion data to evaluate welfare gain due to the coordination mechanism induced by search frictions.

I develop a non-parametric approach to measure the welfare gain; namely, I estimate the ratio between aggregate funding probability with and without dispersion. In the procedure, I show that It is possible to use propensity score or percentile of rate distribution, instead of absolute value, to measure funding probabilities at each period, which reduces the number of parameters needed to estimate and increase estimation efficiency. Accordingly, I estimate that the welfare gain due to the coordination mechanism is about 28% in my sample.

2.8 Appendices

2.8.1 Proofs

2.8.1.1 Proof of Proposition 2.1

First, we have

$$\mathscr{W}_{\text{Dispersed Price}} = L - \delta \mathscr{L} = L - \frac{\delta}{\eta} \mathbb{E}(\ell) I$$

Therefore,

$$\frac{\mathscr{W}_{\text{Dispersed Price}}}{\mathscr{W}_{\text{random matching}}} = \frac{L - \frac{\delta}{\eta} \mathbb{E}(\ell)I}{L - \frac{\delta}{\eta^*} \mathbb{E}(\ell)I}$$

By Lemma 1.6, η solves

$$\frac{\delta}{\eta} \int_0^1 \frac{1}{1 - \left(\frac{\eta \tilde{p}(x)}{\tilde{\eta} p(x) + \delta}\right)^N} dx = \frac{L}{I \mathbb{E}(\ell)}$$

which implies that \tilde{p} is a sufficient static to η . Meanwhile, as shown in the proof of Theorem 1.2, η^* solves

$$\frac{\delta}{\eta^*} \frac{1}{1 - \left(\frac{\eta^* p^*}{\eta^* p^* + \delta}\right)^N} = \frac{1}{\mathbb{E}(\ell)}$$

where $p^* = \frac{1}{\mathbb{E}(\ell)}$. Thus, $\mathbb{E}(\ell)$ is sufficient for η^* . In $H(x) \equiv \tilde{p}(x)\mathbb{E}(\ell)$, by $\tilde{p}(1) = 1$, we have $H(1) = \mathbb{E}(\ell)$, and hence $\tilde{p}(x) = H(x)/H(1)$.

2.8.1.2 Proof of Proposition 2.2

Because

$$P(x) = \left(\frac{\eta \tilde{p}(x)}{\eta \tilde{p}(x) + \delta}\right)^{N}$$

we have

$$\tilde{p}(x) = \frac{\delta}{\eta} \frac{P(x)^{1/N}}{1 - P(x)^{1/N}}$$

Equation (1.3), we have

$$H(x) \equiv \tilde{p}(x)\mathbb{E}(\ell) = \frac{\eta \mathscr{L}}{I} \cdot \frac{\delta}{\eta} \cdot \frac{P(x)^{1/N}}{1 - P(x)^{1/N}}$$

In terms of $\delta \mathscr{L}$, by Equation (1.11), we have

$$\delta \mathscr{L} = \int_0^1 \frac{1}{1 - P(x)} dx$$

2.8.2 Tables

Variable	Mean	Std. dev.	Min	Max
All Listings				
Loan Size	37,489.39	199,656	400	20,000,000
Interest Rates	0.126417	0.0334	10%	20%
Durations	29.31826	40.41698	1	810
Progress	49.07%	0.4448	0	100%
Number of Observations	508,888			
Primary Loans				
Loan Size	3,770,128	2,442,229	1,000,000	20,000,000
Interest Rates	18%	0	18%	18%
Durations	612.787	227.4511	180	810
Progress	100%	0	100%	100%
Number of Observations	939			
Secondary Loans				
Loan Size	30,442.37	48,530.61	400	5,000,000
Interest Rates	12.81%	0.0296	10%	20%
Durations	27.98	30.59	1	360
Progress	47.58%	0.4451	0	100%
Number of Observations	464,718			

Table 2.1: Summary Statistics of All Listings

Note: The data for this table summarize information on 508,888 listings. Moreover, I report the summary statistics for Primary Loans and Secondary Loans. The time horizon of the data is from January 2015 to May 2015. All interest rates are annualized; loan sizes are measured by CNY(1 USD = 6.2 CNY); duration is measured by day.

Variable	Mean	Std. dev.	Min	Max
1-Month Secondary Loans				
Loan Size	310,90.98	50,245.85	400	4,370,000
Interest Rates	14.32%	.0076	10%	20%
Number of Bidders	6.36	6.46	0	342
Progress	51.79%	.4270	0	100%
Number of Observations	297,685			
Number of Successful Loans	126,121			

Table 2.2: Summary Statistics of 1-month Secondary Loans

Rate	Listings	Funded Listings	Funding Probability	Funding Probability (Weighted by Size)
≥10%	789	65	8.24%	4.77%
$\geq 11\%$	1,216	67	5.51%	8.84%
$\geq 12\%$	7,508	1,582	21.07%	43.69%
$\geq 13\%$	29,774	9,579	32.17%	47.27%
$\geq 14\%$	204,445	88,942	43.50%	59.56%
$\geq 15\%$	47,556	21,958	46.17%	54.54%
$\geq 16\%$	6,397	3,928	61.40%	65.24%
Total	297,685	126,121	42.37%	57.34%

Table 2.3: Funding Probability by Interest Rate

Percentile	Funding Probability	Funding Probability (Weighted by Size)
10	7.75%	10.89%
20	32.87%	30.87%
30	36.81%	28.84%
40	40.88%	32.81%
50	49.95%	44.68%
60	50.99%	43.68%
70	62.79%	61.44%
80	66.95%	67.62%
90	75.42%	78.92%
Total	42.37%	57.34%

Table 2.4: Funding Probability by Rate

Note: The funding probability by percentile is computed by the average of hourly statistics.

	Daily	Weekly	Overall
Individual	0.3390%	0.5692%	0.5889%
Aggregate	0.4042%	0.4870%	0.6756%

Table 2.5: Individual-level Rate Dispersion

	Aggregate		Individual	
	10 Percentile	90 Percentile	Min	Max
Daily	13.78%	14.67%	13.93%	14.66%
Weekly	13.71%	14.84%	13.65%	14.89%
Overall	13.71%	15.21%	12.44%	15.88%

Table 2.6: Individual Rate Dispersion (min/max)

Rate Dispersion	Hourly	Daily	Weekly	Overall
All Listings	0.3403%	0.4042%	0.4870%	0.6756%
Successful Listings	0.1268%	0.2186%	0.3498%	0.6218%

Table 2.7: Rate Dispersion over Time

	Whole Sample	Group 1	Group 2
	DT _{it}	DT _{it}	DT _{it}
PROGRESS CUMULATED _{it-1}	-373.1***	1.260	-4045.1***
	(-52.25)	(1.57)	(-50.82)
WEBSITE PAGE _{it}	-6.316***	29.18***	60.47***
	(-9.36)	(297.67)	(14.99)
LISTING AGE _{it}	16.80***	1.209***	17.63***
	(555.25)	(99.88)	(154.25)
SIZE _i	-0.000910***	-0.000189***	-0.00712***
	(-31.33)	(-53.41)	(-25.58)
RATE _i	-8898.7***	-3280.3***	-90183.8***
	(-21.32)	(-69.09)	(-21.20)
L _t	0.0000118***	-0.00000602***	0.0000446*
	(6.40)	(-29.46)	(2.09)
I _t	-0.00000405	-0.00000697***	-0.0000685
	(-1.38)	(-21.40)	(-1.92)
V _{primary}	-0.000000904***	1.15e-08	-0.00000668***
	(-9.78)	(1.12)	(-6.69)
T _{primary}	-0.150***	0.0106***	-1.602***
	(-11.93)	(7.54)	(-11.80)
Daytime	28.71***	5.271***	133.4*
	(5.90)	(9.74)	(2.39)
Weekend	0.446	4.120***	366.3***
	(0.09)	(7.24)	(7.01)
Night	-22.08**	33.53***	171.6*
	(-2.83)	(37.45)	(2.39)
cons	1620.1***	516.7***	15753.1***
	(25.70)	(72.29)	(24.57)
Biddings	900,405	815,518	84,842
Listings	126,149	113,349	12,800
R-sq	0.3235	0.1742	0.3300

t statistics in parentheses * p < 0.05, ** p < 0.01, *** p < 0.001

Table 2.8: Effect of Funding Progress

Note: The funding speed is computed only for fully funded listings. WEBSITE PAGE_{it} is listing i's default page number at the time t; LISTING AGE_{it} is listing i's age at time t measured in minutes; SIZE_i and Rate_i are listing i's size measured in CNY and annualized interest rate, respectively. L_t represents the amount of listing inflow at time t measure in CNY; I_t represents total amount of funding inflow at time t. Other control variables include V_{primary} and T_{primary} measures, which measure the total volume of primary loans and their respective duration measured by day; daytime represents the dummy if the time is between 9am and 5pm; night is the dummy if t is between 1am and 9am; weekday is the dummy if the time is from Monday to Friday. In the rest of the tables, the variables will be identically defined unless stipulated otherwise.

	Hourly	Daily
	r_t	<i>r</i> _t
V _t	1.42e-09*** (16.29)	2.33e-10*** (9.66)
I _t	-7.09e-10*** (-4.98)	-2.75e-10*** (-6.47)
<i>V</i> _{primary}	-3.20e-11*** (-9.55)	-4.89e-11*** (-4.27)
<i>T</i> _{primary}	-0.00000470*** (-8.87)	-0.00000168 (-0.98)
Daytime	0.00175*** (8.48)	
Night	0.00465*** (18.83)	
Weekday	-0.0000769 (-1.82)	0.0000696 (0.52)
cons	0.144*** (323.70)	0.143*** (104.66)
N R-sq	2234 0.2851	94 0.6685

t statistics in parentheses

* p < 0.05, ** p < 0.01, *** p < 0.001

Table 2.9: Average Interest Rate and Market Supply/Demand

Note: r_t is the average rate at time t. In the first panel, t is measured in hours. In the second panel, t is measured in days.

	Hourly	Daily
	d_t	d_t
Z _t	0.000261** (3.28)	-0.0154 (-1.05)
z_t^2	-0.00000265** (-2.82)	0.00650 (1.45)
r _t	0.508*** (24.63)	0.476*** (6.28)
<i>V</i> _{primary}	1.06e-11** (3.05)	1.66e-11 (1.80)
<i>T</i> _{primary}	-0.00000376*** (-6.32)	-0.00000110 (-0.70)
Daytime	-0.000831*** (-3.74)	
Night	-0.000744*** (-3.34)	
Weekday	-0.0000497 (-1.07)	-0.000145 (-1.18)
cons	-0.0638*** (-20.49)	-0.0512*** (-3.78)
N R-sq	2231 0.2709	94 0.6454

t statistics in parentheses

* p < 0.05, ** p < 0.01, *** p < 0.001

Table 2.10: Rate Dispersion and Market Supply/Demand

Note: d_t is the rate dispersion measured by JNS statistics. In the first panel, *t* is measured in hours. In the second panel, *t* is measured in days.

	Hourly	Daily
	r _{it}	r _{it}
Size _{it}	1.64e-08*** (77.77)	1.70e-08*** (75.35)
L_t	0.0000620*** (8.38)	0.00000821*** (3.99)
V_t	5.99e-08*** (13.23)	1.40e-08*** (8.61)
I_t	-6.05e-08*** (-13.25)	-1.42e-08*** (-8.65)
V _{primary}	-2.40e-11*** (-6.69)	-2.41e-11 (-1.81)
T _{primary}	-0.00000709*** (-12.30)	-0.00000288 (-1.51)
Weekday	-0.0000594 (-1.31)	0.0000962 (0.64)
Daytime	0.00128*** (5.99)	
Night	0.00466*** (16.65)	
cons	0.145*** (288.60)	0.139*** (81.55)
N R-sq	106237 0.1505	106237 0.1301

t statistics in parentheses

* p < 0.05, ** p < 0.01, *** p < 0.001

Table 2.11: Correlation between Sizes and Rates

	D 11	TT 1
	Daily	Hourly
	N_t	N_t
Lt	-4.402***	-80.73***
r	(-814.56)	(-427.79)
I _t	0.000266***	0.00468***
ł	(551.48)	(360.14)
Vprimary	0.0000295***	0.0000341***
primary	(113.59)	(64.13)
Vprimary	0.847***	1.551***
primary	(20.40)	(17.39)
Night		-2757.5***
U		(-58.77)
Davtime		-3147.5***
		(-106.39)
Weekday	-413.9***	-407.7***
5	(-123.44)	(-54.65)
cons	29308.5***	30883.6***
	(840.52)	(410.81)
Ν	116	2696
R-sq	0.8031	0.5316

* p < 0.05, ** p < 0.01, *** p < 0.001

Table 2.12: Average Loan Size

Note: N_t measures average number of investors for each successful listings. In the first panel, t is measured in days. In the second panel, t is measured in hours.

	Mean	SD	95% C.I.
0	0.3580	0.0828	[0.1923, 0.5236]
10%	0.4674	0.1020	[0.2633, 0.6715]
20%	0.6311	0.1615	[0.3080, 0.9542]
30%	0.7286	0.1971	[0.3343, 1.1229]
40%	0.8333	0.2144	[0.4045, 1.2622]
50%	0.9772	0.2420	[0.4932, 1.4612]
60%	1.2539	0.3183	[0.6172, 1.8906]
70%	1.2995	0.2788	[0.7417, 1.8572]
80%	1.2999	0.2636	[0.7725, 1.8272]
90%	1.6737	0.3112	[1.0513, 2.2962]
100%	1.7649	0.2851	[1.1946, 2.3351]

Table 2.13: Estimate of H(x) with 10th Percentile

Mean	SD	95% C.I.
0.3844	0.0860	[0.2124, 0.5565]
0.6152	0.1629	[0.2893, 0.9410]
1.0356	0.3664	[0.3027, 1.7684]
1.4727	0.5697	[0.3332, 2.6122]
1.8020	0.5495	[0.7031, 2.9010]

Table 2.14: Estimate of H(x) with 25th Percentile

2.8.3 Figures



Figure 2.1: Volume of Different Types of Loans



Figure 2.2: Volume of Different Types of Loans



Figure 2.3: Rate Distribution vs Supply and Demand

Note: First row reports the change of rate distribution, average rate and rate dispersion w.r.t. different loan size *N*; Second row reports the change of rate distribution, average rate and rate dispersion w.r.t. different supply demand ratio L/I; For parameters, let $\delta = 0.9, r_0 = 0.1, v = 0.18$. For the distribution of ℓ , I apply distribution used in [BM13], letting $Pr(\ell = 1) = 1 - \lambda$; $Pr(\ell = 2) = \lambda$ and choose $\lambda = 0.7$. As shown in Figure 2.5, it is closed to my estimated result.



Figure 2.4: Distribution of Progress Delay

Note: We measured real progresses by cumulatively adding investors' bidding amount prior to the time they bid. The delay is measured by the real progress minus the progress shown on website snapshots at the same time.



Figure 2.5: Estimates of H(x)



Figure 2.6: Calibrated $\frac{\mathscr{W}_{random match}}{\mathscr{W}_{dispersed price}}$

CHAPTER 3

Optimal Mechanism Design with Limited Enforcement

3.1 Introduction

Consider an economy in which firms need external finance to buy capital and produce. The primary impediment of the financial contract emerges from asymmetric information such as risk (e.g., [SW81]) and expected return (e.g., [DMW87]). Many studies have focused on different instruments, such as credit rationing ([SW81]) and collateral ([Bes85]) in resolving the informational problem. In this paper, I am interested in how the optimal mechanism with basic terms is distorted when principal cannot perfectly enforce it.

This paper is based on the idea that, although the monopolistic bank can design a mechanism to screen firms with different qualities, it cannot prevent the firm from absconding with the money (i.e., consume the borrowed funds without producing). Models with only information asymmetry usually show that under the optimal mechanism, bad firms produce lower than optimal level and good firms produce efficiently. I demonstrate that, with limited enforcement, both types' productions are distorted from the efficient levels.

I argue that the enforcement issue is critical in designing financial contracts. Different from markets that implement the cash on delivery (e.g., auction ([Mye81]) and sales ([MR78])), the natural lag between lending and repayment time increases costs on monitoring funds' usage. Besides, if the firm is in some industries such as services and those with more intangible assets, the project's progress and quality of capital might be difficult to verify. My main result is also supported by empirical works concerning law and finance. For instance, [LPLdSSV97] documents a sample of 49 countries and shows that countries with poorer investor protection and weaker legal enforcement have smaller and narrower capital markets.

My model builds on [MR78] model of non-linear pricing mechanism design. In the environment, the firm (agent) has no initial wealth and draws productivity from a commonly known distribution. The monopolistic bank (principal) seeks to mitigate the information asymmetry by offering a menu of contracts, which specify the amount to lend (quantity) and the repayment obligation (price). The firm can apply for any contract. With borrowed funds, the firm can either produce and repay according to contract terms, or abscond with the money and consume it. When a firm chooses to steal, the bank can only catch the firm with some fixed probability uniformly across all applicants and contracts.

Let us ignore the legality of firm behavior. The firm would choose to steal funds if and only if the benefit of theft is larger than the net profit of producing. When the environment has only adverse selection friction, the two-dimensional contracts can separate agents by providing high types both greater quantity and higher repayment obligation. Low types can only afford a contract with both smaller size and repayment obligation. Nevertheless, in a limited enforcement environment, the repayment obligation specified in the contract fails to restrain firms from opting to steal. As the mechanism is observable and there is no credit rationing, the optimal stealing strategy is to apply for the largest loan on the menu. In other words, the benefit from stealing provides the firm an endogenous outside option which depends on the overall scale of the financial mechanism.

Accordingly, the optimal mechanism under limited enforcement has to achieve the balance between (i) separating agents by quantity-price contracts and (ii) mitigating the incentive of stealing. The first aspect is standard in theories of mechanism design (e.g., [Mye81], [MR78]). To resolve the absconding problem, I show that in the optimal mechanism, the loan size designed for the high type is shrunk from the efficient level, because the firm's optimal stealing strategy is to apply for the contract designed for the high-type firm, as monotonicity is required in the separating mechanism. In the general model with continuous type space, I fully characterize the optimal mechanism and compare it with the environment with adverse selection only (second best). First, it is shown that the allocations of highest types are ironed: There is a threshold type above which all firms will be assigned to the same contract. Moreover, the loan size is the same as that of the threshold type in the second best scenario. Besides the pooling on the top, I also show that the optimal mechanism allocates second best quantities to types lower than the benchmark. Moreover, the optimal mechanism cannot deter firms from stealing funds. A fascinating surprise is that the set of the firms that steal is independent of enforcement levels. Compared to the second best scenario, which is the extreme case in my model, the set of firms that steal in my model is the same as the set of firms that will not participate in a typical adverse selection setup. Figure 3.1 visualizes the loan sizes under different environments.



Figure 3.1: Optimal loan sizes across types

The intuition behind the results is as follows. First, recall that the benefit from the optimal stealing strategy is the expected payoff of consuming the largest loan in the mechanism, which is uniform for all types. It creates an endogenous outside option of all agents and is the primary reason of truncating the largest loan size in the optimal mechanism. Now suppose a mechanism is fully separating, the outside option plays no role in designing the

Note: Range I represents firms who will cheat under limited enforcement. Range II shows loan sizes of separating firms. Range III illustrates the pooling on the top.

optimal screening device. Technically, in the spirit of [Mye81], the marginal revenue for separating types (range II in Figure 3.1) is not affected by such option, so I can apply standard point-wise maximization to find optimal allocation. It is indeed true, as stealing and producing are mutually exclusive actions, but the separating device only works for firms that produce. Therefore, given an arbitrary level of outside options, the loan sizes in the optimal mechanism should be the non-decreasing curve closest to the second best marginal value function. Among all the candidate allocation rules, the optimal mechanism employs the one with ironing and pooling on the top. The set of types that steal is independent of enforce-ment level also because marginal revenue contributed by each type is unaffected by limited enforcement. In a pure adverse selection environment, the set of firms that are squeezed out of the market is determined by the cut-off type, whose marginal revenue is zero to the bank. As limited enforcement does not affect marginal value, the cut-off should not vary.

The model also allows conducting some interesting comparative statics on the creation and allocation of social surplus. First, it is evident that both social surplus and the bank's profit are negatively impacted by limited enforcement, as it creates additional distortion on high-quality firms' production. Monopolistic banks' profit is negatively affected due to both the distortion on top and reduction on repayment obligation. When enforcement is perfect, the lowest participating type would be squeezed to zero profit. But when the benefit of stealing funds is positive, the bank loses part of the market power of exploiting low-type firms, thus will deduct the repayment obligation by the exact amount of stealing benefit to incentivize the firm. In fact, I will show that such deduction will be diffused to all types. Thus, the expected profit earned by the firm is increased by the "threat" of stealing. On the other hand, a firm's profit is bounded by total surplus, which is decreasing in enforcement limitation. Thus, a firm's profit and enforcement level have an inverse-U relationship, until a critical point beyond which the bank would choose to quit and the financial market evaporates.

Implementation of the optimal mechanism is straightforward in my model and can be achieved by an initial monetary compensation plus a menu of standard debt contracts specifying sizes and interest rates. Under the compensation-debt scheme, every firm can earn positive payoff, which is equal to that it would obtain by stealing, though some of the firms would not produce at all.

Related Literature

This paper builds on the canonical setup of [MR78] and [Mye81]. The key feature of these models is that when the agents' payoff function satisfies the single-crossing property, the principal is able to use a price-quantity mechanism to let agents reveal information. The survey by [Ril01] summarizes the important studies of screening models. Our work extends the setup by allowing limited enforcement. We argue that this assumption is quite natural in financial markets because principals deliver goods (investments) first and collect payoffs (interest) in the future, which is quite different from auction markets ([Mye81]) and the manufacturing sector ([MR78])¹.

Our work is also closely related to the literature concerning sources of borrowing constraints in debt contract problems. For example, [AH04] analyzes an environment in which a firm can default and liquidize accumulated capital at any time. [Atk91] studies international lending with the risk of debt repudiation. Both works provide firms outside value via strategic defaults. They show that friction limits the scale of loans, which is consistent with our findings. The difference is that with information asymmetry, while lower-quality firms have a higher likelihood of defaulting, it is the higher-quality firms that suffer from the endogenous borrowing constraint.

Regarding methodology, our work is an application of the ironing procedure in mechanism design theories. Both [MR78] and [Mye81] mention the scenarios in which the method is necessary because the regularity condition fails. Later works such as [RC98] formalize the issue and generalize it to a multi-dimensional mechanism. They show that ironing is

¹There is a body of literature that discusses the commitment issue in mechanism design, e.g., [BS01] and [Skr06].

robust due to the existence of second-order incentive compatibility. All of these works require ironing because certain functional or parametric assumptions fail to hold. We show that ironing is a strategic choice by the principal in the presence of limited enforcement even if the regularity condition holds.

The rest of the paper is organized as follows. In section 3.2, we demonstrate a twotype benchmark, which illustrates why distortion on the top is optimal. We generalize the model to continuous type space in section 3.3 and fully characterize the optimal mechanism. Welfare implication and implementation will also be mentioned. We conduct numerical exercise and comparative statics in section 3.4. Section 3.5 summarizes and concludes.

3.2 An Example

3.2.1 The Environment

I start with a simple two-type example. Firm(He) can be of either (H)igh or (L)ow productivity. A monopolistic bank(She) designs loan contracts to screen them in order to maximize expected revenue.

The game has 3 stages t = -1, 0, 1. At t = -1, the bank designs loan contracts based on the distribution of upcoming firm's productivity θ . Let $\theta \in {\theta_L < \theta_H}$ with probabilities $p_L + p_H = 1$. Let a loan contract takes the form (b, T), where *b* is the loan size and *T* is the associated repayment obligation². At t = 0, firm with productivity θ enters the market. Observing all the contracts, the firm takes the most preferred one or exits immediately with outside value normalized to zero. With acquired funds *b*, firm can either buy inputs to produce or abscond with the money. At t = 1, payoffs realize. If firm produced at t = 0, he earns revenue $\theta \cdot f(b)$ and repays obligation *T*. So his net profit is

 $v(b,T;\theta) \equiv \theta \cdot f(b) - T$

²Alternatively, by defining total repayment by $T \equiv (1+r) \cdot b$, a contract can be also represented by loan size and interest rates (b, r).

Bank earns

$$u(b,T) \equiv T - b$$

The total surplus created by production is

$$w(b, \theta) \equiv \theta \cdot f(b) - b$$

If firm chose to cheat at t = 0, his expected payoff at t = 1 is $\lambda \cdot b$. $\lambda \in [0, 1]$ is a parameter that controls the level of contract enforcement. It can be interpreted as the probability that the bank cannot catch the firm or only a fraction of the stolen found can be called back. Note that firm's expected payoff λb is the bank's expected loss, as no surplus is created.

First Best

If the market is of complete information and contracts are perfectly enforceable, then for each θ , the size of the debt $b^*(\theta)$ satisfies that

$$b^*(\theta) = \arg\max_b \theta f(b) - b$$

which indicates the monopolistic bank also maximizes social surplus. Suppose that $f(\cdot)$ is increasing, concave and differentiable. Then the first best loan size satisfies the first order condition,

$$f'(b^*(\theta)) = \frac{1}{\theta}$$
(FB)

By concavity of $f(\cdot)$, $b^*(\theta)$ is increasing in θ .

3.2.2 Second Best

When information is asymmetric but enforcement is perfect, the optimal mechanism may take three forms. First, it consists of two contracts that separate two types of firms (separating mechanism); second, there is one contract that is only affordable to high type (partial-

participating mechanism); third, the mechanism has one contract that is affordable to both types (pooling mechanism).

In the first case, the bank maximizes

$$v_{\text{sep}} = \max_{b_{\theta}, T_{\theta}} p_H(T_H - b_H) + p_L(T_L - b_L)$$

subject to IR and IC constraints.

$$\theta_H \cdot f(b_H) - T_H \ge 0 \tag{IR}_H$$

$$\theta_L \cdot f(b_L) - T_L \ge 0 \tag{IR}_L$$

$$\theta_H \cdot f(b_H) - T_H \ge \theta_H \cdot f(b_L) - T_L \tag{IC}_H$$

$$\theta_L \cdot f(b_L) - T_L \ge \theta_L \cdot f(b_H) - T_H \tag{IC}_L$$

Solving the programming problem is standard. By IR_L , IR_H is redundant. Then, assume IR_L and IC_H are binding, the bank's problem becomes

$$v_{sep} = \max_{b_{\theta}} p_L \cdot [\theta_L \cdot f(b_L) - b_L] + p_H \cdot [\theta_H \cdot f(b_H) - \theta_H \cdot f(b_L) + \theta_L \cdot f(b_L) - b_H]$$

So optimal loan sizes satisfy

$$f'(b_H) = \frac{1}{\theta_H}$$
$$f'(b_L) = \frac{1 - p_H}{\theta_L - p_H \theta_H}$$

By concavity of $f(\cdot)$, we have $b_H > b_L$. Then, it can be verified that IC_L is satisfied automatically. The first order conditions implies that in equilibrium, high type firm produces efficiently, while low type's production is distorted from its first best level. Regarding the other two cases, when $p_H \theta_H > \theta_L$, serving both types will induce too much loss in informational rent. Therefore, the bank will prefer to use partial-participating mechanism. The

problem becomes

$$v_{\text{partial}} = \max_{b_H, T_H} p_H(T_H - b_H)$$

subject to

$$\theta_H f(b_H) - T_H \ge 0$$

Then only high type participates and produces efficiently,

$$f'(b_H) = \frac{1}{\theta_H}$$

Lastly, as for pooling-mechanism, the contract (b,T) should attract both types. So firm's problem is

$$v_{\text{pool}} = \max_{b,T} T - b$$

such that

$$\theta_L f(b) - T \ge 0$$

which is equivalent to

$$v_{\text{pool}} = \max_{b} \theta_L f(b) - b$$

It is obvious that the expected profit earned from pooling mechanism is dominated by that using separating equilibrium because the objective function of separating mechanism can be re-written as

$$v_{sep} = \max_{b_L, b_H} \theta_L f(b_L) - b_L + p_H(\theta_H f(b_H) - \theta_H f(b_L) - b_H + b_L)$$

Then the solution is strictly larger than that in pooling mechanism by letting $b_H = b_L + \varepsilon$ without violating necessary constraints.

In summary, When enforcement is perfect ($\lambda = 0$), we show that (i) high type firm produces at first best level, while low type firm will either produce lower than optimal or be squeezed out of the market; (ii) Pooling mechanism is dominated by separating mechanism, so is never implemented.

3.2.3 Limited Enforcement

Now let's consider the case of limited enforcement ($\lambda > 0$). I will show that first, high type's production will be distorted; second, pooling mechanism might be used. I start with solving the optimal separating mechanism. If the firm steals the acquired fund *b*, his expected payoff will be λb , which is independent of θ . So the benefit of stealing can be considered as firm's endogenous outside value. Also, since the optimal stealing strategy is to apply for the largest loan in the mechanism and separating equilibrium requires monotonicity, firm's outside option in equilibrium will be λb_H . So compared with second best environment, the only difference is that IR constraints become

$$\boldsymbol{\theta} \cdot f(\boldsymbol{b}_{\boldsymbol{\theta}}) - T_{\boldsymbol{\theta}} \ge \boldsymbol{\lambda} \cdot \boldsymbol{b}_H \tag{IR}_{\boldsymbol{\theta}}$$

Following the same procedure, by binding IC_H and IR_L , the bank's objective function can be written as

$$\begin{aligned} v(b_{\theta}, T_{\theta}) &= p_L \cdot \left[\theta_L \cdot f(b_L) - \lambda \cdot b_H - b_L\right] \\ &+ p_H \cdot \left[\theta_H \cdot f(b_H) - \theta_H \cdot f(b_L) + \theta_L \cdot f(b_L) - \lambda \cdot b_H - b_H\right] \end{aligned}$$

So optimal separating loan sizes satisfy

$$f'(b_H) = \frac{p_H + \lambda}{p_H \theta_H}$$
$$f'(b_L) = \frac{p_L}{\theta_L - p_H \theta_H}$$

Notice that the solution is achieved by implicitly assuming that $b_H > b_L$ and IC_L is satisfied. When $\lambda = 0$, the conditions are satisfied automatically. When $\lambda > 0$, the conditions require the enforcement level is large enough, i.e.

$$\lambda \leq rac{p_H(heta_H - heta_L)}{ heta_L - p_H heta_H}$$

Under the circumstance, the conditions for optimal separating mechanism implies that b_L is the same as that in second best case. However, the loan size of good firms will be distorted by λ . It is the main difference and key impact of the limited enforcement. Intuitively, limited enforcement provide incentives for firms to steal money and a rational firm will steal the largest amount when it is profitable. To prevent such stealing, the bank has to impose a ceiling on loan sizes. As the loan size for high type firm is larger, it will be affected first. Models with only asymmetric information (for example, [MR78]) argues that, in Bayesian Nash Equilibrium, the distortion exists only for low type and high type will produce at efficient level. But with limited enforcement, the optimal contract involves an additional distortion on the top. By the result of separating mechanism, it is easy to derive that the condition for partial-participating mechanism remains the same

$$p_H \theta_H > \theta_L$$

That is, when the above inequality fails, it is more profitable to let high type firm participate only. Note that, however, if the bank only provides such contract, low type firm will choose to steal funds. Then the bank's problem becomes

$$v_{\text{partial}} = \max_{\hat{b},\hat{T}} p_H \cdot (T-b) - p_L \cdot \lambda b$$

subject to IR_H . Then the optimal loan size satisfies

$$f'(\hat{b}) = \frac{p_H + \lambda}{p_H \theta_H}$$

which is the same as that in separating mechanism.

Remember that all the above analysis is based on the assumption that the enforcement is not too small, i.e. $\lambda \leq \frac{p_H(\theta_H - \theta_L)}{\theta_L - p_H \theta_H}$. If it is violated, then separating mechanism does not exist, as the ceiling is too low and affect both types. I consider the possibility of pooling mechanism, i.e.

$$v_{\text{pool}} = \max_{\hat{b}, \hat{T}} T - b$$

subject to IR_L . The optimal loan size satisfies

$$f'(b) = rac{1+\lambda}{ heta_L}$$

The above analysis discussed how production and social surplus are distorted by limited enforcement. Briefly speaking, (i) in addition to the distortion of low type's, high type's loan size is also shrunk; (ii) it might be optimal for the bank to provide pooling mechanism which is always dominated in perfect enforcement environment.

The last part of the section discusses how the surplus is split between the two parties. In particular, besides the changes in loan sizes, how firm's obligation and interest rates are affected. For comparison purpose, we only consider the separating mechanism. When enforcement is perfect, the bank will design the contracts such that low type earns zero. However, in my problem, low type's payoff is at least $\lambda b_H > 0$. In separating mechanism, since b_L is independent of λ , an increasing in low type firm's payoff implies a decrease in bank's profit. Second, by the binding of high type' IC constraint, high type firm's equilibrium profit can be expressed by

$$v(\theta_H) = \theta f(b_H) - T_H = \theta_H \cdot f(b_L) - \theta_L \cdot f(b_L) + \lambda b_H$$

Since b_L is free of θ shown above, we conclude that the profit of high type firm is increased by λb_H , even if its production is lower than efficient level. Intuitively, when bank tries to reduce the incentive of cheating, she transmits some market power to the firm. Besides lower type claims a positive profit λb_H , high type's final surplus is also increased by λb_H , which can be directly reflected in the reduction of T_{θ} . As for interest rates, since b_L is not affected by λ , $r_L = T_L/b_L - 1$ becomes smaller. The impact on r_H is ambiguous since both T_H and b_H are reduced.

3.3 Continuous Type Model

In this section, we generate the model to continuous type space. Assume that the firm's productivity takes value $\theta \in \Theta = [\underline{\theta}, \overline{\theta}]$. Denote the CDF and PDF of type distribution as $G(\cdot)$ and $g(\cdot)$, respectively. The firm's production function is denoted as $F(\theta, b)$. Timing is the same as two-type case. Bank designs mechanism in terms of loan sizes and repayment obligations $\{(b,T)\}$ based on type distribution *G*. On the other side, by taking a contract with $\{(b,T)\}$, the firm with θ can either produce and repay according to the contract which is equal to

$$v(\theta, b, T) = F(\theta, b) - T$$

Assumption 3.1. *Production function* $F(\theta, b)$ *satisfies that*

- *1.* $F(\theta, b)$ is continuously differentiable.
- 2. $F(\theta, b)$ is increasing in θ and b.
- *3.* $v(b,T;\theta)$ satisfies Single crossing property.

In addition, he can abscond with acquired funds and earn expected payoff λb , where $\lambda \in [0,1]$ controls the level of enforcement. Given the setup, the first-best loan size $b^*(\theta)$ satisfies that

$$F_2(\boldsymbol{\theta}, b^*(\boldsymbol{\theta})) = 1$$

for all $\theta \in \Theta$. F_i is the partial derivative of F w.r.t. *i*th argument.

3.3.1 Mechanism with Perfect Enforcement

I start with the case of perfect enforcement (second best), i.e. $\lambda = 0$. By revelation principle, the optimal mechanism can be denoted as

$$\widehat{\mathscr{M}} = \{ \hat{b}(\theta), \hat{T}(\theta) \}_{\theta \in \Theta_{sb}^{P}}$$

where Θ_{sb}^{P} is the set of types who produce under the mechanism. It is easy to verify that Θ_{sb}^{P} takes the form $[\theta_*, \bar{\theta}]$ for some threshold type $\theta_* \geq \underline{\theta}$. Denote

$$\tilde{v}(\theta', \theta) \equiv F(\theta, \hat{b}(\theta')) - \hat{T}(\theta')$$

as the payoff of a firm with type θ who reports θ' . The mechanism is truth-telling if and only if

$$\boldsymbol{\theta} = \arg \max_{\boldsymbol{\theta}'} \tilde{v}(\boldsymbol{\theta}', \boldsymbol{\theta})$$

for all $\theta \in \Theta_{sb}^{P}$. Assume the value function is differentiable, then local maximization requires that $\frac{\partial \tilde{v}}{\partial \theta'} = 0$ at $\theta' = \theta$, i.e.

$$F_2(\theta, \hat{b}(\theta)) \cdot \hat{b}'(\theta) = \hat{T}'(\theta)$$

Denote equilibrium payoff of the firm with type θ as

$$\hat{v}(\theta) = F(\theta, \hat{b}(\theta)) - \hat{T}(\theta)$$

so the marginal value

$$\hat{v}'(\boldsymbol{\theta}) = F_1(\boldsymbol{\theta}, \hat{b}(\boldsymbol{\theta}))\hat{b}'(\boldsymbol{\theta}) + F_2(\boldsymbol{\theta}, b(\boldsymbol{\theta})) \cdot b'(\boldsymbol{\theta}) - \hat{T}'(\boldsymbol{\theta})$$

Plug the local maximization condition into $\hat{v}'(\theta)$, the marginal value satisfies

 $\hat{v}'(\theta) = F_1(\theta, \hat{b}(\theta))$

The firm's ex-ante payoff is

$$E[\hat{v}(oldsymbol{ heta})] \equiv \int_{oldsymbol{ heta}\in\Theta} \hat{v}(oldsymbol{ heta}) dG(oldsymbol{ heta})$$

Integrating by part gives us

$$\begin{split} E[\hat{v}(\theta)] &= \int_{\theta \in \Theta} \hat{v}(\theta) d(-(1 - G(\theta))) \\ &= -\hat{v}(\theta)(1 - F(\theta))|_{\theta_*}^{\bar{\theta}} + \int_{\theta_*}^{\bar{\theta}} (1 - G(\theta))\hat{v}'(\theta) d\theta \\ &= \hat{v}(\theta_*) + \int_{\theta_*}^{\bar{\theta}} (1 - G(\theta))\hat{v}'(\theta) d\theta \end{split}$$

As outside option of firm is zero, the equilibrium payoff of lowest participating type is $\hat{v}(\theta_*) = 0$, so we have

$$E[\hat{v}(\theta)] = \int_{\theta_*}^{\bar{\theta}} \frac{1 - G(\theta)}{g(\theta)} F_1(\theta, \hat{b}(\theta)) dG(\theta)$$
(3.1)

As for the bank's problem, the expected revenue is equals to

$$u = \int_{\theta_*}^{\bar{\theta}} \hat{T}(\theta) - \hat{b}(\theta) dG(\theta)$$

which is the expected repayment minus loan quantity. Alternatively, it can be expressed by expected social surplus minus the firm's expected value,

$$u = \int_{\theta_*}^{\bar{\theta}} F(\theta, \hat{b}(\theta)) - \hat{b}(\theta) dG(\theta) - \int_{\theta_*}^{\bar{\theta}} \hat{v}(\theta) dG(\theta)$$

By equation (3.1), we have

$$u = \int_{\theta^*}^{\bar{\theta}} F(\theta, \hat{b}(\theta)) - \hat{b}(\theta) - \frac{1 - G(\theta)}{g(\theta)} F_1(\theta, \hat{b}(\theta)) dG(\theta)$$
(3.2)

The integrand

$$MR(\theta,b) \equiv F(\theta,b) - b - \frac{1 - G(\theta)}{g(\theta)}F_1(\theta,b)$$

is the marginal revenue generated from type θ . Differentiating by *b*,

$$J(\boldsymbol{\theta}, b) \equiv F_2(\boldsymbol{\theta}, b) - 1 - \frac{1 - G(\boldsymbol{\theta})}{g(\boldsymbol{\theta})} F_{12}(\boldsymbol{\theta}, b)$$

Point-wise maximization requires that the optimal loan since $\hat{b}(\theta)$ satisfies

$$J(\boldsymbol{\theta}, \hat{b}(\boldsymbol{\theta})) = 0$$

Here we impose the following standard assumptions.

Assumption 3.2. Assume

- $J(\theta, b)$ is increasing in θ ;
- $MR(\theta, b)$ is pseudo-concave in b for each θ .

The first bullet is the typical regularity assumption mentioned by [Mye81], with which the optimal quantity $\hat{b}(\theta)$ is increasing, so is consistent with incentive compatibility. The Second bullet makes sure that at each θ , the point-wise maximization is equivalent to global maximization, which guarantees uniqueness of the solution. Besides, the assumption implies that for any other quantity b', smaller $|\hat{b}(\theta) - b|$ implies larger $MR(b', \theta)$. Recall that the first best quantity satisfies $F_2(\theta, \hat{b}(\theta)) - 1 = 0$. Compared with point-wise maximization condition above, we can see that the optimal mechanism with private information is efficient when firm's productivity θ at highest level $\bar{\theta}$. Firms with lower productivity will produce below optimal level. To fully solve for the mechanism, we need to pin down the lowest type who chooses to participate θ_* , which satisfies

$$\widehat{MR}(\theta_*) = 0 \tag{3.3}$$

where $\widehat{MR}(\theta) = MR(\theta, \hat{b}(\theta))$. The above analysis can be summarized by the following proposition.

Proposition 3.1. The optimal mechanism $\widehat{\mathcal{M}}$ satisfies that (i) $\hat{b}(\theta)$ point-wisely maximizes $MR(\theta, b)$ for all $\theta \in [\theta_*, \overline{\theta}]$; (ii) θ_* satisfies Equation (3.3).

3.3.2 Contracts with limited enforcement

Now we incorporate limited enforcement into the model. The main question we are interested in is that how the production is influenced by the enforcement problem. As predicted in the two-type benchmark, high type's production is distorted due to the threat of lower types' cheating behavior. The result, intuitively, will preserve in the continuous type model. With continuous type model, we are able to investigate the impact on the overall population. To be specific, we are interested in (i) how the distortion on the top is transmitted to lower types as we have to maintain monotonicity; (ii) which types will be producing and who will steal in the environment. Given λ , denote the optimal mechanism with as

$$\mathcal{M}(\lambda) = \{(b(\theta); \lambda), T(\theta; \lambda)\}_{\theta \in \Theta_P(\lambda)}$$

where $\Theta_P(\lambda)$ represents the set of firms who borrow and produce. For types $\theta \in \Theta \setminus \Theta_P(\lambda)$, instead of exiting the market, they will choose to steal funds. An immediate observation is that if a firm chooses to steal, the optimal strategy is to apply for the contract with the largest size $b_{\max}(\lambda)$. Therefore, all types share the same endogenous IR constraints,

$$v(\boldsymbol{\theta}; \boldsymbol{\lambda}) \ge \boldsymbol{\lambda} \cdot \boldsymbol{b}_{\max}(\boldsymbol{\lambda}) \tag{IR}$$

In addition, single crossing property of $v(b,T;\theta)$ requires that $b(\theta;\lambda)$ is monotone. So it should be the case that $b_{\max}(\lambda) = b(\bar{\theta};\lambda)$. Also, the set of participating types $\Theta_P(\lambda)$ takes the form $[\theta_*(\lambda), \bar{\theta}]$ as firm's payoff is increasing in type. For firms with $\theta \in \Theta_P(\lambda)$, marginal value still takes the form

$$v'(\boldsymbol{\theta};\boldsymbol{\lambda}) = F_1(\boldsymbol{\theta},b(\boldsymbol{\theta};\boldsymbol{\lambda}))$$

Therefore, the bank's expected revenue can be written as

$$u(\lambda) = \int_{\theta_*(\lambda)}^{\bar{\theta}} T(\theta;\lambda) - b(\theta;\lambda) dG(\theta) - \lambda G(\theta_*) b(\bar{\theta};\lambda)$$

The firm's ex-ante payoff is equal to

$$E[v(\theta;\lambda)] = \int_{\theta_*(\lambda)}^{\bar{\theta}} F(\theta, b(\theta;\lambda)) - T(\theta;\lambda) dG(\theta) + G(\theta_*(\lambda)) \cdot \lambda \cdot b(\bar{\theta};\lambda)$$

In the expression, the integral is firm's expected profit conditional on being a producing type, while the term $G(\theta_*(\lambda)) \cdot \lambda \cdot b(\bar{\theta}; \lambda)$ is the firm's expected payoff by taking advantage of limited enforcement and stealing. Integrating by part, we have

$$E[v(\theta;\lambda)] = v(\theta_*)(1 - G(\theta_*(\lambda)) + \int_{\theta_*(\lambda)}^{\bar{\theta}} (1 - G(\theta))v'(\theta;\lambda)d\theta + G(\theta_*(\lambda)) \cdot \lambda \cdot b(\bar{\theta};\lambda)$$

In equilibrium, the lowest producing type is indifferent between producing, $v(\theta_*(\lambda) = \lambda b(\bar{\theta}; \lambda))$ so firm's ex-ante profit is equal to

$$E[v(\theta;\lambda)] = \int_{\theta_*(\lambda)}^{\bar{\theta}} \frac{1 - G(\theta)}{g(\theta)} F_1(\theta, b(\theta;\lambda)) dG(\theta) + \lambda b(\bar{\theta};\lambda)$$

Note that though only a firm with type $\theta \in [\underline{\theta}, \theta_*(\lambda)]$ chooses to cheat, the outside option applies to all types. For those who cheat, $\lambda b(\bar{\theta}; \lambda)$ is their direct payoff of stealing and stealing types earn the value from the reduction in $T(\theta; \lambda)$ by $\lambda b(\bar{\theta}; \lambda)$. Not matter which is the case, bank loses the profit. Hence, the above equation involves the term $\lambda b(\bar{\theta}; \lambda)$. Therefore, bank's expected revenue becomes

$$u(\lambda) = \int_{\theta_*(\lambda)}^{\bar{\theta}} F(\theta, b(\theta; \lambda)) - b(\theta; \lambda) - \frac{1 - G(\theta)}{g(\theta)} F_1(\theta, b(\theta; \lambda)) dG(\theta) - \lambda b(\bar{\theta}; \lambda) \quad (3.4)$$

Different from Equation (3.2) with $\lambda = 0$, the modified problem creates a trade-off faced by the bank. On one hand, the bank has incentive to lend more money to good firms. On the other hand, however, larger $b(\bar{\theta}; \lambda)$ might lead more types to cheat. It seems that bank would decrease overall loan sizes in order to optimize it's profit. Note that Equation (3.4) is achieved under the precondition that $b(\theta; \lambda)$ is non-decreasing. However, due to the expected loss $\lambda b(\bar{\theta}; \lambda)$, point-wise maximization concludes that

$$rac{\partial u(oldsymbol{\lambda})}{\partial b(ar{m{ heta}};oldsymbol{\lambda})} = J(m{ heta},b(m{ heta};oldsymbol{\lambda})dG(m{ heta}) - oldsymbol{\lambda} = -oldsymbol{\lambda} < 0$$

which implies $b(\bar{\theta}; \lambda)$ should be as small as possible, which violates the precondition. The



Figure 3.2: Point-wise maximization violates monotonicity

violation is similar to problems discussed in [MR78] and [Mye81], as monotonicity of quantity is pre-assumed. The main difference of the current model to theirs is that the violation of monotonicity is resulted from the endogenous outside option $\lambda b(\bar{\lambda}; \theta)$, rather than regularity condition. To avoid cheating, the bank can either reduce repayment obligations or shrink loan sizes. However, it is easy to see that the first instrument won't work as both repayment obligation and money stealing are simply transfers between the two parties. Without varying in *b*, the expected decreasing in *T* will generate exact the same change in funds stolen. Regarding optimal design in loan sizes, we solve the problem following a two-step method. First, we fixed the largest loan size, denoted as \bar{b} and find the corresponding optimal size schedule $b(\theta; \lambda, \bar{b})$. Second, we find the optimal \bar{b} and thus the whole optimal mechanism. Step one, for \bar{b} , it is obvious that $\bar{b} < b_{sb}(\bar{\theta})$, as it is the first best quantity for type $\bar{\theta}$. To maximize bank's profit given \bar{b} ,

$$u(\lambda,\bar{b}) = \int_{\theta_*}^{\bar{\theta}} F(\theta,b(\theta;\lambda)) - b(\theta;\lambda) - \frac{1 - G(\theta)}{g(\theta)} F_1(\theta,b(\theta;\lambda)) dG(\theta) - \lambda\bar{b}$$

Under the ceiling \bar{b} , the optimal loan sizes satisfy

Proposition 3.2. Given $\bar{b} < b_{sb}(\bar{\theta})$, the profit maximization $b(\theta)$ satisfies

$$b(\theta; \lambda, \bar{b}) = \min\{\hat{b}(\theta), \bar{b}\}$$

The proposition claims that the most profitable allocation schedule is the second best quantities with ironing on the top, as visualized below, it is the closest non-decreasing curve to the second best allocation. By Assumption 3.2, as $MR(\theta, b)$ is Pseudo-concave, the point-



Figure 3.3: Optimal $b(\theta; \lambda, \overline{b})$ flattens loan sizes on the top

wise maximizer $b_{sb}(\theta)$ is the unique global maximizer, which implies that and the closer the $b(\theta; b^*)$ to second best quantity $\hat{b}(\theta)$, the larger the marginal value earned from type θ . Since all types with $\hat{b}(\theta) < \bar{b}$ remain the loan size at second best level, their marginal value $\widehat{MV}(\theta)$ will be the same as second best environment. Thus, profit maximizing participating set is the same as that in the second best case.
Corollary 3.1. The lowest type θ_* who produces satisfies $\widehat{MV}(\theta_*) = 0$, which is independent of λ .

The above discussion determines the structure of allocation $b(\theta)$ under arbitrary ceiling \overline{b} . The second step is to determine the optimal \overline{b} that maximizes bank's profit. According to Proposition 3.2 and Corollary 3.1, the marginal value accumulated from types $\theta \in [\theta_*, \theta^*]$ is the same as second best $\widehat{MR}(\theta)$, where θ^* satisfies $\hat{b}(\theta^*) = \overline{b}$, so to find optimal b^* , it is equivalent to find θ^* which maximizes

$$u(\lambda) = \int_{\theta_*}^{\theta^*} \widehat{MR}(\theta) dG(\theta) + \int_{\theta^*}^{\bar{\theta}} MR(\theta, \hat{b}(\theta^*)) - \lambda \hat{b}(\theta^*)$$

Thus, the optimal $\theta^*(\lambda)$ can be pinned down by the following first order condition,

$$\int_{\theta^*}^{\bar{\theta}} MR_2(\theta, \hat{b}(\theta^*)) = \lambda$$
(3.5)

The left hand side is the marginal loss in total revenue. As the quantity is fixed at $\hat{b}(\theta^*)$, $MR(\theta, b(\theta^*)$ cannot achieve $\widehat{MR}(\theta)$. The right hand side is the marginal gain from reducing stealing behaviors. I summary the main result demonstrated above as following theorem.

Theorem 3.1. With limited enforcement, the optimal mechanism $\mathcal{M}(\lambda)$ satisfies that (i) $b(\theta; \lambda) = \hat{b}(\theta)$ for $\theta \in [\theta_*, \theta^*]$ where θ_* satisfies Equation (3.3) and θ^* satisfies Equation (3.5); (ii) When $\theta > \theta^*$, the loan size is equal to $b(\theta; \lambda) = \hat{b}(\theta^*)$; (iii) When $\theta < \theta_*$, the firm choose to steal funds and earn $\lambda \hat{b}(\theta^*)$.

3.3.3 Efficiency and Surplus

With limited enforcement and asymmetric information, we show that optimal screening mechanism involves an ironing on the top, which departs from second best solution. Figure 3.4 illustrates the allocations in cases of first best, second best and limited enforcement, respectively. Therefore, the social surplus is further distorted by λ especially for higher



Figure 3.4: Optimal loan sizes across types

Note: Green line is the first best loan sizes; Yellow line is for second best. Red curve is loan sizes with limited enforcement.

types. Regarding the split of surplus, according to Corollary 3.1, the types who produce will not change in equilibrium. But the lowest type θ_* 's outside option becomes

$$v(\theta_*) = \lambda \hat{b}(\theta^*(\lambda))$$

For the rest of types, their value will be equal to

$$v(\theta;\lambda) = \lambda \hat{b}(\theta^*(\lambda)) + \int_{\theta_*}^{\theta} v'(\theta;\lambda) = \lambda b(\theta^*(\lambda)) + \int_{\theta_*}^{\theta} F_1(\theta,b(\theta;\lambda)d(\theta)) + \int_{\theta_*}^{\theta} F_1(\theta,b(\theta;\lambda)d(\theta))$$

For all $\theta \in [\theta_*, \theta^*(\lambda)]$. Since $b(\theta; \lambda) = \hat{b}(\theta)$ when $\theta \leq \theta^*(\theta)$, the only difference is the term $\lambda \hat{b}(\theta^*(\lambda))$.

3.3.4 Implementation

Although the mechanism can be simply implemented as a menu of price-quantity packages as typical non-linear pricing mechanism, it seems unrealistic to provide negative interest rate or compensation for external finance. And cheating for loan could be with risk of being legitimate. A valid and feasible implementation could be a program involves an upon entry subsidy λb^* plus the second best mechanism $\widehat{\mathcal{M}}$. If firm cheats but fails, both subsidy and borrowed fund has to be called back. The sum of the two component is exactly equivalent to the optimal mechanism.

3.4 A Numerical Example

In this section, we numerically simulates the model assuming that

$$F(\theta,b) = \theta b^{\alpha}$$

where $0 < \alpha < 1$. Also, assume $\theta \sim U[0, \overline{\theta}]$. Therefore, by

$$F_2(\theta, \hat{b}(\theta)) - 1 - \frac{1 - G(\theta)}{g(\theta)} F_{12}(\theta, \hat{b}(\theta)) = 0$$

it is easy to derive the second best allocation

$$\hat{b}(\boldsymbol{ heta}) = \left(\boldsymbol{lpha}(2\boldsymbol{ heta} - ar{m{ heta}})
ight)^{rac{1}{1-lpha}}$$

and

$$\widehat{MV}(\theta) = \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right) \cdot (2\theta - \bar{\theta})^{\frac{1}{1-\alpha}}$$

Also, $\widehat{MV}(\theta_*) = 0$ implies that

$$heta_*=rac{ar heta}{2}$$

When enforcement is limited, by Equation (3.5), θ^* is of the form

$$\theta^*(\lambda) = \left(1 - \frac{\lambda \alpha}{1 - \alpha}\right) \cdot \bar{\theta}$$

Suppose $\alpha = 0.5$, then $\theta^*(\lambda) = (1 - \lambda)\overline{\theta}$. As $\theta^*(\lambda) \ge \theta_*$ is necessary, bank operates only when $\lambda < 0.5$, otherwise no matter how she designs the mechanism, the loss induced will be

larger than profit earned. The following graphs are generated under the parameter setting

$$(\bar{\theta},\lambda,\alpha) = (10,0.25,0.5)$$

In optimal mechanism, $\theta_* = 0.5$, $\theta^* = 0.75$ and $b^* = 6.25$. Figure 3.5 shows the optimal loan sizes under mechanism. Figure 3.6 shows surplus created under each scenario. Besides



Figure 3.5: Simulated loan sizes in different scenarios

the net green area, limited enforcement leads to the loss of yellow area. Figure 3.7 illustrates how the surplus is split between the two parties. When $\theta < \theta_*$, there is no production. However, since lowest types can always cheat, their payoff is guaranteed by λb^* , which is also the bank's loss. Meanwhile, types $\theta \in [\theta_*, \theta_{**}]$ produce but the bank suffers loss, too. It implies that bank pays subsidy to the firms. Otherwise, those types would rather cheat than produce. Compared with losing λb^* , paying subsidy makes the bank better-off. Last comparative statics discuss how surplus, payoffs and profits are affected by different level

Note: Green line is the first best loan sizes; Yellow line is for second best. Red curve is loan sizes with limited enforcement with $\lambda = 0.25$.



Figure 3.6: Simulated surplus $w(\theta) = F(\theta, b(\theta)) - b(\theta)$ Note: Although types $\theta > \theta^*$ are pooling but higher type can create higher surplus.



Figure 3.7: Split of surplus between bank and firm

Note:

of enforcement λ . It is trivial both total surplus and bank's expected revenue are decreasing in λ . Regarding firm's profit, there are two forces. First, when λ becomes positive, the benefit earned from deduction in repayment obligation is larger. However, when λ continues increasing, total loss in surplus suppresses the firm's benefit from the outside option. Hence, firm's expected payoff starts decreasing. Until $\lambda = 0.5$, the enforcement will urge bank to shut down and no surplus will be produced.



Figure 3.8: Expected payoff under different λ

Note: Both bank's revenue and total surplus is decreasing in λ . There is a hump-shape relationship between firm's profit and enforcement level.

3.5 Conclusion

The paper studies the optimal non-linear pricing mechanism under limited enforcement. I show that when the principal cannot perfectly enforce a mechanism, agent's participation constraints are endogenous and correlated with the highest type's quantity. The force, in addition to that low productivity agent's output, is distorted by informational rent and induces high type's production to be also distorted by the reduction of loan sizes. By studying a continuous type mechanism design problem, I show that the optimal mechanism under limited enforcement will result in ironing for high productivity firms. The middle-type firms will produce the same as in the second best situation. Meanwhile, there is a positive measure of firms that will take advantage of enforcement limitation inevitably. I also show that as the limited enforcement does not change incentive compatibility constraints, the set of types that cheat in equilibrium is not affected by enforcement level.

Regarding surplus, both social surplus and bank's profit are negatively impacted by enforcement limitation. When enforcement is effective but not perfect, firms benefit from the threat of cheating. When enforcement becomes harder, the social surplus continues to shrink until banks choose to leave the market.

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