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OF $\pi\pi$ AND πK SCATTERING

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November 24, 1965

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ABSTRACT

A method suggested by Balázs is used to calculate the $\pi\pi$ and πK amplitudes in the resonance region. An input force corresponding to vector meson exchange reproduces the vector mesons, and also produces 2^+ mesons at roughly the masses of the $f^0(1250)$, $f^{0'}(1500)$, and $K^{**}(1405)$. Inclusion of an estimate of Pomeranchon exchange in the input improves the output slightly. Regge trajectories and residues are computed, and their intercepts agree approximately with estimates based on high-energy experiments.

I. INTRODUCTION

Balázs has proposed a method for constructing, from a given absorptive part in the crossed channel, a Schrödinger-equation potential that will reproduce the relativistic amplitude for scattering of spinless particles.[1] In this paper I present the results of some calculations of $(\pi\pi)$ and (πK) scattering, using an approximation to this Balázs method; these calculations reproduce many generally accepted features of resonance-energy $(\pi\pi)$ and (πK) scattering.

The Schrödinger-equation potential in this method is local but energy dependent. Writing it as

$$V(r,q^2) = \frac{-1}{2\pi\mu} \int_{t_0}^{\infty} dt \; v(t,q^2) e^{-r\sqrt{t}}/r ,$$

where q is the magnitude of the momentum in the center of mass of the s channel and μ is the reduced mass, Balázs proposed an iterative method for constructing $v(t,q^2)$ from the supposedly-known absorptive part A_t; the details of this construction are in reference 1. The scattering amplitude calculated from this Schrödinger equation would then be identical with the relativistic amplitude of which A_t was the absorptive part. In a problem with exchange potentials, this procedure is to be used for amplitudes of definite J-parity. For the Balázs method to be exact would require that (a) the amplitude obey elastic unitarity at the energy in question, (b) $v(t,q^2) \to 0$ as $t \to \infty$, and (c) A_t be known exactly.

The practical application of this derivation is to justify an approximation scheme, in which the lowest approximation consists of setting

$$v(t,q^2) = 2s^{-1/2} A_t^B(t,s) ,$$

where A_t^B is the Born approximation[2] to A_t, and s is the square of the energy in the center of mass. The scattering amplitude calculated from the Schrödinger equation will be unitary, and in this sense this approximation is similar to the N/D approach. It differs from the N/D approach in several interesting features: first, if A_t^B is constructed from elementary-particle exchange, there is no high-energy cutoff, even if the exchanged particles have spin. And second, although the usual N/D calculation takes the Born approximation to the left-hand cut, solving the Schrödinger equation means in effect doing the entire Mandelstam iteration.[3] Thus in the Balázs approximation, contributions from all orders of this iteration are considered, even though none but the first has relativistic s dependence.

In Section II, I describe the potentials that I have used, and in Sec. III present the results obtained.

II. CONSTRUCTION OF POTENTIALS

A. First ($\pi\pi$) Potential

My first calculation of ($\pi\pi$) scattering is an extension of an example given by Balázs.[1] He considered the force due to exchange of an elementary ρ, and obtained, in the small-width approximation,

$$A_t^B(t,s) = 4\pi\, \beta_{I1}\, (2\ell + 1)\, P_\ell(Z_t)\, \Gamma_{in}\, q_R^2\, \delta(t - m^2)\ , \qquad (1)$$

where β is the isotopic-spin crossing matrix, $\ell = 1$ the spin and m the mass of the exchanged ρ, $q_R^2 = \frac{1}{4} m^2 - m_\pi^2$, and Γ_{in} is the input reduced width. The potential corresponding to (1) is

$$V(r,q^2) = -24\, \beta_{I1}\, \Gamma_{in}\, s^{-1/2}(s + 2q_R^2)r^{-1}\, e^{-mr}\ . \qquad (2)$$

Balázs looked for the ρ resonance in the $\ell = 1$ amplitude obtained from (2), and found an approximately self-consistent solution with $m = 4.2\, m_\pi$ and $\Gamma = 0.47$ (a ρ of mass 750 MeV and width 100 MeV has $m = 5.3\, m_\pi$ and $\Gamma = 0.17$).

However, it turns out that approximate self consistency is not sufficient to determine the value of these parameters, as within 10% there are many self-consistent solutions. In what is referred to below as calculation A, I use the potential given by (2), with the input ρ mass fixed at 750 MeV. I then adjust the input width to obtain the output ρ mass at the same energy; this solution is also self consistent, and has $\Gamma = 0.46$ (corresponding to a width of 270 MeV). This calculation has no cutoff and no strip width; with Γ_{in} determined, there are no other free parameters, and the entire (low-energy) $(\pi\pi)$ amplitude in all three isotopic-spin states can be calculated.

B. Second $(\pi\pi)$ Potential

The results of calculation A, which are described in Sec. III, resembled experimental results sufficiently to encourage me to try to improve the potential by including some estimate of the contribution due to the exchange of the Pomeranchuk trajectory.

Chew[4] has discussed the generalized potential arising from exchange of a Regge trajectory in the t channel, where the generalized potential $V^s(t,s)$ is

$$V^s(t,s) = \frac{1}{\pi} \int_{t_0}^{\infty} \frac{dt' \, A_t^B(t',s)}{t' - t} . \tag{3}$$

He writes V^s as a sum of partial waves in the t channel (even partial waves for trajectories of positive signature) and argues that only the lowest need be kept. He gives approximate expressions for the two lowest terms of the Pomeranchuk potential: the $J = 2$ part looks like the exchange of an elementary f^0 multiplied by a "form factor" in t; the $J = 0$ part, even though it has no associated particle, is more important, and is repulsive. The exact value of the $J = 0$ part depends on a strip parameter, which I have set equal to $200\ m_\pi^2$.

I cannot make direct use of Chew's estimates, since they are intended for only small values of t, and hence do not accurately define A_t^B where I need it; instead, for each of these two partial waves I have replaced A_t^B by a delta function, chosen so that the V^s calculated from Eq. (3) has the same value and derivative at $t = 0$ as do Chew's estimates. It then turns out that the $J = 2$ term is almost completely negligible; the $J = 0$ term is substantially smaller, but has a slightly larger range, than the ρ potential used in calculation A. The complete potential is the sum of the ρ and the Pomeranchuk potentials:[5]

$$V(r,q^2) = V^{\rho}(r,q^2) + V^{P}_{J=0}(r,q^2) + V^{P}_{J=2}(r,q^2) ,$$

where in units in which $m_\pi^{\ 2} = 1$,

$$V^{\rho}(r,q^2) = -24\, \beta_{I1}\, \Gamma_{in}\, s^{-1/2}\, (s + 12)\, e^{-5.3r}\, r^{-1}$$

$$V^{P}_{J=0}(r,q^2) = +180\, \beta_{I0}\, s^{-1/2}\, e^{-5.0r}\, r^{-1}$$

$$V^{P}_{J=2}(r,q^2) = -\beta_{I0}\, s^{-1/2}(12 + 0.94s + 0.013s^2)e^{-5.1r}\, r^{-1} . \tag{4}$$

I again adjust Γ_{in} to get the output ρ mass at 750 MeV, and find $\Gamma_{in} = 0.56$. The results obtained from the potential given by (4) I call calculation B.

Surely the expressions given in (4) have at best a tenuous connection with the correct Pomeranchuk potential, whatever that may be. However, if we accept Chew's arguments, these expressions should be reasonable qualitative estimates, and since they appear as fairly small corrections to the much better established ρ potential, might be expected to improve the results of calculation A . As shown in Sec. III, this apparently is the case.

C. (πK) Potential

The last calculation, which I call calculation C, is of the (πK) scattering amplitude. I have not attempted here to put in the Pomeranchuk potential, but consider only vector meson exchange, in analogy with calculation A. There are now two exchange forces, ρ exchange in the t channel and $K^*(891)$ exchange in the u channel. With the assumption that the couplings $\gamma_{\rho\pi\pi}$ and $\gamma_{\rho K\bar{K}}$ are related to $\gamma_{K^*K\pi}$ as predicted by SU_3, and with the physical values for the input masses used, the potential appropriate for $I = \frac{1}{2}$ and angular momentum ℓ is given by

$$V(r,q^2) = V^{\rho}(r,q^2) + (-1)^{\ell} V^{K^*}(r,q^2) ,$$

$$V^{\rho}(r,q^2) = -32\,\Gamma'_{in}\; s^{-1/2}\,(s + 0.5)\; e^{-5.3r}\, r^{-1}$$

$$V^{K^*}(r,q^2) = -8\,\Gamma'_{in}\; s^{-1/2}\,(s + 12)\; e^{-6.3r}\, r^{-1} . \qquad (5)$$

Here Γ'_{in}, the reduced K^* width, is determined by requiring the output mass of the K^* to be 891 MeV; this requirement gives $\Gamma'_{in} = 0.57$, corresponding to a width of 153 MeV.

Having constructed the potentials for the three calculations, I solved the Schrödinger equation numerically, to find the scattering amplitude and the Regge parameters at physical values of q^2. The computations were done on an IBM 7094 computer, in part with a program written by Burke and Tate.[6]

III. RESULTS

In both calculations A and B, for $I = 0$ there are two Regge trajectories above $\ell = 0$ at theshold; these I identify with the P and P′ trajectories, and the associated particles at $\ell = 2$ with the

$f^{o}(1250)$ and the $f^{o'}(1500)$.[7] For $I = 1$ there is one such trajectory, which I identify with the ρ ; for $I = 2$ the force is repulsive. The real parts of the trajectories continue to rise as the energy is increased, at least until $s = 400\ m_\pi^2$, and in fact do eventually go through higher physical values of ℓ . However, one of the assumptions of the Balázs method is that the amplitude is elastic at the energy in question, and so it must lose its validity as the energy approaches the strip width. As none of these "recurrences" occur below $s = 300\ m_\pi^2$, they could be considered spurious. The masses and the reduced widths of the three output resonances are listed in Table I; the output reduced widths are computed by

$$\Gamma = \left[\frac{\sqrt{s}}{8q^{2\ell+1}} \left(\frac{d\delta}{ds}\right)^{-1} \right]_{s=s_{resonance}}$$

and have units of $(\mathrm{GeV})^{2-2\ell}$.

A calculation of s-wave scattering by this method is less reliable than a calculation of higher partial waves, for the usual reason that the s wave depends more strongly on the shorter-range parts of the potential; --but it might be interesting anyway. The results of this calculation are the strange-looking phase shifts shown in Fig. 1; the fact that these phase shifts wander through 90^{o} has no significance. A large but not necessarily resonant s-wave amplitude near the ρ mass has been previously suspected from the asymmetry in ρ^{o} decay.[8] The s-wave scattering length is $-0.08\ m_\pi^{-1}$ in calculation A and $-0.18\ m_\pi^{-1}$ in calculation B; the negative sign would be expected from the existence of trajectories above $\ell = 0$ at threshold.

Because of the factor $s^{-1/2}$ in the definition of the potential, the Balazs approximation must break down near $s = 0$. Nevertheless it would be interesting to be able to compute the values of the Regge parameters at this point, for then they could be compared with the values obtained from experiments at high energy in the crossed channel. The procedure I have adopted is to calculate the Regge parameters α and $\ell n\, \gamma \equiv \ell n(\beta/q^{2\alpha})$ above threshold, and attempt a straight-line extrapolation to $s = 0$. In most cases this extrapolation seemed possible, and the results for the $(\pi\pi)$ amplitude are given in Table II.

The width of the K^* in calculation C is very nearly equal to the input width, which is about 2.5 times the experimental value. There is one other (πK) trajectory, with the quantum numbers of the $K^{**}(1405)$. The results for these two trajectories are presented in Table III. For the $I = \frac{3}{2}$ (πK) amplitude the force is repulsive.

As can be seen from the tables, the general features of the experimental situation are reproduced quite well. Some of the closer agreements with experiment may well be fortuitous, but the overall pattern, especially the appearance of the 2^+ mesons, could not be. The most glaring discrepancy of my results with experimental results is the fact that the input and output particle widths are too large, but this seems to be a common feature of most dynamical calculations.[9] The results of calculation B seem somewhat better than those of calculation A, although most of the differences are small. The output ρ width is improved substantially, destroying the consistency between input and output.

It is interesting to compare the above calculations, especially calculation A, with the N/D calculation of Collins and Teplitz.[10] They used an input ρ with a width of 0.43, which is about the same as was used in calculation A, but their output ρ trajectory did not quite make it to $\ell = 1$, and no trajectory rose above $\ell = 1.5$. Thus the effective force in the present calculation is much stronger than in the N/D calculation, even though the input forces are similar. It seems reasonable that this is because the method used here does include contributions to the force from higher terms in the Mandelstam iteration. If this conjecture be correct, then the results presented here indicate that a calculation that actually performs the iteration might be expected to be very successful.

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FOOTNOTES AND REFERENCES

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

1. L. A. P. Balázs, Phys. Rev. 137, B1510 (1965).

2. That is, A_t^B represents the contribution of a few elastic partial waves in the t channel, which in the strip approximation (see reference 4) is the Born approximation to A_t.

3. R. Blankenbecler, M. L. Goldberger, N. N. Khuri, and S. B. Trieman, Ann. Phys. 10, 62 (1960).

4. G. F. Chew, Supplment of the Prog. Theor. Phys. (Kyoto), extra number, p. 118 (1965); also Pomeranchuk Repulsion and Resonance Narrowing, Phys. Rev. (to be published).

5. P. D. B. Collins (private communication) assures me that the effect of a Reggeized ρ potential is about the same as that of the elementary ρ potential, which is used here.

6. P. G. Burke and C. Tate, Fortran Program TREGGE, Lawrence Radiation Laboratory Report UCRL-10384, July 1962 (unpublished).

7. V. E. Barnes et al., Phys. Rev. Letters 15, 322 (1965).

8. M. M. Islam and R. Piñon, Phys. Rev. Letters 12, 310 (1964).

9. For example, F. Zachariasen, Phys. Rev. Letters 7, 112 (1961); and J. R. Fulco, G. L. Shaw, and D. Y. Wong, Phys. Rev. 137, B1242 (1965).

10. P. D. B. Collins and V. L. Teplitz, Phys. Rev. 140, B663 (1965).

Table I. Results for the $(\pi\pi)$ amplitude. The force input to calculation A is ρ exchange, and to calculation B is ρ exchange and Pomeranchon exchange.

	ρ_{mass} (MeV)	Γ_ρ	f^0_{mass} (MeV)	Γ_{f^0}	$f^{0'}_{mass}$ (MeV)	$\Gamma_{f^{0'}}$
Calculation A	750	0.50	1070	0.50	1900	0.55
Calculation B	750	0.29	920	0.35	1435	0.95
Experiment	750[a]	0.17(100MeV)[a]	1250[b]	0.25(100MeV)[c]	1500[c]	

a. Ambiguities in the experimental values are not important to this paper; these numbers for the ρ comes from C. Alff et al., Phys. Rev. Letters 9, 322 (1962).

b. W. Selove et al., Phys. Rev. Letters 9, 272 (1962).

c. See reference 7.

Table II. Values of the Regge parameters of the $(\pi\pi)$ amplitude at $s = 0$.

	α_ρ	γ_ρ [a]	α_P	γ_P	$\alpha_{P'}$	$\gamma_{P'}$
Calculation A	0.7	b	1.3	b	0.7	b
Calculation B	0.45	0.05	1.3	0.0036	0.65	0.09
Other estimates	0.54[c]	0.026[d]	1.0[c]	0.006[d]	0.50[c]	0.065[d]

a. γ is defined by $\gamma = \beta/q^{2\alpha}$, where q is in units of m_π.

b. I was unable to make a reliable extrapolation from the physical region.

c. R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965).

d. These values come from an analysis of high-energy πN, NN, and $\bar{N}N$ scattering, and use of the factorization theorem; see R. J. N. Phillips and W. Rarita, Phys. Rev. Letters 14, 502 (1965), and the appendix to Heinz J. Rothe, Evaluation of the I = 0 Pion-Pion Scattering Length Using a Forward Scattering Dispersion Relation, Phys. Rev. (to be published).

Table III. Results for the (πK) amplitude.

	K^{*}_{mass} (MeV)	Γ_{K^*}	α_{K^*}	γ_{K^*} [a]	K^{**}_{mass} (MeV)	$\Gamma_{K^{**}}$	$\alpha_{K^{**}}$	$\gamma_{K^{**}}$
Calculation C	891	0.55	0.4	0.14	1265	0.16	0.75	0.02
Experiment	891[b]	0.22[b] (50MeV)			1405[b]	0.12[b] (95MeV)		

a. See footnote a, Table II.

b. A. H. Rosenfeld et al., Rev. Mod. Phys. 37, 633 (1965).

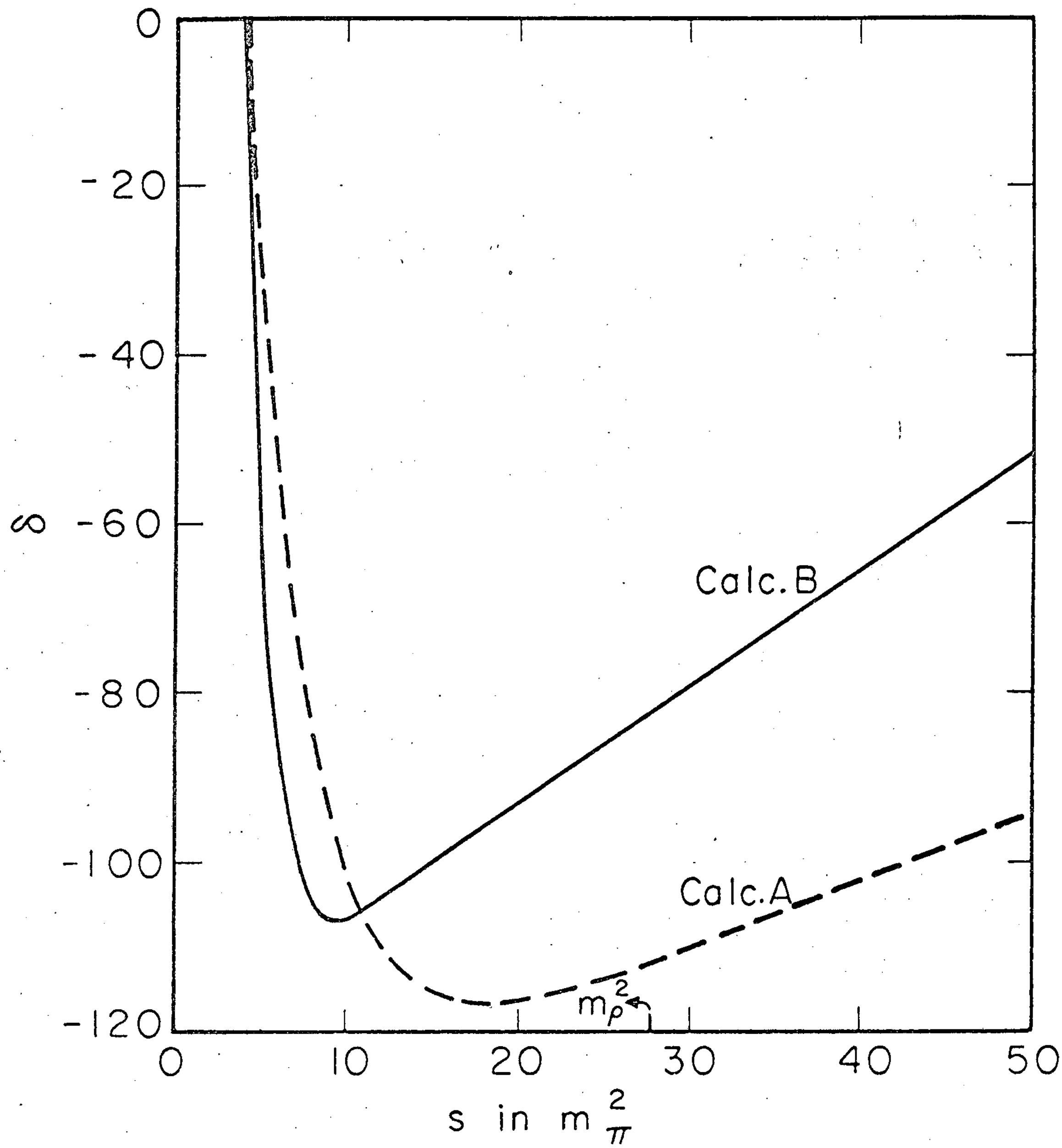

Fig. 1

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