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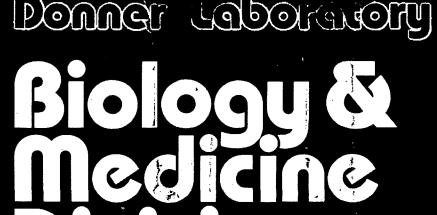## UNIVERSITY OF CALIFORNIA

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Biology &
Medicine
Division

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A NEW FAST ALGORITHM FOR THE EVALUATION OF REGIONS OF INTEREST AND STATISTICAL UNCERTAINTY IN COMPUTED TOMOGRAPHY

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Abstract. A new algorithm for region of interest evaluation in computed tomography has been developed. The new algorithm replaces the conventional technique of repeated image reconstructions with convolution of the projected regions and multiple vector inner products with the raw tomographic data sets. Quantitation of regions of interest is made without the need for reconstruction of tomographic images. The computational advantage of the new algorithm over conventional methods is between a factor of 20 and a factor of 500 for typical applications encountered in medical science studies. The greatest benefit of the new algorithm (and the motivation for its development) is the ease with which the statistical uncertainty of the result is computed. The entire covariance matrix for the evaluation of regions of interest can be calculated with relatively few operations.

1. Introduction

Computed tomography has gained recognition in the last decade as a valuable tool for the noninvasive visualization of the interior of three-dimensional objects. Examples of its use in medicine include CAT (computer assisted tomography) scanning to image the distribution of x-ray linear attenuation coefficients, emission computed tomography to image radiotracer concentrations, and NMR (nuclear magnetic resonance) imaging of nuclear spin densities and relaxation times.

Region of interest evaluation in computed tomography is a technique used to quantitate the tomographic imaging process throughout a volume of particular significance to the investigator. It reduces statistical uncertainty and allows the investigator to analyze the properties of the region of interest at different times and under different physical conditions. Conventional region of interest evaluation is a time consuming process. It requires a tomographic reconstruction and the summation of the contents of the picture elements (pixels) within the significant volume.

In this paper we describe a new algorithm which requires no reconstruction and evaluates the significant volume directly. The new algorithm also gives the statistical uncertainty of the result and the covariance matrix if more than one region is evaluated.

2. Conventional Region of Interest Evaluation

Region of interest evaluation of computed tomographic data is generally performed by reconstructing a transaxial image of the distribution of imaging agent. Region of interest evaluation consists of summing reconstructed pixel values over predetermined regions of the image.

Of the many algorithms for the reconstruction of tomographic images, the most widely used is the convolution method (Bracewell and Riddle 1967, Ramachandran and Lakshminarayanan 1971). In the convolution method, data from each projection angle are filtered (convolved with a particular kernel) and backprojected to form the image. The projection data can arise from either parallel beam or one of several fan beam geometries (Huesman et al. 1977). The algorithm is summarized in equation (1)

$$B_{ij} = \sum_{km} F_{ij}^{km} \sum_{\ell} C_k^{\ell} p_{\ell m} = \sum_{km} F_{ij}^{km} q_{km} , \qquad (1)$$

where $p_{\ell m}$ are the projection data at angle m and bin ℓ, C_k^{ℓ} is the convolution kernel, q_{km} are the convolved data, F_{ij}^{km} are the backprojection factors, and B_{ij} are the reconstructed values for pixel (i,j). The sum over ℓ is convolution in configuration space, and C_k^{ℓ} corresponds to a ramp or modified ramp filter in Fourier space for parallel beam geometry. The sum over k and m is backprojection, and F_{ij}^{km} corresponds to the projection of pixel (i,j) into projection bin k at angle m. Linear interpolation is usually used, and F_{ij}^{km} is non-zero only at the two

nearest bins where the center of pixel (i,j) projects at angle m. $F_{ij}{}^{km}$ is a very sparse matrix and is calculated as needed rather than being precomputed and saved.

The region of interest evaluation is summarized in equation (2)

$$R_\alpha = \sum_{ij \epsilon \alpha} B_{ij} \; , \tag{2}$$

where α refers to a set of (usually contiguous) pixels within the reconstructed image. R_α is the sum of reconstructed pixel values, B_{ij}, over the region specified by α.

Statistical uncertainty for R_α is difficult to calculate in this formulation. While uncertainty for the individual pixel values, B_{ij}, can be computed in a straightforward manner (Huesman et al. 1977) the correlation between neighboring pixel values is high. We know only of approximate formulas for the statistical uncertainty of R_α (Budinger et al. 1978).

3. New Algorithm for Region of Interest Evaluation

We have developed a new algorithm for the evaluation of regions of interest which obviates the need for reconstruction of the image and calculates the final solution directly. The new formulation simplifies the problem rendering the statistical uncertainty trivial to calculate. The correlations between different regions are also straightforward to compute.

We begin by substituting equation (1) into equation (2) to get the full equation for the number of events in the region of interest:

$$R_\alpha = \sum_{ij \epsilon \alpha} \sum_{km} F_{ij}^{km} \sum_{\ell} C_k^\ell \, p_{\ell m} \; . \tag{3}$$

Interchanging the order of summation and rearranging leads to

$$R_\alpha = \sum_{\ell m} p_{\ell m} \sum_k C_k^\ell \sum_{ij \epsilon \alpha} F_{ij}^{km} = \sum_{\ell m} p_{\ell m} \sum_k C_k^\ell \, g_\alpha^{km} \; , \tag{4}$$

where

$$g_\alpha^{km} = \sum_{ij \epsilon \alpha} F_{ij}^{km} \; . \tag{5}$$

Notice that $g_\alpha{}^{km}$ is the sum of backprojection factors of region α for projection bin k and angle m. The elements of g_α are formed by the summation of pixels inside the region of interest along rays corresponding to each projection bin for each angle as shown in figure 1. In other words, g_α corresponds to the projections of the region of interest

containing unit weight per pixel.

Figure 1. A region of interest is evaluated by projecting the uniformly weighted region at each angle. The projected region is convolved, and a vector inner product is formed with the raw tomographic data set.

Further simplification is obtained by defining the vector h_α as

$$h_\alpha^{\ell m} = \sum_k C_k^\ell \, g_\alpha^{km} \, , \tag{6}$$

so that

$$R_\alpha = \sum_{\ell m} h_\alpha^{\ell m} \, p_{\ell m} \, . \tag{7}$$

Since the convolution kernel, $C_k{}^\ell$, is symmetric in k and ℓ, h_α is the convolution of g_α as shown in figure 1. $C_k{}^\ell$ is independent of the

tomographic data, and g_α depends only on the region of interest selected for analysis, so that h_α can be precomputed. This results in a significant computational saving because regions are usually evaluated for more than one data set.

Knowledge of the boundaries of the region is all that is needed to perform the projection operation. Therefore, concept of a pixel can be discarded in the new formulation, since the description of a region by a set of pixels is not necessary. In what follows, we retain the pixel approach and notation in order to maintain correspondence with the conventional method of data analysis from regions of interest after reconstruction.

4. Covariance Matrix for Regions of Interest

Because of the particularly simple form of equation (7), we can write the covariance matrix for regions of interest as follows:

$$\text{covar}\{R_\alpha, R_\beta\} = \sum_{\ell'm'} \sum_{\ell''m''} h_\alpha^{\ell'm'} h_\beta^{\ell''m''} \text{covar}\{p_{\ell'm'}, p_{\ell''m''}\} \ . \tag{8}$$

Since the projection data are generally statistically independent, the covariance matrix of the data is diagonal, and equation (8) simplifies to

$$\text{covar}\{R_\alpha, R_\beta\} = \sum_{\ell m} h_\alpha^{\ell m} h_\beta^{\ell m} \text{var}\{p_{\ell m}\} \ , \tag{9}$$

where $\text{var}\{p_{\ell m}\}$ are the diagonal elements of the covariance matrix of the measured data, $\text{covar}\{p_{\ell'm'}, p_{\ell''m''}\}$.

The diagonal elements of the covariance matrix of equation (9) are the variances of the evaluated regions and are given by

$$\text{var}\{R_\alpha\} = \sum_{\ell m} [h_\alpha^{\ell m}]^2 \text{var}\{p_{\ell m}\} \ . \tag{10}$$

The calculation of the variance of an evaluated region or calculation of the entire covariance matrix is particularly simple and contains relatively few operations.

5. Application of the New Algorithm to Dynamic Data Analysis

Dynamic emission computed tomography is a good example of a field which utilizes region of interest evaluation. For analysis of dynamic emission computed tomographic data, regions of interest are determined by inspection of a high-statistics emission image or a corresponding transmission image. Regions are drawn to include areas of physiological significance in order to determine the uptake and clearance of radiotracer. If possible, regions are also drawn over vascular areas to determine the amount

of radiotracer delivered or made available to the tissue.

The time-course of the residue function (amount of radiotracer in the tissue) is evaluated by calculating the activity in the regions for multiple tomographic data sets acquired sequentially after the radiotracer is introduced. The input function (amount of radiotracer available to the tissue) is evaluated similarly, if possible, or is determined from serial blood samples. With measurements of the residue and input functions (both as functions of time) model parameters can be estimated. The model parameters characterize blood flow and/or physiologic function of the tissue within the region of interest.

The model parameters are most efficiently estimated if the statistical uncertainties and correlations of the residue and input functions are known. Knowledge of the statistical nature of these functions also allows us to calculate the covariance matrix for the model parameters, thus enabling us to quantify our confidence in the estimates of model parameters reflecting physiologic function of the tissue.

To this end, a computationally efficient region of interest evaluation and uncertainty determination is necessary. The new algorithm presented here satisfies this need. After regions of interest have been determined, the computations indicated by equations (5) and (6) are performed, and h_α for each region are stored for later use. Regions of interest and the covariance matrices are then evaluated for each time point as indicated by equations (7) and (9), respectively.

6. Computational Advantage of the New Algorithm

We shall delineate the number of floating point operations (multiplication and addition) required for the new algorithm and compare it to the number of operations required for conventional region of interest evaluation. The derivation of the total number of floating point operations for the new algorithm, the conventional method, and an accelerated conventional algorithm are presented in the appendix. We shall assume that parallel beam geometry and linear interpolation are used. We also assume that the projection data are statistically uncorrelated. The results can easily be extended to fan beam configurations.

For dynamic data analysis, region of interest evaluation is performed for a number of tomographic data sets using the same regions of interest. It is therefore interesting to evaluate the number of operations required to analyze all of these sequential data sets. If the number of data sets (time points) is denoted by T, then the number of operations required for the new algorithm to project and convolve the regions and to form the vector inner product of equation (7) is

$$\#O_n = M \sum_\alpha \left[10N_\alpha + \frac{2K^2}{\sqrt{N}} \sqrt{N_\alpha} + 2TK \right] , \qquad (11)$$

where N_α is the number of pixels in each region of interest, N is the

number of pixels in the reconstruction region, M is the number of projection angles, and K is the number of projection bins for each angle. The number of operations required for the conventional method to reconstruct the image is

$$\#0_c = TM \left[11N + 2K^2\right] ,\tag{12}$$

and the number of operations necessary for the accelerated conventional method to reconstruct image pixels only within the regions of interest is

$$\#0_a = TM \sum_\alpha \left[11N_\alpha + \frac{2K^2}{\sqrt{N}} \sqrt{N_\alpha}\right] .\tag{13}$$

Equations (12) and (13) also assume T data sets and multiple regions of interest and are to be compared to equation (11). The actual region of interest evaluation [equation (2)] is not included in equations (12) and (13) because the number of operations is small compared to the reconstruction. Equations (11) through (13) are derived in the appendix.

As a representative example to compare the new algorithm to the conventional method we take 10,000 pixels (100 x 100 array), 100 angles, 100 bins per angle, and 100 pixels in each regions of interest. Table 1. compares equations (11) through (13) for the combinations of T=20, 40, and 60 time points and 1, 5, 10, and 20 regions of interest.

Table 1. Number of Floating Point Operations (x 10^5) for Region of Interest Evaluation[†]

Number of Regions	Convolved Projected Regions eqn (11)			Reconstructed Image eqn (12)			Reconstructed Regions eqn (13)		
	T=20	T=40	T=60	T=20	T=40	T=60	T=20	T=40	T=60
1	7	11	15	2600	5200	7800	62	124	186
5	35	55	75	2600	5200	7800	310	620	930
10	70	110	150	2600	5200	7800	620	1240	1860
20	140	220	300	2600	5200	7800	1240	2480	3720

† N = 10,000 pixels
  M = 100 angles
  K = 100 bins per angle
  N_α = 100 pixels in each region of interest

There is a substantial computational saving with the new algorithm which increases with larger numbers of tomographic data sets and fewer regions. The computational advantage for this example varies between 19 (for 20 regions and 20 data sets) and 520 (for 1 region and 60 data sets). What is perhaps more important, the statistical uncertainties for the resulting region of interest evaluation can be calculated with 3×10^4 floating point operations for each time point and each region in this example. The remainder of the covariance matrix can also be calculated with 3×10^4 operations per matrix element.

7. Conclusions

A new fast algorithm for the evaluation of regions of interest in computed tomography has been developed. The conventional technique of repeated image reconstruction has been replaced by convolution of the projected regions and multiple vector inner products with the raw tomographic data sets. Reconstruction of the tomographic images is unnecessary as is the concept of pixels, since the projection operation can be performed once the boundaries of the region are known. The new algorithm will also find application in higher dimensional x-ray and NMR tomography, as similar computational efficiency will be found in the evaluation of three-dimensional regions of interest.

The computational saving of the new algorithm is substantial. There is a 20 to 500 fold reduction in the number of floating point operations for typical applications encountered in medical science studies. The nature of the computations imbedded in the new algorithm make it readily amenable to hardwired implementation. Since the region of interest evaluation and the calculation of statistical uncertainty are both vector operations, implementation of the new algorithm on an array processor is also a logical choice.

The greatest benefit of the new algorithm (and the motivation for its development) is the computation of statistical uncertainty for the region of interest quantitation. This results in more efficient estimation of model parameters in subsequent analysis of physiologic function from the dynamic data.

Acknowledgements

Appendix

Derivation of the number of operations required for region of interest evaluation

In this appendix we examine the number of floating point operations required for region of interest evaluation using the new algorithm. Computational details of the new algorithm for region of interest evaluation can be separated into three steps: projection of the region [equation (5)]; convolution of the projected region [equation (6)]; and region of interest evaluation [equation (7)]. We shall delineate the number of operations (multiplication and addition) required for each of these steps and compare them to the number of operations required for conventional region of interest evaluation. We shall assume that parallel beam geometry and linear interpolation are used. We also assume that the projection data are statistically uncorrelated. The results can easily be extended to fan beam configurations.

Projection of a region of interest consists of determining the point of projection of each pixel onto the projection arrays at each angle:

$$z = D (j \cos\theta_m - i \sin\theta_m) + z_{om} ,$$ (A1)

where z specifies the projection point in units of projection bin width; D is the size of a pixel in units of projection bin width; i and j refer to horizontal and vertical pixel location, respectively; m is the angle index; θ_m is the projection angle; and z_{om} is an offset. The quantities $\sin\theta_m$, $\cos\theta_m$, and z_{om} are calculated once for each angle. Letting k be the integer part of z and using linear interpolation we get

$$F_{ij}^{km} = 1 + k - z ,$$ (A2)

$$F_{ij}^{k+1,m} = z - k ,$$ (A3)

and all other F_{ij} for this choice of i, j, and m are zero. Therefore, equation (5) requires three multiplications and seven additions per pixel per angle for step #1, or

$$\#M_1 = 3MN_\alpha ,$$ (A4)

$$\#A_1 = 7MN_\alpha ,$$ (A5)

where M is the number of projection angles, and N_α is the number of pixels in region α.

Convolution of a projected region consists of a standard digital convolution, but the projected region is of limited extent, and the average number of projected bins can be estimated to be approximately equal to D times the square root of the number of pixels in the region. D is about equal to $K/\sqrt{N}$, where K is the number of projection bins for each

angle and N is the total number of pixels in the reconstruction region, so that

$$\#M_2 = MKD\sqrt{N_\alpha} = \frac{MK^2}{\sqrt{N}} \sqrt{N_\alpha} \, , \tag{A6}$$

$$\#A_2 = MKD\sqrt{N_\alpha} = \frac{MK^2}{\sqrt{N}} \sqrt{N_\alpha} \, . \tag{A7}$$

Region of interest evaluation is completed by computing the vector inner product given in equation (7), and the number of operations is

$$\#M_3 = MK \, , \tag{A8}$$

$$\#A_3 = MK \, . \tag{A9}$$

For dynamic data analysis, region of interest evaluation is performed for a number of tomographic data sets using the same regions of interest. We therefore evaluate the number of operations required to analyze all of these sequential data sets. If the number of data sets (time points) is denoted by T, then the number of operations required by the new algorithm is

$$\#M_n = M \sum_\alpha \left[3N_\alpha + \frac{K^2}{\sqrt{N}} \sqrt{N_\alpha} + TK \right] \, , \tag{A10}$$

$$\#A_n = M \sum_\alpha \left[7N_\alpha + \frac{K^2}{\sqrt{N}} \sqrt{N_\alpha} + TK \right] \, . \tag{A11}$$

Adding equations (A10) and (A11) gives the total number of floating point operations for the new algorithm:

$$\#O_n = M \sum_\alpha \left[10N_\alpha + \frac{2K^2}{\sqrt{N}} \sqrt{N_\alpha} + 2TK \right] \, . \tag{11}$$

Notice that steps #1 and #2 need be performed only once for each region of interest, and step #3 is performed for each region and each time point.

The number of operations required for region of interest evaluation using the conventional method can be evaluated by inspection of equation (1). Convolution requires MK^2 multiplications and MK^2 additions. Evaluation of the backprojection factors [equations (A1) through (A3)] and backprojection require 4MN multiplications and 7MN additions. Therefore the number of operations required for the conventional method is

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$$\#M_c = TM\ [4N + K^2]\ ,\qquad\qquad\qquad\qquad (A12)$$

$$\#A_c = TM\ [7N + K^2]\ .\qquad\qquad\qquad\qquad (A13)$$

Adding equations (A12) and (A13) gives the total number of floating point operations for the conventional algorithm:

$$\#0_c = TM\ [11N + 2K^2]\ .\qquad\qquad\qquad\qquad (12)$$

For the conventional method the entire image is reconstructed, even though the regions of interest generally occupy only a small part of the image. If only pixels within the regions of interest are reconstructed, only those need to be backprojected, and therefore only a limited number of bins of the convolved projections need to be calculated. Taking these factors into account, the resulting number of operations necessary for region of interest evaluation using an accelerated conventional method is

$$\#M_a = TM \sum_{\alpha} [4N_\alpha + \frac{K^2}{\sqrt{N}} \sqrt{N_\alpha}]\ ,\qquad\qquad (A14)$$

$$\#A_a = TM \sum_{\alpha} [7N_\alpha + \frac{K^2}{\sqrt{N}} \sqrt{N_\alpha}]\ .\qquad\qquad (A15)$$

Adding equations (A14) and (A15) gives the total number of floating point operations for the accelerated conventional algorithm:

$$\#0_a = TM \sum_{\alpha} [11N_\alpha + \frac{2K^2}{\sqrt{N}} \sqrt{N_\alpha}]\ .\qquad\qquad (13)$$

The actual region of interest evaluation [equation (2)] is not included in equations (12) and (13) because the number of operations is small compared to the reconstruction.

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