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Technical note with Supporting Results for Outlier Accommodation State Estimation: A Risk-Averse Performance-Specified Approach

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This tech note extends the discussion of the numerical results in [1]. The main article should be read first. This technical note includes numerical results that could not fit within the journal page constraints. It presents the statistics of the vertical error for the nonlinear (INS) model and discusses the interplay between the error, risk, and GDOP metrics for portions of an experiment using the linear (PVA) model. The error, risk and PVA metrics are defined in Sections VIII-C and VIII-E of [1].

Each table and figure herein considers five algorithms, as summarized in Section VIII-B of [1]. Four of the algorithms are the NP-(E)KF with four different values of the decision parameter γ. The final algorithm is the RAPS approach.

I. Nonlinear (INS) Model: vertical Error

The vertical position accuracy for the nonlinear (INS aided) system is summarized in Table I. The table is divided into two sections, each containing five rows. The top section presents the statistics for the experiment with $\mu = 2$ and is labeled with NP-EKF1 and RAPS1. The bottom section presents the statistics for the experiment with $\mu = 7$ and is labeled with NP-EKF2 and RAPS2.

RAPS performance is almost the same for both $\mu = 2$ and $\mu = 7$ because it chooses the minimum risk selection vector b at each time from all feasible b vectors. As the magnitude of generated outliers increases, they become easier for the threshold test to detect; therefore, the NP-EKF becomes increasingly successful at removing outliers, which improves its performance. However, its performance varies significantly with respect to outlier magnitude μ and decision threshold γ.

TABLE I: GNSS-INS Vertical Performance Statistics For $\mu = 2$ (top) and $\mu = 7$ (bottom).

Methods	Mean of error (m)	Std. of error (m)	Error < 2 m	Maximum error (m)
NP-EKF1 $\gamma = 5$	1.89	0.52	0.62	3.46
NP-EKF1 $\gamma = 4$	1.92	0.51	0.61	3.46
NP-EKF1 $\gamma = 3$	1.42	0.61	0.83	2.90
NP-EKF1 $\gamma = 2$	0.59	0.47	1	1.85
RAPS1	0.37	0.39	1	1.63
NP-EKF2 $\gamma = 5$	0.39	0.36	1	1.86
NP-EKF2 $\gamma = 4$	0.39	0.36	1	1.86
NP-EKF2 $\gamma = 3$	0.38	0.35	1	1.74
NP-EKF2 $\gamma = 2$	0.36	0.37	1	1.70
RAPS2	0.39	0.40	1	1.82

It is also important to note that the vertical position error behaves well, even though the performance specification included in the RAPS optimization process only constrained the horizontal portion of the position error (see eqn. (44) in [1]). This is because every satellite line-of-sight vector (i.e., h_k^s defined in Section VI.A of [1]) has a non-zero element corresponding to the vertical position error.

II. Linear (PVA) Model Performance Comparison

Fig. 1 in [1] presents a graph indicative of the performance of each estimator for the PVA model as a function of the decision parameter γ, when the value of μ is changed from 0 to 20. That graph was created using Monte Carlo analysis averaging over 10 trials. Fig. 1 herein presents graphs of the horizontal error (top), risk R_k (middle) and GDOP (bottom) for a portion of a single experiment using the GNSS data with the linear PVA model. All subfigures include a curve for each of five mentioned algorithms in Section VIII-A of [1]. At some times, some curves may not be visible due to their overlapping.

Tables IIa and IIb provide vertical and horizontal positioning accuracy statistics, respectively, for the five linear estimation algorithms using the PVA model. In both Tables, the top section is for $\mu = 6$ and the bottom sections is for $\mu = 17$.

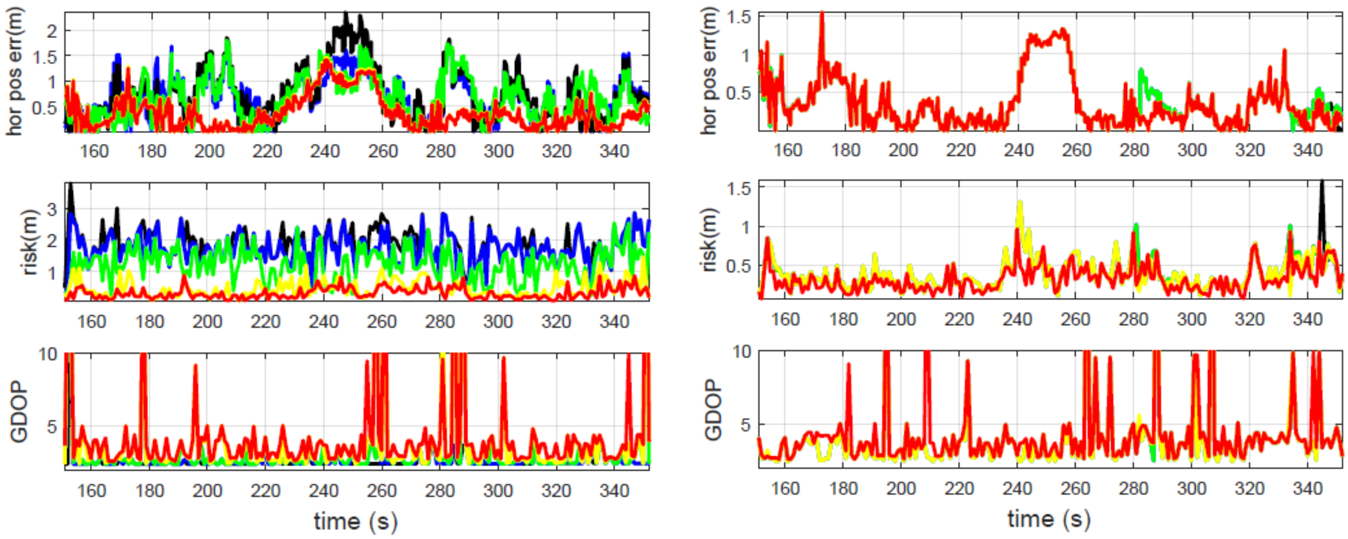
Fig. 1 allows performance comparison between the five algorithms for two different values of the outlier mean magnitude μ. Fig. 1a presents data for $\mu = 6$ (i.e. outliers magnitude distributed in $U[4.5, 7.5]$). Fig. 1b presents data for $\mu = 17$ (i.e. outliers magnitude distributed in $U[15.5, 18.5]$).

For $\mu = 6$, RAPS both achieves the minimum risk at all times and the best horizontal position accuracy at most times. The lowest risk is clearly expected as RAPS minimizes risk. The best positioning performance is less obvious, especially as the GDOP of RAPS is highest. A reason why RAPS achieves the best positioning performance is due to its having the lowest risk of outlier inclusion. For any of the algorithms, once an outlier is included, then both the prior mean and covariance are wrong, which affects the validity of all subsequent estimation and decisions about which measurements to use. Minimizing the risk of outlier inclusion therefore has obvious benefits for accuracy and reliability, especially when there are more measurements available than are required to meet a stated specification.

The performance of NP-KFs improves for $\mu = 17$ relative to the case where $\mu = 6$. This is because outliers with larger magnitude are more likely to be detected for any fixed value of the decision parameter γ. When all the algorithms correctly remove the outliers, their curves are overlapping. RAPS performance is almost the same for both scenarios, because it considers all feasible solutions and selects the one with minimum risk. Hence, its error is robust for different values of μ.

References

[1] E. Aghapour, F. Rahman, and J. A. Farrell, "Outlier accommodation in state estimation: A risk-averse performance-specified approach," *Submitted to IEEE T. on Control Systems Technology*, October 2018.



(a) Horizontal Position Error, Risk, and GDOP for $\mu = 6$

(b) Horizontal Position Error, Risk, and GDOP for $\mu = 17$

Fig. 1: Performance comparison using GNSS data with the linear PVA model. The yellow, green, blue and black curves display the results for NP-KF approach γ =2, 3, 4, and 5, respectively. The red curve shows the RAPS performance.

TABLE II: GNSS-PVA Performance Statistics

(a) Vertical: $\mu = 6$ (top) and $\mu = 17$ (bottom).

Methods	Mean of error (m)	Std. of error (m)	Error < 2 m	Maximum error (m)
NP-KF1 $\gamma = 5$	2.85	0.76	0.12	5.04
NP-KF1 $\gamma = 4$	2.82	0.74	0.10	5.02
NP-KF1 $\gamma = 3$	2.20	0.83	0.38	4.11
NP-KF1 $\gamma = 2$	0.68	0.60	0.96	3.30
RAPS1	0.56	0.46	0.99	2.07
NP-KF1 $\gamma = 5$	0.55	0.48	0.99	2.39
NP-KF1 $\gamma = 4$	0.55	0.48	0.99	2.39
NP-KF1 $\gamma = 3$	0.55	0.48	0.99	2.39
NP-KF1 $\gamma = 2$	0.55	0.48	0.99	2.40
RAPS2	0.56	0.46	0.99	2.09

(b) Horizontal: $\mu = 6$ (top) and $\mu = 17$ (bottom).

Methods	Mean of error (m)	Std. of error (m)	Sub-meter accuracy	Maximum error (m)
NP-KF1 $\gamma = 5$	0.72	0.52	0.74	2.30
NP-KF1 $\gamma = 4$	0.66	0.42	0.76	1.79
NP-KF1 $\gamma = 3$	0.64	0.41	0.78	1.79
NP-KF1 $\gamma = 2$	0.37	0.33	0.92	1.45
RAPS1	0.35	0.31	0.95	1.41
NP-KF2 $\gamma = 5$	0.37	0.32	0.96	1.45
NP-KF2 $\gamma = 4$	0.37	0.32	0.96	1.45
NP-KF2 $\gamma = 3$	0.37	0.32	0.96	1.45
NP-KF2 $\gamma = 2$	0.33	0.31	0.96	1.45
RAPS2	0.33	0.31	0.96	1.45