Lawrence Berkeley National Laboratory

HIFAN 1072 LBNL–46882

Transport of Charged Particle Beams

Edward P. Lee

Lawrence Berkeley National Laboratory/
Virtual National Laboratory

Presented to the Princeton Plasma Physics Laboratory (PPPL)
Plasma Theory Group
September 20, 2000

HIFAN 1072

Transport of Charged Particle Beams

Edward Lee
LBNL/VNL

Presented to the PPPL Plasma Theory Group

Sept. 20, 2000

Transported Charged Particle Beam
$\approx$ Non-Neutral Plasma

- Space Charge + Thermal Pressure Try to Blow Beam Apart
- Use Magnetic or Electrostatic Lenses To Hold it Togeather
- Typical HIF Beam at Low Energy:

10.0 MeV Cs^+ ($M = 133$ amu)

$v/c \approx .013$

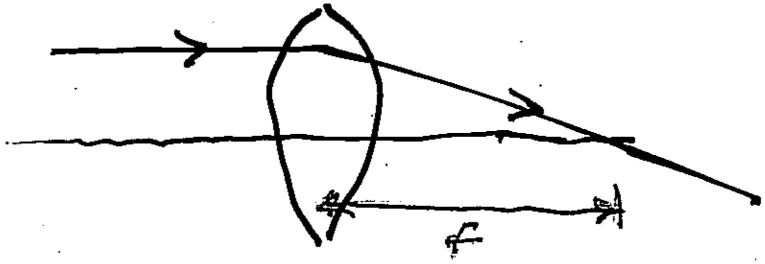
$I = 1.0$ Ampere

Radius $a = 2.0$ cm

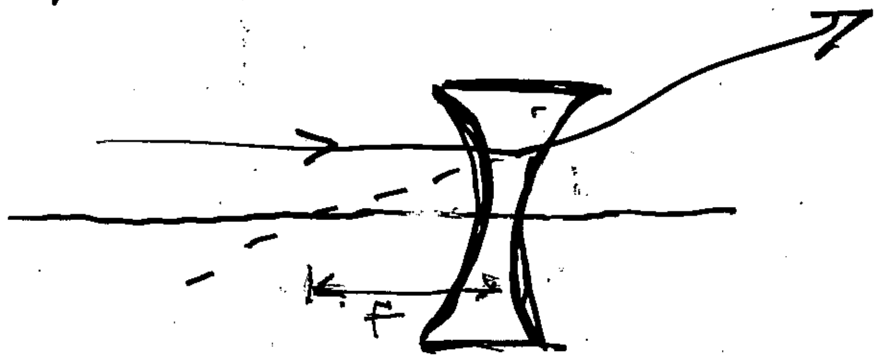
Temperature ≈ 30 eV

Space Charge Potential ≈ 3000 Volts

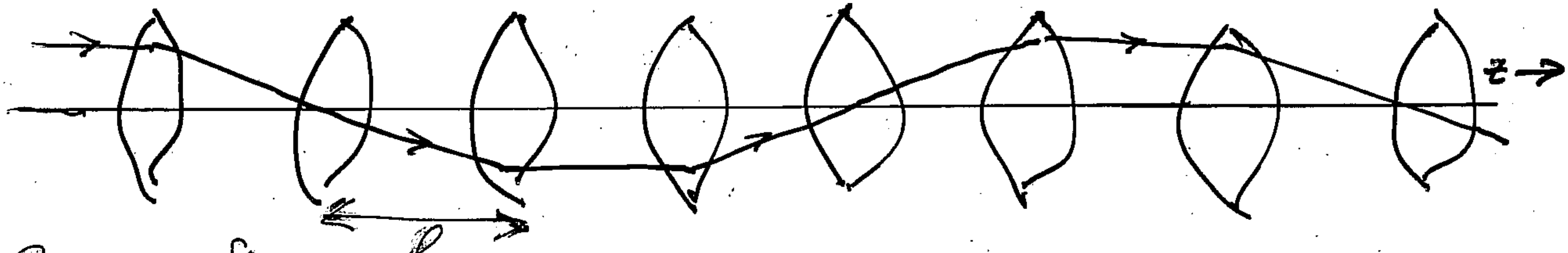
Single Particle In A System of Lenses



focus



defocus



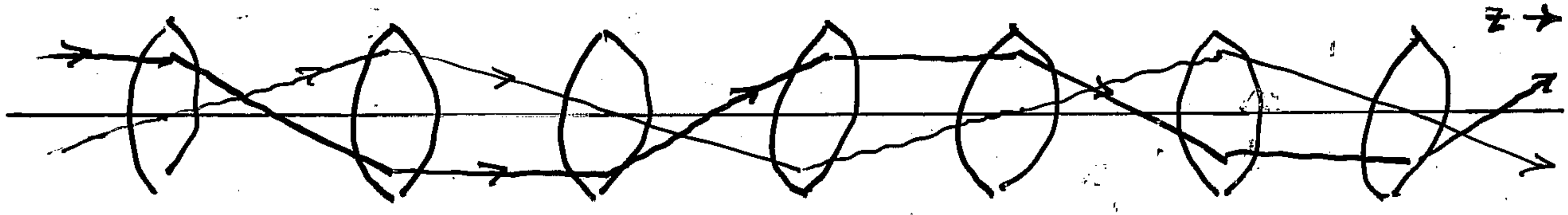
Case of
$\ell = f$

ℓ = space between lens centers

Particle osscillates one full period every 6 lenses

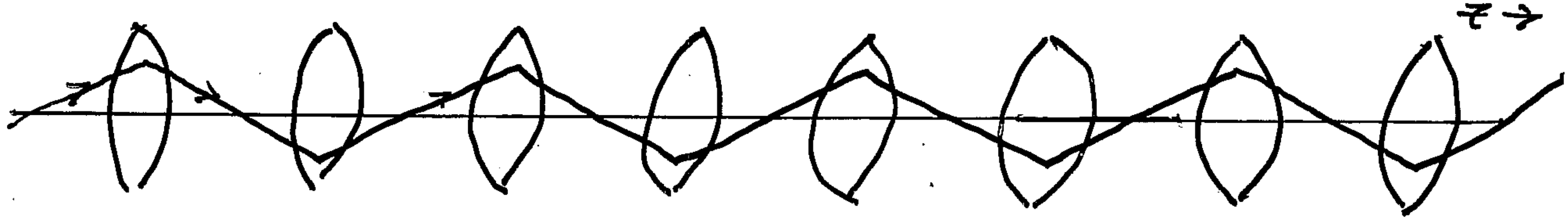
∴ "Tune" = σ_0 = $\dfrac{\text{phase advance}}{\text{per lattice period}}$ = 60°

Case of $f = \frac{1}{2} l$



Tune $\sigma_0 = \dfrac{360^\circ}{4 \text{ lenses}} = 90^\circ$

Case of $f = \frac{1}{4} l$



Tune $\sigma_0 = \dfrac{360^\circ}{2 \text{ lenses}} = 180^\circ$

Looks OK, But Actually There is Trouble!

Note In Above Examples:

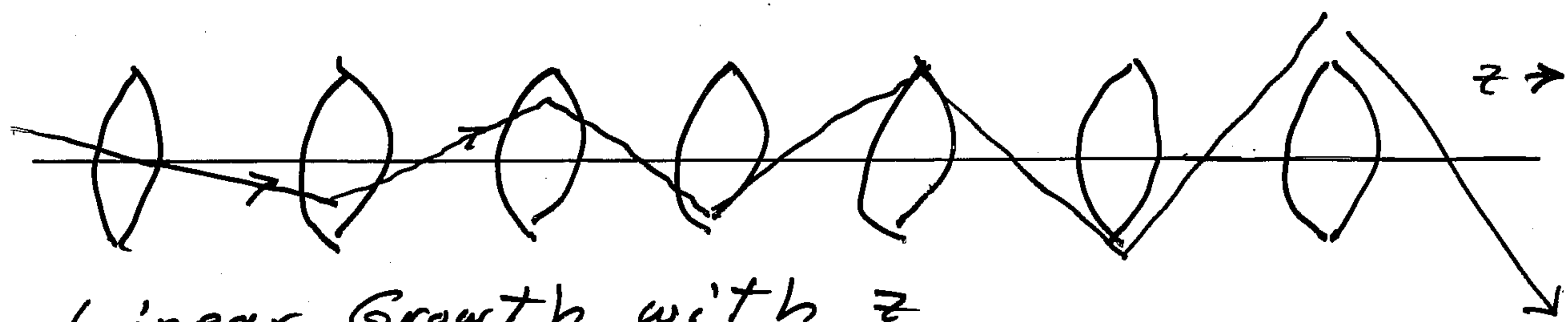
$$\cos(\sigma_0) = 1 - \frac{\ell}{2f}$$

(to be proven later)

$\frac{\ell}{f}$	$1-\frac{\ell}{2f}$	σ_0
1	1/2	60°
2	0	90°
4	−1	180°

- For $f < \frac{1}{4}\ell$ $\cos(\sigma_0) < -1$
  - → σ_0 is complex
  - → an unstable orbit

Look at second orbit for $f = \frac{1}{4}\ell$:



<u>Linear</u> Growth with z

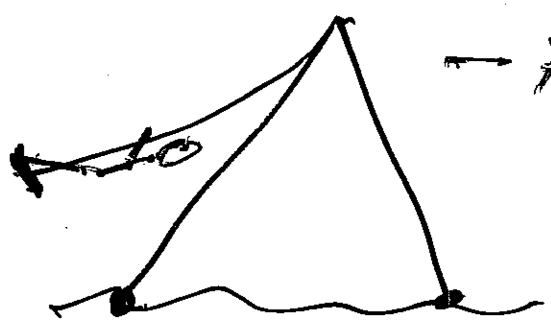
Acts Like a Driven Resonance

We Have a "Parametric Instability"

Natural Particle Frequency $> \frac{1}{2}$ Rate of Lens Spacing

– Note Factor of $\frac{1}{2}$ –   Parametric Drive

Think of Pumping A Swing Standing Up on The Seat



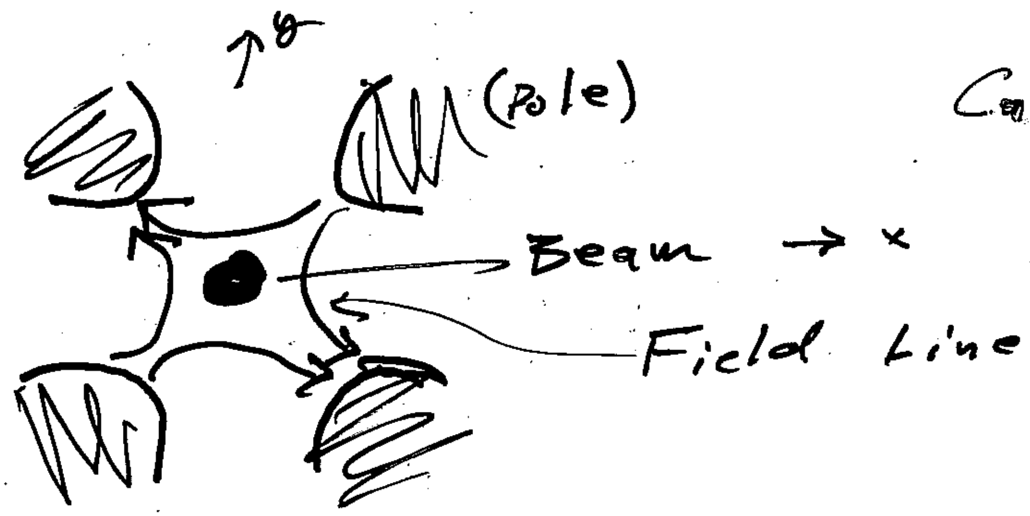
– Pump Twice per swing Period –

For Thin Lenses We Found A "Stop Band" For All $f < \frac{1}{4} l$

For Thick Lenses There Are Additional Pass Bands

Most Accelerators Use Quadrupole Lenses

Called "Strong Focusing"



Defocus in x direction (as drawn)
Focus in y direction

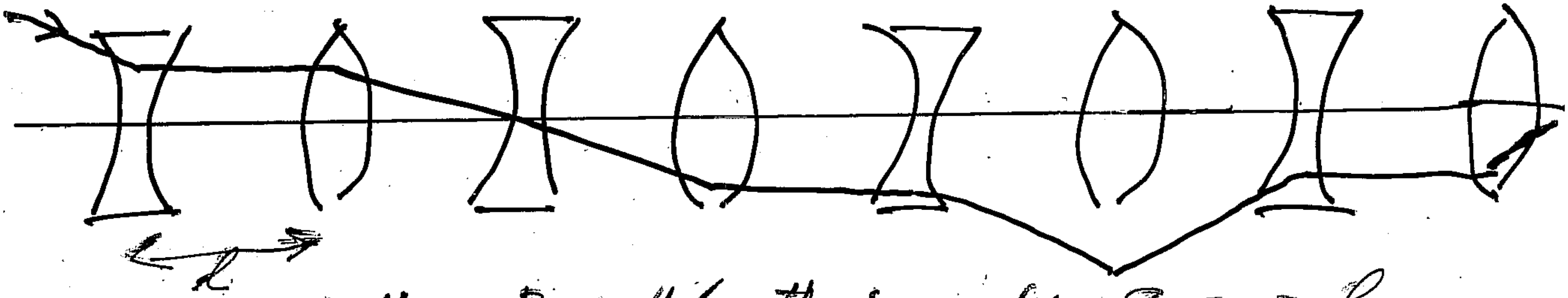
- Solution — Alternate Polarity Along The Lense System

Both Directions Confine Particle On stable Orbit if $\sigma_0 < 180°$

— Invention of Nick Christofilos ~ 1950

Alternating Gradient, Strong Focussing

Case of $f = \ell$



Note Lattice Period Length is now $S = 2\ell$

$$\sigma_0 = \frac{180^\circ}{3 \text{ pairs of lenses}} = 60^\circ$$

More Generally

$$\cos(\sigma_0) = 1 - \frac{\ell^2}{2f^2}$$

– Unstable for $\ell > 2f$

Problem #1

$$\frac{d^2x}{dz^2} = -K(z)\,x$$

$K(z)$ = any Periodic function

$$K(z+S) = K(z)$$

Define "Transfer Matrix" M

$$\begin{pmatrix} x \\ x' \end{pmatrix}_S = M \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$$

$\|M\| = 1$ to conserve phase space area (Hamiltonian System)

Show Eigenvalues of M are

$$\lambda = \frac{\mathrm{Tr}(M)}{2} \pm \sqrt{\left(\frac{\mathrm{Tr}M}{2}\right)^2 - 1}$$

(cont.)

Problem #1 Continued

& For stable case we may set

$$\cos(\sigma_0) = \frac{\mathrm{tr}(M)}{2}$$

& $\lambda = e^{\pm i\sigma_0}$ (σ_0 real)

Return to simple focus lense system:

$$M = \underbrace{\begin{pmatrix} 1 & \ell \\ 0 & 1 \end{pmatrix}}_{\text{drift}} \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}}_{\text{lens}} = \begin{pmatrix} 1-\frac{\ell}{f} & \ell \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$\rightarrow \cos(\sigma_0) = \frac{\mathrm{tr}\,M}{2} = 1 - \frac{\ell}{2f}$$

Problem #2

For Alternating focus, defocus show

$$\cos(\sigma_0) = 1 - \frac{\ell^2}{2f^2}$$

(Multiply 4 appropriate Matrices)

extra

Storage Rings Have Added Constraints

- Keep $\sigma_0 < 180^\circ$ as in linear system (60° – 90° is typical)
- But Total Tune For Full Ring Circumference is Large :

$$x\text{-total tune} = 360^\circ \times \underbrace{\text{large number}}_{\nu_x}$$

$$y\text{-total tune} = 360^\circ \times \underbrace{\text{different large number}}_{\nu_y}$$

- Small Magnet Errors Drive <u>Resonances</u>

$$l\,\nu_x + m\,\nu_y = n$$

$$l,\ m,\ n = \text{integers}$$

- Design (ν_x, ν_y) To Avoid Strong Resonances
- Space Charge can shift (ν_x, ν_y) into resonance

How Do We Construct a Phase Space Distribution ?

$$f(x, x', z) = ?$$

when $\quad x'' = -K(z)\, x$

Recall Simple Harmonic Oscillator

$$x'' = -k^2 x \qquad k = \text{const}$$

$$\rightarrow \quad E = \frac{x'^2 + k^2 x^2}{2} = \text{const}$$

— Try $f = F(E)$ —

Satisfies Vlasov Equation . . .

We Can Do Much Much More Than This

Little Appreciated Fact:

$$x'' = -K(z)\,x \qquad [\text{Any } K(z)]$$

Always Has Two Constants of The Motion!!

Let $x_1(z)$, $x_2(z)$ be any pair of solutions (independent from each other)

Wronskians Are Constant $\Big\}$ $I_1 = x x_1' - x' x_1$

$I_2 = x x_2' - x' x_2$

i.e. $I_1' = \cancel{x' x_1'} + x x_1'' - x'' x_1 - \cancel{x' x_1'}$

$= -x K x_1 + K x x_1 = 0$

In general $f(x, x'; z) =$ function of (I_1, I_2)

Solve for initial conditions $x_0, x_0' =$ functions of I_1, I_2

$\longrightarrow f(x, x'; z) = f\big(x_0(I_1, I_2), x_0'(I_1, I_2), z=0\big)$

What About Space Charge ?

$$\frac{d^2x}{dz^2} \simeq -K(z)\,x + \frac{Q}{a^2(z)}\,x$$

Here Q = perveance of Lawson $= \dfrac{e\lambda}{(4\pi\varepsilon_0)\,Mv^2/2e}$

$\lambda = \dfrac{\rho}{\pi a^2}$ = line charge density

$a(z)$ = beam radius

If We Neglect Thermal Pressure We Have "Laminar Flow"

- Look at Particle at beam edge $\left(x(z) = a(z)\right)$

$$\frac{d^2a}{dz^2} = -K(z)\,a + \frac{Q}{a} \longrightarrow \frac{2Q}{a(z) + b(z)}$$

— Envelope Equation —

Assumes ~ Round Beam Profile ~

Use Smooth Limit For Periodic Focussing

$$-K(z) \rightarrow -\left(\frac{\sigma_0}{S}\right)^2 \qquad \begin{cases} S = \text{lattice period} \\ \sigma_0 = \text{tune per period} \end{cases}$$

$$\frac{d^2 a}{dz^2} = -\left(\frac{\sigma_0}{S}\right)^2 a + \frac{Q}{a}$$

Equilibrium : $\frac{d^2 a}{dz^2} = 0$, $a = a_0$

$$\rightarrow 0 = -\left(\frac{\sigma_0}{S}\right)^2 a_0 + \frac{Q}{a_0}$$

Transportable Perveance $\quad Q = \left(\frac{\sigma_0 a_0}{S}\right)^2$

A Much Better Formula (good to ~ 1%) :

$$Q = 2\left(1 - \cos(\sigma_0)\right)\left(\frac{\bar{a}}{S}\right)^2$$

$\bar{a}$ = mean edge radius

Example – Return to Parameters on pg 1

10 MeV C_s^+

$I = 1.0$ Ampere

$a = 2.0$ cm

$v/c = .013$

$$\lambda = \frac{I}{v} = \frac{1.0}{(.013)(3\times10^8)} = .3\times10^{-6}\ \frac{C}{m}$$

$$Q = \frac{\lambda}{(4\pi\epsilon_0)\, Mv^2/2e} = \frac{(.3\times10^{-6})}{\left(\frac{1}{9\times10^9}\right)(10\times10^6)} = 2.7\times10^{-4}$$

– Q is always small for beams transported in vacuum –

Take $\sigma_0 = 60° \approx 1.0\ \frac{\text{radian}}{\text{period}}$

$$\rightarrow S \equiv \frac{\sigma_0 a}{\sqrt{Q}} = \frac{1.0\times.02}{\sqrt{2.7\times10^{-4}}} \approx 1.2\text{ m}$$

Significance: $a \ll S \rightarrow$ Lenses Well Spaced Apart + Paraxial Approximation Good..!

Perturbation Of Beam Envelope — Breathing Mode —

Set $\quad a = a_0 + \delta a(z)$ $\;\leftarrow$ small

$$\frac{d^2 a}{dz^2} = -\left(\frac{\sigma_0}{S}\right)^2 a + \frac{Q}{a}$$

$$\rightarrow \frac{d^2 \delta a}{dz^2} = -\left(\frac{\sigma_0}{S}\right)^2 \delta a - \frac{Q}{a_0^2}\,\delta a$$

$$\frac{Q}{a_0^2} = \left(\frac{\sigma_0}{S}\right)^2$$

$$= -2\left(\frac{\sigma_0}{S}\right)^2 \delta a$$

$$\therefore \quad \delta a \sim \cos\left(\sqrt{2}\,\frac{\sigma_0 z}{S}\right)$$

Looks Stable, But Smooth Limit Misses Parametrically Driven Phenomena

Note Lattice Frequency $\sim 2\pi/S$

$\rightarrow$ Expect Trouble For $\dfrac{\sqrt{2}\,\sigma_0}{S} > \dfrac{1}{2}\cdot\left(\dfrac{2\pi}{S}\right)$ $\left\{ \begin{array}{l} \sigma_0 > \dfrac{\pi}{\sqrt{2}} \\ \approx 12\text{[illegible]} \end{array} \right.$

We Have A Mystery

No Qualitative Understanding Today!

- Simple Calculation (above):

  $\sigma_0 > 128^\circ \Rightarrow$ Unstable

- Solve Exact Envelope Equations (x, y planes) & Elliptic Profiles:

  $\sigma_0 \gtrsim 120^\circ$ ($\pm$ few°) $\rightarrow$ Unstable

- PIC Simulation (SemiGaussian f):

  $\sigma_0 \gtrsim 85^\circ \Rightarrow$ "Bad Things Happen"
  Thermal Pressure Grows

- LBNL Experiments
  ~1985~

  $\sigma_0 \gtrsim 80^\circ \Rightarrow$ Thermal Pressure Grows