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Author
Dreiner, Herbi K.

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Herbi K. Dreiner[1], Christoph Luhn[2],
Hitoshi Murayama[3,4], Marc Thormeier[5]

[1]Physikalisches Institut der Universitat Bonn,
Nussallee 12, 53115 Bonn, Germany

[2]Institute for Fundamental Theory, Department of Physics,
University of Florida, Gainesville, FL 32611, USA

[3]Theoretical Physics Group,
Ernest Orlando Lawrence Berkeley National Laboratory,
University of California, Berkeley, CA 94720, USA

[4]Department of Physics,
University of California, Berkeley, CA 94720, USA

[5]Service de Physique Theorique, CEA-Saclay,
Orme des Merisiers, 91191 Gif-sur-Yvette Cedex, France

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Proton Hexality from an Anomalous Flavor $U(1)$ and Neutrino Masses – Linking to the String Scale

Herbi K. Dreiner[1],* Christoph Luhn[2],†
Hitoshi Murayama[3,4],‡ Marc Thormeier[5]§

[1] *Physikalisches Institut der Universität Bonn, Nußallee 12, 53115 Bonn, Germany*

[2] *Institute for Fundamental Theory, Department of Physics, University of Florida, Gainesville, FL 32611, USA*

[3] *Theoretical Physics Group, Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley, CA 94720, USA*

[4] *Department of Physics, University of California, Berkeley, CA 94720, USA*

[5] *Service de Physique Théorique, CEA-Saclay, Orme des Merisiers, 91191 Gif-sur-Yvette Cedex, France*

Abstract

We devise minimalistic gauged $U(1)_X$ Froggatt-Nielsen models which at low-energy give rise to the recently suggested discrete gauge $\mathbb{Z}_6$-symmetry, proton hexality, thus stabilizing the proton. Assuming three generations of right-handed neutrinos, with the proper choice of X-charges, we obtain viable neutrino masses. Furthermore, we find scenarios such that no X-charged hidden sector superfields are needed, which from a bottom-up perspective allows the calculation of g_{string}, g_X and G_{SM}'s Kač-Moody levels. The only mass scale apart from M_{grav} is m_{soft}.

*E-mail: `dreiner@th.physik.uni-bonn.de`
†E-mail: `luhn@phys.ufl.edu`
‡E-mail: `murayama@hitoshi.berkeley.edu`
§E-mail: `thor@th.physik.uni-bonn.de`

1 Introduction

In this paper, we consider low-energy discrete symmetries, $\mathbb{Z}_N$, as extensions of the $SU(3)\times SU(2)\times U(1)$ gauge symmetry of the Minimal Supersymmetric Standard Model (MSSM). We focus on the case, where the $\mathbb{Z}_N$ is the remnant of a spontaneously broken local gauge symmetry, in order to avoid potentially harmful gravity effects [1]. Such discrete symmetries originating in a gauge theory are called discrete gauge symmetries (DGSs) [2]. In Refs. [3, 4], a systematic study was performed of all the DGSs resulting from Abelian, anomaly-free gauge symmetries, $U(1)_X$, which leave the MSSM invariant. Specifically, the following assumptions were made in these studies[1]

- The only light, low-energy fields are those of the MSSM. All beyond-the-MSSM fields are heavy.
- At least the following superpotential terms are $\mathbb{Z}_N$-invariant:

$$Q^i H^D \overline{D^j}, \;\; Q^i H^U \overline{U^j}, \;\; L^i H^D \overline{E^j}, \;\; H^D H^U, \;\; L^i H^U L^j H^U \,, \tag{1.1}$$

  where we have made use of the standard notation for the MSSM chiral superfields, see for example [6]. The invariance of the first three terms implies that the $\mathbb{Z}_N$-symmetry, but not necessarily the original $U(1)_X$, is family-universal.

Given these assumptions, the only possible DGS resulting from an anomaly-free $U(1)_X$ are the $\mathbb{Z}_2$-symmetry matter parity (M_p), the $\mathbb{Z}_3$-symmetry baryon triality (B_3) and the $\mathbb{Z}_6$-symmetry proton hexality ($P_6 = M_p \times B_3$) [3, 4]. In Refs. [7, 8], the $U(1)_X$ gauge charges were determined, which lead to a low-energy M_p, B_3, or P_6, respectively. See also Refs. [9, 10] for related work on the conditions for DGSs in GUTs.

It is now of great interest to see whether realistic flavor models for the Standard Model (SM) fermion masses and mixings can be constructed in each case. Employing the original $U(1)_X$ in a minimal Froggatt-Nielsen (FN) scenario [11] and using the Green-Schwarz (GS) mechanism [12] to cancel the $U(1)_X$ anomalies, a successful M_p-model was constructed in Ref. [7] and its implications for suppressed proton decay were discussed in Refs. [13, 14]. Later, a corresponding B_3-model was constructed in Ref. [8], with a detailed discussion of the neutrino masses.

It is the purpose of this note to construct a P_6-FN flavor model, in order to complete this program. Furthermore, from the phenomenological point of view, proton hexality is a very attractive symmetry. It combines the advantages of the M_p and the B_3 models [4]: the lightest supersymmetric particle (LSP) is stable and the dangerous dimension-four and dimension-five proton decay operators are forbidden. We shall proceed analogously to Refs. [7, 8] and refer the reader to these publications for an explanation of our notation and an introduction to for example the Giudice-Masiero/Kim-Nilles (GM/KN) mechanism [15, 16].

There has been extensive previous work on anomalous flavor models employing the Green-Schwarz mechanism and with breaking slightly below the Planck scale, see for

[1] In Ref. [5], the case will be investigated where these points are modified such that massless right-handed neutrinos exist, hence the possible DGSs in combination with Dirac rather than Majorana neutrinos will be explored.

example Refs. [17, 18, 19, 20, 21, 22]. However, we believe this is the first work on such a model aiming for a remnant "gauged" P_6. There are also some non-anomalous flavor models with $U(1)_X$ breaking at the TeV scale [23, 24, 25, 26, 27, 28, 29].

This note is structured as follows: In Sect. 2, we discuss the constraints on the X-charges which are not related to neutrino phenomenology. In Sect. 3, we then focus on the neutrino sector and how it fixes the X-charges; corresponding tables are given in Appendix B. In Sect. 4, we discuss the possibility and the implications of excluding X-charged hidden sector superfields, enabling us to calculate the string coupling constant. We conclude in Sect. 5.

2 Non-Neutrino Constraints on the X-Charges

In the following we proceed as in Refs. [7, 8] and consider only one flavon chiral superfield A, with $U(1)_X$-charge $X_A = -1$. In order to obtain a viable flavor model, the $U(1)_X$ charges of the P_6–FN models must satisfy several phenomenological and consistency constraints. They must

(a) reproduce phenomenologically acceptable charged SM fermion masses and mixings, see Ref. [30],

(b) reproduce phenomenologically acceptable neutrino masses and mixings,

(c) satisfy the Green-Schwarz mixed linear anomaly cancellation conditions (with gauge coupling unification), as well as guarantee that the mixed quadratic anomaly vanishes on its own, *e.g.* Ref. [7],

(d) imply the desired low-energy DGS P_6, *i.e.* give rise to the following discrete family-independent $\mathbb{Z}_6$-charges for the MSSM chiral superfields [4, 31]:

$$z_Q = 0, \quad z_{\overline{D}} = 5, \quad z_{\overline{U}} = 1, \quad z_L = 4, \quad z_{\overline{E}} = 1, \quad z_{H^D} = 1, \quad z_{H^U} = 5\,,$$

and (as will be argued later) $z_{\overline{N}} = 3$ for the additional right-handed neutrino (SM singlet) chiral superfields.

Excluding the conditions (b) and (d) for a moment, it was shown in Table 1 of Ref. [7] that all 20 X-charges of the MSSM+$\overline{N^i}$ superfields can be expressed in terms of nine real numbers. Note that for simplicity, we assume three generations of right-handed neutrinos, unlike in Ref. [7] where only two generations were introduced.

$$\begin{array}{lll} x = 0,1,2,3, & X_{L^1}, & X_{\overline{N^1}}, \\ y = -1,0,1, & \Delta^L_{21} \equiv X_{L^2} - X_{L^1}, & X_{\overline{N^2}}, \\ z = 0,1, & \Delta^L_{31} \equiv X_{L^3} - X_{L^1}, & X_{\overline{N^3}}, \end{array} \tag{2.1}$$

Here X_F denotes the $U(1)_X$-charge of the field F. A few comments are in order:

- Δ^L_{31} and Δ^L_{21} can only take integer values.
- x is related to the ratio of the vacuum expectation values (VEVs) of the two Higgs doublets, $\tan\beta = \frac{v_u}{v_d}$, by $\epsilon^x \sim \frac{m_b}{m_t}\tan\beta$.

- y parameterizes the phenomenologically viable ϵ-structures for the CKM matrix. Our preferred choice is $y = 0$ as it gives a CKM matrix with $U_{12}^{\mathrm{CKM}} \sim \epsilon$, $U_{13}^{\mathrm{CKM}} \sim \epsilon^3$, and $U_{23}^{\mathrm{CKM}} \sim \epsilon^2$, see Ref. [7].

- z is related to the ratio m_e/m_μ. It turns out to equal $-X_{H^U} - X_{H^D}$ and thus deals with the origin and the magnitude of the μ-parameter. For $z = 1$, the bilinear Higgs term is forbidden before $U(1)_X$-breaking. After $U(1)_X$-breaking it is generated via the combination of the FN-mechanism together with the GM/KN-mechanism, resulting in a μ-parameter of the order of the soft supersymmetry breaking scale m_{soft}. So the μ-problem finds a natural solution, unlike in the case for $z = 0$; we will hence assume $z = 1$ throughout this article.

- The X-charges of the first generation lepton doublet L^1 and the three right-handed neutrinos are unconstrained at this stage. We will explain in a moment why the right-handed neutrinos have to be introduced at all.

- Assuming a string-embedded FN framework, the expansion parameter ϵ is a derived quantity which depends on x and z. For $z = 1$ and $x = 0, 1, 2, 3$ we get ϵ within the interval (see Ref. [7] and references therein for details)

$$0.186 \leq \epsilon \leq 0.222\,. \tag{2.2}$$

Let us now include (d), *i.e.* the constraints arising from the requirement of a low-energy DGS P_6. The necessary and sufficient conditions on the X-charges for obtaining P_6 conservation are derived in Ref. [8]. With $p = \pm 1$ they are

$$X_{H^D} - X_{L^1} = -\frac{1}{2} + \text{integer}, \qquad 3X_{Q^1} + X_{L^1} = -\frac{p}{3} + \text{integer}, \tag{2.3}$$

as well as (see the argument in Item 3 in Sect. 3.1) the three X-charges of the right-handed neutrinos being half-odd-integer. Inserting the expression for X_{Q^1} of Table 1 in Ref. [7], we can rewrite this as

$$\Delta^H \equiv X_{L^1} - X_{H^D} - \frac{1}{2}, \qquad 3\zeta + p \equiv \Delta_{21}^L + \Delta_{31}^L - z, \tag{2.4}$$

where $\Delta^H, \zeta \in \mathbb{Z}$. We thus impose proton hexality by trading the parameters X_{L^1} and Δ_{21}^L of Eq. (2.1) for the *integer* parameters Δ^H and $3\zeta + p$. The resulting constrained X-charges are shown in Table 1.

3 Neutrino Constraints on the X-Charges

3.1 The Origin of P_6 Neutrino Masses

Next we take the remaining constraints (b) into account, *i.e.* the experimental data from the neutrino sector. To do so, let us first consider the possible sources of neutrino masses in a P_6 invariant FN scenario.

$$
\begin{array}{rcl}
X_{H^D} &=& \frac{1}{5\ (6+x+z)}\ \Big(6y + x\ (2x + 11 + z - 2\Delta^H) \\
&& -z\ (\frac{11}{2} + 3\Delta^H) - 2\ (6 + 6\Delta^H - \Delta^L_{31}) - \frac{2}{3}\ (6 + x + z)(3\zeta + p)\Big) \\
X_{H^U} &=& -z - \ X_{H^D} \\
X_{Q^1} &=& \frac{1}{3}\Big(\frac{19}{2} - X_{H^D} + x + 2y + z - \Delta^H - \frac{1}{3}(3\zeta + p)\Big) \\
X_{Q^2} &=& X_{Q^1} - 1 - y \\
X_{Q^3} &=& X_{Q^1} - 3 - y \\
X_{\overline{U^1}} &=& X_{H^D} - X_{Q^1} + 8 + z \\
X_{\overline{U^2}} &=& X_{\overline{U^1}} - 3 + y \\
X_{\overline{U^3}} &=& X_{\overline{U^1}} - 5 + y \\
X_{\overline{D^1}} &=& -X_{H^D} - X_{Q^1} + 4 + x \\
X_{\overline{D^2}} &=& X_{\overline{D^1}} - 1 + y \\
X_{\overline{D^3}} &=& X_{\overline{D^1}} - 1 + y \\
X_{L^1} &=& X_{H^D} + \Delta^H + \frac{1}{2} \\
X_{L^2} &=& X_{L^1} - \Delta^L_{31} + z + (3\zeta + p) \\
X_{L^3} &=& X_{L^1} + \Delta^L_{31} \\
X_{\overline{E^1}} &=& -X_{H^D} + 4 - X_{L^1} + x + z \\
X_{\overline{E^2}} &=& X_{\overline{E^1}} - 2 - 2z + \Delta^L_{31} - (3\zeta + p) \\
X_{\overline{E^3}} &=& X_{\overline{E^1}} - 4 - \ z - \Delta^L_{31} \\
X_{\overline{N^1}} &=& \frac{1}{2} + \Delta^{\overline{N}}_1 \\
X_{\overline{N^2}} &=& \frac{1}{2} + \Delta^{\overline{N}}_2 \\
X_{\overline{N^3}} &=& \frac{1}{2} + \Delta^{\overline{N}}_3
\end{array}
$$

Table 1: The constrained X-charges which lead to an acceptable low-energy phenomenology of quark and charged lepton masses and quark mixing. In addition, the GS anomaly cancellation conditions have been implemented as well as the quadratic anomaly condition. Furthermore, P_6 is conserved, *i.e.* Eq. (2.4) has been imposed. x, y, z and p are integers specified in Eqs. (2.1,2.4). Δ^H, Δ^L_{31}, and ζ are integers as well but still unconstrained. The $\Delta^{\overline{N}}_i$ of the right-handed neutrinos are yet-unspecified integers.

1. Neutrino masses can*not* derive from matter parity (M_p) violating operators such as LH^U or $LL\overline{E}$, as these are forbidden by P_6.

2. Therefore, and in the lack of right-handed neutrinos, (Majorana) neutrino masses can only originate from the dimension five superpotential term $L^iH^UL^jH^U$. As-

suming a minimal number of fundamental mass scales, *i.e.* only $m_{\mathrm{soft}} \approx 0.1 - 1\,\mathrm{TeV}$ and $M_{\mathrm{grav}} = 2.4 \cdot 10^{18}\,\mathrm{GeV}$, this operator is suppressed by $\frac{1}{M_{\mathrm{grav}}}$. This results in the following neutrino mass matrix

$$\left[\boldsymbol{M^{(\nu)}_{LH^U LH^U}}\right]_{ij} \;\sim\; \frac{\langle H^U\rangle^2}{M_{\mathrm{grav}}} \cdot \epsilon^{X_{L^i}+X_{L^j}+2X_{H^U}}. \tag{3.1}$$

Since $X_{L^i} + X_{L^j} + 2X_{H^U} \geq 0$ and $\epsilon \approx 0.2$, the absolute neutrino mass scale cannot exceed $\frac{\langle H^U\rangle^2}{M_{\mathrm{grav}}} \approx 1.3 \cdot 10^{-5}\mathrm{eV}$ in this scenario (with $\langle H^U\rangle \sim m_t$). From the observed atmospheric neutrino oscillations, we however know that the absolute mass scale must be at least $5 \cdot 10^{-2}\mathrm{eV}$. Thus the neutrino mass matrix cannot (solely) originate from the non-renormalizable operator $L^i H^U L^j H^U$. This is not the case if we allow for the mass scale which suppresses $L^i H^U L^j H^U$ to be lower than M_{grav}, see *e.g.* the model in Ref. [4]. Note that in the case where $X_{L^i} + X_{L^j} + 2X_{H^U} < 0$, the operator $L^i H^U L^j H^U$ is generated from the Kähler potential via the GM/KN-mechanism in combination with the FN-mechanism, leading to an even stronger suppression by a factor of $\frac{m_{\mathrm{soft}}}{M^2_{\mathrm{grav}}}$.

3. When enlarging the particle spectrum by three generations of right-handed neutrinos $\overline{N^i}$, *i.e.* particles which couple trilinearly to $L^i H^U$, a new possibility for the neutrino mass term arises. Since $L^i H^U L^j H^U$ is P_6-allowed and the term $L^i H^U \overline{N^j}$ by definition as well, but $L^i H^U$ is P_6-forbidden, the right-handed neutrinos must carry a half-odd-integer X-charge. Thus the Majorana mass term $\overline{N^i}\,\overline{N^j}$ is necessarily also P_6-allowed. In Ref. [5], the possibility of DGSs which allow for $L^i H^U \overline{N^j}$ but forbid $\overline{N^i}\,\overline{N^j}$ and $L^i H^U L^j H^U$ will be discussed.

Throughout this article, we consider the third possibility above as the only viable source of neutrino masses in our scenario. The flavon field A and the right-handed neutrinos $\overline{N^i}$ have a lot in common. Apart from their $U(1)_X$-charges, both are uncharged. But there are also certain important differences: 1.) After $U(1)_X$ breaking A will not carry any $\mathbb{Z}$-charge, whereas the $\overline{N^i}$ will. 2.) The flavon field A acquires a VEV, whereas the $\overline{N^i}$ are assumed not to. This is just like the MSSM non-Higgs scalar fields, which are not supposed to acquire a VEV, in order to *e.g.* preserve color and/or electromagnetism. Note that $\langle A\rangle = \epsilon M_{\mathrm{grav}}$, but $\langle\overline{N^i}\rangle = 0$ is consistent with the requirement of SUSY being unbroken at $\epsilon M_{\mathrm{grav}}$, *i.e.* $\langle D_X\rangle = \langle F_A\rangle = \langle F_{\overline{N^i}}\rangle = 0$.

In the discussion of the constraints on the X-charges coming from the neutrino sector, we have to distinguish between four cases. These differ in the origin of the superpotential terms $L^i H^U \overline{N^j}$ and $\overline{N^i}\,\overline{N^j}$. Depending on the overall X-charge, the terms are either of pure FN origin or effectively generated via the GM/KN-mechanism in combination with the FN-mechanism. For the Majorana mass terms, the low-energy effective superpotential terms are[2]

$$X_{\overline{N^i}} + X_{\overline{N^j}} \geq 0: \qquad \frac{1}{2}\, M^{(\mathrm{M})}_{ij} \overline{N^i}\,\overline{N^j} \;\sim\; \frac{1}{2}\, M_{\mathrm{grav}} \cdot \epsilon^{X_{\overline{N^i}} + X_{\overline{N^j}}} \cdot \overline{N^i}\,\overline{N^j}, \tag{3.2}$$

$$X_{\overline{N^i}} + X_{\overline{N^j}} < 0: \qquad \frac{1}{2}\, M^{(\mathrm{M})}_{ij} \overline{N^i}\,\overline{N^j} \;\sim\; \frac{1}{2}\, m_{\mathrm{soft}} \cdot \epsilon^{-X_{\overline{N^i}} - X_{\overline{N^j}}} \cdot \overline{N^i}\,\overline{N^j}, \tag{3.3}$$

while for the Dirac mass terms we have

$$X_{L^i} + X_{H^U} + X_{\overline{N^j}} \geq 0: \qquad \frac{M^{(\mathrm{D})}_{ij}}{\langle H^U \rangle}\, L^i H^U \overline{N^j} \;\sim\; \epsilon^{X_{L^i} + X_{H^U} + X_{\overline{N^j}}} \cdot L^i H^U \overline{N^j}, \tag{3.4}$$

$$X_{L^i} + X_{H^U} + X_{\overline{N^j}} < 0: \qquad \frac{M^{(\mathrm{D})}_{ij}}{\langle H^U \rangle}\, L^i H^U \overline{N^j} \;\sim\; \frac{m_{\mathrm{soft}}}{M_{\mathrm{grav}}} \cdot \epsilon^{-X_{L^i} - X_{H^U} - X_{\overline{N^j}}} \cdot L^i H^U \overline{N^j}. \tag{3.5}$$

The labeling of the four different cases is shown in the following table.

	$X_{\overline{N^i}} + X_{\overline{N^j}} \geq 0$	$X_{\overline{N^i}} + X_{\overline{N^j}} < 0$
$X_{L^i} + X_{H^U} + X_{\overline{N^j}} \geq 0$	I	II
$X_{L^i} + X_{H^U} + X_{\overline{N^j}} < 0$	III	IV

(This can be compared also to Table 5 of Ref. [7]: Case I contains their 1.+2., Case II 6., Case III 3. and Case IV 4.+5.)

When determining the masses of the light neutrino degrees of freedom we have to diagonalize the 6×6 neutrino mass matrix

$$\begin{pmatrix} \mathbf{0} & \boldsymbol{M}^{(\mathbf{D})} \\ \boldsymbol{M}^{(\mathbf{D})T} & \boldsymbol{M}^{(\mathbf{M})} \end{pmatrix}. \tag{3.6}$$

We have approximated the $(1,1)$ entry of the matrix above to be the 3×3 zero matrix, because we already concluded earlier [see below Eq. (3.1)] that $\boldsymbol{M}^{(\boldsymbol{\nu})}_{\boldsymbol{LH^U LH^U}}$ does not contribute substantially enough to the absolute neutrino masses.

Under the assumption that the ϵ-suppression is not able to compensate the gravitational scale M_{grav} such that one arrives at m_{soft} or $\langle H^U \rangle$ (which would be ~ 24 powers of ϵ), we see from Eqs. (3.2-3.5) that automatically $\boldsymbol{M}^{(\mathbf{D})} \ll \boldsymbol{M}^{(\mathbf{M})}$ for the Cases I, III and IV. We can thus directly apply the see-saw formula to calculate the masses of the three light neutrinos. In Case II, there are three possibilities

(i) $\boldsymbol{M}^{(\mathbf{D})} \ll \boldsymbol{M}^{(\mathbf{M})} \;\longrightarrow\;$ standard see-saw,

(ii) $\boldsymbol{M}^{(\mathbf{D})} \approx \boldsymbol{M}^{(\mathbf{M})}$,

(iii) $\boldsymbol{M}^{(\mathbf{D})} \gg \boldsymbol{M}^{(\mathbf{M})} \;\longrightarrow\;$ pseudo Dirac neutrinos.

[2] We assume that *all* entries of the 3×3 mass matrices have the same origin: Either they are all generated by pure FN or all via GM/KN+FN. Allowing otherwise would lead to enormous suppressions between some of the elements of the mass matrices, effectively leading to textures, which for simplicity we prefer to avoid.

For Case (II.iii), the ϵ-suppression must lower $\langle H^U \rangle \sim 200\,\mathrm{GeV}$ down to the neutrino mass scale, in order to be phenomenologically viable. This corresponds to about 20 powers of ϵ and we do not consider it any further. In Case (II.ii) one would naturally, *i.e.* without finetuning among the submatrices $\boldsymbol{M}^{(\mathbf{D})}$ and $\boldsymbol{M}^{(\mathbf{M})}$, expect the neutrino mass matrix to have six singular values (masses) of *the same order*; as for (II.iii), extreme ϵ-suppression is required to obtain three sub-eV neutrinos. Hence, we also discard Case (II.ii). For the rest of this article, we refer to Case (II.i) as Case II.

Regardless of the Case (I - IV), in the following the light neutrino mass matrix is derived from the see-saw mechanism [32, 33, 34, 35] and is given as (discarding the contributions from $L^i H^U L^j H^U$)

$$\boldsymbol{M}^{(\boldsymbol{\nu})} \;=\; -\boldsymbol{M}^{(\mathbf{D})} \cdot {\boldsymbol{M}^{(\mathbf{M})}}^{-1} \cdot {\boldsymbol{M}^{(\mathbf{D})}}^{T}. \tag{3.7}$$

For later convenience we change the basis of the right-handed neutrinos so that $\boldsymbol{M}^{(\mathbf{M})}$ is diagonal. Such a basis transformation is unproblematic after $U(1)_X$ is broken. As discussed in Ref. [8], this basis transformation does not alter the ϵ-structure of $\boldsymbol{M}^{(\mathbf{D})}$ in Eqs. (3.4) and (3.5). It is now straightforward to determine $\boldsymbol{M}^{(\boldsymbol{\nu})}$ for the upper four cases:

$$M_{ij}^{(\nu,\mathrm{I})} \;\sim\; \frac{\langle H^U\rangle^2}{M_{\mathrm{grav}}}\,\epsilon^{2\Delta^H - 2z + 1 + \Delta^L_{i1} + \Delta^L_{j1}}, \tag{3.8}$$

$$M_{ij}^{(\nu,\mathrm{II})} \;\sim\; \frac{\langle H^U\rangle^2}{m_{\mathrm{soft}}}\,\epsilon^{2\Delta^H - 2z + 1 + \Delta^L_{i1} + \Delta^L_{j1}} \times \sum_{a=1}^{3} \epsilon^{4X_{\overline{N^a}}}, \tag{3.9}$$

$$M_{ij}^{(\nu,\mathrm{III})} \;\sim\; \frac{\langle H^U\rangle^2\, m_{\mathrm{soft}}^2}{M_{\mathrm{grav}}^3}\,\epsilon^{-2\Delta^H + 2z - 1 - \Delta^L_{i1} - \Delta^L_{j1}} \times \sum_{a=1}^{3} \epsilon^{-4X_{\overline{N^a}}}, \tag{3.10}$$

$$M_{ij}^{(\nu,\mathrm{IV})} \;\sim\; \frac{\langle H^U\rangle^2\, m_{\mathrm{soft}}}{M_{\mathrm{grav}}^2}\,\epsilon^{-2\Delta^H + 2z - 1 - \Delta^L_{i1} - \Delta^L_{j1}}. \tag{3.11}$$

Here we have made use of Table 1 and the definition $\Delta^L_{i1} \equiv X_{L^i} - X_{L^1}$. Note that the dependence on the X-charges of the right-handed neutrinos drops out in Cases I and IV, as has been shown analytically in Ref. [7]. Thus the masses of the light neutrinos do not depend on the charges $X_{\overline{N^a}}$. For Cases II and III one might naïvely expect that although the *overall mass scale* of the light neutrinos depends on the $X_{\overline{N^a}}$, their *mass ratios* $\widetilde{m}_3 : \widetilde{m}_2 : \widetilde{m}_1$ do not. The latter however is not true, as is shown explicitly for Case II in Appendix A. Making use of the orderings[3] $X_{L^3} \leq X_{L^2} \leq X_{L^1}$ and $X_{\overline{N^3}} \leq X_{\overline{N^2}} \leq X_{\overline{N^1}}$, we obtain

$$\widetilde{m}_3 \;:\; \widetilde{m}_2 \;:\; \widetilde{m}_1 \;\sim\; 1 \;:\; \epsilon^{2(X_{L^2}-X_{L^3})+4(X_{\overline{N^2}}-X_{\overline{N^3}})} \;:\; \epsilon^{2(X_{L^1}-X_{L^3})+4(X_{\overline{N^1}}-X_{\overline{N^3}})}. \tag{3.12}$$

Assuming $X_{\overline{N^2}} - X_{\overline{N^3}} \geq 1$, the second largest neutrino mass would be suppressed by a factor of at least ϵ^4 compared to the heaviest neutrino. Even when including the effects of unknown $\mathcal{O}(1)$ coefficients, this suppression is too large to be consistent with the data (see

[3]The ordering of X_{L^i} is necessary for obtaining a phenomenologically acceptable charged lepton mass matrix (see the discussion in Ref. [8]), while we are free to choose the ordering of $X_{\overline{N^i}}$ without loss of generality.

Sect. 3.3). For Case II, we must therefore constrain the X-charges of the right-handed neutrinos by

$$X_{\overline{N^2}} = X_{\overline{N^3}}, \tag{3.13}$$

$$X_{\overline{N^1}} = X_{\overline{N^2}} = X_{\overline{N^3}}, \tag{3.14}$$

for (normal and inverted) hierarchy and degeneracy, respectively (see Sect. 3.3).

Similarly for Case III: Here one obtains the condition $X_{\overline{N^1}} = X_{\overline{N^2}}$ for (normal and inverted) hierarchical light neutrinos, and $X_{\overline{N^1}} = X_{\overline{N^2}} = X_{\overline{N^3}}$ for degenerate scenarios.

3.2 Constraints from Neutrino Mixing

The ϵ-structure of the light neutrino mass matrix is determined by Δ^L_{21} and Δ^L_{31}. We have $M^{(\nu)}_{ij} \propto \epsilon^{X_{L^i}+X_{L^j}}$ for Cases I & II whereas for Cases III & IV we find $M^{(\nu)}_{ij} \propto \epsilon^{-X_{L^i}-X_{L^j}}$. Both types of matrices are diagonalized by a unitary transformation $\widetilde{U}^{(\nu)}_{ij} \sim \epsilon^{|X_{L^i}-X_{L^j}|}$, so that

$$\widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})^*} \cdot \boldsymbol{M}^{(\boldsymbol{\nu})} \cdot \widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})^\dagger} = \begin{pmatrix} \widetilde{m}_1 & 0 & 0 \\ 0 & \widetilde{m}_2 & 0 \\ 0 & 0 & \widetilde{m}_3 \end{pmatrix}, \tag{3.15}$$

with

$$\text{Case I}: \quad \widetilde{m}_1 : \widetilde{m}_2 : \widetilde{m}_3 \sim 1 : \epsilon^{2\Delta^L_{21}} : \epsilon^{2\Delta^L_{31}}, \tag{3.16}$$

$$\text{Case II}: \quad \widetilde{m}_1 : \widetilde{m}_2 : \widetilde{m}_3 \sim 1 : \epsilon^{2\Delta^L_{21}+4(X_{\overline{N^2}}-X_{\overline{N^1}})} : \epsilon^{2\Delta^L_{31}+4(X_{\overline{N^3}}-X_{\overline{N^1}})}, \tag{3.17}$$

$$\text{Case III}: \quad \widetilde{m}_1 : \widetilde{m}_2 : \widetilde{m}_3 \sim 1 : \epsilon^{-2\Delta^L_{21}-4(X_{\overline{N^2}}-X_{\overline{N^1}})} : \epsilon^{-2\Delta^L_{31}-4(X_{\overline{N^3}}-X_{\overline{N^1}})}, \tag{3.18}$$

$$\text{Case IV}: \quad \widetilde{m}_1 : \widetilde{m}_2 : \widetilde{m}_3 \sim 1 : \epsilon^{-2\Delta^L_{21}} : \epsilon^{-2\Delta^L_{31}}. \tag{3.19}$$

As mentioned above and discussed in greater detail in Appendix A, the ratios of the light neutrino masses depend on the X-charges of the right-handed neutrinos in Cases II and III [see Eqs. (A.16) and (A.17)]. Recalling the orderings $X_{L^3} \leq X_{L^2} \leq X_{L^1}$ and $X_{\overline{N^3}} \leq X_{\overline{N^2}} \leq X_{\overline{N^1}}$ we find

$$\text{Cases I \& II}: \quad \widetilde{m}_1 \leq \widetilde{m}_2 \leq \widetilde{m}_3, \qquad \text{Cases III \& IV}: \quad \widetilde{m}_1 \geq \widetilde{m}_2 \geq \widetilde{m}_3, \tag{3.20}$$

respectively. In order to compare the theoretically derived mixing matrices $\widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}$ with neutrino phenomenology, it is convenient to define the matrix $\boldsymbol{U}^{(\boldsymbol{\nu})} \equiv \boldsymbol{U}^{\mathbf{MNS}\,\dagger}$, so that

$$\boldsymbol{U}^{(\boldsymbol{\nu})^*} \cdot \boldsymbol{M}^{(\boldsymbol{\nu})} \cdot \boldsymbol{U}^{(\boldsymbol{\nu})^\dagger} = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{3.21}$$

Here $m_1 \leq m_2 \leq m_3$ for normal and $m_3 \leq m_1 \leq m_2$ for inverted ordering of the neutrino masses, see *e.g.* Ref. [36]. $\boldsymbol{U}^{\mathbf{MNS}}$ is the Maki-Nakagawa-Sakata matrix [37] for mixing in the lepton sector. Working in a basis with diagonal charged leptons, *cf.* Ref. [8], this mixing is solely due to the neutrino sector. Comparing Eqs. (3.15, 3.21), we can easily determine the relation between $\boldsymbol{U}^{(\boldsymbol{\nu})}$ and $\widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}$ and thus the theoretically predicted structure of the MNS matrix for the various scenarios:

- Considering the Cases I & II and a normal neutrino mass ordering, we simply have

$$\boldsymbol{U}^{(\boldsymbol{\nu})} \;=\; \boldsymbol{T_{123}} \cdot \widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}, \qquad \text{with} \qquad \boldsymbol{T_{123}} \;\equiv\; \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.22}$$

- while an inverted mass ordering leads to

$$\boldsymbol{U}^{(\boldsymbol{\nu})} \;=\; \boldsymbol{T_{231}} \cdot \widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}, \qquad \text{with} \qquad \boldsymbol{T_{231}} \;\equiv\; \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{3.23}$$

- For Cases III & IV, we similarly find that for a normal neutrino mass ordering

$$\boldsymbol{U}^{(\boldsymbol{\nu})} \;=\; \boldsymbol{T_{321}} \cdot \widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}, \qquad \text{with} \qquad \boldsymbol{T_{321}} \;\equiv\; \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{3.24}$$

- and for an inverted mass ordering

$$\boldsymbol{U}^{(\boldsymbol{\nu})} \;=\; \boldsymbol{T_{213}} \cdot \widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}, \qquad \text{with} \qquad \boldsymbol{T_{213}} \;\equiv\; \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.25}$$

Since $\boldsymbol{U}^{\mathbf{MNS}\,\dagger} = \boldsymbol{U}^{(\boldsymbol{\nu})} = \boldsymbol{T_{...}} \cdot \widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}$, with $\widetilde{U}^{(\nu)}_{ij} \sim \epsilon^{|X_{L^i} - X_{L^j}|}$, we obtain severe constraints on the possible values for Δ^L_{i1} from the experimentally allowed ϵ-structure of the MNS matrix [8]

$$\boldsymbol{U}^{\mathbf{MNS}} \quad \sim \quad \begin{pmatrix} \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1,2,...} \\ \epsilon^{0,1,2} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1,2} & \epsilon^{0,1} & \epsilon^{0,1} \end{pmatrix}. \tag{3.26}$$

Here, multiple possibilities for the exponents of ϵ are separated by commas. Depending on $\boldsymbol{T_{...}}$ we have four different equations for

$$\epsilon^{|X_{L^i} - X_{L^j}|} \quad \sim \quad \widetilde{U}^{(\nu)}_{ij} = \left[\boldsymbol{T_{...}}^{\dagger} \cdot \boldsymbol{U}^{\mathbf{MNS}\,\dagger} \right]_{ij}. \tag{3.27}$$

The resulting ϵ-structures of $\widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}$ are shown in Table 2 together with the compatible values for the pairs $(\Delta^L_{21}, \Delta^L_{31})$. Notice that due to the ordering $X_{L^3} \leq X_{L^2} \leq X_{L^1}$, we must have $\Delta^L_{i1} \leq 0$ as well as $\Delta^L_{21} \geq \Delta^L_{31}$.

Having derived the constraints on the parameters Δ^L_{i1} from neutrino mixing, we must also satisfy the second condition of Eq. (2.4), which states that $\Delta^L_{21} + \Delta^L_{31} - z$ must *not* be a multiple of three. As mentioned earlier, we choose to work with $z = 1$ in order to have the μ-term generated by the GM/KN+FN-mechanism. Therefore the choice $(\Delta^L_{21}, \Delta^L_{31}) = (-1, -1)$ is incompatible with the requirement of P_6 conservation, and in the remainder of this article we – of course – do not consider this P_6 violating solution.

We conclude the discussion of the neutrino mixing with some observations regarding the CHOOZ [38] mixing angle, θ_{13}. In our notation this angle is parameterized by the

	Cases I & II	Cases III & IV
normal mass ordering	$\begin{pmatrix} \epsilon^{0,1} & \epsilon^{0,1,2} & \epsilon^{0,1,2} \\ \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1,2,\dots} & \epsilon^{0,1} & \epsilon^{0,1} \end{pmatrix}$	$\begin{pmatrix} \epsilon^{0,1,2,\dots} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1,2} & \epsilon^{0,1,2} \end{pmatrix}$
$(\Delta_{21}^L, \Delta_{31}^L)$	$(0,0),\ (0,-1),\ (-1,-2),$ $(-1,-1)$	$(0,0),\ (0,-1),$ $(-1,-1)$
inverted mass ordering	$\begin{pmatrix} \epsilon^{0,1,2,\dots} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1,2} & \epsilon^{0,1,2} \\ \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \end{pmatrix}$	$\begin{pmatrix} \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1,2} & \epsilon^{0,1,2} \\ \epsilon^{0,1,2,\dots} & \epsilon^{0,1} & \epsilon^{0,1} \end{pmatrix}$
$(\Delta_{21}^L, \Delta_{31}^L)$	$(0,0),\ (0,-1),$ $(-1,-1)$	$(0,0),\ (0,-1),$ $(-1,-1)$

Table 2: The constraints on the values of Δ_{i1}^L originating from the experimentally observed neutrino mixing. The structure of the matrix $\widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})} = \boldsymbol{T}_{\dots}{}^{\dagger} \cdot \boldsymbol{U}^{\mathbf{MNS}\,\dagger}$ is shown. As also $\widetilde{U}_{ij}^{(\nu)} \sim \epsilon^{|X_{L^i} - X_{L^j}|}$ must be satisfied, only a few pairs of $(\Delta_{21}^L, \Delta_{31}^L)$ are possible. Demanding P_6 invariance, the choice $(-1,-1)$ is excluded, see below Eq. (3.27).

entry $\epsilon^{0,1,2,\dots}$ in the mixing matrices $\widetilde{\boldsymbol{U}}^{(\boldsymbol{\nu})}$ of Table 2. As the CHOOZ angle is small, one should try to find solutions in terms of $(\Delta_{21}^L\,,\,\Delta_{31}^L)$ where this entry is ϵ^1 or ϵ^2.

Comparing with the four matrices in Table 2, we see that a normal mass ordering with $(0,-1)$ or $(-1,-2)$ is preferred for Cases I & II, while inverted neutrino masses with $(0,-1)$ are suggested for Cases III & IV. More precisely, $(0,-1)$ leads to $U_{13}^{\text{MNS}} \sim \epsilon$ for normal ordered Cases I & II and inverted ordered Cases III & IV, while $(-1,-2)$ analogously results in $U_{13}^{\text{MNS}} \sim \epsilon^2$. By choosing Δ_{31}^L appropriately, one can understand the smallness of the CHOOZ angle in terms of the flavor group $U(1)_X$.

There exist of course other possible explanations for the smallness of θ_{13}. For example, in Ref. [39] this is achieved by separating the effective neutrino mass matrix as a sum of two parts; each contains only a 2×2 block and is of rank one. Alternatively, there is a plethora of models adopting non-Abelian discrete symmetries like *e.g.* A_4 [40, 41, 42, 43], $\Delta(27)$ [44, 45], S_3 [46, 47], S_4 [48, 49], $\mathbb{Z}_7 \rtimes \mathbb{Z}_3$ [50], $PSL_2(7)$ [51] to give rise to the tri-bimaximal mixing pattern [52], in which θ_{13} is exactly zero.

3.3 Constraints from Neutrino Masses

Before discussing the Cases I - IV individually, some general remarks concerning the magnitude of the three light neutrino masses are in order. We shall combine the results of the solar [53, 54], atmospheric [55], reactor [56], and accelerator [57] neutrino oscillation experiments,[4] as well as the upper bound on the absolute neutrino mass scale originating from the kinematic mass measurements [60]. This leads to three possible scenarios, see *e.g.* Refs. [36, 61]:

$$\begin{aligned} m_1 < m_2 \ll m_3 \approx 0.05\,\text{eV}, &\qquad \text{normal hierarchical,} \\ m_3 \ll m_1 < m_2 \approx 0.05\,\text{eV}, &\qquad \text{inverted hierarchical,} \\ 0.05\,\text{eV} \ll m_1 \approx m_2 \approx m_3 < 2.2\,\text{eV}, &\qquad \text{degenerate.} \end{aligned}$$

Assuming a (normal or inverted) hierarchical scenario, the absolute upper neutrino mass scale $m^{\nu}_{\text{abs}} \equiv \max(m_1, m_2, m_3)$ is about 0.05 eV, a value which is consistent with the cosmological upper bound on the sum of the neutrino masses, $\sum_i m_i \leq 0.7\,\text{eV}$ [62, 63]. For an inverted hierarchy, *two* neutrinos must have a mass around this scale, while the third neutrino is much lighter. As the suppression between the masses of the two heavier neutrinos is given by [*cf.* Eqs. (3.16-3.19), respectively]

$$\text{Case I}: \qquad \frac{\widetilde{m}_2}{\widetilde{m}_3} \sim \epsilon^{2(\Delta^L_{21}-\Delta^L_{31})}, \tag{3.28}$$

$$\text{Case II}: \qquad \frac{\widetilde{m}_2}{\widetilde{m}_3} \sim \epsilon^{2(\Delta^L_{21}-\Delta^L_{31})+4(X_{\overline{N2}}-X_{\overline{N3}})}, \tag{3.29}$$

$$\text{Case III}: \qquad \frac{\widetilde{m}_2}{\widetilde{m}_1} \sim \epsilon^{-2\Delta^L_{21}+4(X_{\overline{N1}}-X_{\overline{N2}})}, \tag{3.30}$$

$$\text{Case IV}: \qquad \frac{\widetilde{m}_2}{\widetilde{m}_1} \sim \epsilon^{-2\Delta^L_{21}}, \tag{3.31}$$

the inverted hierarchical scenario is not possible for all pairs $(\Delta^L_{21}, \Delta^L_{31})$: For Cases I & II we need $(0,0)$ whereas for III & IV $(0,0)$ as well as $(0,-1)$ are acceptable.

For the degenerate case, m^{ν}_{abs} can take values within the range $[0.2\,\text{eV}, 2.2\,\text{eV}]$, where the lower end of the interval is estimated such that it satisfies the condition $0.05\,\text{eV} \ll m^{\nu}_{\text{abs}}$. Concerning the cosmological bound, high values for the neutrino masses are more or less disfavored, depending on which cosmological observations are included in the derivation of the bound [62, 63]. We return to this issue in the discussion of our results. Within our P_6 FN-framework, the degenerate scenario is only possible if we have $\Delta^L_{21} = \Delta^L_{31} = 0$. This in turn requires a certain amount of finetuning among the $\mathcal{O}(1)$ coefficients in order to get correct neutrino masses and mixing.

We now turn to the discussion of each of the individual Cases I - IV. In our calculations we take $M_{\text{grav}} = 2.4 \cdot 10^{18}\,\text{GeV}$,

$$100\,\text{GeV} \leq m_{\text{soft}} \leq 1000\,\text{GeV}, \tag{3.32}$$

and $\langle H^U \rangle \sim m_t = 175\,\text{GeV}$.[5] In addition we assume $z = 1$, as well as Eq. (2.2).

[4] We disregard the result of the LSND experiment [58], which could not be confirmed by MiniBooNE [59].

[5] Of course, one only knows $\sqrt{\langle H^U \rangle^2 + \langle H^D \rangle^2}$ and not $\langle H^U \rangle$ alone. However, the latter depends only weakly on $\tan\beta$ (and hence x) in the range $2 \leq \tan\beta \leq 50$.

(I) From Eq. (3.8) and the ordering $\Delta^L_{31} \leq \Delta^L_{21} \leq \Delta^L_{11} = 0$, we get the absolute neutrino mass scale as

$$m^\nu_{\mathrm{abs}} \sim \frac{m_t^2}{M_{\mathrm{grav}}}\,\epsilon^{2\Delta^H + 2\Delta^L_{31} - 1}. \tag{3.33}$$

Solving for the exponent yields

$$2\Delta^H + 2\Delta^L_{31} - 1 \sim \frac{1}{\ln\epsilon}\cdot\ln\left(\frac{m^\nu_{\mathrm{abs}} M_{\mathrm{grav}}}{m_t^2}\right). \tag{3.34}$$

– For a normal or inverted hierarchical scenario $m^\nu_{\mathrm{abs}} \approx 0.05\,\mathrm{eV}$. Inserting this and the limiting values for ϵ, we arrive at the following allowed range

$$-2\Delta^H - 2\Delta^L_{31} \in [3.9\,,\,4.5], \tag{3.35}$$

where the lower value of the interval is obtained for small values of x. Since the left-hand side is necessarily an (even) integer, the hierarchical Case I slightly prefers small x. However, due to possible unknown $\mathcal{O}(1)$ coefficients we cannot rule out large x. Furthermore, Eq. (3.35) determines Δ^H as

$$\Delta^H = -2 - \Delta^L_{31}\,. \tag{3.36}$$

– Considering the degenerate case, which is only possible for $\Delta^L_{21} = \Delta^L_{31} = 0$, the absolute mass scale m^ν_{abs} should be within the interval $[0.2\,\mathrm{eV}\,,\,2.2\,\mathrm{eV}]$. With this we are similarly lead to

$$-2\Delta^H \in [4.7\,,\,7], \tag{3.37}$$

where the lower value corresponds to both small x and small m^ν_{abs}. Thus we have for the degenerate neutrino scenario

$$\Delta^H = -3, \tag{3.38}$$

a value which is compatible with all $x = 0, 1, 2, 3$. $x = 0$ leads to a neutrino mass scale of $m^\nu_{\mathrm{abs}} \approx 1.7\,\mathrm{eV}$ and $x = 3$ to $m^\nu_{\mathrm{abs}} \approx 0.5\,\mathrm{eV}$. Taken at face value, both are in conflict with the cosmological upper bound on the sum of the neutrino masses. However, $\mathcal{O}(1)$ coefficients can alleviate this tension. In the comment column of Table 6 we give the naïve sum of the neutrino masses assuming all $\mathcal{O}(1)$ coefficients are exactly one.

All possible sets of parameters $(\Delta^L_{21}, \Delta^L_{31}, 3\zeta + p, \Delta^H, x)$ are summarized in Table 3. The compatibility with the various neutrino mass scenarios is denoted by the symbol ✓. Note that by virtue of Eq. (2.4), the first three parameters are not independent of each other. As pointed out earlier, we assume $z = 1$. The allowed values for $y = -1, 0, 1$ remain unconstrained by the neutrino sector. Altogether we can find $4 \times 4 \times 3 = 48$ distinct sets of X-charge assignments (including also less favored possibilities), which fulfill the constraints of Tables 1+3. They are given in Appendix B, Table 5.

Δ^L_{21}	Δ^L_{31}	$3\zeta+p$	Δ^H	x	normal hier.	inverted hier.	degenerate
0	0	-1	-3	$0,1,2,3$			✓
0	0	-1	-2	$0,1,(2,3)$	✓	✓	
0	-1	-2	-1	$0,1,(2,3)$	✓		
-1	-2	-4	0	$0,1,(2,3)$	✓		

Table 3: The sets of parameters which are compatible with neutrino phenomenology in Case I, where the terms $L^iH^U\overline{N^j}$ and $\overline{N^i}\,\overline{N^j}$ have pure FN origin. We assume $z=1$. The hierarchical scenarios slightly prefer small x and disfavor large (denoted by the parentheses). The parameter $y=-1,0,1$ remains unconstrained.

For Case I, the X-charges of the right-handed neutrinos are not directly constrained by neutrino phenomenology. Recall however that this case requires by definition $X_{L^i}+X_{H^U}+X_{\overline{N^j}}\geq 0$ and $X_{\overline{N^i}}+X_{\overline{N^j}}\geq 0$ for all $i,j=1,2,3$. With Table 1 and $z=1$ this translates into

$$\Delta^{\overline{N}}_i \;\;\geq\;\; -\Delta^L_{31}-\Delta^H, \tag{3.39}$$

leading to $\Delta^{\overline{N}}_i\geq 3$ in the degenerate case and $\Delta^{\overline{N}}_i\geq 2$ for hierarchical scenarios.

On the other hand, there exists also an upper bound on $\Delta^{\overline{N}}_i$. Qualitatively, very high X-charge for the right-handed neutrinos would suppress the Majorana mass matrix in Eq. (3.2) so that its mass scale becomes comparable to or even smaller than the Dirac masses of Eq. (3.4). Thus the see-saw formula would no longer apply. Requiring that $M^{(\mathrm{D})}_{33}\ll M^{(\mathrm{M})}_{11}$ yields the condition

$$2\Delta^{\overline{N}}_1-\Delta^{\overline{N}}_3 \;\;<\;\; \frac{1}{\ln\epsilon}\cdot\ln\left(\frac{\langle H^U\rangle}{M_{\mathrm{grav}}}\right)\;-\;z+\Delta^L_{31}+\Delta^H. \tag{3.40}$$

Depending on ϵ, the first term on the right-hand side is numerically between 22.1 and 24.7. With the latter, *i.e.* for the case where $\epsilon=0.222$, we arrive at the upper bounds of $2\Delta^{\overline{N}}_1-\Delta^{\overline{N}}_3\leq 20$ for the degenerate and $2\Delta^{\overline{N}}_1-\Delta^{\overline{N}}_3\leq 21$ for the hierarchical case, respectively. In Sect. 4, we will constrain the $\Delta^{\overline{N}}_i$ by requiring the absence of X-charged hidden sector superfields.

It is worth noting that thermal leptogenesis requires the lightest right-handed neutrino to be not too light: $M^{(\mathrm{M})}_{11}\gtrsim 4\times10^8$ GeV if the spectrum is hierarchical (no close states) but otherwise with rather conservative assumptions [64]. Even though the considerations here do not determine the X-charges of $\overline{N^i}$, and hence their masses, we do obtain quite restrictive constraints once we require that all anomalies are canceled without introducing additional (hidden) fields charged only under $U(1)_X$ but not the standard model. See Appendix B for more details.

(II) Proceeding with Case II, we obtain from Eq. (3.9) and the orderings $\Delta^L_{31}\leq\Delta^L_{21}\leq$

$\Delta^L_{11} = 0$ and $X_{\overline{N^3}} \leq X_{\overline{N^2}} \leq X_{\overline{N^1}}$ that

$$m^\nu_{\rm abs} \quad \sim \quad \frac{m_t^2}{m_{\rm soft}}\, \epsilon^{2\Delta^H + 2\Delta^L_{31} - 1 + 4X_{\overline{N^3}}}. \tag{3.41}$$

The hierarchical scenarios require

$$2\Delta^H + 2\Delta^L_{31} + 4X_{\overline{N^3}} \quad \in \quad [17.1\,,\, 20.6], \tag{3.42}$$

the left boundary of the interval corresponds to small ϵ [see Eq. (2.2)] and *large* $m_{\rm soft}$ [see Eq. (3.32)]. Thus Δ^H is given by

$$\Delta^H \quad = \quad -\Delta^L_{31} - 2X_{\overline{N^3}} + \left\{ \begin{array}{ll} 9, & x = 0, 1, (2)\,, \\ 10, & x = 2, 3\,. \end{array} \right. \tag{3.43}$$

Here and in the following, values in parentheses are acceptable only if we rely on suitable $\mathcal{O}(1)$ coefficients to satisfy phenomenological conditions similar to Eq. (3.42) with the above specified parameter ranges. For instance, without any $\mathcal{O}(1)$ coefficients in Eq. (3.41), the value $x = (2)$ leads to $m_{\rm soft} = 1990\,{\rm GeV}$ which is outside of the initially assumed range for the soft supersymmetry breaking scale.

The three possible values of x in the first line of Eq. (3.43) yield $m_{\rm soft} \approx 230\,{\rm GeV}$, $m_{\rm soft} \approx 680\,{\rm GeV}$, and $m_{\rm soft} \approx 1990\,{\rm GeV}$, respectively. For the second line, we find analogously 90 GeV and 230 GeV, for $x = 2, 3$. As pointed out above, these "predictions" of the soft supersymmetry breaking scale do not take into account the variation due to the unknown $\mathcal{O}(1)$ coefficients in any FN model. Allowing for such a factor to be anything within the interval $[\frac{1}{\sqrt{10}}\,,\, \sqrt{10}]$, there is actually no hard constraint on $m_{\rm soft}$, except for the case with $x = 2$ which prefers large $m_{\rm soft}$ in the first line and low $m_{\rm soft}$ in the second.

For degenerate neutrinos, the possible variation of the absolute mass scale within the interval $[0.2\,{\rm eV}\,,\, 2.2\,{\rm eV}]$ leads to a further widening of the allowed range for Δ^H, in addition to flexibility in ϵ and $m_{\rm soft}$. Since $\Delta^L_{21} = \Delta^L_{31} = 0$, we have

$$2\Delta^H + 4X_{\overline{N^3}} \quad \in \quad [14.9\,,\, 19.6], \tag{3.44}$$

which results in the possible values

$$\Delta^H \quad = \quad -2X_{\overline{N^3}} + \left\{ \begin{array}{ll} 8, & x = 0, 1, 2, (3), \\ 9, & x = 1, 2, 3. \end{array} \right. \tag{3.45}$$

Again, there is no significant constraint on $m_{\rm soft}$. However, the first line of Eq. (3.45) with $x = 2, (3)$ prefers a large soft breaking scale while the second line with $x = 1$ suggests low $m_{\rm soft}$. Due to the constraints on the $U(1)_X$-charges given in Table 1, we can define an integer n as

$$n \quad \equiv \quad -X_{\overline{N^3}} - \frac{1}{2}. \tag{3.46}$$

Since $X_{\overline{N^i}} + X_{\overline{N^j}} < 0$, the X-charges of the right-handed neutrinos must be negative, hence $n \geq 0$. Another condition is that

$$X_{L^i} + X_{H^U} + X_{\overline{N^j}} = \Delta^L_{i1} + \Delta^H - \frac{1}{2} + X_{\overline{N^j}} \geq \Delta^L_{31} + \Delta^H - \frac{1}{2} + X_{\overline{N^3}} \geq 0\,. \tag{3.47}$$

Inserting respectively Eqs. (3.43,3.45) shows that this is automatically satisfied. However, there is yet another relation to be met. Recall that for the see-saw mechanism we require $\boldsymbol{M^{(\mathrm{D})}} \ll \boldsymbol{M^{(\mathrm{M})}}$. This provides us with a lower bound on $X_{\overline{N^i}}$, as can be seen in the following. From Eqs. (3.3) and (3.4), the lightest right-handed neutrino has a Majorana mass of the order $m_{\text{soft}}\, \epsilon^{-2X_{\overline{N^3}}}$ and the heaviest Dirac mass is of order $\langle H^U \rangle\, \epsilon^{X_{L^3}+X_{H^U}+X_{\overline{N^3}}}$. Therefore we require

$$\frac{m_{\text{soft}}}{\langle H^U \rangle} \;\gg\; \epsilon^{X_{L^3}+X_{H^U}+3X_{\overline{N^3}}}. \tag{3.48}$$

As a conservative estimate, we take $m_{\text{soft}} = 1000\,\text{GeV}$, yielding $\frac{m_{\text{soft}}}{\langle H^U \rangle} \approx \epsilon^{-1}$ for the left-hand side. Therefore

$$\Delta^L_{31} + \Delta^H - \frac{1}{2} + 3X_{\overline{N^3}} = X_{L^3} + X_{H^U} + 3X_{\overline{N^3}} \;\;>\;\; -1. \tag{3.49}$$

For the hierarchical cases, we insert Eq. (3.43) into Eq. (3.49). Expressing $X_{\overline{N^3}}$ in terms of $n \geq 0$ we arrive at the conditions

$$0 \leq n \leq \begin{cases} 8, & x = 0, 1, (2), \\ 9, & x = 2, 3, \end{cases} \tag{3.50}$$

where the two lines correspond to the two possibilities for Δ^H in Eq. (3.43).

For the degenerate case, where $\Delta^L_{31} = 0$, we similarly obtain with Eq. (3.45)

$$0 \leq n \leq \begin{cases} 7, & x = 0, 1, 2, (3), \\ 8, & x = 1, 2, 3. \end{cases} \tag{3.51}$$

In Table 4, we give all sets of parameters $(\Delta^L_{21}, \Delta^L_{31}, 3\zeta + p, \Delta^H, x, n)$, which comply with the phenomenology of neutrino masses and mixings for Case II. We assume $z = 1$, and the parameter $y = -1, 0, 1$ remains unaffected by the neutrino sector. Compared to the analogous table for Case I, we have added the parameter $n \in \mathbb{N}$, which is defined by the X-charge of the right-handed neutrino $\overline{N^3}$ [*cf.* Eq. (3.46)] and determines the parameter Δ^H. Limiting ourselves to Case II restricts the allowed values for n. Altogether we can thus find $[(4 \times 8 + 3 \times 9) + (3 \times 9 + 2 \times 10) + (3 \times 9 + 2 \times 10) + (3 \times 9 + 2 \times 10)] \times 3 = 600$ sets of X-charge assignments, including also less favored possibilities. Some of these charge assignments, however, are identical due to the first two rows of Table 4. There are 504 distinct sets of X-charges. A selected subset resulting from Table 4 is given in Appendix B, Table 7. For the relevant criteria see the next Section.

It is worth noting that there is a constraint from neutrino oscillation due to the presence of right-handed states. The most stringent limit comes from appearance experiment searches: $\nu_\mu \to \nu_\ell$, for $\ell = e, \tau$. Because we required the right-handed neutrino masses to be much higher than the light neutrino masses, Eq. (3.48), the oscillation probability is averaged out, and hence we obtain an upper limit on the mixing angle. The effective "$\sin^2 2\theta$" must be less than about 3×10^{-4} [65, 66, 67, 68, 59]. In terms of the mixing matrices, the limit is therefore $|U_{\mu i} U^*_{\ell i}| \approx$

Δ^L_{21}	Δ^L_{31}	$3\zeta+p$	Δ^H	x	n	normal	inverted	degenerate
0	0	-1	$2n+\left\{\begin{array}{r}9\\10\end{array}\right.$	$0,1,2,(3)$ $1,2,3$	$0\leq n\leq 7$ $0\leq n\leq 8$			✓
0	0	-1	$2n+\left\{\begin{array}{r}10\\11\end{array}\right.$	$0,1,(2)$ $2,3$	$0\leq n\leq 8$ $0\leq n\leq 9$	✓	✓	
0	-1	-2	$2n+\left\{\begin{array}{r}11\\12\end{array}\right.$	$0,1,(2)$ $2,3$	$0\leq n\leq 8$ $0\leq n\leq 9$	✓		
-1	-2	-4	$2n+\left\{\begin{array}{r}12\\13\end{array}\right.$	$0,1,(2)$ $2,3$	$0\leq n\leq 8$ $0\leq n\leq 9$	✓		

Table 4: The sets of parameters which are compatible with neutrino phenomenology in Case II where the term $L^iH^U\overline{N^j}$ has pure FN origin while $\overline{N^i}\,\overline{N^j}$ is generated via GM/KN. We assume $z=1$. The parameter $y=-1,0,1$ remains unconstrained, n can take only positive integer values which are restricted as shown in the table.

$\frac{M^{(\mathrm{D})}_{\mu i}M^{(\mathrm{D})}_{\ell i}}{M^{(\mathrm{M})\,2}_{ii}} \lesssim \epsilon^2$, where we assumed $m_{\mathrm{soft}}\sim\langle H^U\rangle$. Therefore, we obtain $2\Delta_H+\Delta^L_{21}+\Delta^L_{\ell i}-6n\ >\ 5$. This restricts the allowed ranges of n in Table 4 slightly more: All upper limits on n are reduced by 1 to 6, 7, 7, 8, 7, 8, 7, 8, respectively.

(III) For Case III the scale of the Dirac mass matrix $M^{(\mathrm{D})}_{ij}$ in Eq. (3.5) is given by the $(1,1)$ entry. Since $X_{L^1}+X_{H^U}+X_{\overline{N^1}}<0$, this mass scale has an upper bound

$$M^{(\mathrm{D})}_{11}\ <\ \frac{\langle H^U\rangle\, m_{\mathrm{soft}}}{M_{\mathrm{grav}}}\,. \tag{3.52}$$

Calculating the light neutrino mass matrix by the see-saw formula, Eq. (3.7), can only generate an absolute neutrino mass scale m^ν_{abs} which is smaller than $M^{(\mathrm{D})}_{11}$. Furthermore,

$$0.05\ \mathrm{eV}\ \leq\ m^\nu_{\mathrm{abs}}\ <\ M^{(\mathrm{D})}_{11}\ <\ \frac{\langle H^U\rangle\, m_{\mathrm{soft}}}{M_{\mathrm{grav}}}\,, \tag{3.53}$$

and thus the soft scale has to be extraordinarily large, at least 500 TeV. This renders Case III highly unattractive. We will therefore not elaborate on the possibility of the Dirac mass matrix being generated by GM/KN+FN any further.

(IV) As for Case III.

4 An X-charged Hidden Sector?

The GS cancellation of chiral anomalies often requires the introduction of further X-charged matter fields, which are singlets under the Standard Model gauge group, *i.e.* hidden sector superfields, for examples see Refs. [7, 8, 69].[6] But as we now explain, in our P_6 conserving FN study, it is possible to have GS anomaly cancellation without exotic, hidden sector, matter. In such a case, anomaly considerations open up a window on the underlying string theory. It should be stressed that the condition of no further X-charged matter is an option which does not affect any of the previous considerations.

Two of the GS conditions are given as[7] [7]

$$\frac{\mathcal{A}_{CCX}}{k_C} = \frac{\mathcal{A}_{GGX}}{24} = \frac{\mathcal{A}_{XXX}}{k_X}, \tag{4.1}$$

where the positive real parameters $k_{...}$ are the affine or Kač-Moody levels, which take integer values for non-Abelian gauge groups. $\mathcal{A}_{...}$ denote the anomaly coefficients, with G standing for "gravity", C for $SU(3)_C$, and X for $U(1)_X$. The $k_{...}$ are related to the corresponding gauge coupling constants at the unification scale

$$g_C^2 k_C = g_X^2 k_X = 2 g_{\text{string}}^2. \tag{4.2}$$

These $2+2$ equations give

$$\begin{aligned} g_{\text{string}} &= g_C \sqrt{\frac{12 \cdot A_{CCX}}{A_{GGX}}}, & g_X &= g_C \sqrt{\frac{A_{CCX}}{A_{XXX}}}\,, \\ k_X &= \frac{24 \cdot A_{XXX}}{A_{GGX}}, & k_C &= \frac{24 \cdot A_{CCX}}{A_{GGX}}\,. \end{aligned} \tag{4.3}$$

Assuming, as in deriving Table 1, that all non-MSSM superfields are color singlets, we have

$$\mathcal{A}_{CCX} = \tfrac{1}{2} \sum_{i=1}^{3} \left(2 X_{Q^i} + X_{\overline{U^i}} + X_{\overline{D^i}} \right), \tag{4.4}$$

$$\begin{aligned} \mathcal{A}_{GGX} &= \sum_{i=1}^{3} \left(6 X_{Q^i} + 3 X_{\overline{U^i}} + 3 X_{\overline{D^i}} + 2 X_{L^i} + X_{\overline{E^i}} + X_{\overline{N^i}} \right) \\ &\quad + 2 \left(X_{H^D} + X_{H^U} \right) \ + \ X_A + \mathcal{A}_{GGX}^{\text{hidden}}, \end{aligned} \tag{4.5}$$

$$\begin{aligned} \mathcal{A}_{XXX} &= \sum_{i=1}^{3} \left(6 {X_{Q^i}}^3 + 3 {X_{\overline{U^i}}}^3 + 3 {X_{\overline{D^i}}}^3 + 2 {X_{L^i}}^3 + {X_{\overline{E^i}}}^3 + {X_{\overline{N^i}}}^3 \right) \\ &\quad + 2 \left({X_{H^D}}^3 + {X_{H^U}}^3 \right) \ + \ {X_A}^3 + \mathcal{A}_{XXX}^{\text{hidden}}. \end{aligned} \tag{4.6}$$

Here and in Eq. (4.2), we have used the standard GUT-normalization of non-Abelian groups with generators t_a such that $\mathrm{tr}[t_a t_b] = \frac{1}{2}\delta_{ab}$. With Table 1, we get *e.g.*

$$\mathcal{A}_{CCX} = \tfrac{3}{2}(6 + x + z), \tag{4.7}$$

$$\mathcal{A}_{GGX} = 62 + 12x + 8z + \Delta_1^{\overline{N}} + \Delta_2^{\overline{N}} + \Delta_3^{\overline{N}} + \Delta_{21}^L + \Delta_{31}^L + 3\Delta^H + \mathcal{A}_{GGX}^{\text{hidden}}. \tag{4.8}$$

So despite the 17 MSSM X-charges being known, *cf.* Tables 3 and 4, we cannot give numerical values for $\{g_{\text{string}},\ g_X,\ k_X,\ k_C\}$, since the $\Delta_i^{\overline{N}}$, $\mathcal{A}_{GGX}^{\text{hidden}}$ and $\mathcal{A}_{XXX}^{\text{hidden}}$ are still

[6] However, in Ref. [7], with three instead of two generations of right-handed neutrinos and $k_C = 3$ the GS anomaly cancellation conditions could also have been satisfied without exotic matter.

[7] We differ from Ref. [7] by a factor of 3 in the denominator of the third ratio.

unknown. *But* now let us suppose that the left-chiral MSSM superfields, as well as the $\overline{N^i}$ and the flavon A are the only X-charged superfields. Hence $\mathcal{A}_{GGX}^{\text{hidden}}$ and $\mathcal{A}_{XXX}^{\text{hidden}}$ vanish.[8] We can then scan all 48+504 X-charge assignments, defined by the parameters $\{x, z, \Delta_{21}^L, \Delta_{31}^L, \Delta^H\}$, for solutions to the fourth equality of Eq. (4.3) with the requirement of k_C *being an integer*:

$$k_C = \frac{36(6+x+z)}{62+12x+8z+\Delta_1^{\overline{N}}+\Delta_2^{\overline{N}}+\Delta_3^{\overline{N}}+\Delta_{21}^L+\Delta_{31}^L+3\Delta^H} \,. \tag{4.9}$$

As pointed out above, the integers $\Delta_i^{\overline{N}}$ are already constrained. Besides the required ordering $\Delta_3^{\overline{N}} \leq \Delta_2^{\overline{N}} \leq \Delta_1^{\overline{N}}$ we have

- For Case I, see below Eqs. (3.39) and (3.40),

$$\begin{aligned} &\text{hierarchical :} \qquad 2 \leq \Delta_3^{\overline{N}}\,, \qquad 2\Delta_1^{\overline{N}} - \Delta_3^{\overline{N}} \leq 21\,, &(4.10)\\ &\text{degenerate :} \qquad 3 \leq \Delta_3^{\overline{N}}\,, \qquad 2\Delta_1^{\overline{N}} - \Delta_3^{\overline{N}} \leq 20\,, &(4.11)\end{aligned}$$

- and for Case II, see Eqs. (3.13,3.14,3.46), with n given in Table 4,

$$\begin{aligned} &\text{hierarchical :} \qquad -n-1 \;=\; \Delta_3^{\overline{N}} = \Delta_2^{\overline{N}} \leq \Delta_1^{\overline{N}} < 0\,, &(4.12)\\ &\text{degenerate :} \qquad -n-1 \;=\; \Delta_3^{\overline{N}} = \Delta_2^{\overline{N}} = \Delta_1^{\overline{N}}\,. &(4.13)\end{aligned}$$

We then find that the 48 sets of Case I are all in accord with $k_C = 3$. The required values for $\sum_i \Delta_i^{\overline{N}}$ are given in Table 6. The conditions on $\Delta_i^{\overline{N}}$ however do *not* determine the X-charges of the right-handed neutrinos uniquely; see Appendix B for a complete list of the remaining possibilities in each case. On the other hand, there exist six cases (# $25, 26, 27, 37, 38, 39$) which are also compatible with $k_C = 2$. In these models, the constraints on $\Delta_i^{\overline{N}}$ fix their individual values uniquely, *cf.* Table 6.

Turning to Case II, the X-charges of the right-handed neutrinos have to satisfy stronger constraints due to Eqs. (4.12,4.13). Demanding Eq. (4.9), only 24 of the 504 models in Table 4 survive; they are displayed in Table 7. In all 24 cases we have $k_C = 2$, and $\Delta_i^{\overline{N}}$ is fixed uniquely as given in Table 8.

A brief comment about the number of possible models before and after imposing Eq. (4.9) is in order. Excluding the right-handed neutrinos, we start with 48 distinct sets of X-charge assignments in Case I and 504 in Case II. This huge difference is due to the fact that in Case II the dependence of the effective neutrino mass matrix $\boldsymbol{M^{(\nu)}}$ on the right-handed neutrinos $\overline{N^i}$, see Eq. (3.9), allows for a variation of Δ^H parameterized by n. In Case I, such a dependence and thus a similar parameter is absent. Taking the right-handed neutrinos into account, the dependence of $\boldsymbol{M^{(\nu)}}$ on $\overline{N^i}$ strongly limits the possible X-charges for $\overline{N^i}$ in Case II [*cf.* Eqs. (4.12,4.13)], whereas for Case I, $X_{\overline{N^i}}$ can be chosen from an interval [*cf.* Eqs. (4.10,4.11)]. When it comes to finding solutions to Eq. (4.9), this freedom of assigning $X_{\overline{N^i}}$ in Case I allows each of the 48 sets of X-charges to be consistent without an X-charged hidden sector. In Case II, the situation is much more constrained, reducing 504 models to only 24 viable ones.

[8] $\mathcal{A}_{GGX}^{\text{hidden}}$ and $\mathcal{A}_{XXX}^{\text{hidden}}$ also vanish if the additional exotic particles are vector-like.

Having determined the Kač-Moody levels k_C which are consistent with the assumption of no exotic X-charged matter, we can calculate the string coupling constant g_{string} from Eq. (4.2). Inserting $g_C[M_{\text{string}}] \approx g_C[M_{\text{GUT}}] = 0.72$ we get

$$g_{\text{string}} \sim 0.88\,, \qquad \text{for} \quad k_C = 3\,, \tag{4.14}$$

$$g_{\text{string}} \sim 0.72\,, \qquad \text{for} \quad k_C = 2\,. \tag{4.15}$$

From k_C we can obtain the other Kač-Moody levels of G_{SM} from the gauge coupling unification relation[9]

$$k_C \;=\; k_W \;=\; \frac{3}{5}\,k_Y\,, \tag{4.16}$$

which adopts the Y-normalization with $Y_L = 1/2$, and has already been implemented when deriving Table 1, *cf.* Ref. [7]. Thus, the models of Case I with $k_C = 3$ have $k_W = 3$ and $k_Y = 5$, while those with $k_C = 2$ (*i.e.* six models of Case I and all models of Case II) demand $k_W = 2$ and $k_Y = 10/3$.

The question arises whether Kač-Moody levels k_C and k_W higher than 1 can be obtained from string model building. Actually, such models have been considered, *e.g.* [73, 74, 75], but a systematic investigation of this issue is lacking. Nevertheless, there are indications that higher Kač-Moody levels might occur rather generically, see *e.g.* Ref. [76]. Also, from the phenomenological point of view, models with higher levels have already been discussed, *e.g.* in Ref. [77]. This is important regarding the possible representations for the Higgs fields in the theory [78, 79, 80].

In addition to the Kač-Moody levels of G_{SM}, we can, from a bottom-up perspective, calculate the $U(1)_X$ gauge coupling constant g_X in those cases, where the $\Delta_i^{\overline{N}}$ are uniquely fixed, *i.e.* for all models with $k_C = 2$. Evaluating the second equality of Eq. (4.3) yields values within the interval

$$g_X \;\in\; [0.0085\,,\, 0.0145]\,, \tag{4.17}$$

which in turn enables us to calculate the mass of the heavy $U(1)_X$ vector boson B'

$$m_{B'} \;\sim\; g_X \cdot \epsilon \cdot M_{\text{grav}} \;\approx\; 5 \times 10^{15}\,\text{GeV}. \tag{4.18}$$

The results for each of the 6+24 models with uniquely fixed X-charge assignments are listed in Tables 6+8. We point out that the k_X corresponding to the above determined g_X are quite high integers, *e.g.* 8839 for # 6 of Case II. This underlines that the scenarios without X-charged exotic matter are to be taken more as an existence proof rather than concrete models.

5 Discussion and Conclusion

In this note, we have devised FN models in which the anomalous $U(1)_X$ gauge symmetry is broken down to the discrete $\mathbb{Z}_6$-symmetry, proton hexality. The masses of the light neutrino states are generated by introducing right-handed neutrinos $\overline{N^i}$ and applying

[9] A non-standard gauge coupling unification with $k_C = k_W = \frac{3}{4}k_Y$ was put forward in Refs. [70, 71] and has been recently applied to FN models in Ref. [72].

the see-saw mechanism. For Case I, the Majorana mass terms of $\overline{N^i}$ originate only from the FN-mechanism, while for Case II they result effectively from a combination of the FN- and the GM/KN-mechanism. Requiring phenomenologically acceptable fermion masses and mixings, the GS mixed anomaly cancellation conditions with gauge coupling unification, as well as the low-energy remnant discrete symmetry P_6, we are led to 48 X-charge assignments for Case I (*cf.* Table 3) and 504 X-charge assignments for Case II (*cf.* Table 4).

Under the assumption of no exotic X-charged particles, all 48 sets of Case I, but only 24 of the 504 sets of Case II are compatible with the GS anomaly cancellation conditions. The X-charges of the resulting 48+24 sets are shown in Tables 5 and 7. Furthermore, we can determine the Kač-Moody levels of G_{SM} in these models. For $k_C = 2$, the X-charges of the right-handed neutrinos are fixed uniquely. This enables us to calculate the gauge coupling constant g_X of $U(1)_X$ in these cases.

All results are listed in Tables 6 and 8 together with the obtained light neutrino mass spectrum, the maximal denominator of the X-charges, as well as some additional comments on each of the models. We emphasize here that all are phenomenologically acceptable because the unknown $\mathcal{O}(1)$ coefficients allow a certain flexibility. However, if asked to select "preferred" models, one can consider the following three criteria:

(1) "nice" CKM matrix,

(2) naturally small CHOOZ mixing angle,

(3) small maximal denominator for the X-charges.

Sets with $y = 0$ lead to our preferred ϵ-structure of the CKM matrix, see Sect. 2. These amount to one third of all the models. The CHOOZ mixing angle corresponds to the $(1,3)$ entry of the MNS matrix. This is naturally suppressed in our models if $\Delta^L_{31} = -2$ ($U^{\mathrm{MNS}}_{13} \sim \epsilon^2$) or $\Delta^L_{31} = -1$ ($U^{\mathrm{MNS}}_{13} \sim \epsilon$), see the end of Sect. 3.2. Altogether 24+21 sets lead to a naturally small CHOOZ angle by virtue of the $U(1)_X$ charge assignments. Finally, we have labeled the 10+3 models with a maximal denominator ≤ 54 by "denom." in the comments. From the aesthetical viewpoint, the most appealing set is # 6 of Case II (Table 8) where all X-charges are multiples of $1/6$. This model features a small CHOOZ angle but, unfortunately, a not so nice CKM matrix. With regard to criterion (3), we however emphasize that models with highly-fractional X-charges are very common, especially when fulfilling phenomenological constraints, see Ref. [81].

Looking for models which satisfy all of the above three criteria, we find that – remarkably enough – only one remains: namely # 32 of Case I (Table 6). This model has a normal hierarchical neutrino mass spectrum with $U^{\mathrm{MNS}}_{13} \sim \epsilon$, the maximal denominator of the X-charges is 30. Without X-charged hidden sector matter, $k_C = 3$ and $\sum_i \Delta^{\overline{N}}_i = 18$, leading to 16 distinct X-charge assignments for the right-handed neutrinos, *cf.* Appendix B.

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Appendix

A $X_{\overline{N^a}}$-Dependence of the Neutrino Masses

For Case II [*cf.* Eq. (3.9)], the Dirac and the Majorana mass matrices can be written as

$$M_{ij}^{(\mathrm{D})} \;=\; A\cdot\alpha_{ij}\,\epsilon^{X_{L^i}+X_{\overline{N^j}}}, \qquad M_{ij}^{(\mathrm{M})} \;=\; B\cdot\beta_{ij}\,\epsilon^{-X_{\overline{N^i}}-X_{\overline{N^j}}}, \tag{A.1}$$

with $A \equiv \langle H^U\rangle\,\epsilon^{X_{H^U}}$ and $B \equiv m_{3/2}$. The dimensionless coefficients α_{ij} and β_{ij} are of order one. In our basis, $M_{ij}^{(\mathrm{M})}$ and thus β_{ij} is diagonal. With this notation the effective light neutrino mass matrix reads

$$\begin{aligned} M_{ij}^{(\nu,\mathrm{II})} &= -\frac{A^2}{B}\cdot\sum_k \frac{\alpha_{ik}\,\alpha_{jk}}{\beta_{kk}}\,\epsilon^{X_{L^i}+X_{L^j}+4X_{\overline{N^k}}} \\ &= -\frac{A^2}{B}\cdot\sum_k a_{ik}\,a_{jk}. \end{aligned} \tag{A.2}$$

In the last step we have defined $a_{ik} \equiv \frac{\alpha_{ik}}{\sqrt{\beta_{kk}}}\,\epsilon^{X_{L^i}+2X_{\overline{N^k}}}$. The light neutrino masses $\widetilde{m} = \frac{A^2}{B}\lambda$ can now be obtained from the characteristic polynomial[10] of $\boldsymbol{M^{(\nu,\mathrm{II})}}$,

$$C_3\lambda^3 + C_2\lambda^2 + C_1\lambda + C_0 \;=\; 0, \tag{A.3}$$

where

$$C_3 \;=\; p_1p_2p_3, \tag{A.4}$$

[10] $\boldsymbol{M^{(\nu)}}$ can be diagonalized by a unitary matrix $\boldsymbol{V}$. From $\boldsymbol{V^T}\cdot\boldsymbol{M^{(\nu)}}\cdot\boldsymbol{V} = \boldsymbol{M^{(\nu)}_{\mathrm{diag}}}$ we obtain the equation $\boldsymbol{M^{(\nu)}}\vec{v} = \widetilde{m}\,\vec{v}^* = \widetilde{m}\,\boldsymbol{P}\vec{v}$. Here, $\vec{v}$ is one of the three normalized vectors of $\boldsymbol{V}$, and $\boldsymbol{P}$ is a diagonal matrix with $P_{ii} = p_i = \frac{v_i^*}{v_i}$. The singular values $\widetilde{m}$ of $\boldsymbol{M^{(\nu)}}$ are determined by the condition $\det(\boldsymbol{M^{(\nu)}} - \widetilde{m}\,\boldsymbol{P}) = 0$, which – up to the phase factors p_i – is just the characteristic polynomial.

$$C_2 = p_1p_2(a_{33}^2+a_{32}^2+a_{31}^2)+p_1p_3(a_{23}^2+a_{22}^2+a_{21}^2)+p_2p_3(a_{13}^2+a_{12}^2+a_{11}^2), \quad (A.5)$$

$$\begin{aligned} C_1 = \; & p_1\left[(a_{33}a_{22}-a_{32}a_{23})^2+(a_{31}a_{23}-a_{33}a_{21})^2+(a_{32}a_{21}-a_{31}a_{22})^2\right] \\ & +p_2\left[(a_{13}a_{32}-a_{12}a_{33})^2+(a_{11}a_{33}-a_{13}a_{31})^2+(a_{12}a_{31}-a_{11}a_{32})^2\right] \\ & +p_3\left[(a_{23}a_{12}-a_{22}a_{13})^2+(a_{21}a_{13}-a_{23}a_{11})^2+(a_{22}a_{11}-a_{21}a_{12})^2\right], \end{aligned} \quad (A.6)$$

$$C_0 = (a_{33}a_{22}a_{11}+a_{31}a_{23}a_{12}+a_{32}a_{21}a_{13}-a_{33}a_{21}a_{12}-a_{31}a_{22}a_{13}-a_{32}a_{23}a_{11})^2. \quad (A.7)$$

As $a_{ik} \sim \epsilon^{X_{L^i}+2X_{\overline{N^k}}}$, the *order* of the coefficients $C_{\dots}$ can be readily determined. With $X_{L^3} \leq X_{L^2} \leq X_{L^1}$ and $X_{\overline{N^3}} \leq X_{\overline{N^2}} \leq X_{\overline{N^1}}$ we get

$$\begin{aligned} C_3 &= c_3, && (A.8)\\ C_2 &= c_2\,\epsilon^{2X_{L^3}+4X_{\overline{N^3}}}, && (A.9)\\ C_1 &= c_1\,\epsilon^{2X_{L^2}+2X_{L^3}+4X_{\overline{N^2}}+4X_{\overline{N^3}}}, && (A.10)\\ C_0 &= c_0\,\epsilon^{2X_{L^1}+2X_{L^2}+2X_{L^3}+4X_{\overline{N^1}}+4X_{\overline{N^2}}+4X_{\overline{N^3}}}, && (A.11) \end{aligned}$$

where c_3, c_2, c_1, c_0 are $\mathcal{O}(1)$ coefficients. Inserting these expressions into Eq. (A.3), the three singular values λ can be obtained. The order of the largest λ depends only on the cubic and the quadratic term: Assuming [this is justified in hindsight[11] from the result Eq. (A.13)]

$$C_3\lambda^3, C_2\lambda^2 > C_1\lambda, C_0, \quad (A.12)$$

we get $C_3\lambda + C_2 = 0$, which yields

$$\lambda_3 = -\frac{c_2}{c_3}\,\epsilon^{2X_{L^3}+4X_{\overline{N^3}}} + \text{equal/higher orders}, \quad (A.13)$$

where "equal" applies only if $X_{L^2} = X_{L^3}$ and $X_{\overline{N^2}} = X_{\overline{N^3}}$. Similarly, the order of the second singular value is derived from the quadratic and the linear term of Eq. (A.3)

$$\lambda_2 = -\frac{c_1}{c_2}\,\epsilon^{2X_{L^2}+4X_{\overline{N^2}}} + \text{equal/higher orders}, \quad (A.14)$$

where "equal" applies only if either $X_{L^2} = X_{L^3}$ and $X_{\overline{N^2}} = X_{\overline{N^3}}$ or $X_{L^1} = X_{L^2}$ and $X_{\overline{N^1}} = X_{\overline{N^2}}$. Finally, the order of λ_1 is obtained from the linear and the constant term

$$\lambda_1 = -\frac{c_0}{c_1}\,\epsilon^{2X_{L^1}+4X_{\overline{N^1}}} + \text{equal/higher orders}, \quad (A.15)$$

where "equal" applies only if $X_{L^1} = X_{L^2}$ and $X_{\overline{N^1}} = X_{\overline{N^2}}$. This yields the following ratios for the light neutrino masses

$$\widetilde{m}_3 : \widetilde{m}_2 : \widetilde{m}_1 \sim \epsilon^{2X_{L^3}+4X_{\overline{N^3}}} : \epsilon^{2X_{L^2}+4X_{\overline{N^2}}} : \epsilon^{2X_{L^1}+4X_{\overline{N^1}}}. \quad (A.16)$$

Analogously, we obtain for Case III that

$$\widetilde{m}_1 : \widetilde{m}_2 : \widetilde{m}_3 \sim \epsilon^{-2X_{L^1}-4X_{\overline{N^1}}} : \epsilon^{-2X_{L^2}-4X_{\overline{N^2}}} : \epsilon^{-2X_{L^3}-4X_{\overline{N^3}}}. \quad (A.17)$$

[11] This method is akin to the slow roll approximation in inflationary cosmology: The Klein-Gordon equation for a homogeneous scalar field φ reads $\ddot\varphi + 3H\dot\varphi + m^2\varphi = 0$, H being the Hubble parameter. Assuming slow roll, *i.e.* $\ddot\varphi \ll \{H\dot\varphi, m^2\varphi\}$, yields $3H\dot\varphi + m^2\varphi = 0$, which's solution in hindsight justifies the slow roll approximation.

B Tables of X-Charges

Combining Table 3 with Table 1 leads to the X-charge assignments of Table 5 (Case I): *e.g.* the choice $\Delta^L_{31} = 0$, $3\zeta + p = -1$ and $\Delta^H = -3$, with $z = 1$, yields for instance

$$X_{H^D} \;=\; \frac{4x\cdot(3x+28)+36y+193}{30(x+7)}.$$

Then one picks a value for x and a value for y. Table 6 displays some features of these 48 possibilities. In search for a low fractionality for the X-charges one finds with $y = 0$ cases # 8 (54), # 17 (48), # 32 (30), with $y = 1$ cases # 21 (30), # 27 (42), with $y = -1$ cases # 19 (18), # 22 (12), # 25 (30), # 31 (18), # 43 (30). The numbers in parentheses give the maximal denominators of the X-charges, in the normalization where $X_A = -1$.

Assuming no X-charged hidden sector superfields, one can determine the Kač-Moody levels k_C and the *sum* of the $\Delta^{\overline{N}}_i$. Recalling the constraints of Eqs. (4.10,4.11), we find for $k_C = 3$ that $(\Delta^{\overline{N}}_1, \Delta^{\overline{N}}_2, \Delta^{\overline{N}}_3)$ can take the following values, respectively:

$\sum_i \Delta^{\overline{N}}_i = 23$: $(8,8,7)$, $(9,7,7)$, $(9,8,6)$, $(9,9,5)$, $(10,7,6)$, $(10,8,5)$, $(10,9,4)$, $(10,10,3)$, $(11,6,6)$, $(11,7,5)$, $(11,8,4)$, $(11,9,3)$, $(12,6,5)$, $(12,7,4)$,

$\sum_i \Delta^{\overline{N}}_i = 20$: $(7,7,6)$, $(8,6,6)$, $(8,7,5)$, $(8,8,4)$, $(9,6,5)$, $(9,7,4)$, $(9,8,3)$, $(9,9,2)$, $(10,5,5)$, $(10,6,4)$, $(10,7,3)$, $(10,8,2)$, $(11,5,4)$, $(11,6,3)$, $(11,7,2)$, $(12,4,4)$, $(12,5,3)$,

$\sum_i \Delta^{\overline{N}}_i = 18$: $(6,6,6)$, $(7,6,5)$, $(7,7,4)$, $(8,5,5)$, $(8,6,4)$, $(8,7,3)$, $(8,8,2)$, $(9,5,4)$, $(9,6,3)$, $(9,7,2)$, $(10,4,4)$, $(10,5,3)$, $(10,6,2)$, $(11,4,3)$, $(11,5,2)$, $(12,3,3)$,

$\sum_i \Delta^{\overline{N}}_i = 17$: $(6,6,5)$, $(7,5,5)$, $(7,6,4)$, $(7,7,3)$, $(8,5,4)$, $(8,6,3)$, $(8,7,2)$, $(9,4,4)$, $(9,5,3)$, $(9,6,2)$, $(10,4,3)$, $(10,5,2)$, $(11,3,3)$, $(11,4,2)$.

For $k_C = 2$, the $\Delta^{\overline{N}}_i$ are uniquely fixed and given in Table 6.

It is interesting to note that the lower limit from thermal leptogenesis, $M^{(\mathrm{M})}_{11} \gtrsim 4\times 10^8$ GeV [64], requires $1 + 2\Delta^{\overline{N}}_1 \lesssim 15$, and hence $\Delta^{\overline{N}}_1 \lesssim 7$. We observe that there is only a small number of combinations allowed within this limit [*e.g.* for $\sum_i \Delta^{\overline{N}}_i = 20$ only $(7,7,6)$ is okay]. On the other hand, some of the solutions above predict no hierarchy between $\overline{N^1}$ and $\overline{N^2}$, and the bound may be less severe, *e.g.* 2×10^7 GeV in Ref. [82]. In the extreme case of resonant enhancement, one can allow for even TeV scale right-handed neutrinos [83].

Case II is treated similarly. However, displaying explicitly the 504 sets of X-charges which are hinted at in Table 4 would fill more than 12 pages. We content ourselves with presenting those 24 models which are consistent without X-charged exotic matter. They are given in Tables 7 and 8. Small maximal denominators of the X-charges are obtained for cases # 6 (6), # 7 (30), # 9 (42).

#	X_{HD}	X_{HU}	X_{Q1}	X_{Q2}	X_{Q3}	$X_{\overline{U1}}$	$X_{\overline{U2}}$	$X_{\overline{U3}}$	$X_{\overline{D1}}$	$X_{\overline{D2}}$	$X_{\overline{D3}}$	X_{L1}	X_{L2}	X_{L3}	$X_{\overline{E1}}$	$X_{\overline{E2}}$	$X_{\overline{E3}}$
1	$\frac{157}{210}$	$-\frac{367}{210}$	$\frac{388}{105}$	$\frac{388}{105}$	$\frac{178}{105}$	$\frac{1271}{210}$	$\frac{431}{210}$	$\frac{11}{210}$	$-\frac{31}{70}$	$-\frac{171}{70}$	$-\frac{171}{70}$	$-\frac{184}{105}$	$-\frac{184}{105}$	$-\frac{184}{105}$	$\frac{1261}{210}$	$\frac{631}{210}$	$\frac{211}{210}$
2	$\frac{193}{210}$	$-\frac{403}{210}$	$\frac{452}{105}$	$\frac{347}{105}$	$\frac{137}{105}$	$\frac{393}{70}$	$\frac{183}{70}$	$\frac{43}{70}$	$-\frac{257}{210}$	$-\frac{467}{210}$	$-\frac{467}{210}$	$-\frac{166}{105}$	$-\frac{166}{105}$	$-\frac{166}{105}$	$\frac{1189}{210}$	$\frac{559}{210}$	$\frac{139}{210}$
3	$\frac{229}{210}$	$-\frac{439}{210}$	$\frac{172}{35}$	$\frac{102}{35}$	$\frac{32}{35}$	$\frac{1087}{210}$	$\frac{667}{210}$	$\frac{247}{210}$	$-\frac{421}{210}$	$-\frac{421}{210}$	$-\frac{421}{210}$	$-\frac{148}{105}$	$-\frac{148}{105}$	$-\frac{148}{105}$	$\frac{1117}{210}$	$\frac{487}{210}$	$\frac{67}{210}$
4	$\frac{281}{240}$	$-\frac{521}{240}$	$\frac{311}{80}$	$\frac{311}{80}$	$\frac{151}{80}$	$\frac{377}{60}$	$\frac{137}{60}$	$\frac{17}{60}$	$-\frac{7}{120}$	$-\frac{247}{120}$	$-\frac{247}{120}$	$-\frac{319}{240}$	$-\frac{319}{240}$	$-\frac{319}{240}$	$\frac{739}{120}$	$\frac{379}{120}$	$\frac{139}{120}$
5	$\frac{317}{240}$	$-\frac{557}{240}$	$\frac{1081}{240}$	$\frac{841}{240}$	$\frac{361}{240}$	$\frac{349}{60}$	$\frac{169}{60}$	$\frac{49}{60}$	$-\frac{33}{40}$	$-\frac{73}{40}$	$-\frac{73}{40}$	$-\frac{283}{240}$	$-\frac{283}{240}$	$-\frac{283}{240}$	$\frac{703}{120}$	$\frac{343}{120}$	$\frac{103}{120}$
6	$\frac{353}{240}$	$-\frac{593}{240}$	$\frac{1229}{240}$	$\frac{749}{240}$	$\frac{269}{240}$	$\frac{107}{20}$	$\frac{67}{20}$	$\frac{27}{20}$	$-\frac{191}{120}$	$-\frac{191}{120}$	$-\frac{191}{120}$	$-\frac{247}{240}$	$-\frac{247}{240}$	$-\frac{247}{240}$	$\frac{667}{120}$	$\frac{307}{120}$	$\frac{67}{120}$
7	$\frac{143}{90}$	$-\frac{233}{90}$	$\frac{551}{135}$	$\frac{551}{135}$	$\frac{281}{135}$	$\frac{1757}{270}$	$\frac{677}{270}$	$\frac{137}{270}$	$\frac{89}{270}$	$-\frac{451}{270}$	$-\frac{451}{270}$	$-\frac{41}{45}$	$-\frac{41}{45}$	$-\frac{41}{45}$	$\frac{569}{90}$	$\frac{299}{90}$	$\frac{119}{90}$
8	$\frac{31}{18}$	$-\frac{49}{18}$	$\frac{127}{27}$	$\frac{100}{27}$	$\frac{46}{27}$	$\frac{325}{54}$	$\frac{163}{54}$	$\frac{55}{54}$	$-\frac{23}{54}$	$-\frac{77}{54}$	$-\frac{77}{54}$	$-\frac{7}{9}$	$-\frac{7}{9}$	$-\frac{7}{9}$	$\frac{109}{18}$	$\frac{55}{18}$	$\frac{19}{18}$
9	$\frac{167}{90}$	$-\frac{257}{90}$	$\frac{719}{135}$	$\frac{449}{135}$	$\frac{179}{135}$	$\frac{1493}{270}$	$\frac{953}{270}$	$\frac{413}{270}$	$-\frac{319}{270}$	$-\frac{319}{270}$	$-\frac{319}{270}$	$-\frac{29}{45}$	$-\frac{29}{45}$	$-\frac{29}{45}$	$\frac{521}{90}$	$\frac{251}{90}$	$\frac{71}{90}$
10	$\frac{601}{300}$	$-\frac{901}{300}$	$\frac{1283}{300}$	$\frac{1283}{300}$	$\frac{683}{300}$	$\frac{1009}{150}$	$\frac{409}{150}$	$\frac{109}{150}$	$\frac{18}{25}$	$-\frac{32}{25}$	$-\frac{32}{25}$	$-\frac{149}{300}$	$-\frac{149}{300}$	$-\frac{149}{300}$	$\frac{487}{75}$	$\frac{262}{75}$	$\frac{112}{75}$
11	$\frac{637}{300}$	$-\frac{937}{300}$	$\frac{1471}{300}$	$\frac{1171}{300}$	$\frac{571}{300}$	$\frac{311}{50}$	$\frac{161}{50}$	$\frac{61}{50}$	$-\frac{2}{75}$	$-\frac{77}{75}$	$-\frac{77}{75}$	$-\frac{113}{300}$	$-\frac{113}{300}$	$-\frac{113}{300}$	$\frac{469}{75}$	$\frac{244}{75}$	$\frac{94}{75}$
12	$\frac{673}{300}$	$-\frac{973}{300}$	$\frac{553}{100}$	$\frac{353}{100}$	$\frac{153}{100}$	$\frac{857}{150}$	$\frac{557}{150}$	$\frac{257}{150}$	$-\frac{58}{75}$	$-\frac{58}{75}$	$-\frac{58}{75}$	$-\frac{77}{300}$	$-\frac{77}{300}$	$-\frac{77}{300}$	$\frac{451}{75}$	$\frac{226}{75}$	$\frac{76}{75}$
13	$\frac{67}{210}$	$-\frac{277}{210}$	$\frac{368}{105}$	$\frac{368}{105}$	$\frac{158}{105}$	$\frac{407}{70}$	$\frac{127}{70}$	$-\frac{13}{70}$	$\frac{37}{210}$	$-\frac{383}{210}$	$-\frac{383}{210}$	$-\frac{124}{105}$	$-\frac{124}{105}$	$-\frac{124}{105}$	$\frac{1231}{210}$	$\frac{601}{210}$	$\frac{181}{210}$
14	$\frac{103}{210}$	$-\frac{313}{210}$	$\frac{144}{35}$	$\frac{109}{35}$	$\frac{39}{35}$	$\frac{1129}{210}$	$\frac{499}{210}$	$\frac{79}{210}$	$-\frac{127}{210}$	$-\frac{337}{210}$	$-\frac{337}{210}$	$-\frac{106}{105}$	$-\frac{106}{105}$	$-\frac{106}{105}$	$\frac{1159}{210}$	$\frac{529}{210}$	$\frac{109}{210}$
15	$\frac{139}{210}$	$-\frac{349}{210}$	$\frac{496}{105}$	$\frac{286}{105}$	$\frac{76}{105}$	$\frac{1037}{210}$	$\frac{617}{210}$	$\frac{197}{210}$	$-\frac{97}{70}$	$-\frac{97}{70}$	$-\frac{97}{70}$	$-\frac{88}{105}$	$-\frac{88}{105}$	$-\frac{88}{105}$	$\frac{1087}{210}$	$\frac{457}{210}$	$\frac{37}{210}$
16	$\frac{179}{240}$	$-\frac{419}{240}$	$\frac{887}{240}$	$\frac{887}{240}$	$\frac{407}{240}$	$\frac{121}{20}$	$\frac{41}{20}$	$\frac{1}{20}$	$\frac{67}{120}$	$-\frac{173}{120}$	$-\frac{173}{120}$	$-\frac{181}{240}$	$-\frac{181}{240}$	$-\frac{181}{240}$	$\frac{721}{120}$	$\frac{361}{120}$	$\frac{121}{120}$
17	$\frac{43}{48}$	$-\frac{91}{48}$	$\frac{69}{16}$	$\frac{53}{16}$	$\frac{21}{16}$	$\frac{67}{12}$	$\frac{31}{12}$	$\frac{7}{12}$	$-\frac{5}{24}$	$-\frac{29}{24}$	$-\frac{29}{24}$	$-\frac{29}{48}$	$-\frac{29}{48}$	$-\frac{29}{48}$	$\frac{137}{24}$	$\frac{65}{24}$	$\frac{17}{24}$
18	$\frac{251}{240}$	$-\frac{491}{240}$	$\frac{1183}{240}$	$\frac{703}{240}$	$\frac{223}{240}$	$\frac{307}{60}$	$\frac{187}{60}$	$\frac{67}{60}$	$-\frac{39}{40}$	$-\frac{39}{40}$	$-\frac{39}{40}$	$-\frac{109}{240}$	$-\frac{109}{240}$	$-\frac{109}{240}$	$\frac{649}{120}$	$\frac{289}{120}$	$\frac{49}{120}$
19	$\frac{7}{6}$	$-\frac{13}{6}$	$\frac{35}{9}$	$\frac{35}{9}$	$\frac{17}{9}$	$\frac{113}{18}$	$\frac{41}{18}$	$\frac{5}{18}$	$\frac{17}{18}$	$-\frac{19}{18}$	$-\frac{19}{18}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{37}{6}$	$\frac{19}{6}$	$\frac{7}{6}$
20	$\frac{13}{10}$	$-\frac{23}{10}$	$\frac{203}{45}$	$\frac{158}{45}$	$\frac{68}{45}$	$\frac{521}{90}$	$\frac{251}{90}$	$\frac{71}{90}$	$\frac{17}{90}$	$-\frac{73}{90}$	$-\frac{73}{90}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$-\frac{1}{5}$	$\frac{59}{10}$	$\frac{29}{10}$	$\frac{9}{10}$
21	$\frac{43}{30}$	$-\frac{73}{30}$	$\frac{77}{15}$	$\frac{47}{15}$	$\frac{17}{15}$	$\frac{53}{10}$	$\frac{33}{10}$	$\frac{13}{10}$	$-\frac{17}{30}$	$-\frac{17}{30}$	$-\frac{17}{30}$	$-\frac{1}{15}$	$-\frac{1}{15}$	$-\frac{1}{15}$	$\frac{169}{30}$	$\frac{79}{30}$	$\frac{19}{30}$
22	$\frac{19}{12}$	$-\frac{31}{12}$	$\frac{49}{12}$	$\frac{49}{12}$	$\frac{25}{12}$	$\frac{13}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{19}{3}$	$\frac{10}{3}$	$\frac{4}{3}$
23	$\frac{511}{300}$	$-\frac{811}{300}$	$\frac{471}{100}$	$\frac{371}{100}$	$\frac{171}{100}$	$\frac{899}{150}$	$\frac{449}{150}$	$\frac{149}{150}$	$\frac{44}{75}$	$-\frac{31}{75}$	$-\frac{31}{75}$	$\frac{61}{300}$	$\frac{61}{300}$	$\frac{61}{300}$	$\frac{457}{75}$	$\frac{232}{75}$	$\frac{82}{75}$
24	$\frac{547}{300}$	$-\frac{847}{300}$	$\frac{1601}{300}$	$\frac{1001}{300}$	$\frac{401}{300}$	$\frac{823}{150}$	$\frac{523}{150}$	$\frac{223}{150}$	$-\frac{4}{25}$	$-\frac{4}{25}$	$-\frac{4}{25}$	$\frac{97}{300}$	$\frac{97}{300}$	$\frac{97}{300}$	$\frac{439}{75}$	$\frac{214}{75}$	$\frac{64}{75}$
25	$-\frac{1}{30}$	$-\frac{29}{30}$	$\frac{17}{5}$	$\frac{17}{5}$	$\frac{7}{5}$	$\frac{167}{30}$	$\frac{47}{30}$	$-\frac{13}{30}$	$\frac{19}{30}$	$-\frac{41}{30}$	$-\frac{41}{30}$	$-\frac{8}{15}$	$-\frac{8}{15}$	$-\frac{23}{15}$	$\frac{167}{30}$	$\frac{77}{30}$	$\frac{47}{30}$
26	$\frac{29}{210}$	$-\frac{239}{210}$	$\frac{421}{105}$	$\frac{316}{105}$	$\frac{106}{105}$	$\frac{359}{70}$	$\frac{149}{70}$	$\frac{9}{70}$	$-\frac{31}{210}$	$-\frac{241}{210}$	$-\frac{241}{210}$	$-\frac{38}{105}$	$-\frac{38}{105}$	$-\frac{143}{105}$	$\frac{1097}{210}$	$\frac{467}{210}$	$\frac{257}{210}$
27	$\frac{13}{42}$	$-\frac{55}{42}$	$\frac{97}{21}$	$\frac{55}{21}$	$\frac{13}{21}$	$\frac{197}{42}$	$\frac{113}{42}$	$\frac{29}{42}$	$-\frac{13}{14}$	$-\frac{13}{14}$	$-\frac{13}{14}$	$-\frac{4}{21}$	$-\frac{4}{21}$	$-\frac{25}{21}$	$\frac{205}{42}$	$\frac{79}{42}$	$\frac{37}{42}$
28	$\frac{97}{240}$	$-\frac{337}{240}$	$\frac{287}{80}$	$\frac{287}{80}$	$\frac{127}{80}$	$\frac{349}{60}$	$\frac{109}{60}$	$-\frac{11}{60}$	$\frac{121}{120}$	$-\frac{119}{120}$	$-\frac{119}{120}$	$-\frac{23}{240}$	$-\frac{23}{240}$	$-\frac{263}{240}$	$\frac{683}{120}$	$\frac{323}{120}$	$\frac{203}{120}$
29	$\frac{133}{240}$	$-\frac{373}{240}$	$\frac{1009}{240}$	$\frac{769}{240}$	$\frac{289}{240}$	$\frac{107}{20}$	$\frac{47}{20}$	$\frac{7}{20}$	$\frac{29}{120}$	$-\frac{91}{120}$	$-\frac{91}{120}$	$\frac{13}{240}$	$\frac{13}{240}$	$-\frac{227}{240}$	$\frac{647}{120}$	$\frac{287}{120}$	$\frac{167}{120}$
30	$\frac{169}{240}$	$-\frac{409}{240}$	$\frac{1157}{240}$	$\frac{677}{240}$	$\frac{197}{240}$	$\frac{293}{60}$	$\frac{173}{60}$	$\frac{53}{60}$	$-\frac{21}{40}$	$-\frac{21}{40}$	$-\frac{21}{40}$	$\frac{49}{240}$	$\frac{49}{240}$	$-\frac{191}{240}$	$\frac{611}{120}$	$\frac{251}{120}$	$\frac{131}{120}$
31	$\frac{5}{6}$	$-\frac{11}{6}$	$\frac{34}{9}$	$\frac{34}{9}$	$\frac{16}{9}$	$\frac{109}{18}$	$\frac{37}{18}$	$\frac{1}{18}$	$\frac{25}{18}$	$-\frac{11}{18}$	$-\frac{11}{18}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{35}{6}$	$\frac{17}{6}$	$\frac{11}{6}$
32	$\frac{29}{30}$	$-\frac{59}{30}$	$\frac{22}{5}$	$\frac{17}{5}$	$\frac{7}{5}$	$\frac{167}{30}$	$\frac{77}{30}$	$\frac{17}{30}$	$\frac{19}{30}$	$-\frac{11}{30}$	$-\frac{11}{30}$	$\frac{7}{15}$	$\frac{7}{15}$	$-\frac{8}{15}$	$\frac{167}{30}$	$\frac{77}{30}$	$\frac{47}{30}$
33	$\frac{11}{10}$	$-\frac{21}{10}$	$\frac{226}{45}$	$\frac{136}{45}$	$\frac{46}{45}$	$\frac{457}{90}$	$\frac{277}{90}$	$\frac{97}{90}$	$-\frac{11}{90}$	$-\frac{11}{90}$	$-\frac{11}{90}$	$\frac{3}{5}$	$\frac{3}{5}$	$-\frac{2}{5}$	$\frac{53}{10}$	$\frac{23}{10}$	$\frac{13}{10}$
34	$\frac{377}{300}$	$-\frac{677}{300}$	$\frac{397}{100}$	$\frac{397}{100}$	$\frac{197}{100}$	$\frac{943}{150}$	$\frac{343}{150}$	$\frac{43}{150}$	$\frac{133}{75}$	$-\frac{17}{75}$	$-\frac{17}{75}$	$\frac{227}{300}$	$\frac{227}{300}$	$-\frac{73}{300}$	$\frac{449}{75}$	$\frac{224}{75}$	$\frac{149}{75}$
35	$\frac{413}{300}$	$-\frac{713}{300}$	$\frac{1379}{300}$	$\frac{1079}{300}$	$\frac{479}{300}$	$\frac{289}{50}$	$\frac{139}{50}$	$\frac{39}{50}$	$\frac{77}{75}$	$\frac{2}{75}$	$\frac{2}{75}$	$\frac{263}{300}$	$\frac{263}{300}$	$-\frac{37}{300}$	$\frac{431}{75}$	$\frac{206}{75}$	$\frac{131}{75}$
36	$\frac{449}{300}$	$-\frac{749}{300}$	$\frac{1567}{300}$	$\frac{967}{300}$	$\frac{367}{300}$	$\frac{791}{150}$	$\frac{491}{150}$	$\frac{191}{150}$	$\frac{7}{25}$	$\frac{7}{25}$	$\frac{7}{25}$	$\frac{299}{300}$	$\frac{299}{300}$	$\frac{1}{300}$	$\frac{413}{75}$	$\frac{188}{75}$	$\frac{113}{75}$
37	$-\frac{53}{210}$	$-\frac{157}{210}$	$\frac{353}{105}$	$\frac{353}{105}$	$\frac{143}{105}$	$\frac{377}{70}$	$\frac{97}{70}$	$-\frac{43}{70}$	$\frac{187}{210}$	$-\frac{233}{210}$	$-\frac{233}{210}$	$\frac{26}{105}$	$-\frac{79}{105}$	$-\frac{184}{105}$	$\frac{1051}{210}$	$\frac{631}{210}$	$\frac{421}{210}$
38	$-\frac{17}{210}$	$-\frac{193}{210}$	$\frac{139}{35}$	$\frac{104}{35}$	$\frac{34}{35}$	$\frac{1039}{210}$	$\frac{409}{210}$	$-\frac{11}{210}$	$\frac{23}{210}$	$-\frac{187}{210}$	$-\frac{187}{210}$	$\frac{44}{105}$	$-\frac{61}{105}$	$-\frac{166}{105}$	$\frac{979}{210}$	$\frac{559}{210}$	$\frac{349}{210}$
39	$\frac{19}{210}$	$-\frac{229}{210}$	$\frac{481}{105}$	$\frac{271}{105}$	$\frac{61}{105}$	$\frac{947}{210}$	$\frac{527}{210}$	$\frac{107}{210}$	$-\frac{47}{70}$	$-\frac{47}{70}$	$-\frac{47}{70}$	$\frac{62}{105}$	$-\frac{43}{105}$	$-\frac{148}{105}$	$\frac{907}{210}$	$\frac{487}{210}$	$\frac{277}{210}$
40	$\frac{47}{240}$	$-\frac{287}{240}$	$\frac{851}{240}$	$\frac{851}{240}$	$\frac{371}{240}$	$\frac{113}{20}$	$\frac{33}{20}$	$-\frac{7}{20}$	$\frac{151}{120}$	$-\frac{89}{120}$	$-\frac{89}{120}$	$\frac{167}{240}$	$-\frac{73}{240}$	$-\frac{313}{240}$	$\frac{613}{120}$	$\frac{373}{120}$	$\frac{253}{120}$
41	$\frac{83}{240}$	$-\frac{323}{240}$	$\frac{333}{80}$	$\frac{253}{80}$	$\frac{93}{80}$	$\frac{311}{60}$	$\frac{131}{60}$	$\frac{11}{60}$	$\frac{59}{120}$	$-\frac{61}{120}$	$-\frac{61}{120}$	$\frac{203}{240}$	$-\frac{37}{240}$	$-\frac{277}{240}$	$\frac{577}{120}$	$\frac{337}{120}$	$\frac{217}{120}$
42	$\frac{119}{240}$	$-\frac{359}{240}$	$\frac{1147}{240}$	$\frac{667}{240}$	$\frac{187}{240}$	$\frac{283}{60}$	$\frac{163}{60}$	$\frac{43}{60}$	$-\frac{11}{40}$	$-\frac{11}{40}$	$-\frac{11}{40}$	$\frac{239}{240}$	$-\frac{1}{240}$	$-\frac{241}{240}$	$\frac{541}{120}$	$\frac{301}{120}$	$\frac{181}{120}$
43	$\frac{19}{30}$	$-\frac{49}{30}$	$\frac{56}{15}$	$\frac{56}{15}$	$\frac{26}{15}$	$\frac{59}{10}$	$\frac{19}{10}$	$-\frac{1}{10}$	$\frac{49}{30}$	$-\frac{11}{30}$	$-\frac{11}{30}$	$\frac{17}{15}$	$\frac{2}{15}$	$-\frac{13}{15}$	$\frac{157}{30}$	$\frac{97}{30}$	$\frac{67}{30}$
44	$\frac{23}{30}$	$-\frac{53}{30}$	$\frac{196}{45}$	$\frac{151}{45}$	$\frac{61}{45}$	$\frac{487}{90}$	$\frac{217}{90}$	$\frac{37}{90}$	$\frac{79}{90}$	$-\frac{11}{90}$	$-\frac{11}{90}$	$\frac{19}{15}$	$\frac{4}{15}$	$-\frac{11}{15}$	$\frac{149}{30}$	$\frac{89}{30}$	$\frac{59}{30}$
45	$\frac{9}{10}$	$-\frac{19}{10}$	$\frac{224}{45}$	$\frac{134}{45}$	$\frac{44}{45}$	$\frac{443}{90}$	$\frac{263}{90}$	$\frac{83}{90}$	$\frac{11}{90}$	$\frac{11}{90}$	$\frac{11}{90}$	$\frac{7}{5}$	$\frac{2}{5}$	$-\frac{3}{5}$	$\frac{47}{10}$	$\frac{27}{10}$	$\frac{17}{10}$
46	$\frac{319}{300}$	$-\frac{619}{300}$	$\frac{1177}{300}$	$\frac{1177}{300}$	$\frac{577}{300}$	$\frac{307}{50}$	$\frac{107}{50}$	$\frac{7}{50}$	$\frac{151}{75}$	$\frac{1}{75}$	$\frac{1}{75}$	$\frac{469}{300}$	$\frac{169}{300}$	$-\frac{131}{300}$	$\frac{403}{75}$	$\frac{253}{75}$	$\frac{178}{75}$
47	$\frac{71}{60}$	$-\frac{131}{60}$	$\frac{91}{20}$	$\frac{71}{20}$	$\frac{31}{20}$	$\frac{169}{30}$	$\frac{79}{30}$	$\frac{19}{30}$	$\frac{19}{15}$	$\frac{4}{15}$	$\frac{4}{15}$	$\frac{101}{60}$	$\frac{41}{60}$	$-\frac{19}{60}$	$\frac{77}{15}$	$\frac{47}{15}$	$\frac{32}{15}$
48	$\frac{391}{300}$	$-\frac{691}{300}$	$\frac{1553}{300}$	$\frac{953}{300}$	$\frac{353}{300}$	$\frac{769}{150}$	$\frac{469}{150}$	$\frac{169}{150}$	$\frac{13}{25}$	$\frac{13}{25}$	$\frac{13}{25}$	$\frac{541}{300}$	$\frac{241}{300}$	$-\frac{59}{300}$	$\frac{367}{75}$	$\frac{217}{75}$	$\frac{142}{75}$

Table 5: The numerical results for the 48 possible X-charge assignments of Case I, determined from Tables 3+1.

#	Δ^L_{21}	Δ^L_{31}	$3\zeta+p$	Δ^H	x	y	spectrum	maximal denominator	anomalies : $k_C, \sum_i \Delta_i^{\overline{N}}$	comments
1	0	0	-1	-3	0	-1	deg.	210	3 23	$(\sum_i m_i \approx 5.0\,\mathrm{eV})$
2	0	0	-1	-3	0	0	deg.	210	3 23	CKM $(\sum_i m_i \approx 5.0\,\mathrm{eV})$
3	0	0	-1	-3	0	1	deg.	210	3 23	$(\sum_i m_i \approx 5.0\,\mathrm{eV})$
4	0	0	-1	-3	1	-1	deg.	240	3 23	$(\sum_i m_i \approx 3.2\,\mathrm{eV})$
5	0	0	-1	-3	1	0	deg.	240	3 23	CKM $(\sum_i m_i \approx 3.2\,\mathrm{eV})$
6	0	0	-1	-3	1	1	deg.	240	3 23	$(\sum_i m_i \approx 3.2\,\mathrm{eV})$
7	0	0	-1	-3	2	-1	deg.	270	3 23	$(\sum_i m_i \approx 2.1\,\mathrm{eV})$
8	0	0	-1	-3	2	0	deg.	54	3 23	CKM, denom. $(\sum_i m_i \approx 2.1\,\mathrm{eV})$
9	0	0	-1	-3	2	1	deg.	270	3 23	$(\sum_i m_i \approx 2.1\,\mathrm{eV})$
10	0	0	-1	-3	3	-1	deg.	300	3 23	$(\sum_i m_i \approx 1.4\,\mathrm{eV})$
11	0	0	-1	-3	3	0	deg.	300	3 23	CKM $(\sum_i m_i \approx 1.4\,\mathrm{eV})$
12	0	0	-1	-3	3	1	deg.	300	3 23	$(\sum_i m_i \approx 1.4\,\mathrm{eV})$
13	0	0	-1	-2	0	-1	inv. & nor. hier.	210	3 20	
14	0	0	-1	-2	0	0	inv. & nor. hier.	210	3 20	CKM
15	0	0	-1	-2	0	1	inv. & nor. hier.	210	3 20	
16	0	0	-1	-2	1	-1	inv. & nor. hier.	240	3 20	
17	0	0	-1	-2	1	0	inv. & nor. hier.	48	3 20	CKM, denom.
18	0	0	-1	-2	1	1	inv. & nor. hier.	240	3 20	
19	0	0	-1	-2	2	-1	inv. & nor. hier.	18	3 20	denom.
20	0	0	-1	-2	2	0	inv. & nor. hier.	90	3 20	CKM
21	0	0	-1	-2	2	1	inv. & nor. hier.	30	3 20	denom.
22	0	0	-1	-2	3	-1	inv. & nor. hier.	12	3 20	denom.
23	0	0	-1	-2	3	0	inv. & nor. hier.	300	3 20	CKM
24	0	0	-1	-2	3	1	inv. & nor. hier.	300	3 20	
25	0	-1	-2	-1	0	-1	nor. hier.	30	2 60 3 18	CHOOZ, denom., $\underline{\Delta^{\overline{N}}_{1,2,3} = 20, g_X = 0.0141}$
26	0	-1	-2	-1	0	0	nor. hier.	210	2 60 3 18	CHOOZ, CKM, $\underline{\Delta^{\overline{N}}_{1,2,3} = 20, g_X = 0.0142}$
27	0	-1	-2	-1	0	1	nor. hier.	42	2 60 3 18	CHOOZ, denom., $\underline{\Delta^{\overline{N}}_{1,2,3} = 20, g_X = 0.0141}$
28	0	-1	-2	-1	1	-1	nor. hier.	240	3 18	CHOOZ
29	0	-1	-2	-1	1	0	nor. hier.	240	3 18	CHOOZ, CKM
30	0	-1	-2	-1	1	1	nor. hier.	240	3 18	CHOOZ
31	0	-1	-2	-1	2	-1	nor. hier.	18	3 18	CHOOZ, denom.
32	0	-1	-2	-1	2	0	nor. hier.	30	3 18	CHOOZ, CKM, denom.
33	0	-1	-2	-1	2	1	nor. hier.	90	3 18	CHOOZ
34	0	-1	-2	-1	3	-1	nor. hier.	300	3 18	CHOOZ
35	0	-1	-2	-1	3	0	nor. hier.	300	3 18	CHOOZ, CKM
36	0	-1	-2	-1	3	1	nor. hier.	300	3 18	CHOOZ
37	-1	-2	-4	0	0	-1	nor. hier.	210	2 59 3 17	CHOOZ, $\underline{\Delta^{\overline{N}}_{1,2} = 20, \Delta^{\overline{N}}_3 = 19, g_X = 0.0145}$
38	-1	-2	-4	0	0	0	nor. hier.	210	2 59 3 17	CHOOZ, CKM, $\underline{\Delta^{\overline{N}}_{1,2} = 20, \Delta^{\overline{N}}_3 = 19, g_X = 0.0145}$
39	-1	-2	-4	0	0	1	nor. hier.	210	2 59 3 17	CHOOZ, $\underline{\Delta^{\overline{N}}_{1,2} = 20, \Delta^{\overline{N}}_3 = 19, g_X = 0.0145}$
40	-1	-2	-4	0	1	-1	nor. hier.	240	3 17	CHOOZ
41	-1	-2	-4	0	1	0	nor. hier.	240	3 17	CHOOZ, CKM
42	-1	-2	-4	0	1	1	nor. hier.	240	3 17	CHOOZ
43	-1	-2	-4	0	2	-1	nor. hier.	30	3 17	CHOOZ, denom.
44	-1	-2	-4	0	2	0	nor. hier.	90	3 17	CHOOZ, CKM
45	-1	-2	-4	0	2	1	nor. hier.	90	3 17	CHOOZ
46	-1	-2	-4	0	3	-1	nor. hier.	300	3 17	CHOOZ
47	-1	-2	-4	0	3	0	nor. hier.	60	3 17	CHOOZ, CKM
48	-1	-2	-4	0	3	1	nor. hier.	300	3 17	CHOOZ

Table 6: The features of the X-charge assignments in Table 5 (Case I). In the comments we state the reason for preferring individual cases: "CKM" means that this model naturally exhibits a nice CKM matrix, *i.e.* $y = 0$. "CHOOZ" refers to a naturally small CHOOZ angle: $\sin\theta_{13} \approx \epsilon^{|\Delta^L_{31}|}$, with $|\Delta^L_{31}| = 1, 2$. We write "denom." to label cases where the X-charges have a maximal denominator $\leq$ 54. For the degenerate scenarios we show the naïve sum of the neutrino masses, $\sum_i m_i$, without $\mathcal{O}(1)$ coefficients. Assuming no exotic matter, the three $\Delta_i^{\overline{N}}$ are uniquely fixed for $k_C = 2$, unlike for $k_C = 3$.

#	X_{HD}	X_{HU}	X_{Q^1}	X_{Q^2}	X_{Q^3}	$X_{\overline{U^1}}$	$X_{\overline{U^2}}$	$X_{\overline{U^3}}$	$X_{\overline{D^1}}$	$X_{\overline{D^2}}$	$X_{\overline{D^3}}$	X_{L^1}	X_{L^2}	X_{L^3}	$X_{\overline{E^1}}$	$X_{\overline{E^2}}$	$X_{\overline{E^3}}$
1	$-\frac{2453}{210}$	$\frac{2243}{210}$	$-\frac{64}{35}$	$-\frac{64}{35}$	$-\frac{134}{35}$	$-\frac{179}{210}$	$-\frac{1019}{210}$	$-\frac{1439}{210}$	$\frac{3677}{210}$	$\frac{3257}{210}$	$\frac{3257}{210}$	$\frac{1556}{105}$	$\frac{1556}{105}$	$\frac{1556}{105}$	$\frac{391}{210}$	$-\frac{239}{210}$	$-\frac{659}{210}$
2	$-\frac{2417}{210}$	$\frac{2207}{210}$	$-\frac{128}{105}$	$-\frac{233}{105}$	$-\frac{443}{105}$	$-\frac{271}{210}$	$-\frac{901}{210}$	$-\frac{1321}{210}$	$\frac{1171}{70}$	$\frac{1101}{70}$	$\frac{1101}{70}$	$\frac{1574}{105}$	$\frac{1574}{105}$	$\frac{1574}{105}$	$\frac{319}{210}$	$-\frac{311}{210}$	$-\frac{731}{210}$
3	$-\frac{2381}{210}$	$\frac{2171}{210}$	$-\frac{64}{105}$	$-\frac{274}{105}$	$-\frac{484}{105}$	$-\frac{121}{70}$	$-\frac{261}{70}$	$-\frac{401}{70}$	$\frac{3349}{210}$	$\frac{3349}{210}$	$\frac{3349}{210}$	$\frac{1592}{105}$	$\frac{1592}{105}$	$\frac{1592}{105}$	$\frac{247}{210}$	$-\frac{383}{210}$	$-\frac{803}{210}$
4	$-\frac{2347}{210}$	$\frac{2137}{210}$	$-\frac{163}{105}$	$-\frac{163}{105}$	$-\frac{373}{105}$	$-\frac{131}{210}$	$-\frac{971}{210}$	$-\frac{1391}{210}$	$\frac{1171}{70}$	$\frac{1031}{70}$	$\frac{1031}{70}$	$\frac{1504}{105}$	$\frac{1504}{105}$	$\frac{1399}{105}$	$\frac{389}{210}$	$-\frac{241}{210}$	$-\frac{451}{210}$
5	$-\frac{2311}{210}$	$\frac{2101}{210}$	$-\frac{33}{35}$	$-\frac{68}{35}$	$-\frac{138}{35}$	$-\frac{223}{210}$	$-\frac{853}{210}$	$-\frac{1273}{210}$	$\frac{3349}{210}$	$\frac{3139}{210}$	$\frac{3139}{210}$	$\frac{1522}{105}$	$\frac{1522}{105}$	$\frac{1417}{105}$	$\frac{317}{210}$	$-\frac{313}{210}$	$-\frac{523}{210}$
6	$-\frac{65}{6}$	$\frac{59}{6}$	$-\frac{1}{3}$	$-\frac{7}{3}$	$-\frac{13}{3}$	$-\frac{3}{2}$	$-\frac{7}{2}$	$-\frac{11}{2}$	$\frac{91}{6}$	$\frac{91}{6}$	$\frac{91}{6}$	$\frac{44}{3}$	$\frac{44}{3}$	$\frac{41}{3}$	$\frac{7}{6}$	$-\frac{11}{6}$	$-\frac{17}{6}$
7	$-\frac{361}{30}$	$\frac{331}{30}$	$-\frac{29}{15}$	$-\frac{29}{15}$	$-\frac{59}{15}$	$-\frac{11}{10}$	$-\frac{51}{10}$	$-\frac{71}{10}$	$\frac{539}{30}$	$\frac{479}{30}$	$\frac{479}{30}$	$\frac{232}{15}$	$\frac{232}{15}$	$\frac{217}{15}$	$\frac{47}{30}$	$-\frac{43}{30}$	$-\frac{73}{30}$
8	$-\frac{2491}{210}$	$\frac{2281}{210}$	$-\frac{139}{105}$	$-\frac{244}{105}$	$-\frac{454}{105}$	$-\frac{323}{210}$	$-\frac{953}{210}$	$-\frac{1373}{210}$	$\frac{1203}{70}$	$\frac{1133}{70}$	$\frac{1133}{70}$	$\frac{1642}{105}$	$\frac{1642}{105}$	$\frac{1537}{105}$	$\frac{257}{210}$	$-\frac{373}{210}$	$-\frac{583}{210}$
9	$-\frac{491}{42}$	$\frac{449}{42}$	$-\frac{5}{7}$	$-\frac{19}{7}$	$-\frac{33}{7}$	$-\frac{83}{42}$	$-\frac{167}{42}$	$-\frac{251}{42}$	$\frac{689}{42}$	$\frac{689}{42}$	$\frac{689}{42}$	$\frac{332}{21}$	$\frac{332}{21}$	$\frac{311}{21}$	$\frac{37}{42}$	$-\frac{89}{42}$	$-\frac{131}{42}$
10	$-\frac{1103}{90}$	$\frac{1013}{90}$	$-\frac{296}{135}$	$-\frac{296}{135}$	$-\frac{566}{135}$	$-\frac{287}{270}$	$-\frac{1367}{270}$	$-\frac{1907}{270}$	$\frac{5521}{270}$	$\frac{4981}{270}$	$\frac{4981}{270}$	$\frac{821}{45}$	$\frac{821}{45}$	$\frac{776}{45}$	$\frac{91}{90}$	$-\frac{179}{90}$	$-\frac{269}{90}$
11	$-\frac{1091}{90}$	$\frac{1001}{90}$	$-\frac{212}{135}$	$-\frac{347}{135}$	$-\frac{617}{135}$	$-\frac{419}{270}$	$-\frac{1229}{270}$	$-\frac{1769}{270}$	$\frac{5317}{270}$	$\frac{5047}{270}$	$\frac{5047}{270}$	$\frac{827}{45}$	$\frac{827}{45}$	$\frac{782}{45}$	$\frac{67}{90}$	$-\frac{203}{90}$	$-\frac{293}{90}$
12	$-\frac{1079}{90}$	$\frac{989}{90}$	$-\frac{128}{135}$	$-\frac{398}{135}$	$-\frac{668}{135}$	$-\frac{551}{270}$	$-\frac{1091}{270}$	$-\frac{1631}{270}$	$\frac{5113}{270}$	$\frac{5113}{270}$	$\frac{5113}{270}$	$\frac{833}{45}$	$\frac{833}{45}$	$\frac{788}{45}$	$\frac{43}{90}$	$-\frac{227}{90}$	$-\frac{317}{90}$
13	$-\frac{2393}{210}$	$\frac{2183}{210}$	$-\frac{167}{105}$	$-\frac{167}{105}$	$-\frac{377}{105}$	$-\frac{169}{210}$	$-\frac{1009}{210}$	$-\frac{1429}{210}$	$\frac{1189}{70}$	$\frac{1049}{70}$	$\frac{1049}{70}$	$\frac{1586}{105}$	$\frac{1481}{105}$	$\frac{1376}{105}$	$\frac{271}{210}$	$-\frac{149}{210}$	$-\frac{359}{210}$
14	$-\frac{2357}{210}$	$\frac{2147}{210}$	$-\frac{103}{105}$	$-\frac{208}{105}$	$-\frac{418}{105}$	$-\frac{87}{70}$	$-\frac{297}{70}$	$-\frac{437}{70}$	$\frac{3403}{210}$	$\frac{3193}{210}$	$\frac{3193}{210}$	$\frac{1604}{105}$	$\frac{1499}{105}$	$\frac{1394}{105}$	$\frac{199}{210}$	$-\frac{221}{210}$	$-\frac{431}{210}$
15	$-\frac{2321}{210}$	$\frac{2111}{210}$	$-\frac{13}{35}$	$-\frac{83}{35}$	$-\frac{153}{35}$	$-\frac{353}{210}$	$-\frac{773}{210}$	$-\frac{1193}{210}$	$\frac{3239}{210}$	$\frac{3239}{210}$	$\frac{3239}{210}$	$\frac{1622}{105}$	$\frac{1517}{105}$	$\frac{1412}{105}$	$\frac{127}{210}$	$-\frac{293}{210}$	$-\frac{503}{210}$
16	$-\frac{2573}{210}$	$\frac{2363}{210}$	$-\frac{69}{35}$	$-\frac{69}{35}$	$-\frac{139}{35}$	$-\frac{269}{210}$	$-\frac{1109}{210}$	$-\frac{1529}{210}$	$\frac{3827}{210}$	$\frac{3407}{210}$	$\frac{3407}{210}$	$\frac{1706}{105}$	$\frac{1601}{105}$	$\frac{1496}{105}$	$\frac{211}{210}$	$-\frac{209}{210}$	$-\frac{419}{210}$
17	$-\frac{2537}{210}$	$\frac{2327}{210}$	$-\frac{143}{105}$	$-\frac{248}{105}$	$-\frac{458}{105}$	$-\frac{361}{210}$	$-\frac{991}{210}$	$-\frac{1411}{210}$	$\frac{1221}{70}$	$\frac{1151}{70}$	$\frac{1151}{70}$	$\frac{1724}{105}$	$\frac{1619}{105}$	$\frac{1514}{105}$	$\frac{139}{210}$	$-\frac{281}{210}$	$-\frac{491}{210}$
18	$-\frac{2501}{210}$	$\frac{2291}{210}$	$-\frac{79}{105}$	$-\frac{289}{105}$	$-\frac{499}{105}$	$-\frac{151}{70}$	$-\frac{291}{70}$	$-\frac{431}{70}$	$\frac{3499}{210}$	$\frac{3499}{210}$	$\frac{3499}{210}$	$\frac{1742}{105}$	$\frac{1637}{105}$	$\frac{1532}{105}$	$\frac{67}{210}$	$-\frac{353}{210}$	$-\frac{563}{210}$
19	$-\frac{2809}{240}$	$\frac{2569}{240}$	$-\frac{437}{240}$	$-\frac{437}{240}$	$-\frac{917}{240}$	$-\frac{53}{60}$	$-\frac{293}{60}$	$-\frac{413}{60}$	$\frac{741}{40}$	$\frac{661}{40}$	$\frac{661}{40}$	$\frac{4031}{240}$	$\frac{3791}{240}$	$\frac{3551}{240}$	$\frac{109}{120}$	$-\frac{131}{120}$	$-\frac{251}{120}$
20	$-\frac{2773}{240}$	$\frac{2533}{240}$	$-\frac{289}{240}$	$-\frac{529}{240}$	$-\frac{1009}{240}$	$-\frac{27}{20}$	$-\frac{87}{20}$	$-\frac{127}{20}$	$\frac{2131}{120}$	$\frac{2011}{120}$	$\frac{2011}{120}$	$\frac{4067}{240}$	$\frac{3827}{240}$	$\frac{3587}{240}$	$\frac{73}{120}$	$-\frac{167}{120}$	$-\frac{287}{120}$
21	$-\frac{2737}{240}$	$\frac{2497}{240}$	$-\frac{47}{80}$	$-\frac{207}{80}$	$-\frac{367}{80}$	$-\frac{109}{60}$	$-\frac{229}{60}$	$-\frac{349}{60}$	$\frac{2039}{120}$	$\frac{2039}{120}$	$\frac{2039}{120}$	$\frac{4103}{240}$	$\frac{3863}{240}$	$\frac{3623}{240}$	$\frac{37}{120}$	$-\frac{203}{120}$	$-\frac{323}{120}$
22	$-\frac{1121}{90}$	$\frac{1031}{90}$	$-\frac{302}{135}$	$-\frac{302}{135}$	$-\frac{572}{135}$	$-\frac{329}{270}$	$-\frac{1409}{270}$	$-\frac{1949}{270}$	$\frac{5587}{270}$	$\frac{5047}{270}$	$\frac{5047}{270}$	$\frac{857}{45}$	$\frac{812}{45}$	$\frac{767}{45}$	$\frac{37}{90}$	$-\frac{143}{90}$	$-\frac{233}{90}$
23	$-\frac{1109}{90}$	$\frac{1019}{90}$	$-\frac{218}{135}$	$-\frac{353}{135}$	$-\frac{623}{135}$	$-\frac{461}{270}$	$-\frac{1271}{270}$	$-\frac{1811}{270}$	$\frac{5383}{270}$	$\frac{5113}{270}$	$\frac{5113}{270}$	$\frac{863}{45}$	$\frac{818}{45}$	$\frac{773}{45}$	$\frac{13}{90}$	$-\frac{167}{90}$	$-\frac{257}{90}$
24	$-\frac{1097}{90}$	$\frac{1007}{90}$	$-\frac{134}{135}$	$-\frac{404}{135}$	$-\frac{674}{135}$	$-\frac{593}{270}$	$-\frac{1133}{270}$	$-\frac{1673}{270}$	$\frac{5179}{270}$	$\frac{5179}{270}$	$\frac{5179}{270}$	$\frac{869}{45}$	$\frac{824}{45}$	$\frac{779}{45}$	$-\frac{11}{90}$	$-\frac{191}{90}$	$-\frac{281}{90}$

Table 7: The numerical results for the X-charge assignments of Case II which allow no further matter to be introduced. These 24 models are obtained from the 504 distinct sets of Table 4.

#	Δ^L_{21}	Δ^L_{31}	$3\zeta+p$	Δ^H	x	n	y	spectrum	max. denom.	k_C	$\Delta^{\overline{N}}_1$	$\Delta^{\overline{N}}_2$	$\Delta^{\overline{N}}_3$	g_X	comments
1	0	0	−1	26	0	8	−1	inv. & nor. hier.	210	2	−4	−9	−9	0.0100	
2	0	0	−1	26	0	8	0	inv. & nor. hier.	210	2	−4	−9	−9	0.0100	CKM
3	0	0	−1	26	0	8	1	inv. & nor.hier.	210	2	−4	−9	−9	0.0100	
4	0	−1	−2	25	0	7	−1	nor. hier.	210	2	−2	−8	−8	0.0107	CHOOZ
5	0	−1	−2	25	0	7	0	nor. hier.	210	2	−2	−8	−8	0.0108	CHOOZ, CKM
6	0	−1	−2	25	0	7	1	nor. hier.	6	2	−2	−8	−8	0.0108	CHOOZ, denom.
7	0	−1	−2	27	0	8	−1	nor. hier.	30	2	−6	−9	−9	0.0096	CHOOZ, denom.
8	0	−1	−2	27	0	8	0	nor. hier.	210	2	−6	−9	−9	0.0096	CHOOZ, CKM
9	0	−1	−2	27	0	8	1	nor. hier.	42	2	−6	−9	−9	0.0096	CHOOZ, denom.
10	0	−1	−2	30	2	9	−1	nor. hier.	270	2	−1	−10	−10	0.0086	CHOOZ
11	0	−1	−2	30	2	9	0	nor. hier.	270	2	−1	−10	−10	0.0086	CHOOZ, CKM
12	0	−1	−2	30	2	9	1	nor. hier.	270	2	−1	−10	−10	0.0086	CHOOZ
13	−1	−2	−4	26	0	7	−1	nor. hier.	210	2	−3	−8	−8	0.0105	CHOOZ
14	−1	−2	−4	26	0	7	0	nor. hier.	210	2	−3	−8	−8	0.0105	CHOOZ, CKM
15	−1	−2	−4	26	0	7	1	nor. hier.	210	2	−3	−8	−8	0.0105	CHOOZ
16	−1	−2	−4	28	0	8	−1	nor. hier.	210	2	−7	−9	−9	0.0094	CHOOZ
17	−1	−2	−4	28	0	8	0	nor. hier.	210	2	−7	−9	−9	0.0094	CHOOZ, CKM
18	−1	−2	−4	28	0	8	1	nor. hier.	210	2	−7	−9	−9	0.0094	CHOOZ
19	−1	−2	−4	28	1	8	−1	nor. hier.	240	2	−1	−9	−9	0.0096	CHOOZ
20	−1	−2	−4	28	1	8	0	nor. hier.	240	2	−1	−9	−9	0.0097	CHOOZ, CKM
21	−1	−2	−4	28	1	8	1	nor. hier.	240	2	−1	−9	−9	0.0097	CHOOZ
22	−1	−2	−4	31	2	9	−1	nor. hier.	270	2	−2	−10	−10	0.0085	CHOOZ
23	−1	−2	−4	31	2	9	0	nor. hier.	270	2	−2	−10	−10	0.0085	CHOOZ, CKM
24	−1	−2	−4	31	2	9	1	nor. hier.	270	2	−2	−10	−10	0.0085	CHOOZ

Table 8: The features of the X-charge assignments in Table 7 (Case II). In the comments we state the reason for preferring individual cases: "CKM" refers to a nice CKM matrix, "CHOOZ" to a naturally small CHOOZ angle ($|\Delta^L_{31}| = 1, 2$), and "denom." labels models where the X-charges have a maximal denominator $\leq$ 42.

References

[1] L. M. Krauss and F. Wilczek. *Phys. Rev. Lett.*, 62:1221, 1989.

[2] F. J. Wegner. *J. Math. Phys.*, 12:2259–2272, 1971.

[3] L. E. Ibáñez and G. G. Ross. *Nucl. Phys.*, B368:3–37, 1992.

[4] H. K. Dreiner, C. Luhn, and M. Thormeier. *Phys. Rev.*, D73:075007, 2006, hep-ph/0512163.

[5] C. Luhn and M. Thormeier. 2007, arXiv:0711.0756.

[6] B. C. Allanach, A. Dedes, and H. K. Dreiner. *Phys. Rev.*, D69:115002, 2004, hep-ph/0309196.

[7] H. K. Dreiner, H. Murayama, and M. Thormeier. *Nucl. Phys.*, B729:278–316, 2005, hep-ph/0312012.

[8] H. K. Dreiner, C. Luhn, H. Murayama, and M. Thormeier. *Nucl. Phys.*, B774:127, 2007, hep-ph/0610026.

[9] S. P. Martin. *Phys. Rev.*, D46:2769–2772, 1992, hep-ph/9207218.

[10] R. N. Mohapatra and M. Ratz. *Phys. Rev.*, D76:095003, 2007, arXiv:0707.4070.

[11] C. D. Froggatt and H. B. Nielsen. *Nucl. Phys.*, B147:277, 1979.

[12] M. B. Green and J. H. Schwarz. *Phys. Lett.*, B149:117–122, 1984.

[13] R. Harnik, D. T. Larson, H. Murayama, and M. Thormeier. *Nucl. Phys.*, B706:372, 2005, hep-ph/0404260.

[14] D. T. Larson. 2004, hep-ph/0410035.

[15] G. F. Giudice and A. Masiero. *Phys. Lett.*, B206:480–484, 1988.

[16] J. E. Kim and H.-P. Nilles. *Mod. Phys. Lett.*, A9:3575–3584, 1994, hep-ph/9406296.

[17] L. E. Ibáñez and G. G. Ross. *Phys. Lett.*, B332:100–110, 1994, hep-ph/9403338.

[18] V. Jain and R. Shrock. *Phys. Lett.*, B352:83, 1995, hep-ph/9412367.

[19] E. Dudas, S. Pokorski, and C. A. Savoy. *Phys. Lett.*, B356:45–55, 1995, hep-ph/9504292.

[20] P. Binétruy, S. Lavignac, and P. Ramond. *Nucl. Phys.*, B477:353–377, 1996, hep-ph/9601243.

[21] N. Irges, S. Lavignac, and P. Ramond. *Phys. Rev.*, D58:035003, 1998, hep-ph/9802334.

[22] N. Maekawa. *Prog. Theor. Phys.*, 106:401–418, 2001, hep-ph/0104200.

[23] M. Cvetič et al. *Phys. Rev.*, D56:2861–2885, 1997, hep-ph/9703317.

[24] P. Langacker and J. Wang. *Phys. Rev.*, D58:115010, 1998, hep-ph/9804428.

[25] D. A. Demir, G. L. Kane, and T. T. Wang. *Phys. Rev.*, D72:015012, 2005, hep-ph/0503290.

[26] M.-C. Chen, A. de Gouvea, and B. A. Dobrescu. *Phys. Rev.*, D75:055009, 2007, hep-ph/0612017.

[27] H.-S. Lee, K. T. Matchev, and T. T. Wang. 2007, arXiv:0709.0763.

[28] H.-S. Lee, C. Luhn, and K. T. Matchev. *Work in progress*.

[29] C. A. Savoy and M. Thormeier. *Work in progress*.

[30] G. Ross and M. Serna. 2007, arXiv:0704.1248.

[31] K. S. Babu, I. Gogoladze, and K. Wang. *Phys. Lett.*, B570:32–38, 2003, hep-ph/0306003.

[32] P. Minkowski. *Phys. Lett.*, B67:421, 1977.

[33] M. Gell-Mann, P. Ramond, and R. Slansky. in Sanibel Symposium, February, 1979: CALT 68–709, retroprint: hep-ph/9809459.

[34] Y. Yanagida. *Proc. of the Workshop on the Unified Theory*, 1979, edited by O. Sawada and A. Sugamoto: KEK Report N0. 79–18.

[35] R. N. Mohapatra and G. Senjanovic. *Phys. Rev. Lett.*, 44:912, 1980.

[36] C. Giunti. *Nucl. Phys. Proc. Suppl.*, 145:231, 2005, hep-ph/0412148.

[37] Z. Maki, M. Nakagawa, and S. Sakata. *Prog. Theor. Phys.*, 28:870, 1962.

[38] M. Apollonio et al. *Phys. Lett.*, B466:415, 1999, hep-ex/9907037.

[39] Q. Shafi and Z. Tavartkiladze. *Nucl. Phys.*, B772:133, 2007, hep-ph/0611377.

[40] E. Ma and G. Rajasekaran. *Phys. Rev.*, D64:113012, 2001, hep-ph/0106291.

[41] K. S. Babu, E. Ma, and J. W. F. Valle. *Phys. Lett.*, B552:207, 2003, hep-ph/0206292.

[42] A. Zee. *Phys. Lett.*, B630:58, 2005, hep-ph/0508278.

[43] G. Altarelli and F. Feruglio. *Nucl. Phys.*, B720:64, 2005, hep-ph/0504165.

[44] I. de Medeiros Varzielas, S. F. King, and G. G. Ross. *Phys. Lett.*, B648:201, 2007, hep-ph/0607045.

[45] C. Luhn, S. Nasri, and P. Ramond. *J. Math. Phys.*, 48:073501, 2007, hep-th/0701188.

[46] W. Grimus and L. Lavoura. *JHEP*, 01:018, 2006, hep-ph/0509239.

[47] R. N. Mohapatra, S. Nasri, and H.-B. Yu. *Phys. Lett.*, B639:318, 2006, hep-ph/0605020.

[48] T. Brown, S. Pakvasa, H. Sugawara, and Y. Yamanaka. *Phys. Rev.*, D30:255, 1984.

[49] C. Hagedorn, M. Lindner, and R. N. Mohapatra. *JHEP*, 06:042, 2006, hep-ph/0602244.

[50] C. Luhn, S. Nasri, and P. Ramond. *Phys. Lett.*, B652:27, 2007, arXiv:0706.2341.

[51] C. Luhn, S. Nasri, and P. Ramond. 2007, arXiv:0709.1447.

[52] P. F. Harrison, D. H. Perkins, and W. G. Scott. *Phys. Lett.*, B530:167, 2002, hep-ph/0202074.

[53] S. N. Ahmed et al. *Phys. Rev. Lett.*, 92:181301, 2004, nucl-ex/0309004.

[54] M. B. Smy et al. *Phys. Rev.*, D69:011104, 2004, hep-ex/0309011.

[55] Y. Ashie et al. *Phys. Rev.*, D71:112005, 2005, hep-ex/0501064.

[56] T. Araki et al. *Phys. Rev. Lett.*, 94:081801, 2005, hep-ex/0406035.

[57] M. H. Ahn et al. *Phys. Rev. Lett.*, 90:041801, 2003, hep-ex/0212007.

[58] C. Athanassopoulos et al. *Phys. Rev. Lett.*, 77:3082, 1996, nucl-ex/9605003.

[59] A. A. Aguilar-Arevalo et al. *Phys. Rev. Lett.*, 98:231801, 2007, arXiv:0704.1500.

[60] J. Bonn et al. *Nucl. Phys. Proc. Suppl.*, 91:273, 2001.

[61] M. C. Gonzalez-Garcia and M. Maltoni. 2007, arXiv:0704.1800.

[62] R. N. Mohapatra et al. 2005, hep-ph/0510213.

[63] M. Fukugita, K. Ichikawa, M. Kawasaki, and O. Lahav. *Phys. Rev.*, D74:027302, 2006, astro-ph/0605362.

[64] W. Buchmüller, P. Di Bari, and M. Plümacher. *Phys. Lett.*, B547:128, 2002, hep-ph/0209301.

[65] E. Eskut et al. *Phys. Lett.*, B497:8, 2001.

[66] P. Astier et al. *Nucl. Phys.*, B611:3, 2001, hep-ex/0106102.

[67] B. Armbruster et al. *Phys. Rev.*, D65:112001, 2002, hep-ex/0203021.

[68] P. Astier et al. *Phys. Lett.*, B570:19, 2003, hep-ex/0306037.

[69] A. H. Chamseddine and H. K. Dreiner. *Nucl. Phys.*, B458:65, 1996, hep-ph/ 9504337.

[70] V. Barger, J. Jiang, P. Langacker, and T. Li. *Phys. Lett.*, B624:233, 2005, hep-ph/0503226.

[71] V. Barger, J. Jiang, P. Langacker, and T. Li. *Nucl. Phys.*, B726:149, 2005, hep-ph/0504093.

[72] I. Gogoladze, C.-A. Lee, T. Li, and Q. Shafi. 2007, arXiv:0704.3568.

[73] D. C. Lewellen. *Nucl. Phys.*, B337:61, 1990.

[74] G. Aldazabal, A. Font, L. E. Ibáñez, and A. M. Uranga. *Nucl. Phys.*, B452:3, 1995, hep-th/9410206.

[75] K. R. Dienes, A. E. Faraggi, and J. March-Russell. *Nucl. Phys.*, B467:44, 1996, hep-th/9510223.

[76] G. Lopes Cardoso, D. Lüst, and T. Mohaupt. *Nucl. Phys.*, B450:115, 1995, hep-th/9412209.

[77] K. S. Babu, I. Gogoladze, and K. Wang. *Nucl. Phys.*, B660:322–342, 2003, hep-ph/0212245.

[78] H. K. Dreiner, J. L. Lopez, D. V. Nanopoulos, and D. B. Reiss. *Nucl. Phys.*, B320:401, 1989.

[79] H. Dreiner, J. Lopez, D. Nanopoulos, and D. Reiss. *Phys. Lett.*, B216:283, 1989.

[80] I. Antoniadis, J. Ellis, J. Hagelin, and D. Nanopoulos. *Phys. Lett.*, B231:65, 1989.

[81] H. K. Dreiner and M. Thormeier. *Phys. Rev.*, D69:053002, 2004, hep-ph/0305270.

[82] G. F. Giudice, A. Notari, M. Raidal, A. Riotto, and A. Strumia. *Nucl. Phys.*, B685:89, 2004, hep-ph/0310123.

[83] A. Pilaftsis and T. E. J. Underwood. *Nucl. Phys.*, B692:303, 2004, hep-ph/0309342.