Title
Solution of division by access to multiplication: Evidence from eye tracking

Journal
Proceedings of the Annual Meeting of the Cognitive Science Society, 38(0)

Authors
Tan, Shawn
Muldner, Kasia
LeFevre, Jo-Anne

Publication Date
2016

Peer reviewed

Solution of division by access to multiplication: Evidence from eye tracking

Shawn Tan (shawn.tan@carleton.ca)
Institute of Cognitive Science, Carleton University

Kasia Muldner (kasia.muldner@carleton.ca)
Institute of Cognitive Science, Carleton University

Jo-Anne LeFevre (jo-anne.lefevre@carleton.ca)
Institute of Cognitive Science, Carleton University

Abstract

People report solving division problems by mentally recasting division problems as multiplication (e.g., 72 ÷ 8 → 8 × [?] = 72). Mediation of division by multiplication occurs mainly on larger problems. Eye tracking data was used to determine whether patterns of gaze durations on division problems provided support for mediation. Adults solved division problems in two formats: traditional (e.g., 72 ÷ 8 = [ ]) and recasted (e.g. 72 = 8 × [ ]). Processing of individual problem elements was compared across formats. Results provide support for mediation. Processing patterns for traditionally-formatted problems were more similar to those for traditional division in earlier work (72 ÷ 8) whereas problems in recasted format (72 = 8 × [ ]) were more similar to patterns found when participants solved multiplication problems (e.g., 8 × 9). These findings provide a novel source of support for differential processing of problems across presentation formats.

Keywords: mental arithmetic; strategies; numerical cognition; mathematical cognition; division; eye tracking; gaze duration

Introduction

Mental arithmetic involves the coordination of memory retrieval, strategy selection, and decision processes. Much of the work on mental arithmetic involves basic problems such as 9 + 6, 18 ÷ 6 or 7 × 5. Even for these basic problems, participants report using a variety of solution processes. Although memory retrieval is the most commonly reported strategy, addition and subtraction problems may also be solved using counting (LeFevre et al., 1996a; 2006), and multiplication problems may be solved via reference to a related fact (e.g., solving 9 × 6 as 10 × 6 – 6; Campbell & Xue, 2001; LeFevre et al., 1996b). Our goal in the present research was to use eye tracking to provide a detailed description of the processes involved in solving division problems. We focused on simple division, that is, the set of problems which are defined by the inverse of multiplication problems with operands ranging from 2 to 9 (i.e., 2 × 2 to 9 × 9), such as 12 ÷ 4, 28 ÷ 7, and 54 ÷ 9.

Participants report using two strategies to solve simple division problems, that is, *retrieval* and *recasting*. LeFevre and Morris (1999) found that when participants were presented with division problems (e.g., 72 ÷ 9), they reported first mentally recasting these as multiplication problems (e.g., 9 × [ ] = 72), and subsequently retrieving the answer associated with the implied multiplication problem (e.g., 9 × 8).

In general, people solve problems with smaller operands more quickly and accurately than they solve problems with larger operands (Zbrodoff & Logan, 2005). Although this *problem-size effect* is extensively documented, the causes of it are not yet fully understood (Ashcraft & Guillaume, 2009). One possibility is that the problem-size effect arises, in part, from individual differences in strategy use across different types of problems. Accordingly, LeFevre and Morris (1999) found that strategy reports varied with the size of the operands. For division problems with small operands, direct retrieval was the most frequently reported strategy, whereas recasting was mainly reported for division problems with dividends larger than 25 (referred to as *large*) and was less frequently reported for problems with dividends of 25 or less (referred to as *small*). The additional step of recasting could explain why division problems took longer to solve. However, LeFevre and Morris (1999) did not explicitly manipulate the format of division problems.

Manipulating the format of basic arithmetic problems offers insight into the representations of arithmetic facts and the types of cognitive processes used by problem solvers. Mauro, LeFevre and Morris (2003) found that participants took longer to solve problems in division formats than in recasted multiplication formats, particularly for large problems. Based on these results, Mauro et al. proposed the *mediation hypothesis*: Presenting division in recasted formats (e.g., 42 = 7 × [ ]) provided problem solvers with a visual representation that is more compatible with the process of activation and retrieval of associated multiplication facts than the traditional division formats (e.g., 42 ÷ 7 = [ ]). They also proposed that solving division-format problems produces longer latencies than solving the corresponding problem in multiplication format because solvers recast the problem before retrieving the solution. This recasting stage is not needed for division problems that are shown in recasted formats or for division problems in traditional format that have small operands because they are solved through direct retrieval. Similar patterns have been identified for subtraction problems, which participants report recasting into addition formats to access their knowledge of addition (Campbell, 2008).

Although Mauro et al. (2003) found evidence for the mediation hypothesis, they relied on broad behavioral

Table 1: Examples of formats, elements and spatial positions of division problems used in the present experiment. Of primary interest are the recasted format (b) and the traditional format (d), shown in bold.

	Spatial Positions				
	1	2	3	4	5
	Recasted format				
(a)	72	=	[ ]	×	9
(b)	**72**	**=**	**9**	×	**[ ]**
	Traditional format				
(c)	72	÷	[ ]	=	9
(d)	**72**	÷	**9**	**=**	**[ ]**

measures (reaction time, error rate). We propose that more fine-grained information about how attention is allocated across formats could shed further light on the mediation hypothesis. Accordingly, in the present work we used eye tracking to record participants' gaze patterns as they solved division problems in traditional division and recasted multiplication formats. In general, eye tracking allows the identification of potential interest areas in a visual display that capture participants' attention as they process the display in response to a specific stimulus. This method has been used extensively to study participants' focus of attention during performance of tasks such as reading or scene processing (Rayner, 2009). However, use of eye tracking techniques is still novel in the field of numerical cognition (reviewed by Hartmann, 2015), and thus there are relatively few studies that have used eye tracking to study simple arithmetic (cf. Moeller, Klein, & Nuerk, 2011; Zhou, Zhao, Chen, & Zhou, 2012). Curtis, Huebner, and LeFevre (in press) compared patterns of eye tracking on single-digit addition and multiplication problems, and the corresponding inverse subtraction and division problems. For division problems such as 56 ÷ 8, they found that participants spent more time processing the dividend (i.e., 56) compared to the divisor (i.e., 8). In contrast, on the inverse multiplication problems (e.g., 8 × 7), processing time was divided evenly between the two operands.

In the present study, to further investigate the mediation hypothesis, participants' gaze information was recorded as they solved problems in either a traditional division format (e.g., 72 ÷ 8 = [ ]) or in a recasted multiplication format (e.g., 72 = 8 × [ ]). Interactions of problem elements and elements' location within the structure of the problem can influence dwell time durations and resulting gaze patterns. Given that changing the location of one problem element entailed a change in location of other elements in the problem, it was therefore necessary to control the location of a specific element across all problem formats. Since prior work suggests that problem size is a factor in determining whether participants use recasting (LeFevre and Morris, 1999; LeFevre, Mauro and Morris, 2003), we controlled for confounds between problem size and spatial location by maintaining the dividend at the first spatial position across formats (note that the dividend determines problem size). Gaze times upon each problem element and overall gaze patterns were analyzed to obtain information on the processing of problems in recasted vs. traditional formats and over large and small problems. We subsequently compared eye-tracking patterns from division problems in our study with eye-tracking patterns from division and multiplication problems reported in Curtis et al. (in press) consisting of three elements (e.g., 72 ÷ 9, 8 × 9).

Methods

Stimuli

As shown in Table 1, we created division problems in two formats: traditional (c and d) and recasted (a and b). The dividend (i.e., the larger number) was always located in the first position whereas the missing element was located at either the third or fifth position. Thus, there were four possible problem formats. Although participants solved problems presented in all four formats, we were most interested in comparing the most familiar traditional format, that is format (d), to the recasted format (b). This comparison matched the positions of the dividend and the divisor, such that only the location of the symbols varied across the two formats.

For division formats, we created 128 division problems by combining divisors ranging from 2 to 9 with quotients ranging from 2 to 9 and varying the location of the missing element at either the third or the fifth position. In this set, there were 112 non-tie problems in which divisors and quotients are not equal. These were divided into two sets of 56 such that reciprocal problems appeared in different sets (e.g., 6 ÷ 2 = 3, 6 ÷ 3 = 2). There were 8 tie problems with equal divisors and quotients (e.g., 9 ÷ 3 = 3). Tie problems were presented twice in each set. Thus, there were 56 non-tie and 16 tie problems totaling to 72 problems in division format for each set. The procedure was repeated for multiplication formats. This resulted in a total of 288 problems. We divided the problems into two experimental sets, counterbalanced by format, problem size, and location of missing element. Participants were randomly assigned to one of the experimental sets.

We also created 16 practice problems by combining dividends ranging from 2 to 9 with a divisor of 1, located at either the 3rd or 5th position respectively. The missing element was located at the alternative location (5th or 3rd position).

Problems were presented in black 60 point Arial font on a white background. Five interest areas were defined, one around each problem element. Each interest area measured 6 cm by 4 cm and was centered on the problem element (see Figure 1). The borders of each interest area were connected but did not overlap.

Participants

Thirty-three students (13 males, 20 females) from Carleton University participated in the experiment for course credit.

All participants had normal or corrected-to-normal vision, and ranged in age from 16 to 33 years (*Mdn* = 20).

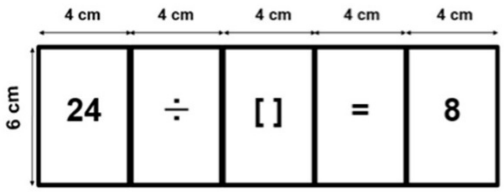

Figure 1: Dimensions of interest regions centered on problem elements for a typical problem in the experiment.

Procedure

Participants were seated at a distance of approximately 60cm in front of a desktop computer with a 15-inch by 13-inch monitor. The computer was linked to an SR-Eyelink 1000 eye tracker. The experiment began with the practice block followed by the 144 problems (36 in each of the four formats) divided into four blocks, with 36 problems in each block. Participants were allowed to take a break after completing each block. Each block started with a calibration phase followed by presentation of the problems.

At the start of each trial, a 1.25 cm by 1 cm rectangle randomly appeared at one of the four corners of the screen. Participants were instructed to look at this rectangle, and once the eye tracker detected a fixation upon the rectangle, the problem appeared. This procedure was used to ensure that participants' attention would not be focused on the central interest area upon initial presentation of the problem, because this could result in artificial inflation of fixation times on the central interest area. Participants were asked to verbally state their answer into a microphone placed directly in front of them and their responses were connected to an ASIO voice trigger accurate to +/- 1 ms. Problems remained on the screen until the microphone recorded a response. Once this occurred, the problem disappeared and the experimenter recorded the response using a keyboard. Eye gaze information was recorded by the eye tracker. The experiment lasted for about 50 minutes.

Results

Trials were discarded if participants made an irrelevant sound before providing a solution or if the apparatus failed to detect a response. This resulted in 338 (7.1%) of the trials being discarded. Participants made errors on a further 300 (6.8%) of the remaining trials. These were not included in the analyses of response latencies.

Overall Performance

To get a sense of overall performance, we analyzed median latencies and percentage of errors for all participants in two separate 2(format: multiplication vs. division) by 2(problem size: large vs. small) by 2(position of missing element: 3rd vs. 5th) repeated-measures ANOVAs. Means (of medians) across participants are shown in Table 2. Small problems had dividends of 25 or less; large problems had dividends larger than 25.

As expected, participants were faster at solving small problems than large problems (1393 vs. 1760 ms), $F(1,32) = 40.6$, $p < .001$, $\eta_p^2 = .56$. The location of the missing element also had an effect: Participants responded more slowly when the third element was missing than when the fifth element was missing (1605 vs. 1547 ms), $F(1,32) = 5.61$, $p = .024$, $\eta_p^2 = .15$. Although participants were faster on recasted than on traditional formats (1560 vs. 1593 ms), the difference was not significant, $F(1,32) = 1.6$, $p = .21$, $\eta_p^2 = .05$.

Mirroring the results for response time, participants made fewer errors on small than on large problems (4.7% vs. 9.4%), $F(1,32) = 16.8$, $p < .001$, $\eta_p^2 = .34$. No other main effects or interactions were significant.

In summary, the overall analyses were consistent with our expectations. Participants were faster and more accurate on small than on large problems. We also replicated the trend for recasted formats to be solved more quickly than traditional formats. However, the location of the missing element also influenced solution times. Thus, further analyses were focused on the comparison of the most familiar traditional format, (d) in Table 1, to the recasted format with the same structure, (b) in Table 1. For this pair, large problems in recasted format were solved more quickly than those in traditional format (1689 vs. 1735 ms).

Table 2: Mean latencies, standard errors, and percentages.

Format	*M* (ms)		*SE* (ms)		*Error* (%)	
	Small	Large	Small	Large	Small	Large
Traditional Format						
72÷[ ]=9	1444	1808	74.3	128.3	4.2	10.4
72÷9=[ ]	1385	1735	66.2	119.8	5.0	9.6
Recasted Format						
72=[ ]×9	1364	1805	62.4	109.7	3.6	7.7
72=9×[ ]	1380	1689	69.6	97.1	5.9	9.8

Eye gaze patterns

Gaze duration. Gaze duration, defined as the sum of all fixation times in an area of interest was calculated and analyzed for each of the five interest areas (as shown in Figure 1). Analyses of the total number of fixations in each area of interest produced similar results and so are not described in further detail.

To obtain information on participants' attention patterns for the interest areas in the traditional format (e.g., 72 ÷ 9 = [ ]) and the corresponding recasted format (e.g., 72 = 9 × [ ]), we analyzed median gaze durations for each participant in a given interest area in a 5(interest area: left number, symbol-1, middle number, symbol-2, missing element) ×

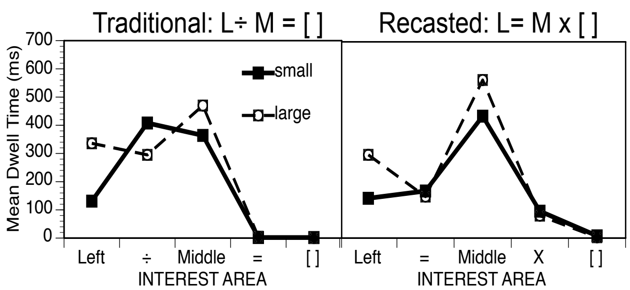

Figure 2: Mean gaze durations (total dwell time) for each interest area by format and problem size.

2(format: traditional, vs. recasted) × 2 (problem size: small vs. large) repeated-measures ANOVA. The analysis showed main effects of problem size $F(1,32) = 33.69$, $p < .001$, $\eta_p^2 = .513$, and interest area, $F(4,29) = 105.34$, $p < .001$, $\eta_p^2 = .936$, interactions between format and interest area, $F(4,29) = 15.63$, $p < .001$, $\eta_p^2 = .683$; interest area and problem size, $F(4,29) = 17.23$, $p < .001$, $\eta_p^2 = .704$; and among format, interest area, and problem size, $F(4,29) = 4.09$, $p < .01$, $\eta_p^2 = .361$ (see Figure 2).

As shown in Figure 2, participants spent very little time on the last two interest areas, presumably because these contained no important information (e.g., the blank was always in the last position in the two formats that are the target of this analysis). In the traditional division format, participants looked longest at the division sign and the middle number (the divisor). Compared to small problems, processing time on large problems was also allocated to the left number (the dividend). In contrast, in the recasted format, participants spent more time looking at the middle interest area (i.e., the divisor), with relatively little time spent looking at the equal sign or the left number. On large problems, somewhat more time was spent looking at the left number, but this was still considerably less than gaze durations on the middle number in this format.

In some respects, these results are similar to those found by Curtis et al. (2016), who used traditional division problems but presented these without the equal sign or the symbol for the missing element (e.g., 72 ÷ 9). Specifically, in both studies, gaze durations were longer for the dividend when solving large as compared to small problems, accounting for the overall effect of problem size. However, in contrast to Curtis et al., in our data there was a tendency for fixations in the division format to be shifted rightwards toward the middle number, possibly because the additional symbols in our problems resulted in an overall shift of gaze to the centre of the display. Curtis et al. observed longer gaze durations to the operator, which in their stimuli appeared in the middle of the display, on small problems for all operations. Although these results suggest that processing of arithmetic problems is directed to the centre of the display, regardless of the underlying symbols, this pattern does not always hold. Specifically, Curtis et al. found that on large division problems, participants spent longer fixating on the left operand (the dividend) than on either the operation sign or the divisor. As stated above, this was not the case in the present research, where gaze durations to the dividend were longer on large traditional problems than in the other conditions, but participants nevertheless fixated most on the middle operand (the divisor). Thus, there were similarities between the traditional division format in this research and in Curtis et al., and between recasted formats and multiplication problems in Curtis et al.

The pattern of results we obtained, coupled with previous findings, provides some support for the mediation hypothesis. However, gaze durations for arithmetic problems also varied depending on the specific elements included in the problem display. Even though participants did not spend much time processing the equal sign or the missing element in the traditional format, the presence of these elements may have influenced the overall processing patterns. Such findings suggest that gaze patterns may need to be interpreted carefully across different display conditions.

Distribution of attention over time. To explore gaze patterns over the course of solving a division problem (i.e., gaze profiles), we normalized the distribution of processing times for each problem. This normalization was necessary because participants varied in terms of the total time spent to solve a given problem. For each problem, the total time taken to solve the problem was divided into six equal time intervals. Within each time interval, gaze duration on each interest area was calculated by summing the duration of all fixations for that interest area. We then calculated the median gaze duration for each interest area and time interval for each participant, and then calculated means across all participants. Figure 3 shows these mean distributions of gaze durations in each interest area across all time intervals for small and large problems in traditional and recasted formats.

As shown in Figure 3, problem format and problem size influenced participants' distribution of attention over time. However, in comparing the graphs in Figure 3, some common patterns of attention can still be observed. First, there were longer mean durations on the first symbol than the first and middle operands during time interval 1 for all four graphs, suggesting that initial processing on all problems was directed more towards the first symbol (dashed red line in Figure 3) than towards either operand. This pattern may indicate that participants initially fixate to the left of center (see Schneider, Maruyama, Dehaene, & Sigman, 2012, for a similar finding). The left of center area (i.e., the second area of interest) contains the symbol relevant for discriminating between the formats: A division sign in the second area of interest indicates a traditional problem whereas an equal sign indicates a problem in recasted format. Gaze duration on this interest area declines quickly in the next two intervals for all except small problems in traditional formats (e.g., 10 ÷ 2 = [ ]; 18 ÷ 3 = [ ]). For all problem types, gaze duration in the middle of the display (i.e., the single-digit divisor, see green line in Figure 3) increases in time interval 2.

For the recasted format, problem size did not have much effect on gaze profiles. Following focus of attention on the division sign during the early stages of the trial, gaze duration on the middle interest area dominated the remainder of the trial (green line in Figure 3, lower panels). There was stable amount of processing to the larger number (i.e., the dividend, blue dotted line in Figure 3) that was greater on large than on small problems. On both large and small problems, the problem size effect appeared to be distributed equally over the left and middle operands (see also Figure 2). The similarity in patterns of processing between small and large problems observed here for the recasted format was also reported by Curtis et al. (in press) for traditional multiplication problems (e.g., 3 × 9, 8 × 7). There were longer gaze durations on the dividend for large problems than small problems but this did not influence distribution patterns over time.

In contrast to the findings for the recasted format, problem size did influence distribution of attention patterns over time for the traditional format. On small problems, gaze durations were similar for the middle operand and the division sign after time interval 2. Gaze duration on the left operand (the dividend) decreased after time interval 1, suggesting that participants allocated less attention to the dividend after the beginning of the trial. In contrast, for large problems, gaze duration on the operation sign declined after the first interval. The left operand (dividend) and middle operand had similar gaze durations for the first half of the trial, with a gradual shift toward a focus on the middle operand in the last half of the trial. This pattern may reflect recasting of the problem in the first half of the trial, followed by retrieval based on multiplication knowledge. Curtis et al. found that the dividend was processed longer on large than on small division problems. We also found a similar result with longer gaze durations for large than small problems over all time intervals.

In summary, the analyses of gaze durations across the course of the trial support the mediation hypothesis in that gaze distribution patterns over time varied across formats. We observed similar patterns of gaze distribution over time for large and small problems in recasted format as compared to different patterns of gaze distribution over time across large and small division problems in traditional format. These differences occurred even though the location of the numerical information was identical in the two formats.

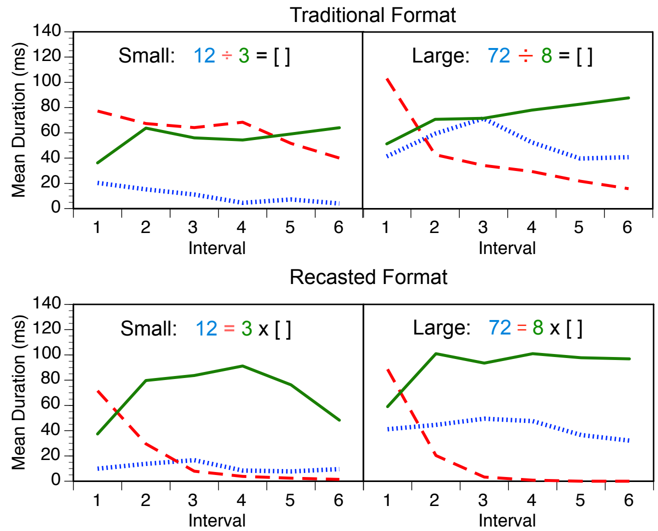

Figure 3: Gaze durations normalized across time intervals. Processing time on the dividend (i.e., the largest number) is shown by the blue dotted line, the first symbol (i.e., the division sign in the traditional format and the equal sign in the recasted format) is shown by the red dashed line, and the divisor (the smaller number) is shown by the green solid line. Because very little time was spent on the second symbol or the missing element (see Figure 2), these were not included in the graphs.

Discussion

In this research we used eye tracking measures to assess whether strategy differences reported in LeFevre and Morris (1999) on division problems (i.e., retrieval and recasting) would be reflected in gaze patterns. Based on earlier studies (LeFevre and Morris, 1999; Mauro et al., 2003), retrieval from memory was the most common strategy for small problems, as well as for problems in recasted formats, whereas for large problems in traditional formats, participants reported mentally transforming the problem into a multiplication format before retrieving the solution (e.g., mentally changing 72 ÷ 8 into 8 × [ ] = 72). We hypothesized that eye patterns would reflect, at least in part, the mental activities going on during problem solution (Rayner, 2009) and that differences observed in eye-tracking measures across formats would be associated with a change in strategy that would influence gaze patterns. The present research supports this hypothesis by showing that gaze patterns revealed by eye tracking data vary both across formats and problem size.

Gaze patterns for problems presented in recasted format (i.e., 72 = 8 × [ ]) were similar to patterns obtained from participants who solved traditional multiplication problems (i.e., 8 × 9) reported by Curtis et al. (in press), suggesting that in our study, participants were recasting the division problems into a multiplication format. In both cases, processing time was concentrated on the central element of the presented stimulus, with relatively less time spent processing the other problem elements. Curtis et al. suggested that participants' focus on the centre of the display (i.e., the operation sign) reflected mental processing occurring during retrieval of the answer, because it continued after the two operands had been processed. In the present research, the focus on the center of the display (the single-digit divisor) may also indicate that participants are using this number to access multiplication knowledge, as part of mental processing involved in retrieval of an answer. In contrast, when the same problems were presented in the traditional division format, processing time was distributed more evenly across the first, second, and third interest areas, with additional processing time on the dividend (first interest area) for large as compared to small problems. Curtis et al. also observed this problem-size effect on the dividend for division problems in the traditional format.

Notably, we also identified some factors that may be important for interpreting gaze patterns. Comparisons between the results of Curtis et al. (in press) and the present research suggest that varying the way problems are displayed may influence processing patterns. Division problems in Curtis et al. were presented without equal signs or missing answers (e.g., 72 ÷ 8). Although there were similarities in the distribution of processing time across that format and the format used in the present research (e.g., 72 ÷ 8 = [ ]), there were also differences that appear to be related to the presence of the additional problem elements. These findings suggest that a larger range of formats that balance the position of specific elements and control for format familiarity will be necessary in studies that use eye tracking to study mental arithmetic.

Despite this qualification, the present research demonstrates the utility of using eye-tracking measures to extend and corroborate findings from more traditional dependent variables. We also showed the importance of comparisons across experiments with varying problem structures and formats. Comparing gaze profiles over time further extends inferences from dwell time analyses to mental activity over the time course of the problem solving process. In summary, gaze durations and time-course profiles provided theoretically and empirically relevant information about mental arithmetic.

Acknowledgments

This research was supported by the Natural Sciences and Engineering Research Council of Canada through Discovery Grants to J. LeFevre and K. Muldner.

References

Ashcraft, M. H., & Guillaume, M. M. (2009). Mathematical cognition and the problem size effect. In B. Ross (Ed.), *Psychology of Learning and Motivation, Vol. 51* (Vol. 51, pp. 121–151). Burlington: Academic Press.

Campbell, J. I. D. (2008). Subtraction by addition. *Memory & Cognition*, *36*(6), 1094–102.

Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General*, *130*(2), 299–315. doi:10.1037//0096-3445.130.2.299

Curtis, E., Huebner, M. G., & LeFevre, J. (in press). The relationship between problem size and fixation patterns during addition, subtraction, multiplication, and division. *Journal of Numerical Cognition*.

Hartmann, M., Mast, F. W., & Fischer, M. H. (2015). Spatial biases during mental arithmetic: evidence from eye movements on a blank screen. *Frontiers in Psychology, 6,* 1-8.doi: 10.3389/fpsyg.2015.00012

LeFevre, J.A., Sadesky, G. S., & Bisanz, J. (1996a). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *22*(1), 216–230. doi:10.1037//0278-7393.22.1.216

LeFevre, J.A., Bisanz, J., Daley, K. E., Buffone, L., Greenham, S. L., & Sadesky, G. S. (1996b). Multiple routes to solution of single-digit multiplication problems. *Journal of Experimental Psychology: General*, *125*(3), 284–306. doi:10.1037/0096-3445.125.3.284

LeFevre, J. A., DeStefano, D., Penner-Wilger, M., & Daley, K. E. (2006). Selection of Procedures in Mental Subtraction. Canadian Journal of Experimental Psychology, 60(3), 209–220. doi:10.1037/cjep2006020

LeFevre, J. A., & Morris, J. (1999). More on the relation between division and multiplication in simple arithmetic: Evidence for mediation of division solutions via multiplication. *Memory and Cognition, 27,* 803-812.

Mauro, D. G., LeFevre, J. A., & Morris, J. (2003). Effects of problem format on division and manipulation performance: Division facts are mediated via multiplication-based representations. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 29,* 163-170.

Moeller, K., Klein, E., & Nuerk, H.-C. (2011). Three processes underlying the carry effect in addition--evidence from eye tracking. *British Journal of Psychology*, *102*, 623–645. doi:10.1111/j.2044-8295.2011.02034.x

Rayner, K. (2009). Eye movements and attention in reading, scene perception, and visual search. *Quarterly Journal of Experimental Psychology*, *62*(8), 1457–1506.

Schneider, E., Maruyama, M., Dehaene, S., & Sigman, M. (2012). Eye gaze reveals a fast, parallel extraction of the syntax of arithmetic formulas. *Cognition*, *125*(3), 475–490. doi:10.1016/j.cognition.2012.06.015

Zbrodoff, N. J., & Logan, G. D. (2005). What everyone finds: The problem-size effect. In *Handbook of mathematical cognition* (pp. 345–508). Psychology Press, New York, NY.

Zhou, F., Zhao, Q., Chen, C., & Zhou, X. (2012). Mental representations of arithmetic facts: Evidence from eye movement recordings supports the preferred operand-order-specific representation hypothesis. *Quarterly Journal of Experimental Psychology*, *65*(4), 661–74. doi:10.1080/17470218.2011.61621.