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ABSTRACT

If two laser beams have a difference frequency nearly equal to the plasma frequency, nonlinear interaction resonantly excites longitudinal plasma oscillations. These then induce transitions to other transverse modes. Nonlinear damping of the longitudinal mode heats the plasma. The process is optimized by having parallel beams, equal laser intensities, and damping equal to the frequency mismatch.

We propose a new method of heating a plasma, utilizing the excitation of longitudinal plasma waves by resonance with the difference frequencies of a set of transverse waves. The energy is provided by two (nearly) parallel laser beams, with frequencies (ω_L, ω_{L-1}) differing by approximately the plasma frequency: $\omega_L - \omega_{L-1} \equiv \omega_p + \Delta_L$, with the mismatch Δ_L small (say, $10^{-2}\ \omega_p$), and $\omega_L \gg \omega_p$. The nonlinear interaction of transverse and longitudinal waves [see Eq. (2) and the insets of Fig. 1] excites a longitudinal wave, with wave-vector $\underset{\sim}{k}_p = \underset{\sim}{k}_L - \underset{\sim}{k}_{L-1}$, which is nonlinearly damped, if its amplitude is sufficiently large. (There is no Landau damping, since $\omega_p/k_p \approx c$; collisional damping is too weak for our purposes.) This wave in turn interacts with each of the two transverse waves (L, L-1) to produce two more,

at $\underset{\sim}{k}_{L-2} = \underset{\sim}{k}_{L-1} - \underset{\sim}{k}_p$ and $\underset{\sim}{k}_{L+1} = \underset{\sim}{k}_L + \underset{\sim}{k}_p$. When $\underset{\sim}{k}_L$, $\underset{\sim}{k}_{L-1}$ are nearly parallel, the new mismatches $\Delta_\ell \equiv \omega_\ell - \omega_{\ell-1} - \omega_p$, determined by the dispersion relation $\omega_\ell^2 = k_\ell^2 c^2 + \omega_p^2$, are also small; otherwise they are not.

In quantum language, a coherent set of photons L undergo induced (by L - 1) decay into photons L - 1 and plasmons. The damping of the plasmons deposits energy irreversibly into the plasma. Some of the plasmons, before they are absorbed, engage in further three-wave interactions, inducing the decay of the photons L - 1 into photons L - 2, and so on, coherently cascading the photon frequency downward. Others induce transitions upward in frequency, by converting L into L + 1, and so on. Because energy is conserved in these interactions, and also the number of photons is conserved, the process must be preferentially downward, to allow for the plasma heating. (See curve 5 of Fig. 1a for an example.) For maximum efficiency, the downward rate should be maximized relative to the upward spreading. This is accomplished if the two laser intensities are roughly equal, and if the damping rate approximates the mismatch [see Eq. (5)].

The resonant interaction between two transverse waves and one longitudinal mode has been studied by Kroll et al.,[1] Tsytovich,[2] and Wolff,[3] among others. The fundamental equations for the interaction of the scalar potential $\phi(z,t)$ of the longitudinal wave and the vector potential $A_x(z,t)$ of a set of parallel, linearly x-polarized transverse waves is

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\phi(z,t) = -\frac{e\omega_p^2}{2mc^2} A_x^2 , \qquad (1a)$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2 - c^2\frac{\partial^2}{\partial z^2}\right)A_x(z,t) = \frac{e}{m}\frac{\partial^2\phi}{\partial z^2} A_x ; \qquad (1b)$$

where we treat the unperturbed electron plasma as cold, uniform, and stationary, and ignore dissipation for the time being. To derive these equations, we use the invariance of the canonical x-momentum[3] to obtain $v_x = -eA_x/mc$. The Lorentz force along z is thus $(e/c)v_xB_y = -(e^2/2mc^2)\partial A_x{}^2/\partial z$, whence $4\pi n_0 e\dot{v}_z = -(\partial/\partial z)\omega_p{}^2[\phi + (e/2mc^2)A_x{}^2]$. This is then inserted into the last term of $(\partial^2/\partial t^2)(\partial^2\phi/\partial z^2) = -(\partial^2/\partial t^2)4\pi ne = 4\pi e(\partial^2/\partial t\partial z)nv_z \doteq (\partial/\partial z)4\pi n_0 e\dot{v}_z$; we have used the Poisson and continuity equations, and have dropped harmonic-producing[3] terms. Integration with respect to z then yields Eq. (1a). For Eq. (1b), we use the wave equation $\Box A_x = -4\pi j_x/c = (4\pi e/c)(n_0 + \delta n)(eA_x/mc)$, and replace δn by $-(4\pi e)^{-1}\,\partial^2\phi/\partial z^2$.

The coupled equations for the wave amplitudes are obtained from (1) by setting $A_x(z,t) = \Sigma_\ell A_\ell(z,t)\exp i(k_\ell z - \omega_\ell t) + \text{c.c.}$, and $\phi(z,t) = -i\phi_1(z,t)\exp i(k_p z - \omega_p t) + \text{c.c.}$ Assuming that the amplitudes vary slowly, we obtain

$$\left(\frac{\partial}{\partial t} + \gamma\right)\phi_1 = \kappa \sum_\ell A_\ell A^*_{\ell-1}\exp(-i\Delta_\ell t)\ , \qquad (2a)$$

$$\left(\frac{\partial}{\partial t} + c_\ell\frac{\partial}{\partial z}\right)A_\ell = (\kappa/\ell)[A_{\ell+1}\phi_1{}^*\exp(-i\Delta_{\ell+1}t) - A_{\ell-1}\phi_1\exp(i\Delta_\ell t)]\ , \qquad (2b)$$

where $\kappa \equiv e\omega_p/2mc^2$, $c_\ell \equiv k_\ell c^2/\omega_\ell \approx c$ is the group velocity of mode ℓ, we have set $\omega_\ell \approx \ell\omega_p$ in the coefficient of (2b), and we have introduced a phenomenological damping coefficient γ in (2a). The corresponding wave-energy densities are $W_p \equiv [\omega_p\partial\epsilon/\partial\omega]k_p{}^2|\phi_1|^2/4\pi = \omega_p{}^2|\phi_1|^2/2\pi c^2$ and $W_\ell \equiv \omega_\ell{}^{-1}[\partial(\omega^2\epsilon)/\partial\omega](\omega_\ell/c)^2|A_\ell|^2/4\pi = \omega_\ell{}^2|A_\ell|^2/2\pi c^2$, where $\epsilon(\omega) = 1 - (\omega_p{}^2/\omega^2)$. We note that Eq. (2b) conserves transverse action (photon number): $(\partial/\partial t)\,\Sigma_\ell W_\ell/\omega_\ell = -(\partial/\partial z)\Sigma_\ell c_\ell W_\ell/\omega_\ell$; while the set (2a), (2b) conserves energy (with dissipation):

$(\partial/\partial t)(W_p + \Sigma_\ell W_\ell) = -(\partial/\partial z)\Sigma_\ell c_\ell W_\ell - 2\gamma W_p$. (Note that the longitudinal mode has zero group velocity.)

We study here (a) the boundary-value problem, in which two laser beams with steady intensities $cW_L{}^o$ and $cW_{L-1}{}^o$ are incident on a semi-infinite $(z > 0)$ plasma, and we look for quasi-steady state solutions as functions of z; and (b) the initial-value problem, in which two laser modes are present at $t = 0$, uniform in space, with energy densities $W_L{}^o$ and $W_{L-1}{}^o$, and we look for the evolution as a function only of time. The full space-time problem will be studied in a later publication, together with the important effect of plasma non-uniformity.

In the steady-state problem, the dissipation rate is $Q = 2\gamma W_p = -(d/dz)\Sigma_\ell c_\ell W_\ell$, from the energy conservation law. Dividing both sides by the constant action flux density $\Sigma_\ell c_\ell W_\ell/\omega_\ell \equiv J$, we find that $Q/J = -\omega_p\, d\langle\ell\rangle/dz$, where $\langle\ell\rangle(z)$ is the action-weighted mean mode number. Thus it is desired to have $\langle\ell\rangle$ decrease as rapidly as possible. Its derivative is given by Eq. (5). Similar considerations apply to the initial-value problem, where $-d\langle\ell\rangle/dt$ is to be maximized.

The set (2) has the characteristic rate[4] $\Gamma_0 \equiv \kappa A_L{}^0 = [W_L{}^o/(8nmc^2)]^{\frac{1}{2}}(\omega_p/L)$, but this is not the actual rate, which is found below. In dimensionless variables $[t' \equiv \Gamma_0 t,\ z' \equiv \Gamma_0 z/c,\ \Delta' \equiv \Delta/\Gamma_0,\ \gamma' \equiv \gamma/\Gamma_0,\ \phi_1' \equiv \phi_1/A_1{}^o,\ A_\ell' \equiv A_\ell/A_L{}^o]$, the equations (2) retain the same form [denoted (2)' below], but with κ deleted. The boundary (or initial) conditions are $A_L'(0) = 1$, $A_{L-1}'(0) \equiv \alpha\, e^{i\theta}$ (α positive).

An explicit solution for the boundary-value problem may be found, if all the mismatches are set equal $(\Delta_\ell \to \Delta)$; this is equivalent to ignoring dispersion, which is not too bad if $\ell \gg 1$. To be consistent, we then set $c_\ell \to c$, and $\ell \to L$ in the coefficient of (2b). For time-independent A_ℓ, the steady-state solution of (2a)' is $\phi_1'(z',t') = (\gamma' - i\Delta')^{-1}B\exp(-i\Delta' t')$, where $B \equiv \Sigma_\ell A_\ell'(z')A_{\ell-1}'^{*}(z')$. Thus $\phi(z,t)$ is driven at the (common) beat frequency $\omega_\ell - \omega_{\ell-1} = \omega_p + \Delta$, not at its natural frequency ω_p. Substituting ϕ_1' into (2b)', we obtain

$$L\, dA_\ell'/dz' = (\gamma' + i\Delta')^{-1} B^* A_{\ell+1}' - (\gamma' - i\Delta')^{-1} B A_{\ell-1}'. \tag{3}$$

We use (3) to show that dB/dz' vanishes, i.e., $B(z') = B(0) = \alpha e^{-i\theta}$. Introducing $\rho \equiv \tan^{-1}(\Delta/\gamma)$ and $\zeta \equiv 2\alpha L^{-1}(\gamma'^2 + \Delta'^2)^{-\frac{1}{2}} z' \equiv 2\alpha L^{-1} c^{-1}(\gamma^2 + \Delta^2)^{-\frac{1}{2}} \Gamma_0^2 z$, and setting $A_\ell'' \equiv A_\ell' \exp i\ell(\theta - \rho)$, we can write (3) as $2\, dA_\ell''/d\zeta = A_{\ell+1}'' - A_{\ell-1}''$. This is the recursion relation for Bessel functions, except for sign. Hence the solution of (3) satisfying the boundary conditions yields

$$\begin{aligned} |A_{L+n}'|^2 &= J_n^2 + \alpha^2 J_{n+1}^2 - 2\alpha(\cos\rho)J_n J_{n+1}, \\ |A_{L-1-n}'|^2 &= \alpha^2 J_n^2 + J_{n+1}^2 + 2\alpha(\cos\rho)J_n J_{n+1}, \end{aligned} \tag{4}$$

where ζ is the argument of the Bessel functions.

From Eq. (3), we may directly calculate the evolution of mean[5] mode number $\langle\ell\rangle(z) \equiv \Sigma_\ell\, \ell|A_\ell|^2(z)/\Sigma_\ell |A_\ell|^2(z)$. In dimensional variables, we find the cascade rate

$$\Gamma \equiv -c\frac{d\langle\ell\rangle}{dz} = \frac{1}{4L^3}\cdot\frac{\gamma\,\omega_p^2}{\gamma^2 + \Delta^2}\cdot\frac{\alpha^2}{(1+\alpha^2)^2}\cdot\frac{W^o}{nmc^2}, \tag{5}$$

where $W^o \equiv W_L^o + W_{L-1}^o$ is the total input energy density. We recall that Γ represents the rate of plasma heating, and is to be maximized. (Note that in contrast to Γ_0, it varies linearly with W^o.) For given W^o, it vanishes as $\alpha^2 \to 0$ or ∞, and is maximized at $\alpha = 1$.

This is illustrated in Fig. 1, which presents $|A_\ell'|^2$ versus ℓ, at several ζ, for the two cases $\alpha = 1$ ($W_{L-1}^o = W_L^o$) and $\alpha = 10$ ($W_{L-1}^o = 100\, W_L^o$). The latter case evolves almost symmetrically about $L - 1$, and little heating results. The former case is quite asymmetric, which is desired. The dependence of Γ on the damping rate γ is similar, Γ being maximized when $\gamma = \Delta$. This is evident from Eqs. (4), where the asymmetry between higher and lower modes is seen to be proportional to $\cos\rho = \gamma/(\gamma^2 + \Delta^2)^{\frac{1}{2}}$.

Formula (5) leads to an estimate of required laser intensity, for a criterion that $\langle\ell\rangle$ change by unity in one centimeter, say. Taking $L \sim 10$, $\gamma \sim \Delta \sim 10^{-2}\omega_p$, $\omega_p \sim 2 \times 10^{13}\ \mathrm{sec}^{-1}$, $\alpha \sim 1$, $n \sim 10^{17}\ \mathrm{cm}^{-3}$, we obtain $W^o c \sim 10^{14}$ watt cm^{-2}. The longitudinal field produced is then sufficient to produce damping by parametric instability,[6] with γ of the order assumed.[7]

To study the effects of variable mismatch Δ_ℓ (caused by dispersion), we have numerically integrated Eqs. (2) for the uniform case ($\partial/\partial z \equiv 0$). Of the several cases studied, we report only the following: ω_L was chosen to be $1.8 \times 10^{14}\ \mathrm{sec}^{-1}$ (CO_2 laser), and $L = 10$. The initial power density P (expressed in $\mathrm{W/cm}^2$) was in modes 10 and 9, with $\alpha^2 = 0.1$. The damping coefficient was chosen to be $\gamma = 10^4 P^{\frac{1}{2}}\ \mathrm{sec}^{-1}$, corresponding to $\gamma' = 0.3$, and $\gamma/\omega_p = 0.005$ at $10^{14}\ \mathrm{W/cm}^2$.

The mismatch Δ_L was adjusted for optimum energy transfer. Because the mismatch Δ_ℓ decreases algebraically with ℓ, due to the plasma dispersion, it is desirable to choose Δ_L positive. Then for higher modes, the mismatch increases, and thus coupling to those modes is inhibited; while for lower modes, the coupling is enhanced as Δ_ℓ decreases and passes through zero to negative values. The best choice of Δ_L was in the range $0.01 < \Delta_L/\omega_p < 0.03$. With this choice, energy transferred to higher modes was blocked, and eventually made to cascade back down, with little energy remaining in those modes. Figure 2 shows the fractional energy transfer after a time interval $t = 5\,\gamma^{-1}$. (For later times, the transfer rate becomes relatively slow.) We note that the effective threshold power density is 10^{14} W/cm^2; for weaker power, the mismatch prevents appreciable energy transfer.

When the calculation was repeated with Δ_ℓ held constant (i.e., neglecting dispersion), typically 0.3 to 0.4 of the energy remained in the higher modes $\ell > L$, and did not cascade down. In this and other respects, there was agreement between the time-dependent uniform case and the space-dependent steady-state case.

When the initial laser beams are not (nearly) parallel, spontaneous frequency conversion to other transverse modes will not occur because of the large mismatches. The longitudinal mode then catalyzes the complete transfer of action from L to L - 1. For the anti-parallel case, for example, the longitudinal mode has $\underset{\sim}{k}_p = \underset{\sim}{k}_L - \underset{\sim}{k}_{L-1}$ as before, but now $|\underset{\sim}{k}_p| \simeq 2k_L \simeq 2\,\omega_L/c$, whereas in the parallel case $k_p \simeq \omega_p/c \ll 2\,\omega_L/c$. As a result, the wave couping on the right side of Eq. (1b), proportional to $k_p^{\,2}$, is greatly enhanced. To balance the advantage of enhanced coupling are two disadvantages and one further advantage. First, the damping rate may be greatly enhanced by Landau damping (since now $\omega_p/k_p \ll c$), beyond the optimum $\gamma \sim \Delta$. Secondly, the further transition to L - 2 cannot be induced by the longitudinal mode $\underset{\sim}{k}_p$, since this would require $\underset{\sim}{k}_{L-2} = \underset{\sim}{k}_{L-1} - \underset{\sim}{k}_p = 2\underset{\sim}{k}_{L-1} - \underset{\sim}{k}_L$, or $k_{L-2} \approx 3k_L$, violating the dispersion relation. This means that the further decay must be induced instead by a third laser beam L - 2 in any desired direction, and the corresponding longitudinal

wave excited $\underset{\sim}{k}_p{}' = \underset{\sim}{k}_{L-1} - \underset{\sim}{k}_{L-2}$ is not the same as $\underset{\sim}{k}_p$. Thirdly, no energy is lost on up-conversion, since each transition must be seeded by its own laser beam.

The idea of using laser beats to heat a plasma was suggested to us by M. Rosenbluth and N. Kroll. Discussions with Y.-R. Shen and C. Townes were helpful for orientation. S. Bodner, W. Kunkel, D. Nicholson, and R. Riddell provided us with good advice and encouragement.

REFERENCES AND FOOTNOTES

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3. P. Wolff, Proceedings Second International Conference on Light Scattering in Solids, edited by M. Balkanski (Flammarion, Paris, 1971), page 180.
4. We may take A_L^o real and positive.
5. The weighting has been changed, because the replacement $\ell \to L$ changes the invariant from J to $\Sigma_\ell |A_\ell|^2$.
6. For thresholds, see e.g., P. Kaw and J. Dawson, Phys. Fluids 12, 2586 (1969); for a bibliography, see Kaufman, Kaw, and Kruer, Comments on Plasma Physics and Controlled Fusion 1, 39 (1972).
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FIGURE CAPTIONS

Fig. 1. Mode energy as a function of mode number $\ell \equiv \omega_\ell/\omega_p$, at several positions ζ [defined below Eq. (3)]. In case (a), the laser intensities are equal ($\alpha = 1$). In case (b), they are very unequal ($\alpha = 10$). The damping rate is comparable to the mismatch ($\cos \rho = 0.5$).

Fig. 2. Fractional energy transfer as a function of laser power density.

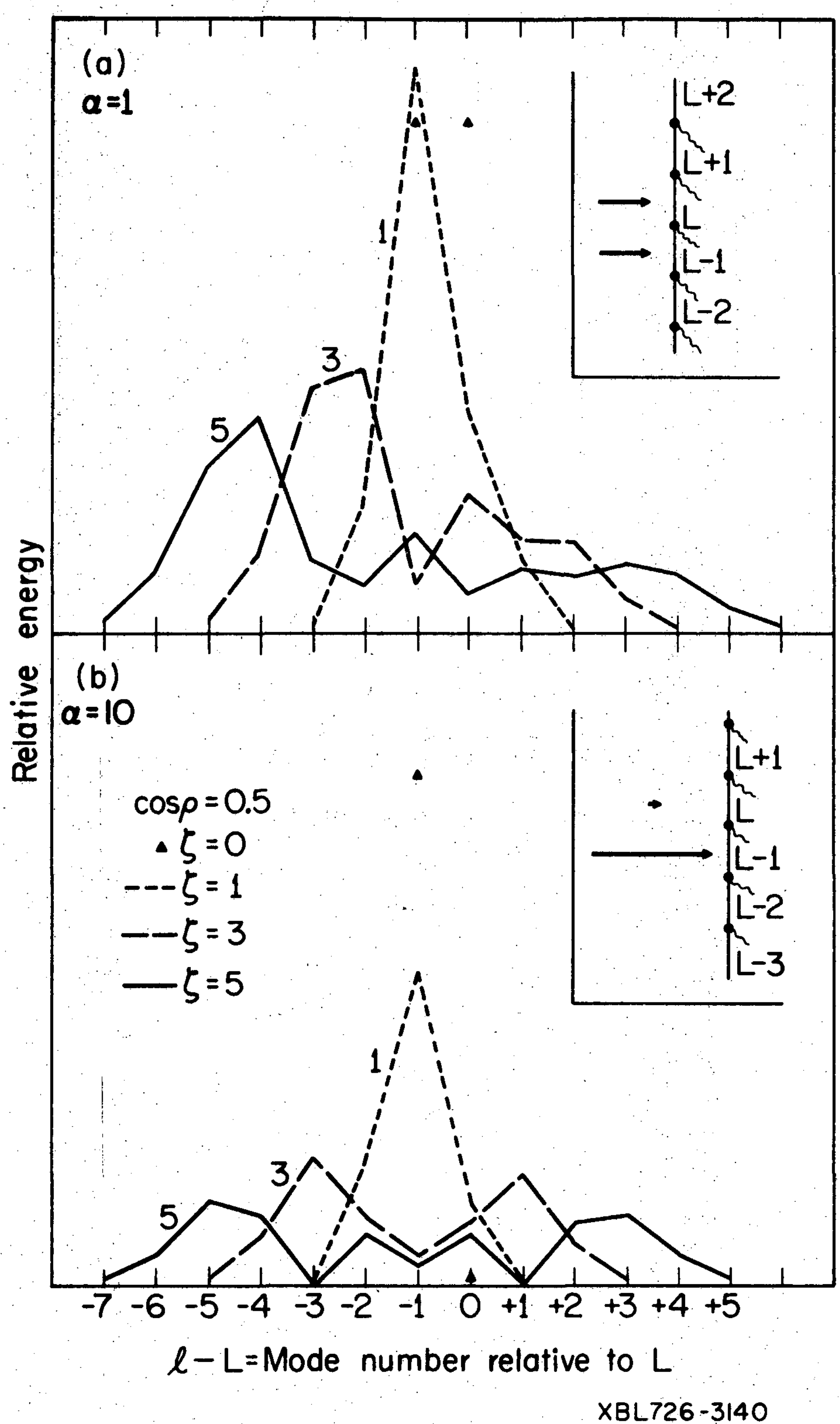
(a)
α=1
L+2
L+1
L
L-1
L-2
1
3
5
(b)
α=10
cosρ=0.5
ζ=0
ζ=1
ζ=3
ζ=5
L+1
L
L-1
L-2
L-3
Relative energy
-7 -6 -5 -4 -3 -2 -1 0 +1 +2 +3 +4 +5
ℓ−L=Mode number relative to L
XBL726-3140

Fig. 1

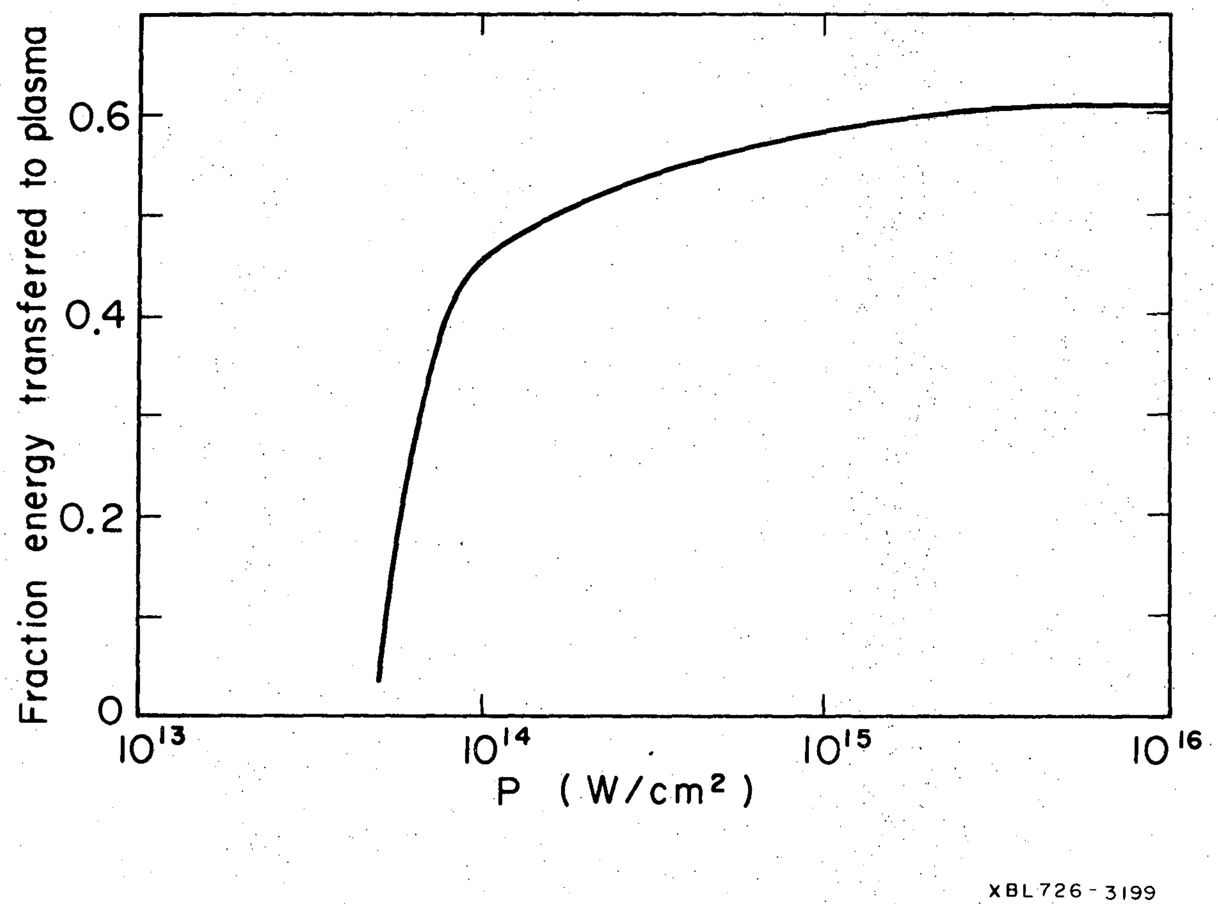
Fraction energy transferred to plasma
0
0.2
0.4
0.6
10^13
10^14
10^15
10^16
P (W/cm²)
XBL 726-3199

Fig. 2

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