Title

Connectivity of vehicular ad hoc networks with continuous node distribution patterns

Authors
Jin, W L
Wang, Bruce

Publication Date
2010-08-01

University of California Transportation Center
UCTC-FR-2010-30

Connectivity of vehicular ad hoc networks with continuous node distribution patterns

Wen-Long Jin and Bruce Wang
University of California, Irvine
August 2010

Connectivity of vehicular ad hoc networks with continuous node distribution patterns

Wen-Long Jin [*] and Bruce Wang [†]

April 20, 2010

Abstract

The connectivity of vehicular ad hoc networks (VANets) can be affected by the special distribution patterns, usually dependent and non-uniform, of vehicles in a transportation network. In this study, we introduce a new framework for computing the connectivity in a VANet for continuous distribution patterns of communication nodes on a line in a transportation network. Such distribution patterns can be estimated from traffic densities obtained through loop detectors or other detectors. When communication nodes follow homogeneous Poisson distributions, we obtain a new closed-form solution to connectivity; when distribution patterns of communication nodes are given by spatial renewal processes, we derive an approximate closed-form solution to the connectivity; and when communication nodes follow non-homogeneous Poisson distributions, we propose a recursive model of connectivity. For a shock-wave traffic, we demonstrate the consistency between analytical results with those simulated with ns-2, a

[*] Department of Civil and Environmental Engineering, California Institute for Telecommunications and Information Technology, Institute of Transportation Studies, 4000 Anteater Instruction and Research Bldg, University of California, Irvine, CA 92697-3600. Tel: 949-824-1672. Fax: 949-824-8385. Email: wjin@uci.edu. Corresponding author

[†] Zachry Department of Civil Engineering, Texas A&M University, College Station, TX; Tel: 979-845-9901. Email: bwang@civil.tamu.com or bwang@tamu.edu.

communication simulator. With the developed models, we also discuss the impacts on connectivity of road-side stations and different distribution patterns of vehicles. Given continuous traffic conditions, the connectivity model could be helpful for designing routing protocols in VANets and implementing vehicle-infrastructure integration systems. Limitations and future research related to this study are discussed in the conclusion section.

Keywords: Vehicular ad hoc networks; Inter-vehicle communications; Instantaneous connectivity; Traffic density; Homogeneous Poisson processes; Renewal processes; Non-homogeneous Poisson processes

1 Introduction

Rapid developments in various frontiers of telecommunications and information technologies could enable the development of next-generation Intelligent Transportation Systems (ITS) that rely on inter-vehicle communications (IVC) to disseminate time-critical and location-based traffic information. In 2004, US Department of Transportation initiated efforts in developing Vehicle Infrastructure Integration (VII) systems (USDOT, 2004; Dong et al., 2006). In a VII system or a Vehicular Ad hoc Network (VANet) shown in **Figure** 1, information can be exchanged among IVC-enabled vehicles, traffic management centers, various elements of road infrastructure including traffic signals, message signs, bus stops, and other safety hardware. If we call both IVC-enabled vehicles and road-side stations as nodes, only a portion of the vehicles are nodes in a transportation network. Compared with existing centralized transportation information systems, IVC-based systems are less costly to deploy and use and more resilient to natural disasters. However, we are also facing many challenges for developing such systems (Blum et al., 2004).

As in other mobile ad hoc networks, multihop connectivity is a fundamental performance measure of VANets. Estimating connectivity a priori is essential for determining specifications of appropriate communication devices, routing protocols, database management schemes, and the range of effective applications. In the literature, there have been extensive studies on multihop connectivity of various types of radio networks. Studied radio networks can be one-dimensional (Cheng and Robertazzi, 1989; Piret, 1991) or two-dimensional (Philips et al., 1989; Gupta and Kumar, 1998; Desai and Manjunath, 2002). Research methodologies include theoretical analysis of asymptotic connectivity (Dousse et al., 2002) based on percolation theory (Gilbert, 1961; Meester and Roy, 1996) as well as Monte Carlo simulations (Tang et al., 2003). Performance measures of connectivity include expected propagation distance (Cheng and Robertazzi, 1989), the probability for having at least one communication path between two nodes (Hartenstein et al., 2001), the k-connectivity (Penrose, 1999; Bettstetter, 2002), or the critical transmission range for asymptotic cases (Philips et al., 1989; Piret, 1991; Gupta and Kumar, 1998).

There have also been some studies of connectivity properties of VANets in recent years. In (Hartenstein et al., 2001), the probability of establishing a communication path between two nodes are studied for mobile bidirectional traffic with simulations. In (Wu et al., 2004), an analytical model was proposed for estimating connectivity for vehicles following a Poisson distribution and moving randomly and independently. In (Wu et al., 2005; Yang and Recker, 2005), the propagation distance was studied also with traffic simulations. In (Jin and Recker, 2006, 2010, 2007), the instantaneous success rate and connectivity of VANets were modeled by considering most-forwarded within-range (MFR) information propagation chains when vehicles' positions are given in a uniform or non-uniform traffic stream. In (Wang, 2007), instantaneous information propagation was modeled as a Markov chain in a uniform traffic stream where vehicles' positions follow a Poisson distribution. In (Ukkusuri and Du, 2008), the geometric connectivity of one-dimensional VANets is studied for positions of vehicles following independent distributions but with some disturbance. In (Wang et al., 2009), a convolution connectivity model was developed for a general distribution of the spacing between two consecutive vehicles. In (Jin and Recker, 2009), a connectivity model was developed for the connectivity between two nodes, given arbitrary locations of vehicles and probabilities for vehicles to nodes. In (Schönhof et al., 2006), connectivity of VANets for dynamic, uniform traffic is studied while store-and-forward information propagation is considered.

Indeed, connectivity and other performance of VANets are very hard to estimate due to the high mobility of communication nodes and signal interferences. In this study, we focus on another issue, namely, the non-uniform distribution patterns of vehicles and nodes in transportation networks. As we know, if vehicles' positions are assumed to follow a homogeneous Poisson distribution, then vehicles' movements are independent, and the traffic density is location-independent and uniform. However, on a line in a transportation network, e.g., the red line shown in **Figure** 1, on which there can be highways, arterial roads, surface streets, and other kinds of roadways, the density of vehicles and, therefore, the density of nodes, can vary dramatically due to driving behaviors and restrictions of network geometry. For examples, with higher travel demands and more lanes, freeways usually carry much higher density of vehicles than local streets; around a lane-drop or merging area, traffic density is usually significantly higher in the upstream section with the formation of queues; when a shock wave forms (Lighthill and Whitham, 1955), traffic density is higher in the downstream part; vehicles tend to form clusters in sparse traffic; and traffic signals can cause gaps between vehicle platoons. Therefore, it is important to consider the impacts of the special distribution patterns and mobility patterns of vehicles in transportation networks when estimating the connectivity between two nodes.

In a road network, traffic patterns can be represented by discrete vehicle positions as in car-following models (Gazis et al., 1961), distribution patterns of the spacings between two consecutive vehicles (Haight, 1963), or continuous densities as in kinematic wave models (Lighthill and Whitham, 1955; Richards, 1956; Jin, 2003). In general, the distribution of communication nodes on a road network is non-homogeneous and dynamic. To the best of our knowledge, there has been no analytical model of connectivity that can incorporate general, dynamic traffic. To achieve such a model, we believe that it is essential for us to understand stationary but general distributions of nodes. This has been the major motivation of this study. With loop detectors, it is possible to detect in real time continuous traffic conditions on a road network. For example, the PeMS project (CalTrans, 2010) can provide traffic densities, speeds, and flow-rates on most of California freeways. Such traffic densities can be used to estimate the distribution pattern of communication nodes, given market penetration rates.

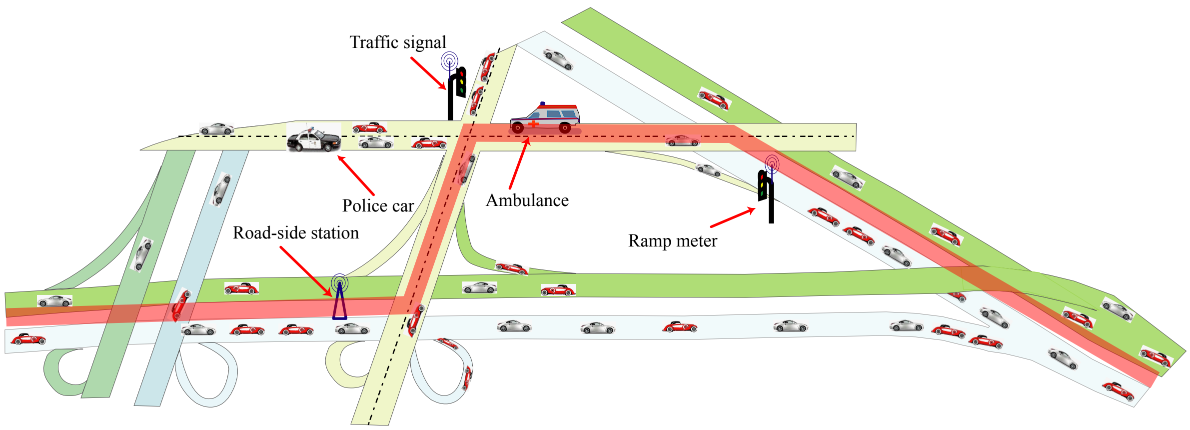

Figure 1: An illustration of vehicular ad hoc networks

In this study, we present a framework for computing multihop connectivity along a communication path in a road network when the distribution patterns of communication nodes are given by spatial Poisson processes or renewal processes (Grimmett and Stirzaker, 2001). That is, we are interested in the probability of establishing a communication path between two nodes, given traffic density and market penetration rate along the path. For example, we can use the developed model to estimate the probability of establishing a communication path marked by red in **Figure** 1. An efficient estimation of such connectivity is of practical importance: first, routing protocols would be able to find better information propagation routes by taking routes with higher connectivity; second, connectivity can be used as one metric when determining where and when to deploy stationary or mobile road-side stations. In our study, we are particularly interested in "instantaneous" connectivity at any time instant. Roughly, instantaneous connectivity is determined by the number of communication nodes, their relative locations, and the transmission range of wireless units. Here we denote location-dependent instantaneous traffic density by $\rho(x)$, market penetration rate by $\mu(x)$, and node density by $\kappa(x)$. Then we have that $\kappa(x) = \rho(x)\mu(x)$. In reality, it is possible that both traffic density and market penetration rate of communication nodes are known at any location, but the exact locations of equipped vehicles are usually unknown. Therefore, communication nodes follow random distributions with known, location-dependent densities. Note that, in our study of instantaneous connectivity, vehicles of both directions along a communication path are considered to have equivalent contributions. Also we do not differentiate lanes, since for an American standard lane of 12 feet (or 3.7 m) the lateral distances between vehicles on a bidirectional, multilane freeway can be omitted for a Dedicated Short-Range Communication (DSRC) transmission range of up to 1000 m (FCC, 1999). That is, in our study, we consider a communication path to be a line of communication nodes.

In this study, we will develop a model for the connectivity of VANets along a line of vehicles in a transportation network. We omit the impacts of the mobility of nodes on the connectivity and consider the instantaneous multihop connectivity as in (Jin and Recker, 2006, 2010, 2009). Different from most existing studies, we do not assume either independent or uniform distributions of nodes. Different from studies in (Jin and Recker, 2006, 2010, 2009), where the positions of vehicles are discrete, we consider continuous distributions of vehicles, where either the distribution function of spacings between two consecutive nodes or the location-dependent density of nodes are given. We will first develop relationships between the *end node probability*, which is the probability for a node to be the last or end node of a communication path starting from a sender, and *connectivity*, which is the probability for existing a communication path between a node and the sender. Then we can derive a simplified model for computing connectivity between two nodes. With the model, we will be able to discuss how the distribution patterns of vehicles and traffic dynamics can affect connectivity properties of an IVC system. Although the results are more general since the distribution patterns of communication nodes are arbitrary, the results can also be applied for the case when communication nodes follow Poisson distributions.

The rest of the paper is organized as follows. In Section 2, we give a detailed explanation of the conceptual framework and definitions for our new model and derive an analytical recursive model of connectivity for general traffic with arbitrary distribution of nodes. In Section 3, we study connectivity of VANets when the communication nodes follow homogeneous Poisson distributions. In Section 4, when communication nodes follow a renewal process, we approximate the renewal process by a Poisson process and obtain a closed formula for connectivity of VANets. In Section 5, when communication nodes follow non-homogeneous Poisson processes when location-dependent densities are given, we obtain a recursive formula for computing connectivity. In Section 6, we compare the analytical results with those of ns-2 simulations for a shock-wave traffic scenario. In Section 7, we make some conclusions.

2 A recursive model of connectivity for general traffic

We consider vehicles and road-side stations along a line in a transportation network and assume that information propagates in the direction of x-axis. At a time instant, we take a snap-shot of traffic flow on a line and analyze IVC information propagation along the line. The continuous patterns of nodes in spacing distributions or densities, obtained through observations or simulations, can be arbitrarily uniform or non-uniform. Here we assume that all nodes have the same DSRC transmission range r and apply a simple communication model, in which two nodes can communicate with each other when they are within each other's transmission range, and do not consider the effect of signal interferences (Gupta and Kumar, 2000).

2.1 Model derivation

For the information propagation process starting from the sender at $x = x_0$ to any location $x \geq x_0$, we denote the farthest reach of a message through multihop relays by a random variable X. That is, the message can only travel as far as X after multiple hops. Let $p(x_0, x)$ $(x > x_0)$ be the probability density function of the farthest reach of a message sending from x_0; i.e., $p(x_0, x) = Prob(X = x)$.

Therefore, $p(x_0,x)\Delta x$ can be considered as the *end node probability* in $[x, x+\Delta x)$ as in (Jin and Recker, 2009). If the probability that there does not exist another node within the transmission range of the sender is $P(x_0)$, then the probability density function on the whole road, $\bar{p}(x_0,x)$, has a spike (or discrete component) at x_0 of height $P(x_0)$. Thus $\bar{p}(x_0,x)$ is a mixed probability density function (Haight, 1963)

$$\bar{p}(x_0,x) = \begin{cases} P(x_0)\delta(x-x_0) + p(x_0,x), & x \geq x_0, \\ 0, & x < x_0, \end{cases}$$

where $\delta(x-x_0)$ is a Dirac function. Note that $\bar{p}(x_0,x)$ is 0 for $x < x_0$, since we do not consider information propagation in the opposite direction of x-axis. If $\mathbf{P}(x_0,x) = Prob(X \leq x)$ is the cumulative distribution of the farthest reach, then

$$\mathbf{P}(x_0,x) = \int_{-\infty}^{x} \bar{p}(x_0,y)dy = \begin{cases} P(x_0) + \int_{y=x_0}^{x} p(x_0,y)dy, & x \geq x_0, \\ 0, & x < x_0. \end{cases} \quad (1)$$

Thus $\mathbf{P}(x_0,x_0) = P(x_0)$, and $\mathbf{P}(x_0,x)$ can be considered as the probability for a message to cover a region of $[x_0,x]$. Therefore, $\mathbf{S}(x_0,x) = Prob(X > x) = 1 - \mathbf{P}(x_0,x)$ is the success rate for a message to travel beyond x.

Let $\mathbf{C}(x_0,x)$ be the connectivity between two nodes at x_0 and x, i.e., the probability for a message to reach a receiver at x. Then we have that

$$\mathbf{C}(x_0,x) = 1 - \mathbf{P}(x_0,x-r) = \mathbf{S}(x_0,x-r), \quad (2)$$

since two nodes at x_0 and x are connected if and only if a message can travel beyond $x-r$ from x_0. Obviously, $\mathbf{C}(x_0,x) = 1$ when $x \in [x_0, x_0+r)$, but $\mathbf{C}(x_0,x_0+r) = 1 - \mathbf{P}(x_0,x_0) = 1 - P(x_0)$. Thus $\mathbf{C}(x_0,x)$ is not continuous at x_0+r. From (2) we have for $x > x_0$

$$p(x_0,x) = -\frac{\mathrm{d}\mathbf{C}(x_0,x+r)}{\mathrm{d}x}. \quad (3)$$

We denote the probability for existing at least one node in $[x,x+\Delta x]$ by $\mathscr{P}_1([x,x+\Delta x])$ and the probability for existing no node in $(x+\Delta x, x+r+\Delta x]$ by $\mathscr{P}_0((x+\Delta x, x+r+\Delta x])$. The farthest reach of a message is in $[x, x+\Delta x]$ if and only if (i) there exists at least one node in $[x, x+\Delta x]$, (ii) the node is connected to the sender, and (iii) there is no node in $(x+\Delta x, x+r+\Delta x]$. Since all these three events are independent, the probability for the farthest reach in $[x, x+\Delta x]$ is

$$\bar{p}(x_0, x)\Delta x = \mathscr{P}_1([x, x+\Delta x])\mathbf{C}(x_0, x)\mathscr{P}_0((x+\Delta x, x+r+\Delta x]). \tag{4}$$

Letting $\Delta x \to 0$, we thus have

$$\bar{p}(x_0, x) = \mathbf{C}(x_0, x)\mathscr{P}_0((x, x+r]) \lim_{\Delta x \to 0} \frac{\mathscr{P}_1([x, x+\Delta x])}{\Delta x} = \mathbf{C}(x_0, x)\lambda(x, r), \tag{5}$$

where $\lambda(x, r) = \mathscr{P}_0((x, x+r]) \lim_{\Delta x \to 0} \frac{\mathscr{P}_1([x, x+\Delta x])}{\Delta x}$. In particular, we have

$$\lambda(x_0, r) = \mathscr{P}_0((x_0, x_0+r])\delta(x-x_0), \tag{6}$$

since the sender is always equipped and $\mathscr{P}_1([x_0, x_0+\Delta x]) = 1$. From (1) we have

$$P(x_0) = \mathscr{P}_0((x_0, x_0+r]). \tag{7}$$

Thus we have $\lambda(x_0, r) = P(x_0)\delta(x-x_0)$.

Combining (1)-(2) and (5), we have the following models of connectivity and probability density function of farthest reach

$$\mathbf{C}(x_0, x) = \begin{cases} 1 - \int_{y=x_0}^{x-r} \mathbf{C}(x_0, y)\lambda(y, r)dy, & x \geq x_0 + r, \\ 1, & x < x_0 + r. \end{cases} \tag{8}$$

$$\bar{p}(x_0, x) = \begin{cases} (1 - \int_{y=x_0}^{x-r} \bar{p}(x_0, y)dy)\lambda(x, r), & x \geq x_0, \\ 0, & x < x_0. \end{cases} \tag{9}$$

In particular, when $x = x_0 + r$, we have

$$\mathbf{C}(x_0, x_0+r) = 1 - \int_{y=x_0}^{x_0} \mathbf{C}(x_0, y)\lambda(y, r)dy.$$

From (6) and (7), we have $\mathbf{C}(x_0, x_0+r) = 1 - P(x_0)$, which is consistent with (2). In this model, $\lambda(x, r)$ depends on the distribution of nodes along a line and are determined by traffic conditions and the market penetration of equipped vehicles. Further, both equations are recursive, in the sense that both $\mathbf{C}(x_0,x)$ and $p(x_0,x)$ can be computed from the corresponding quantities in $[x_0, x-r]$. In addition, from (3) and (5), we can obtain a first-order linear delay differential equation for $x - r > x_0$

$$\frac{\mathrm{d}\mathbf{C}(x_0,x)}{\mathrm{d}x} = -\lambda(x-r,r)\mathbf{C}(x_0,x-r). \tag{10}$$

From (8) and (9) we can see that the connectivity between the sender at x_0 and a receiver at x only depends on the distribution of nodes in $[x_0,x]$, and $p(x_0,x)$ depends on the distribution of nodes in $[x_0,x+r]$. In addition, $\mathbf{C}(x_0,x)$ is non-increasing with x, but $\bar{p}(x_0,x)$ may not be. The connectivity function is symmetric with respect to the information propagation direction; i.e., $\mathbf{C}(x_0,x) = \mathbf{C}(x,x_0)$.

2.2 Improving connectivity with road-side stations

In VANets, as shown in **Figure** 1, stationary road-side stations can be installed by transportation authorities for the purpose of collecting and disseminating traffic information or by other parties for commercial or other purposes. In emergency situations, police cars, ambulance, and other emergency vehicles could be deployed along a road and serve as road-side stations. Here we assume that the road-side stations have the same communication units as vehicles and are not inter-connected through wired or wireless communications other than inter-vehicle communications. This assumption is particularly true when road-side stations are also deployed in an ad hoc fashion. When road-side stations are inter-connected, the results are quite straightforward.

Theorem 2.1 *If there is a road side station at* $x = x_1 \in (x_0,x_2)$*; i.e., if* $\lambda(x_1,r) = \mathscr{P}_0((x_1,x_1+r])\delta(x-x_1)$*, then*

$$\mathbf{C}(x_0,x_2) = \mathbf{C}(x_0,x_1)\mathbf{C}(x_1,x_2). \tag{11}$$

Proof. If $x_2 < x_0 + r$, it is obvious, since $\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1) = \mathbf{C}(x_1, x_2) = 1$. For $x_2 \geq x_0 + r$, we have from (8)

$$\begin{aligned}\mathbf{C}(x_0, x_2) &= 1 - \int_{y=x_0}^{x_1 - r} \mathbf{C}(x_0, y)\lambda(y, r)dy \\ &\quad - \int_{y=x_1-r}^{x_1^-} \mathbf{C}(x_0, y)\lambda(y, r)dy \\ &\quad - \int_{y=x_1}^{x_2 - r} \mathbf{C}(x_0, y)\lambda(y, r)dy.\end{aligned}$$

Since we can always find a node, i.e., the road-side station, in $(y, y+r]$, $\lambda(y, r) = \mathscr{P}_0((y, y+r]) \lim_{\Delta x \to 0} \frac{\mathscr{P}_1([y, y+\Delta x])}{\Delta x} = 0$ for $y \in [x_1 - r, x_1)$, we have

$$\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1) - \int_{y=x_1}^{x_2 - r} \mathbf{C}(x_0, y)\lambda(y, r)dy.$$

When $x_2 < x_1 + r$, from the equation above we have $\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1)$. Since $\mathbf{C}(x_1, x_2) = 1$ for $x_2 < x_1 + r$, we have $\mathbf{C}(x_0, x_2) = \mathbf{C}(x_0, x_1)\mathbf{C}(x_1, x_2)$. For $x_2 \geq x_1 + r$, assuming that $\mathbf{C}(x_0, y) = \mathbf{C}(x_0, x_1)\mathbf{C}(x_1, y)$ for $y \in [x_1, x_2 - r]$, from the equation above we have

$$\begin{aligned}\mathbf{C}(x_0, x_2) &= \mathbf{C}(x_0, x_1) - \int_{y=x_1}^{x_2 - r} \mathbf{C}(x_0, x_1)\mathbf{C}(x_1, y)\lambda(y, r)dy \\ &= \mathbf{C}(x_0, x_1)(1 - \int_{y=x_1}^{x_2 - r} \mathbf{C}(x_1, y)\lambda(y, r)dy) \\ &= \mathbf{C}(x_0, x_1)\mathbf{C}(x_1, x_2).\end{aligned}$$

By mathematical induction, we conclude that (11) is always true. ■

For given distributions of nodes, i.e., $\lambda(x, r)$ for $x \in [x_0, x_2]$, the solution of the following optimization problem

$$\max_{x \in [x_0, x_2]} \mathbf{C}(x_0, x)\mathbf{C}(x, x_2)$$

will yield the best location for deploying a road-side station along $[x_0, x_2]$. For more than one road-side station, we can also formulate the problem similarly. Thus, the solution of the optimization problem will determine the best locations of deploying emergency vehicles when disasters occur.

3 Homogeneous Poisson distributions of nodes

In this section, we consider the case when communication nodes follow spatial Poisson distributions, either homogeneous or non-homogeneous. Here we assume that the sender is at $x_0 = 0$ and simply denote $P_0 = P(x_0)$, $\mathbf{C}(x) = \mathbf{C}(x_0, x)$, $\mathbf{P}(x) = \mathbf{P}(x_0, x)$, $p(x) = p(x_0, x)$, and $\bar{p}(x) = \bar{p}(x_0, x)$. If nodes follow the Poisson distribution with a constant node density κ, then the nodes are uniformly and independently distributed on a line, and the spacing between two consecutive nodes follows a negative exponential distribution. Note that $\kappa = \rho(x)\mu(x)$, where $\rho(x)$ is the instantaneous traffic densities, and $\mu(x)$ the instantaneous market penetration rates. Thus we can have varying traffic densities and market penetration rates but constant node densities.

3.1 A closed-form solution

When the distributions of communication nodes are given by a homogeneous Poisson process with a location-independent average density κ, we have that $\mathscr{P}_0((x, x+r]) = e^{-\kappa r}$, and $\lim_{\Delta x \to 0} \frac{\mathscr{P}_1([x, x+\Delta x])}{\Delta x} = \kappa$. Therefore, $P_0 = e^{-\kappa r}$, and $\lambda(x, r) \equiv \lambda(r) = \kappa e^{-\kappa r}$. That is $\lambda(x, r)$ is location-independent with homogeneous distributions of communication nodes. Then the recursive connectivity model (8) can be written as

$$\mathbf{C}(x) = \begin{cases} 1, & x < r, \\ \mathbf{C}(r) \equiv 1 - P_0, & x = r, \\ \mathbf{C}(r) - \lambda(r) \int_{y=0^+}^{x-r} \mathbf{C}(y) dy, & x > r. \end{cases} \tag{12}$$

Theorem 3.1 *We have that for a natural number $n \geq 1$*

$$\mathbf{C}(x) = \begin{cases} 1, & x \in [0, r), \\ \sum_{m=0}^{n} \mathbf{C}(mr) \frac{(-\lambda(r)(x-nr))^{n-m}}{(n-m)!}, & x \in [nr, (n+1)r], n = 1, 2, \cdots \end{cases} \tag{13}$$

The proof of this theorem is given in Appendix A. The derivation of (13) is based on the observation of (12) that $\mathbf{C}(x)$ is constant in $[0, r)$, linear in $[r, 2r)$, quadratic in $[2r, 3r)$, and so on.

Note that for $x \in [0, r)$, we can also have $\mathbf{C}(x) = \sum_{m=0}^{n} \mathbf{C}(mr) \frac{(-\lambda(r)(x-nr))^{n-m}}{(n-m)!}$ with $n = 0$, but we have it separately to emphasize that there is a discontinuity at $x = r$. That is, $\mathbf{C}(x)$ is continuous at any $x > r$.

From (13), in particular, we have $C(0) = 1$ and

$$
\begin{aligned}
\mathbf{C}((n+1)r) &= \begin{cases} 1 - P_0, & n = 0, \\ \sum_{m=0}^{n} \mathbf{C}(mr) \frac{(-\lambda(r)r)^{n-m}}{(n-m)!}, & n \geq 1. \end{cases} \\
&= \begin{cases} 1 - P_0, & n = 0, \\ \sum_{m=0}^{n} \mathbf{C}((n-m)r) \frac{(-\lambda(r)r)^{m}}{m!}, & n \geq 1. \end{cases}
\end{aligned} \tag{14}
$$

It is straightforward to show that (13) is equivalent to the closed-form solution in (Dousse et al., 2002). Also it can be shown that $\mathbf{C}((n+1)r) \leq \mathbf{C}(nr)$ and $\mathbf{C}(x) \in [\mathbf{C}((n+1)r), \mathbf{C}(nr)]$ for $x \in [nr, (n+1)r]$.

3.2 Properties

From (10) we obtain for $x > r$

$$
\frac{\mathrm{d}\mathbf{C}(x)}{\mathrm{d}x} = -p(x-r) = -\lambda(r)\mathbf{C}(x-r). \tag{15}
$$

According to (Dousse et al., 2002), we have for $x \geq 2r$

$$
\mathbf{C}(r)e^{-(x-2r)\lambda(r)} - \lambda(r) \leq \mathbf{C}(x) \leq \mathbf{C}(r)e^{-(x-r)\lambda(r)}. \tag{16}
$$

Here we can study the behavior of connectivity by assuming the following critical transmission range

$$
r_c = \alpha \log(x\kappa)/\kappa, \tag{17}
$$

where $\alpha > 0$. That is, when a transmission range r is greater than r_c, then the VANet along the traffic stream is totally connected; when r is smaller than r_c, the VANet is not connected. Then

$\lambda(r_c) = \kappa(x\kappa)^{-\alpha}$, $\mathbf{C}(r_c) = 1-(x\kappa)^{-\alpha}$, and

$$\mathbf{C}(r_c)e^{-(x-2r)\kappa(x\kappa)^{-\alpha}} - \kappa(x\kappa)^{-\alpha} \le \mathbf{C}(x) \le \mathbf{C}(r_c)e^{-(x-r)\kappa(x\kappa)^{-\alpha}}.$$

For fixed κ and $x \to \infty$, we have

$$\begin{aligned} \lim_{x\to\infty} \mathbf{C}(r_c)e^{-(x-2r)\kappa(x\kappa)^{-\alpha}} - \kappa(x\kappa)^{-\alpha} &= \\ \lim_{x\to\infty} \mathbf{C}(r_c)e^{-(x-r)\kappa(x\kappa)^{-\alpha}} &= \begin{cases} 1, & \alpha > 1, \\ 0, & \alpha < 1. \end{cases} \end{aligned}$$

Therefore,

$$\lim_{x\to\infty} \mathbf{C}(x) = \begin{cases} 1, & \alpha > 1, \\ 0, & \alpha < 1, \end{cases}$$

which is consistent with the results in (Piret, 1991). For fixed x and $\kappa \to \infty$, we have similar results. In (Gupta and Kumar, 1998), a less strict bound on r_c for two-dimensional wireless networks and $\kappa \to \infty$ was presented, and it is as expected since better connectivity can be achieved in a two-dimensional network.

From (13) and (2), we have the cumulative distribution of farthest reach as $x \in [(n-1)r, nr]$

$$\mathbf{P}(x) = 1 - \sum_{m=0}^{n} \mathbf{C}(mr) \frac{(-\lambda(r)(x+r-nr))^{n-m}}{(n-m)!}. \tag{18}$$

Further from (5) and (15), we have $n = 1, 2, \cdots$

$$p(x) = \lambda(r)\mathbf{C}(x) = \begin{cases} \lambda(r), & x \in (0, r), \\ \lambda(r)\sum_{m=0}^{n} \mathbf{C}(mr)\frac{(-\lambda(r)(x-nr))^{n-m}}{(n-m)!}, & x \in [nr, (n+1)r]. \end{cases}$$

Then the average distance for information propagation along an infinitely long traffic stream is

$$\begin{aligned} E(X) &= \int_{x=0}^{\infty} p(x)xdx = -\int_{x=0}^{\infty} xd\mathbf{C}(x+r) \\ &= -x\mathbf{C}(x+r)|_0^{\infty} + \int_{x=0}^{\infty} \mathbf{C}(x+r)dx \\ &= \int_{x=0}^{\infty} \mathbf{C}(x+r)dx = -\frac{\mathbf{C}(x+2r)}{\lambda(r)}|_{x=0}^{\infty} \\ &= \frac{\mathbf{C}(2r)}{\lambda(r)} = \frac{\mathbf{C}(r)}{\lambda(r)} - r, \end{aligned} \tag{19}$$

according to (15) and (14). Therefore,

$$E(X) \quad = \quad \frac{e^{\kappa r}-1}{\kappa} - r.$$

Similarly, we can get the variance of the information propagation distance as

$$V(X) = E(X^2) - [E(X)]^2 = \frac{2}{\lambda(r)^2}C(3r) - [E(x)]^2 = \frac{e^{2\kappa r} - 2\kappa r e^{\kappa r} - 1}{\kappa^2}.$$

Both results are consistent with those in (Wang, 2007).

In **Figure** 2, we show the impact of the location of a road-side station on the connectivity between a sender and a receiver which are 10 km away with $\kappa = 4$ nodes/km and $r = 1$ km. Here the connectivity is computed with (13) and (11). From the figure we can see that the best locations of the road-side station are just within the transmission range of either the sender or the receiver. Here the discontinuity in the connectivity curve is caused by that in $\mathbf{C}(x)$ in (13) at $x = r$.

4 An approximate connectivity model for renewal processes of communication nodes

When the distribution patterns of communication nodes are governed by renewal processes, we denote the spacing between two communication consecutive nodes by a random variable S. Let $f(s)$ be the probability density function of S, so that $F(s) = Prob(S \leq s)$ is the cumulative distribution of the spacing. As we know, on a multilane road, $s \in [0, \infty)$. In this case, the distribution of nodes is still location-independent, but may lead to non-uniform traffic density.

4.1 A convolution model

For any spacing distribution function $f(s)$, we have for $x \geq r$ (Dousse et al., 2002; Wang et al., 2009)

$$\mathbf{C}(x) \quad = \quad \int_{x-r}^{x} \mathbf{C}(y) f(x-y) dy = \int_{0}^{r} \mathbf{C}(x-s) f(s) ds. \tag{20}$$

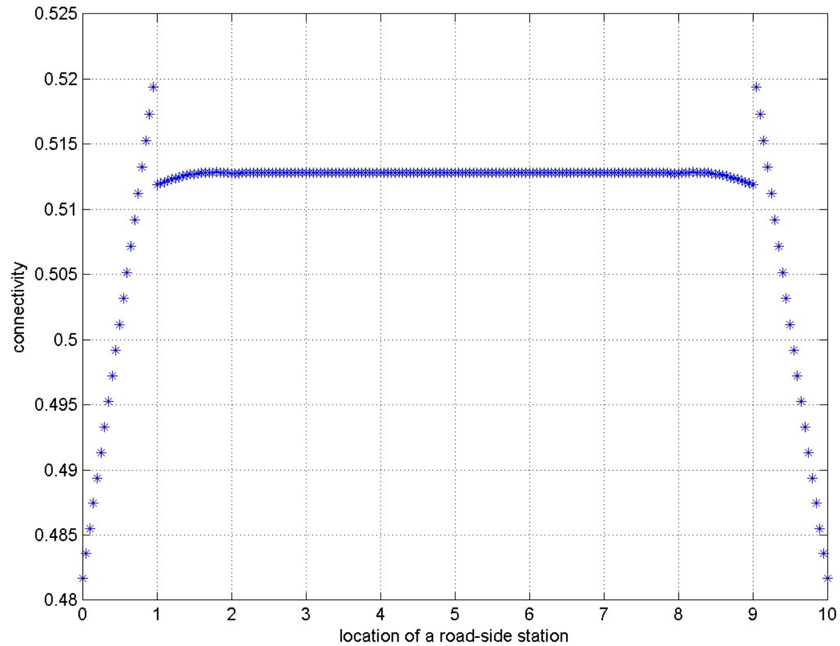

Figure 2: Impact of the location of a road-side station

This yields $\mathbf{C}(r) = \int_0^r f(s)ds$, and

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x) &= \int_0^r \frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x-s)f(s)ds = -\int_0^r f(s)d\mathbf{C}(x-s) \\ &= -f(s)\mathbf{C}(x-s)|_0^r + \int_0^r \mathbf{C}(x-s)f'(s)ds \\ &= -f(r)\mathbf{C}(x-r) + f(0)\mathbf{C}(x) + \int_0^r \mathbf{C}(x-s)f'(s)ds\end{aligned}$$

If we can still apply the recursive model (8) with a location-independent $\lambda(r)$, then we have the following:

$$\mathbf{C}(x) = \begin{cases} 1, & x < r, \\ \mathbf{C}(r) \equiv 1 - P_0 = \int_0^r f(s)ds, & x = r, \\ \mathbf{C}(r) - \lambda(r)\int_{y=0^+}^{x-r} \mathbf{C}(y)dy, & x > r, \end{cases} \tag{21}$$

and $\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x) = -\lambda(r)\mathbf{C}(x-r)$ for $x > r$, which leads to $\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x)|_{r^+} = -\lambda(r)$. For $f(s) = \kappa^2 e^{-\kappa s} s$, we have $f(0)=0$, and $\frac{\mathrm{d}}{\mathrm{d}x}\mathbf{C}(x)|_{r^+}=-f(r)\mathbf{C}(0)+f(r)=0$. In addition, $\lim_{\Delta x\to 0}\frac{\mathscr{P}_1([x,x+\Delta x])}{\Delta x}=0$. Therefore, $f(r)=0$, and we cannot obtain a recursive formulation as in (21).

4.2 An approximate closed-form solution

We derive an approximate solution of (20) in the form of (21) and choose $\lambda(r)$ such that the average information propagation distance is the same. From (19), we have

$$E(x) = \frac{\int_0^r f(s)ds}{\lambda(r)} - r.$$

The average information propagation distance for (20) is (Wang et al., 2009)

$$E(x) = \frac{\int_0^r sf(s)ds}{1-\int_0^r f(s)ds}.$$

Therefore, we choose

$$\lambda(r) = \frac{\int_0^r f(s)ds(1-\int_0^r f(s)ds)}{r-r\int_0^r f(s)ds+\int_0^r sf(s)ds}. \tag{22}$$

Then the closed-form solution of (21) is the same as in (13) with $\lambda(r)$ given in (22).

When $f(s)=\kappa^2 e^{-\kappa s}s$, $r=1$ km, and $\kappa=5.8$ nodes/km, the results of the connectivity at x obtained by the accurate model in (20) and the approximate model in (13) with (22) are shown in **Figure** 3, from which we can see the approximate model is quite accurate.

5 Non-homogeneous Poisson distributions of nodes

In a continuous traffic flow theory, one can track the changes in density $\rho(x,t)$ at location x and time t. Further, if we assume that the probability for vehicles to be equipped as $\mu(x,t)$, then the density of nodes is $\kappa(x)=\rho(x)\mu(x)$, where time t is omitted for instantaneous information propagation in a continuous traffic stream. In this case, the distribution patterns of communication nodes follow a

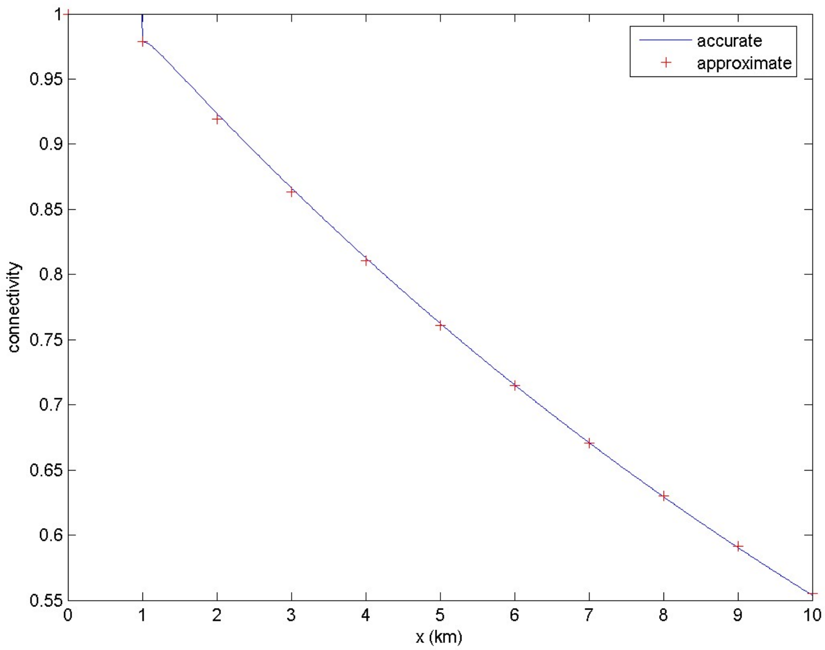

Figure 3: Accurate and approximate connectivity computed from (20) and (14), respectively

non-homogeneous Poisson process with location-dependent densities. From the definition of $\kappa(x)$, we can see that

$$\kappa(x) = \lim_{\Delta x \to 0} \frac{\mathscr{P}_1([x, x+\Delta x])}{\Delta x}. \tag{23}$$

That is, $\kappa(x)\Delta x$ is the probability for finding at least one equipped vehicle in $[x, x+\Delta x]$. Since the sender is at $x = x_0$ and always equipped, we have a spike at $x = x_0$ and

$$\bar{\kappa}(x) = \delta(x - x_0) + \kappa(x).$$

5.1 A recursive model

For such a continuous traffic stream, we do not have a location-independent spacing distribution function and thus cannot use the convolution formulation in (20). However, since the probability for existing no node in $(x^+, x+r]$ is $\mathscr{P}_0((x^+, x+r]) = e^{-\int_{x^+}^{x+r}\kappa(y)dy}$, we obtain a recursive model in (8) and the differential equation in (10) with

$$\lambda(x,r) = \kappa(x)e^{-\int_{x^+}^{x+r}\kappa(y)dy}. \tag{24}$$

The correctness of the model (8) with (24) can be proved with the discrete model developed in (Jin and Recker, 2009) as follows. We approximate $\kappa(x)$ with a piece-wise constant function as

$$\bar{\kappa}(x) = \begin{cases} \delta(x-x_0), & x = x_0 \\ \kappa_i \equiv \int_{x_0+(i-1)\Delta x}^{x_0+i\Delta x}\kappa(x)dx, & x - x_0 \in ((i-1)\Delta x, i\Delta x] \end{cases}$$

Then in cell i for $x \in (x_0+(i-1)\Delta x, x_0+i\Delta x]$, where $\Delta x = (x-x_0)/m$, we place vehicles at $x_{i,j} = x_0+(i-1)\Delta x + \frac{j-1/2}{n}\Delta x$ for $j = 1, \cdots, n$, and each vehicle has the probability of $\mu_i = \frac{\kappa_i \Delta x}{n}$ to be a node. For this discrete traffic stream, we can apply the results in (Jin and Recker, 2009) and obtain the connectivity of $\mathbf{C}_d(x_0, x; m, n)$. Then $\mathbf{C}(x_0, x) = \lim_{m,n\to\infty}\mathbf{C}_d(x_0, x; m, n)$ satisfies (8) and (24).

We can compute $\mathbf{C}(x_0, x)$ numerically as follows. We divide $[x_0, x]$ into cells with a length of $\Delta x = r/n$ and denote $\mathbf{C}_i \equiv \mathbf{C}(x_0, x_i)$ with $x_i = x_0 + i\Delta x$. We compute $\lambda(x_i, r)$ in (24) as

$$\lambda(x_i, r) = \kappa(x_i)e^{-\sum_{j=i+1}^{i+n}\kappa(x_j)\Delta x}.$$

In particular, $\lambda(x_0, r) = e^{-\sum_{j=1}^{n}\kappa(x_j)\Delta x}$. Further we can compute $\mathbf{C}_i$ from (8) for $i \geq n$

$$\mathbf{C}_i = \begin{cases} 1, & i < n, \\ 1 - \lambda(x_0, r), & i = n, \\ \mathbf{C}_{i-1} - \mathbf{C}_{i-n}\lambda(x_{i-n}, r)\Delta x, & i > n. \end{cases}$$

5.2 Impacts of node distributions on connectivity

In this subsection, we consider a road section with a length of 10 km, on which a sender and a receiver are at the boundaries. The average density of the road section is ρ, and vehicles are uniformly distributed with density ρ_1 and ρ_2 on the first and second halves of the road, respectively. Thus, $\rho_1+\rho_2=2\rho$. When the probability for all vehicles inside the road section to be equipped is the same as $\mu=0.1$, and the transmission range for all nodes is the same as $r=1$ km, we have

$$\kappa(x) = \begin{cases} \rho_1\mu, & x\in[0,5], \\ \rho_2\mu, & x\in(5,10]. \end{cases}$$

When we divide a transmission range into $n=100$ cells, the connectivity between the sender and the receiver is shown in **Figure** 4, where the average densities, ρ, vary from 40 veh/km to 60 veh/km, and the densities in the first half, ρ_1, vary from $\frac{\rho}{2}+1$ to $\frac{3\rho}{2}-1$. From the figure, we can see that, when the total number of nodes inside the road section is the same, the distribution patterns of vehicles can significantly affect the connectivity, and the connectivity reaches its maximum when $\rho_1=\rho$; i.e., when vehicles are uniformly distributed on the whole road section. In addition, we can see that the connectivity is symmetric in ρ_1. Here, we make the following conjecture: For the same number of nodes between a sender and a receiver, when all nodes have the same transmission range, then the connectivity between the sender and the receiver reaches its maximum when these nodes are uniformly distributed between the sender and the receiver.

6 Instantaneous success rate in a shock-wave traffic

In this section, we apply the new modeling framework to estimate success rates of instantaneous information propagation through a shock-wave traffic. Traffic dynamics are governed by the Lighthill-Whitham-Richards (LWR) model(Lighthill and Whitham, 1955; Richards, 1956) and a triangular fundamental diagram (Munjal et al., 1971; Haberman, 1977; Newell, 1993):

$$Q(\rho) = \min\{v_f\rho, \frac{1}{4}v_f(\rho_j-\rho)\},$$

where the free flow speed $v_f=104$ km/h, and jam density $\rho_j=150$ veh/km/lane. The speed-density relation can be written as $V(\rho)=Q(\rho)/\rho$.

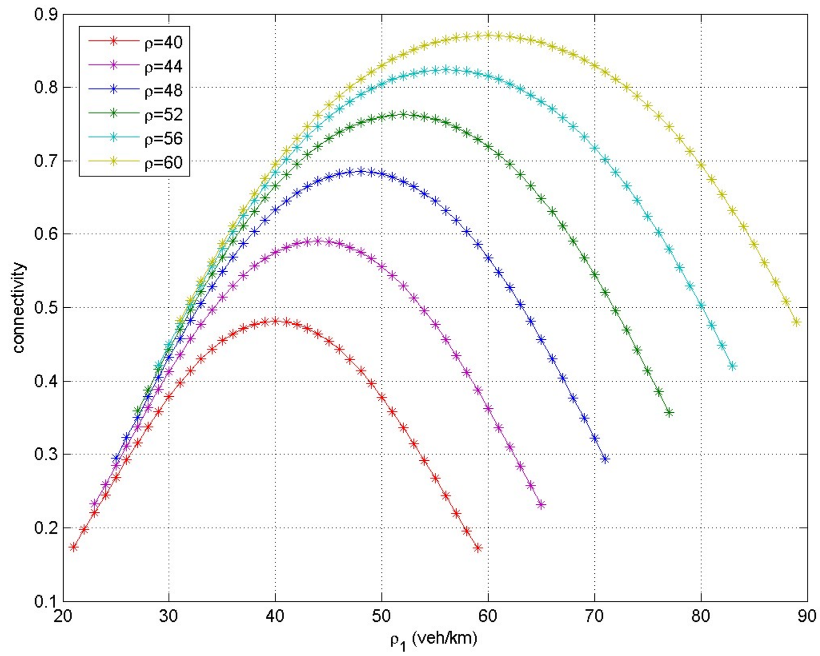

Figure 4: Impacts of node distribution patterns on connectivity

Here we consider unidirectional traffic on a two-lane road, on which, the initial traffic density on the road is (unit: veh/km)

$$\rho(x, t=0) = \begin{cases} 60, & x < 0; \\ 80, & x > 0. \end{cases}$$

Thus the initial speeds are (unit: km/h)

$$v(x, t=0) = \begin{cases} 104, & x < 0; \\ 71.5, & x > 0. \end{cases}$$

From the LWR model, we can see that a shock wave emanates from $x = 0$ and travels upstream at a speed of $v_s = -26$ km/h. Solution of traffic density is given by (unit: veh/km)

$$\rho(x, t) = \begin{cases} 60, & x < -26t; \\ 80, & x > -26t. \end{cases}$$

In this example, we assume that an information source travels with a vehicle initially at $x_0 = -10$ km. We can see that the information source meets the shock wave at $t_c = |x_0|/(104+26) = \frac{1}{13}$ hr≈ 4.6 min. Then the trajectory of the information source is

$$x(t) = \begin{cases} -10+104t, & t < \frac{1}{13} \text{ hr}, \\ -2+71.5(t-\frac{1}{13}), & t \geq \frac{1}{13} \text{ hr}. \end{cases}$$

If we denote $s(t)$ as the distance between the shock wave interface and the information source, then

$$s(t) = -26t - x(t) = \begin{cases} -130(t-\frac{1}{13}), & t < \frac{1}{13} \text{ hr}, \\ -97.5(t-\frac{1}{13}), & t \geq \frac{1}{13} \text{ hr}. \end{cases}$$

Then at any time t, traffic density at the downstream direction of the information source is

$$\rho_+(d(t)) = \begin{cases} 60, & d(t) < s(t); \\ 80, & d(t) \geq s(t), \end{cases}$$

where $d(t) \geq 0$ is the absolute distance from a downstream point to the information source. Similarly, traffic density at the upstream direction is

$$\rho_-(d(t)) = \begin{cases} 80, & d(t) < -s(t); \\ 60, & d(t) \geq -s(t), \end{cases}$$

where $d(t) \geq 0$ is the absolute distance from an upstream point to the information source.

For this traffic scenario, we set $\Delta x = 0.01$ km, market penetration rate $\mu = 0.1$, and transmission range $r = 1$ km. We then compute the success rate of instantaneous information propagation at any time instant t with the formulas in Section 5.1 and (2). In **Figure** 5, we show the success rate for the information to travel beyond a point in both upstream and downstream directions for t=0, 2.3, 4.6, and 9.9 min. In the figure, we also show the corresponding results simulated in ns-2 with a flooding scheme. Details for the ns-2 simulations are discussed in (Jung et al., 2010). From the figure, we can clearly see the impacts of the shock wave on information propagation: we can achieve relatively higher success rates when information propagates in the downstream part of a

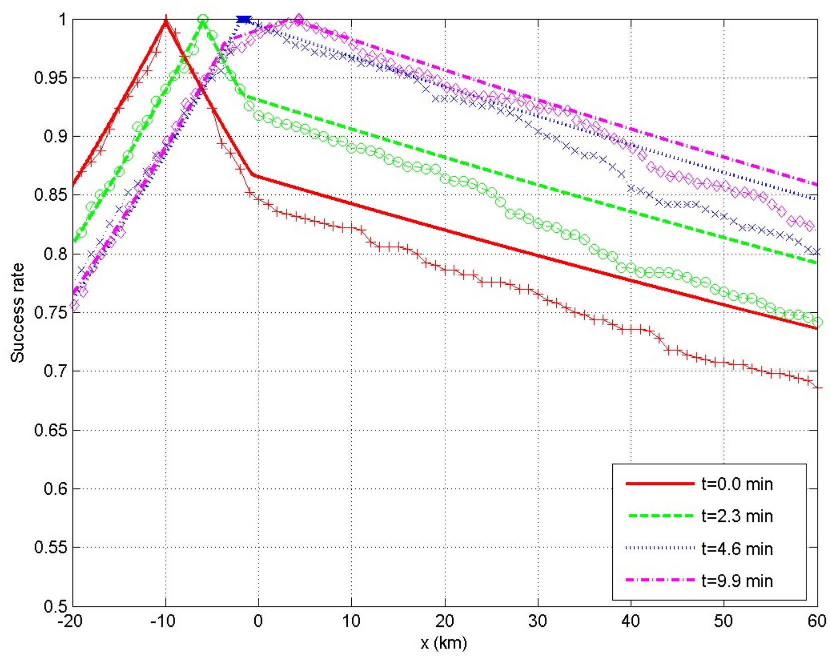

Figure 5: Instantaneous success rate in a shock wave traffic: marked curves are obtained from ns-2 simulations in (Jung et al., 2010)

shock wave, where the traffic density is higher. Furthermore, from the figure, we can see that theoretical results of connectivity can match the simulation results quite well, even though traffic dynamics and signal interference are considered in ns-2 simulations. It is also as expected that, for a longer distance of information propagation, the differences between theoretical and simulation results get larger.

7 Conclusion

In this paper we presented a recursive model for the connectivity in a VANet with continuous node distributions governed by Poisson or renewal processes and considered the improvement of connectivity by road-side stations. Given homogeneous Poisson distributions of nodes, we discussed the asymptotic properties of the connectivity and obtained a closed-form formulation of the connectivity. Given non-homogeneous Poisson distributions of nodes, we applied the recursive model to study the impact of variations in node densities. Given renewal processes of nodes, we showed that there does not exist an exact recursive model but presented an approximate solution. With the developed models, we also discussed the impacts on connectivity of road-side stations and different distribution patterns of vehicles. Results in Section 6 demonstrate that the developed theoretical models can serve as a first-order estimation of connectivity in a realistic VANet, where vehicles are mobile and signal transmissions interfere with each other. From this study, we can clearly see that the instantaneous distribution patterns of vehicles on the road can have significant impacts on connectivity and other performance of VANets. By incorporating the impacts of vehicle distribution patterns, more efficient broadcasting schemes can be developed (Chen et al., 2010a,b).

In a traffic stream, the evolution of traffic density on a homogeneous road-link can be described by the following LWR theory (Lighthill and Whitham, 1955; Richards, 1956)

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial Q(\rho(x,t))}{\partial x} = 0,$$

where $Q(\rho(x,t))$ is a unimodal flux-density relation, or a fundamental diagram. Thus we can also study the dynamics of the instantaneous connectivity when we know the probability for vehicles to be equipped, $\mu(x,t)$, such that the density of nodes $\kappa(x,t) = \rho(x,t)\mu(x,t)$. In addition, mobility of vehicles can improve the connectivity of VANets (Grossglauser and Tse, 2002), and we will be interested in studying the impact of traffic dynamics on the connectivity during a time interval. The model could be used to develop more efficient communication routing protocols based on connectivity estimations Yang et al. (2008) and design vehicle-infrastructure integration systems Dong et al. (2006).

In the developed model, we assume that a traveler or system operator can have full information on $\rho(x)$, the traffic density, and $\mu(x)$, the probability that vehicles are equipped with wireless communication units, at x along a communication path. Such an assumption can be too strict in reality. In the future, we will investigate how the accuracy in estimating traffic densities can impact the estimation of connectivity.

The developed modeling framework also bears certain limitations. First, in this model, all nodes are assumed to have the same transmission range; it will be interesting to check how heterogeneous transmission ranges would impact connectivity. Second, we omit the impacts of lateral distances between vehicles on bidirectional, multilane roadways; this assumption may not be acceptable when transmission ranges are much smaller than 1000 m. Third, a very simple underlying communication model is assumed for this study; i.e., two nodes are connected when they are within the transmission range. But at the physical level the connectivity between two nodes is a random process determined by transmission power, and at the MAC level communications between two nodes may not be established due to signal contention and other interferences. In the future, we will also be interested in studying how transportation network topology and vehicle mobility would impact connectivity. In particular, in dynamic traffic flow, road-side stations, which can also store and forward messages, could significantly improve VANet connectivity.

Acknowledgments

The author would like to thank Will Recker, Amelia Regan and Rex Chen at the University of California, Irvine for their helpful comments and discussions. Several anonymous reviewers' comments are also appreciated. This research was supported in part by a grant from the University of California Transportation Center (UCTC). The views and results herein are the authors' alone.

References

Bettstetter, C. (2002). On the minimum node degree and connectivity of a wireless multihop network. In *MobiHoc*, pages 80–91.

Blum, J. J., Eskandarian, A., and Hoffman, L. J. (2004). Challenges of intervehicle ad hoc networks. *IEEE Transactions on Intelligent Transportation Systems*, 5(4).

CalTrans (2010). Performance Measurement System (PeMS). Accessed April 20, 2010.

Chen, R., Jin, W.-L., and Regan, A. (2010a). Broadcasting in Vehicular Networks: Issues and Approaches. *IEEE Network Magazine*, 24(1):20–25.

Chen, R., Jin, W.-L., and Regan, A. (2010b). Multi-Hop Broadcasting in Vehicular Ad Hoc Networks with Shockwave Traffic. In *IEEE Intelligent Vehicular Communications System Workshop*.

Cheng, Y.-C. and Robertazzi, T. G. (1989). Critical connectivity phenomena in multihop radio models. *IEEE Transactions on Communications*, 37(7):770–777.

Desai, M. and Manjunath, D. (2002). On the connectivity in finite ad hoc networks. *IEEE Communications Letters*, 6(10):437–439.

Dong, X., Li, K., Misener, J., Varayia, P., and Zhang, W. (2006). Expediting Vehicle Infrastructure Integration (EVII). Technical report. UCB-ITS-PRR-2006-20.

Dousse, O., Thiran, P., and Hasler, M. (2002). Connectivity in ad-hoc and hybrid networks. In *Proc. IEEE Infocom*, New York.

FCC (1999). FCC allocates spectrum in 5.9 ghz range for Intelligent Transportation Systems uses. Accessed September 26, 2007.

Gazis, D. C., Herman, R., and Rothery, R. W. (1961). Nonlinear follow-the-leader models of traffic flow. *Operations Research*, 9(4):545–567.

Gilbert, E. N. (1961). Random plane networks. *SIAM Journal on Applied Mathematics*, 9(4):533–543.

Grimmett, G. and Stirzaker, D. (2001). *Probability and random processes*. Oxford University Press, USA.

Grossglauser, M. and Tse, D. (2002). Mobility increases the capacity of ad hoc wireless networks. *IEEE/ACM Transactions on Networking (TON)*, 10(4):477–486.

Gupta, P. and Kumar, P. R. (1998). Critical power for asymptotic connectivity in wireless networks. In McEneaney, W. M., Yin, G., and Zhang, Q., editors, *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W.H. Fleming*. Birkhäuser, Boston.

Gupta, P. and Kumar, P. R. (2000). The capacity of wireless networks. *IEEE Transactions on Information Theory*, 46(2):388–404.

Haberman, R. (1977). *Mathematical models*. Prentice Hall, Englewood Cliffs, NJ.

Haight, F. A. (1963). *Mathematical Theories of Traffic Flow*. Academic Press New York.

Hartenstein, H., Bochow, B., Ebner, A., Lott, M., Radimirsch, M., and Vollmer, D. (2001). Position-aware ad hoc wireless networks for inter-vehicle communications: the fleetnet project. In *Proceedings of the 2nd ACM international symposium on Mobile ad hoc networking & computing*, pages 259 – 262, Long Beach, CA, USA.

Jin, W.-L. (2003). *Kinematic Wave Models of Network Vehicular Traffic*. PhD thesis, University of California, Davis. http://arxiv.org/abs/math.DS/0309060.

Jin, W.-L. and Recker, W. W. (2006). Instantaneous information propagation in a traffic stream through inter-vehicle communication. *Transportation Research Part B*, 40(3):230–250.

Jin, W.-L. and Recker, W. W. (2007). A Monte Carlo Simulation Model of Inter-Vehicle Communication. *Transportation Research Record: Journal of the Transportation Research Board*, 2000:8–15.

Jin, W.-L. and Recker, W. W. (2009). Instantaneous connectivity of one-dimensional inter-vehicle communication networks for general traffic conditions. *Proceedings of Transportation Research Board Annual Meeting*.

Jin, W.-L. and Recker, W. W. (2010). An analytical model of multihop connectivity of inter-vehicle communication systems. *IEEE Transactions on Wireless Communications*, 9(1):106–112.

Jung, J., Chen, R., Jin, W.-L., Jayakrishnan, R., and Regan, A. C. (2010). An empirical study of inter-vehicle communication performance using ns-2. Working paper.

Lighthill, M. J. and Whitham, G. B. (1955). On kinematic waves: II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London A*, 229(1178):317–345.

Meester, R. and Roy, R. (1996). *Continuum Percolation*. Cambridge University Press.

Munjal, P. K., Hsu, Y. S., and Lawrence, R. L. (1971). Analysis and validation of lane-drop effects of multilane freeways. *Transportation Research*, 5:257–266.

Newell, G. F. (1993). A simplified theory of kinematic waves in highway traffic I: General theory. II: Queuing at freeway bottlenecks. III: Multi-destination flows. *Transportation Research B*, 27:281–313.

Penrose, M. D. (1999). On k-connectivity for a geometric random graph. *Random Structures and Algorithms*, 15(2):145 – 164.

Philips, T. J., Panwar, S. S., and Tantawi, A. N. (1989). Connectivity properties of a packet radio network model. *IEEE Transactions on Information Theory*, 35(5):1044–1047.

Piret, P. (1991). On the connectivity of radio networks. *IEEE Transactions on Information Theory*, 37(5):1490–1492.

Richards, P. I. (1956). Shock waves on the highway. *Operations Research*, 4:42–51.

Schönhof, M., Kesting, A., Treiber, M., and Helbing, D. (2006). Coupled vehicle and information flows: Message transport on a dynamic vehicle network. *Physica A: Statistical Mechanics and its Applications*, 363(1):73–81.

Tang, A., Florens, C., and Low, S. H. (2003). An empirical study on the connectivity of ad hoc networks. In *IEEE Proceedings Aerospace Conference*, volume 3, pages 1333–1338.

Ukkusuri, S. and Du, L. (2008). Geometric connectivity of vehicular ad hoc networks: analytical characterization. *Transportation Research Part C*, 16(5):615–634.

USDOT (2004). USDOT'S ITS Program - Major Initiatives — 2004. Accessed January 30, 2007.

Wang, B., Adams, T., Jin, W.-L., and Meng, Q. (2009). The process of information propagation in a traffic stream with a general vehicle headway: A revisit. *Transportation Research Part C*.

Wang, X. (2007). Modeling the process of information relay through inter-vehicle communication. *Transportation Research Part B*, 41(6):684–700.

Wu, H., Fujimoto, R., and Riley, G. (2004). Analytical models for information propagation in vehicle-to-vehicle networks. *Vehicular Technology Conference*, 6:4548 – 4552.

Wu, H., Lee, J., Hunter, M., Fujimoto, R., Guensler, R., and Ko, J. (2005). Simulated Vehicle-to-Vehicle Message Propagation Efficiency on Atlanta's I-75 Corridor. *Transportation Research Record (TRR)*.

Yang, Q., Lim, A., Li, S., Fang, J., and Agrawal, P. (2008). ACAR: Adaptive Connectivity Aware Routing Protocol for Vehicular Ad Hoc Networks. In *Computer Communications and Networks, 2008. ICCCN'08. Proceedings of 17th International Conference on*, pages 1–6.

Yang, X. and Recker, W. W. (2005). Simulation studies of information propagation in a self-organizing distributed traffic information system. *Transportation Research Part C*, 13(5-6):370–290.

Appendix A: Proof of Theorem 3.1

Proof. In the following we show that (13) satisfies (15). First, when $x = nr$ for $n = 1, 2, \cdots$, we have

$$\mathbf{C}(nr) \;=\; \sum_{m=0}^{n} \mathbf{C}(mr) \frac{(-\lambda(r)(nr - nr))^{n-m}}{(n-m)!} = \mathbf{C}(nr).$$

Thus the function $\mathbf{C}(x)$ is well-defined.

Second, when $x > r$, from (13) we have $n = 1, 2, \cdots$

$$\frac{\mathrm{d}\mathbf{C}(x)}{\mathrm{d}x} \;=\; -\lambda(r) \sum_{m=0}^{n-1} \mathbf{C}(mr) \frac{(-\lambda(r)(x - nr))^{n-m-1}}{(n-m-1)!}.$$

When $n = 1$, we have $x \in (r, 2r)$ and

$$\frac{\mathrm{d}\mathbf{C}(x)}{\mathrm{d}x} \;=\; -\lambda(r)\mathbf{C}(0) = -\lambda(r)\mathbf{C}(x-r).$$

When $n = 2, 3, \cdots$, we have $x \geq 2r$ and

$$\begin{aligned}
\frac{\mathrm{d}\mathbf{C}(x)}{\mathrm{d}x} &= -\lambda(r) \sum_{m=0}^{n-1} \mathbf{C}(mr) \frac{(-\lambda(r)[(x-r) - (n-1)r])^{n-m-1}}{(n-m-1)!} \\
&= -\lambda(r) \sum_{m=0}^{n'} \mathbf{C}(mr) \frac{(-\lambda(r)[(x-r) - n'r])^{n'-m}}{(n'-m)!},
\end{aligned}$$

where $n' = 1, 2, \cdots$. Thus we have

$$\frac{\mathrm{d}\mathbf{C}(x)}{\mathrm{d}x} \;=\; -\lambda(r)\mathbf{C}(x-r).$$

This is consistent with (15). ∎