Title
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Publication Date
1984-06-01

LBL-17936

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

Presented at the International Conference on Fusion Reactions Below the Coulomb Barrier, Cambridge, MA, June 13-15, 1984

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June 1984

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

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LBL-17936

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This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

+On leave of absence from Hahn Meitner Institut fuer Kernforschung Berlin, D-100 Berlin 39, West-Germany.

WHAT CAN WE LEARN FROM HEAVY-ION SUB-BARRIER FUSION EXCITATION FUNCTIONS?*

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Over the last years, a large amount of heavy ion fusion data has been collected for energies around and well below the Coulomb barrier. As to their theoretical interpretation, the state of the art may be summarized as follows:

For lighter systems, roughly $Z_1Z_2 \leqslant 80$, a description of fusion as penetration through a one-dimensional, more or less standard potential barrier yields a satisfactory interpretation of the experimental data.

For heavier systems such an attempt fails dramatically, underestimating the sub-barrier data by orders of magnitude.

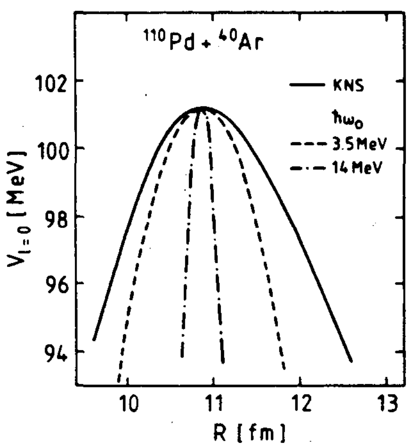

Fig. 1. Effective s-wave interaction potential for $^{40}Ar + ^{110}Pd$. Curves are explained in the text.

Fig. 1 may serve for demonstrating the failure of the one-dimensional approach. It shows, for $^{40}Ar + ^{110}Pd$, the s-wave potential barrier, ref. [1], together with the inverted parabola of the same height and curvature. The latter determines the Hill-Wheeler approximation for the transmission probability. A fit to the data by varying the width of this parabola would produce the very thin, obviously unphysical parabola in the middle of the figure. Similar findings are reported in ref. [2]. Assuming one-dimensional barrier penetration, the authors offer an inversion procedure to determine the shape of the barrier from the experimental data. Whereas for light systems the extracted barriers are in agreement

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with the standard pictures, the results for heavy systems are obviously unreasonable.

The conclusion is that heavy-ion sub-barrier fusion represents a many-dimensional dynamical problem involving other degrees of freedom besides the orbital motion of the two ions.

The time scales typical for these additional degrees of freedom have to be comparable to or shorter than the one governing the orbital motion (represented, for instance, by the $\hbar\omega_B$ characterizing the inverted s-wave barrier). Fast degrees of freedom, allowing an adiabatic treatment, could be absorbed in an effective one-dimensional picture.

The nature of the relevant degrees of freedom is still very much debated. Several candidates have been proposed in the literature. These are rotation of well deformed nuclei [3], surface vibrations [4], neck formation [5] and nucleon transfer modes [6].

It should, however, be stressed that for whatever degrees of freedom, coupling them to the relative motion effects the fusion excitation function in qualitatively very much the same way:

Instead of penetrating through the one-dimensional "frozen density" potential barrier, the system faces now a potential "surface," i.e. a barrier ridge. As an example, fig. 2 shows the potential surface underlying the two-dimensional model of ref. [5]. The minimum in height of this ridge corresponds to the completely relaxed configuration, the adiabatic barrier. An incoming wave packet probes at least part of that ridge and, in general, the transmission probability will be enhanced as compared to the one-dimensional frozen density result.

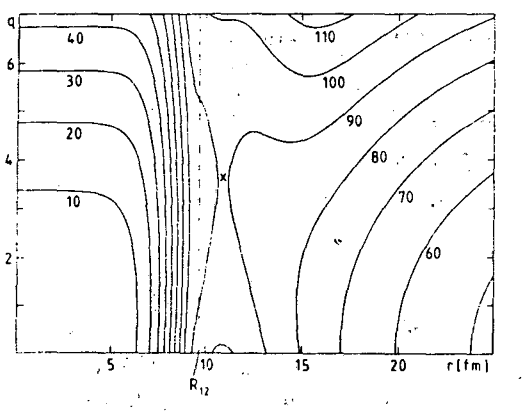

Fig. 2 Countour plot for the potential energy surface as used in ref. [5] for $^{40}Ar + ^{110}Pd$.

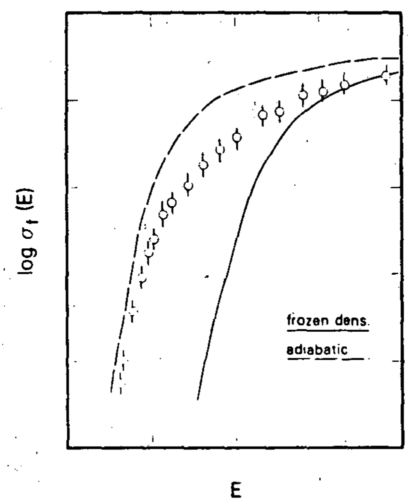

Fig. 3 Schematic representation of the frozen density and an adiabatic calculation in relation to experimental data.

For very low energies, the adiabatic path, crossing the ridge at the minimum, will more and more dominate the excitation function. Fig. 3 displays schematically how in general the frozen density and the adiabatic calculation will approach the data.

This consideration holding for any additional degree(s) of freedom, one has to conclude that the energy dependence of the sub-barrier fusion excitation function will be rather unspecific to details of a dynamical model. Testing a given model by reproducing the experimental data remains rather inconclusive (in particular when an exact, ab initio calculation is either not available or prohibitive).

One might turn the problem around and ask what features of a given model are in fact sensitively tested by a comparison to the experimental $\sigma_f(E)$. In the spirit of the above discussion the variation of barrier heights over the ridge seems to be of central importance.

In order to investigate this point, the following parametrization might be helpful.

For the frozen density potential choose the Yukawa plus exponential potential of ref. [1].

In order to allow for a variation of the barrier height, subtract a Gaussian

$$V_G(r) = V_S(r_B)\exp\left\{-(r - r_B)^2 \frac{V_S''(r_B)}{2V_S(r_B)}\right\} \qquad (1)$$

with the same position, height and curvature as the frozen density s-wave barrier

$$V_S(r) = V_{KNS}(r) + V_{coul}(r) \quad , \qquad (2)$$

$$V_{\ell=0}(r,\alpha) = V_S(r) - \alpha V_G(r) \quad . \qquad (3)$$

α is a positive number smaller than 1. For ^{40}Ar + ^{110}Pd, V_S and V_G are plotted in fig. 4.

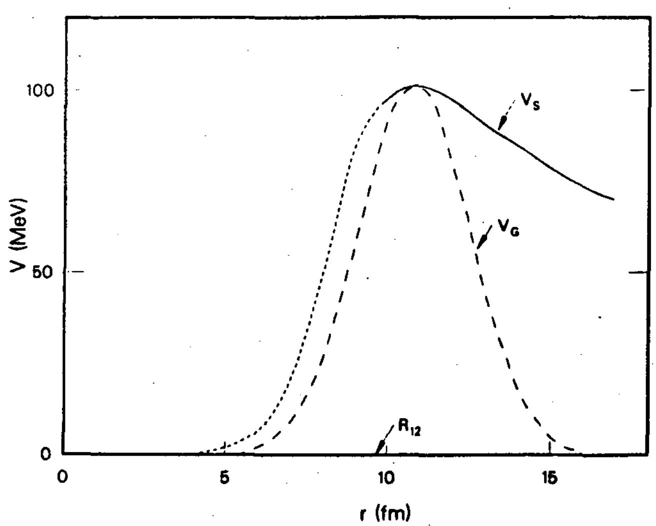

Fig. 4 The potentials V_S (full curve) and V_G (dashed curve) for ^{40}Ar + ^{110}Pd. The dotted curve gives the exponential continuation of V_S beyond R_{12}.

For a given α calculate the transmission coefficients $t_\ell(\alpha,E)$.

Two technical remarks are in order here.

(i) For the results presented below, the effective frozen density potential $V_{KNS}(r) + \ell(\ell+1)/2mr^2 + V_{Coul}(r)$ is exponentially continued for $R < R_{12} = R_1 + R_2$, ref. [5], and the wave function calculated for the boundary condition that for $r \rightarrow -\infty$ there is outgoing flux only. This corresponds to an ingoing wave boundary condition.

(ii) It is essential to calculate the transmission coefficients quantumechanically exactly. The most commonly invoked approximations, Hill-Wheeler and WKB, both fail in the present context. A Hill-Wheeler approximation obviously breaks down well below the barrier. On the other hand, the falloff of $\sigma_f(E)$ with decreasing E is dominantly determined by the rate with which successive partial waves fall below the respective barriers, i.e. the region around the top of the barriers. Naive WKB does not apply here.

Once the transmission coefficients $t_\ell(\alpha,E)$ are calculated, take the average

$$\bar{t}_\ell(E) = \left| \frac{1}{\alpha_0} \int_0^{\alpha_0} d\alpha \; t_\ell^{1/2}(\alpha,E) \right|^2 . \qquad (4)$$

and evaluate the fusion cross section

$$\sigma_f(E) = \frac{\pi}{k^2} \sum_\ell (2\ell+1)\bar{t}_\ell(E) \quad . \qquad (5)$$

The average (4) has the following properties. For large E all $t_\ell(\alpha,E)$ are equal to 1, independent of α. So is $\bar{t}_\ell(E)$. For decreasing E the integral is more and more dominated by the largest values of α, i.e. the lowest barriers, the vicinity of the adiabatic path.

Averaging over amplitudes $t_\ell^{1/2}$ rather than probabilities t_ℓ should to some extent simulate the coherent superposition of contributions due to different barrier heights. In a sudden situation averaging over probabilities should be more appropriate. Expression (4) is, therefore, not proposed for the coupling of collective rotations.

The upper limit of the integral, α_0, is a parameter and adjusted in order to reproduce a given experimental $\sigma_f(E)$. However, the fit is restricted to a reproduction of the shape of $\sigma_f(E)$ and an overall scale factor is allowed for. This procedure is adopted because

(i) The absolute scale of the experimental data is, at least in some cases, uncertain within 10% or so.

(ii) For most of the data not all the decay channels of the fused system and therefore only a partial fusion cross section is measured. Therefore, the absolute scale of the cross sections seems to be physically less significant than the shapes.

Fig. 5 shows the fits to a number of experimental data. The obtained values

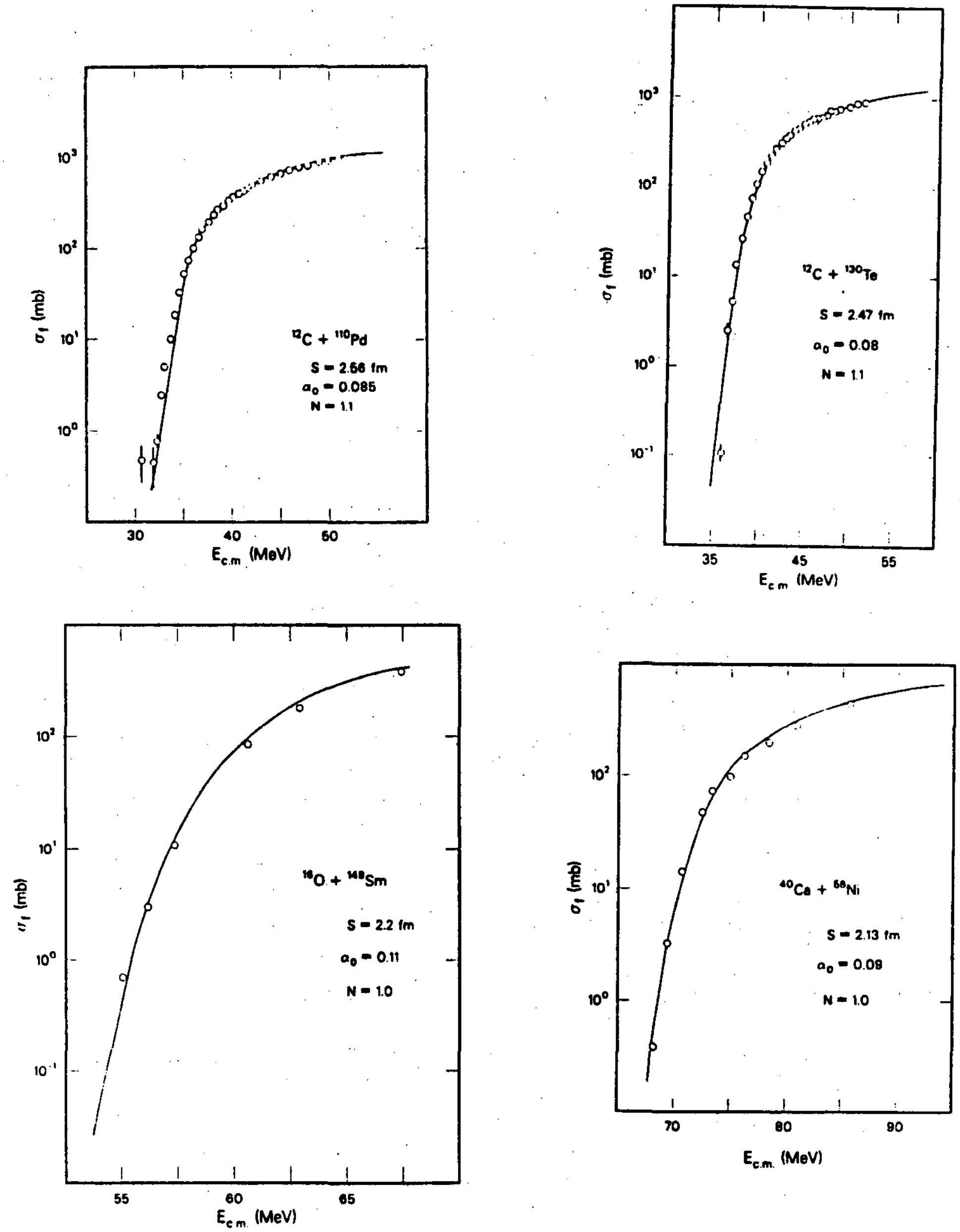

Fig.5 Comparison of equs (4,5) with data. Data are taken from refs. 9,9, 10, 11,12, 13, 9, 14, 15, respectively.

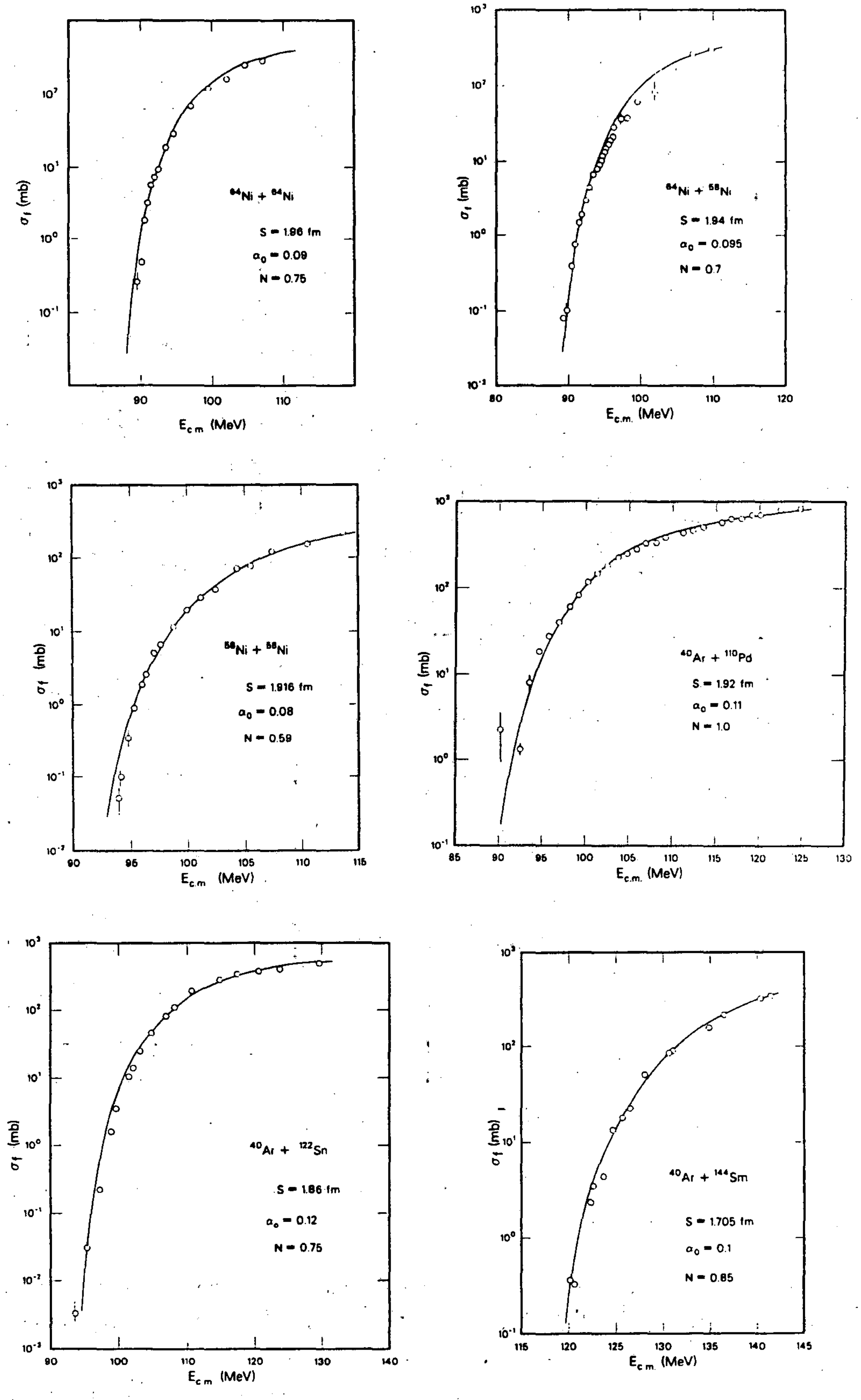

Fig. 5 continued

for α_0 and the rescaling N are indicated. (It should be noted that in some cases N is suspiciously different from 1.)

One might conclude that the range of accessible barrier heights is indeed an, or even the, essential ingredient determining the energy dependence of $\sigma_f(E)$.

Fig. 6 displays the variation of barrier heights $\Delta B = \alpha_0 V_G(r_B)$ versus the surface-surface distance at the barrier [7]

$$s = r_B - C_1 - C_2 \quad , \tag{6}$$

$$C_i = R_i - 1/R_i \quad , \quad R_i(1.28\, A^{1/3} - 0.76 + 0.8\, A^{-1/3})\mathrm{fm} \quad . \tag{7}$$

There is a clear trend with s. Local fluctuations are more apparent when the exponential dependence of the frozen density barrier upon s is taken out and α_0 is plotted directly. This is shown in fig. 6b (full dots). Still, variations around some average trend are not dramatic.

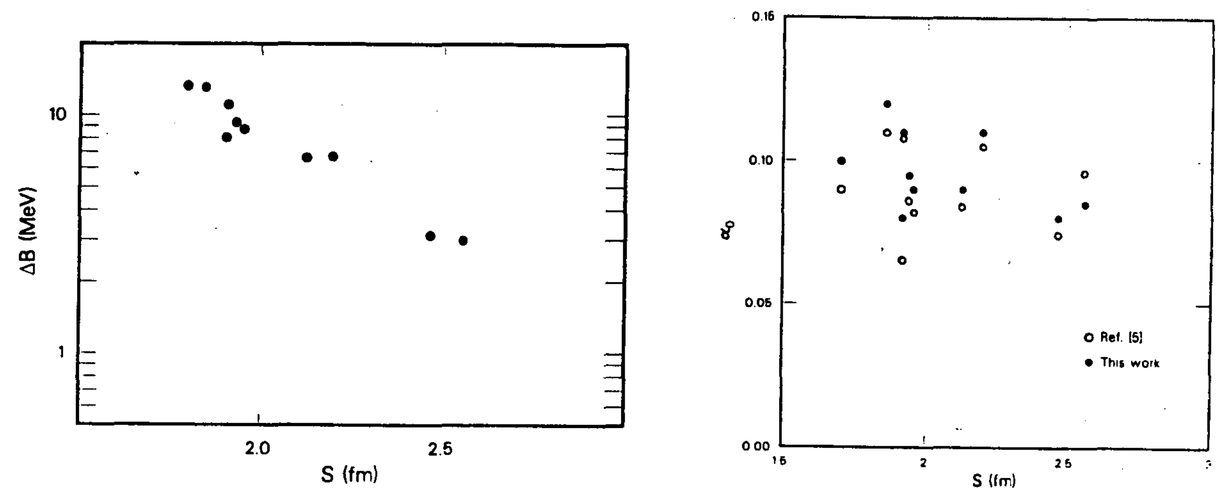

Fig. 6. (a) The variaton of the barrier height, ΔB versus the surface-surface distance s. (b) The parameter α_0 versus s.

In order to check the physical significance of these results, the obtained values for ΔB should be compared to those of explicit model calculations describing the same data.

In ref. [5] a model was investigated which coupled penetration through a KNS barrier to a harmonic oscillator degree of freedom in a linear fashion. The coupling form factor is a Gaussian similar to (1). Its width is chosen in such a way that the curvature of the adiabatic barrier equals the one of the frozen density barrier. (Fig. 2 resembles the potential energy surface of this model for ^{40}Ar + ^{110}Pd.) The difference $\widetilde{\Delta B}$ between the two barriers and the $\hbar\omega$ of the harmonic oscillator are fit parameters $\bar{h}$. Again the absolute scale is adjusted. A typical fit is shown in fig. 7. (The dotted line demonstrates that for $\hbar\omega$ = 1 to 2 MeV a sudden (zero point motion) approximation is not justified.)

The model is meant to simulate, at least to some extent, coupling to a neck degree of freedom. The coupling is concentrated at rather small ion-ion distances. The fitted $\widetilde{\Delta B}$-values roughly coincide with those extractable from liquid drop potential surfaces for orbital motion coupled to a neck degree of freedom.

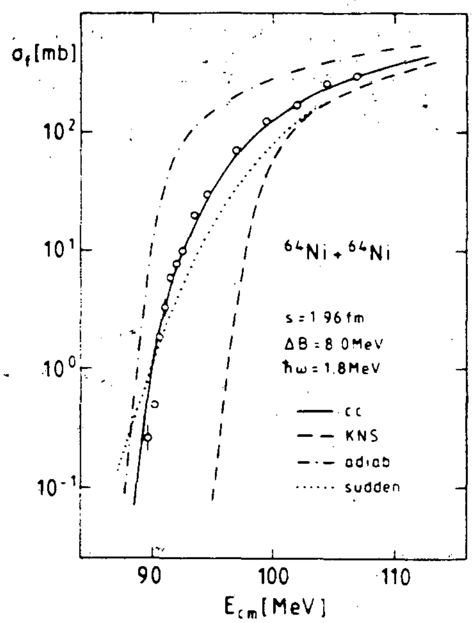

Fig. 7. The result of ref. [5] for ^{64}Ni + ^{64}Ni. cc marks the exact result of the two-dimensional model.

$\widetilde{\Delta B}$ converted to α_0, the results of ref. [5] are given in fig. 6 as open circles. Agreement with the present parametrization is reasonable. This supports the conclusion that indeed the range of accessible barrier heights is the essential feature reflected in the sub-barrier fusion excitation functions.

Based on a rather restricted set of data, this conclusion is certainly still preliminary. For further corroboration work should proceed in two directions:

(i) The parametrization should be applied to all available data in order to check its applicability and the systematics of $\alpha_0(s)$. It will be interesting to check if the parametrization is sensitive to time scales. One might expect that for fusion of well deformed nuclei, collective rotation allowing for a sudden approximation, instead of eq. (4) an average over t itself, i.e. over probabilities proves to be more appropriate.

(ii) The obtained ΔB should be compared to all available detailed model calculations fitting the same data.

As for the relevance of neck formation, the systematics of the parameters α_o, or $\tilde{\Delta B}$ and $\hbar\omega$ in ref. [5], as well as the findings of ref. [8], strongly suggest that a liquid drop like property of the combined system is responsible at least for the smooth trend in these results. At least for the heavier systems it is somehow hard to imagine that modes of the isolated nuclei are the dominant degrees of freedom determining the fusion cross section. The model of ref. [5] provides some arguments in favour of a neck degree of freedom. However, a more realistic modelling of neck formation is obviously and badly needed in order to allow for more definite conclusions.

This work is in part based on results obtained in collaboration with H. J. Krappe, M. C. Nemes ad H. Rossner. The author is very much indebted to these colleagues. The hospitality of the Lawrence Berkeley Laboratory, extended to the author during the last year, is gratefully acknowledged.

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