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Author

Gatto, R.

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R. Gatto

Radiation Laboratory
University of California
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Abstract

A discussion is given of the $K_{\mu 3}$ and $K_{e 3}$ decay modes, making use of the invariance properties of the theory, under the general hypothesis of parity nonconservation and lepton nonconservation. The assumption of local interaction between the final fermions is examined in detail. Detection of a possible up-down asymmetry, with respect to the $K_{\mu 3}$ decay plane, of the electrons from the subsequent μ decay is suggested as a possible test of time reversal.

INVARIANTS IN $K_{\mu 3}$ AND $K_{e 3}$ DECAY*

R. Gatto†

Radiation Laboratory
University of California
Berkeley, California

September 25, 1957

The $K_{\mu 3}$ and $K_{e 3}$ decay modes have recently been discussed by Furuichi et al.¹ and by Pais and Treiman.² We present here some general results for such decays which can be directly derived from the invariance properties of the theory. We consider the general case with parity nonconservation and with non-conservation of leptons. The discussion will be limited to local interaction between the two final fermions, and a detailed examination of such a hypothesis will be given.

Following the methods used by Furuichi et al. and by Pais and Treiman, we write the transition-matrix element for a decay process $K \rightarrow l + \pi + \nu$, where l denotes either μ or e , in the form

$$T_{fi} = (\bar{l}(C_S + C'_S \gamma_5) \nu) + \frac{i}{m_K} (\bar{l} \gamma_\alpha V'_\alpha (C_V + C'_V \gamma_5) \nu) + \frac{1}{m_K^2} (\bar{l} \sigma_{\alpha\beta} V'_\alpha U'_\beta (C_T + C'_T \gamma_5) \nu) + \\ + (\bar{l} (B_S + B'_S \gamma_5) \nu^C) + \frac{i}{m_K} (\bar{l} \gamma_\alpha V'_\alpha (B_V + B'_V \gamma_5) \nu^C) + \frac{1}{m_K^2} (\bar{l} \sigma_{\alpha\beta} V'_\alpha U'_\beta (B_T + B'_T \gamma_5) \nu^C).$$

In Eq. (1) we have denoted the free-particle spinors with the same symbol as used for the corresponding particle; V'_α , U'_α , V'_α , and U'_α are linear combinations of the independent four-momenta in the decay; the scalars C , C' , B , and B' are also formed from the independent four-momenta; m_K is the mass of the decaying heavy meson. We use the Pauli notations; in particular $\bar{l} = l^* \gamma_4$, $\sigma_{\alpha\beta} = \frac{i}{2} [\gamma_\alpha, \gamma_\beta]$

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¹Furuichi, Kodama, Ogawa, Sugahara, Wakasa, Yonezawa, Progr. Theoret. Phys. 17, 89 (1957).

²A. Pais and S.B. Treiman, Phys. Rev. 105, 1616 (1957).

and $\nu^C = C^{-1}\bar{\nu}$, where C is the charge-conjugation matrix, satisfying $C\gamma_\mu C^{-1} = -\gamma_\mu^T$ and $CC^T = 1$. The 4 available four-momenta are restricted by conservation of the total four-momentum. In both References 1 and 2 it is assumed that the four-momenta of l and of ν appear in T only in the combination $P^{(l)} + P^{(\nu)}$. This assumption rests on the hypothesis that both leptons are produced at the same local vertex and on the neglect of electromagnetic corrections. We shall refer to this assumption as the locality hypothesis. It must be stressed that such locality here refers only to the weak interaction producing the lepton pair. Because of total momentum conservation and of the locality hypothesis, only two four-momenta are independent, and we choose as such the K four-momentum P_α and the π four-momentum p_α .³ Making use of the locality hypothesis, of the Dirac equation for the leptons, and of the antisymmetry of $\sigma_{\alpha\beta}$, we can write Eq. (1) in the form

$$T_{fi} = (\bar{l}(G_S + G'_S \gamma_5)\nu) + \frac{1}{M} (\bar{l}\gamma_\alpha P_\alpha (G_V + G'_V \gamma_5)\nu) - \frac{1}{M^2} (\bar{l}\sigma_{\alpha\beta} p'_\alpha P_\beta (G_T + G'_T \gamma_5)\nu) + \quad (2)$$

$$+ (\bar{l}(F_S + F'_S \gamma_5)\nu^C) + \frac{1}{M} (\bar{l}\gamma_\alpha P_\alpha (F_V + F'_V \gamma_5)\nu^C) - \frac{1}{M^2} (\bar{l}\sigma_{\alpha\beta} p'_\alpha P_\beta (F_T + F'_T \gamma_5)\nu)$$

In Eq. (2) G_S is of the form $C_S + (\text{constant}) (m_l/m_K) C_V$, where m_l is the mass of the emitted lepton, and similar relations hold for G'_S , F_S , and F'_S . Therefore, as remarked by Treiman and Pais,² even in the absence of scalar interaction at the lepton vertex, the transition matrix will contain a scalar term proportional to m_l if any vector interaction is present. This term, however, is negligible for K_{e3} decay because of the smallness of the electron mass.

As first shown by Pauli and by Pursey, under the assumption of zero rest mass for the neutrino, there exist two commuting subgroups of transformations of the neutrino field which leave the free-particle equation and the commutation relations invariant and transform the interaction Hamiltonian into

³For the comparison with the experimental data the most convenient frame of reference is that in which the two leptons have equal and opposite momenta. An analysis of K_3 decays in such frame of reference will be found in a forthcoming paper by B. McCormick.

a physically equivalent Hamiltonian.^{4, 5, 6, 7} The transformations of the first subgroup are

$$(I) \quad v' = \alpha v + \beta \gamma_5 C^{-1} \bar{v}, \quad \bar{v}' = \alpha^* \bar{v} + \beta^* v C \gamma_5$$

with α and β constants, satisfying

$$|\alpha|^2 + |\beta|^2 = 1.$$

The transformations of the second subgroup are

$$(II) \quad v' = \exp \left[\frac{1}{2} i \xi + \frac{1}{2} i \eta \gamma_5 \right] v, \quad \bar{v}' = \bar{v} \exp \left[-\frac{1}{2} i \xi + \frac{1}{2} i \eta \gamma_5 \right]$$

with ξ and η real. A Transformation (I) is equivalent to a substitution on the scalar quantities G_j , G_j' , F_j , and F_j' of Eq. (2) according to

$$G_j \rightarrow G_j \alpha^* + F_j' \beta^*, \quad G_j' \rightarrow G_j' \alpha^* + F_j \beta^*, \quad (3)$$

$$F_j \rightarrow F_j \alpha - G_j' \beta, \quad F_j' \rightarrow F_j' \alpha - G_j \beta. \quad (3')$$

A Transformation (II) is equivalent to

$$G_j + G_j' \rightarrow (G_j + G_j') e^{i \frac{1}{2} \eta + i \frac{1}{2} \xi}, \quad G_j - G_j' \rightarrow (G_j - G_j') e^{-i \frac{1}{2} \eta + i \frac{1}{2} \xi} \quad (4)$$

$$F_j + F_j' \rightarrow (F_j + F_j') e^{i \frac{1}{2} \eta - i \frac{1}{2} \xi}, \quad F_j - F_j' \rightarrow (F_j - F_j') e^{-i \frac{1}{2} \eta - i \frac{1}{2} \xi}. \quad (4')$$

Since we shall be concerned with the product transformation of (I) and (II), we can choose $\xi = 0$, because an arbitrary phase factor is already contained in (I). Following the method in References 4, 5, and 6, we construct the quantities

⁴Pauli, Nuovo cimento 6, 204 (1957).

⁵D. L. Pursey, Nuovo cimento, 6, 266 (1957).

⁶Gerhart Luders, On the Pursey-Pauli Invariants in the theory of β decay, August 1957, UCRL-3903 (Nuovo cimento, to be published).

⁷R. Gatto and G. Luders, Invariants in μ decay. UCRL-3935; September 1957 (Nuovo cimento, to be published).

$$V_j^{(+)} = \begin{bmatrix} G_j^{(+)} \\ F_j^{(+)} \end{bmatrix} \quad V_j^{(-)} = \begin{bmatrix} F_j^{(-)} \\ G_j^{(-)*} \end{bmatrix} \quad (5)$$

where $G_j^{(\pm)} = G_j \pm G_j'$, $F_j^{(\pm)} = F_j \pm F_j'$. Such quantities transform under the product of (I) and (II) according to

$$V_j^{(\pm)} \rightarrow M V_j^{(\pm)} ; \quad (6)$$

the matrix M is given by

$$M = e^{i \frac{1}{2} \eta} \begin{bmatrix} \alpha^* & \beta^* \\ -\beta & \alpha \end{bmatrix} \quad (7)$$

Regarding $V_j^{(\pm)}$ as vectors in a two-dimensional space (with complex coordinates), one sees that they are transformed according to a unitary transformation. The invariants under the group of Transformations (7) are given by the scalar products

$$V_j^{(+)\dagger} V_l^{(+)} = G_j^{(+)*} G_l^{(+)} + F_j^{(+)*} F_l^{(+)} \quad (8)$$

$$V_j^{(-)\dagger} V_l^{(-)} = G_j^{(-)} G_l^{(-)*} + F_j^{(-)} F_l^{(-)*} \quad (8')$$

$$V_j^{(+)\dagger} V_l^{(-)} = G_j^{(+)*} F_l^{(-)*} + F_j^{(+)*} G_l^{(-)*} \quad (8'')$$

and their complex conjugates. Only invariants of the forms (8) and (8') can appear in the description of the decay process. It will be convenient to substitute for them an equivalent set of invariants which have the property of remaining unchanged under one of the operations T, C, P, or under all three such operations. To this purpose one has only to substitute into Eqs. (8) and (8') the expressions

$$\frac{1}{2} \Re [V_j^{(+)\dagger} V_l^{(+)} \pm V_l^{(-)\dagger} V_j^{(-)}] \quad (9)$$

$$\frac{1}{2} \Im [V_j^{(+)\dagger} V_l^{(+)} \pm V_l^{(-)\dagger} V_j^{(-)}] \quad (9')$$

One finds that eighteen different invariants can be constructed in this way.

The list is as follows:

$$E_j = |G_j|^2 + |G_j'|^2 + |F_j|^2 + |F_j'|^2, \quad (10)$$

$$D_j = \Re [G_j^* G_j' + F_j^* F_j'] , \quad (10')$$

$$E_{j\ell} = \Re [G_j^* G_\ell + G_j'^* G_\ell' + F_j^* F_\ell + F_j'^* F_\ell'] = E_{\ell j} \quad (10'')$$

$$D_{j\ell} = \Re [G_j^* G_\ell' + G_j'^* G_\ell + F_j^* F_\ell' + F_j'^* F_\ell] = D_{\ell j} \quad (10''')$$

$$Q_{j\ell} = \Im [G_j^* G_\ell + G_j'^* G_\ell' + F_j^* F_\ell + F_j'^* F_\ell'] = -Q_{\ell j} \quad (10''')$$

$$N_{j\ell} = \Im [G_j^* G_\ell' + G_j'^* G_\ell + F_j^* F_\ell' + F_j'^* F_\ell] = -N_{\ell j} \quad (10''')$$

For each of the above six classes, there are three independent invariants. These invariants are not all independent, but have to satisfy relations and inequalities that follow from their algebraic structure.

In the K_{e3} decay modes it is a good approximation to neglect the electron mass in comparison with the large available energy. In this approximation, and for experiments that do not involve any observation of the possible transverse polarization of the electron, the transition probabilities can only depend on those invariants which remain unchanged under the group of transformations induced by the substitutions

$$(III) \quad e' = \exp \left[\frac{1}{2} i \epsilon + \frac{1}{2} i \phi \gamma_5 \right] e, \quad \bar{e}' = \bar{e} \exp \left[-\frac{1}{2} i \epsilon + \frac{1}{2} i \phi \gamma_5 \right]$$

on the electron-field operators. The group of transformations on the scalars G , G' , F , and F' induced by (III) is given by

$$G_j(\pm) \rightarrow G_j(\pm) e^{\pm i \frac{\phi}{2} - i \frac{\epsilon}{2}}, \quad F_j(\pm) \rightarrow F_j(\pm) e^{\pm i \frac{\phi}{2} - i \frac{\epsilon}{2}} \quad \text{for } j = S, T, \quad (11)$$

$$G_j(\pm) \rightarrow G_j(\pm) e^{\mp i \frac{\phi}{2} - i \frac{\epsilon}{2}}, \quad F_j(\pm) \rightarrow F_j(\pm) e^{\mp i \frac{\phi}{2} - i \frac{\epsilon}{2}} \quad \text{for } j = V. \quad (11')$$

Invariance under this group is therefore equivalent to requiring that there is no interference of S and T with V. Therefore only S-T interference terms, and no S-V or T-V interference terms, can be measured in experiments in which no measurement is carried out of the transverse polarization of the emitted electron (i. e., only intensities or longitudinal polarizations are measured). The only

invariants that can occur in the description of such experiments are E_j , D_j , E_{ST} , D_{ST} , Q_{ST} , and N_{ST} .

The invariants D_j , $D_{j\ell}$, and $N_{j\ell}$ change sign under space reflection. In the expression for the transition probability they must be multiplied by pseudoscalars formed from the observed momenta and spins: They cannot appear in the description of experiments where no pseudoscalars are observed. Invariance under time reversal implies the relation

$$(\psi_f, T\psi_i) = (\tilde{\psi}_i, T\tilde{\psi}_f), \quad (12)$$

where $\tilde{\psi}$ is the time-reversed state of the state ψ . If one can show that the T matrix behaves for the particular transition as a Hermitian matrix, one then can substitute for Eq. (12) the equation

$$(\psi_f, T\psi_i) = (\tilde{\psi}_f, T\tilde{\psi}_i)^* \quad (13)$$

-- which in our case for instance implies specific phase relations between the quantities G , G' , F , and F' . The transition matrix is Hermitian if calculated at first order in perturbation theory. Relation (13), however, also follows from time reversal, independent of the order of perturbation theory, for transitions such that contributions from intermediate states on the energy shell are absent or negligible (this condition is obviously satisfied in our case). If a weak interaction is essentially involved in the transition and if final-state interactions can be neglected, a general proof follows from the unitarity of the S matrix which is expressed by

$$T^\dagger - T = i TT^\dagger. \quad (14)$$

The largest contribution to a matrix element $(\psi_f TT^\dagger \psi_i)$ comes from terms, for instance, of the kind $(\psi_f T\psi_{f'}) (\psi_{f'} T^\dagger \psi_i)$, where $(\psi_f T\psi_{f'})$ is due to the final-state interaction and $(\psi_{f'} T^\dagger \psi_i)$ to the weak interaction. Therefore T behaves as Hermitian as long as final-state interactions can be neglected. Such a condition is verified in our case, for which the only final-state interaction is electromagnetic, and therefore the right-hand side in Eq. (14) is, for the transition, smaller by a factor $\sim e^2$ than the left-hand side. To construct the time-reversed states we recall that time reversal corresponds to replacement of c numbers by their charge conjugates, and, for spinor fields, to the substitutions of

$\epsilon U \psi$ for ψ and $\epsilon^* \psi U^{-1}$ for $\bar{\psi}$,

in which ϵ is a phase factor depending on the particular field and U is defined (apart from a phase factor) by⁸

$$U^{-1} \gamma_{\mu}^T U = \gamma_{\mu}, \quad UU^{\dagger} = 1 \quad (15)$$

It follows that $UU^* = -1$. A state of a fermion described by the spinor u can be represented by

$$\phi_u = \int \bar{\psi} \gamma_4 u dr \phi_0,$$

where ϕ_0 is the vacuum. Applying time reversal and using Eq. (15), we find that the time-reversed state is

$$\phi_u = \epsilon^* \int \bar{\psi} \gamma_4 U^{-1} u^* dr \phi_0.$$

One finds in this way that under time reversal $u^* \rightarrow -\epsilon^* U u$ and $u = -\epsilon u U^{-1}$.

By a similar argument, and using the commutation relation $CU^* = UC^*$, one finds

$$u^C \rightarrow -\epsilon^* U u^C.$$

We should also remark that under time reversal the four-momenta P_a and p_a of Eq. (2) change sign (in fact for the time-reversed state

$$P_a \rightarrow (-P_1, P_4),$$

and by complex conjugation

$$P_a \rightarrow (P_1, -P_4).$$

With such substitutions,⁹ and making use of Eq. (15), one finds that in order for Eq. (13) to hold (invariance under time reversal) $G_S, G'_S, G_V,$ and G'_V must have the same relative phases, and similarly G_T and G'_T must have the same relative phase, but their phase relative to $G_S, G'_S, G_V,$ and G'_V is $\pi/2$. The same holds

⁸Our definition of U is the same as in G. Lüders, Det. Kon. Dan. Vidensk. Selsk. 28, 5, (1954); however, our definition of C is the same as Pauli's.⁴

⁹The additional phase factors arising from the time-reversal transformation on the wave functions of the bosons are, of course, irrelevant for the discussion. We have also omitted consideration of the plane-wave exponentials. For K^0 mesons we make use of the experimental result that the two lifetimes are very different. This implies directly that the two physical particles are rather accurately represented by the two combinations $K^0 + \bar{K}^0$ and $K^0 - \bar{K}^0$, as can be seen from the results of Lee, Oehme, and Yang, Phys. Rev. 106, 340 (1957).

for the scalars F and F' . Making use of the TCP theorem, one then finds that measurements of the different invariants imply violations of P, T, and C according to the following table (where we cross the symbol corresponding to that symmetry property which is violated):

$$E_j : P, T, C$$

$$D_j : \cancel{P}, T, \cancel{C}$$

$$E_{jl} : P, T, C \text{ for } j = S, l = V; P, \cancel{T}, \cancel{C} \text{ for } j = S, V, l = T$$

$$D_{jl} : \cancel{P}, T, \cancel{C} \text{ for } j = S, l = V; \cancel{P}, \cancel{T}, C \text{ for } j = S, V, l = T$$

$$Q_{jl} : P, \cancel{T}, \cancel{C} \text{ for } j = S, l = V; P, T, C \text{ for } j = S, V, l = T$$

$$N_{jl} : \cancel{P}, \cancel{T}, C \text{ for } j = S, l = V; \cancel{P}, T, \cancel{C} \text{ for } j = S, V, l = T$$

In the decay probability, invariants that violate P can appear only as factors of pseudoscalars (number of momenta odd), invariants that violate T only as factors of pseudoscalars under time inversion (total number of momenta and of σ factors odd), and invariants that violate C as factors of pseudoscalars under charge conjugation (number of σ factors odd). For $K \rightarrow l + \pi + \nu$ the most general distribution will be of the form

$$W \sim a + b (\vec{p}_l \cdot \vec{\sigma}_l) + c (\vec{p}_\pi \cdot \vec{\sigma}_l) + d (\vec{\sigma}_l \wedge \vec{p}_l \cdot \vec{p}_\pi) \quad (16)$$

where a, b, c, d can depend only on \vec{p}_l^2 , \vec{p}_π^2 and $(\vec{p}_l \cdot \vec{p}_\pi)$. Accordingly only E_j , E_{SV} , Q_{ST} , and Q_{VT} can appear in the term a; D_j , D_{SV} , N_{ST} , and N_{VT} in b and c; and E_{ST} , E_{VT} , and Q_{SV} in d. If l is an electron, in the approximation in which the electron mass is neglected, only E_j and Q_{ST} can appear in a, and only D_j and N_{ST} in b, while no further general restrictions can be given for the other terms (involving transverse polarizations).

Observation of the last term in Eq. (16) would be a proof of noninvariance under time reversal. From our discussion it follows that such a term will generally be expected to occur in the distribution, provided that in the leptonic vertex two of the invariants S, V, and T are present, or else V alone. The experiment consists in detecting a possible polarization of the emitted lepton normal to the plane of decay. Detection of such polarization would imply noninvariance under time reversal. In $K^\pm \rightarrow \mu^\pm + \pi^0 + \nu$ and in $K^0 \rightarrow \mu^\pm + \pi + \nu$ the muon polarization can be inferred from the angular distributions of the electrons from the decay of the muon. In the cascade $K \rightarrow \mu + \pi + \nu$, $\mu \rightarrow e + \nu + \nu$ the

electron distribution has the form

$$1 + a \langle \vec{\sigma} \rangle_{\mu} \cdot \vec{p}_e = 1 + A(\vec{p}_{\pi} \cdot \vec{p}_e) + B(\vec{p}_{\mu} \cdot \vec{p}_e) + C(\vec{p}_{\pi} \cdot \vec{p}_{\mu}) \cdot \vec{p}_e,$$

when A, B, C depend on \vec{p}_e^2 and on \vec{p}_{π}^2 , \vec{p}_{μ}^2 , and $(\vec{p}_{\pi} \cdot \vec{p}_{\mu})$. Therefore, any forward-backward asymmetry of the decay electrons with respect either to the pion direction or to the μ direction implies violation of P and of C; any up-down asymmetry with respect to the decay plane implies violation of T and of C.

We want finally to discuss the assumption of locality, which was explicitly introduced to derive Expression (2) for T_{fi} . By such assumption we mean that the two momenta $P^{(l)}$ and $P^{(\nu)}$ of the two final leptons always appear in the form $P^{(l)} + P^{(\nu)}$ in the transition amplitude. If electromagnetic corrections in the final state are neglected such a requirement is satisfied if both leptons are emitted from the same local vertex. Lee and Yang have recently proposed to account for the deviation of ρ from $3/4$ in μ decay with a nonlocal structure for the interaction between the four fermions.¹⁰ We shall consider two different possibilities as to the origin of such nonlocality: (a) that an intermediate heavy boson is involved in the process; (b) that an intermediate pair of fermions, formed by a neutrino and a heavy fermion, is involved. This second possibility was suggested by Lee¹¹ as a possibility for relating parity violation to the neutrino interactions only. In both cases the decay process would be a second-order process in the semiweak interactions of the intermediate bosons or of the intermediate fermion pair with the two spinor fields. Now, if in K decay the two emitted leptons interact with the intermediate boson, or fermion pair, at the same semiweak vertex, the locality hypothesis is of course satisfied. Therefore we have only to consider the possibility that they interact each at a different semiweak vertex. This is impossible however, in Case (a), because two other leptons would have to be emitted, one for each semiweak vertex, and their reabsorption would cause the process to be of higher order. In Case (b) a neutrino would have to be emitted at the μ vertex. Because of conservation of the nucleonic number, the baryon-antibaryon pair must be absorbed at the same vertex, and necessarily, at the ν vertex. Therefore, the ν emitted at the μ vertex must be reabsorbed at the μ

¹⁰T. D. Lee and C. N. Yang (to be published).

¹¹T. D. Lee, Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics, 1957 (Interscience, New York, 1957).

vertex again. Diagrams of this kind are, however, usually assumed to give zero contribution because of the antisymmetrization of the interaction Hamiltonian with respect to all fermion operators. In conclusion: for both the above possibilities (a) and (b), it appears that no conceivable deviation from the locality assumption should occur, apart from electromagnetic interactions in the final state.

The author is indebted to Gerhart L^uders for interesting discussions.